Detailed derivations pertaining to the paper “Fermion as a nonlocal particle-hole excitation”

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1 Free theory equations of motion

Fermions are described by annihilation and creation operators $c_{\mathbf{p}}, c^{\dagger}_{\mathbf{p}}$. These fermions have a Fermi surface at zero temperature, described by $E_F = \epsilon_{\mathbf{p}}$. At zero temperature they have the momentum distribution $< c^{\dagger}_{\mathbf{p}} c_{\mathbf{p}} > \equiv n_F(\mathbf{p}) \equiv \theta(E_F - \epsilon_{\mathbf{p}})$. Here $\theta(X > 0) = 1, \theta(X < 0) = 0$ and $\theta(0) = 1/2$ is the Heaviside step function. We define $c_{\mathbf{p},<} \equiv n_F(\mathbf{p})\, c_{\mathbf{p}}$ and $c_{\mathbf{p},>} \equiv (1 - n_F(\mathbf{p}))\, c_{\mathbf{p}}$. Define,

$$N_>(t) = \sum_{\mathbf{k}} c_{\mathbf{k},<}(t) c^{\dagger}_{\mathbf{k},<}(t); \quad N^{'}_>(t) = \sum_{\mathbf{k}} c^{\dagger}_{\mathbf{k},>}(t) c_{\mathbf{k},>}(t)$$

They represent different version of the number of particle hole pairs. For notational simplicity, in the rest of the description below we assume $\epsilon_{\mathbf{k}} \equiv \frac{k^2}{2m}$. This means $\frac{(\mathbf{k}-\mathbf{q}/2)\cdot\mathbf{q}}{m}$ is shorthand for $\epsilon_{\mathbf{k}} - \epsilon_{\mathbf{k}-\mathbf{q}}$ and so on. In other words, the discussion below is completely general and applicable to any $\epsilon_{\mathbf{k}}$.

$$\begin{aligned}
i\partial_{t_1} < T\ e^{-\lambda N_>(t)}\ e^{-\lambda^{'} N^{'}_>(t)}\ c_{\mathbf{k}-\mathbf{q},>}(t) a^{\dagger}_{\mathbf{k}-\mathbf{q}/2}(-\mathbf{q},t_1)\ c^{\dagger}_{\mathbf{k},<}(t^{'}) > \ &= \ \frac{(\mathbf{k}-\mathbf{q}/2).\mathbf{q}}{m}\ < T\ e^{-\lambda N_>(t)}\ e^{-\lambda^{'} N^{'}_>(t)}\ c_{\mathbf{k}-\mathbf{q},>}(t) a^{\dagger}_{\mathbf{k}-\mathbf{q}/2}(-\mathbf{q},t_1)\ c^{\dagger}_{\mathbf{k},<}(t^{'}) > \\
&+(e^{\lambda+\lambda^{'}} - 1)\ < T\ e^{-\lambda N_>(t)}\ e^{-\lambda^{'} N^{'}_>(t)}\ a^{\dagger}_{\mathbf{k}-\mathbf{q}/2}(-\mathbf{q},t)\ c_{\mathbf{k}-\mathbf{q},>}(t) c^{\dagger}_{\mathbf{k},<}(t^{'}) > \ i\ \delta(t_1 - t) \\
&-(1 - n_F(\mathbf{k}-\mathbf{q}))\ < T\ e^{-\lambda N_>(t)}\ e^{-\lambda^{'} N^{'}_>(t)}\ c_{\mathbf{k},<}(t) c^{\dagger}_{\mathbf{k},<}(t^{'}) > \ i\ \delta(t_1 - t) \\
&+n_F(\mathbf{k})\ < T\ e^{-\lambda N_>(t)}\ e^{-\lambda^{'} N^{'}_>(t)}\ c_{\mathbf{k}-\mathbf{q},>}(t) c^{\dagger}_{\mathbf{k}-\mathbf{q},>}(t^{'}) > \ i\ \delta(t_1 - t^{'})
\end{aligned}$$

$$\begin{aligned}
i\partial_{t_1} < T\ e^{-\lambda N_>(t)}\ e^{-\lambda^{'} N^{'}_>(t)}\ c_{\mathbf{k}-\mathbf{q},<}(t) a_{\mathbf{k}-\mathbf{q}/2}(\mathbf{q},t_1)\ c^{\dagger}_{\mathbf{k},>}(t^{'}) > \ &= \ \frac{(\mathbf{k}-\mathbf{q}/2).\mathbf{q}}{m}\ < T\ e^{-\lambda N_>(t)}\ e^{-\lambda^{'} N^{'}_>(t)}\ c_{\mathbf{k}-\mathbf{q},<}(t) a_{\mathbf{k}-\mathbf{q}/2}(\mathbf{q},t_1)\ c^{\dagger}_{\mathbf{k},>}(t^{'}) > \\
&+(e^{-\lambda-\lambda^{'}} - 1)\ < T\ e^{-\lambda N_>(t)}\ e^{-\lambda^{'} N^{'}_>(t)}\ a_{\mathbf{k}-\mathbf{q}/2}(\mathbf{q},t)\ c_{\mathbf{k}-\mathbf{q},<}(t) c^{\dagger}_{\mathbf{k},>}(t^{'}) > \ i\delta(t_1 - t) \\
&-n_F(\mathbf{k}-\mathbf{q})\ < T\ e^{-\lambda N_>(t)}\ e^{-\lambda^{'} N^{'}_>(t)}\ c_{\mathbf{k},>}(t) c^{\dagger}_{\mathbf{k},>}(t^{'}) > \ i\delta(t_1 - t) \\
&+ < T\ e^{-\lambda N_>(t)}\ e^{-\lambda^{'} N^{'}_>(t)}\ c_{\mathbf{k}-\mathbf{q},<}(t)\ c^{\dagger}_{\mathbf{k}-\mathbf{q},<}(t^{'}) > \ (1 - n_F(\mathbf{k}))\ i\delta(t_1 - t^{'})
\end{aligned}$$

2 Solutions of equations of motion

Write,

$$\begin{aligned}
< T\ e^{-\lambda N_>(t)}\ e^{-\lambda^{'} N^{'}_>(t)}\ c_{\mathbf{k}-\mathbf{q},>}(t) a^{\dagger}_{\mathbf{k}-\mathbf{q}/2}(-\mathbf{q},t_1)\ c^{\dagger}_{\mathbf{k},<}(t^{'}) > \ &= \ < T\ e^{-\lambda N_>(t)}\ e^{-\lambda^{'} N^{'}_>(t)}\ a^{\dagger}_{\mathbf{k}-\mathbf{q}/2}(-\mathbf{q},t_1) c_{\mathbf{k}-\mathbf{q},>}(t)\ c^{\dagger}_{\mathbf{k},<}(t^{'}) > \ = \\
< T\ e^{-\lambda N_>(t)}\ e^{-\lambda^{'} N^{'}_>(t)}\ a^{\dagger}_{\mathbf{k}-\mathbf{q}/2}(-\mathbf{q},t_1)\ c_{\mathbf{k}-\mathbf{q},>}(t) c^{\dagger}_{\mathbf{k},<}(t^{'}) >_> \ \theta(t_1 - t) \ &+ \ < T\ e^{-\lambda N_>(t)}\ e^{-\lambda^{'} N^{'}_>(t)}\ a^{\dagger}_{\mathbf{k}-\mathbf{q}/2}(-\mathbf{q},t_1)\ c_{\mathbf{k}-\mathbf{q},>}(t) c^{\dagger}_{\mathbf{k},<}(t^{'}) >_< \ \theta(t - t_1)
\end{aligned}$$

and

$$\begin{aligned}
< T\ e^{-\lambda N_>(t)}\ e^{-\lambda^{'} N^{'}_>(t)}\ c_{\mathbf{k}-\mathbf{q},<}(t) a_{\mathbf{k}-\mathbf{q}/2}(\mathbf{q},t_1)\ c^{\dagger}_{\mathbf{k},>}(t^{'}) > \ &= \ < T\ e^{-\lambda N_>(t)}\ e^{-\lambda^{'} N^{'}_>(t)}\ a_{\mathbf{k}-\mathbf{q}/2}(\mathbf{q},t_1) c_{\mathbf{k}-\mathbf{q},<}(t)\ c^{\dagger}_{\mathbf{k},>}(t^{'}) > \ = \\
< T\ e^{-\lambda N_>(t)}\ e^{-\lambda^{'} N^{'}_>(t)}\ a_{\mathbf{k}-\mathbf{q}/2}(\mathbf{q},t_1)\ c_{\mathbf{k}-\mathbf{q},<}(t) c^{\dagger}_{\mathbf{k},>}(t^{'}) >_> \ \theta(t_1 - t) \ &+ \ < T\ e^{-\lambda N_>(t)}\ e^{-\lambda^{'} N^{'}_>(t)}\ a_{\mathbf{k}-\mathbf{q}/2}(\mathbf{q},t_1)\ c_{\mathbf{k}-\mathbf{q},<}(t) c^{\dagger}_{\mathbf{k},>}(t^{'}) >_< \ \theta(t - t_1)
\end{aligned}$$

so that,

$$\begin{aligned}
i\partial_{t_1} < T\ e^{-\lambda N_>(t)}\ e^{-\lambda^{'} N^{'}_>(t)}\ a^{\dagger}_{\mathbf{k}-\mathbf{q}/2}(-\mathbf{q},t_1)\ c_{\mathbf{k}-\mathbf{q},>}(t) c^{\dagger}_{\mathbf{k},<}(t^{'}) > \ &= \ \frac{(\mathbf{k}-\mathbf{q}/2).\mathbf{q}}{m}\ < T\ e^{-\lambda N_>(t)}\ e^{-\lambda^{'} N^{'}_>(t)}\ a^{\dagger}_{\mathbf{k}-\mathbf{q}/2}(-\mathbf{q},t_1)\ c_{\mathbf{k}-\mathbf{q},>}(t) c^{\dagger}_{\mathbf{k},<}(t^{'}) > \\
&+(e^{\lambda+\lambda^{'}} - 1)\ < T\ e^{-\lambda N_>(t)}\ e^{-\lambda^{'} N^{'}_>(t)}\ a^{\dagger}_{\mathbf{k}-\mathbf{q}/2}(-\mathbf{q},t)\ c_{\mathbf{k}-\mathbf{q},>}(t) c^{\dagger}_{\mathbf{k},<}(t^{'}) > \ i\ \delta(t_1 - t) \\
&-(1 - n_F(\mathbf{k}-\mathbf{q}))\ < T\ e^{-\lambda N_>(t)}\ e^{-\lambda^{'} N^{'}_>(t)}\ c_{\mathbf{k},<}(t) c^{\dagger}_{\mathbf{k},<}(t^{'}) > \ i\ \delta(t_1 - t) \\
&+n_F(\mathbf{k})\ < T\ e^{-\lambda N_>(t)}\ e^{-\lambda^{'} N^{'}_>(t)}\ c_{\mathbf{k}-\mathbf{q},>}(t) c^{\dagger}_{\mathbf{k}-\mathbf{q},>}(t^{'}) > \ i\ \delta(t_1 - t^{'})
\end{aligned}$$

$$\begin{aligned}
i\partial_{t_1} < T\ e^{-\lambda N_>(t)}\ e^{-\lambda^{'} N^{'}_>(t)}\ a_{\mathbf{k}-\mathbf{q}/2}(\mathbf{q},t_1)\ c_{\mathbf{k}-\mathbf{q},<}(t) c^{\dagger}_{\mathbf{k},>}(t^{'}) > \ &= \ \frac{(\mathbf{k}-\mathbf{q}/2).\mathbf{q}}{m}\ < T\ e^{-\lambda N_>(t)}\ e^{-\lambda^{'} N^{'}_>(t)}\ a_{\mathbf{k}-\mathbf{q}/2}(\mathbf{q},t_1)\ c_{\mathbf{k}-\mathbf{q},<}(t) c^{\dagger}_{\mathbf{k},>}(t^{'}) > \\
&+(e^{-\lambda-\lambda^{'}} - 1)\ < T\ e^{-\lambda N_>(t)}\ e^{-\lambda^{'} N^{'}_>(t)}\ a_{\mathbf{k}-\mathbf{q}/2}(\mathbf{q},t)\ c_{\mathbf{k}-\mathbf{q},<}(t) c^{\dagger}_{\mathbf{k},>}(t^{'}) > \ i\delta(t_1 - t) \\
&-n_F(\mathbf{k}-\mathbf{q})\ < T\ e^{-\lambda N_>(t)}\ e^{-\lambda^{'} N^{'}_>(t)}\ c_{\mathbf{k},>}(t) c^{\dagger}_{\mathbf{k},>}(t^{'}) > \ i\delta(t_1 - t) \\
&+ < T\ e^{-\lambda N_>(t)}\ e^{-\lambda^{'} N^{'}_>(t)}\ c_{\mathbf{k}-\mathbf{q},<}(t)\ c^{\dagger}_{\mathbf{k}-\mathbf{q},<}(t^{'}) > \ (1 - n_F(\mathbf{k}))\ i\delta(t_1 - t^{'})
\end{aligned}$$

$$i\partial_{t_1} < T\ e^{-\lambda N_>(t)}\ e^{-\lambda' N'_>(t)}\ a^{\dagger}_{\mathbf{k-q/2}}(-\mathbf{q},t_1)\ c_{\mathbf{k-q},>}(t) c^{\dagger}_{\mathbf{k},<}(t') >> \quad =$$

$$\frac{(\mathbf{k-q/2).q}}{m}\ < T\ e^{-\lambda N_>(t)}\ e^{-\lambda' N'_>(t)}\ a^{\dagger}_{\mathbf{k-q/2}}(-\mathbf{q},t_1)\ c_{\mathbf{k-q},>}(t) c^{\dagger}_{\mathbf{k},<}(t') >> + n_F(\mathbf{k})\ < T\ e^{-\lambda N_>(t)}\ e^{-\lambda' N'_>(t)}\ c_{\mathbf{k-q},>}(t) c^{\dagger}_{\mathbf{k-q},>}(t') >\ i\ \delta(t_1 - t')$$

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$$i\partial_{t_1} < T\ e^{-\lambda N_>(t)}\ e^{-\lambda' N'_>(t)}\ a^{\dagger}_{\mathbf{k-q/2}}(-\mathbf{q},t_1)\ c_{\mathbf{k-q},>}(t) c^{\dagger}_{\mathbf{k},<}(t') >_< \quad =$$

$$\frac{(\mathbf{k-q/2).q}}{m}\ < T\ e^{-\lambda N_>(t)}\ e^{-\lambda' N'_>(t)}\ a^{\dagger}_{\mathbf{k-q/2}}(-\mathbf{q},t_1)\ c_{\mathbf{k-q},>}(t) c^{\dagger}_{\mathbf{k},<}(t') >_< + n_F(\mathbf{k})\ < T\ e^{-\lambda N_>(t)}\ e^{-\lambda' N'_>(t)}\ c_{\mathbf{k-q},>}(t) c^{\dagger}_{\mathbf{k-q},>}(t') >\ i\ \delta(t_1 - t')$$

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$$i\partial_{t_1} < T\ e^{-\lambda N_>(t)}\ e^{-\lambda' N'_>(t)}\ a_{\mathbf{k-q/2}}(\mathbf{q},t_1)\ c_{\mathbf{k-q},<}(t) c^{\dagger}_{\mathbf{k},>}(t') >> \quad =$$

$$\frac{(\mathbf{k-q/2).q}}{m}\ < T\ e^{-\lambda N_>(t)}\ e^{-\lambda' N'_>(t)}\ a_{\mathbf{k-q/2}}(\mathbf{q},t_1)\ c_{\mathbf{k-q},<}(t) c^{\dagger}_{\mathbf{k},>}(t') >> + < T\ e^{-\lambda N_>(t)}\ e^{-\lambda' N'_>(t)}\ c_{\mathbf{k-q},<}(t)\ c^{\dagger}_{\mathbf{k-q},<}(t') >\ (1 - n_F(\mathbf{k}))\ i\delta(t_1 - t')$$

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$$i\partial_{t_1} < T\ e^{-\lambda N_>(t)}\ e^{-\lambda' N'_>(t)}\ a_{\mathbf{k-q/2}}(\mathbf{q},t_1)\ c_{\mathbf{k-q},<}(t) c^{\dagger}_{\mathbf{k},>}(t') >_< \quad =$$

$$\frac{(\mathbf{k-q/2).q}}{m}\ < T\ e^{-\lambda N_>(t)}\ e^{-\lambda' N'_>(t)}\ a_{\mathbf{k-q/2}}(\mathbf{q},t_1)\ c_{\mathbf{k-q},<}(t) c^{\dagger}_{\mathbf{k},>}(t') >_< + < T\ e^{-\lambda N_>(t)}\ e^{-\lambda' N'_>(t)}\ c_{\mathbf{k-q},<}(t)\ c^{\dagger}_{\mathbf{k-q},<}(t') >\ (1 - n_F(\mathbf{k}))\ i\delta(t_1 - t')$$

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$$< T\ a^{\dagger}_{\mathbf{k}-\mathbf{q}/2}(-\mathbf{q},t_1)\ e^{-\lambda N_>(t)}\ e^{-\lambda' N'_>(t)}\ c_{\mathbf{k}-\mathbf{q},>}(t)c^{\dagger}_{\mathbf{k},<}(t') > \ =$$

$$e^{\frac{i\mathbf{q}.(-2\mathbf{k}t_1+\mathbf{q}(t_1+i\beta))}{2m}}(f_{max}\ e^{\frac{\beta\mathbf{k}.\mathbf{q}}{m}} + in_F(\mathbf{k})\ < T\ e^{-\lambda N_>(t)}\ e^{-\lambda' N'_>(t)}\ c_{\mathbf{k}-\mathbf{q},>}(t)c^{\dagger}_{\mathbf{k}-\mathbf{q},>}(t') >\ i\ \theta(t'-t_1)e^{\frac{q(2ikt'+q(\beta-it'))}{2m}})\ \theta(t_1-t)$$

$$+\ e^{-\frac{it_1\mathbf{q}.(2\mathbf{k}-\mathbf{q})}{2m}}(f_{max} - in_F(\mathbf{k})\ < T\ e^{-\lambda N_>(t)}\ e^{-\lambda' N'_>(t)}\ c_{\mathbf{k}-\mathbf{q},>}(t)c^{\dagger}_{\mathbf{k}-\mathbf{q},>}(t') >\ i\ \theta(t_1-t')\ e^{\frac{it'\mathbf{q}.(2\mathbf{k}-\mathbf{q})}{2m}})\ \theta(t-t_1)$$

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$$< T\ a_{\mathbf{k}-\mathbf{q}/2}(\mathbf{q},t_1)\ e^{-\lambda N_>(t)}\ e^{-\lambda' N'_>(t)}\ c_{\mathbf{k}-\mathbf{q},<}(t)c^{\dagger}_{\mathbf{k},>}(t') > \ =$$

$$e^{\frac{i\mathbf{q}.(-2\mathbf{k}t_1+\mathbf{q}(t_1+i\beta))}{2m}}(g_{max}\ e^{\frac{\beta\mathbf{k}.\mathbf{q}}{m}} + i(1-n_F(\mathbf{k}))\ < T\ e^{-\lambda N_>(t)}\ e^{-\lambda' N'_>(t)}\ c_{\mathbf{k}-\mathbf{q},<}(t)c^{\dagger}_{\mathbf{k}-\mathbf{q},<}(t') >\ i\ \theta(t'-t_1)e^{\frac{(2i\mathbf{k}.\mathbf{q}t'+q^2(\beta-it'))}{2m}})\ \theta(t_1-t)$$

$$+\ e^{-\frac{it_1\mathbf{q}.(2\mathbf{k}-\mathbf{q})}{2m}}(g_{max} - i(1-n_F(\mathbf{k}))\ < T\ e^{-\lambda N_>(t)}\ e^{-\lambda' N'_>(t)}\ c_{\mathbf{k}-\mathbf{q},<}(t)c^{\dagger}_{\mathbf{k}-\mathbf{q},<}(t') >\ i\ \theta(t_1-t')\ e^{\frac{it'\mathbf{q}.(2\mathbf{k}-\mathbf{q})}{2m}})\ \theta(t-t_1)$$

==

$$< T\ e^{-\lambda N_>(t)}\ e^{-\lambda' N'_>(t)}\ N_>(t)\ c_{\mathbf{k},>}(t)\ c^{\dagger}_{\mathbf{k},>}(t') > \ = \ \sum_{\mathbf{q}}\ e^{-\frac{it\mathbf{q}.(2\mathbf{k}-\mathbf{q})}{2m}}(g_{max}-i(1-n_F(\mathbf{k}))\ < T\ e^{-\lambda N_>(t)}\ e^{-\lambda' N'_>(t)}\ c_{\mathbf{k}-\mathbf{q},<}(t)c^{\dagger}_{\mathbf{k}-\mathbf{q},<}(t') >\ i\ \theta(t-t')\ e^{\frac{it'\mathbf{q}.(2\mathbf{k}-\mathbf{q})}{2m}})$$

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$$< T\ e^{-\lambda N_>(t)}\ e^{-\lambda' N'_>(t)}\ \ e^{\lambda N_>(t)}e^{\lambda' N'_>(t)}\ c^{\dagger}_{\mathbf{k}-\mathbf{q},>}(t)c_{\mathbf{k},<}(t)\ e^{-\lambda N_>(t)}\ e^{-\lambda' N'_>(t)}\ c_{\mathbf{k}-\mathbf{q},>}(t)c^{\dagger}_{\mathbf{k},<}(t') > \ =$$

$$e^{\frac{i\mathbf{q}.(-2\mathbf{k}t+\mathbf{q}(t+i\beta))}{2m}}(f_{max}\ e^{\frac{\beta\mathbf{k}.\mathbf{q}}{m}} + in_F(\mathbf{k})\ < T\ e^{-\lambda N_>(t)}\ e^{-\lambda' N'_>(t)}\ c_{\mathbf{k}-\mathbf{q},>}(t)c^{\dagger}_{\mathbf{k}-\mathbf{q},>}(t') >\ i\ \theta(t'-t)e^{\frac{q(2ikt'+q(\beta-it'))}{2m}})$$

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$$u_{\lambda'} = e^{\lambda' N'_>(t)}c^{\dagger}_{\mathbf{k}-\mathbf{q},>}(t)e^{-\lambda' N'_>(t)}$$

$$\partial_{\lambda'}u_{\lambda'} = u_{\lambda'}$$

$$u_{\lambda'} = e^{\lambda'}\ \ c^{\dagger}_{\mathbf{k}-\mathbf{q},>}(t)$$

$$w_{\lambda} = e^{\lambda N_>(t)}c_{\mathbf{k},<}(t)\ e^{-\lambda N_>(t)} = e^{\lambda}\ \ c_{\mathbf{k},<}(t)$$

$$\partial_{\lambda}w_{\lambda} = w_{\lambda}$$

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$$e^{\lambda N_>(t)}e^{\lambda' N'_>(t)}\ c^{\dagger}_{\mathbf{k}-\mathbf{q},>}(t)c_{\mathbf{k},<}(t)\ e^{-\lambda N_>(t)}\ e^{-\lambda' N'_>(t)}\ = \ e^{\lambda+\lambda'}\ \ c^{\dagger}_{\mathbf{k}-\mathbf{q},>}(t)c_{\mathbf{k},<}(t)$$

==

$$e^{\lambda N_>(t)}e^{\lambda' N'_>(t)}\ c^{\dagger}_{\mathbf{k}-\mathbf{q},>}(t)c_{\mathbf{k},<}(t)\ e^{-\lambda N_>(t)}\ e^{-\lambda' N'_>(t)}\ = \ e^{\lambda+\lambda'}\ \ c^{\dagger}_{\mathbf{k}-\mathbf{q},>}(t)c_{\mathbf{k},<}(t)$$

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$$< T\ e^{-\lambda N_>(t)}\ e^{-\lambda' N'_>(t)}\ N_>(t)\ c_{\mathbf{k},>}(t)\ c^{\dagger}_{\mathbf{k},>}(t') > \ =$$

$$\sum_{\mathbf{q}}\ e^{-\frac{it\mathbf{q}.(2\mathbf{k}-\mathbf{q})}{2m}}(g_{max} - i(1-n_F(\mathbf{k}))\ < T\ e^{-\lambda N_>(t)}\ e^{-\lambda' N'_>(t)}\ c_{\mathbf{k}-\mathbf{q},<}(t)c^{\dagger}_{\mathbf{k}-\mathbf{q},<}(t') >\ i\ \theta(t-t')\ e^{\frac{it'\mathbf{q}.(2\mathbf{k}-\mathbf{q})}{2m}})$$

==

$$< T\ e^{-\lambda N_>(t)}\ e^{-\lambda' N'_>(t)}\ \ N'_>(t)\ c_{\mathbf{k},<}(t)c^{\dagger}_{\mathbf{k},<}(t') > \ =$$

$$-\sum_{\mathbf{q}}\ e^{-\lambda-\lambda'}\ \ e^{\frac{i\mathbf{q}.(-2\mathbf{k}t+\mathbf{q}(t+i\beta))}{2m}}(f_{max}\ e^{\frac{\beta\mathbf{k}.\mathbf{q}}{m}} + in_F(\mathbf{k})\ < T\ e^{-\lambda N_>(t)}\ e^{-\lambda' N'_>(t)}\ c_{\mathbf{k}-\mathbf{q},>}(t)c^{\dagger}_{\mathbf{k}-\mathbf{q},>}(t') >\ i\ \theta(t'-t)e^{\frac{\mathbf{q}.(2ikt'+\mathbf{q}(\beta-it'))}{2m}})$$

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2.1 Finding f_{max} and g_{max}

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$$< T\ a^{\dagger}_{\mathbf{k}-\mathbf{q}/2}(-\mathbf{q},t_1)\ e^{-\lambda N_>(t)}\ e^{-\lambda' N'_>(t)}\ c_{\mathbf{k}-\mathbf{q},>}(t)c^{\dagger}_{\mathbf{k},<}(t') > \ =$$

$$e^{\frac{i\mathbf{q}.(-2\mathbf{k}t_1+\mathbf{q}(t_1+i\beta))}{2m}}(f_{max}\ e^{\frac{\beta\mathbf{k}.\mathbf{q}}{m}} + in_F(\mathbf{k})\ < T\ e^{-\lambda N_>(t)}\ e^{-\lambda' N'_>(t)}\ c_{\mathbf{k}-\mathbf{q},>}(t)c^{\dagger}_{\mathbf{k}-\mathbf{q},>}(t') >\ i\ \theta(t'-t_1)\ e^{\frac{\mathbf{q}.(2ikt'+\mathbf{q}(\beta-it'))}{2m}})\ \theta(t_1-t)$$

$$+\ e^{-\frac{it_1\mathbf{q}.(2\mathbf{k}-\mathbf{q})}{2m}}(f_{max} - in_F(\mathbf{k})\ < T\ e^{-\lambda N_>(t)}\ e^{-\lambda' N'_>(t)}\ c_{\mathbf{k}-\mathbf{q},>}(t)c^{\dagger}_{\mathbf{k}-\mathbf{q},>}(t') >\ i\ \theta(t_1-t')\ e^{\frac{it'\mathbf{q}.(2\mathbf{k}-\mathbf{q})}{2m}})\ \theta(t-t_1)$$

==

$$< T\ a_{\mathbf{k}-\mathbf{q}/2}(\mathbf{q},t_1)\ e^{-\lambda N_>(t)}\ e^{-\lambda' N'_>(t)}\ c_{\mathbf{k}-\mathbf{q},<}(t)c^{\dagger}_{\mathbf{k},>}(t') > \ =$$

$$e^{\frac{i\mathbf{q}.(-2\mathbf{k}t_1+\mathbf{q}(t_1+i\beta))}{2m}}(g_{max}\ e^{\frac{\beta\mathbf{k}.\mathbf{q}}{m}} + i(1-n_F(\mathbf{k}))\ < T\ e^{-\lambda N_>(t)}\ e^{-\lambda' N'_>(t)}\ c_{\mathbf{k}-\mathbf{q},<}(t)c^{\dagger}_{\mathbf{k}-\mathbf{q},<}(t') >\ i\ \theta(t'-t_1)e^{\frac{(2i\mathbf{k}.\mathbf{q}t'+\mathbf{q}^2(\beta-it'))}{2m}})\ \theta(t_1-t)$$

$$+\ e^{-\frac{it_1\mathbf{q}.(2\mathbf{k}-\mathbf{q})}{2m}}(g_{max} - i(1-n_F(\mathbf{k}))\ < T\ e^{-\lambda N_>(t)}\ e^{-\lambda' N'_>(t)}\ c_{\mathbf{k}-\mathbf{q},<}(t)c^{\dagger}_{\mathbf{k}-\mathbf{q},<}(t') >\ i\ \theta(t_1-t')\ e^{\frac{it'\mathbf{q}.(2\mathbf{k}-\mathbf{q})}{2m}})\ \theta(t-t_1)$$

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$$< T\ a^{\dagger}_{\mathbf{k}-\mathbf{q}/2}(-\mathbf{q},t)\ e^{-\lambda N_{>}(t)}\ e^{-\lambda^{'} N^{'}_{>}(t)}\ c_{\mathbf{k}-\mathbf{q},>}(t)c^{\dagger}_{\mathbf{k},<}(t^{'}) > \ = \ e^{\frac{i\mathbf{q}.(-2\mathbf{k}t+\mathbf{q}(t+i\beta))}{2m}}(f_{max}\ e^{\frac{\beta \mathbf{k}.\mathbf{q}}{m}}+in_{F}(\mathbf{k})\ < T\ e^{-\lambda N_{>}(t)}\ e^{-\lambda^{'} N^{'}_{>}(t)}\ c_{\mathbf{k}-\mathbf{q},>}(t)c^{\dagger}_{\mathbf{k}-\mathbf{q},>}(t^{'}) > \ i\ \theta(t^{'}-t)\ e^{\frac{\mathbf{q}.(2i\mathbf{k}}{}}$$

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$$< T\ e^{-\lambda N_{>}(t)}\ e^{-\lambda^{'} N^{'}_{>}(t)}\ c_{\mathbf{k}-\mathbf{q},>}(t)a^{\dagger}_{\mathbf{k}-\mathbf{q}/2}(-\mathbf{q},t)\ c^{\dagger}_{\mathbf{k},<}(t^{'}) > \ = \ e^{-\frac{it\mathbf{q}.(2\mathbf{k}-\mathbf{q})}{2m}}(f_{max}-in_{F}(\mathbf{k})\ < T\ e^{-\lambda N_{>}(t)}\ e^{-\lambda^{'} N^{'}_{>}(t)}\ c_{\mathbf{k}-\mathbf{q},>}(t)c^{\dagger}_{\mathbf{k}-\mathbf{q},>}(t^{'}) > \ i\ \theta(t-t^{'})\ e^{\frac{it^{'}\mathbf{q}.(2\mathbf{k}-\mathbf{q})}{2m}})$$

==

$$< T\ a_{\mathbf{k}-\mathbf{q}/2}(\mathbf{q},t)\ e^{-\lambda N_{>}(t)}\ e^{-\lambda^{'} N^{'}_{>}(t)}\ c_{\mathbf{k}-\mathbf{q},<}(t)c^{\dagger}_{\mathbf{k},>}(t^{'}) > \ = \ e^{\frac{i\mathbf{q}.(-2\mathbf{k}t+\mathbf{q}(t+i\beta))}{2m}}(g_{max}\ e^{\frac{\beta \mathbf{k}.\mathbf{q}}{m}}+i(1-n_{F}(\mathbf{k}))\ < T\ e^{-\lambda N_{>}(t)}\ e^{-\lambda^{'} N^{'}_{>}(t)}\ c_{\mathbf{k}-\mathbf{q},<}(t)c^{\dagger}_{\mathbf{k}-\mathbf{q},<}(t^{'}) > \ i\ \theta(t^{'}-t)e^{\frac{(2i\mathbf{k}}{}}$$

==

$$< T\ e^{-\lambda N_{>}(t)}\ e^{-\lambda^{'} N^{'}_{>}(t)}\ c_{\mathbf{k}-\mathbf{q},<}(t)\ a_{\mathbf{k}-\mathbf{q}/2}(\mathbf{q},t)c^{\dagger}_{\mathbf{k},>}(t^{'}) > \ = \ e^{-\frac{it\mathbf{q}.(2\mathbf{k}-\mathbf{q})}{2m}}(g_{max}-i(1-n_{F}(\mathbf{k}))\ < T\ e^{-\lambda N_{>}(t)}\ e^{-\lambda^{'} N^{'}_{>}(t)}\ c_{\mathbf{k}-\mathbf{q},<}(t)c^{\dagger}_{\mathbf{k}-\mathbf{q},<}(t^{'}) > \ i\ \theta(t-t^{'})\ e^{\frac{it^{'}\mathbf{q}.(2\mathbf{k}-\mathbf{q})}{2m}})$$

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$$< T\ a^{\dagger}_{\mathbf{k-q}/2}(-\mathbf{q},t)\ e^{-\lambda^{'} N^{'}_{>}(t)}\ e^{-\lambda N_{>}(t)}\ c_{\mathbf{k-q},>}(t) c^{\dagger}_{\mathbf{k},<}(t^{'}) >$$

$$= \ e^{\frac{i\mathbf{q}.(-2\mathbf{k}t+\mathbf{q}(t+i\beta))}{2m}} (f_{max}\ e^{\frac{\beta \mathbf{k}.\mathbf{q}}{m}} + i n_F(\mathbf{k})\ \ < T\ e^{-\lambda N_{>}(t)}\ e^{-\lambda^{'} N^{'}_{>}(t)}\ c_{\mathbf{k-q},>}(t) c^{\dagger}_{\mathbf{k-q},>}(t^{'}) >\ i\ \theta(t^{'}-t)\ e^{\frac{\mathbf{q}.(2i\mathbf{k}t^{'}+\mathbf{q}(\beta-it^{'}))}{2m}})$$

==

$$< T\ e^{-\lambda N_{>}(t)}\ e^{-\lambda^{'} N^{'}_{>}(t)}\ c_{\mathbf{k-q},>}(t) a^{\dagger}_{\mathbf{k-q}/2}(-\mathbf{q},t)\ c^{\dagger}_{\mathbf{k},<}(t^{'}) > \ =$$

$$e^{-\frac{it\mathbf{q}.(2\mathbf{k}-\mathbf{q})}{2m}} (f_{max} - i n_F(\mathbf{k})\ \ < T\ e^{-\lambda N_{>}(t)}\ e^{-\lambda^{'} N^{'}_{>}(t)}\ c_{\mathbf{k-q},>}(t) c^{\dagger}_{\mathbf{k-q},>}(t^{'}) >\ i\ \theta(t-t^{'})\ e^{\frac{it^{'}\mathbf{q}.(2\mathbf{k}-\mathbf{q})}{2m}})$$

==

$$< T\ a_{\mathbf{k-q}/2}(\mathbf{q},t)\ e^{-\lambda N_{>}(t)}\ e^{-\lambda^{'} N^{'}_{>}(t)}\ c_{\mathbf{k-q},<}(t) c^{\dagger}_{\mathbf{k},>}(t^{'}) > \ =$$

$$e^{\frac{i\mathbf{q}.(-2\mathbf{k}t+\mathbf{q}(t+i\beta))}{2m}} (g_{max}\ e^{\frac{\beta \mathbf{k}.\mathbf{q}}{m}} + i(1-n_F(\mathbf{k}))\ \ < T\ e^{-\lambda N_{>}(t)}\ e^{-\lambda^{'} N^{'}_{>}(t)}\ c_{\mathbf{k-q},<}(t) c^{\dagger}_{\mathbf{k-q},<}(t^{'}) >\ i\ \theta(t^{'}-t) e^{\frac{(2i\mathbf{k}.\mathbf{q}t^{'}+\mathbf{q}^2(\beta-it^{'}))}{2m}})$$

==

$$< T\ e^{-\lambda N_{>}(t)}\ e^{-\lambda^{'} N^{'}_{>}(t)}\ c_{\mathbf{k-q},<}(t)\ a_{\mathbf{k-q}/2}(\mathbf{q},t) c^{\dagger}_{\mathbf{k},>}(t^{'}) > \ =$$

$$e^{-\frac{it\mathbf{q}.(2\mathbf{k}-\mathbf{q})}{2m}} (g_{max} - i(1-n_F(\mathbf{k}))\ \ < T\ e^{-\lambda N_{>}(t)}\ e^{-\lambda^{'} N^{'}_{>}(t)}\ c_{\mathbf{k-q},<}(t) c^{\dagger}_{\mathbf{k-q},<}(t^{'}) >\ i\ \theta(t-t^{'})\ e^{\frac{it^{'}\mathbf{q}.(2\mathbf{k}-\mathbf{q})}{2m}})$$

==

$$e^{\lambda N_{>}(t)}\ e^{\lambda^{'} N^{'}_{>}(t)}\ a^{\dagger}_{\mathbf{k-q}/2}(-\mathbf{q},t)\ e^{-\lambda^{'} N^{'}_{>}(t)}\ e^{-\lambda N_{>}(t)}\ =$$

$$e^{\lambda^{'} N^{'}_{>}(t)}\ c^{\dagger}_{\mathbf{k-q},>}(t)\ e^{-\lambda^{'} N^{'}_{>}(t)}$$

$$e^{\lambda N_{>}(t)}\ c_{\mathbf{k},<}(t)\ e^{-\lambda N_{>}(t)}$$

$$e^{\lambda N_{>}(t)}\ c_{\mathbf{k},<}(t)\ e^{-\lambda N_{>}(t)} = e^{\lambda}\ c_{\mathbf{k},<}(t)$$

$$e^{\lambda^{'} N^{'}_{>}(t)}\ c^{\dagger}_{\mathbf{k-q},>}(t)\ e^{-\lambda^{'} N^{'}_{>}(t)} = e^{\lambda^{'}}\ c^{\dagger}_{\mathbf{k-q},>}(t)$$

$$e^{\lambda N_{>}(t)}\ e^{\lambda^{'} N^{'}_{>}(t)}\ a^{\dagger}_{\mathbf{k-q}/2}(-\mathbf{q},t)\ e^{-\lambda^{'} N^{'}_{>}(t)}\ e^{-\lambda N_{>}(t)}\ =\ e^{\lambda+\lambda^{'}}\ a^{\dagger}_{\mathbf{k-q}/2}(-\mathbf{q},t)$$

$$e^{\lambda+\lambda^{'}} \;< T\; e^{-\lambda N_{>}(t)}\; e^{-\lambda^{'} N^{'}_{>}(t)}\; a^{\dagger}_{\mathbf{k-q}/2}(-\mathbf{q},t)\; c_{\mathbf{k-q},>}(t) c^{\dagger}_{\mathbf{k},<}(t^{'}) > \;=$$

$$e^{\frac{i\mathbf{q}.(-2\mathbf{k}t+\mathbf{q}(t+i\beta))}{2m}}(f_{max}\; e^{\frac{\beta \mathbf{k.q}}{m}} + in_{F}(\mathbf{k})\; < T\; e^{-\lambda N_{>}(t)}\; e^{-\lambda^{'} N^{'}_{>}(t)}\; c_{\mathbf{k-q},>}(t) c^{\dagger}_{\mathbf{k-q},>}(t^{'}) >\; i\; \theta(t^{'}-t)\; e^{\frac{\mathbf{q}.(2i\mathbf{k}t^{'}+\mathbf{q}(\beta-it^{'}))}{2m}})$$

and

$$< T\; e^{-\lambda N_{>}(t)}\; e^{-\lambda^{'} N^{'}_{>}(t)}\; c_{\mathbf{k-q},>}(t) a^{\dagger}_{\mathbf{k-q}/2}(-\mathbf{q},t)\; c^{\dagger}_{\mathbf{k},<}(t^{'}) > \;=$$

$$e^{-\frac{it\mathbf{q}.(2\mathbf{k}-\mathbf{q})}{2m}}(f_{max} - in_{F}(\mathbf{k})\; < T\; e^{-\lambda N_{>}(t)}\; e^{-\lambda^{'} N^{'}_{>}(t)}\; c_{\mathbf{k-q},>}(t) c^{\dagger}_{\mathbf{k-q},>}(t^{'}) >\; i\; \theta(t-t^{'})\; e^{\frac{it^{'}\mathbf{q}.(2\mathbf{k}-\mathbf{q})}{2m}})$$

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$$e^{-\lambda-\lambda^{'}} \;< T\; e^{-\lambda N_{>}(t)}\; e^{-\lambda^{'} N^{'}_{>}(t)}\; a_{\mathbf{k-q}/2}(\mathbf{q},t)\; c_{\mathbf{k-q},<}(t) c^{\dagger}_{\mathbf{k},>}(t^{'}) > \;=$$

$$e^{\frac{i\mathbf{q}.(-2\mathbf{k}t+\mathbf{q}(t+i\beta))}{2m}}(g_{max}\; e^{\frac{\beta \mathbf{k.q}}{m}} + i(1-n_{F}(\mathbf{k}))\; < T\; e^{-\lambda N_{>}(t)}\; e^{-\lambda^{'} N^{'}_{>}(t)}\; c_{\mathbf{k-q},<}(t) c^{\dagger}_{\mathbf{k-q},<}(t^{'}) >\; i\; \theta(t^{'}-t) e^{\frac{(2i\mathbf{k.q}t^{'}+\mathbf{q}^{2}(\beta-it^{'}))}{2m}})$$

and

$$< T\; e^{-\lambda N_{>}(t)}\; e^{-\lambda^{'} N^{'}_{>}(t)}\; c_{\mathbf{k-q},<}(t)\; a_{\mathbf{k-q}/2}(\mathbf{q},t) c^{\dagger}_{\mathbf{k},>}(t^{'}) > \;=$$

$$e^{-\frac{it\mathbf{q}.(2\mathbf{k}-\mathbf{q})}{2m}}(g_{max} - i(1-n_{F}(\mathbf{k}))\; < T\; e^{-\lambda N_{>}(t)}\; e^{-\lambda^{'} N^{'}_{>}(t)}\; c_{\mathbf{k-q},<}(t) c^{\dagger}_{\mathbf{k-q},<}(t^{'}) >\; i\; \theta(t-t^{'})\; e^{\frac{it^{'}\mathbf{q}.(2\mathbf{k}-\mathbf{q})}{2m}})$$

———

$$< T\ e^{-\lambda N_>(t)}\ e^{-\lambda^{'} N^{'}_>(t)}\ [a^{\dagger}_{\mathbf{k-q}/2}(-\mathbf{q},t), c_{\mathbf{k-q},>}(t)] c^{\dagger}_{\mathbf{k},<}(t^{'}) > \ =$$

$$e^{-\lambda-\lambda^{'}}\ e^{\frac{i\mathbf{q}.(-2\mathbf{k}t+\mathbf{q}(t+i\beta))}{2m}} (f_{max}\ e^{\frac{\beta\mathbf{k}.\mathbf{q}}{m}} + in_F(\mathbf{k})\ < T\ e^{-\lambda N_>(t)}\ e^{-\lambda^{'} N^{'}_>(t)}\ c_{\mathbf{k-q},>}(t) c^{\dagger}_{\mathbf{k-q},>}(t^{'}) >\ i\ \theta(t^{'}-t)\ e^{\frac{\mathbf{q}.(2i\mathbf{k}t^{'}+\mathbf{q}(\beta-it^{'}))}{2m}})$$

$$-e^{-\frac{it\mathbf{q}.(2\mathbf{k-q})}{2m}} (f_{max} - in_F(\mathbf{k})\ < T\ e^{-\lambda N_>(t)}\ e^{-\lambda^{'} N^{'}_>(t)}\ c_{\mathbf{k-q},>}(t) c^{\dagger}_{\mathbf{k-q},>}(t^{'}) >\ i\ \theta(t-t^{'})\ e^{\frac{it^{'}\mathbf{q}.(2\mathbf{k-q})}{2m}})$$

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==

$$< T\ e^{-\lambda N_>(t)}\ e^{-\lambda^{'} N^{'}_>(t)}\ [a_{\mathbf{k-q}/2}(\mathbf{q},t), c_{\mathbf{k-q},<}(t)] c^{\dagger}_{\mathbf{k},>}(t^{'}) > \ =$$

$$e^{\lambda+\lambda^{'}}\ e^{\frac{i\mathbf{q}.(-2\mathbf{k}t+\mathbf{q}(t+i\beta))}{2m}} (g_{max}\ e^{\frac{\beta\mathbf{k}.\mathbf{q}}{m}} + i(1-n_F(\mathbf{k}))\ < T\ e^{-\lambda N_>(t)}\ e^{-\lambda^{'} N^{'}_>(t)}\ c_{\mathbf{k-q},<}(t) c^{\dagger}_{\mathbf{k-q},<}(t^{'}) >\ i\ \theta(t^{'}-t) e^{\frac{(2i\mathbf{k}.\mathbf{q}t^{'}+\mathbf{q}^2(\beta-it^{'}))}{2m}})$$

$$-e^{-\frac{it\mathbf{q}.(2\mathbf{k-q})}{2m}} (g_{max} - i(1-n_F(\mathbf{k}))\ < T\ e^{-\lambda N_>(t)}\ e^{-\lambda^{'} N^{'}_>(t)}\ c_{\mathbf{k-q},<}(t) c^{\dagger}_{\mathbf{k-q},<}(t^{'}) >\ i\ \theta(t-t^{'})\ e^{\frac{it^{'}\mathbf{q}.(2\mathbf{k-q})}{2m}})$$

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$$-(1-n_F(\mathbf{k}-\mathbf{q}))\ \ < T\ e^{-\lambda N_>(t)}\ e^{-\lambda^{'} N^{'}_>(t)}\ c_{\mathbf{k},<}(t)c^{\dagger}_{\mathbf{k},<}(t^{'})>\ \ =$$

$$e^{-\lambda-\lambda^{'}}\ e^{\frac{i\mathbf{q}.(-2\mathbf{k}t+\mathbf{q}(t+i\beta))}{2m}}(f_{max}\ e^{\frac{\beta\mathbf{k}.\mathbf{q}}{m}}+in_F(\mathbf{k})\ < T\ e^{-\lambda N_>(t)}\ e^{-\lambda^{'} N^{'}_>(t)}\ c_{\mathbf{k}-\mathbf{q},>}(t)c^{\dagger}_{\mathbf{k}-\mathbf{q},>}(t^{'})>\ i\ \theta(t^{'}-t)\ e^{\frac{\mathbf{q}.(2i\mathbf{k}t^{'}+\mathbf{q}(\beta-it^{'}))}{2m}})$$

$$-e^{-\frac{it\mathbf{q}.(2\mathbf{k}-\mathbf{q})}{2m}}(f_{max}-in_F(\mathbf{k})\ < T\ e^{-\lambda N_>(t)}\ e^{-\lambda^{'} N^{'}_>(t)}\ c_{\mathbf{k}-\mathbf{q},>}(t)c^{\dagger}_{\mathbf{k}-\mathbf{q},>}(t^{'})>\ i\ \theta(t-t^{'})\ e^{\frac{it^{'}\mathbf{q}.(2\mathbf{k}-\mathbf{q})}{2m}})$$

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$$-n_F(\mathbf{k}-\mathbf{q})\ \ < T\ e^{-\lambda N_>(t)}\ e^{-\lambda^{'} N^{'}_>(t)}\ c_{\mathbf{k},>}(t)c^{\dagger}_{\mathbf{k},>}(t^{'})>\ \ =$$

$$e^{\lambda+\lambda^{'}}\ e^{\frac{i\mathbf{q}.(-2\mathbf{k}t+\mathbf{q}(t+i\beta))}{2m}}(g_{max}\ e^{\frac{\beta\mathbf{k}.\mathbf{q}}{m}}+i(1-n_F(\mathbf{k}))\ < T\ e^{-\lambda N_>(t)}\ e^{-\lambda^{'} N^{'}_>(t)}\ c_{\mathbf{k}-\mathbf{q},<}(t)c^{\dagger}_{\mathbf{k}-\mathbf{q},<}(t^{'})>\ i\ \theta(t^{'}-t)e^{\frac{(2i\mathbf{k}.\mathbf{q}t^{'}+\mathbf{q}^2(\beta-it^{'}))}{2m}})$$

$$-e^{-\frac{it\mathbf{q}.(2\mathbf{k}-\mathbf{q})}{2m}}(g_{max}-i(1-n_F(\mathbf{k}))\ < T\ e^{-\lambda N_>(t)}\ e^{-\lambda^{'} N^{'}_>(t)}\ c_{\mathbf{k}-\mathbf{q},<}(t)c^{\dagger}_{\mathbf{k}-\mathbf{q},<}(t^{'})>\ i\ \theta(t-t^{'})\ e^{\frac{it^{'}\mathbf{q}.(2\mathbf{k}-\mathbf{q})}{2m}})$$

or,

$$f_{max}\ =\ \frac{< T\ e^{-\lambda N_>(t)}\ e^{-\lambda^{'} N^{'}_>(t)}\ c_{\mathbf{k}-\mathbf{q},>}(t)c^{\dagger}_{\mathbf{k}-\mathbf{q},>}(t^{'})>\ n_F(\mathbf{k})e^{\frac{it^{'}(2\mathbf{k}.\mathbf{q}-q^2)}{2m}}\ \left(e^{\lambda+\lambda^{'}}\theta(t-t^{'})+\theta(t^{'}-t)\right)+< T\ e^{-\lambda N_>(t)}\ e^{-\lambda^{'} N^{'}_>(t)}\ c_{\mathbf{k},<}(t)c^{\dagger}_{\mathbf{k},<}(t^{'})>\ (n_F(\mathbf{k}-\mathbf{q})-1)e^{\frac{it(}{}}}{e^{\frac{\beta(2\mathbf{k}.\mathbf{q}-q^2)}{2m}}-e^{\lambda+\lambda^{'}}}$$

and

$$g_{max}\ =\ \frac{e^{-\frac{iq^2t^{'}}{2m}}\left(< T\ e^{-\lambda N_>(t)}\ e^{-\lambda^{'} N^{'}_>(t)}\ c_{\mathbf{k}-\mathbf{q},<}(t)c^{\dagger}_{\mathbf{k}-\mathbf{q},<}(t^{'})>\ (n_F(\mathbf{k})-1)\ \left(-e^{\frac{i\mathbf{k}.\mathbf{q}}{m}t^{'}}\right)\left(e^{\lambda+\lambda^{'}}\ \theta(t^{'}-t)+\theta(t-t^{'})\right)-< T\ e^{-\lambda N_>(t)}\ e^{-\lambda^{'} N^{'}_>(t)}\ c_{\mathbf{k},>}(t)c^{\dagger}_{\mathbf{k},>}(t^{'})>\ n_F(\right.}{e^{\frac{\beta(2\mathbf{k}.\mathbf{q}-q^2)}{2m}+\lambda+\lambda^{'}}-1}$$

$$
\begin{aligned}
f_{max} &= \frac{< T\ e^{-\lambda N_>(t)}\ e^{-\lambda^{'} N^{'}_>(t)}\ c_{\mathbf{k}-\mathbf{q},>}(t) c^{\dagger}_{\mathbf{k}-\mathbf{q},>}(t^{'}) >\ n_F(\mathbf{k}) e^{\frac{it^{'}(2\mathbf{k}.\mathbf{q}-q^2)}{2m}} \left(e^{\lambda+\lambda^{'}} \theta(t-t^{'}) + \theta(t^{'}-t) \right)}{e^{\frac{\beta(2\mathbf{k}.\mathbf{q}-q^2)}{2m}} - e^{\lambda+\lambda^{'}}} \\
&+ \frac{< T\ e^{-\lambda N_>(t)}\ e^{-\lambda^{'} N^{'}_>(t)}\ c_{\mathbf{k},<}(t) c^{\dagger}_{\mathbf{k},<}(t^{'}) >\ (n_F(\mathbf{k}-\mathbf{q}) - 1) e^{\frac{it(2\mathbf{k}.\mathbf{q}-q^2)}{2m} + \lambda + \lambda^{'}}}{e^{\frac{\beta(2\mathbf{k}.\mathbf{q}-q^2)}{2m}} - e^{\lambda+\lambda^{'}}}
\end{aligned}
$$

and

$$
\begin{aligned}
g_{max} &= \frac{e^{-\frac{iq^2 t^{'}}{2m}}\ < T\ e^{-\lambda N_>(t)}\ e^{-\lambda^{'} N^{'}_>(t)}\ c_{\mathbf{k}-\mathbf{q},<}(t) c^{\dagger}_{\mathbf{k}-\mathbf{q},<}(t^{'}) >\ (n_F(\mathbf{k}) - 1) \left(-e^{\frac{i\mathbf{k}.\mathbf{q}}{m} t^{'}} \right) \left(e^{\lambda+\lambda^{'}}\ \theta(t^{'}-t) + \theta(t-t^{'}) \right)}{e^{\frac{\beta(2\mathbf{k}.\mathbf{q}-q^2)}{2m} + \lambda + \lambda^{'}} - 1} \\
&+ \frac{-e^{-\frac{iq^2 t^{'}}{2m}}\ < T\ e^{-\lambda N_>(t)}\ e^{-\lambda^{'} N^{'}_>(t)}\ c_{\mathbf{k},>}(t) c^{\dagger}_{\mathbf{k},>}(t^{'}) >\ n_F(\mathbf{k}-\mathbf{q})\ e^{\frac{i(2\mathbf{k}.\mathbf{q}t + q^2(t^{'}-t))}{2m}}}{e^{\frac{\beta(2\mathbf{k}.\mathbf{q}-q^2)}{2m} + \lambda + \lambda^{'}} - 1}
\end{aligned}
$$

3 Closed equations

$$< T\ e^{-\lambda N_>(t)}\ e^{-\lambda^{'} N^{'}_>(t)}\ N_>(t)\ c_{\mathbf{k},>}(t) c^{\dagger}_{\mathbf{k},>}(t^{'}) > \ =$$

$$\sum_{\mathbf{q}}\ e^{-\frac{it\mathbf{q}.(2\mathbf{k}-\mathbf{q})}{2m}} (g_{max} - i(1 - n_F(\mathbf{k}))\ < T\ e^{-\lambda N_>(t)}\ e^{-\lambda^{'} N^{'}_>(t)}\ c_{\mathbf{k}-\mathbf{q},<}(t) c^{\dagger}_{\mathbf{k}-\mathbf{q},<}(t^{'}) > \ i\ \theta(t - t^{'})\ e^{\frac{it^{'}\mathbf{q}.(2\mathbf{k}-\mathbf{q})}{2m}})$$

and

$$< T\ e^{-\lambda N_>(t)}\ e^{-\lambda^{'} N^{'}_>(t)}\ N^{'}_>(t)\ c_{\mathbf{k},<}(t) c^{\dagger}_{\mathbf{k},<}(t^{'}) > \ =$$

$$-e^{-\lambda-\lambda^{'}}\ \sum_{\mathbf{q}}\ e^{\frac{i\mathbf{q}.(-2\mathbf{k}t+\mathbf{q}(t+i\beta))}{2m}} (f_{max}\ e^{\frac{\beta\mathbf{k}.\mathbf{q}}{m}} + in_F(\mathbf{k})\ < T\ e^{-\lambda N_>(t)}\ e^{-\lambda^{'} N^{'}_>(t)}\ c_{\mathbf{k}-\mathbf{q},>}(t) c^{\dagger}_{\mathbf{k}-\mathbf{q},>}(t^{'}) > \ i\ \theta(t^{'} - t)\ e^{\frac{\mathbf{q}.(2i\mathbf{k}t^{'}+\mathbf{q}(\beta-it^{'}))}{2m}})$$

==

$$-\partial_{\lambda}\ < T\ e^{-\lambda N_>(t)}\ e^{-\lambda^{'} N^{'}_>(t)}\ c_{\mathbf{k},>}(t) c^{\dagger}_{\mathbf{k},>}(t^{'}) > \ =$$

$$\sum_{\mathbf{q}}\ e^{-\frac{i\mathbf{q}.(2\mathbf{k}-\mathbf{q})(t-t^{'})}{2m}}\ \frac{< T\ e^{-\lambda N_>(t)}\ e^{-\lambda^{'} N^{'}_>(t)}\ c_{\mathbf{k}-\mathbf{q},<}(t) c^{\dagger}_{\mathbf{k}-\mathbf{q},<}(t^{'}) > \ (1 - n_F(\mathbf{k}))(e^{\lambda+\lambda^{'}}\ \theta(t^{'} - t) + \theta(t - t^{'}))}{e^{\frac{\beta(2\mathbf{k}.\mathbf{q}-q^2)}{2m}+\lambda+\lambda^{'}} - 1}$$

$$+\sum_{\mathbf{q}}\ e^{-\frac{i\mathbf{q}.(2\mathbf{k}-\mathbf{q})(t-t^{'})}{2m}}\ (1 - n_F(\mathbf{k}))\ < T\ e^{-\lambda N_>(t)}\ e^{-\lambda^{'} N^{'}_>(t)}\ c_{\mathbf{k}-\mathbf{q},<}(t) c^{\dagger}_{\mathbf{k}-\mathbf{q},<}(t^{'}) > \ \theta(t - t^{'})$$

$$-\sum_{\mathbf{q}}\ \frac{< T\ e^{-\lambda N_>(t)}\ e^{-\lambda^{'} N^{'}_>(t)}\ c_{\mathbf{k},>}(t) c^{\dagger}_{\mathbf{k},>}(t^{'}) > \ n_F(\mathbf{k} - \mathbf{q})}{e^{\frac{\beta(2\mathbf{k}.\mathbf{q}-q^2)}{2m}+\lambda+\lambda^{'}} - 1}$$

and

$$-\partial_{\lambda^{'}}\ < T\ e^{-\lambda N_>(t)}\ e^{-\lambda^{'} N^{'}_>(t)}\ c_{\mathbf{k},<}(t) c^{\dagger}_{\mathbf{k},<}(t^{'}) > \ =$$

$$\sum_{\mathbf{q}}\ e^{\frac{i\mathbf{q}.(\mathbf{q}-2\mathbf{k})(t-t^{'})}{2m}-\lambda-\lambda^{'}}\ \frac{< T\ e^{-\lambda N_>(t)}\ e^{-\lambda^{'} N^{'}_>(t)}\ c_{\mathbf{k}-\mathbf{q},>}(t) c^{\dagger}_{\mathbf{k}-\mathbf{q},>}(t^{'}) >}{e^{-\frac{\mathbf{q}.(2\mathbf{k}-\mathbf{q})\beta}{2m}}\ e^{\lambda+\lambda^{'}} - 1}\ n_F(\mathbf{k}) \left(e^{\lambda+\lambda^{'}}\theta(t - t^{'}) + \theta(t^{'} - t)\right)$$

$$+\sum_{\mathbf{q}} e^{\frac{i\mathbf{q}.(\mathbf{q}-2\mathbf{k})(t-t^{'})}{2m}-\lambda-\lambda^{'}}\ n_F(\mathbf{k})\ < T\ e^{-\lambda N_>(t)}\ e^{-\lambda^{'} N^{'}_>(t)}\ c_{\mathbf{k}-\mathbf{q},>}(t) c^{\dagger}_{\mathbf{k}-\mathbf{q},>}(t^{'}) > \ \theta(t^{'} - t)$$

$$+\sum_{\mathbf{q}} e^{\frac{\beta\mathbf{q}.(2\mathbf{k}-\mathbf{q})}{2m}}\ \frac{< T\ e^{-\lambda N_>(t)}\ e^{-\lambda^{'} N^{'}_>(t)}\ c_{\mathbf{k},<}(t) c^{\dagger}_{\mathbf{k},<}(t^{'}) >}{e^{\frac{\beta(2\mathbf{k}.\mathbf{q}-q^2)}{2m}} - e^{\lambda+\lambda^{'}}}\ (1 - n_F(\mathbf{k} - \mathbf{q}))$$

==

==

$$-\partial_{\lambda}\ \ < T\ e^{-\lambda N_{>}(t)}\ e^{-\lambda^{'} N^{'}_{>}(t)}\ c_{\mathbf{k},>}(t) c^{\dagger}_{\mathbf{k},>}(t^{'})>\ \ =$$

$$\sum_{\mathbf{q}}\ e^{\lambda+\lambda^{'}} e^{-\frac{i\mathbf{q}.(2\mathbf{k}-\mathbf{q})(t-t^{'})}{2m}}\ \frac{< T\ e^{-\lambda N_{>}(t)}\ e^{-\lambda^{'} N^{'}_{>}(t)}\ c_{\mathbf{k}-\mathbf{q},<}(t) c^{\dagger}_{\mathbf{k}-\mathbf{q},<}(t^{'})>\ (1-n_{F}(\mathbf{k}))}{e^{\frac{\beta(2\mathbf{k}.\mathbf{q}-q^{2})}{2m}+\lambda+\lambda^{'}}-1}\ \left(\theta(t^{'}-t)+\theta(t-t^{'}) e^{\frac{\beta(2\mathbf{k}.\mathbf{q}-q^{2})}{2m}}\right)$$

$$-\sum_{\mathbf{q}}\ \frac{< T\ e^{-\lambda N_{>}(t)}\ e^{-\lambda^{'} N^{'}_{>}(t)}\ c_{\mathbf{k},>}(t) c^{\dagger}_{\mathbf{k},>}(t^{'})>\ n_{F}(\mathbf{k}-\mathbf{q})}{e^{\frac{\beta(2\mathbf{k}.\mathbf{q}-q^{2})}{2m}+\lambda+\lambda^{'}}-1}$$

and

$$-\partial_{\lambda^{'}}\ \ < T\ e^{-\lambda N_{>}(t)}\ e^{-\lambda^{'} N^{'}_{>}(t)}\ c_{\mathbf{k},<}(t) c^{\dagger}_{\mathbf{k},<}(t^{'})>\ \ =$$

$$\sum_{\mathbf{q}}\ e^{\frac{i\mathbf{q}.(\mathbf{q}-2\mathbf{k})(t-t^{'})}{2m}}\ \ \frac{< T\ e^{-\lambda N_{>}(t)}\ e^{-\lambda^{'} N^{'}_{>}(t)}\ c_{\mathbf{k}-\mathbf{q},>}(t) c^{\dagger}_{\mathbf{k}-\mathbf{q},>}(t^{'})>}{e^{-\frac{\mathbf{q}.(2\mathbf{k}-\mathbf{q})\beta}{2m}}\ e^{\lambda+\lambda^{'}}-1}\ n_{F}(\mathbf{k})\left(\theta(t-t^{'})+e^{-\frac{\mathbf{q}.(2\mathbf{k}-\mathbf{q})\beta}{2m}}\ \theta(t^{'}-t)\right)$$

$$-\sum_{\mathbf{q}}\frac{< T\ e^{-\lambda N_{>}(t)}\ e^{-\lambda^{'} N^{'}_{>}(t)}\ c_{\mathbf{k},<}(t) c^{\dagger}_{\mathbf{k},<}(t^{'})>}{e^{-\frac{\beta\mathbf{q}.(2\mathbf{k}-\mathbf{q})}{2m}}\ e^{\lambda+\lambda^{'}}-1}\ (1-n_{F}(\mathbf{k}-\mathbf{q}))$$

==

== Set $\mathbf{k}-\mathbf{q}=\mathbf{p}$.

$$-\partial_{\lambda}\ \ < T\ e^{-\lambda N_{>}(t)}\ e^{-\lambda^{'} N^{'}_{>}(t)}\ c_{\mathbf{k},>}(t) c^{\dagger}_{\mathbf{k},>}(t^{'})>\ \ =$$

$$\sum_{\mathbf{p}}\frac{e^{\lambda+\lambda^{'}}\ e^{i(t-t^{'})(\epsilon_{\mathbf{p}}-\epsilon_{\mathbf{k}})}\ < T\ e^{-\lambda N_{>}(t)}\ e^{-\lambda^{'} N^{'}_{>}(t)}\ c_{\mathbf{p},<}(t) c^{\dagger}_{\mathbf{p},<}(t^{'})>\ (1-n_{F}(\mathbf{k}))\ \left(\theta(t^{'}-t)+\theta(t-t^{'}) e^{\beta(\epsilon_{\mathbf{k}}-\epsilon_{\mathbf{p}})}\right)-< T\ e^{-\lambda N_{>}(t)}\ e^{-\lambda^{'} N^{'}_{>}(t)}\ c_{\mathbf{k},>}(t) c^{\dagger}_{\mathbf{k},>}(t^{'})>\ n_{F}(\mathbf{p})}{e^{\beta(\epsilon_{\mathbf{k}}-\epsilon_{\mathbf{p}})} e^{\lambda+\lambda^{'}}-1}$$

and

$$-\partial_{\lambda^{'}}\ \ < T\ e^{-\lambda N_{>}(t)}\ e^{-\lambda^{'} N^{'}_{>}(t)}\ c_{\mathbf{k},<}(t) c^{\dagger}_{\mathbf{k},<}(t^{'})>\ \ =$$

$$\sum_{\mathbf{p}}\frac{e^{i(t-t^{'})(\epsilon_{\mathbf{p}}-\epsilon_{\mathbf{k}})}\ < T\ e^{-\lambda N_{>}(t)}\ e^{-\lambda^{'} N^{'}_{>}(t)}\ c_{\mathbf{p},>}(t) c^{\dagger}_{\mathbf{p},>}(t^{'})>\ n_{F}(\mathbf{k})\left(\theta(t-t^{'})+e^{\beta(\epsilon_{\mathbf{p}}-\epsilon_{\mathbf{k}})}\ \theta(t^{'}-t)\right)-< T\ e^{-\lambda N_{>}(t)}\ e^{-\lambda^{'} N^{'}_{>}(t)}\ c_{\mathbf{k},<}(t) c^{\dagger}_{\mathbf{k},<}(t^{'})>\ (1-n_{F}(\mathbf{p}))}{e^{\beta(\epsilon_{\mathbf{p}}-\epsilon_{\mathbf{k}})} e^{\lambda+\lambda^{'}}-1}$$

4 Time evolution

We may see from the above equations that the time evolution of the operators is a simple exponential.

$$c_{\mathbf{p}}(t) = \tilde{c}_{\mathbf{p}}(t)\ e^{-i\epsilon_{\mathbf{p}}\ t}$$

The piece-wise constant reduced correlation functions obey these equations,

$$-\partial_{\lambda}\ \ < T\ e^{-\lambda N_{>}(t)}\ e^{-\lambda^{'} N^{'}_{>}(t)}\ \tilde{c}_{\mathbf{k},>}(t) \tilde{c}^{\dagger}_{\mathbf{k},>}(t^{'})>\ \ =$$

$$\sum_{\mathbf{p}}\frac{e^{\lambda+\lambda^{'}}\ \ < T\ e^{-\lambda N_{>}(t)}\ e^{-\lambda^{'} N^{'}_{>}(t)}\ \tilde{c}_{\mathbf{p},<}(t) \tilde{c}^{\dagger}_{\mathbf{p},<}(t^{'})>\ (1-n_{F}(\mathbf{k}))\ \left(\theta(t^{'}-t)+\theta(t-t^{'}) e^{\beta(\epsilon_{\mathbf{k}}-\epsilon_{\mathbf{p}})}\right)-< T\ e^{-\lambda N_{>}(t)}\ e^{-\lambda^{'} N^{'}_{>}(t)}\ \tilde{c}_{\mathbf{k},>}(t) \tilde{c}^{\dagger}_{\mathbf{k},>}(t^{'})>\ n_{F}(\mathbf{p})}{e^{\beta(\epsilon_{\mathbf{k}}-\epsilon_{\mathbf{p}})} e^{\lambda+\lambda^{'}}-1}$$

and

$$-\partial_{\lambda^{'}}\ \ < T\ e^{-\lambda N_{>}(t)}\ e^{-\lambda^{'} N^{'}_{>}(t)}\ \tilde{c}_{\mathbf{k},<}(t) \tilde{c}^{\dagger}_{\mathbf{k},<}(t^{'})>\ \ =$$

$$\sum_{\mathbf{p}}\frac{< T\ e^{-\lambda N_{>}(t)}\ e^{-\lambda^{'} N^{'}_{>}(t)}\ \tilde{c}_{\mathbf{p},>}(t) \tilde{c}^{\dagger}_{\mathbf{p},>}(t^{'})>\ n_{F}(\mathbf{k})\left(\theta(t-t^{'})+e^{\beta(\epsilon_{\mathbf{p}}-\epsilon_{\mathbf{k}})}\ \theta(t^{'}-t)\right)-< T\ e^{-\lambda N_{>}(t)}\ e^{-\lambda^{'} N^{'}_{>}(t)}\ \tilde{c}_{\mathbf{k},<}(t) \tilde{c}^{\dagger}_{\mathbf{k},<}(t^{'})>\ (1-n_{F}(\mathbf{p}))}{e^{\beta(\epsilon_{\mathbf{p}}-\epsilon_{\mathbf{k}})} e^{\lambda+\lambda^{'}}-1}$$

5 Separability condition

$$e^{\lambda+\lambda^{'}}\ \ < T\ e^{-\lambda N_{>}(t)}\ e^{-\lambda^{'} N^{'}_{>}(t)}\ \tilde{c}_{\mathbf{p},<}(t) \tilde{c}^{\dagger}_{\mathbf{p},<}(t^{'})>\ (1-n_{F}(\mathbf{k}))\ \left(\theta(t^{'}-t)+\theta(t-t^{'}) e^{\beta(\epsilon_{\mathbf{k}}-\epsilon_{\mathbf{p}})}\right)-< T\ e^{-\lambda N_{>}(t)}\ e^{-\lambda^{'} N^{'}_{>}(t)}\ \tilde{c}_{\mathbf{k},>}(t) \tilde{c}^{\dagger}_{\mathbf{k},>}(t^{'})>\ n_{F}(\mathbf{p})\ =$$

$$(e^{\beta(\epsilon_{\mathbf{k}}-\epsilon_{\mathbf{p}})} e^{\lambda+\lambda^{'}}-1)\ L_{>}(\mathbf{k};\lambda,\lambda^{'};t-t^{'}) R_{<}(\mathbf{p};\lambda,\lambda^{'};t-t^{'})$$

and

$$< T\ e^{-\lambda N_{>}(t)}\ e^{-\lambda^{'} N^{'}_{>}(t)}\ \tilde{c}_{\mathbf{p},>}(t) \tilde{c}^{\dagger}_{\mathbf{p},>}(t^{'})>\ n_{F}(\mathbf{k})\left(\theta(t-t^{'})+e^{\beta(\epsilon_{\mathbf{p}}-\epsilon_{\mathbf{k}})}\ \theta(t^{'}-t)\right)-< T\ e^{-\lambda N_{>}(t)}\ e^{-\lambda^{'} N^{'}_{>}(t)}\ \tilde{c}_{\mathbf{k},<}(t) \tilde{c}^{\dagger}_{\mathbf{k},<}(t^{'})>\ (1-n_{F}(\mathbf{p}))\ =$$

$$(e^{\beta(\epsilon_{\mathbf{p}}-\epsilon_{\mathbf{k}})} e^{\lambda+\lambda^{'}}-1)\ L_{<}(\mathbf{k};\lambda,\lambda^{'};t-t^{'}) R_{>}(\mathbf{p};\lambda,\lambda^{'};t-t^{'})$$

==

6 Compact equations to be solved

$$-\partial_{\lambda} \;\; < T\; e^{-\lambda N_>(t)}\; e^{-\lambda' N'_>(t)}\; \tilde{c}_{\mathbf{k},>}(t)\tilde{c}^{\dagger}_{\mathbf{k},>}(t') > \;=\; L_>(\mathbf{k};\lambda,\lambda';t-t')\sum_{\mathbf{p}} R_<(\mathbf{p};\lambda,\lambda';t-t')$$

and

$$-\partial_{\lambda'} \;\; < T\; e^{-\lambda N_>(t)}\; e^{-\lambda' N'_>(t)}\; \tilde{c}_{\mathbf{k},<}(t)\tilde{c}^{\dagger}_{\mathbf{k},<}(t') > \;=\; L_<(\mathbf{k};\lambda,\lambda';t-t')\sum_{\mathbf{p}} R_>(\mathbf{p};\lambda,\lambda';t-t')$$

7 Green function in terms of the separated functions

$$e^{\lambda+\lambda'} \;\; < T\; e^{-\lambda N_>(t)}\; e^{-\lambda' N'_>(t)}\; \tilde{c}_{\mathbf{p},<}(t)\tilde{c}^{\dagger}_{\mathbf{p},<}(t') > \; (1-n_F(\mathbf{k}))\; \left(\theta(t'-t)+\theta(t-t')e^{\beta(\epsilon_{\mathbf{k}}-\epsilon_{\mathbf{p}})}\right) - < T\; e^{-\lambda N_>(t)}\; e^{-\lambda' N'_>(t)}\; \tilde{c}_{\mathbf{k},>}(t)\tilde{c}^{\dagger}_{\mathbf{k},>}(t') > \; n_F(\mathbf{p}) \;=$$

$$(e^{\beta(\epsilon_{\mathbf{k}}-\epsilon_{\mathbf{p}})}e^{\lambda+\lambda'}-1)\; L_>(\mathbf{k};\lambda,\lambda';t-t')R_<(\mathbf{p};\lambda,\lambda';t-t')$$

and

$$< T\; e^{-\lambda N_>(t)}\; e^{-\lambda' N'_>(t)}\; \tilde{c}_{\mathbf{k},>}(t)\tilde{c}^{\dagger}_{\mathbf{k},>}(t') > \; n_F(\mathbf{p})\left(\theta(t-t')+e^{\beta(\epsilon_{\mathbf{k}}-\epsilon_{\mathbf{p}})}\;\theta(t'-t)\right) - < T\; e^{-\lambda N_>(t)}\; e^{-\lambda' N'_>(t)}\; \tilde{c}_{\mathbf{p},<}(t)\tilde{c}^{\dagger}_{\mathbf{p},<}(t') > \; (1-n_F(\mathbf{k})) \;=$$

$$(e^{\beta(\epsilon_{\mathbf{k}}-\epsilon_{\mathbf{p}})}e^{\lambda+\lambda'}-1)\; L_<(\mathbf{p};\lambda,\lambda';t-t')R_>(\mathbf{k};\lambda,\lambda';t-t')$$

==

$$< T\; e^{-\lambda N_>(t)}\; e^{-\lambda' N'_>(t)}\; \tilde{c}_{\mathbf{p},<}(t)\tilde{c}^{\dagger}_{\mathbf{p},<}(t') > \; (1-n_F(\mathbf{k})) \;=\; L_<(\mathbf{p};\lambda,\lambda';t-t')R_>(\mathbf{k};\lambda,\lambda';t-t') + L_>(\mathbf{k};\lambda,\lambda';t-t')R_<(\mathbf{p};\lambda,\lambda';t-t')\; (\theta(t'-t)e^{\beta(\epsilon_{\mathbf{k}}-\epsilon_{\mathbf{p}})}+\theta(t-t'))$$

and

$$< T\; e^{-\lambda N_>(t)}\; e^{-\lambda' N'_>(t)}\; \tilde{c}_{\mathbf{k},>}(t)\tilde{c}^{\dagger}_{\mathbf{k},>}(t') > \; n_F(\mathbf{p}) \;=\; L_>(\mathbf{k};\lambda,\lambda';t-t')R_<(\mathbf{p};\lambda,\lambda';t-t') + L_<(\mathbf{p};\lambda,\lambda';t-t')R_>(\mathbf{k};\lambda,\lambda';t-t')\; (e^{\beta(\epsilon_{\mathbf{k}}-\epsilon_{\mathbf{p}})}\theta(t-t')+\theta(t'-t))\; e^{\lambda+\lambda'}$$

OR,

$$< T\; e^{-\lambda N_>(t)}\; e^{-\lambda' N'_>(t)}\; \tilde{c}_{\mathbf{p},<}(t)\tilde{c}^{\dagger}_{\mathbf{p},<}(t') > \; (1-n_F(\mathbf{k})) \;=\; L_<(\mathbf{p};\lambda,\lambda';t-t')R_>(\mathbf{k};\lambda,\lambda';t-t') + L_>(\mathbf{k};\lambda,\lambda';t-t')R_<(\mathbf{p};\lambda,\lambda';t-t')\; e^{\theta(t'-t)\beta(\epsilon_{\mathbf{k}}-\epsilon_{\mathbf{p}})}$$

and

$$< T\; e^{-\lambda N_>(t)}\; e^{-\lambda' N'_>(t)}\; \tilde{c}_{\mathbf{k},>}(t)\tilde{c}^{\dagger}_{\mathbf{k},>}(t') > \; n_F(\mathbf{p}) \;=\; L_>(\mathbf{k};\lambda,\lambda';t-t')R_<(\mathbf{p};\lambda,\lambda';t-t') + L_<(\mathbf{p};\lambda,\lambda';t-t')R_>(\mathbf{k};\lambda,\lambda';t-t')\; e^{\theta(t-t')\beta(\epsilon_{\mathbf{k}}-\epsilon_{\mathbf{p}})}\; e^{\lambda+\lambda'}$$

8 Reduction

$$< T\; e^{-\lambda N_>(t)}\; e^{-\lambda' N'_>(t)}\; \tilde{c}_{\mathbf{p},<}(t)\tilde{c}^{\dagger}_{\mathbf{p},<}(t') > \; (1-n_F(\mathbf{k})) \;=\; L_<(\mathbf{p};\lambda,\lambda';t-t')R_>(\mathbf{k};\lambda,\lambda';t-t') + L_>(\mathbf{k};\lambda,\lambda';t-t')\; e^{\theta(t'-t)\beta\epsilon_{\mathbf{k}}}\; R_<(\mathbf{p};\lambda,\lambda';t-t')\; e^{-\theta(t'-t)\beta\epsilon_{\mathbf{p}}}$$

and

$$< T\; e^{-\lambda N_>(t)}\; e^{-\lambda' N'_>(t)}\; \tilde{c}_{\mathbf{k},>}(t)\tilde{c}^{\dagger}_{\mathbf{k},>}(t') > \; n_F(\mathbf{p}) \;=\; L_>(\mathbf{k};\lambda,\lambda';t-t')R_<(\mathbf{p};\lambda,\lambda';t-t') + L_<(\mathbf{p};\lambda,\lambda';t-t')\; e^{-\theta(t-t')\beta\epsilon_{\mathbf{p}}}\, R_>(\mathbf{k};\lambda,\lambda';t-t')\; e^{\theta(t-t')\beta\epsilon_{\mathbf{k}}}\; e^{\lambda+\lambda'}$$

==

Set,

$$c_<(\lambda,\lambda';t-t')\; L_<(\mathbf{p};\lambda,\lambda';t-t') \;=\; R_<(\mathbf{p};\lambda,\lambda';t-t')\; e^{-\theta(t'-t)\beta\epsilon_{\mathbf{p}}}$$

$$c_>(\lambda,\lambda';t-t')\; L_>(\mathbf{k};\lambda,\lambda';t-t') \;=\; R_>(\mathbf{k};\lambda,\lambda';t-t')\; e^{\theta(t-t')\beta\epsilon_{\mathbf{k}}}$$

==

$$< T\; e^{-\lambda N_>(t)}\; e^{-\lambda' N'_>(t)}\; \tilde{c}_{\mathbf{p},<}(t)\tilde{c}^{\dagger}_{\mathbf{p},<}(t') > \; (1-n_F(\mathbf{k})) \;=\; L_<(\mathbf{p};\lambda,\lambda';t-t')\; R_>(\mathbf{k};\lambda,\lambda';t-t')(1+e^{\beta\epsilon_{\mathbf{k}}}\; c_>^{-1}(\lambda,\lambda';t-t')\; c_<(\lambda,\lambda';t-t'))$$

and

$$< T\; e^{-\lambda N_>(t)}\; e^{-\lambda' N'_>(t)}\; \tilde{c}_{\mathbf{k},>}(t)\tilde{c}^{\dagger}_{\mathbf{k},>}(t') > \; n_F(\mathbf{p}) \;=\; L_>(\mathbf{k};\lambda,\lambda';t-t')R_<(\mathbf{p};\lambda,\lambda';t-t')\; (1+e^{-\beta\epsilon_{\mathbf{p}}}c_<^{-1}(\lambda,\lambda';t-t')c_>(\lambda,\lambda';t-t')\; e^{\lambda+\lambda'})$$

From this we may conclude,

$$R_<(\mathbf{p};\lambda,\lambda';t-t') \;=\; g_<(\lambda,\lambda';t-t')\; n_F(\mathbf{p})\; \left(1+e^{-\beta\epsilon_{\mathbf{p}}}c_<^{-1}(\lambda,\lambda';t-t')c_>(\lambda,\lambda';t-t')\; e^{\lambda+\lambda'}\right)^{-1}$$

and

$$R_>(\mathbf{k};\lambda,\lambda';t-t') \;=\; g_>(\lambda,\lambda';t-t')\; (1-n_F(\mathbf{k}))\; \left(1+e^{\beta\epsilon_{\mathbf{k}}}\; c_>^{-1}(\lambda,\lambda';t-t')\; c_<(\lambda,\lambda';t-t')\right)^{-1}$$

and

$$L_<(\mathbf{p};\lambda,\lambda';t-t') \;=\; c_<^{-1}(\lambda,\lambda';t-t')\; g_<(\lambda,\lambda';t-t')\; n_F(\mathbf{p})\; \left(1+e^{-\beta\epsilon_{\mathbf{p}}}c_<^{-1}(\lambda,\lambda';t-t')c_>(\lambda,\lambda';t-t')\; e^{\lambda+\lambda'}\right)^{-1}\; e^{-\theta(t'-t)\beta\epsilon_{\mathbf{p}}}$$

and

$$L_>(\mathbf{k};\lambda,\lambda';t-t') \;=\; c_>^{-1}(\lambda,\lambda';t-t')\;\; g_>(\lambda,\lambda';t-t')\; (1-n_F(\mathbf{k}))\; \left(1+e^{\beta\epsilon_{\mathbf{k}}}\; c_>^{-1}(\lambda,\lambda';t-t')\; c_<(\lambda,\lambda';t-t')\right)^{-1}\; e^{\theta(t-t')\beta\epsilon_{\mathbf{k}}}$$

or,

$$< T\; e^{-\lambda N_>(t)}\; e^{-\lambda' N'_>(t)}\; \tilde{c}_{\mathbf{p},<}(t)\tilde{c}^{\dagger}_{\mathbf{p},<}(t') > \;=\; c_<^{-1}(\lambda,\lambda';t-t')\; g_<(\lambda,\lambda';t-t')g_>(\lambda,\lambda';t-t')\; n_F(\mathbf{p})\; \left(1+e^{-\beta\epsilon_{\mathbf{p}}}c_<^{-1}(\lambda,\lambda';t-t')c_>(\lambda,\lambda';t-t')\; e^{\lambda+\lambda'}\right)^{-1}\; e^{-\theta(t'-t)\beta\epsilon_{\mathbf{p}}}$$

and

$$< T\; e^{-\lambda N_>(t)}\; e^{-\lambda' N'_>(t)}\; \tilde{c}_{\mathbf{k},>}(t)\tilde{c}^{\dagger}_{\mathbf{k},>}(t') > \;=\; c_>^{-1}(\lambda,\lambda';t-t')\;\; g_>(\lambda,\lambda';t-t')g_<(\lambda,\lambda';t-t')\; (1-n_F(\mathbf{k}))\; \left(1+e^{\beta\epsilon_{\mathbf{k}}}\; c_>^{-1}(\lambda,\lambda';t-t')\; c_<(\lambda,\lambda';t-t')\right)^{-1}\; e^{\theta(t-t')\beta\epsilon_{\mathbf{k}}}$$

9 Finding the remaining unknowns

$$-\partial_\lambda \;< T\, e^{-\lambda N_>(t)}\, e^{-\lambda' N'_>(t)}\, \tilde{c}_{\mathbf{k},>}(t)\tilde{c}^\dagger_{\mathbf{k},>}(t') > \;=\; L_>(\mathbf{k};\lambda,\lambda';t-t')\sum_{\mathbf{p}} R_<(\mathbf{p};\lambda,\lambda';t-t')$$

and

$$-\partial_{\lambda'} \;< T\, e^{-\lambda N_>(t)}\, e^{-\lambda' N'_>(t)}\, \tilde{c}_{\mathbf{p},<}(t)\tilde{c}^\dagger_{\mathbf{p},<}(t') > \;=\; L_<(\mathbf{p};\lambda,\lambda';t-t')\sum_{\mathbf{k}} R_>(\mathbf{k};\lambda,\lambda';t-t')$$

==

$$-\partial_\lambda\; c_>^{-1}(\lambda,\lambda';t-t')\;\; g_>(\lambda,\lambda';t-t')g_<(\lambda,\lambda';t-t')\;(1-n_F(\mathbf{k}))\;\left(1+e^{\beta\epsilon_{\mathbf{k}}}\;c_>^{-1}(\lambda,\lambda';t-t')\;c_<(\lambda,\lambda';t-t')\right)^{-1}\;e^{\theta(t-t')\beta\epsilon_{\mathbf{k}}}$$

$$=\;c_>^{-1}(\lambda,\lambda';t-t')\;\;g_>(\lambda,\lambda';t-t')\;(1-n_F(\mathbf{k}))\;\left(1+e^{\beta\epsilon_{\mathbf{k}}}\;c_>^{-1}(\lambda,\lambda';t-t')\;c_<(\lambda,\lambda';t-t')\right)^{-1}\;e^{\theta(t-t')\beta\epsilon_{\mathbf{k}}}\sum_{\mathbf{p}}\;g_<(\lambda,\lambda';t-t')\;n_F(\mathbf{p})\;\left(1+e^{-\beta\epsilon_{\mathbf{p}}}c_<^{-1}(\lambda,\lambda';t-t')c_>(\lambda,\lambda';t-t')\;e\right.$$

and

$$-\partial_{\lambda'}\;c_<^{-1}(\lambda,\lambda';t-t')\;g_<(\lambda,\lambda';t-t')g_>(\lambda,\lambda';t-t')\;n_F(\mathbf{p})\;\left(1+e^{-\beta\epsilon_{\mathbf{p}}}c_<^{-1}(\lambda,\lambda';t-t')c_>(\lambda,\lambda';t-t')\;e^{\lambda+\lambda'}\right)^{-1}\;e^{-\theta(t'-t)\beta\epsilon_{\mathbf{p}}}$$

$$=\;c_<^{-1}(\lambda,\lambda';t-t')\;g_<(\lambda,\lambda';t-t')\;n_F(\mathbf{p})\;\left(1+e^{-\beta\epsilon_{\mathbf{p}}}c_<^{-1}(\lambda,\lambda';t-t')c_>(\lambda,\lambda';t-t')\;e^{\lambda+\lambda'}\right)^{-1}\;e^{-\theta(t'-t)\beta\epsilon_{\mathbf{p}}}\sum_{\mathbf{k}}\;g_>(\lambda,\lambda';t-t')\;(1-n_F(\mathbf{k}))\;\left(1+e^{\beta\epsilon_{\mathbf{k}}}\;c_>^{-1}(\lambda,\lambda';t-t')\;c_<(\lambda,\lambda';\right.$$

==

$$-\partial_\lambda\;c_>^{-1}(\lambda,\lambda';t-t')\;\;g_>(\lambda,\lambda';t-t')g_<(\lambda,\lambda';t-t')\;\left(1+e^{\beta\epsilon_{\mathbf{k}}}\;c_>^{-1}(\lambda,\lambda';t-t')\;c_<(\lambda,\lambda';t-t')\right)^{-1}$$

$$=\;c_>^{-1}(\lambda,\lambda';t-t')\;\;g_>(\lambda,\lambda';t-t')g_<(\lambda,\lambda';t-t')\left(1+e^{\beta\epsilon_{\mathbf{k}}}\;c_>^{-1}(\lambda,\lambda';t-t')\;c_<(\lambda,\lambda';t-t')\right)^{-1}\;\sum_{\mathbf{p}}\;n_F(\mathbf{p})\;\left(1+e^{-\beta\epsilon_{\mathbf{p}}}c_<^{-1}(\lambda,\lambda';t-t')c_>(\lambda,\lambda';t-t')\;e^{\lambda+\lambda'}\right)^{-1}$$

and

$$-\partial_{\lambda'}\;c_<^{-1}(\lambda,\lambda';t-t')\;g_<(\lambda,\lambda';t-t')g_>(\lambda,\lambda';t-t')\;\left(1+e^{-\beta\epsilon_{\mathbf{p}}}c_<^{-1}(\lambda,\lambda';t-t')c_>(\lambda,\lambda';t-t')\;e^{\lambda+\lambda'}\right)^{-1}$$

$$=\;c_<^{-1}(\lambda,\lambda';t-t')\;g_<(\lambda,\lambda';t-t')g_>(\lambda,\lambda';t-t')\left(1+e^{-\beta\epsilon_{\mathbf{p}}}c_<^{-1}(\lambda,\lambda';t-t')c_>(\lambda,\lambda';t-t')\;e^{\lambda+\lambda'}\right)^{-1}\;\sum_{\mathbf{k}}\;(1-n_F(\mathbf{k}))\;\left(1+e^{\beta\epsilon_{\mathbf{k}}}\;c_>^{-1}(\lambda,\lambda';t-t')\;c_<(\lambda,\lambda';t-t')\right)^{-1}$$

==

$$\Gamma_>(\lambda,\lambda';t-t')\;=\;c_>^{-1}(\lambda,\lambda';t-t')\;\;g_>(\lambda,\lambda';t-t')g_<(\lambda,\lambda';t-t')$$

and

$$\Gamma_<(\lambda,\lambda';t-t')\;=\;c_<^{-1}(\lambda,\lambda';t-t')\;g_<(\lambda,\lambda';t-t')g_>(\lambda,\lambda';t-t')$$

and

$$\gamma(\lambda,\lambda';t-t')\;=\;c_>^{-1}(\lambda,\lambda';t-t')\;c_<(\lambda,\lambda';t-t')$$

==

$$-\partial_\lambda\;\Gamma_>(\lambda,\lambda';t-t')\;\left(1+e^{\beta\epsilon_{\mathbf{k}}}\;\gamma(\lambda,\lambda';t-t')\right)^{-1}\;=\;\Gamma_>(\lambda,\lambda';t-t')\;\left(1+e^{\beta\epsilon_{\mathbf{k}}}\;\gamma(\lambda,\lambda';t-t')\right)^{-1}\;\sum_{\mathbf{p}}\;n_F(\mathbf{p})\;\left(1+e^{-\beta\epsilon_{\mathbf{p}}}\;\frac{e^{\lambda+\lambda'}}{\gamma(\lambda,\lambda';t-t')}\right)^{-1}$$

and

$$-\partial_{\lambda'}\;\Gamma_<(\lambda,\lambda';t-t')\;\left(1+e^{-\beta\epsilon_{\mathbf{p}}}\;\frac{e^{\lambda+\lambda'}}{\gamma(\lambda,\lambda';t-t')}\right)^{-1}\;=\;\Gamma_<(\lambda,\lambda';t-t')\;\left(1+e^{-\beta\epsilon_{\mathbf{p}}}\;\frac{e^{\lambda+\lambda'}}{\gamma(\lambda,\lambda';t-t')}\right)^{-1}\;\sum_{\mathbf{k}}\;(1-n_F(\mathbf{k}))\;\left(1+e^{\beta\epsilon_{\mathbf{k}}}\;\gamma(\lambda,\lambda';t-t')\right)^{-1}$$

==

$$\Gamma_>(\lambda,\lambda';t-t')\;\left(1+e^{\beta\epsilon_{\mathbf{k}}}\;\gamma(\lambda,\lambda';t-t')\right)^{-1}\;=\;\Gamma_>(\lambda_0,\lambda';t-t')\;\left(1+e^{\beta\epsilon_{\mathbf{k}}}\;\gamma(\lambda_0,\lambda')\right)^{-1}\;e^{-\int_{\lambda_0}^{\lambda}ds\;\sum_{\mathbf{p}}\;\frac{n_F(\mathbf{p})}{\left(1+e^{-\beta\epsilon_{\mathbf{p}}}\;\frac{e^{s+\lambda'}}{\gamma(s,\lambda';t-t')}\right)}}$$

and

$$\Gamma_<(\lambda,\lambda';t-t')\;\left(1+e^{-\beta\epsilon_{\mathbf{p}}}\;\frac{e^{\lambda+\lambda'}}{\gamma(\lambda,\lambda';t-t')}\right)^{-1}\;=\;\Gamma_<(\lambda,\lambda_0;t-t')\;\left(1+e^{-\beta\epsilon_{\mathbf{p}}}\;\frac{e^{\lambda+\lambda_0}}{\gamma(\lambda,\lambda_0;t-t')}\right)^{-1}\;e^{-\int_{\lambda_0}^{\lambda'}ds\sum_{\mathbf{k}}\;\frac{(1-n_F(\mathbf{k}))}{\left(1+e^{\beta\epsilon_{\mathbf{k}}}\;\gamma(\lambda,s;t-t')\right)}}$$

==

$$\Gamma_>(\lambda,\lambda';t-t')\;\left(1+e^{\beta\epsilon_{\mathbf{k}}}\;\gamma(\lambda,\lambda';t-t')\right)^{-1}\;=\;\Gamma_>(0,\lambda';t-t')\;\left(1+e^{\beta\epsilon_{\mathbf{k}}}\;\gamma(0,\lambda';t-t')\right)^{-1}\;e^{-\int_0^{\lambda}ds\;\sum_{\mathbf{p}}\;\frac{n_F(\mathbf{p})}{\left(1+e^{-\beta\epsilon_{\mathbf{p}}}\;\frac{e^{s+\lambda'}}{\gamma(s,\lambda';t-t')}\right)}}$$

and

$$\Gamma_<(\lambda,\lambda';t-t')\;\left(1+e^{-\beta\epsilon_{\mathbf{p}}}\;\frac{e^{\lambda+\lambda'}}{\gamma(\lambda,\lambda';t-t')}\right)^{-1}\;=\;\Gamma_<(\lambda,0;t-t')\;\left(1+e^{-\beta\epsilon_{\mathbf{p}}}\;\frac{e^{\lambda}}{\gamma(\lambda,0;t-t')}\right)^{-1}\;e^{-\int_0^{\lambda'}ds\sum_{\mathbf{k}}\;\frac{(1-n_F(\mathbf{k}))}{\left(1+e^{\beta\epsilon_{\mathbf{k}}}\;\gamma(\lambda,s;t-t')\right)}}$$

==

==

$$\Gamma_{>}(\lambda,\lambda^{'};t-t^{'})\ \left(1+e^{\beta\epsilon_{\mathbf{k}}}\ \gamma(0,\lambda^{'};t-t^{'})\right)\ =\ \Gamma_{>}(0,\lambda^{'};t-t^{'})\ \left(1+e^{\beta\epsilon_{\mathbf{k}}}\ \gamma(\lambda,\lambda^{'};t-t^{'})\right)\ e^{-\int_0^{\lambda}ds\ \sum_{\mathbf{p}}\ \frac{n_F(\mathbf{p})}{\left(1+e^{-\beta\epsilon_{\mathbf{p}}}\ \frac{e^{s+\lambda^{'}}}{\gamma(s,\lambda^{'};t-t^{'})}\right)}}$$

and

$$\Gamma_{<}(\lambda,\lambda^{'};t-t^{'})\ \left(1+e^{-\beta\epsilon_{\mathbf{p}}}\ \frac{e^{\lambda}}{\gamma(\lambda,0;t-t^{'})}\right)\ =\ \Gamma_{<}(\lambda,0;t-t^{'})\ \left(1+e^{-\beta\epsilon_{\mathbf{p}}}\ \frac{e^{\lambda+\lambda^{'}}}{\gamma(\lambda,\lambda^{'};t-t^{'})}\right)\ e^{-\int_0^{\lambda^{'}}ds\sum_{\mathbf{k}}\ \frac{(1-n_F(\mathbf{k}))}{\left(1+e^{\beta\epsilon_{\mathbf{k}}}\ \gamma(\lambda,s;t-t^{'})\right)}}$$

==
==

From this we may conclude,

$$\gamma(0,\lambda^{'};t-t^{'})\ =\ \gamma(\lambda,\lambda^{'};t-t^{'})$$

and,

$$\frac{e^{\lambda}}{\gamma(\lambda,0;t-t^{'})}\ =\ \frac{e^{\lambda+\lambda^{'}}}{\gamma(\lambda,\lambda^{'};t-t^{'})}$$

==

$$\Gamma_{>}(\lambda,\lambda^{'};t-t^{'})\ =\ \Gamma_{>}(0,\lambda^{'};t-t^{'})\ e^{-\int_0^{\lambda}ds\ \sum_{\mathbf{p}}\ \frac{n_F(\mathbf{p})}{\left(1+e^{-\beta\epsilon_{\mathbf{p}}}\ \frac{e^{s+\lambda^{'}}}{\gamma(s,\lambda^{'};t-t^{'})}\right)}}$$

and

$$\Gamma_{<}(\lambda,\lambda^{'};t-t^{'})\ =\ \Gamma_{<}(\lambda,0;t-t^{'})\ e^{-\int_0^{\lambda^{'}}ds\sum_{\mathbf{k}}\ \frac{(1-n_F(\mathbf{k}))}{\left(1+e^{\beta\epsilon_{\mathbf{k}}}\ \gamma(\lambda,s;t-t^{'})\right)}}$$

==

$$\gamma(0,\lambda^{'};t-t^{'})\ =\ \gamma(\lambda,\lambda^{'};t-t^{'})$$

and,

$$\gamma(\lambda,\lambda^{'};t-t^{'})\ =\ e^{\lambda^{'}}\ \gamma(\lambda,0;t-t^{'})$$

or,

$$\gamma(0,0;t-t^{'})\ =\ \gamma(\lambda,0;t-t^{'})$$

or,

$$\gamma(\lambda,\lambda^{'};t-t^{'})\ =\ e^{\lambda^{'}}\ \gamma(0,0;t-t^{'});\ Set\ \gamma(0,0;t-t^{'})\equiv e^{-\beta\mu};\ \gamma(\lambda,\lambda^{'};t-t^{'})\ =\ e^{\lambda^{'}}\ e^{-\beta\mu}$$

==

$$\Gamma_{>}(\lambda,\lambda^{'};t-t^{'})\ =\ \Gamma_{>}(0,\lambda^{'};t-t^{'})\ e^{-\int_0^{\lambda}ds\ \sum_{\mathbf{p}}\ \frac{n_F(\mathbf{p})}{\left(1+e^{-\beta(\epsilon_{\mathbf{p}}-\mu)}\ e^{s}\right)}}$$

and

$$\Gamma_{<}(\lambda,\lambda^{'};t-t^{'})\ =\ \Gamma_{<}(\lambda,0;t-t^{'})\ e^{-\int_0^{\lambda^{'}}ds\sum_{\mathbf{k}}\ \frac{(1-n_F(\mathbf{k}))}{\left(1+e^{\beta(\epsilon_{\mathbf{k}}-\mu)}\ e^{s}\right)}}$$

==

$$\Gamma_{>}(\lambda,\lambda^{'};t-t^{'})\ =\ c_{>}^{-1}(\lambda,\lambda^{'};t-t^{'})\ \ g_{>}(\lambda,\lambda^{'};t-t^{'})g_{<}(\lambda,\lambda^{'};t-t^{'})$$

and

$$\Gamma_{<}(\lambda,\lambda^{'};t-t^{'})\ =\ c_{<}^{-1}(\lambda,\lambda^{'};t-t^{'})\ g_{<}(\lambda,\lambda^{'};t-t^{'})g_{>}(\lambda,\lambda^{'};t-t^{'})$$

==

$$<T\ e^{-\lambda N_{>}(t)}\ e^{-\lambda^{'}N^{'}_{>}(t)}\ \tilde{c}_{\mathbf{p},<}(t)\tilde{c}^{\dagger}_{\mathbf{p},<}(t^{'})>\ \ =\ \Gamma_{<}(\lambda,\lambda^{'};t-t^{'})\ \frac{n_F(\mathbf{p})\ e^{-\theta(t^{'}-t)\beta\epsilon_{\mathbf{p}}}}{\left(1+e^{-\beta\epsilon_{\mathbf{p}}}\ \frac{e^{\lambda+\lambda^{'}}}{\gamma(\lambda,\lambda^{'};t-t^{'})}\right)}$$

and

$$<T\ e^{-\lambda N_{>}(t)}\ e^{-\lambda^{'}N^{'}_{>}(t)}\ \tilde{c}_{\mathbf{k},>}(t)\tilde{c}^{\dagger}_{\mathbf{k},>}(t^{'})>\ \ =\ \Gamma_{>}(\lambda,\lambda^{'};t-t^{'})\ \frac{(1-n_F(\mathbf{k}))e^{\theta(t-t^{'})\beta\epsilon_{\mathbf{k}}}}{\left(1+e^{\beta\epsilon_{\mathbf{k}}}\ \gamma(\lambda,\lambda^{'};t-t^{'})\right)}$$

==

$$<T\ e^{-\lambda N_{>}(t)}\ e^{-\lambda^{'}N^{'}_{>}(t)}\ \tilde{c}_{\mathbf{p},<}(t)\tilde{c}^{\dagger}_{\mathbf{p},<}(t^{'})>\ \ =\ \Gamma_{<}(\lambda,0;t-t^{'})\ \frac{n_F(\mathbf{p})\ e^{-\theta(t^{'}-t)\beta\epsilon_{\mathbf{p}}}}{\left(1+e^{-\beta(\epsilon_{\mathbf{p}}-\mu)}\ e^{\lambda}\right)}\ e^{-\int_0^{\lambda^{'}}ds\sum_{\mathbf{k}}\ \frac{(1-n_F(\mathbf{k}))}{\left(1+e^{\beta(\epsilon_{\mathbf{k}}-\mu)}\ e^{s}\right)}}$$

and

$$<T\ e^{-\lambda N_{>}(t)}\ e^{-\lambda^{'}N^{'}_{>}(t)}\ \tilde{c}_{\mathbf{k},>}(t)\tilde{c}^{\dagger}_{\mathbf{k},>}(t^{'})>\ \ =\ \Gamma_{>}(0,\lambda^{'};t-t^{'})\ \frac{(1-n_F(\mathbf{k}))e^{\theta(t-t^{'})\beta\epsilon_{\mathbf{k}}}}{\left(1+e^{\beta(\epsilon_{\mathbf{k}}-\mu)}\ e^{\lambda^{'}}\right)}\ e^{-\int_0^{\lambda}ds\ \sum_{\mathbf{p}}\ \frac{n_F(\mathbf{p})}{\left(1+e^{-\beta(\epsilon_{\mathbf{p}}-\mu)}\ e^{s}\right)}}$$

10 Solving for the remaining coefficients

We now have to solve for the remaining coefficients viz. $\Gamma_{>}(0,\lambda^{'})$ and $\Gamma_{<}(\lambda,0)$. For this we have to examine the complementary equations we have so far neglected. Consider,

$$-\partial_{\lambda^{'}} \; < T \; e^{-\lambda N_{>}(t)} \; e^{-\lambda^{'} N^{'}_{>}(t)} \; \tilde{c}_{\mathbf{p},<}(t)\tilde{c}^{\dagger}_{\mathbf{p},<}(t^{'}) > \;\; = \;\; \Gamma_{<}(\lambda,0;t-t^{'}) \; \frac{n_F(\mathbf{p}) \; e^{-\theta(t^{'}-t)\beta\epsilon_{\mathbf{p}}}}{\left(1+e^{-\beta(\epsilon_{\mathbf{p}}-\mu)} \; e^{\lambda}\right)} \; e^{-\int_0^{\lambda^{'}} ds \sum_{\mathbf{k}} \frac{(1-n_F(\mathbf{k}))}{\left(1+e^{\beta(\epsilon_{\mathbf{k}}-\mu)} \; e^{s}\right)}} \left(\sum_{\mathbf{k}} \; \frac{(1-n_F(\mathbf{k}))}{\left(1+e^{\beta(\epsilon_{\mathbf{k}}-\mu)} \; e^{\lambda^{'}}\right)} \right)$$

and

$$-\partial_{\lambda} \; < T \; e^{-\lambda N_{>}(t)} \; e^{-\lambda^{'} N^{'}_{>}(t)} \; \tilde{c}_{\mathbf{k},>}(t)\tilde{c}^{\dagger}_{\mathbf{k},>}(t^{'}) > \;\; = \;\; \Gamma_{>}(0,\lambda^{'};t-t^{'}) \; \frac{(1-n_F(\mathbf{k}))e^{\theta(t-t^{'})\beta\epsilon_{\mathbf{k}}}}{\left(1+e^{\beta(\epsilon_{\mathbf{k}}-\mu)} \; e^{\lambda^{'}}\right)} \; e^{-\int_0^{\lambda} ds \; \sum_{\mathbf{p}} \frac{n_F(\mathbf{p})}{\left(1+e^{-\beta(\epsilon_{\mathbf{p}}-\mu)} \; e^{s}\right)}} \left(\sum_{\mathbf{p}} \; \frac{n_F(\mathbf{p})}{\left(1+e^{-\beta(\epsilon_{\mathbf{p}}-\mu)} \; e^{\lambda}\right)} \right)$$

==

But on the other hand,

$$-\partial_{\lambda^{'}} \; < T \; e^{-\lambda N_{>}(t)} \; e^{-\lambda^{'} N^{'}_{>}(t)} \; \tilde{c}_{\mathbf{p},<}(t)\tilde{c}^{\dagger}_{\mathbf{p},<}(t^{'}) > \;\; =$$

$$< T \; e^{-\lambda N_{>}(t)} \; e^{-\lambda^{'} N^{'}_{>}(t)} \; N^{'}_{>}(t)\tilde{c}_{\mathbf{p},<}(t)\tilde{c}^{\dagger}_{\mathbf{p},<}(t^{'}) > \;\; = \;\; \sum_{\mathbf{k}} \; < T \; e^{-\lambda N_{>}(t)} \; e^{-\lambda^{'} N^{'}_{>}(t)} \; \tilde{c}^{\dagger}_{\mathbf{k},>}(t)\tilde{c}_{\mathbf{k},>}(t)\tilde{c}_{\mathbf{p},<}(t)\tilde{c}^{\dagger}_{\mathbf{p},<}(t^{'}) >$$

Now assume $t > t^{'} \rightarrow t$ then the above becomes,

$$\sum_{\mathbf{k}} \;\; < e^{-\lambda N_{>}(t)} \; \tilde{c}^{\dagger}_{\mathbf{k},>}(t)\tilde{c}_{\mathbf{p},<}(t)\tilde{c}^{\dagger}_{\mathbf{p},<}(t)\tilde{c}_{\mathbf{k},>}(t) > \;\; = \;\; \Gamma_{<}(\lambda,0;-i\epsilon) \; \frac{n_F(\mathbf{p})}{\left(1+e^{-\beta(\epsilon_{\mathbf{p}}-\mu)} \; e^{\lambda}\right)} \left(\sum_{\mathbf{k}} \; \frac{(1-n_F(\mathbf{k}))}{\left(1+e^{\beta(\epsilon_{\mathbf{k}}-\mu)}\right)} \right)$$

and

$$\sum_{\mathbf{p}} \;\; < e^{-\lambda^{'} N^{'}_{>}(t)} \; \tilde{c}_{\mathbf{p},<}(t)\tilde{c}^{\dagger}_{\mathbf{p},<}(t)\tilde{c}_{\mathbf{k},>}(t)\tilde{c}^{\dagger}_{\mathbf{k},>}(t) > \;\; = \;\; \Gamma_{>}(0,\lambda^{'};-i\epsilon) \; \frac{(1-n_F(\mathbf{k}))e^{\beta\epsilon_{\mathbf{k}}}}{\left(1+e^{\beta(\epsilon_{\mathbf{k}}-\mu)} \; e^{\lambda^{'}}\right)} \left(\sum_{\mathbf{p}} \; \frac{n_F(\mathbf{p})}{\left(1+e^{-\beta(\epsilon_{\mathbf{p}}-\mu)}\right)} \right)$$

We now want to evaluate the left hand side of the above equations in terms of bosonic algebra. Before we do this, we need a minor rearrangement. Even though this rearrangement uses Fermi algebra, the remainder of the evaluation is going to be done purely using bosonic algebra. We write,

$$\tilde{c}_{\mathbf{p},<}(t)\tilde{c}^{\dagger}_{\mathbf{p},<}(t) = -\tilde{c}^{\dagger}_{\mathbf{p},<}(t)\tilde{c}_{\mathbf{p},<}(t) + n_F(\mathbf{p})$$

and

$$\tilde{c}^{\dagger}_{\mathbf{p},<}\tilde{c}_{\mathbf{k},>} \;\; \equiv \;\; a_{\frac{1}{2}(\mathbf{p}+\mathbf{k})}(\mathbf{k}-\mathbf{p}) \; ; \quad \tilde{c}^{\dagger}_{\mathbf{k},>}\tilde{c}_{\mathbf{p},<} \;\; \equiv \;\; a^{\dagger}_{\frac{1}{2}(\mathbf{p}+\mathbf{k})}(\mathbf{k}-\mathbf{p})$$

or,

$$\sum_{\mathbf{k}} \;\; < e^{-\lambda N_{>}(t)} \; a^{\dagger}_{\frac{1}{2}(\mathbf{p}+\mathbf{k})}(\mathbf{k}-\mathbf{p},t)a_{\frac{1}{2}(\mathbf{p}+\mathbf{k})}(\mathbf{k}-\mathbf{p},t) > \;\; = \;\; \Gamma_{<}(\lambda,0;-i\epsilon) \; \frac{n_F(\mathbf{p})}{\left(1+e^{-\beta(\epsilon_{\mathbf{p}}-\mu)} \; e^{\lambda}\right)} \left(\sum_{\mathbf{k}} \; \frac{(1-n_F(\mathbf{k}))}{\left(1+e^{\beta(\epsilon_{\mathbf{k}}-\mu)}\right)} \right)$$

and

$$N^0 \;\; < e^{-\lambda^{'} N^{'}_{>}(t)} \; \tilde{c}_{\mathbf{k},>}(t)\tilde{c}^{\dagger}_{\mathbf{k},>}(t) > - \sum_{\mathbf{p}} \;\; < e^{-\lambda^{'} N^{'}_{>}(t)} \; a_{\frac{1}{2}(\mathbf{p}+\mathbf{k})}(\mathbf{k}-\mathbf{p})a^{\dagger}_{\frac{1}{2}(\mathbf{p}+\mathbf{k})}(\mathbf{k}-\mathbf{p}) > \;\; = \;\; \Gamma_{>}(0,\lambda^{'};-i\epsilon) \; \frac{(1-n_F(\mathbf{k}))e^{\beta\epsilon_{\mathbf{k}}}}{\left(1+e^{\beta(\epsilon_{\mathbf{k}}-\mu)} \; e^{\lambda^{'}}\right)} \left(\sum_{\mathbf{p}} \; \frac{n_F(\mathbf{p})}{\left(1+e^{-\beta(\epsilon_{\mathbf{p}}-\mu)}\right)} \right)$$

$$\sum_{\mathbf{k}} \; < e^{-\lambda N_>(t)} \; a^{\dagger}_{\frac{1}{2}(\mathbf{p}+\mathbf{k})}(\mathbf{k}-\mathbf{p},t)a_{\frac{1}{2}(\mathbf{p}+\mathbf{k})}(\mathbf{k}-\mathbf{p},t) > \; = \; \Gamma_<(\lambda,0;-i\epsilon) \; \frac{n_F(\mathbf{p})}{\left(1+e^{-\beta(\epsilon_{\mathbf{p}}-\mu)} \; e^{\lambda}\right)} \; \left(\sum_{\mathbf{k}} \; \frac{(1-n_F(\mathbf{k}))}{\left(1+e^{\beta(\epsilon_{\mathbf{k}}-\mu)}\right)} \right)$$

and

$$N^0 \; < e^{-\lambda^{'} N^{'}_>(t)} \; \tilde{c}_{\mathbf{k},>}(t)\tilde{c}^{\dagger}_{\mathbf{k},>}(t) > - \sum_{\mathbf{p}} \; < e^{-\lambda^{'} N^{'}_>(t)} \; a_{\frac{1}{2}(\mathbf{p}+\mathbf{k})}(\mathbf{k}-\mathbf{p})a^{\dagger}_{\frac{1}{2}(\mathbf{p}+\mathbf{k})}(\mathbf{k}-\mathbf{p}) > \; = \; \Gamma_>(0,\lambda^{'};-i\epsilon) \; \frac{(1-n_F(\mathbf{k}))e^{\beta\epsilon_{\mathbf{k}}}}{\left(1+e^{\beta(\epsilon_{\mathbf{k}}-\mu)} \; e^{\lambda^{'}}\right)} \; \left(\sum_{\mathbf{p}} \; \frac{n_F(\mathbf{p})}{\left(1+e^{-\beta(\epsilon_{\mathbf{p}}-\mu)}\right)} \right)$$

but,

$$< T \; e^{-\lambda^{'} N^{'}_>(t)} \; \tilde{c}_{\mathbf{k},>}(t)\tilde{c}^{\dagger}_{\mathbf{k},>}(t^{'}) > \; = \; \Gamma_>(0,\lambda^{'};t-t^{'}) \; \frac{(1-n_F(\mathbf{k}))e^{\theta(t-t^{'})\beta\epsilon_{\mathbf{k}}}}{\left(1+e^{\beta(\epsilon_{\mathbf{k}}-\mu)} \; e^{\lambda^{'}}\right)}$$

Thus we have to solve,

$$\sum_{\mathbf{k}} \; < e^{-\lambda N_>(t)} \; a^{\dagger}_{\frac{1}{2}(\mathbf{p}+\mathbf{k})}(\mathbf{k}-\mathbf{p})a_{\frac{1}{2}(\mathbf{p}+\mathbf{k})}(\mathbf{k}-\mathbf{p}) > \; = \; \Gamma_<(\lambda,0;-i\epsilon) \; \frac{n_F(\mathbf{p})}{\left(1+e^{-\beta(\epsilon_{\mathbf{p}}-\mu)} \; e^{\lambda}\right)} \; \left(\sum_{\mathbf{k}} \; \frac{(1-n_F(\mathbf{k}))}{\left(1+e^{\beta(\epsilon_{\mathbf{k}}-\mu)}\right)} \right)$$

and

$$\sum_{\mathbf{p}} \; < e^{-\lambda^{'} N^{'}_>(t)} \; a_{\frac{1}{2}(\mathbf{p}+\mathbf{k})}(\mathbf{k}-\mathbf{p})a^{\dagger}_{\frac{1}{2}(\mathbf{p}+\mathbf{k})}(\mathbf{k}-\mathbf{p}) > \; = \; \Gamma_>(0,\lambda^{'};-i\epsilon) \; \frac{(1-n_F(\mathbf{k}))e^{\beta\epsilon_{\mathbf{k}}}}{\left(1+e^{\beta(\epsilon_{\mathbf{k}}-\mu)} \; e^{\lambda^{'}}\right)} \; \left(N^0 - \sum_{\mathbf{p}} \; \frac{n_F(\mathbf{p})}{\left(1+e^{-\beta(\epsilon_{\mathbf{p}}-\mu)}\right)} \right)$$

The left hand side of the above equations may be evaluated using boson-like algebra.

$$< e^{-\lambda N_>(t)} \; a^{\dagger}_{\frac{1}{2}(\mathbf{p}+\mathbf{k})}(\mathbf{k}-\mathbf{p})a_{\frac{1}{2}(\mathbf{p}+\mathbf{k})}(\mathbf{k}-\mathbf{p}) > \; \equiv \; \frac{Tr(e^{-\beta(H-\mu N)}e^{-\lambda N_>(t)} \; a^{\dagger}_{\frac{1}{2}(\mathbf{p}+\mathbf{k})}(\mathbf{k}-\mathbf{p})a_{\frac{1}{2}(\mathbf{p}+\mathbf{k})}(\mathbf{k}-\mathbf{p}))}{Tr(e^{-\beta(H-\mu N)})}$$

Using the cyclic property of trace we get,

$$(e^{\lambda N_>(t)}e^{\beta(H-\mu N)}a_{\frac{1}{2}(\mathbf{p}+\mathbf{k})}(\mathbf{k}-\mathbf{p})e^{-\beta(H-\mu N)}e^{-\lambda N_>(t)}) \; = \; a_{\frac{1}{2}(\mathbf{p}+\mathbf{k})}(\mathbf{k}-\mathbf{p}) \; e^{-\lambda} \; e^{-\beta(\epsilon_{\mathbf{k}}-\epsilon_{\mathbf{p}})}$$

or,

$$< e^{-\lambda N_>(t)} \; a^{\dagger}_{\frac{1}{2}(\mathbf{p}+\mathbf{k})}(\mathbf{k}-\mathbf{p})a_{\frac{1}{2}(\mathbf{p}+\mathbf{k})}(\mathbf{k}-\mathbf{p}) > \; \equiv \; e^{-\lambda} \; e^{-\beta(\epsilon_{\mathbf{k}}-\epsilon_{\mathbf{p}})} \frac{Tr(e^{-\beta(H-\mu N)}e^{-\lambda N_>(t)} \; a_{\frac{1}{2}(\mathbf{p}+\mathbf{k})}(\mathbf{k}-\mathbf{p})a^{\dagger}_{\frac{1}{2}(\mathbf{p}+\mathbf{k})}(\mathbf{k}-\mathbf{p}))}{Tr(e^{-\beta(H-\mu N)})}$$

Now,

$$a_{\frac{1}{2}(\mathbf{p}+\mathbf{k})}(\mathbf{k}-\mathbf{p})a^{\dagger}_{\frac{1}{2}(\mathbf{p}+\mathbf{k})}(\mathbf{k}-\mathbf{p}) \; = \; a^{\dagger}_{\frac{1}{2}(\mathbf{p}+\mathbf{k})}(\mathbf{k}-\mathbf{p})a_{\frac{1}{2}(\mathbf{p}+\mathbf{k})}(\mathbf{k}-\mathbf{p}) + [a_{\frac{1}{2}(\mathbf{p}+\mathbf{k})}(\mathbf{k}-\mathbf{p}), a^{\dagger}_{\frac{1}{2}(\mathbf{p}+\mathbf{k})}(\mathbf{k}-\mathbf{p})]$$

$$[a_{\frac{1}{2}(\mathbf{p}+\mathbf{k})}(\mathbf{k}-\mathbf{p}), a^{\dagger}_{\frac{1}{2}(\mathbf{p}+\mathbf{k})}(\mathbf{k}-\mathbf{p})] \; = \; [c^{\dagger}_{\mathbf{p},<}c_{\mathbf{k},>}, c^{\dagger}_{\mathbf{k},>}c_{\mathbf{p},<}]$$

$$[a_{\frac{1}{2}(\mathbf{p}+\mathbf{k})}(\mathbf{k}-\mathbf{p}), a^{\dagger}_{\frac{1}{2}(\mathbf{p}+\mathbf{k})}(\mathbf{k}-\mathbf{p})] \; = \; (1-n_F(\mathbf{k}))c^{\dagger}_{\mathbf{p},<}c_{\mathbf{p},<} - c^{\dagger}_{\mathbf{k},>}c_{\mathbf{k},>}n_F(\mathbf{p})$$

$$c^{\dagger}_{\mathbf{p},<}c_{\mathbf{p},<} = -c_{\mathbf{p},<}c^{\dagger}_{\mathbf{p},<} + n_F(\mathbf{p})$$

$$c^{\dagger}_{\mathbf{k},>}c_{\mathbf{k},>} = -c_{\mathbf{k},>}c^{\dagger}_{\mathbf{k},>} + (1-n_F(\mathbf{k}))$$

$$[a_{\frac{1}{2}(\mathbf{p}+\mathbf{k})}(\mathbf{k}-\mathbf{p}), a^{\dagger}_{\frac{1}{2}(\mathbf{p}+\mathbf{k})}(\mathbf{k}-\mathbf{p})] \; = \; -(1-n_F(\mathbf{k})) \; c_{\mathbf{p},<}c^{\dagger}_{\mathbf{p},<} + c_{\mathbf{k},>}c^{\dagger}_{\mathbf{k},>} \; n_F(\mathbf{p})$$

or,

$$< e^{-\lambda N_>(t)} \; a^{\dagger}_{\frac{1}{2}(\mathbf{p}+\mathbf{k})}(\mathbf{k}-\mathbf{p})a_{\frac{1}{2}(\mathbf{p}+\mathbf{k})}(\mathbf{k}-\mathbf{p}) > \; \equiv \; e^{-\lambda} \; e^{-\beta(\epsilon_{\mathbf{k}}-\epsilon_{\mathbf{p}})} \frac{Tr(e^{-\beta(H-\mu N)}e^{-\lambda N_>(t)} \; (a^{\dagger}_{\frac{1}{2}(\mathbf{p}+\mathbf{k})}(\mathbf{k}-\mathbf{p})a_{\frac{1}{2}(\mathbf{p}+\mathbf{k})}(\mathbf{k}-\mathbf{p}) - (1-n_F(\mathbf{k})) \; c_{\mathbf{p},<}c^{\dagger}_{\mathbf{p},<} + c_{\mathbf{k},>}c^{\dagger}_{\mathbf{k},>} \; n_F(\mathbf{p})))}{Tr(e^{-\beta(H-\mu N)})}$$

and

$$< e^{-\lambda N_>(t)} \; a^{\dagger}_{\frac{1}{2}(\mathbf{p}+\mathbf{k})}(\mathbf{k}-\mathbf{p})a_{\frac{1}{2}(\mathbf{p}+\mathbf{k})}(\mathbf{k}-\mathbf{p}) > \; \equiv \; \frac{< e^{-\lambda N_>(t)} \; (-(1-n_F(\mathbf{k})) \; c_{\mathbf{p},<}c^{\dagger}_{\mathbf{p},<} + c_{\mathbf{k},>}c^{\dagger}_{\mathbf{k},>} \; n_F(\mathbf{p})) >}{(e^{\lambda} \; e^{\beta(\epsilon_{\mathbf{k}}-\epsilon_{\mathbf{p}})} - 1)}$$

==

$$< e^{-\lambda^{'} N^{'}_>(t)} \; a_{\frac{1}{2}(\mathbf{p}+\mathbf{k})}(\mathbf{k}-\mathbf{p})a^{\dagger}_{\frac{1}{2}(\mathbf{p}+\mathbf{k})}(\mathbf{k}-\mathbf{p}) > \; = \; e^{\lambda^{'}} \; e^{\beta(\epsilon_{\mathbf{k}}-\epsilon_{\mathbf{p}})} \; < e^{-\lambda^{'} N^{'}_>(t)} \; a^{\dagger}_{\frac{1}{2}(\mathbf{p}+\mathbf{k})}(\mathbf{k}-\mathbf{p})a_{\frac{1}{2}(\mathbf{p}+\mathbf{k})}(\mathbf{k}-\mathbf{p}) >$$

and,

$$a_{\frac{1}{2}(\mathbf{p}+\mathbf{k})}(\mathbf{k}-\mathbf{p})a^{\dagger}_{\frac{1}{2}(\mathbf{p}+\mathbf{k})}(\mathbf{k}-\mathbf{p}) + (1-n_F(\mathbf{k})) \; c_{\mathbf{p},<}c^{\dagger}_{\mathbf{p},<} - c_{\mathbf{k},>}c^{\dagger}_{\mathbf{k},>} \; n_F(\mathbf{p}) \; = \; a^{\dagger}_{\frac{1}{2}(\mathbf{p}+\mathbf{k})}(\mathbf{k}-\mathbf{p})a_{\frac{1}{2}(\mathbf{p}+\mathbf{k})}(\mathbf{k}-\mathbf{p})$$

and,

$$< e^{-\lambda^{'} N^{'}_>(t)} \; a_{\frac{1}{2}(\mathbf{p}+\mathbf{k})}(\mathbf{k}-\mathbf{p})a^{\dagger}_{\frac{1}{2}(\mathbf{p}+\mathbf{k})}(\mathbf{k}-\mathbf{p}) > \; = \; \frac{< e^{-\lambda^{'} N^{'}_>(t)} \; ((1-n_F(\mathbf{k})) \; c_{\mathbf{p},<}c^{\dagger}_{\mathbf{p},<} - c_{\mathbf{k},>}c^{\dagger}_{\mathbf{k},>} \; n_F(\mathbf{p})) >}{(e^{-\lambda^{'}} \; e^{\beta(\epsilon_{\mathbf{p}}-\epsilon_{\mathbf{k}})} - 1)}$$

and,

$$\sum_{\mathbf{p}} \; < e^{-\lambda^{'} N^{'}_>(t)} \; a_{\frac{1}{2}(\mathbf{p}+\mathbf{k})}(\mathbf{k}-\mathbf{p})a^{\dagger}_{\frac{1}{2}(\mathbf{p}+\mathbf{k})}(\mathbf{k}-\mathbf{p}) > \; = \; \Gamma_>(0,\lambda^{'};-i\epsilon) \; \frac{(1-n_F(\mathbf{k}))e^{\beta\epsilon_{\mathbf{k}}}}{\left(1+e^{\beta(\epsilon_{\mathbf{k}}-\mu)} \; e^{\lambda^{'}}\right)} \; \left(N^0 - \sum_{\mathbf{p}} \; \frac{n_F(\mathbf{p})}{\left(1+e^{-\beta(\epsilon_{\mathbf{p}}-\mu)}\right)} \right)$$

==

$$< e^{-\lambda N_>(t)}\ a^{\dagger}_{\frac{1}{2}(\mathbf{p}+\mathbf{k})}(\mathbf{k}-\mathbf{p})\, a_{\frac{1}{2}(\mathbf{p}+\mathbf{k})}(\mathbf{k}-\mathbf{p}) > \ \equiv\ -\frac{(1-n_F(\mathbf{k}))}{(e^{\lambda}\ e^{\beta(\epsilon_{\mathbf{k}}-\epsilon_{\mathbf{p}})}-1)}\ < e^{-\lambda N_>(t)}\ c_{\mathbf{p},<}c^{\dagger}_{\mathbf{p},<} > + \frac{n_F(\mathbf{p})}{(e^{\lambda}\ e^{\beta(\epsilon_{\mathbf{k}}-\epsilon_{\mathbf{p}})}-1)}\ < e^{-\lambda N_>(t)}\ c_{\mathbf{k},>}c^{\dagger}_{\mathbf{k},>} >$$

and,

$$< e^{-\lambda^{'} N^{'}_>(t)}\ a_{\frac{1}{2}(\mathbf{p}+\mathbf{k})}(\mathbf{k}-\mathbf{p})\, a^{\dagger}_{\frac{1}{2}(\mathbf{p}+\mathbf{k})}(\mathbf{k}-\mathbf{p}) > \ =\ \frac{(1-n_F(\mathbf{k}))}{(e^{-\lambda^{'}}\ e^{\beta(\epsilon_{\mathbf{p}}-\epsilon_{\mathbf{k}})}-1)}\ < e^{-\lambda^{'} N^{'}_>(t)}\ c_{\mathbf{p},<}c^{\dagger}_{\mathbf{p},<} > - \frac{n_F(\mathbf{p})}{(e^{-\lambda^{'}}\ e^{\beta(\epsilon_{\mathbf{p}}-\epsilon_{\mathbf{k}})}-1)}\ < e^{-\lambda^{'} N^{'}_>(t)}\ c_{\mathbf{k},>}c^{\dagger}_{\mathbf{k},>} >$$

$$\sum_{\mathbf{k}}\ < e^{-\lambda N_>(t)}\ a^{\dagger}_{\frac{1}{2}(\mathbf{p}+\mathbf{k})}(\mathbf{k}-\mathbf{p})\, a_{\frac{1}{2}(\mathbf{p}+\mathbf{k})}(\mathbf{k}-\mathbf{p}) > \ =\ \Gamma_<(\lambda,0;-i\epsilon)\ \frac{n_F(\mathbf{p})}{\left(1+e^{-\beta(\epsilon_{\mathbf{p}}-\mu)}\ e^{\lambda}\right)}\ \sum_{\mathbf{k}}\ \frac{(1-n_F(\mathbf{k}))}{\left(1+e^{\beta(\epsilon_{\mathbf{k}}-\mu)}\right)}$$

and

$$\sum_{\mathbf{p}}\ < e^{-\lambda^{'} N^{'}_>(t)}\ a_{\frac{1}{2}(\mathbf{p}+\mathbf{k})}(\mathbf{k}-\mathbf{p})\, a^{\dagger}_{\frac{1}{2}(\mathbf{p}+\mathbf{k})}(\mathbf{k}-\mathbf{p}) > \ =\ \Gamma_>(0,\lambda^{'};-i\epsilon)\ \frac{(1-n_F(\mathbf{k}))e^{\beta\epsilon_{\mathbf{k}}}}{\left(1+e^{\beta(\epsilon_{\mathbf{k}}-\mu)}\ e^{\lambda^{'}}\right)}\ \sum_{\mathbf{p}}\ \frac{n_F(\mathbf{p})}{e^{\beta(\epsilon_{\mathbf{p}}-\mu)}+1}$$

$$< T\ e^{-\lambda N_>(t)}\ \tilde{c}_{\mathbf{p},<}(t)\tilde{c}^{\dagger}_{\mathbf{p},<}(t) > \ =\ \Gamma_<(\lambda,0;-i\epsilon)\ \frac{n_F(\mathbf{p})}{\left(1+e^{-\beta(\epsilon_{\mathbf{p}}-\mu)}\ e^{\lambda}\right)}$$

and

$$< T\ e^{-\lambda N_>(t)}\tilde{c}_{\mathbf{k},>}(t)\tilde{c}^{\dagger}_{\mathbf{k},>}(t) > \ =\ \Gamma_>(0,0;-i\epsilon)\ \frac{(1-n_F(\mathbf{k}))e^{\beta\epsilon_{\mathbf{k}}}}{\left(1+e^{\beta(\epsilon_{\mathbf{k}}-\mu)}\right)}\ e^{-\int_0^{\lambda} ds\ \sum_{\mathbf{p}}\ \frac{n_F(\mathbf{p})}{\left(1+e^{-\beta(\epsilon_{\mathbf{p}}-\mu)}\ e^{s}\right)}}$$

$$< T\ e^{-\lambda^{'} N^{'}_>(t)}\ \tilde{c}_{\mathbf{p},<}(t)\tilde{c}^{\dagger}_{\mathbf{p},<}(t) > \ =\ \Gamma_<(0,0;-i\epsilon)\ \frac{n_F(\mathbf{p})}{\left(1+e^{-\beta(\epsilon_{\mathbf{p}}-\mu)}\right)}\ e^{-\int_0^{\lambda^{'}} ds \sum_{\mathbf{k}}\ \frac{(1-n_F(\mathbf{k}))}{\left(1+e^{\beta(\epsilon_{\mathbf{k}}-\mu)}\ e^{s}\right)}}$$

and

$$< T\ e^{-\lambda^{'} N^{'}_>(t)}\ \tilde{c}_{\mathbf{k},>}(t)\tilde{c}^{\dagger}_{\mathbf{k},>}(t) > \ =\ \Gamma_>(0,\lambda^{'};-i\epsilon)\ \frac{(1-n_F(\mathbf{k}))e^{\beta\epsilon_{\mathbf{k}}}}{\left(1+e^{\beta(\epsilon_{\mathbf{k}}-\mu)}\ e^{\lambda^{'}}\right)}$$

11 Reduced equations for $\Gamma_>(0,\lambda^{'};t-t^{'})$ and $\Gamma_>(\lambda,0;t-t^{'})$

$$\sum_{\mathbf{k}}\ (-\frac{(1-n_F(\mathbf{k}))}{(e^{\lambda}\ e^{\beta(\epsilon_{\mathbf{k}}-\epsilon_{\mathbf{p}})}-1)}\ < e^{-\lambda N_>(t)}\ c_{\mathbf{p},<}c^{\dagger}_{\mathbf{p},<} > + \frac{n_F(\mathbf{p})}{(e^{\lambda}\ e^{\beta(\epsilon_{\mathbf{k}}-\epsilon_{\mathbf{p}})}-1)}\ < e^{-\lambda N_>(t)}\ c_{\mathbf{k},>}c^{\dagger}_{\mathbf{k},>} >)\ =\ \Gamma_<(\lambda,0;-i\epsilon)\ \frac{n_F(\mathbf{p})}{\left(1+e^{-\beta(\epsilon_{\mathbf{p}}-\mu)}\ e^{\lambda}\right)}\ \sum_{\mathbf{k}}\ \frac{(1-n_F(\mathbf{k}))}{\left(1+e^{\beta(\epsilon_{\mathbf{k}}-\mu)}\right)}$$

and

$$\sum_{\mathbf{p}}\ (\frac{(1-n_F(\mathbf{k}))}{(e^{-\lambda^{'}}\ e^{\beta(\epsilon_{\mathbf{p}}-\epsilon_{\mathbf{k}})}-1)}\ < e^{-\lambda^{'} N^{'}_>(t)}\ c_{\mathbf{p},<}c^{\dagger}_{\mathbf{p},<} > - \frac{n_F(\mathbf{p})}{(e^{-\lambda^{'}}\ e^{\beta(\epsilon_{\mathbf{p}}-\epsilon_{\mathbf{k}})}-1)}\ < e^{-\lambda^{'} N^{'}_>(t)}\ c_{\mathbf{k},>}c^{\dagger}_{\mathbf{k},>} >)\ =\ \Gamma_>(0,\lambda^{'};-i\epsilon)\ \frac{(1-n_F(\mathbf{k}))e^{\beta\epsilon_{\mathbf{k}}}}{\left(1+e^{\beta(\epsilon_{\mathbf{k}}-\mu)}\ e^{\lambda^{'}}\right)}\ \sum_{\mathbf{p}}\ \frac{n_F(\mathbf{p})}{e^{\beta(\epsilon_{\mathbf{p}}-\mu)}+1}$$

$$\sum_{\mathbf{k}}\ \frac{n_F(\mathbf{p})}{(e^{\lambda}\ e^{\beta(\epsilon_{\mathbf{k}}-\epsilon_{\mathbf{p}})}-1)}\ \Gamma_>(0,0;-i\epsilon)\ \frac{(1-n_F(\mathbf{k}))e^{\beta\epsilon_{\mathbf{k}}}}{\left(1+e^{\beta(\epsilon_{\mathbf{k}}-\mu)}\right)}\ e^{-\int_0^{\lambda} ds\ \sum_{\mathbf{p}}\ \frac{n_F(\mathbf{p})}{\left(1+e^{-\beta(\epsilon_{\mathbf{p}}-\mu)}\ e^{s}\right)}}$$

$$=\ \sum_{\mathbf{k}}\ \left(\Gamma_<(\lambda,0;-i\epsilon)\ \frac{n_F(\mathbf{p})}{\left(1+e^{-\beta(\epsilon_{\mathbf{p}}-\mu)}\ e^{\lambda}\right)}\ \frac{(1-n_F(\mathbf{k}))}{\left(1+e^{\beta(\epsilon_{\mathbf{k}}-\mu)}\right)} + \frac{(1-n_F(\mathbf{k}))}{(e^{\lambda}\ e^{\beta(\epsilon_{\mathbf{k}}-\epsilon_{\mathbf{p}})}-1)}\ \Gamma_<(\lambda,0;-i\epsilon)\ \frac{n_F(\mathbf{p})}{\left(1+e^{-\beta(\epsilon_{\mathbf{p}}-\mu)}\ e^{\lambda}\right)}\right)$$

and

$$\Gamma_<(0,0;-i\epsilon)\ \sum_{\mathbf{p}}\ \frac{(1-n_F(\mathbf{k}))}{(e^{-\lambda^{'}}\ e^{\beta(\epsilon_{\mathbf{p}}-\epsilon_{\mathbf{k}})}-1)}\ \frac{n_F(\mathbf{p})}{\left(1+e^{-\beta(\epsilon_{\mathbf{p}}-\mu)}\right)}\ e^{-\int_0^{\lambda^{'}} ds \sum_{\mathbf{k}}\ \frac{(1-n_F(\mathbf{k}))}{\left(1+e^{\beta(\epsilon_{\mathbf{k}}-\mu)}\ e^{s}\right)}}\ =$$

$$\Gamma_>(0,\lambda^{'};-i\epsilon)\ \sum_{\mathbf{p}}\ \left(\frac{(1-n_F(\mathbf{k}))e^{\beta\epsilon_{\mathbf{k}}}}{\left(1+e^{\beta(\epsilon_{\mathbf{k}}-\mu)}\ e^{\lambda^{'}}\right)}\ \frac{n_F(\mathbf{p})}{e^{\beta(\epsilon_{\mathbf{p}}-\mu)}+1} + \frac{n_F(\mathbf{p})}{(e^{-\lambda^{'}}\ e^{\beta(\epsilon_{\mathbf{p}}-\epsilon_{\mathbf{k}})}-1)}\ \frac{(1-n_F(\mathbf{k}))e^{\beta\epsilon_{\mathbf{k}}}}{\left(1+e^{\beta(\epsilon_{\mathbf{k}}-\mu)}\ e^{\lambda^{'}}\right)}\right)$$

===

$$\sum_{\mathbf{k}} \frac{n_F(\mathbf{p})}{(e^{\lambda}\ e^{\beta(\epsilon_{\mathbf{k}}-\epsilon_{\mathbf{p}})}-1)}\ \Gamma_{>}(0,0;-i\epsilon)\ \frac{(1-n_F(\mathbf{k}))e^{\beta\epsilon_{\mathbf{k}}}}{\left(1+e^{\beta(\epsilon_{\mathbf{k}}-\mu)}\right)}\ e^{-\int_0^{\lambda} ds\ \sum_{\mathbf{p}} \frac{n_F(\mathbf{p})}{\left(1+e^{-\beta(\epsilon_{\mathbf{p}}-\mu)}\ e^{s}\right)}}$$

$$= \sum_{\mathbf{k}} \left(\Gamma_{<}(\lambda,0;-i\epsilon)\ \frac{n_F(\mathbf{p})}{\left(1+e^{-\beta(\epsilon_{\mathbf{p}}-\mu)}\ e^{\lambda}\right)}\ \frac{(1-n_F(\mathbf{k}))}{\left(1+e^{\beta(\epsilon_{\mathbf{k}}-\mu)}\right)} + \frac{(1-n_F(\mathbf{k}))}{(e^{\lambda}\ e^{\beta(\epsilon_{\mathbf{k}}-\epsilon_{\mathbf{p}})}-1)}\ \Gamma_{<}(\lambda,0;-i\epsilon)\ \frac{n_F(\mathbf{p})}{\left(1+e^{-\beta(\epsilon_{\mathbf{p}}-\mu)}\ e^{\lambda}\right)} \right)$$

and

$$\Gamma_{<}(0,0;-i\epsilon)\ \sum_{\mathbf{p}} \frac{(1-n_F(\mathbf{k}))}{(e^{-\lambda'}\ e^{\beta(\epsilon_{\mathbf{p}}-\epsilon_{\mathbf{k}})}-1)}\ \frac{n_F(\mathbf{p})}{\left(1+e^{-\beta(\epsilon_{\mathbf{p}}-\mu)}\right)}\ e^{-\int_0^{\lambda'} ds \sum_{\mathbf{k}} \frac{(1-n_F(\mathbf{k}))}{\left(1+e^{\beta(\epsilon_{\mathbf{k}}-\mu)}\ e^{s}\right)}} =$$

$$\Gamma_{>}(0,\lambda';-i\epsilon)\ \sum_{\mathbf{p}} \left(\frac{(1-n_F(\mathbf{k}))e^{\beta\epsilon_{\mathbf{k}}}}{\left(1+e^{\beta(\epsilon_{\mathbf{k}}-\mu)}\ e^{\lambda'}\right)}\ \frac{n_F(\mathbf{p})}{e^{\beta(\epsilon_{\mathbf{p}}-\mu)}+1} + \frac{n_F(\mathbf{p})}{(e^{-\lambda'}\ e^{\beta(\epsilon_{\mathbf{p}}-\epsilon_{\mathbf{k}})}-1)}\ \frac{(1-n_F(\mathbf{k}))e^{\beta\epsilon_{\mathbf{k}}}}{\left(1+e^{\beta(\epsilon_{\mathbf{k}}-\mu)}\ e^{\lambda'}\right)} \right)$$

This means,

===

$$\Gamma_{>}(0,0;-i\epsilon)\ e^{-\int_0^{\lambda} ds\ \sum_{\mathbf{p}} \frac{n_F(\mathbf{p})}{\left(1+e^{-\beta(\epsilon_{\mathbf{p}}-\mu)}\ e^{s}\right)}} \sum_{\mathbf{k}} \frac{n_F(\mathbf{p})}{(e^{\lambda}\ e^{\beta(\epsilon_{\mathbf{k}}-\epsilon_{\mathbf{p}})}-1)}\ \frac{(1-n_F(\mathbf{k}))e^{\beta\epsilon_{\mathbf{k}}}}{\left(1+e^{\beta(\epsilon_{\mathbf{k}}-\mu)}\right)} = \Gamma_{<}(\lambda,0;-i\epsilon)e^{-\beta\mu} \sum_{\mathbf{k}} \frac{n_F(\mathbf{p})}{\left(e^{\beta(\epsilon_{\mathbf{k}}-\epsilon_{\mathbf{p}})+\lambda}-1\right)}\ \frac{(1-n_F(\mathbf{k}))e^{\beta\epsilon_{\mathbf{k}}}}{\left(1+e^{\beta(\epsilon_{\mathbf{k}}-\mu)}\right)}$$

and

$$\Gamma_{<}(0,0;-i\epsilon)e^{-\int_0^{\lambda'} ds \sum_{\mathbf{k}} \frac{(1-n_F(\mathbf{k}))}{\left(1+e^{\beta(\epsilon_{\mathbf{k}}-\mu)}\ e^{s}\right)}} \sum_{\mathbf{p}} \frac{(1-n_F(\mathbf{k}))}{(e^{-\lambda'}\ e^{\beta(\epsilon_{\mathbf{p}}-\epsilon_{\mathbf{k}})}-1)}\ \frac{n_F(\mathbf{p})}{\left(1+e^{-\beta(\epsilon_{\mathbf{p}}-\mu)}\right)} = e^{-\lambda'}e^{\beta\mu}\ \Gamma_{>}(0,\lambda';-i\epsilon) \sum_{\mathbf{p}} \frac{(1-n_F(\mathbf{k}))}{\left(e^{-\lambda'}e^{\beta(\epsilon_{\mathbf{p}}-\epsilon_{\mathbf{k}})}-1\right)}\ \frac{n_F(\mathbf{p})}{\left(1+e^{-\beta(\epsilon_{\mathbf{p}}-\mu)}\right)}$$

This means,

11.1 The solution to the coefficients

$$\Gamma_{<}(\lambda,0;-i\epsilon) = e^{\beta\mu}\ \Gamma_{>}(0,0;-i\epsilon)\ e^{-\int_0^{\lambda} ds\ \sum_{\mathbf{p}} \frac{n_F(\mathbf{p})}{\left(1+e^{-\beta(\epsilon_{\mathbf{p}}-\mu)}\ e^{s}\right)}}$$

and

$$\Gamma_{>}(0,\lambda';-i\epsilon) = e^{\lambda'}e^{-\beta\mu}\ \Gamma_{<}(0,0;-i\epsilon)\ e^{-\int_0^{\lambda'} ds \sum_{\mathbf{k}} \frac{(1-n_F(\mathbf{k}))}{\left(1+e^{\beta(\epsilon_{\mathbf{k}}-\mu)}\ e^{s}\right)}}$$

This also means,

$$\Gamma_{<}(0,0;-i\epsilon) = e^{\beta\mu}\ \Gamma_{>}(0,0;-i\epsilon)$$

We make the assertion that this is also valid for general time differences. This means,

$$\Gamma_{<}(\lambda,0;t-t') = e^{\beta\mu}\ \Gamma_{>}(0,0;t-t')\ e^{-\int_0^{\lambda} ds\ \sum_{\mathbf{p}} \frac{n_F(\mathbf{p})}{\left(1+e^{-\beta(\epsilon_{\mathbf{p}}-\mu)}\ e^{s}\right)}}$$

and

$$\Gamma_{>}(0,\lambda';t-t') = e^{\lambda'}e^{-\beta\mu}\ \Gamma_{<}(0,0;t-t')\ e^{-\int_0^{\lambda'} ds \sum_{\mathbf{k}} \frac{(1-n_F(\mathbf{k}))}{\left(1+e^{\beta(\epsilon_{\mathbf{k}}-\mu)}\ e^{s}\right)}}$$

This also means,

$$\Gamma_{<}(0,0;t-t') = e^{\beta\mu}\ \Gamma_{>}(0,0;t-t')$$

This means,

$$< T\ e^{-\lambda N_{>}(t)}\ e^{-\lambda' N'_{>}(t)}\ \tilde{c}_{\mathbf{p},<}(t)\tilde{c}^{\dagger}_{\mathbf{p},<}(t') > = e^{\beta\mu}\ \Gamma_{>}(0,0;t-t')\ e^{-\int_0^{\lambda} ds\ \sum_{\mathbf{p}} \frac{n_F(\mathbf{p})}{\left(1+e^{-\beta(\epsilon_{\mathbf{p}}-\mu)}\ e^{s}\right)}}\ \frac{n_F(\mathbf{p})\ e^{-\theta(t'-t)\beta\epsilon_{\mathbf{p}}}}{\left(1+e^{-\beta(\epsilon_{\mathbf{p}}-\mu)}\ e^{\lambda}\right)}\ e^{-\int_0^{\lambda'} ds \sum_{\mathbf{k}} \frac{(1-n_F(\mathbf{k}))}{\left(1+e^{\beta(\epsilon_{\mathbf{k}}-\mu)}\ e^{s}\right)}}$$

and

$$< T\ e^{-\lambda N_{>}(t)}\ e^{-\lambda' N'_{>}(t)}\ \tilde{c}_{\mathbf{k},>}(t)\tilde{c}^{\dagger}_{\mathbf{k},>}(t') > = e^{\lambda'}e^{-\beta\mu}\ \Gamma_{<}(0,0;t-t')\ e^{-\int_0^{\lambda'} ds \sum_{\mathbf{k}} \frac{(1-n_F(\mathbf{k}))}{\left(1+e^{\beta(\epsilon_{\mathbf{k}}-\mu)}\ e^{s}\right)}}\ \frac{(1-n_F(\mathbf{k}))e^{\theta(t-t')\beta\epsilon_{\mathbf{k}}}}{\left(1+e^{\beta(\epsilon_{\mathbf{k}}-\mu)}\ e^{\lambda'}\right)}\ e^{-\int_0^{\lambda} ds\ \sum_{\mathbf{p}} \frac{n_F(\mathbf{p})}{\left(1+e^{-\beta(\epsilon_{\mathbf{p}}-\mu)}\ e^{s}\right)}}$$

The coefficient $\Gamma_{>}(0,0;t-t')$ (or $\Gamma_{<}(0,0;t-t')$) is the only that remains to be fixed. We now set $\lambda,\lambda'=0$ to get,

$$< T\ \tilde{c}_{\mathbf{p},<}(t)\tilde{c}^{\dagger}_{\mathbf{p},<}(t') > = e^{\beta\mu}\ \Gamma_{>}(0,0;t-t')\ \frac{e^{-\theta(t'-t)\beta\epsilon_{\mathbf{p}}}}{\left(1+e^{-\beta(\epsilon_{\mathbf{p}}-\mu)}\right)}\ n_F(\mathbf{p})$$

and

$$< T\ \tilde{c}_{\mathbf{k},>}(t)\tilde{c}^{\dagger}_{\mathbf{k},>}(t') > = \Gamma_{>}(0,0;t-t')\ \frac{e^{\theta(t-t')\beta\epsilon_{\mathbf{k}}}}{\left(1+e^{\beta(\epsilon_{\mathbf{k}}-\mu)}\right)}\ (1-n_F(\mathbf{k}))$$

==

$$< T\ \tilde{c}_{\mathbf{p},<}(t)\tilde{c}^{\dagger}_{\mathbf{p},<}(t^{'}) > \ = \ \Gamma_{>}(0,0;t-t^{'})\ \frac{e^{\theta(t-t^{'})\beta\epsilon_{\mathbf{p}}}}{\left(1+e^{\beta(\epsilon_{\mathbf{p}}-\mu)}\right)}\ n_F(\mathbf{p})$$

and

$$< T\ \tilde{c}_{\mathbf{k},>}(t)\tilde{c}^{\dagger}_{\mathbf{k},>}(t^{'}) > \ = \ \Gamma_{>}(0,0;t-t^{'})\ \frac{e^{\theta(t-t^{'})\beta\epsilon_{\mathbf{k}}}}{\left(1+e^{\beta(\epsilon_{\mathbf{k}}-\mu)}\right)}\ (1-n_F(\mathbf{k}))$$

Now,

$$< T\ \tilde{c}_{\mathbf{p},<}(t)\tilde{c}^{\dagger}_{\mathbf{p},<}(t^{'}) > \ + \ < T\ \tilde{c}_{\mathbf{p},>}(t)\tilde{c}^{\dagger}_{\mathbf{p},>}(t^{'}) > \ = \ < T\ \tilde{c}_{\mathbf{p}}(t)\tilde{c}^{\dagger}_{\mathbf{p}}(t^{'}) >$$

Thus,

$$< T\ \tilde{c}_{\mathbf{p}}(t)\tilde{c}^{\dagger}_{\mathbf{p}}(t^{'}) > \ = \ \Gamma_{>}(0,0;t-t^{'})\ \frac{e^{\theta(t-t^{'})\beta\epsilon_{\mathbf{p}}}}{\left(1+e^{\beta(\epsilon_{\mathbf{p}}-\mu)}\right)}$$

$$< \tilde{c}_{\mathbf{p}}(t)\tilde{c}^{\dagger}_{\mathbf{p}}(t) > \ = \ \Gamma_{>}(0,0;-i\epsilon)\ \frac{e^{\beta\epsilon_{\mathbf{p}}}}{\left(1+e^{\beta(\epsilon_{\mathbf{p}}-\mu)}\right)}$$

$$-< \tilde{c}^{\dagger}_{\mathbf{p}}(t)\tilde{c}_{\mathbf{p}}(t) > \ = \ \Gamma_{>}(0,0;i\epsilon)\ \frac{1}{\left(1+e^{\beta(\epsilon_{\mathbf{p}}-\mu)}\right)}$$

A choice,

$$\Gamma_{>}(0,0;t-t^{'}) \ = \ sgn(t-t^{'})\ e^{-\theta(t-t^{'})\beta\mu}$$

and

$$\Gamma_{<}(0,0;t-t^{'}) \ = \ e^{\beta\mu}\ \Gamma_{>}(0,0;t-t^{'})$$

ensures

$$< \tilde{c}_{\mathbf{p}}(t)\tilde{c}^{\dagger}_{\mathbf{p}}(t) > \ + \ < \tilde{c}^{\dagger}_{\mathbf{p}}(t)\tilde{c}_{\mathbf{p}}(t) > \ = \ 1$$

Lastly, the meaning of the constant μ that was introduced as a proxy for $\gamma(0,0;t-t^{'})$ through $\gamma(0,0;t-t^{'}) \equiv e^{-\beta\mu}$, is identical to the usual chemical potential since we must ensure that the average total number of fermions is conserved.

$$\sum_{\mathbf{p}} \frac{1}{e^{\beta(\epsilon_{\mathbf{p}}-\mu)}+1} \ = \ N^0 \ = \ \sum_{\mathbf{p}} n_F(\mathbf{p})$$

12 Final answer for the fermion correlation function derived using boson algebra

$$< T\ e^{-\lambda N_{>}(t)}\ e^{-\lambda^{'} N^{'}_{>}(t)}\ \tilde{c}_{\mathbf{p},<}(t)\tilde{c}^{\dagger}_{\mathbf{p},<}(t^{'}) > \ = \ sgn(t-t^{'})\ \frac{n_F(\mathbf{p})\ e^{-\theta(t^{'}-t)\beta(\epsilon_{\mathbf{p}}-\mu)}}{\left(1+e^{-\beta(\epsilon_{\mathbf{p}}-\mu)}\ e^{\lambda}\right)}\ e^{-\int_0^{\lambda} ds\ \sum_{\mathbf{p}} \frac{n_F(\mathbf{p})}{\left(1+e^{-\beta(\epsilon_{\mathbf{p}}-\mu)}\ e^{s}\right)}}\ e^{-\int_0^{\lambda^{'}} ds \sum_{\mathbf{k}} \frac{(1-n_F(\mathbf{k}))}{\left(1+e^{\beta(\epsilon_{\mathbf{k}}-\mu)}\ e^{s}\right)}}$$

and

$$< T\ e^{-\lambda N_{>}(t)}\ e^{-\lambda^{'} N^{'}_{>}(t)}\ \tilde{c}_{\mathbf{k},>}(t)\tilde{c}^{\dagger}_{\mathbf{k},>}(t^{'}) > \ = \ e^{\lambda^{'}}\ sgn(t-t^{'})\ \frac{(1-n_F(\mathbf{k}))e^{\theta(t-t^{'})\beta(\epsilon_{\mathbf{k}}-\mu)}}{\left(1+e^{\beta(\epsilon_{\mathbf{k}}-\mu)}\ e^{\lambda^{'}}\right)}\ e^{-\int_0^{\lambda} ds\ \sum_{\mathbf{p}} \frac{n_F(\mathbf{p})}{\left(1+e^{-\beta(\epsilon_{\mathbf{p}}-\mu)}\ e^{s}\right)}}\ e^{-\int_0^{\lambda^{'}} ds \sum_{\mathbf{k}} \frac{(1-n_F(\mathbf{k}))}{\left(1+e^{\beta(\epsilon_{\mathbf{k}}-\mu)}\ e^{s}\right)}}$$

13 Result using direct trace over fermion states

$$< T\ e^{-\lambda N_{>}(t)}\ e^{-\lambda^{'} N^{'}_{>}(t)}\ \tilde{c}_{\mathbf{p},<}(t)\tilde{c}^{\dagger}_{\mathbf{p},<}(t^{'}) > \ =$$

$$\theta(t-t^{'})\ < e^{-\lambda \sum_{\mathbf{k}\neq\mathbf{p}}\ \tilde{c}_{\mathbf{k},<}\tilde{c}^{\dagger}_{\mathbf{k},<}} > \ < e^{-\lambda\ \tilde{c}_{\mathbf{p},<}\tilde{c}^{\dagger}_{\mathbf{p},<}}\ \tilde{c}_{\mathbf{p},<}\tilde{c}^{\dagger}_{\mathbf{p},<} >< \ e^{-\lambda^{'} \sum_{\mathbf{k}}\ \tilde{c}^{\dagger}_{\mathbf{k},>}\tilde{c}_{\mathbf{k},>}} \ >$$

$$-\theta(t^{'}-t)\ < \tilde{c}^{\dagger}_{\mathbf{p},<}\ e^{-\lambda\ \tilde{c}_{\mathbf{p},<}\tilde{c}^{\dagger}_{\mathbf{p},<}}\ \tilde{c}_{\mathbf{p},<} >< e^{-\lambda \sum_{\mathbf{k}\neq\mathbf{p}}\ \tilde{c}_{\mathbf{k},<}\tilde{c}^{\dagger}_{\mathbf{k},<}} \ > \ < e^{-\lambda^{'} \sum_{\mathbf{k}}\ \tilde{c}^{\dagger}_{\mathbf{k},>}\tilde{c}_{\mathbf{k},>}} \ >$$

$$= \ \theta(t-t^{'})\ < e^{-\lambda \sum_{\mathbf{k}\neq\mathbf{p}}\ \tilde{c}_{\mathbf{k},<}\tilde{c}^{\dagger}_{\mathbf{k},<}} \ > \ < e^{-\lambda\ \tilde{c}_{\mathbf{p},<}\tilde{c}^{\dagger}_{\mathbf{p},<}}\ \tilde{c}_{\mathbf{p},<}\tilde{c}^{\dagger}_{\mathbf{p},<} >< \ e^{-\lambda^{'} \sum_{\mathbf{k}}\ \tilde{c}^{\dagger}_{\mathbf{k},>}\tilde{c}_{\mathbf{k},>}} \ >$$

$$-\theta(t^{'}-t)\ < \tilde{c}^{\dagger}_{\mathbf{p},<}\ e^{-\lambda\ \tilde{c}_{\mathbf{p},<}\tilde{c}^{\dagger}_{\mathbf{p},<}}\ \tilde{c}_{\mathbf{p},<} >< e^{-\lambda \sum_{\mathbf{k}\neq\mathbf{p}}\ \tilde{c}_{\mathbf{k},<}\tilde{c}^{\dagger}_{\mathbf{k},<}} \ > \ < e^{-\lambda^{'} \sum_{\mathbf{k}}\ \tilde{c}^{\dagger}_{\mathbf{k},>}\tilde{c}_{\mathbf{k},>}} \ >$$

$$= \ \theta(t-t^{'})\ (\prod_{\mathbf{k}\neq\mathbf{p}} \frac{Tr(e^{-\beta(\epsilon_{\mathbf{k}}-\mu)\tilde{c}^{\dagger}_{\mathbf{k},<}\tilde{c}_{\mathbf{k},<}}e^{-\lambda\ \tilde{c}_{\mathbf{k},<}\tilde{c}^{\dagger}_{\mathbf{k},<}})}{Tr(e^{-\beta(\epsilon_{\mathbf{k}}-\mu)\tilde{c}^{\dagger}_{\mathbf{k},<}\tilde{c}_{\mathbf{k},<}})})\ \frac{Tr(e^{-\beta(\epsilon_{\mathbf{p}}-\mu)\tilde{c}^{\dagger}_{\mathbf{p},<}\tilde{c}_{\mathbf{p},<}}e^{-\lambda\ \tilde{c}_{\mathbf{p},<}\tilde{c}^{\dagger}_{\mathbf{p},<}}\ \tilde{c}_{\mathbf{p},<}\tilde{c}^{\dagger}_{\mathbf{p},<})}{Tr(e^{-\beta(\epsilon_{\mathbf{p}}-\mu)\tilde{c}^{\dagger}_{\mathbf{p},<}\tilde{c}_{\mathbf{p},<}})}\ (\prod_{\mathbf{k}} \frac{Tr(e^{-\beta(\epsilon_{\mathbf{k}}-\mu)\ \tilde{c}^{\dagger}_{\mathbf{k},>}\tilde{c}_{\mathbf{k},>}}e^{-\lambda^{'}\ \tilde{c}^{\dagger}_{\mathbf{k},>}\tilde{c}_{\mathbf{k},>}})}{Tr(e^{-\beta(\epsilon_{\mathbf{k}}-\mu)\ \tilde{c}^{\dagger}_{\mathbf{k},>}\tilde{c}_{\mathbf{k},>}})})$$

$$-\theta(t^{'}-t)\ \frac{Tr(e^{-\beta(\epsilon_{\mathbf{p}}-\mu)\tilde{c}^{\dagger}_{\mathbf{p},<}\tilde{c}_{\mathbf{p},<}}\tilde{c}^{\dagger}_{\mathbf{p},<}\ e^{-\lambda\ \tilde{c}_{\mathbf{p},<}\tilde{c}^{\dagger}_{\mathbf{p},<}}\ \tilde{c}_{\mathbf{p},<})}{Tr(e^{-\beta(\epsilon_{\mathbf{p}}-\mu)\tilde{c}^{\dagger}_{\mathbf{p},<}\tilde{c}_{\mathbf{p},<}})}(\prod_{\mathbf{k}\neq\mathbf{p}} \frac{Tr(e^{-\beta(\epsilon_{\mathbf{k}}-\mu)\tilde{c}^{\dagger}_{\mathbf{k},<}\tilde{c}_{\mathbf{k},<}}e^{-\lambda\tilde{c}_{\mathbf{k},<}\tilde{c}^{\dagger}_{\mathbf{k},<}})}{Tr(e^{-\beta(\epsilon_{\mathbf{k}}-\mu)\tilde{c}^{\dagger}_{\mathbf{k},<}\tilde{c}_{\mathbf{k},<}})}\ \prod_{\mathbf{k}} \frac{Tr(e^{-\beta(\epsilon_{\mathbf{k}}-\mu)\tilde{c}^{\dagger}_{\mathbf{k},>}\tilde{c}_{\mathbf{k},>}}e^{-\lambda^{'}\ \tilde{c}^{\dagger}_{\mathbf{k},>}\tilde{c}_{\mathbf{k},>}})}{Tr(e^{-\beta(\epsilon_{\mathbf{k}}-\mu)\tilde{c}^{\dagger}_{\mathbf{k},>}\tilde{c}_{\mathbf{k},>}})}$$

$$= \ \theta(t-t^{'})\frac{(e^{-\lambda}\ n_F(\mathbf{p}))}{(e^{-\lambda}+e^{-\beta(\epsilon_{\mathbf{p}}-\mu)})}\ (\prod_{\mathbf{k}} \frac{(e^{-\lambda n_F(\mathbf{k})}+e^{-\beta(\epsilon_{\mathbf{k}}-\mu)n_F(\mathbf{k})})}{(1+e^{-\beta(\epsilon_{\mathbf{k}}-\mu)n_F(\mathbf{k})})})\ (\prod_{\mathbf{k}} \frac{(1+e^{-\beta(\epsilon_{\mathbf{k}}-\mu)\ (1-n_F(\mathbf{k}))}e^{-\lambda^{'}\ (1-n_F(\mathbf{k}))})}{(1+e^{-\beta(\epsilon_{\mathbf{k}}-\mu)\ (1-n_F(\mathbf{k}))})})$$

$$-\theta(t^{'}-t)\ e^{-\beta(\epsilon_{\mathbf{p}}-\mu)}\ \frac{n_F(\mathbf{p})\ e^{-\lambda}}{(e^{-\lambda}+e^{-\beta(\epsilon_{\mathbf{p}}-\mu)})}\ (\prod_{\mathbf{k}} \frac{(e^{-\lambda n_F(\mathbf{k})}+e^{-\beta(\epsilon_{\mathbf{k}}-\mu)n_F(\mathbf{k})})}{(1+e^{-\beta(\epsilon_{\mathbf{k}}-\mu)n_F(\mathbf{k})})})\ \prod_{\mathbf{k}} \frac{(1+e^{-\beta(\epsilon_{\mathbf{k}}-\mu)(1-n_F(\mathbf{k}))}e^{-\lambda^{'}\ (1-n_F(\mathbf{k}))})}{(1+e^{-\beta(\epsilon_{\mathbf{k}}-\mu)(1-n_F(\mathbf{k}))})}$$

$$< T \; e^{-\lambda N_{>}(t)} \; e^{-\lambda^{'} N^{'}_{>}(t)} \; \tilde{c}_{\mathbf{p},<}(t)\tilde{c}^{\dagger}_{\mathbf{p},<}(t^{'}) > \;\; =$$

$$sgn(t-t^{'}) \; e^{-\theta(t^{'}-t)\beta(\epsilon_{\mathbf{p}}-\mu)} \; \frac{n_F(\mathbf{p}) \; e^{-\lambda}}{(e^{-\lambda}+e^{-\beta(\epsilon_{\mathbf{p}}-\mu)})} \; (\prod_{\mathbf{k}} \frac{(e^{-\lambda n_F(\mathbf{k})}+e^{-\beta(\epsilon_{\mathbf{k}}-\mu)n_F(\mathbf{k})})}{(1+e^{-\beta(\epsilon_{\mathbf{k}}-\mu)n_F(\mathbf{k})})}) \; \prod_{\mathbf{k}} \frac{(1+e^{-\beta(\epsilon_{\mathbf{k}}-\mu)(1-n_F(\mathbf{k}))}e^{-\lambda^{'}\;(1-n_F(\mathbf{k}))})}{(1+e^{-\beta(\epsilon_{\mathbf{k}}-\mu)(1-n_F(\mathbf{k}))})}$$

which agrees with the above result from bosonic algebra.

Similarly,

$$< T \; e^{-\lambda N_{>}(t)} \; e^{-\lambda^{'} N^{'}_{>}(t)} \; \tilde{c}_{\mathbf{k},>}(t)\tilde{c}^{\dagger}_{\mathbf{k},>}(t^{'}) > \;\; = \;\; \frac{Tr(e^{-\beta\sum_{\mathbf{p}}(\epsilon_{\mathbf{p}}-\mu)c^{\dagger}_{\mathbf{p}}c_{\mathbf{p}}}[\theta(t-t^{'}) \; e^{-\lambda N_{>}(t)} \; e^{-\lambda^{'} N^{'}_{>}(t)} \; \tilde{c}_{\mathbf{k},>}(t)\tilde{c}^{\dagger}_{\mathbf{k},>}(t^{'}) - \theta(t^{'}-t) \; \tilde{c}^{\dagger}_{\mathbf{k},>}(t^{'})e^{-\lambda N_{>}(t)} \; e^{-\lambda^{'} N^{'}_{>}(t)} \; \tilde{c}_{\mathbf{k},>}(t)])}{Tr(e^{-\beta\sum_{\mathbf{p}}(\epsilon_{\mathbf{p}}-\mu)c^{\dagger}_{\mathbf{p}}c_{\mathbf{p}}})}$$

$$= \frac{Tr(e^{-\beta\sum_{\mathbf{p}}(\epsilon_{\mathbf{p}}-\mu)c^{\dagger}_{\mathbf{p}}c_{\mathbf{p}}}[\theta(t-t^{'}) \; e^{-\sum_{\mathbf{p}}(\lambda \; c_{\mathbf{p},<}c^{\dagger}_{\mathbf{p},<}+\lambda^{'} \; c^{\dagger}_{\mathbf{p},>}c_{\mathbf{p},>})} \; \tilde{c}_{\mathbf{k},>}(t)\tilde{c}^{\dagger}_{\mathbf{k},>}(t^{'}) - \theta(t^{'}-t) \; \tilde{c}^{\dagger}_{\mathbf{k},>}(t^{'}) \; e^{-\sum_{\mathbf{p}}(\lambda \; c_{\mathbf{p},<}c^{\dagger}_{\mathbf{p},<}+\lambda^{'} \; c^{\dagger}_{\mathbf{p},>}c_{\mathbf{p},>})} \; \tilde{c}_{\mathbf{k},>}(t)])}{Tr(e^{-\beta\sum_{\mathbf{p}}(\epsilon_{\mathbf{p}}-\mu)c^{\dagger}_{\mathbf{p}}c_{\mathbf{p}}})}$$

$$= \; \theta(t-t^{'}) \; \frac{Tr(e^{-\beta\sum_{\mathbf{p}}(\epsilon_{\mathbf{p}}-\mu)c^{\dagger}_{\mathbf{p}}c_{\mathbf{p}}} e^{-\sum_{\mathbf{p}}(\lambda \; c_{\mathbf{p},<}c^{\dagger}_{\mathbf{p},<}+\lambda^{'} \; c^{\dagger}_{\mathbf{p},>}c_{\mathbf{p},>})} \; \tilde{c}_{\mathbf{k},>}(t)\tilde{c}^{\dagger}_{\mathbf{k},>}(t^{'}))}{Tr(e^{-\beta\sum_{\mathbf{p}}(\epsilon_{\mathbf{p}}-\mu)c^{\dagger}_{\mathbf{p}}c_{\mathbf{p}}})}$$

$$- \; \theta(t^{'}-t) \; \frac{Tr(e^{-\beta\sum_{\mathbf{p}}(\epsilon_{\mathbf{p}}-\mu)c^{\dagger}_{\mathbf{p}}c_{\mathbf{p}}} \; \tilde{c}^{\dagger}_{\mathbf{k},>}(t^{'}) \; e^{-\sum_{\mathbf{p}}(\lambda \; c_{\mathbf{p},<}c^{\dagger}_{\mathbf{p},<}+\lambda^{'} \; c^{\dagger}_{\mathbf{p},>}c_{\mathbf{p},>})} \; \tilde{c}_{\mathbf{k},>}(t))}{Tr(e^{-\beta\sum_{\mathbf{p}}(\epsilon_{\mathbf{p}}-\mu)c^{\dagger}_{\mathbf{p}}c_{\mathbf{p}}})}$$

$$= \; \theta(t-t^{'}) \; \frac{Tr(e^{-\beta\sum_{\mathbf{p}}(\epsilon_{\mathbf{p}}-\mu)c^{\dagger}_{\mathbf{p}}c_{\mathbf{p}}} e^{-\sum_{\mathbf{p}}(\lambda \; c_{\mathbf{p},<}c^{\dagger}_{\mathbf{p},<}+\lambda^{'} \; c^{\dagger}_{\mathbf{p},>}c_{\mathbf{p},>})} \; \tilde{c}_{\mathbf{k},>}(t)\tilde{c}^{\dagger}_{\mathbf{k},>}(t^{'}))}{Tr(e^{-\beta\sum_{\mathbf{p}}(\epsilon_{\mathbf{p}}-\mu)c^{\dagger}_{\mathbf{p}}c_{\mathbf{p}}})}$$

$$- \; \theta(t^{'}-t) \; \frac{Tr(e^{-\beta\sum_{\mathbf{p}}(\epsilon_{\mathbf{p}}-\mu)c^{\dagger}_{\mathbf{p}}c_{\mathbf{p}}} \; e^{-\sum_{\mathbf{p}}(\lambda \; c_{\mathbf{p},<}c^{\dagger}_{\mathbf{p},<}+\lambda^{'} \; c^{\dagger}_{\mathbf{p},>}c_{\mathbf{p},>})} \; e^{\lambda^{'}} \; \tilde{c}^{\dagger}_{\mathbf{k},>}(t^{'}) \; \tilde{c}_{\mathbf{k},>}(t))}{Tr(e^{-\beta\sum_{\mathbf{p}}(\epsilon_{\mathbf{p}}-\mu)c^{\dagger}_{\mathbf{p}}c_{\mathbf{p}}})}$$

$$= \; [\theta(t-t^{'}) \; \frac{Tr(e^{-\beta(\epsilon_{\mathbf{k}}-\mu)c^{\dagger}_{\mathbf{k}}c_{\mathbf{k}}} e^{-(\lambda \; c_{\mathbf{k},<}c^{\dagger}_{\mathbf{k},<}+\lambda^{'} \; c^{\dagger}_{\mathbf{k},>}c_{\mathbf{k},>})} \; \tilde{c}_{\mathbf{k},>}(t)\tilde{c}^{\dagger}_{\mathbf{k},>}(t^{'}))}{Tr(e^{-\beta(\epsilon_{\mathbf{k}}-\mu)c^{\dagger}_{\mathbf{k}}c_{\mathbf{k}}})} \; - \; \theta(t^{'}-t) \; \frac{Tr(e^{-\beta(\epsilon_{\mathbf{k}}-\mu)c^{\dagger}_{\mathbf{k}}c_{\mathbf{k}}} \; e^{-(\lambda \; c_{\mathbf{k},<}c^{\dagger}_{\mathbf{k},<}+\lambda^{'} \; c^{\dagger}_{\mathbf{k},>}c_{\mathbf{k},>})} \; e^{\lambda^{'}} \; \tilde{c}^{\dagger}_{\mathbf{k},>}(t^{'}) \; \tilde{c}_{\mathbf{k},>}(t))}{Tr(e^{-\beta(\epsilon_{\mathbf{k}}-\mu)c^{\dagger}_{\mathbf{k}}c_{\mathbf{k}}})}]$$

$$\prod_{\mathbf{p}\neq\mathbf{k}} \; \frac{Tr(e^{-\beta(\epsilon_{\mathbf{p}}-\mu)c^{\dagger}_{\mathbf{p}}c_{\mathbf{p}}} e^{-(\lambda \; c_{\mathbf{p},<}c^{\dagger}_{\mathbf{p},<}+\lambda^{'} \; c^{\dagger}_{\mathbf{p},>}c_{\mathbf{p},>})})}{Tr(e^{-\beta(\epsilon_{\mathbf{p}}-\mu)c^{\dagger}_{\mathbf{p}}c_{\mathbf{p}}})}$$

$$= \; (1-n_F(\mathbf{k})) \; [\frac{\theta(t-t^{'})}{(1+e^{-\beta(\epsilon_{\mathbf{k}}-\mu)}e^{-\lambda^{'}})} \; - \; \frac{\theta(t^{'}-t)}{(e^{\beta(\epsilon_{\mathbf{k}}-\mu)}+e^{-\lambda^{'}})}] \; \prod_{\mathbf{p}} \; \frac{(e^{-\beta n_F(\mathbf{p})(\epsilon_{\mathbf{p}}-\mu)}+e^{-\lambda \; n_F(\mathbf{p})})}{(1+e^{-\beta \; n_F(\mathbf{p})(\epsilon_{\mathbf{p}}-\mu)})} \; \prod_{\mathbf{p}} \; \frac{(e^{-\beta(1-n_F(\mathbf{p}))(\epsilon_{\mathbf{p}}-\mu)}e^{-\lambda^{'}\;(1-n_F(\mathbf{p}))}+1)}{(1+e^{-\beta(\epsilon_{\mathbf{p}}-\mu)\;(1-n_F(\mathbf{p}))})}$$

But,

$$(1-n_F(\mathbf{k})) \; [\frac{\theta(t-t^{'})}{(1+e^{-\beta(\epsilon_{\mathbf{k}}-\mu)}e^{-\lambda^{'}})} \; - \; \frac{\theta(t^{'}-t)}{(e^{\beta(\epsilon_{\mathbf{k}}-\mu)}+e^{-\lambda^{'}})}] \; = \; e^{\lambda^{'}} \; sgn(t-t^{'}) \; \frac{(1-n_F(\mathbf{k}))e^{\theta(t-t^{'})\beta(\epsilon_{\mathbf{k}}-\mu)}}{\left(1+e^{\beta(\epsilon_{\mathbf{k}}-\mu)} \; e^{\lambda^{'}}\right)}$$

This means,

$$< T \; e^{-\lambda N_{>}(t)} \; e^{-\lambda^{'} N^{'}_{>}(t)} \; \tilde{c}_{\mathbf{k},>}(t)\tilde{c}^{\dagger}_{\mathbf{k},>}(t^{'}) > \;\; =$$

$$e^{\lambda^{'}} \; sgn(t-t^{'}) \; \frac{(1-n_F(\mathbf{k}))e^{\theta(t-t^{'})\beta(\epsilon_{\mathbf{k}}-\mu)}}{\left(1+e^{\beta(\epsilon_{\mathbf{k}}-\mu)} \; e^{\lambda^{'}}\right)} \; \prod_{\mathbf{p}} \; \frac{(e^{-\beta n_F(\mathbf{p})(\epsilon_{\mathbf{p}}-\mu)}+e^{-\lambda \; n_F(\mathbf{p})})}{(1+e^{-\beta \; n_F(\mathbf{p})(\epsilon_{\mathbf{p}}-\mu)})} \; \prod_{\mathbf{p}} \; \frac{(e^{-\beta(1-n_F(\mathbf{p}))(\epsilon_{\mathbf{p}}-\mu)}e^{-\lambda^{'}\;(1-n_F(\mathbf{p}))}+1)}{(1+e^{-\beta(\epsilon_{\mathbf{p}}-\mu)\;(1-n_F(\mathbf{p}))})}$$

which agrees with the above result from bosonic algebra.