Nature of the problem

Given Jacobian:Type equation here.

$$determinant: -abc\rho^2\sin(\varphi)$$

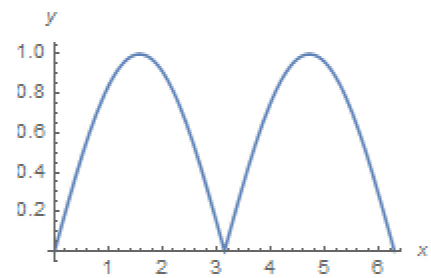
Triple Integral:

- The integrand can be broken up into two parts, the first part will never be negative, the second can be:

$$|-abc\rho^2\sin(\varphi)| = |abc\rho^2(-\sin(\varphi))| = |abc\rho^2| * |-\sin(\varphi)|$$

$|abc|$ Is just a constant and $|\rho^2|$ will always range from 0 to 1 and won't be negative

$|-\sin(\varphi)|$ On the other hand is different from $-\sin(\varphi)$ and looks like this:



- Triple integral:

$$\int_0^{2\pi}\int_0^{\pi}\int_0^{1}[(abc)\rho^2\sin(\varphi)]\,d\rho d\varphi d\theta$$

- From $0\ to\ \pi$, $|-\sin(\varphi)|$ has the same area under the curve as $\sin(\varphi)$. This is why multiplying the jacobian by -1 instead of taking the absolute value did not yield an incorrect volume. It is also why you can just use $\sin(\varphi)$ instead of $|-\sin(\varphi)|$ in the integrand.

$$V = 2abc\int_0^{\pi}\int_0^{\pi} sin(\varphi)\int_0^{1}[\rho^2]\,d\rho d\varphi d\theta$$

$$V = 2abc\int_0^{\pi}\int_0^{\pi} \sin(\varphi)\int_0^{1}[\rho^2]\,d\rho d\varphi d\theta$$

$$V = 2abc\int_0^{\pi}\int_0^{\pi} \sin(\varphi)\,d\varphi d\theta\left[\left(\frac{1}{3}\right)p^3\right]\Bigg|_0^1$$

$$V = \left(\frac{2}{3}\right)abc\int_0^{\pi}\int_0^{\pi} \sin(\varphi)\,d\varphi d\theta$$

$$V = \left(\frac{2}{3}\right)abc\int_0^{\pi} d\theta[-\cos(\varphi)]\Bigg|_0^{\pi}$$

$$V = \left(\frac{2}{3}\right) abc(-(-1) + (1)) \int_0^{\pi} d\theta$$

$$V = \left(\frac{2}{3}\right) abc(2)[\theta]\Big|_0^{\pi}$$

$$V = \left(\tfrac{4}{3}\right) abc[\pi - 0]$$

$$V = \frac{4\pi}{3} abc$$

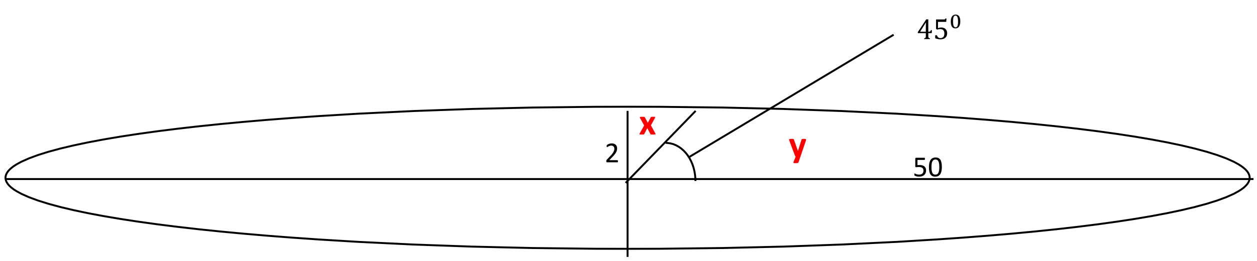
Unfortunately, this still does not solve the problem for values other than increments of $\frac{\pi}{2}$. For example if we integrated the ellipsoid using this jacobian from $0\ to \frac{\pi}{2}$ for φ and $0\ to \frac{\pi}{4}$ for θ, we do not get an accurate volume for that portion of an ellipsoid. This means the way I am approaching this problem is fundamentally wrong.

$$V_y = \int_0^{\frac{\pi}{4}} \int_0^{\frac{\pi}{2}} \int_0^1 [(abc)\rho^2 \sin(\varphi)]\, d\rho d\varphi d\theta = \frac{\pi}{12} abc$$

This is only true if $a = b = c$

$$V_x = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \int_0^{\frac{\pi}{2}} \int_0^1 [(abc)\rho^2 \sin(\varphi)]\, d\rho d\varphi d\theta = \frac{\pi}{12} abc$$

$$\boxed{But\ what\ if\ a = 50, b = 2\ and\ c = 2?}$$



$$\boxed{x \neq y}$$

But according to the above integration, it does. This is what I need to fix.

Below is the start of an integration that I cannot finish. It is possible it can be done through iterations but it would be best if a symbolic definite integral can be found

Unfinished triple integral of ellipsoid without using jacobian transformation:

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$$

$$\frac{\rho^2\sin^2(\varphi)\sin^2(\theta)}{a^2}+\frac{\rho^2\sin^2(\varphi)\cos^2(\theta)}{b^2}+\frac{\rho^2\cos^2(\varphi)}{c^2}=1$$

$$\rho^2\left[\frac{\sin^2(\varphi)\sin^2(\theta)}{a^2}+\frac{\sin^2(\varphi)\cos^2(\theta)}{b^2}+\frac{\cos^2(\varphi)}{c^2}\right]=1$$

$$\rho=\pm\sqrt{a^2\csc^2(\varphi)\csc^2(\theta)+b^2\csc^2(\varphi)\sec^2(\theta)+c^2\sec^2(\varphi)}$$

$$V=\int_{\theta_1}^{\theta_2}\int_{\varphi_1}^{\varphi_2}\int_0^{\sqrt{a^2\csc^2(\varphi)\csc^2(\theta)+b^2\csc^2(\varphi)\sec^2(\theta)+c^2\sec^2(\varphi)}} d\rho d\varphi d\theta$$

$$V=\int_{\theta_1}^{\theta_2}\int_{\varphi_1}^{\varphi_2}\sqrt{a^2\csc^2(\varphi)\csc^2(\theta)+b^2\csc^2(\varphi)\sec^2(\theta)+c^2\sec^2(\varphi)}\,d\varphi d\theta$$

$$V=\int_{\theta_1}^{\theta_2}\int_{\varphi_1}^{\varphi_2}\sqrt{\csc^2(\varphi)[a^2\csc^2(\theta)+b^2\sec^2(\theta)]+c^2\sec^2(\varphi)}\,\mathrm{d\varphi d\theta}$$

This is where I get stuck.