Solution to the equation of motion

Rishi Paresh Joshi

1 Solution to the equation of motion Eq. (28 and 29):

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The lesser than equation is:

$$< T\ a^{\dagger}_{\mathbf{k-q}/2}(-\mathbf{q},t_1)\ e^{-\lambda N_{>}(t)}\ e^{-\lambda^{'} N^{'}_{>}(t)}\ c_{\mathbf{k-q},>}(t) c^{\dagger}_{\mathbf{k},<}(t^{'}) > \ =$$

$$e^{\frac{i\mathbf{q}.(-2\mathbf{k}t_1+\mathbf{q}(t_1+i\beta))}{2m}} (f_{max}\ e^{\frac{\beta \mathbf{k}.\mathbf{q}}{m}} + i n_F(\mathbf{k})\ < T\ e^{-\lambda N_{>}(t)}\ e^{-\lambda^{'} N^{'}_{>}(t)}\ c_{\mathbf{k-q},>}(t) c^{\dagger}_{\mathbf{k-q},>}(t^{'}) >\ i\ \theta(t^{'}-t_1)\ e^{\frac{\mathbf{q}.(2i\mathbf{k}t^{'}+\mathbf{q}(\beta-it^{'}))}{2m}})\ \theta(t_1-t)$$

$$+\ e^{-\frac{it_1\mathbf{q}.(2\mathbf{k}-\mathbf{q})}{2m}} (f_{max} - i n_F(\mathbf{k})\ < T\ e^{-\lambda N_{>}(t)}\ e^{-\lambda^{'} N^{'}_{>}(t)}\ c_{\mathbf{k-q},>}(t) c^{\dagger}_{\mathbf{k-q},>}(t^{'}) >\ i\ \theta(t_1-t^{'})\ e^{\frac{it^{'}\mathbf{q}.(2\mathbf{k}-\mathbf{q})}{2m}})\ \theta(t-t_1)$$

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The greater than equation is:

$$< T\ a_{\mathbf{k-q}/2}(\mathbf{q},t_1)\ e^{-\lambda N_{>}(t)}\ e^{-\lambda^{'} N^{'}_{>}(t)}\ c_{\mathbf{k-q},<}(t) c^{\dagger}_{\mathbf{k},>}(t^{'}) > \ =$$

$$e^{\frac{i\mathbf{q}.(-2\mathbf{k}t_1+\mathbf{q}(t_1+i\beta))}{2m}} (g_{max}\ e^{\frac{\beta \mathbf{k}.\mathbf{q}}{m}} + i(1-n_F(\mathbf{k}))\ < T\ e^{-\lambda N_{>}(t)}\ e^{-\lambda^{'} N^{'}_{>}(t)}\ c_{\mathbf{k-q},<}(t) c^{\dagger}_{\mathbf{k-q},<}(t^{'}) >\ i\ \theta(t^{'}-t_1) e^{\frac{(2i\mathbf{k}.\mathbf{q}t^{'}+\mathbf{q}^2(\beta-it^{'}))}{2m}})\ \theta(t_1-t)$$

$$+\ e^{-\frac{it_1\mathbf{q}.(2\mathbf{k}-\mathbf{q})}{2m}} (g_{max} - i(1-n_F(\mathbf{k}))\ < T\ e^{-\lambda N_{>}(t)}\ e^{-\lambda^{'} N^{'}_{>}(t)}\ c_{\mathbf{k-q},<}(t) c^{\dagger}_{\mathbf{k-q},<}(t^{'}) >\ i\ \theta(t_1-t^{'})\ e^{\frac{it^{'}\mathbf{q}.(2\mathbf{k}-\mathbf{q})}{2m}})\ \theta(t-t_1)$$

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2 Solutions for f max and g max are:

$$f_{max}\ =\ \frac{< T\ e^{-\lambda N_{>}(t)}\ e^{-\lambda^{'} N^{'}_{>}(t)}\ c_{\mathbf{k-q},>}(t) c^{\dagger}_{\mathbf{k-q},>}(t^{'}) >\ n_F(\mathbf{k}) e^{\frac{it^{'}(2\mathbf{k}.\mathbf{q}-q^2)}{2m}}\ \left(e^{\lambda+\lambda^{'}}\theta(t-t^{'})+\theta(t^{'}-t)\right)}{e^{\frac{\beta(2\mathbf{k}.\mathbf{q}-q^2)}{2m}} - e^{\lambda+\lambda^{'}}}$$

$$+\ \frac{< T\ e^{-\lambda N_{>}(t)}\ e^{-\lambda^{'} N^{'}_{>}(t)}\ c_{\mathbf{k},<}(t) c^{\dagger}_{\mathbf{k},<}(t^{'}) >\ (n_F(\mathbf{k-q})-1) e^{\frac{it(2\mathbf{k}.\mathbf{q}-q^2)}{2m}+\lambda+\lambda^{'}}}{e^{\frac{\beta(2\mathbf{k}.\mathbf{q}-q^2)}{2m}} - e^{\lambda+\lambda^{'}}}$$

and

$$g_{max}\ =\ \frac{e^{-\frac{iq^2t^{'}}{2m}}\ < T\ e^{-\lambda N_{>}(t)}\ e^{-\lambda^{'} N^{'}_{>}(t)}\ c_{\mathbf{k-q},<}(t) c^{\dagger}_{\mathbf{k-q},<}(t^{'}) >\ (n_F(\mathbf{k})-1)\ \left(-e^{\frac{i\mathbf{k}.\mathbf{q}}{m}t^{'}}\right) \left(e^{\lambda+\lambda^{'}}\ \theta(t^{'}-t)+\theta(t-t^{'})\right)}{e^{\frac{\beta(2\mathbf{k}.\mathbf{q}-q^2)}{2m}+\lambda+\lambda^{'}} - 1}$$

$$+\ \frac{-e^{-\frac{iq^2t^{'}}{2m}}\ < T\ e^{-\lambda N_{>}(t)}\ e^{-\lambda^{'} N^{'}_{>}(t)}\ c_{\mathbf{k},>}(t) c^{\dagger}_{\mathbf{k},>}(t^{'}) >\ n_F(\mathbf{k-q})\ e^{\frac{i(2\mathbf{k}.\mathbf{q}t+q^2(t^{'}-t))}{2m}}}{e^{\frac{\beta(2\mathbf{k}.\mathbf{q}-q^2)}{2m}+\lambda+\lambda^{'}} - 1}$$

3 Simplification

Using a more compact notation,

$$
\begin{aligned}
G_{\mathbf{k},>}(\lambda,\lambda^{'};t,t^{'}) &\equiv \; < Te^{-\lambda N_>(t)}e^{-\lambda^{'}N^{'}_>(t)}c_{\mathbf{k},>}(t)c^{\dagger}_{\mathbf{k},>}(t^{'})>; \\
G_{\mathbf{k},<}(\lambda,\lambda^{'};t,t^{'}) &\equiv \; < Te^{-\lambda N_>(t)}e^{-\lambda^{'}N^{'}_>(t)}c_{\mathbf{k},<}(t)c^{\dagger}_{\mathbf{k},<}(t^{'})>
\end{aligned}
\tag{1}
$$

and,

$$
\begin{aligned}
F_{\mathbf{k}<}(\mathbf{q};t_1) &= \langle Te^{-\lambda N_>(t)-\lambda' N'_>(t)} \; c_{\mathbf{k+q}>}(t) \; a^{\dagger}_{\mathbf{k}+\frac{\mathbf{q}}{2}}(\mathbf{q},t_1) \; c^{\dagger}_{\mathbf{k}<}(t')\rangle \\
F_{\mathbf{k}>}(\mathbf{q};t_1) &= \langle Te^{-\lambda N_>(t)-\lambda' N'_>(t)} \; c_{\mathbf{k-q}<}(t) \; a_{\mathbf{k}-\frac{\mathbf{q}}{2}}(\mathbf{q},t_1) \; c^{\dagger}_{\mathbf{k}>}(t')\rangle
\end{aligned}
\tag{2}
$$

We get,

$$
\begin{aligned}
f_{max} \;=\; & \frac{G_{\mathbf{k-q},>}(\lambda,\lambda^{'};t,t^{'}) \;\; n_F(\mathbf{k})e^{\frac{it^{'}(2\mathbf{k}.\mathbf{q}-q^2)}{2m}} \;\; \left(e^{\lambda+\lambda^{'}}\theta(t-t^{'})+\theta(t^{'}-t)\right)}{e^{\frac{\beta(2\mathbf{k}.\mathbf{q}-q^2)}{2m}}-e^{\lambda+\lambda^{'}}} \\
& + \; \frac{G_{\mathbf{k},<}(\lambda,\lambda^{'};t,t^{'}) \;\; (n_F(\mathbf{k-q})-1)e^{\frac{it(2\mathbf{k}.\mathbf{q}-q^2)}{2m}+\lambda+\lambda^{'}}}{e^{\frac{\beta(2\mathbf{k}.\mathbf{q}-q^2)}{2m}}-e^{\lambda+\lambda^{'}}}
\end{aligned}
$$

and,

$$
\begin{aligned}
g_{max} \;=\; & \frac{e^{-\frac{iq^2t^{'}}{2m}} \; G_{\mathbf{k-q},<}(\lambda,\lambda^{'};t,t^{'}) \;\; (n_F(\mathbf{k})-1) \;\; \left(-e^{\frac{i\mathbf{k}.\mathbf{q}}{m}t^{'}}\right)\left(e^{\lambda+\lambda^{'}} \; \theta(t^{'}-t)+\theta(t-t^{'})\right)}{e^{\frac{\beta(2\mathbf{k}.\mathbf{q}-q^2)}{2m}+\lambda+\lambda^{'}}-1} \\
& + \; \frac{-e^{-\frac{iq^2t^{'}}{2m}} \;\; G_{\mathbf{k},>}(\lambda,\lambda^{'};t,t^{'}) \; n_F(\mathbf{k-q}) \; e^{\frac{i(2\mathbf{k}.\mathbf{q}t+q^2(t^{'}-t))}{2m}}}{e^{\frac{\beta(2\mathbf{k}.\mathbf{q}-q^2)}{2m}+\lambda+\lambda^{'}}-1}
\end{aligned}
$$

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The lesser than equation is

$$
\begin{aligned}
F_{\mathbf{k}<}(-\mathbf{q};t_1) \;=\; & e^{\frac{i\mathbf{q}.(-2\mathbf{k}t_1+\mathbf{q}(t_1+i\beta))}{2m}}(f_{max} \; e^{\frac{\beta\mathbf{k}.\mathbf{q}}{m}}+in_F(\mathbf{k}) \; G_{\mathbf{k-q},>}(\lambda,\lambda^{'};t,t^{'}) \; i \; \theta(t^{'}-t_1) \; e^{\frac{\mathbf{q}.(2i\mathbf{k}t^{'}+\mathbf{q}(\beta-it^{'}))}{2m}}) \; \theta(t_1-t) \\
& + \; e^{-\frac{it_1\mathbf{q}.(2\mathbf{k}-\mathbf{q})}{2m}}(f_{max}-in_F(\mathbf{k}) \; G_{\mathbf{k-q},>}(\lambda,\lambda^{'};t,t^{'}) \; i \; \theta(t_1-t^{'}) \; e^{\frac{it^{'}\mathbf{q}.(2\mathbf{k}-\mathbf{q})}{2m}}) \; \theta(t-t_1)
\end{aligned}
$$

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The greater than equation is:

$$
\begin{aligned}
F_{\mathbf{k}>}(\mathbf{q};t_1) \;=\; & e^{\frac{i\mathbf{q}.(-2\mathbf{k}t_1+\mathbf{q}(t_1+i\beta))}{2m}}(g_{max} \; e^{\frac{\beta\mathbf{k}.\mathbf{q}}{m}}+i(1-n_F(\mathbf{k})) \; G_{\mathbf{k-q},<}(\lambda,\lambda^{'};t,t^{'}) \;\; i \; \theta(t^{'}-t_1)e^{\frac{(2i\mathbf{k}.\mathbf{q}t^{'}+\mathbf{q}^2(\beta-it^{'}))}{2m}}) \; \theta(t_1-t) \\
& + \; e^{-\frac{it_1\mathbf{q}.(2\mathbf{k}-\mathbf{q})}{2m}}(g_{max}-i(1-n_F(\mathbf{k})) \; G_{\mathbf{k-q},<}(\lambda,\lambda^{'};t,t^{'}) \;\; i \; \theta(t_1-t^{'}) \; e^{\frac{it^{'}\mathbf{q}.(2\mathbf{k}-\mathbf{q})}{2m}}) \; \theta(t-t_1)
\end{aligned}
$$

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4 $F_<(-q;t_1)$ and $F_>(q;t_1)$ derived

$$< T\ e^{-\lambda N_>(t)}\ e^{-\lambda^{'} N^{'}_>(t)}\ c_{\mathbf{k-q},>}(t) a^\dagger_{\mathbf{k-q}/2}(-\mathbf{q},t_1)\ c^\dagger_{\mathbf{k},<}(t^{'}) > \ = $$

$$e^{-i(t_1-t)(\epsilon_\mathbf{k}-\epsilon_\mathbf{k-q})}\Big[(e^{\lambda+\lambda^{'}}-1)\ < T\ e^{-\lambda N_>(t)}\ e^{-\lambda^{'} N^{'}_>(t)} c_{\mathbf{k-q},>}(t)\ a^\dagger_{\mathbf{k-q}/2}(-\mathbf{q},t)\ c^\dagger_{\mathbf{k},<}(t^{'}) >$$

$$-(1-n_F(\mathbf{k-q}))\ e^{\lambda+\lambda^{'}}\ < T\ e^{-\lambda N_>(t)}\ e^{-\lambda^{'} N^{'}_>(t)}\ c_{\mathbf{k},<}(t) c^\dagger_{\mathbf{k},<}(t^{'}) > \Big]\ \left(\theta(t_1-t) - \frac{1}{1-e^{\beta(\epsilon_\mathbf{k}-\epsilon_\mathbf{k-q})}}\right)$$

$$+ n_F(\mathbf{k}) e^{-i(t_1-t^{\prime})(\epsilon_\mathbf{k}-\epsilon_\mathbf{k-q})}\ < T\ e^{-\lambda N_>(t)}\ e^{-\lambda^{'} N^{'}_>(t)}\ c_{\mathbf{k-q},>}(t) c^\dagger_{\mathbf{k-q},>}(t^{'}) > \ \left(\theta(t_1-t^{\prime}) - \frac{1}{1-e^{\beta(\epsilon_\mathbf{k}-\epsilon_\mathbf{k-q})}}\right)$$

and

$$< T\ e^{-\lambda N_>(t)}\ e^{-\lambda^{'} N^{'}_>(t)}\ c_{\mathbf{k-q},<}(t) a_{\mathbf{k-q}/2}(\mathbf{q},t_1)\ c^\dagger_{\mathbf{k},>}(t^{'}) > \ = $$

$$e^{-i(t_1-t)(\epsilon_\mathbf{k}-\epsilon_\mathbf{k-q})}\Big[(e^{-\lambda-\lambda^{'}}-1)\ < T\ e^{-\lambda N_>(t)}\ e^{-\lambda^{'} N^{'}_>(t)}\ c_{\mathbf{k-q},<}(t) a_{\mathbf{k-q}/2}(\mathbf{q},t) c^\dagger_{\mathbf{k},>}(t^{'}) >$$

$$- n_F(\mathbf{k-q})\ e^{-\lambda-\lambda^{'}}\ < T\ e^{-\lambda N_>(t)}\ e^{-\lambda^{'} N^{'}_>(t)}\ c_{\mathbf{k},>}(t) c^\dagger_{\mathbf{k},>}(t^{'}) > \ \Big]\left(\theta(t_1-t) - \frac{1}{1-e^{\beta(\epsilon_\mathbf{k}-\epsilon_\mathbf{k-q})}}\right)$$

$$+ e^{-i(t_1-t^{\prime})(\epsilon_\mathbf{k}-\epsilon_\mathbf{k-q})} < T\ e^{-\lambda N_>(t)}\ e^{-\lambda^{'} N^{'}_>(t)}\ c_{\mathbf{k-q},<}(t)\ c^\dagger_{\mathbf{k-q},<}(t^{'}) > \ (1-n_F(\mathbf{k}))\ \left(\theta(t_1-t^{\prime}) - \frac{1}{1-e^{\beta(\epsilon_\mathbf{k}-\epsilon_\mathbf{k-q})}}\right)$$

Using a more compact notation,

$$F_{\mathbf{k}<}(\mathbf{q};t_1) \ = \langle Te^{-\lambda N_>(t) - \lambda^{\prime} N^{\prime}_>(t)}\ c_{\mathbf{k+q}>}(t)\ a^\dagger_{\mathbf{k}+\frac{\mathbf{q}}{2}}(\mathbf{q},t_1)\ c^\dagger_{\mathbf{k}<}(t^{\prime})\rangle \tag{3}$$

$$F_{\mathbf{k}>}(\mathbf{q};t_1) \ = \langle Te^{-\lambda N_>(t) - \lambda^{\prime} N^{\prime}_>(t)}\ c_{\mathbf{k-q}<}(t)\ a_{\mathbf{k}-\frac{\mathbf{q}}{2}}(\mathbf{q},t_1)\ c^\dagger_{\mathbf{k}>}(t^{\prime})\rangle \tag{4}$$

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Lesser than derived equation is,

$$F_{\mathbf{k}<}(-\mathbf{q};t_1) \ = e^{-i(t_1-t)(\epsilon_\mathbf{k}-\epsilon_\mathbf{k-q})}\Big[(e^{\lambda+\lambda^{'}}-1)\ F_{\mathbf{k}<}(-\mathbf{q};t)\ -(1-n_F(\mathbf{k-q}))\ e^{\lambda+\lambda^{'}}\ \ G_{\mathbf{k},<}(\lambda,\lambda^{'};t,t^{'})\ \Big]\ \left(\theta(t_1-t) - \frac{1}{1-e^{\beta(\epsilon_\mathbf{k}-\epsilon_\mathbf{k-q})}}\right)$$

$$+n_F(\mathbf{k}) e^{-i(t_1-t^{\prime})(\epsilon_\mathbf{k}-\epsilon_\mathbf{k-q})}\ G_{\mathbf{k-q},>}(\lambda,\lambda^{'};t,t^{'})\ \ \left(\theta(t_1-t^{\prime}) - \frac{1}{1-e^{\beta(\epsilon_\mathbf{k}-\epsilon_\mathbf{k-q})}}\right)$$

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and,

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greater than derived equation is,

$$F_{\mathbf{k}>}(\mathbf{q};t_1) \ = e^{-i(t_1-t)(\epsilon_\mathbf{k}-\epsilon_\mathbf{k-q})}\Big[(e^{-\lambda-\lambda^{'}}-1)\ F_{\mathbf{k}>}(\mathbf{q};t)\ - n_F(\mathbf{k-q})\ e^{-\lambda-\lambda^{'}} G_{\mathbf{k},>}(\lambda,\lambda^{'};t,t^{'})\ \ \Big]\left(\theta(t_1-t) - \frac{1}{1-e^{\beta(\epsilon_\mathbf{k}-\epsilon_\mathbf{k-q})}}\right)$$

$$+e^{-i(t_1-t^{\prime})(\epsilon_\mathbf{k}-\epsilon_\mathbf{k-q})} G_{\mathbf{k-q},<}(\lambda,\lambda^{'};t,t^{'})\ (1-n_F(\mathbf{k}))\ \left(\theta(t_1-t^{\prime}) - \frac{1}{1-e^{\beta(\epsilon_\mathbf{k}-\epsilon_\mathbf{k-q})}}\right).$$

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5 Simplification of $F_<(-q;t_1)$ and $F_>(q;t_1)$ derived equations

5.1 $F_<(-q;t_1)$ derived simplified equation

Since we have $F_<(-q;t)$ and $F_>(q;t)$ in the RHS of the derived equations, we need to replace them with $G>$ and $G<$, so that we can match with answer after substituting f_{max} and g_{max} in the solution to the equations of motion.

In $\lim t_1 \to t^-$, the lesser than derived equation is,

$$F_{\mathbf{k}<}(-\mathbf{q};t) \;=-\Big[(e^{\lambda+\lambda'}-1)\;F_{\mathbf{k}<}(\mathbf{q};t)\;-(1-n_F(\mathbf{k}-\mathbf{q}))\;e^{\lambda+\lambda'}\;\;G_{\mathbf{k},<}(\lambda,\lambda';t,t')\;\Big]\;\left(\frac{1}{1-e^{\beta(\epsilon_{\mathbf{k}}-\epsilon_{\mathbf{k}-\mathbf{q}})}}\right)$$
$$+n_F(\mathbf{k})e^{-i(t-t')(\epsilon_{\mathbf{k}}-\epsilon_{\mathbf{k}-\mathbf{q}})}\;G_{\mathbf{k}-\mathbf{q},>}(\lambda,\lambda';t,t')\;\;\left(\theta(t-t')-\frac{1}{1-e^{\beta(\epsilon_{\mathbf{k}}-\epsilon_{\mathbf{k}-\mathbf{q}})}}\right)$$

Taking the $F_{\mathbf{k}<}(-\mathbf{q};t)$ terms on RHS,

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$F_{\mathbf{k}<}(-\mathbf{q};t)$ is:

$$F_{\mathbf{k}<}(-\mathbf{q};t) \;=\frac{1-e^{\beta(\epsilon_{\mathbf{k}}-\epsilon_{\mathbf{k}-\mathbf{q}})}}{e^{\lambda+\lambda'}-e^{\beta(\epsilon_{\mathbf{k}}-\epsilon_{\mathbf{k}-\mathbf{q}})}}\Bigg[(1-n_F(\mathbf{k}-\mathbf{q}))\;\frac{e^{\lambda+\lambda'}}{1-e^{\beta(\epsilon_{\mathbf{k}}-\epsilon_{\mathbf{k}-\mathbf{q}})}}\;G_{\mathbf{k},<}(\lambda,\lambda';t,t')$$
$$+n_F(\mathbf{k})e^{-i(t-t')(\epsilon_{\mathbf{k}}-\epsilon_{\mathbf{k}-\mathbf{q}})}\;G_{\mathbf{k}-\mathbf{q},>}(\lambda,\lambda';t,t')\;\;\left(\theta(t-t')-\frac{1}{1-e^{\beta(\epsilon_{\mathbf{k}}-\epsilon_{\mathbf{k}-\mathbf{q}})}}\right)\Bigg]$$

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We need to substitute this into the lesser than derived equation.

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Lesser than derived equation in terms $G_<$ and $G_>$is,

$$F_{\mathbf{k}<}(-\mathbf{q};t_1)\;=e^{-i(t_1-t)(\epsilon_{\mathbf{k}}-\epsilon_{\mathbf{k}-\mathbf{q}})}\Bigg[(e^{\lambda+\lambda'}-1)\;\frac{1-e^{\beta(\epsilon_{\mathbf{k}}-\epsilon_{\mathbf{k}-\mathbf{q}})}}{e^{\lambda+\lambda'}-e^{\beta(\epsilon_{\mathbf{k}}-\epsilon_{\mathbf{k}-\mathbf{q}})}}\Bigg((1-n_F(\mathbf{k}-\mathbf{q}))\;\frac{e^{\lambda+\lambda'}}{1-e^{\beta(\epsilon_{\mathbf{k}}-\epsilon_{\mathbf{k}-\mathbf{q}})}}\;G_{\mathbf{k},<}(\lambda,\lambda';t,t')$$
$$+n_F(\mathbf{k})e^{-i(t-t')(\epsilon_{\mathbf{k}}-\epsilon_{\mathbf{k}-\mathbf{q}})}\;G_{\mathbf{k}-\mathbf{q},>}(\lambda,\lambda';t,t')\;\;\left(\theta(t-t')-\frac{1}{1-e^{\beta(\epsilon_{\mathbf{k}}-\epsilon_{\mathbf{k}-\mathbf{q}})}}\right)\Bigg)$$
$$-(1-n_F(\mathbf{k}-\mathbf{q}))\;e^{\lambda+\lambda'}\;\;G_{\mathbf{k},<}(\lambda,\lambda';t,t')\;\Bigg]\;\left(\theta(t_1-t)-\frac{1}{1-e^{\beta(\epsilon_{\mathbf{k}}-\epsilon_{\mathbf{k}-\mathbf{q}})}}\right)$$
$$+n_F(\mathbf{k})e^{-i(t_1-t')(\epsilon_{\mathbf{k}}-\epsilon_{\mathbf{k}-\mathbf{q}})}\;G_{\mathbf{k}-\mathbf{q},>}(\lambda,\lambda';t,t')\;\;\left(\theta(t_1-t')-\frac{1}{1-e^{\beta(\epsilon_{\mathbf{k}}-\epsilon_{\mathbf{k}-\mathbf{q}})}}\right)$$

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5.2 $F_>(q;t_1)$ derived simplified equation

Similarly, in $\lim t_1 \to t^-$, the greater than derived equation is,

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$$F_{\mathbf{k}>}(\mathbf{q};t) \ = \left(-\frac{1}{1-e^{\beta(\epsilon_{\mathbf{k}}-\epsilon_{\mathbf{k-q}})}}\right)\left[(e^{-\lambda-\lambda^{'}}-1)\ F_{\mathbf{k}>}(\mathbf{q};t)\ -n_F(\mathbf{k-q})\ e^{-\lambda-\lambda^{'}}G_{\mathbf{k},>}(\lambda,\lambda^{'};t,t^{'})\ \right]$$
$$+e^{-i(t-t')(\epsilon_{\mathbf{k}}-\epsilon_{\mathbf{k-q}})}G_{\mathbf{k-q},<}(\lambda,\lambda^{'};t,t^{'})\ (1-n_F(\mathbf{k}))\ \left(\theta(t-t')-\frac{1}{1-e^{\beta(\epsilon_{\mathbf{k}}-\epsilon_{\mathbf{k-q}})}}\right).$$

Taking $F_{\mathbf{k}>}(\mathbf{q};t)$ to the right hand side,

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$F_{\mathbf{k}>}(\mathbf{q};t)$ is:

$$F_{\mathbf{k}>}(\mathbf{q};t) \ = \left(\frac{1-e^{\beta(\epsilon_{\mathbf{k}}-\epsilon_{\mathbf{k-q}})}}{e^{-\lambda-\lambda'}-e^{\beta(\epsilon_{\mathbf{k}}-\epsilon_{\mathbf{k-q}})}}\right)\left[n_F(\mathbf{k-q})\ e^{-\lambda-\lambda^{'}}G_{\mathbf{k},>}(\lambda,\lambda^{'};t,t^{'})\right.$$
$$\left.+e^{-i(t-t')(\epsilon_{\mathbf{k}}-\epsilon_{\mathbf{k-q}})}G_{\mathbf{k-q},<}(\lambda,\lambda^{'};t,t^{'})\ (1-n_F(\mathbf{k}))\ \left(\theta(t-t')-\frac{1}{1-e^{\beta(\epsilon_{\mathbf{k}}-\epsilon_{\mathbf{k-q}})}}\right)\right].$$

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We need to substitute this into the greater than derived equation.

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Greater than derived equation in terms $G_<$ and $G_>$is,

$$F_{\mathbf{k}>}(\mathbf{q};t_1) \ = e^{-i(t_1-t)(\epsilon_{\mathbf{k}}-\epsilon_{\mathbf{k-q}})}\left[(e^{-\lambda-\lambda^{'}}-1)\ \left(\left(\frac{1-e^{\beta(\epsilon_{\mathbf{k}}-\epsilon_{\mathbf{k-q}})}}{e^{-\lambda-\lambda'}-e^{\beta(\epsilon_{\mathbf{k}}-\epsilon_{\mathbf{k-q}})}}\right)\left[n_F(\mathbf{k-q})\ e^{-\lambda-\lambda^{'}}G_{\mathbf{k},>}(\lambda,\lambda^{'};t,t^{'})\right.\right.\right.$$
$$\left.\left.+e^{-i(t-t')(\epsilon_{\mathbf{k}}-\epsilon_{\mathbf{k-q}})}G_{\mathbf{k-q},<}(\lambda,\lambda^{'};t,t^{'})\ (1-n_F(\mathbf{k}))\ \left(\theta(t-t')-\frac{1}{1-e^{\beta(\epsilon_{\mathbf{k}}-\epsilon_{\mathbf{k-q}})}}\right)\right]\right)$$
$$\left.-\ n_F(\mathbf{k-q})\ e^{-\lambda-\lambda^{'}}G_{\mathbf{k},>}(\lambda,\lambda^{'};t,t^{'})\ \right]\left(\theta(t_1-t)-\frac{1}{1-e^{\beta(\epsilon_{\mathbf{k}}-\epsilon_{\mathbf{k-q}})}}\right)$$
$$+e^{-i(t_1-t')(\epsilon_{\mathbf{k}}-\epsilon_{\mathbf{k-q}})}G_{\mathbf{k-q},<}(\lambda,\lambda^{'};t,t^{'})\ (1-n_F(\mathbf{k}))\ \left(\theta(t_1-t')-\frac{1}{1-e^{\beta(\epsilon_{\mathbf{k}}-\epsilon_{\mathbf{k-q}})}}\right).$$

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6 RHS of F less equation

$$e^{\frac{i\mathbf{q}.(-2\mathbf{k}t_1+\mathbf{q}(t_1+i\beta))}{2m}}(f_{max}\ e^{\frac{\beta\mathbf{k}.\mathbf{q}}{m}}+in_F(\mathbf{k})\ G_{\mathbf{k}-\mathbf{q},>}(\lambda,\lambda^{'};t,t^{'})\ i\ \theta(t^{'}-t_1)\ e^{\frac{\mathbf{q}.(2i\mathbf{k}t^{'}+\mathbf{q}(\beta-it^{'}))}{2m}})\ \theta(t_1-t)$$
$$+\ e^{-\frac{it_1\mathbf{q}.(2\mathbf{k}-\mathbf{q})}{2m}}(f_{max}-in_F(\mathbf{k})\ G_{\mathbf{k}-\mathbf{q},>}(\lambda,\lambda^{'};t,t^{'})\ i\ \theta(t_1-t^{'})\ e^{\frac{it^{'}\mathbf{q}.(2\mathbf{k}-\mathbf{q})}{2m}})\ \theta(t-t_1)$$

where,

$$f_{max} = \frac{G_{\mathbf{k}-\mathbf{q},>}(\lambda,\lambda^{'};t,t^{'})\ \ n_F(\mathbf{k})e^{\frac{it^{'}(2\mathbf{k}.\mathbf{q}-q^2)}{2m}}\ \left(e^{\lambda+\lambda^{'}}\theta(t-t^{'})+\theta(t^{'}-t)\right)}{e^{\frac{\beta(2\mathbf{k}.\mathbf{q}-q^2)}{2m}}-e^{\lambda+\lambda'}}$$
$$+\ \frac{G_{\mathbf{k},<}(\lambda,\lambda^{'};t,t^{'})\ \ (n_F(\mathbf{k}-\mathbf{q})-1)e^{\frac{it(2\mathbf{k}.\mathbf{q}-q^2)}{2m}+\lambda+\lambda^{'}}}{e^{\frac{\beta(2\mathbf{k}.\mathbf{q}-q^2)}{2m}}-e^{\lambda+\lambda'}}$$

The RHS is

$$e^{\frac{i\mathbf{q}.(-2\mathbf{k}t_1+\mathbf{q}(t_1+i\beta))}{2m}}\Bigg[\Bigg(\frac{G_{\mathbf{k}-\mathbf{q},>}(\lambda,\lambda^{'};t,t^{'})\ \ n_F(\mathbf{k})e^{\frac{it^{'}(2\mathbf{k}.\mathbf{q}-q^2)}{2m}}\ \left(e^{\lambda+\lambda^{'}}\theta(t-t^{'})+\theta(t^{'}-t)\right)}{e^{\frac{\beta(2\mathbf{k}.\mathbf{q}-q^2)}{2m}}-e^{\lambda+\lambda'}}$$
$$+\ \frac{G_{\mathbf{k},<}(\lambda,\lambda^{'};t,t^{'})\ \ (n_F(\mathbf{k}-\mathbf{q})-1)e^{\frac{it(2\mathbf{k}.\mathbf{q}-q^2)}{2m}+\lambda+\lambda^{'}}}{e^{\frac{\beta(2\mathbf{k}.\mathbf{q}-q^2)}{2m}}-e^{\lambda+\lambda'}}\Bigg)\ e^{\frac{\beta\mathbf{k}.\mathbf{q}}{m}}$$
$$+in_F(\mathbf{k})\ G_{\mathbf{k}-\mathbf{q},>}(\lambda,\lambda^{'};t,t^{'})\ i\ \theta(t^{'}-t_1)\ e^{\frac{\mathbf{q}.(2i\mathbf{k}t^{'}+\mathbf{q}(\beta-it^{'}))}{2m}}\Bigg]\ \theta(t_1-t)$$
$$+\ e^{-\frac{it_1\mathbf{q}.(2\mathbf{k}-\mathbf{q})}{2m}}\Bigg[\Bigg(\frac{G_{\mathbf{k}-\mathbf{q},>}(\lambda,\lambda^{'};t,t^{'})\ \ n_F(\mathbf{k})e^{\frac{it^{'}(2\mathbf{k}.\mathbf{q}-q^2)}{2m}}\ \left(e^{\lambda+\lambda^{'}}\theta(t-t^{'})+\theta(t^{'}-t)\right)}{e^{\frac{\beta(2\mathbf{k}.\mathbf{q}-q^2)}{2m}}-e^{\lambda+\lambda'}}$$
$$+\ \frac{G_{\mathbf{k},<}(\lambda,\lambda^{'};t,t^{'})\ \ (n_F(\mathbf{k}-\mathbf{q})-1)e^{\frac{it(2\mathbf{k}.\mathbf{q}-q^2)}{2m}+\lambda+\lambda^{'}}}{e^{\frac{\beta(2\mathbf{k}.\mathbf{q}-q^2)}{2m}}-e^{\lambda+\lambda'}}\Bigg)$$
$$-in_F(\mathbf{k})\ G_{\mathbf{k}-\mathbf{q},>}(\lambda,\lambda^{'};t,t^{'})\ i\ \theta(t_1-t^{'})\ e^{\frac{it^{'}\mathbf{q}.(2\mathbf{k}-\mathbf{q})}{2m}}\Bigg]\ \theta(t-t_1)$$

7 RHS of F greater equation

$$e^{\frac{i\mathbf{q}.(-2\mathbf{k}t_1+\mathbf{q}(t_1+i\beta))}{2m}}(g_{max}\ e^{\frac{\beta\mathbf{k}.\mathbf{q}}{m}}+i(1-n_F(\mathbf{k}))\ G_{\mathbf{k}-\mathbf{q},<}(\lambda,\lambda^{'};t,t^{'})\ \ i\ \theta(t^{'}-t_1)e^{\frac{(2i\mathbf{k}.\mathbf{q}t^{'}+\mathbf{q}^2(\beta-it^{'}))}{2m}})\ \theta(t_1-t)$$
$$+\ e^{-\frac{it_1\mathbf{q}.(2\mathbf{k}-\mathbf{q})}{2m}}(g_{max}-i(1-n_F(\mathbf{k}))\ G_{\mathbf{k}-\mathbf{q},<}(\lambda,\lambda^{'};t,t^{'})\ \ i\ \theta(t_1-t^{'})\ e^{\frac{it^{'}\mathbf{q}.(2\mathbf{k}-\mathbf{q})}{2m}})\ \theta(t-t_1)$$

where,

$$g_{max}\ =\ \frac{e^{-\frac{iq^2t^{'}}{2m}}\ G_{\mathbf{k}-\mathbf{q},<}(\lambda,\lambda^{'};t,t^{'})\ \ (n_F(\mathbf{k})-1)\ \left(-e^{\frac{i\mathbf{k}.\mathbf{q}}{m}t^{'}}\right)\left(e^{\lambda+\lambda'}\ \theta(t^{'}-t)+\theta(t-t^{'})\right)}{e^{\frac{\beta(2\mathbf{k}.\mathbf{q}-q^2)}{2m}+\lambda+\lambda'}-1}$$
$$+\ \frac{-e^{-\frac{iq^2t^{'}}{2m}}\ G_{\mathbf{k},>}(\lambda,\lambda^{'};t,t^{'})\ n_F(\mathbf{k}-\mathbf{q})\ e^{\frac{i(2\mathbf{k}.\mathbf{q}t+q^2(t^{'}-t))}{2m}}}{e^{\frac{\beta(2\mathbf{k}.\mathbf{q}-q^2)}{2m}+\lambda+\lambda'}-1}$$

The RHS is,

$$e^{\frac{i\mathbf{q}.(-2\mathbf{k}t_1+\mathbf{q}(t_1+i\beta))}{2m}}\Bigg[\Bigg(\frac{e^{-\frac{iq^2t^{'}}{2m}}\ G_{\mathbf{k}-\mathbf{q},<}(\lambda,\lambda^{'};t,t^{'})\ \ (n_F(\mathbf{k})-1)\ \left(-e^{\frac{i\mathbf{k}.\mathbf{q}}{m}t^{'}}\right)\left(e^{\lambda+\lambda'}\ \theta(t^{'}-t)+\theta(t-t^{'})\right)}{e^{\frac{\beta(2\mathbf{k}.\mathbf{q}-q^2)}{2m}+\lambda+\lambda'}-1}$$
$$+\ \frac{-e^{-\frac{iq^2t^{'}}{2m}}\ G_{\mathbf{k},>}(\lambda,\lambda^{'};t,t^{'})\ n_F(\mathbf{k}-\mathbf{q})\ e^{\frac{i(2\mathbf{k}.\mathbf{q}t+q^2(t^{'}-t))}{2m}}}{e^{\frac{\beta(2\mathbf{k}.\mathbf{q}-q^2)}{2m}+\lambda+\lambda'}-1}\Bigg)e^{\frac{\beta\mathbf{k}.\mathbf{q}}{m}}$$
$$+i(1-n_F(\mathbf{k}))\ G_{\mathbf{k}-\mathbf{q},<}(\lambda,\lambda^{'};t,t^{'})\ \ i\ \theta(t^{'}-t_1)e^{\frac{(2i\mathbf{k}.\mathbf{q}t^{'}+\mathbf{q}^2(\beta-it^{'}))}{2m}}\Bigg]\ \theta(t_1-t)$$
$$+\ e^{-\frac{it_1\mathbf{q}.(2\mathbf{k}-\mathbf{q})}{2m}}\Bigg[\Bigg(\frac{e^{-\frac{iq^2t^{'}}{2m}}\ G_{\mathbf{k}-\mathbf{q},<}(\lambda,\lambda^{'};t,t^{'})\ \ (n_F(\mathbf{k})-1)\ \left(-e^{\frac{i\mathbf{k}.\mathbf{q}}{m}t^{'}}\right)\left(e^{\lambda+\lambda'}\ \theta(t^{'}-t)+\theta(t-t^{'})\right)}{e^{\frac{\beta(2\mathbf{k}.\mathbf{q}-q^2)}{2m}+\lambda+\lambda'}-1}$$
$$+\ \frac{-e^{-\frac{iq^2t^{'}}{2m}}\ G_{\mathbf{k},>}(\lambda,\lambda^{'};t,t^{'})\ n_F(\mathbf{k}-\mathbf{q})\ e^{\frac{i(2\mathbf{k}.\mathbf{q}t+q^2(t^{'}-t))}{2m}}}{e^{\frac{\beta(2\mathbf{k}.\mathbf{q}-q^2)}{2m}+\lambda+\lambda'}-1}\Bigg)$$
$$-i(1-n_F(\mathbf{k}))\ G_{\mathbf{k}-\mathbf{q},<}(\lambda,\lambda^{'};t,t^{'})\ \ i\ \theta(t_1-t^{'})\ e^{\frac{it^{'}\mathbf{q}.(2\mathbf{k}-\mathbf{q})}{2m}}\Bigg]\ \theta(t-t_1)$$

8 Verification of lesser than derived equation and Eq. (28) after substituting f max

We need to show,

$$
\begin{aligned}
& e^{-i(t_1-t)(\epsilon_{\mathbf{k}}-\epsilon_{\mathbf{k-q}})}\Bigg[(e^{\lambda+\lambda'}-1)\ \frac{1-e^{\beta(\epsilon_{\mathbf{k}}-\epsilon_{\mathbf{k-q}})}}{e^{\lambda+\lambda'}-e^{\beta(\epsilon_{\mathbf{k}}-\epsilon_{\mathbf{k-q}})}}\Bigg((1-n_F(\mathbf{k-q}))\ \frac{e^{\lambda+\lambda'}}{1-e^{\beta(\epsilon_{\mathbf{k}}-\epsilon_{\mathbf{k-q}})}}\ G_{\mathbf{k},<}(\lambda,\lambda';t,t') \\
& \quad + n_F(\mathbf{k})e^{-i(t-t')(\epsilon_{\mathbf{k}}-\epsilon_{\mathbf{k-q}})}\ G_{\mathbf{k-q},>}(\lambda,\lambda';t,t')\ \left(\theta(t-t')-\frac{1}{1-e^{\beta(\epsilon_{\mathbf{k}}-\epsilon_{\mathbf{k-q}})}}\right)\Bigg) \\
& \quad - (1-n_F(\mathbf{k-q}))\ e^{\lambda+\lambda'}\ \ G_{\mathbf{k},<}(\lambda,\lambda';t,t')\ \Bigg]\ \left(\theta(t_1-t)-\frac{1}{1-e^{\beta(\epsilon_{\mathbf{k}}-\epsilon_{\mathbf{k-q}})}}\right) \\
& \quad + n_F(\mathbf{k})e^{-i(t_1-t')(\epsilon_{\mathbf{k}}-\epsilon_{\mathbf{k-q}})}\ G_{\mathbf{k-q},>}(\lambda,\lambda';t,t')\ \left(\theta(t_1-t')-\frac{1}{1-e^{\beta(\epsilon_{\mathbf{k}}-\epsilon_{\mathbf{k-q}})}}\right)
\end{aligned}
$$

and

$$
\begin{aligned}
& e^{\frac{i\mathbf{q}.(-2\mathbf{k}t_1+\mathbf{q}(t_1+i\beta))}{2m}}\Bigg[\Bigg(\frac{G_{\mathbf{k-q},>}(\lambda,\lambda';t,t')\ \ n_F(\mathbf{k})e^{\frac{it'(2\mathbf{k}.\mathbf{q}-q^2)}{2m}}\ \left(e^{\lambda+\lambda'}\theta(t-t')+\theta(t'-t)\right)}{e^{\frac{\beta(2\mathbf{k}.\mathbf{q}-q^2)}{2m}}-e^{\lambda+\lambda'}} \\
& \quad + \ \frac{G_{\mathbf{k},<}(\lambda,\lambda';t,t')\ \ (n_F(\mathbf{k-q})-1)e^{\frac{it(2\mathbf{k}.\mathbf{q}-q^2)}{2m}+\lambda+\lambda'}}{e^{\frac{\beta(2\mathbf{k}.\mathbf{q}-q^2)}{2m}}-e^{\lambda+\lambda'}}\Bigg)\ e^{\frac{\beta\mathbf{k}.\mathbf{q}}{m}} \\
& \quad + in_F(\mathbf{k})\ G_{\mathbf{k-q},>}(\lambda,\lambda';t,t')\ i\ \theta(t'-t_1)\ e^{\frac{\mathbf{q}.(2i\mathbf{k}t'+\mathbf{q}(\beta-it'))}{2m}}\Bigg]\ \theta(t_1-t) \\
& + \ e^{-\frac{it_1\mathbf{q}.(2\mathbf{k}-\mathbf{q})}{2m}}\Bigg[\Bigg(\frac{G_{\mathbf{k-q},>}(\lambda,\lambda';t,t')\ \ n_F(\mathbf{k})e^{\frac{it'(2\mathbf{k}.\mathbf{q}-q^2)}{2m}}\ \left(e^{\lambda+\lambda'}\theta(t-t')+\theta(t'-t)\right)}{e^{\frac{\beta(2\mathbf{k}.\mathbf{q}-q^2)}{2m}}-e^{\lambda+\lambda'}} \\
& \quad + \ \frac{G_{\mathbf{k},<}(\lambda,\lambda';t,t')\ \ (n_F(\mathbf{k-q})-1)e^{\frac{it(2\mathbf{k}.\mathbf{q}-q^2)}{2m}+\lambda+\lambda'}}{e^{\frac{\beta(2\mathbf{k}.\mathbf{q}-q^2)}{2m}}-e^{\lambda+\lambda'}}\Bigg) \\
& \quad - in_F(\mathbf{k})\ G_{\mathbf{k-q},>}(\lambda,\lambda';t,t')\ i\ \theta(t_1-t')\ e^{\frac{it'\mathbf{q}.(2\mathbf{k}-\mathbf{q})}{2m}}\Bigg]\ \theta(t-t_1)
\end{aligned}
$$

are the same.

9 Verification of greater than derived equation and Eq. (29) after substituting g max

We need to show,

$$e^{-i(t_1-t)(\epsilon_{\mathbf{k}}-\epsilon_{\mathbf{k-q}})}\Bigg[(e^{-\lambda-\lambda^{'}}-1)\ \Bigg(\left(\frac{1-e^{\beta(\epsilon_{\mathbf{k}}-\epsilon_{\mathbf{k-q}})}}{e^{-\lambda-\lambda^{'}}-e^{\beta(\epsilon_{\mathbf{k}}-\epsilon_{\mathbf{k-q}})}}\right)\Bigg[n_F(\mathbf{k-q})\ e^{-\lambda-\lambda^{'}}G_{\mathbf{k},>}(\lambda,\lambda^{'};t,t^{'})$$

$$+e^{-i(t-t')(\epsilon_{\mathbf{k}}-\epsilon_{\mathbf{k-q}})}G_{\mathbf{k-q},<}(\lambda,\lambda^{'};t,t^{'})\ (1-n_F(\mathbf{k}))\ \left(\theta(t-t')-\frac{1}{1-e^{\beta(\epsilon_{\mathbf{k}}-\epsilon_{\mathbf{k-q}})}}\right)\Bigg]\Bigg)$$

$$-\,n_F(\mathbf{k-q})\ e^{-\lambda-\lambda^{'}}G_{\mathbf{k},>}(\lambda,\lambda^{'};t,t^{'})\ \Bigg]\left(\theta(t_1-t)-\frac{1}{1-e^{\beta(\epsilon_{\mathbf{k}}-\epsilon_{\mathbf{k-q}})}}\right)$$

$$+e^{-i(t_1-t')(\epsilon_{\mathbf{k}}-\epsilon_{\mathbf{k-q}})}G_{\mathbf{k-q},<}(\lambda,\lambda^{'};t,t^{'})\ (1-n_F(\mathbf{k}))\ \left(\theta(t_1-t')-\frac{1}{1-e^{\beta(\epsilon_{\mathbf{k}}-\epsilon_{\mathbf{k-q}})}}\right).$$

and,

$$e^{\frac{i\mathbf{q}.(-2\mathbf{k}t_1+\mathbf{q}(t_1+i\beta))}{2m}}\Bigg[\Bigg(\frac{e^{-\frac{iq^2t^{'}}{2m}}\ G_{\mathbf{k-q},<}(\lambda,\lambda^{'};t,t^{'})\ \ (n_F(\mathbf{k})-1)\ \left(-e^{\frac{i\mathbf{k}.\mathbf{q}}{m}t^{'}}\right)\left(e^{\lambda+\lambda^{'}}\ \theta(t^{'}-t)+\theta(t-t^{'})\right)}{e^{\frac{\beta(2\mathbf{k}.\mathbf{q}-q^2)}{2m}+\lambda+\lambda^{'}}-1}$$

$$+\ \ \frac{-e^{-\frac{iq^2t^{'}}{2m}}\ G_{\mathbf{k},>}(\lambda,\lambda^{'};t,t^{'})\ n_F(\mathbf{k-q})\ e^{\frac{i(2\mathbf{k}.\mathbf{q}t+q^2(t^{'}-t))}{2m}}}{e^{\frac{\beta(2\mathbf{k}.\mathbf{q}-q^2)}{2m}+\lambda+\lambda^{'}}-1}\Bigg)e^{\frac{\beta\mathbf{k}.\mathbf{q}}{m}}$$

$$+\,i(1-n_F(\mathbf{k}))\ G_{\mathbf{k-q},<}(\lambda,\lambda^{'};t,t^{'})\ \ i\ \theta(t^{'}-t_1)e^{\frac{(2i\mathbf{k}.\mathbf{q}t^{'}+\mathbf{q}^2(\beta-it^{'}))}{2m}}\Bigg]\ \theta(t_1-t)$$

$$+\ \ e^{-\frac{it_1\mathbf{q}.(2\mathbf{k}-\mathbf{q})}{2m}}\Bigg[\Bigg(\frac{e^{-\frac{iq^2t^{'}}{2m}}\ G_{\mathbf{k-q},<}(\lambda,\lambda^{'};t,t^{'})\ \ (n_F(\mathbf{k})-1)\ \left(-e^{\frac{i\mathbf{k}.\mathbf{q}}{m}t^{'}}\right)\left(e^{\lambda+\lambda^{'}}\ \theta(t^{'}-t)+\theta(t-t^{'})\right)}{e^{\frac{\beta(2\mathbf{k}.\mathbf{q}-q^2)}{2m}+\lambda+\lambda^{'}}-1}$$

$$+\ \ \frac{-e^{-\frac{iq^2t^{'}}{2m}}\ G_{\mathbf{k},>}(\lambda,\lambda^{'};t,t^{'})\ n_F(\mathbf{k-q})\ e^{\frac{i(2\mathbf{k}.\mathbf{q}t+q^2(t^{'}-t))}{2m}}}{e^{\frac{\beta(2\mathbf{k}.\mathbf{q}-q^2)}{2m}+\lambda+\lambda^{'}}-1}\Bigg)$$

$$-i(1-n_F(\mathbf{k}))\ G_{\mathbf{k-q},<}(\lambda,\lambda^{'};t,t^{'})\ \ i\ \theta(t_1-t^{'})\ e^{\frac{it^{'}\mathbf{q}.(2\mathbf{k}-\mathbf{q})}{2m}}\Bigg]\ \theta(t-t_1)$$

are the same.

10 $\lim t_1 \to t^-$ LHS of lesser than and greater than equations

10.1 $\lim t_1 \to t^-$ LHS of the lesser than equation

Now, we take $t_1 \to t^-$. Expanding $a^\dagger_{\mathbf{k}-\mathbf{q}/2}(-\mathbf{q}, t)$, taking a sum over $\mathbf{q}$ and writing in terms of $N^{'}_{>}$, the LHS of the lesser than equation becomes,

$$\sum_q < T\ e^{-\lambda N_>(t)}\ e^{-\lambda^{'} N^{'}_>(t)}\ c_{\mathbf{k}-\mathbf{q},>}(t) a^\dagger_{\mathbf{k}-\mathbf{q}/2}(-\mathbf{q}, t)\ c^\dagger_{\mathbf{k},<}(t^{'}) >$$

$$= \sum_q < T\ e^{-\lambda N_>(t)}\ e^{-\lambda^{'} N^{'}_>(t)}\ c_{\mathbf{k}-\mathbf{q},>}(t) c^\dagger_{\mathbf{k}-\mathbf{q},>}(t)\ c_{\mathbf{k},<}(t) c^\dagger_{\mathbf{k},<}(t^{'}) >$$

$$= \sum_q (1 - n_F(\mathbf{k}-\mathbf{q})) < T\ e^{-\lambda N_>(t)}\ e^{-\lambda^{'} N^{'}_>(t)}\ c_{\mathbf{k},<}(t) c^\dagger_{\mathbf{k},<}(t^{'}) >$$

$$- < T\ e^{-\lambda N_>(t)}\ e^{-\lambda^{'} N^{'}_>(t)}\ N^{'}_>(t) c_{\mathbf{k},<}(t) c^\dagger_{\mathbf{k},<}(t^{'}) >$$

==

Hence, the LHS of the lesser than equation is:

$$\sum_q < T\ e^{-\lambda N_>(t)}\ e^{-\lambda^{'} N^{'}_>(t)}\ c_{\mathbf{k}-\mathbf{q},>}(t) a^\dagger_{\mathbf{k}-\mathbf{q}/2}(-\mathbf{q}, t)\ c^\dagger_{\mathbf{k},<}(t^{'}) >$$

$$= \sum_q (1 - n_F(\mathbf{k}-\mathbf{q})) G_{\mathbf{k},<}(\lambda, \lambda^{'}; t, t^{'})\ + \partial_{\lambda^{'}} G_{\mathbf{k},<}(\lambda, \lambda^{'}; t, t^{'})$$

==

10.2 $\lim t_1 \to t^-$ LHS of the greater than equation

Similarly, the LHS of the greater than equation becomes,

$$\sum_q < T\ e^{-\lambda N_>(t)}\ e^{-\lambda^{'} N^{'}_>(t)}\ c_{\mathbf{k}-\mathbf{q},<}(t) a_{\mathbf{k}-\mathbf{q}/2}(\mathbf{q}, t)\ c^\dagger_{\mathbf{k},>}(t^{'}) >$$

$$= \sum_q < T\ e^{-\lambda N_>(t)}\ e^{-\lambda^{'} N^{'}_>(t)}\ c_{\mathbf{k}-\mathbf{q},<}(t) c^\dagger_{\mathbf{k}-\mathbf{q},<}(t)\ c_{\mathbf{k},>}(t) c^\dagger_{\mathbf{k},>}(t^{'}) >$$

$$= < T\ e^{-\lambda N_>(t)}\ e^{-\lambda^{'} N^{'}_>(t)}\ N_>(t)\ c_{\mathbf{k},>}(t) c^\dagger_{\mathbf{k},>}(t^{'}) >$$

==

Hence, the LHS of the greater than equation is:

$$\sum_q < T\ e^{-\lambda N_>(t)}\ e^{-\lambda^{'} N^{'}_>(t)}\ c_{\mathbf{k}-\mathbf{q},<}(t) a_{\mathbf{k}-\mathbf{q}/2}(\mathbf{q}, t)\ c^\dagger_{\mathbf{k},>}(t^{'}) >$$

$$= -\partial_\lambda G_{\mathbf{k},>}(\lambda, \lambda^{'}; t, t^{'})$$

==

11 $\lim t_1 \to t^-$ RHS of the lesser than and greater than equations

11.1 $\lim t_1 \to t^-$ RHS of the lesser than equation

Take $t_1 \to t^-$. Taking a sum over $\mathbf{q}$, makes the RHS terms in the lesser than equation,

$$e^{\frac{i\mathbf{q}.(-2\mathbf{k}t_1+\mathbf{q}(t_1+i\beta))}{2m}}(f_{max}\ e^{\frac{\beta\mathbf{k}.\mathbf{q}}{m}} + in_F(\mathbf{k})\ G_{\mathbf{k-q},>}(\lambda,\lambda^{'};t,t^{'})\ i\ \theta(t^{'}-t_1)\ e^{\frac{\mathbf{q}.(2i\mathbf{k}t^{'}+\mathbf{q}(\beta-it^{'}))}{2m}})\ \theta(t_1-t)$$

$$+\ e^{-\frac{it_1\mathbf{q}.(2\mathbf{k}-\mathbf{q})}{2m}}(f_{max} - in_F(\mathbf{k})\ G_{\mathbf{k-q},>}(\lambda,\lambda^{'};t,t^{'})\ i\ \theta(t_1-t^{'})\ e^{\frac{it^{'}\mathbf{q}.(2\mathbf{k}-\mathbf{q})}{2m}})\ \theta(t-t_1),$$

now, become,

$$RHS = \sum_{\mathbf{q}} e^{-\frac{it_1\mathbf{q}.(2\mathbf{k}-\mathbf{q})}{2m}}(f_{max} - in_F(\mathbf{k})\ G_{\mathbf{k-q},>}(\lambda,\lambda^{'};t,t^{'})\ i\ \theta(t-t^{'})\ e^{\frac{it^{'}\mathbf{q}.(2\mathbf{k}-\mathbf{q})}{2m}})$$

$$= \sum_{\mathbf{q}} e^{-\frac{it\mathbf{q}.(2\mathbf{k}-\mathbf{q})}{2m}}\Bigg(\frac{G_{\mathbf{k-q},>}(\lambda,\lambda^{'};t,t^{'})\ \ n_F(\mathbf{k})e^{\frac{it^{'}(2\mathbf{k}.\mathbf{q}-q^2)}{2m}}\ \left(e^{\lambda+\lambda^{'}}\theta(t-t^{'})+\theta(t^{'}-t)\right)}{e^{\frac{\beta(2\mathbf{k}.\mathbf{q}-q^2)}{2m}}-e^{\lambda+\lambda^{'}}}$$

$$+\ \frac{G_{\mathbf{k},<}(\lambda,\lambda^{'};t,t^{'})\ \ (n_F(\mathbf{k}-\mathbf{q})-1)e^{\frac{it(2\mathbf{k}.\mathbf{q}-q^2)}{2m}+\lambda+\lambda^{'}}}{e^{\frac{\beta(2\mathbf{k}.\mathbf{q}-q^2)}{2m}}-e^{\lambda+\lambda^{'}}}$$

$$-in_F(\mathbf{k})\ G_{\mathbf{k-q},>}\lambda,\lambda^{'};t,t^{'})\ i\ \theta(t-t^{'})\ e^{\frac{it^{'}\mathbf{q}.(2\mathbf{k}-\mathbf{q})}{2m}}\Bigg).$$

$$= \sum_{\mathbf{q}} e^{-\frac{it\mathbf{q}.(2\mathbf{k}-\mathbf{q})}{2m}}\ \frac{G_{\mathbf{k-q},>}(\lambda,\lambda^{'};t,t^{'})\ \ n_F(\mathbf{k})e^{\frac{it^{'}(2\mathbf{k}.\mathbf{q}-q^2)}{2m}}\ \left(e^{\lambda+\lambda^{'}}\theta(t-t^{'})+\theta(t^{'}-t)\right)}{e^{\frac{\beta(2\mathbf{k}.\mathbf{q}-q^2)}{2m}}-e^{\lambda+\lambda^{'}}}$$

$$+\sum_{\mathbf{q}}\ \frac{G_{\mathbf{k},<}(\lambda,\lambda^{'};t,t^{'})\ \ (n_F(\mathbf{k}-\mathbf{q})-1)e^{\lambda+\lambda^{'}}}{e^{\frac{\beta(2\mathbf{k}.\mathbf{q}-q^2)}{2m}}-e^{\lambda+\lambda^{'}}}$$

$$+\sum_{\mathbf{q}} n_F(\mathbf{k})\ G_{\mathbf{k-q},>}(\lambda,\lambda^{'};t,t^{'})\ \ \theta(t-t^{'})\ e^{\frac{i(t^{'}-t)\mathbf{q}.(2\mathbf{k}-\mathbf{q})}{2m}}).$$

$$= \sum_{\mathbf{q}} e^{-\frac{it\mathbf{q}.(2\mathbf{k}-\mathbf{q})}{2m}}\ \frac{G_{\mathbf{k-q},>}(\lambda,\lambda^{'};t,t^{'})\ \ n_F(\mathbf{k})e^{\frac{it^{'}(2\mathbf{k}.\mathbf{q}-q^2)}{2m}}\ \left(e^{\frac{\beta(2\mathbf{k}.\mathbf{q}-q^2)}{2m}}\theta(t-t^{'})+\theta(t^{'}-t)\right)}{e^{\frac{\beta(2\mathbf{k}.\mathbf{q}-q^2)}{2m}}-e^{\lambda+\lambda^{'}}}$$

$$+\sum_{\mathbf{q}}\ \frac{G_{\mathbf{k},<}(\lambda,\lambda^{'};t,t^{'})\ \ (n_F(\mathbf{k}-\mathbf{q})-1)e^{\lambda+\lambda^{'}}}{e^{\frac{\beta(2\mathbf{k}.\mathbf{q}-q^2)}{2m}}-e^{\lambda+\lambda^{'}}}$$

==

Hence, RHS of the lesser than equation is:

$$RHS = \sum_{\mathbf{q}} e^{-\frac{it\mathbf{q}.(2\mathbf{k}-\mathbf{q})}{2m}}\ \frac{G_{\mathbf{k-q},>}(\lambda,\lambda^{'};t,t^{'})\ \ n_F(\mathbf{k})e^{\frac{it^{'}(2\mathbf{k}.\mathbf{q}-q^2)}{2m}}\ \left(e^{\frac{\beta(2\mathbf{k}.\mathbf{q}-q^2)}{2m}}\theta(t-t^{'})+\theta(t^{'}-t)\right)}{e^{\frac{\beta(2\mathbf{k}.\mathbf{q}-q^2)}{2m}}-e^{\lambda+\lambda^{'}}}$$

$$+\sum_{\mathbf{q}}\ \frac{G_{\mathbf{k},<}(\lambda,\lambda^{'};t,t^{'})\ \ (n_F(\mathbf{k}-\mathbf{q})-1)e^{\lambda+\lambda^{'}}}{e^{\frac{\beta(2\mathbf{k}.\mathbf{q}-q^2)}{2m}}-e^{\lambda+\lambda^{'}}}$$

==

12 $\lim t_1 \to t^-$ of the simplified lesser than equation

$$\sum_q (1-n_F(\mathbf{k}-\mathbf{q}))G_{\mathbf{k},<}(\lambda,\lambda^{'};t,t^{'}) \ + \partial_{\lambda^{'}} G_{\mathbf{k},<}(\lambda,\lambda^{'};t,t^{'})$$

$$= \sum_{\mathbf{q}} e^{-\frac{it\mathbf{q}.(2\mathbf{k}-\mathbf{q})}{2m}} \ \frac{G_{\mathbf{k}-\mathbf{q},>}(\lambda,\lambda^{'};t,t^{'}) \ \ n_F(\mathbf{k}) e^{\frac{it^{'}(2\mathbf{k}.\mathbf{q}-q^2)}{2m}} \ \ \left(e^{\frac{\beta(2\mathbf{k}.\mathbf{q}-q^2)}{2m}}\theta(t-t^{'})+\theta(t^{'}-t)\right)}{e^{\frac{\beta(2\mathbf{k}.\mathbf{q}-q^2)}{2m}} - e^{\lambda+\lambda^{'}}}$$

$$+ \sum_{\mathbf{q}} \ \frac{G_{\mathbf{k},<}(\lambda,\lambda^{'};t,t^{'}) \ \ (n_F(\mathbf{k}-\mathbf{q})-1)e^{\lambda+\lambda^{'}}}{e^{\frac{\beta(2\mathbf{k}.\mathbf{q}-q^2)}{2m}} - e^{\lambda+\lambda^{'}}}$$

Taking $\mathbf{p} \to \mathbf{k}-\mathbf{q}$, we verified the equations.

12.1 $\lim t_1 \to t^-$ of the RHS of the greater than equation

Take $t_1 \to t^-$. Taking a sum over $\mathbf{q}$, makes the RHS terms in the greater than equation,

$$e^{\frac{i\mathbf{q}.(-2\mathbf{k}t_1+\mathbf{q}(t_1+i\beta))}{2m}}(g_{max}\ e^{\frac{\beta\mathbf{k}.\mathbf{q}}{m}} + i(1-n_F(\mathbf{k}))\ G_{\mathbf{k}-\mathbf{q},<}(\lambda,\lambda^{'};t,t^{'})\ \ i\ \theta(t^{'}-t_1)e^{\frac{(2i\mathbf{k}.\mathbf{q}t^{'}+\mathbf{q}^2(\beta-it^{'}))}{2m}})\ \theta(t_1-t)$$

$$+\ e^{-\frac{it\mathbf{q}.(2\mathbf{k}-\mathbf{q})}{2m}}(g_{max} - i(1-n_F(\mathbf{k}))\ G_{\mathbf{k}-\mathbf{q},<}(\lambda,\lambda^{'};t,t^{'})\ \ i\ \theta(t_1-t^{'})\ e^{\frac{it^{'}\mathbf{q}.(2\mathbf{k}-\mathbf{q})}{2m}})\ \theta(t-t_1)$$

now, become,

$$RHS = \sum_{\mathbf{q}}\ e^{-\frac{it\mathbf{q}.(2\mathbf{k}-\mathbf{q})}{2m}}(g_{max} - i(1-n_F(\mathbf{k}))\ G_{\mathbf{k}-\mathbf{q},<}(\lambda,\lambda^{'};t,t^{'})\ \ i\ \theta(t-t^{'})\ e^{\frac{it^{'}\mathbf{q}.(2\mathbf{k}-\mathbf{q})}{2m}})$$

$$= \sum_{\mathbf{q}}\ e^{-\frac{it\mathbf{q}.(2\mathbf{k}-\mathbf{q})}{2m}}\Bigg(\frac{e^{-\frac{iq^2t^{'}}{2m}}\ G_{\mathbf{k}-\mathbf{q},<}(\lambda,\lambda^{'};t,t^{'})\ \ (n_F(\mathbf{k})-1)\ \left(-e^{\frac{i\mathbf{k}.\mathbf{q}}{m}t^{'}}\right)\left(e^{\lambda+\lambda^{'}}\ \theta(t^{'}-t)+\theta(t-t^{'})\right)}{e^{\frac{\beta(2\mathbf{k}.\mathbf{q}-q^2)}{2m}+\lambda+\lambda^{'}}-1}$$

$$+\ \frac{-e^{-\frac{iq^2t^{'}}{2m}}\ G_{\mathbf{k},>}(\lambda,\lambda^{'};t,t^{'})\ n_F(\mathbf{k}-\mathbf{q})\ e^{\frac{i(2\mathbf{k}.\mathbf{q}t+q^2(t^{'}-t))}{2m}}}{e^{\frac{\beta(2\mathbf{k}.\mathbf{q}-q^2)}{2m}+\lambda+\lambda^{'}}-1}$$

$$-i(1-n_F(\mathbf{k}))\ G_{\mathbf{k}-\mathbf{q},<}(\lambda,\lambda^{'};t,t^{'})\ \ i\ \theta(t-t^{'})\ e^{\frac{it^{'}\mathbf{q}.(2\mathbf{k}-\mathbf{q})}{2m}}\Bigg)$$

13 of $\lim t_1 \to t^-$ of the simplified greater than equation

$$-\partial_\lambda G_{\mathbf{k},>}(\lambda,\lambda^{'};t,t^{'})$$

$$= \sum_{\mathbf{q}}\ e^{-\frac{it\mathbf{q}.(2\mathbf{k}-\mathbf{q})}{2m}}\Bigg(\frac{e^{-\frac{iq^2t^{'}}{2m}}\ G_{\mathbf{k}-\mathbf{q},<}(\lambda,\lambda^{'};t,t^{'})\ \ (n_F(\mathbf{k})-1)\ \left(-e^{\frac{i\mathbf{k}.\mathbf{q}}{m}t^{'}}\right)\left(e^{\lambda+\lambda^{'}}\ \theta(t^{'}-t)+\theta(t-t^{'})\right)}{e^{\frac{\beta(2\mathbf{k}.\mathbf{q}-q^2)}{2m}+\lambda+\lambda^{'}}-1}$$

$$+\ \frac{-e^{-\frac{iq^2t^{'}}{2m}}\ G_{\mathbf{k},>}(\lambda,\lambda^{'};t,t^{'})\ n_F(\mathbf{k}-\mathbf{q})\ e^{\frac{i(2\mathbf{k}.\mathbf{q}t+q^2(t^{'}-t))}{2m}}}{e^{\frac{\beta(2\mathbf{k}.\mathbf{q}-q^2)}{2m}+\lambda+\lambda^{'}}-1}$$

$$+(1-n_F(\mathbf{k}))\ G_{\mathbf{k}-\mathbf{q},<}(\lambda,\lambda^{'};t,t^{'})\ \ \theta(t-t^{'})\ e^{\frac{it^{'}\mathbf{q}.(2\mathbf{k}-\mathbf{q})}{2m}}\Bigg)$$