Algorithm breakdown for Kelly and Alan

Given algorithm:

$$Volume = -\left(\frac{1}{3}\right)abc(\cos(\varphi_2) - \cos(\varphi_1))(\theta_2 - \theta_1)$$

Symbolic equation for the volume of an entire ellipsoid using the algorithm

- θ will range from 0 $to\ 2\pi$
- ϕ will range from 0 to π
- ρ will range from 0 to 1 (it is eliminated in the triple integral which is why it's not present in the symbolic definite integral, which is the algorithm)

$$Volume = -\left(\frac{1}{3}\right)abc[\cos(\pi) - \cos(0)][2\pi - 0]$$

$$Volume = -\left(\frac{1}{3}\right)abc[(-1) - (1)][2\pi]$$

$$Volume = -\left(\frac{1}{3}\right)abc[-2][2\pi]$$

$$Volume = \frac{4\pi}{3}abc$$

Plugging in bounds for ϕ and θ that are increments of $\frac{\pi}{2}$ will also yield a positive accurate answer.

Say for example a user wanted to calculate the upper left quarter of the ellipsoid

- θ will range from $\frac{\pi}{2}\ to \frac{3\pi}{2}$
- ϕ will range from 0 to $\frac{\pi}{2}$
- ρ will range from 0 to 1 (Again, eliminated from the symbolic definite integral)

$$Volume = -\left(\frac{1}{3}\right)abc\left[\cos\left(\frac{\pi}{2}\right) - \cos(0)\right]\left[\frac{3\pi}{2} - \frac{\pi}{2}\right]$$

$$Volume = -\left(\frac{1}{3}\right)abc[0 - 1][\pi]$$

$$Volume = \frac{\pi}{3}abc$$

****Please see document "Integration of determinant of jacobian for Kelly and Alan" for jacobian determinant reduction and triple integral****

I hope this helps explain the way the algorithm works