A different way to arrive at the same algorithm:

Equation of an ellipsoid: $$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

Transformation into u, v and w space:

- $u = \frac{x}{a}, v = \frac{y}{b}, w = \frac{z}{c}$ *which yields the equation* $u^2 + v^2 + w^2 = 1$

This is now the equation of a sphere with a radius of 1 in u, v and w space.

Solve the equations for x, y *and* z you have:

$$ua = x, \qquad vb = y \quad and \quad wc = z$$

Now for the jacobian transformation:

- $$\begin{bmatrix} \frac{\partial x}{\partial u} & \frac{\partial y}{\partial u} & \frac{\partial z}{\partial u} \\ \frac{\partial x}{\partial v} & \frac{\partial y}{\partial v} & \frac{\partial z}{\partial v} \\ \frac{\partial x}{\partial w} & \frac{\partial y}{\partial w} & \frac{\partial z}{\partial w} \end{bmatrix} = \begin{bmatrix} \mathrm{a} & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix} \ determinant = abc$$

Conversion of $u^2 + v^2 + w^2 = 1$ in to spherical coordinates:

$$u = \rho \sin(\varphi)\cos(\theta), v = \rho \sin(\varphi)\sin(\theta), and\ w = \rho\cos(\varphi)$$

Convert equation:

$$\rho^2 \sin^2(\varphi)\sin^2(\theta) + \rho^2 \sin^2(\varphi)\cos^2(\theta) + \rho^2 \cos^2(\varphi) = 1$$

Simplify equation:

$$\rho^2 \sin^2(\varphi)[\sin^2(\theta) + \cos^2(\theta)] + \rho^2\cos^2(\varphi) = 1$$

$$\rho^2 \sin^2(\varphi) + \rho^2 \cos^2(\varphi) = 1$$

$$\rho^2(\sin^2(\varphi) + \cos^2(\varphi)) = 1$$

$$\rho^2 = 1$$

$$\rho = 1$$

Triple integral after conversion to spherical coordinates:

- The integrand $\rho^2 \sin(\varphi)$ is derived from a second jacobian but generally, anytime one converts to spherical coordinates, it can be placed in the integrand

$$Volume = \iiint_Q \rho^2 \sin(\varphi)|jacobian|dV$$

$$Volume = \int_{\theta_1}^{\theta_2} \int_{\varphi_1}^{\varphi_2} \int_0^1 \rho^2 \sin(\varphi)|abc|d\rho d\varphi d\theta$$

$$V = abc \int_{\theta_1}^{\theta_2} \int_{\varphi_1}^{\varphi_2} sin(\varphi) \int_0^1 [\rho^2]\, d\rho d\varphi d\theta$$

$$V = abc \int_{\theta_1}^{\theta_2} \int_{\varphi_1}^{\varphi_2} \sin(\varphi) \int_0^1 [\rho^2]\, d\rho d\varphi d\theta$$

$$V = abc \int_{\theta_1}^{2\pi} \int_{\varphi_1}^{\varphi_2} \sin(\varphi)\, d\varphi d\theta \left[\left(\frac{1}{3}\right) p^3\right]$$

$$V = \left(\frac{1}{3}\right) abc \int_{\theta_1}^{\theta_2} \int_{\varphi_1}^{\varphi_2} \sin(\varphi)\, d\varphi d\theta$$

$$V = \left(\frac{1}{3}\right) abc \int_{\theta_1}^{\theta_2} d\theta[-\cos(\varphi)]$$

$$V = \left(\frac{1}{3}\right) abc(-\cos(\varphi_2) + \cos(\varphi_1)) \int_{\theta_1}^{\theta_2} d\theta$$

$$V = -\left(\frac{1}{3}\right) abc(\cos(\varphi_2) - \cos(\varphi_1)) \int_{\theta_1}^{\theta_2} d\theta$$

$$\boxed{V = -\left(\frac{1}{3}\right) abc[\cos(\varphi_2) - \cos(\varphi_1)][\theta_2 - \theta_1]}$$