Integration of determinant of jacobian for Kelly and Alan

Determinant incorrectly multiplied by -1 instead of taking the absolute value:

(This was a mistake I made a while ago, but it still worked out. See the document **Nature of the Problem** for more details)

$$(abc)\rho^2 \cos^2(\varphi)\cos^2(\theta)\sin(\varphi) + (abc)\rho^2 \cos^2(\varphi)\sin^2(\theta)\sin(\varphi) + (abc)\rho^2 \cos^2(\theta)\sin^3(\varphi) + (abc)\rho^2 \sin^3(\varphi)\sin^2(\theta)$$

Simplifying using trig identifies:

$$(abc)\rho^2 \sin(\varphi)\cos^2(\varphi)[\cos^2(\theta)+\sin^2(\theta)] + (abc)\rho^2 \sin^3(\varphi)[\cos^2(\theta)+\sin^2(\theta)]$$

$$(abc)\rho^2 \sin(\varphi)\cos^2(\varphi) + (abc)\rho^2 \sin^3(\varphi)$$

$$(abc)\rho^2 \sin(\varphi)\,[\cos^2(\varphi)+\sin^2(\varphi)]$$

$$(abc)\rho^2 \sin(\varphi)$$

Triple Integral:

$$Volume = \int_{\theta_1}^{\theta_2}\int_{\varphi_1}^{\varphi_2}\int_0^1 [(abc)\rho^2 \sin(\varphi)]\, d\rho d\varphi d\theta$$

$$V = abc\int_{\theta_1}^{\theta_2}\int_{\varphi_1}^{\varphi_2} sin(\varphi)\int_0^1 [\rho^2]\, d\rho d\varphi d\theta$$

$$V = abc\int_{\theta_1}^{\theta_2}\int_{\varphi_1}^{\varphi_2} \sin(\varphi)\int_0^1 [\rho^2]\, d\rho d\varphi d\theta$$

$$V = abc\int_{\theta_1}^{2\pi}\int_{\varphi_1}^{\varphi_2} \sin(\varphi)\, d\varphi d\theta \left[\left(\frac{1}{3}\right)p^3\right]\Bigg|_0^1$$

$\left[\left(\frac{1}{3}\right)(1)^3 - \left(\frac{1}{3}\right)(0)^3\right] = \frac{1}{3}$ This is why ρ goes away

$$V = \left(\frac{1}{3}\right)abc\int_{\theta_1}^{\theta_2}\int_{\varphi_1}^{\varphi_2} \sin(\varphi)\, d\varphi d\theta$$

$$V = \left(\frac{1}{3}\right)abc\int_{\theta_1}^{\theta_2} d\theta[-\cos(\varphi)]\Bigg|_{\varphi_1}^{\varphi_2}$$

$$V = \left(\frac{1}{3}\right)abc(-\cos(\varphi_2)+\cos(\varphi_1))\int_{\theta_1}^{\theta_2} d\theta$$

Symbolic integration

$$V = -\left(\frac{1}{3}\right)abc(\cos(\varphi_2)-\cos(\varphi_1))\int_{\theta_1}^{\theta_2} d\theta$$

Factor out a -1

$$V = -\left(\frac{1}{3}\right) abc[\cos(\varphi_2) - \cos(\varphi_1)][\theta_2 - \theta_1]$$ Final algorithm

Please see the document titled "Nature of the Problem" to see how to properly integrate the absolute value of the determinant and then an explanation about what trouble I'm running into with this strategy all together