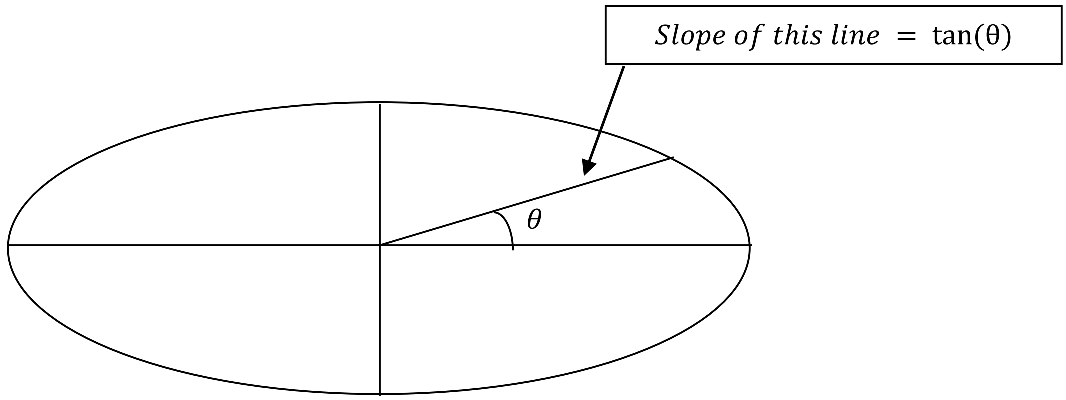
Partial Solution to ellipsoid Problem

Given:

- user HAS to specify a φ value of $\frac{\pi}{2}$ **or** π
- User specifies a value for θ with the idea of spherical coordinates in mind



Equation of the line from the origin

- $y = \tan(\theta)\, x$
- $\frac{1}{x} = \frac{\tan(\theta)}{y}$
- $x = \frac{y}{\tan(\theta)}$ Lower bound for x

Equation of ellipse in $xy\ plane$:

- $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Solving the equation for x in the $xy\ plane$:

- $\frac{x^2}{a^2} = 1 - \frac{y^2}{b^2}$
- $x = \pm a\sqrt{1 - \frac{y^2}{b^2}}$ Upper bound for x

Solving for x bounds:

- $x = m(y^2) + a$
- $0 = m(b^2) + a$ When x is $0, y$ is b^2
- $-\frac{a}{b^2} = m$ Solve for m
- $x = -\frac{a}{b^2}(y^2) + a$

Setting the equations equal to each other to find the $y\ value$ where the two equations intersect

- $\frac{y}{\tan(\theta)} = a\sqrt{1 - \frac{y^2}{b^2}}$
- $\frac{y^2}{\tan^2(\theta)} = a^2\left(1 - \frac{y^2}{b^2}\right)$
- $\frac{y^2}{\mathrm{a}^2\tan^2(\theta)} = \left(1 - \frac{y^2}{b^2}\right)$
- $\frac{y^2}{\mathrm{a}^2\tan^2(\theta)} + \frac{y^2}{b^2} = 1$
- $y^2\left(\frac{1}{a^2\tan^2(\theta)} + \frac{1}{b^2}\right) = 1$
- $y^2\left(\frac{b^2}{a^2b^2\tan^2(\theta)} + \frac{a^2\tan^2(\theta)}{a^2b^2\tan^2(\theta)}\right) = 1$
- $y^2 = \frac{a^2b^2\tan^2(\theta)}{b^2+a^2\tan^2(\theta)}$
- $y = \frac{ab(\tan(\theta))}{\sqrt{b^2+a^2\tan^2(\theta)}}$ Upper bound for y

Triple integral:

$$\int_0^{\frac{ab(\tan(\theta))}{\sqrt{b^2+a^2\tan^2(\theta)}}} \int_{\frac{y}{\tan(\theta)}}^{-\frac{a}{b^2}(y^2)+a} \int_0^{c\sqrt{1-\frac{x^2}{a^2}-\frac{y^2}{b^2}}} dzdxdy$$

This definite integral will be the algorithm