Jacobian

	u	v	w
ρ	$a\sin(\phi)\cos(\theta)$	$b\sin(\phi)\sin(\theta)$	$c\cos(\phi)$
θ	$-a\rho\sin(\phi)\sin(\theta)$	$b\rho\sin(\phi)\cos(\theta)$	0
ϕ	$a\rho\cos(\phi)\cos(\theta)$	$b\rho\cos(\phi)\sin(\theta)$	$-c\rho\sin(\phi)$

equation of ellipsoid $= \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$

equation in u,v,w space $= u^2 + v^2 + w^2 = 1$

where $u = \frac{x}{a}$ $v = \frac{y}{b}$ $w = \frac{z}{c}$

Jacobian transformation

Turn equation into spherical coordinates $=$

u: $\rho\sin\phi\cos\theta = \frac{x}{a}$ v: $\rho\sin\phi\sin\theta = \frac{y}{b}$

w: $\rho\cos(\phi) = \frac{z}{c}$

Determinant

$$\int_{\phi_1}^{\phi_2}\int_{\theta_1}^{\theta_2}\int_{0}^{1} |\text{Jacobian}|\, d\rho\, d\theta\, d\phi = \text{Volume} \Rightarrow$$

Definite interval

User specifies values for $a, b, c, \phi_1, \phi_2, \theta_1, \theta_2$

$$\text{Volume} = -\frac{1}{3}\left[abc\left(\cos(\phi_2) - \cos(\phi_1)\right)\right]\left[\theta_2 - \theta_1\right]$$

```
clear
clear all
clc

syms phi theta p a b c phiStart phiEnd thetaStart thetaEnd

x = (p*sin(phi)*cos(theta))*a
y = (p*sin(phi)*sin(theta))*b
z = (p*cos(phi))*c



jacobian = [diff(x, p) diff(x, theta) diff(x, phi); diff(y, p) ...
    diff(y, theta) diff(y, phi); diff(z, p) diff(z, theta) diff(z, phi);]

% absolute value of the determinant (does not work with absolute value)
determ = -1 * det(jacobian)

% limits always from 0 to 1 for p
inner = int(determ, p, 0, 1);

% user specified limits phi
middle = int(inner, phi, phiStart, phiEnd);

% user specified limits theta
outer = int(middle, theta, thetaStart, thetaEnd)
```

```
clear
clear all
clc

syms phi theta p a b c phiStart phiEnd thetaStart thetaEnd

u = (p*sin(phi)*cos(theta))*a
v = (p*sin(phi)*sin(theta))*b
w = (p*cos(phi))*c



jacobian = [diff(u, p) diff(u, theta) diff(u, phi); diff(v, p) ...
    diff(v, theta) diff(v, phi); diff(w, p) diff(w, theta) diff(w, phi);]

% absolute value of the determinant (does not work with absolute value)
determ = -1 * det(jacobian)

% limits always from 0 to 1 for p
inner = int(determ, p, 0, 1);

% user specified limits phi
middle = int(inner, phi, phiStart, phiEnd);

% user specified limits theta
outer = int(middle, theta, thetaStart, thetaEnd)
```