

Nature of the problem

Given Jacobian:

$$\text{determinant: } -abc\rho^2\sin(\varphi)$$

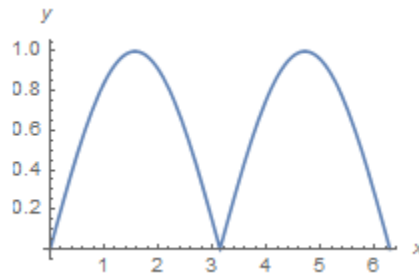
Triple Integral:

- The triple integral will need to be broken up into 2 integrals

$$|abc\rho^2(-\sin(\varphi))| = |abc\rho^2| * |-\sin(\varphi)|$$

$|abc|$ is just a constant and $|\rho^2|$ will always range from 0 to 1 and won't be negative

$|-\sin(\varphi)|$ On the other hand is different from $-\sin(\varphi)$ and looks like this:



- Therefore it needs to be broken up into two integrals, one from 0 to π and the other from π to 2π
- Luckily, due to symmetry, you have:

$$2 \int_0^\pi \int_0^\pi \int_0^1 [(abc)\rho^2 \sin(\varphi)] d\rho d\varphi d\theta$$

$$V = 2abc \int_0^\pi \int_0^\pi \sin(\varphi) \int_0^1 [\rho^2] d\rho d\varphi d\theta$$

$$V = 2abc \int_0^\pi \int_0^\pi \sin(\varphi) \int_0^1 [\rho^2] d\rho d\varphi d\theta$$

$$V = 2abc \int_0^\pi \int_0^\pi \sin(\varphi) d\varphi d\theta \left[\left(\frac{1}{3} \right) \rho^3 \right] \bigg|_0^1$$

$$V = \left(\frac{2}{3} \right) abc \int_0^\pi \int_0^\pi \sin(\varphi) d\varphi d\theta$$

$$V = \left(\frac{2}{3} \right) abc \int_0^\pi d\theta [-\cos(\varphi)] \bigg|_0^\pi$$

$$V = \left(\frac{2}{3} \right) abc(-(-1) + (1)) \int_0^\pi d\theta$$

$$V = \left(\frac{2}{3}\right) abc(2)[\theta] \bigg|_0^\pi$$

$$V = \left(\frac{4}{3}\right) abc[\pi - 0]$$

$$V = \frac{4\pi}{3} abc$$

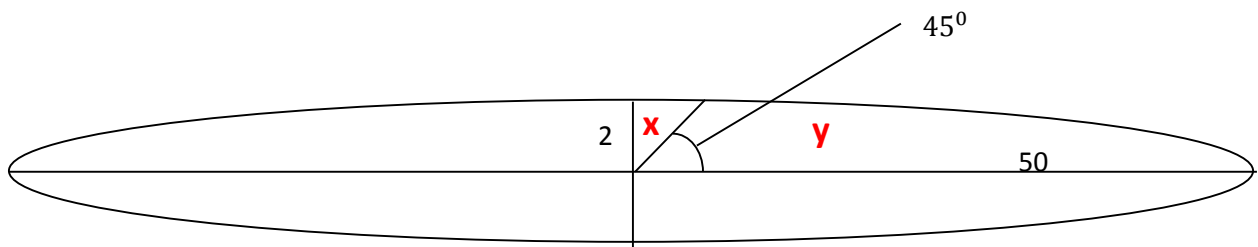
Unfortunately, this still does not solve the problem for values other than increments of $\frac{\pi}{2}$. For example if we integrated the ellipsoid using this jacobian from 0 to $\frac{\pi}{2}$ for φ and 0 to $\frac{\pi}{4}$ for θ , I do not get an accurate volume for that portion of an ellipsoid. This means the way I am approaching this problem is fundamentally wrong.

$$V_y = \int_0^{\frac{\pi}{4}} \int_0^{\frac{\pi}{2}} \int_0^1 [(abc)\rho^2 \sin(\varphi)] d\rho d\varphi d\theta = \frac{\pi}{12} abc$$

This is only true if $a = b = c$

$$V_x = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \int_0^{\frac{\pi}{2}} \int_0^1 [(abc)\rho^2 \sin(\varphi)] d\rho d\varphi d\theta = \frac{\pi}{12} abc$$

But what if $a = 50, b = 2$ and $c = 2$?



$$x \neq y$$

But according to the above integration, it does. This is what I need to fix.