Definitions of Sea bosons and free particle Hamiltonian

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1 Definitions:

The following are definitions in the paper "Single-particle Green functions in exactly solvable models of Bose and Fermi liquids" by Girish S. Setlur and Yia-Chung Chang (1998).

The Sea-displacement operator,

$$A_k(q) = \frac{1}{\sqrt{n_{k-q/2}}} c^{\dagger}_{k-q/2} \left(\frac{n^{\beta}(k-q/2)}{\langle N \rangle} \right)^{1/2} e^{i\theta(k,q)} c_{k+q/2}. \tag{1}$$

With this definition, we get the kinetic energy to be,

$$KE = \sum_{k,q} \frac{k \cdot q}{m} \Lambda_k(-q) A^{\dagger}_k(q) A_k(q) + N\epsilon_0. \tag{2}$$

where,

$$\Lambda_k(q) = \sqrt{n_{k+q/2}\left(1 - n_{k-q/2}\right)}.$$

The final Hamiltonian for the Fermi system becomes,

$$H = \sum_{k,q} \omega_k(q) A^{\dagger}_k(q) A_k(q)$$

$$+ \sum_{q \neq 0} \frac{v_q}{2V} \sum_{k,k'} \left[\Lambda_k(q) A_k(-q) + \Lambda_k(-q) A^{\dagger}_k(-q) \right] \times \left[\Lambda_{k'}(-q) A_{k'}(q) + \Lambda_{k'}(q) A^{\dagger}_{k'}(-q) \right],$$

where,

$$v_q = \int V(r) \exp\left(-iq \cdot \frac{r}{\hbar}\right) dr \ ??$$

and,

$$\omega_k(q) = \left(\frac{k \cdot q}{m}\right) \Lambda_k(-q).$$

2 The definition of Sea-Boson:

Given 0 K momentum distribution,

$$n_F(\mathbf{k}) = \Theta\left(k_F - |\mathbf{k}|\right) \quad . \tag{3}$$

Fermi Fock space operator were defined by us in momentum basis as,

$$c^{\dagger}_{\mathbf{p},<} = n_F(\mathbf{p}) c^{\dagger}_{\mathbf{p}}, \tag{4}$$

$$c_{\mathbf{p},<} = n_F(\mathbf{p}) c_{\mathbf{p}} \tag{5}$$

and,

$$c^{\dagger}_{\mathbf{p},>} = (1 - n_F)(\mathbf{p}) c^{\dagger}_{\mathbf{p}}, \tag{6}$$

$$c_{\mathbf{p},>} = (1 - n_F)(\mathbf{p}) c_{\mathbf{p}}. \tag{7}$$

with the particle-hole operator,

$$\hat{N}_{>} = \sum_{\mathbf{k}} c_{\mathbf{k},<} c^{\dagger}_{\mathbf{k},<} \tag{8}$$

and the Sea-displacement operators as,

$$A_{\mathbf{k}}(\mathbf{q}) = c^{\dagger}_{\mathbf{k}-\frac{\mathbf{q}}{2},<} \, \frac{1}{\sqrt{\hat{N}_{>}}} \, c_{\mathbf{k}+\frac{\mathbf{q}}{2},>} \tag{9}$$

and the Sea-bosons,

$$a^{\dagger}_{\mathbf{p}}(\mathbf{q}) = c^{\dagger}_{\mathbf{p}+\mathbf{q}/2,>} c_{\mathbf{p}-\mathbf{q}/2,<} \ . \tag{10}$$

We have derived the kinetic energy to be,

$$KE = \sum_{\mathbf{p}} \frac{|\mathbf{p}|^2}{2m} n_F(\mathbf{p}) + \sum_{\mathbf{k},\mathbf{q}} \frac{\mathbf{k}\cdot\mathbf{q}}{m} A^{\dagger}_{\mathbf{k}}(\mathbf{q}) A_{\mathbf{k}}(\mathbf{q}) \tag{11}$$

$$KE = \sum_{\mathbf{p}} \frac{|\mathbf{p}|^2}{2m} n_F(\mathbf{p}) + \sum_{\mathbf{k},\mathbf{q}} \frac{\mathbf{k}\cdot\mathbf{q}}{m} \frac{1}{N_{>}} a^{\dagger}_{\mathbf{k}}(\mathbf{q}) a_{\mathbf{k}}(\mathbf{q}) \tag{12}$$