Jacobian

	u	v	w
ρ	$\frac{\sin(\phi)\cos(\theta)}{a}$	$\frac{\sin(\phi)\sin(\theta)}{b}$	$\frac{\cos(\phi)}{c}$
θ	$\frac{-\rho\sin(\phi)\sin(\theta)}{a}$	$\frac{\rho\sin(\phi)\cos(\theta)}{b}$	0
ϕ	$\frac{\rho\cos(\phi)\cos(\theta)}{a}$	$\frac{\rho\cos(\phi)\sin(\theta)}{b}$	$\frac{-\rho\sin(\phi)}{c}$

Determinant

$$\int_{\phi_1}^{\phi_2}\int_{\theta_1}^{\theta_2}\int_{0}^{1}\left[\rho^2\sin(\phi)\right]\left|\text{Jacobian}\right| d\rho\, d\theta\, d\phi = \text{Volume}$$

Equation of ellipsoid $= \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$

Equation is u,v,w space $= u^2 + v^2 + w^2 = 1$

where $u = \frac{x}{a}$ $v = \frac{y}{b}$ $w = \frac{z}{c}$

Jacobian transformation

Turn equation into spherical coordinates $= u = \frac{\rho\sin\phi\cos\theta}{a}$, $v = \frac{\rho\sin\phi\sin\theta}{b}$, $w = \rho\cos\phi$

☆ User specifies $\theta_1, \theta_2, \phi_1, \phi_2$, want to integrate these values as constants

☆ looking for definite integral

```
clear
clear all
clc

syms phi theta p a b c phiStart phiEnd thetaStart thetaEnd

u = (p*sin(phi)*cos(theta))/a
v = (p*sin(phi)*sin(theta))/b
w = (p*cos(phi))/c

% thetaStart = 0;
% thetaEnd = 2*pi;
% phiStart = 0;
% phiEnd = pi;
% a = 3;
% b = 3;
% c = 2;

jacobian = [diff(u, p) diff(v, p) diff(w, p); diff(u, theta) ...
    diff(v, theta) diff(w, theta); diff(u, phi) diff(v, phi) diff(w, phi);]

% absolute value of the determinant
determ = abs(det(jacobian))

% limits always from 0 to 1 for p
inner = int((p^2*sin(phi))*determ, p, 0, 1);

% user specified limits phi
middle = int(inner, phi, phiStart, phiEnd);

% user specified limits theta
outer = int(middle, theta, thetaStart, thetaEnd)
```

```
u =

(p*cos(theta)*sin(phi))/a


v =

(p*sin(phi)*sin(theta))/b


w =

(p*cos(phi))/c


jacobian =

[     (cos(theta)*sin(phi))/a,    (sin(phi)*sin(theta))/b,       cos(phi)/c]
[ -(p*sin(phi)*sin(theta))/a, (p*cos(theta)*sin(phi))/b,                   0]
[   (p*cos(phi)*cos(theta))/a, (p*cos(phi)*sin(theta))/b, -(p*sin(phi))/c]


determ =

abs(p^2*cos(phi)^2*cos(theta)^2*sin(phi) + p^2*cos(phi)^2*sin(phi)*sin(theta)^2 + p^2*cos↙
(theta)^2*sin(phi)^3 + p^2*sin(phi)^3*sin(theta)^2)/(abs(a)*abs(b)*abs(c))

Warning: Explicit integral could not be found.
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outer =

int(int(int((p^2*abs(p^2*cos(phi)^2*cos(theta)^2*sin(phi) + p^2*cos(phi)^2*sin(phi)*sin↙
(theta)^2 + p^2*cos(theta)^2*sin(phi)^3 + p^2*sin(phi)^3*sin(theta)^2)*sin(phi))/(abs(a)↙
*abs(b)*abs(c)), p == 0..1), phi == phiStart..phiEnd), theta == thetaStart..thetaEnd)

>>
```

Can someone find the explicit integral?