$$\text{equation of ellipsoid} = \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

$$\text{equation in } u,v,w \text{ space} = u^2 + v^2 + w^2 = 1$$

$$\text{where } u = \frac{x}{a} \quad v = \frac{y}{b} \quad w = \frac{z}{c}$$

Jacobian transformation

Turn equation into spherical coordinates =

$$u \quad \rho\sin\phi\cos\theta = \frac{x}{a} \qquad v \quad \rho\sin\phi\sin\theta = \frac{y}{b}$$

$$w \quad \rho\cos(\phi) = \frac{z}{c}$$

Jacobian

	u	v	w
ρ	$a\sin(\phi)\cos(\theta)$	$b\sin(\phi)\sin(\theta)$	$c\cos(\phi)$
θ	$-a\sin(\phi)\sin(\theta)$	$b\rho\sin(\phi)\cos(\theta)$	0
ϕ	$a\rho\cos(\phi)\cos(\theta)$	$b\rho\cos(\phi)\sin(\theta)$	$-c\rho\sin(\phi)$

Determinant

$\longrightarrow$ Jacobian

$$\int_{\theta_1}^{\theta_2}\int_{\phi_1}^{\phi_2}\int_0^1 |Jacobian|\, d\rho\, d\theta\, d\phi = Volume$$

Definite integral

User specifies values for $a, b, c, \phi_1, \phi_2, \theta_1, \theta_2$

$$Volume = -\frac{1}{3}\Big[abc(\cos(\phi_2) - \cos(\phi_1))\Big]\big[\theta_2 - \theta_1\big]$$

```
u =

a*p*cos(theta)*sin(phi)


v =

b*p*sin(phi)*sin(theta)


w =

c*p*cos(phi)


jacobian =

[ a*cos(theta)*sin(phi), -a*p*sin(phi)*sin(theta), a*p*cos(phi)*cos(theta)]
[ b*sin(phi)*sin(theta),  b*p*cos(theta)*sin(phi), b*p*cos(phi)*sin(theta)]
[           c*cos(phi),                         0,            -c*p*sin(phi)]


determ =

a*b*c*p^2*cos(phi)^2*cos(theta)^2*sin(phi) + a*b*c*p^2*cos(phi)^2*sin(phi)*sin(theta)^2 +↙
a*b*c*p^2*cos(theta)^2*sin(phi)^3 + a*b*c*p^2*sin(phi)^3*sin(theta)^2


outer =

-(a*b*c*(cos(phiEnd) - cos(phiStart))*(thetaEnd - thetaStart))/3

>>
```

```matlab
clear
clear all
clc

syms phi theta p a b c phiStart phiEnd thetaStart thetaEnd

u = (p*sin(phi)*cos(theta))*a
v = (p*sin(phi)*sin(theta))*b
w = (p*cos(phi))*c




jacobian = [diff(u, p) diff(u, theta) diff(u, phi); diff(v, p) ...
    diff(v, theta) diff(v, phi); diff(w, p) diff(w, theta) diff(w, phi);]

% absolute value of the determinant (does not work with absolute value)
determ = -1 * det(jacobian)

% limits always from 0 to 1 for p
inner = int(determ, p, 0, 1);

% user specified limits phi
middle = int(inner, phi, phiStart, phiEnd);

% user specified limits theta
outer = int(middle, theta, thetaStart, thetaEnd)
```