A different way to arrive at the same algorithm:

Equation of an ellipsoid: $$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$$

Transformation into u, v and w space:

- $u=\frac{x}{a}, v=\frac{y}{b}, w=\frac{z}{c}$ *which yields the equation* $u^2+v^2+w^2=1$

This is now the equation of a sphere with a radius of 1 in u, v and w space.

Solve the equations for x, y *and* z you have:

$$ua=x, \qquad vb=y \quad and \quad wc=z$$

Now for the jacobian transformation:

- $$\begin{bmatrix} \frac{\partial x}{\partial u} & \frac{\partial y}{\partial u} & \frac{\partial z}{\partial u} \\ \frac{\partial x}{\partial v} & \frac{\partial y}{\partial v} & \frac{\partial z}{\partial v} \\ \frac{\partial x}{\partial w} & \frac{\partial y}{\partial w} & \frac{\partial z}{\partial w} \end{bmatrix} = \begin{bmatrix} \mathrm{a} & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix} \ determinant = abc$$

Conversion of $u^2+v^2+w^2=1$ in to spherical coordinates:

$$u=\rho\sin(\varphi)\cos(\theta), v=\rho\sin(\varphi)\sin(\theta), and\ w= \rho\cos(\varphi)$$

Convert equation:

$$\rho^2\sin^2(\varphi)\sin^2(\theta)+\rho^2\sin^2(\varphi)\cos^2(\theta)+\rho^2\cos^2(\varphi)=1$$

Simplify equation:

$$\rho^2\sin^2(\varphi)[\sin^2(\theta)+\cos^2(\theta)]+\rho^2\cos^2(\varphi)=1$$

$$\rho^2\sin^2(\varphi)+\rho^2\cos^2(\varphi)=1$$

$$\rho^2(\sin^2(\varphi)+\cos^2(\varphi))=1$$

$$\rho^2=1$$

$$\rho=1$$

Triple integral after conversion to spherical coordinates:

- The integrand $\rho^2 \sin(\varphi)$ is derived from a second jacobian but generally, anytime one converts to spherical coordinates, it can be placed in the integrand

$$Volume = \iiint_Q \rho^2 \sin(\varphi)|jacobian|dV$$

$$Volume = \int_0^{2\pi} \int_0^{\pi} \int_0^{1} \rho^2 \sin(\varphi)|abc|d\rho d\varphi d\theta$$

$$Volume = abc \int_0^{2\pi} \int_0^{\pi} \sin(\varphi) \int_0^{1} \rho^2 d\rho d\varphi d\theta$$

$$Volume = \left(\frac{1}{3}\right) abc \int_0^{2\pi} \int_0^{\pi} \sin(\varphi) d\varphi d\theta$$

$$Volume = \left(\frac{1}{3}\right) abc[-\cos(\pi) + \cos(0)] \int_0^{2\pi} d\theta$$

$$Volume = \left(\frac{1}{3}\right) abc[2][2\pi - 0]$$

$$Volume = \frac{4\pi}{3} abc$$