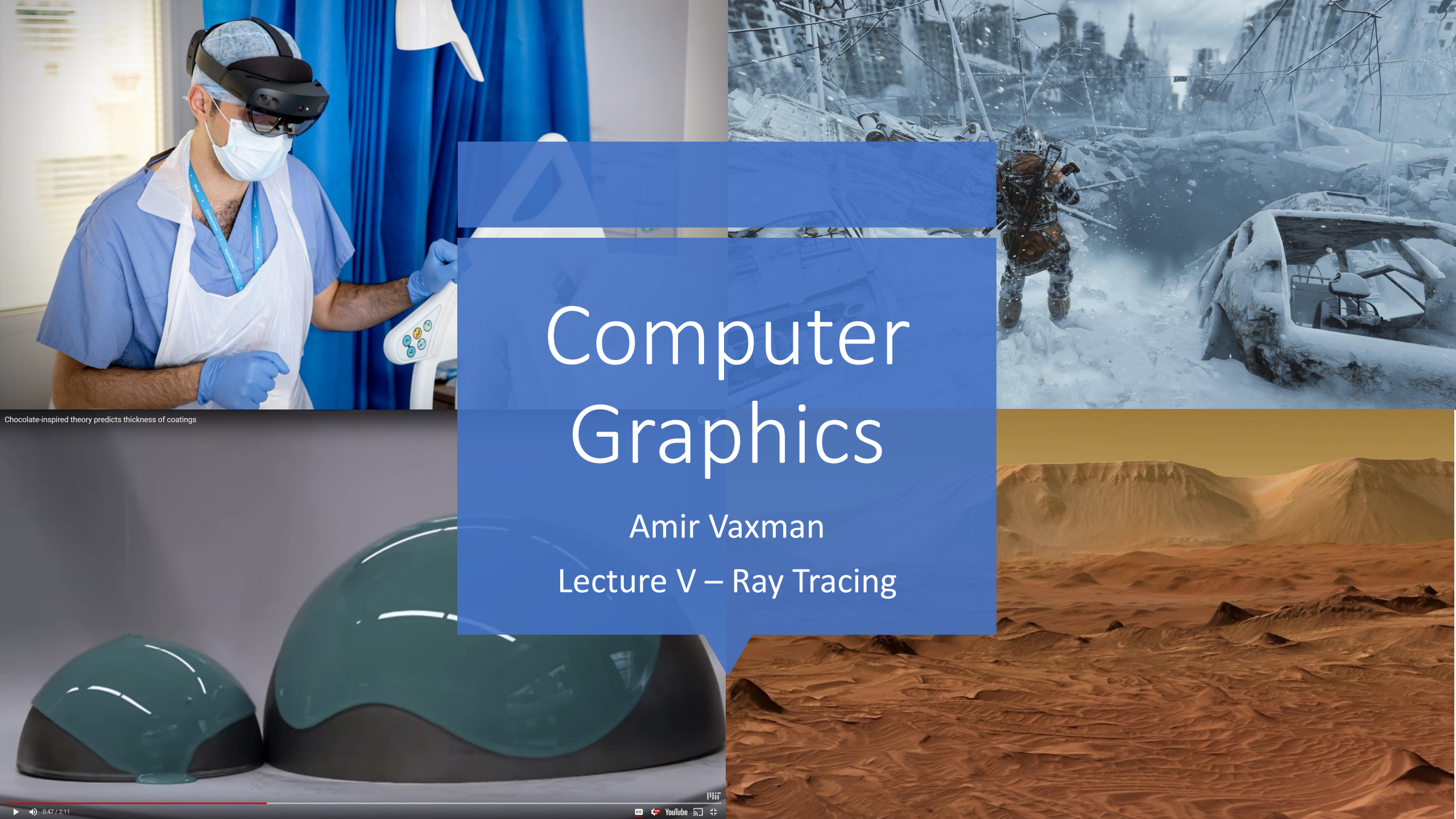


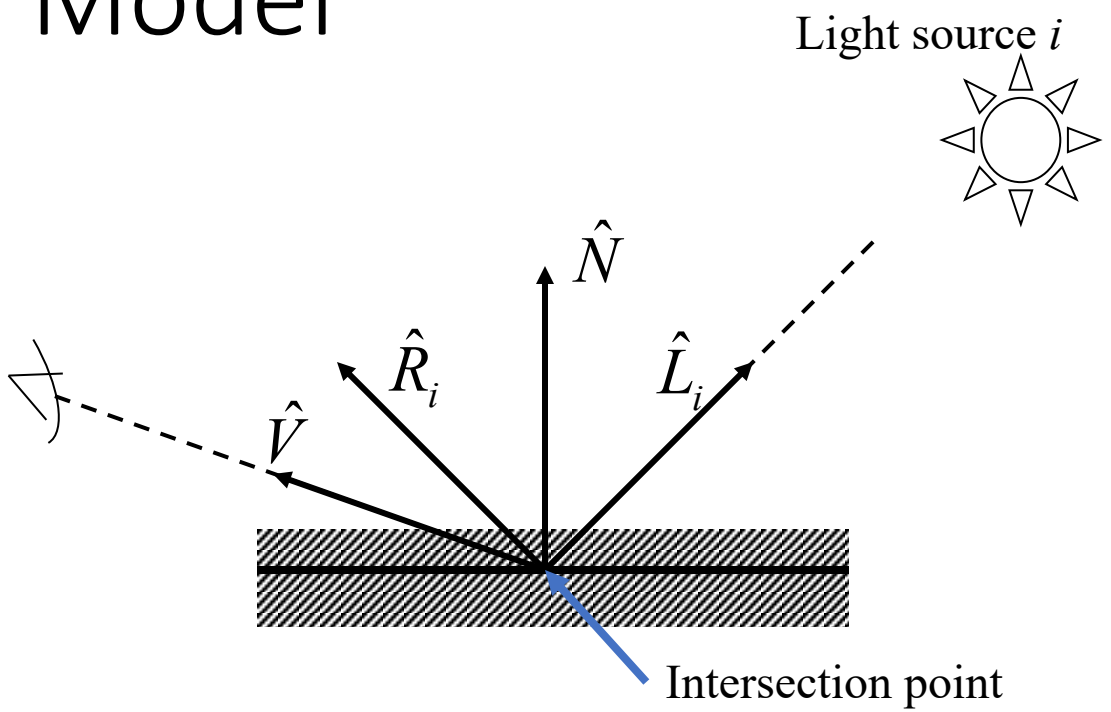
Computer Graphics

Amir Vaxman

Lecture V – Ray Tracing

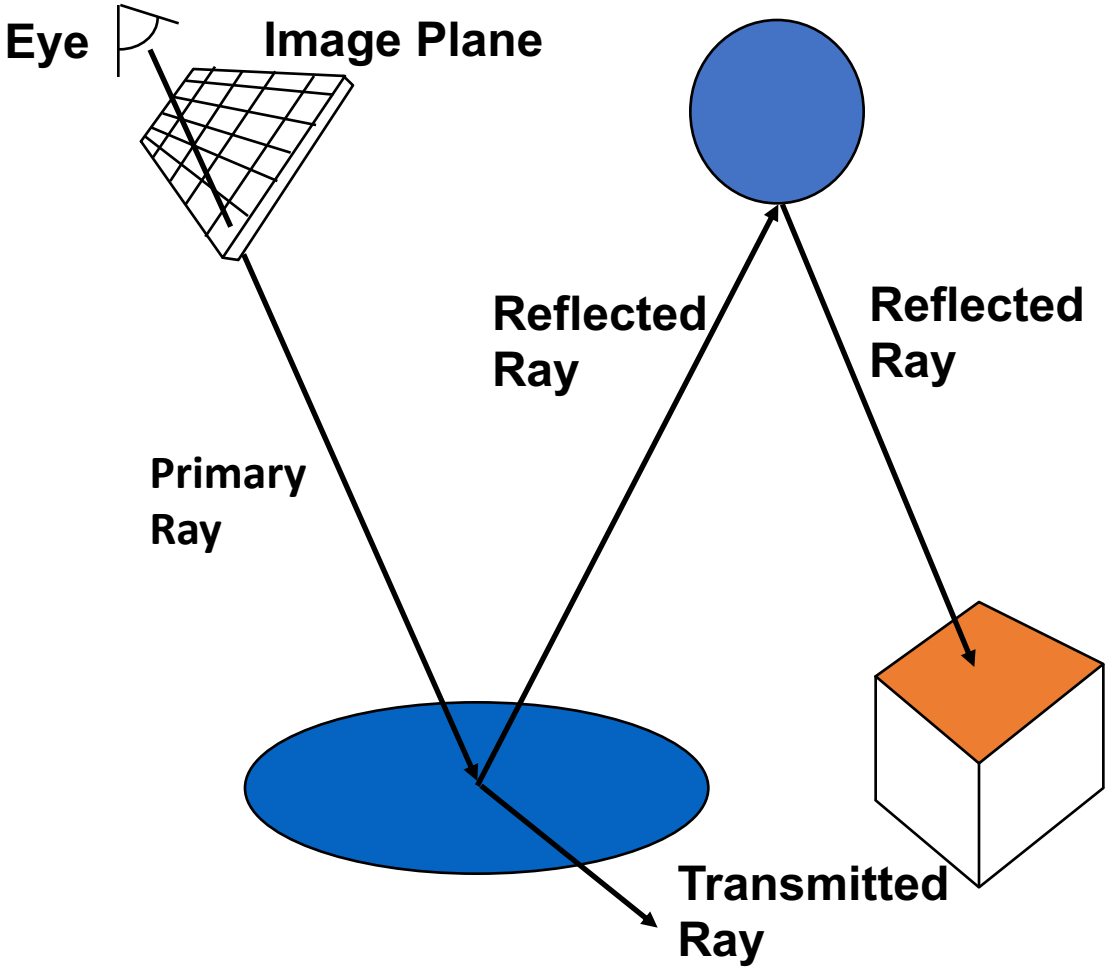
Simple Illumination Model

- Coefficients:
  - k_a – ambient
  - k_d – diffuse
  - k_s – specular
  - n – specular highlight
- L_i – intensity of source i



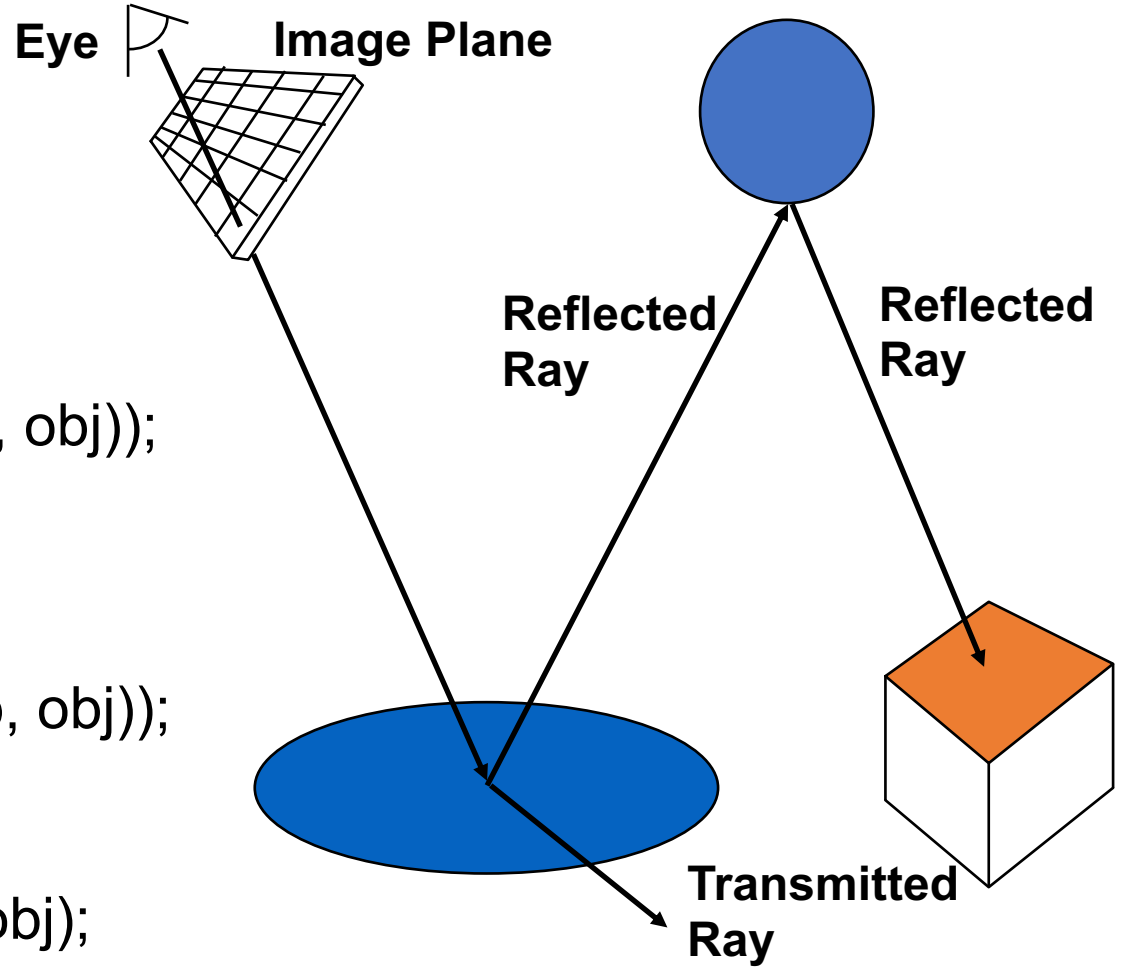
$$I = k_a I_a + k_d \sum_i I_i (\hat{N} \cdot \hat{L}_i) + k_s \sum_i I_i (\hat{R}_i \cdot \hat{V})^n$$

Ray-Tracing Algorithm



A Basic Ray-Tracing Algorithm

```
RayTrace(r, scene)
<obj, p> := FirstIntersection(r, scene)
if (no obj)  return BackgroundColor;
else begin
    if ( Reflect(obj) ) then
       ReflectColor := RayTrace(ReflectRay(r, p, obj));
    else
       ReflectColor := Black;
   if ( Transparent(obj) ) then
       RefractColor := RayTrace(RefractRay(r, p, obj));
    else
       RefractColor := Black;
   return Shade(ReflectColor, RefractColor, p, obj);
end;
```

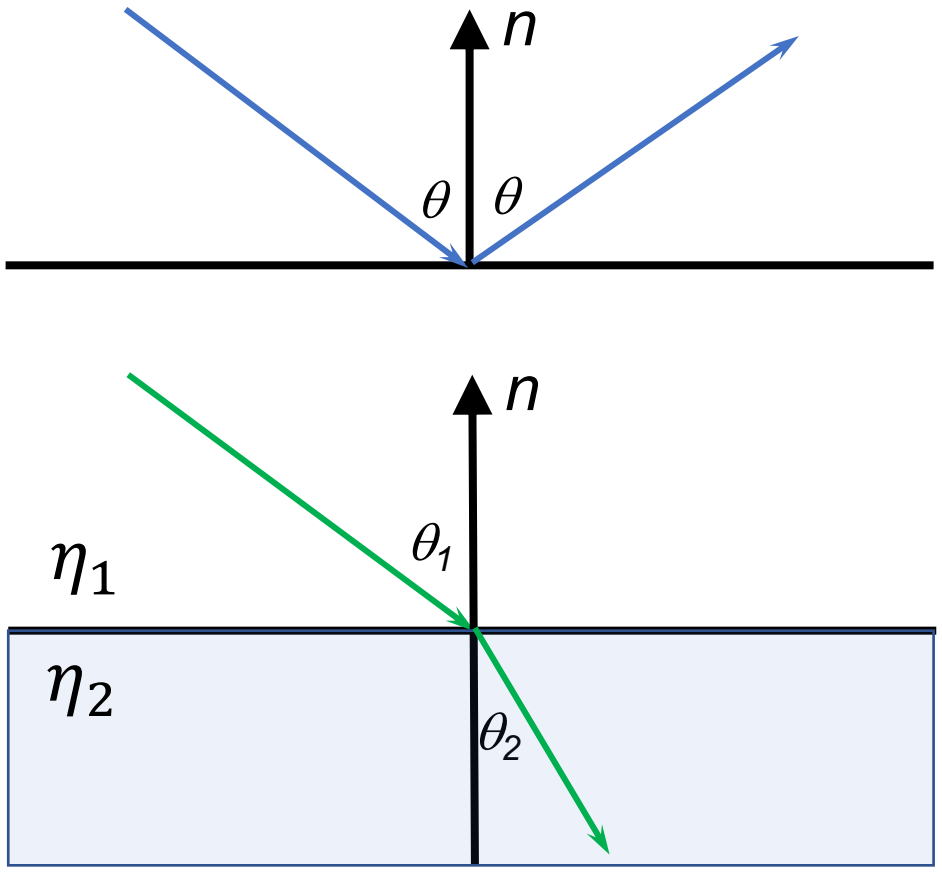


Reflection and Refraction

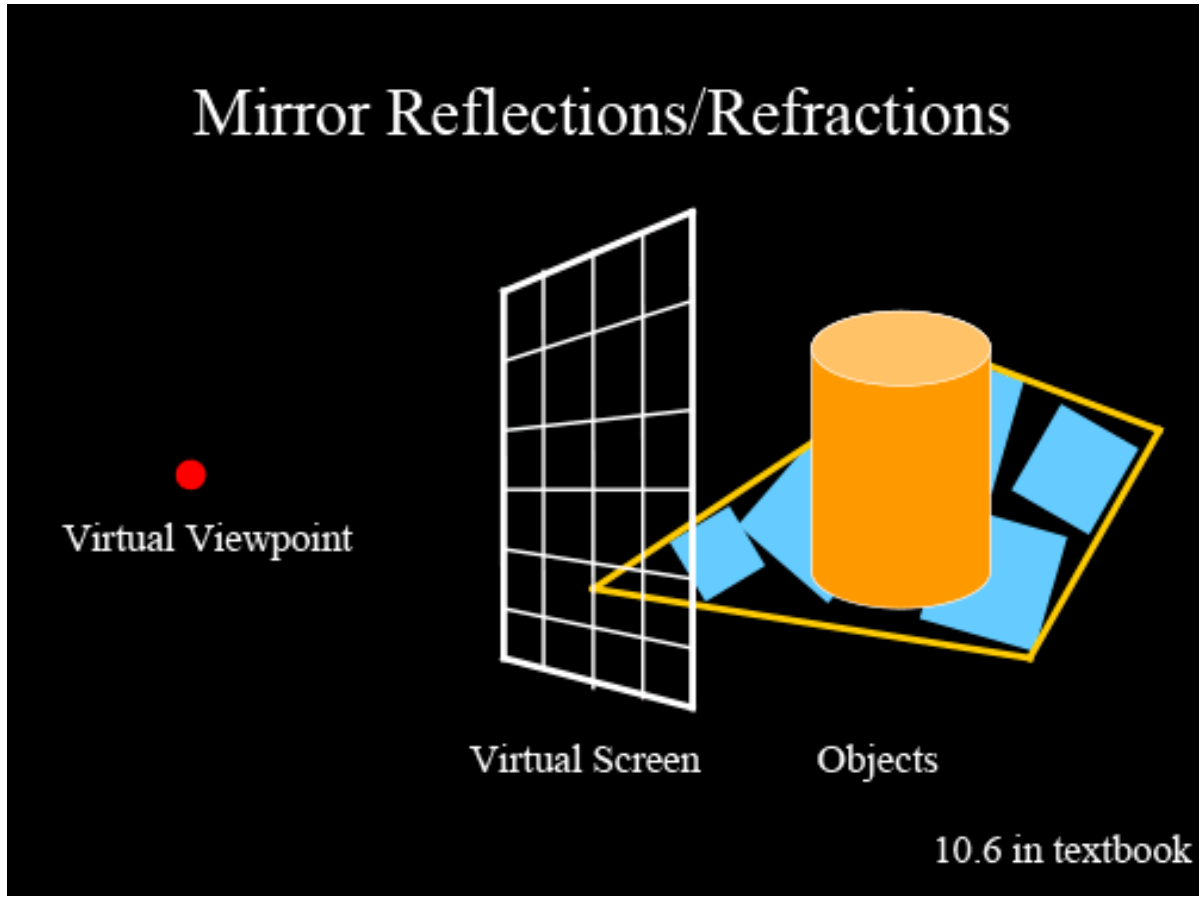
- Reflaction: symmetric to normal
- Refraction: Snell's law

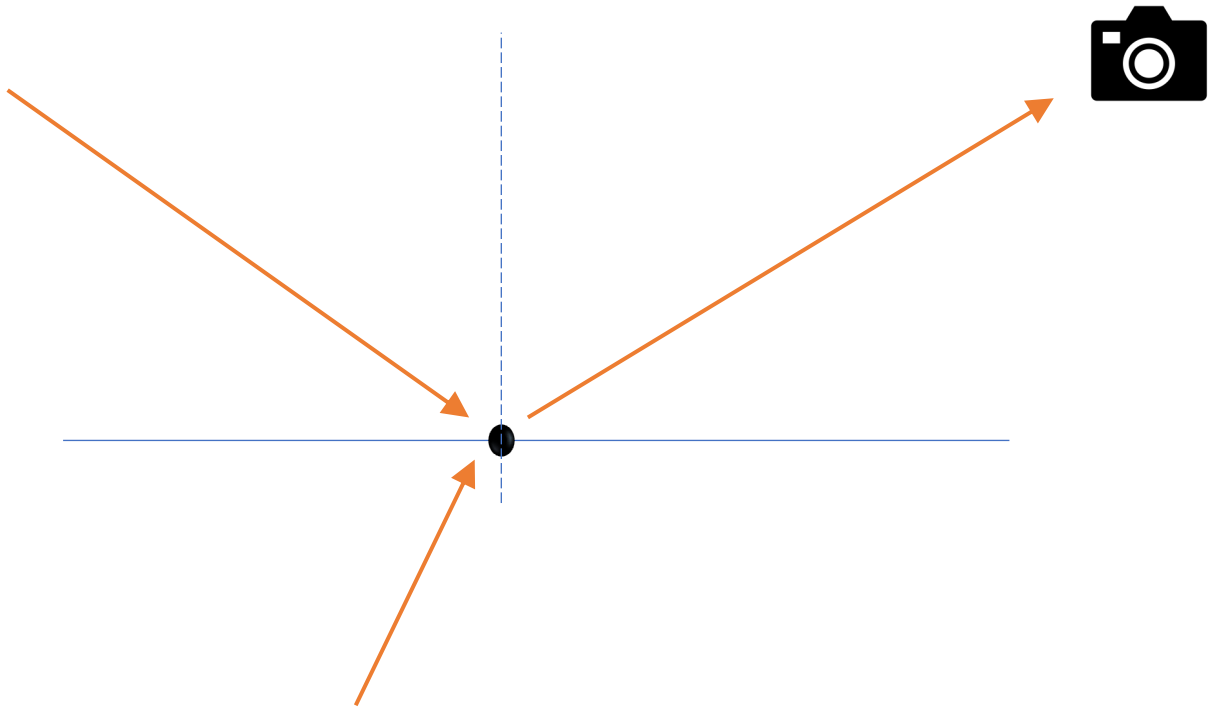
$$\frac{\sin(\theta_1)}{\sin(\theta_2)} = \frac{\eta_2}{\eta_1}$$

- η_2/η_1 : Relative refractive index
  - Depends on the materials



Mirror (specular) reflection/refraction

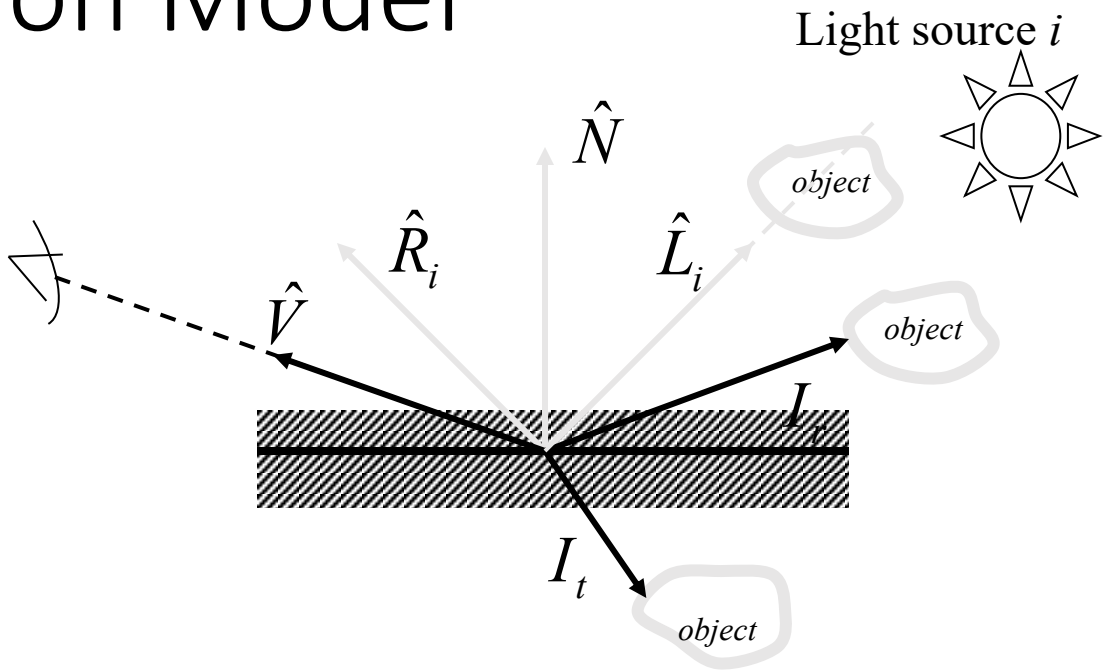




http://studiomaven.org/Context__Overview_of_Ray-Traced_Rendering.html

Extended Illumination Model

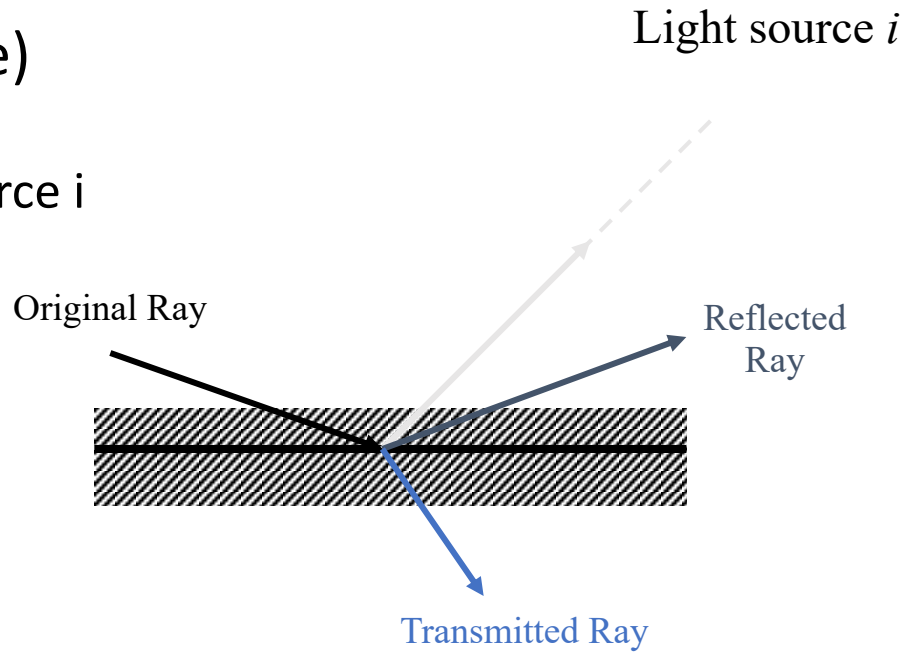
- New terms:
- Shadow
  - L_i – intensity of source i, or 0 if source not visible
- Reflection
  - k_r – reflection coefficient
  - I_r – reflected ray intensity
- Transmission
  - k_t – transmission coefficient
  - I_t – transmitted ray intensity



$$I = k_a I_a + k_d \sum_i I_i (\hat{N} \cdot \hat{L}_i) + k_s \sum_i I_i (\hat{R}_i \cdot \hat{V})^n + k_r I_r + k_t I_t$$

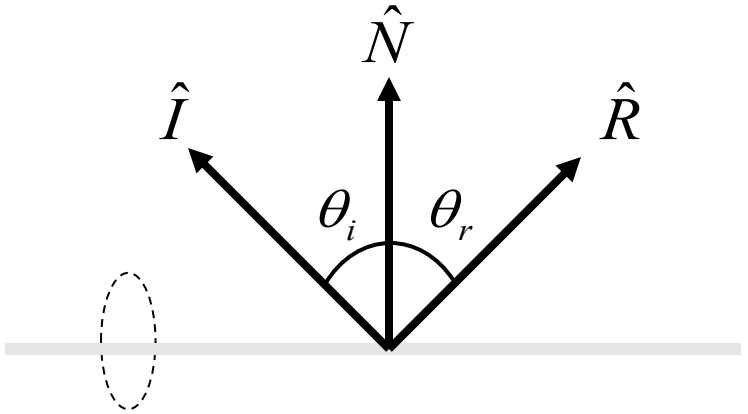
Implementation

- Shadow
  - Fire shadow rays (for each light source)
    - Origin: intersection point
    - Direction: intersection point → Light source i
- Reflection
  - Fire a reflected ray
    - Origin: intersection point
    - Direction: reflection vector
- Transmission
  - Fire a transmitted ray
    - Origin: intersection point
    - Direction: transmission vector



The Reflection Vector

- Unit vectors
  - N – normal
  - I – incident
  - R – reflected



- I, R and N are coplanar

$$\theta_i = \theta_r \Rightarrow \hat{I} \cdot \hat{N} = \hat{R} \cdot \hat{N}$$

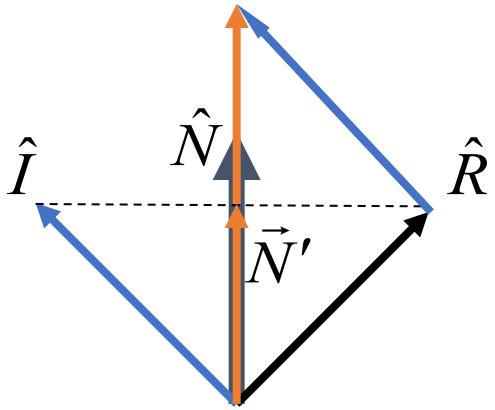
Reflection Vector

$$\vec{N}' = (\hat{I} \cdot \hat{N})\hat{N}$$

$$\hat{R} + \hat{I} = 2\vec{N}'$$

$$\Downarrow$$

$$\hat{R} = 2\vec{N}' - \hat{I} = 2(\hat{I} \cdot \hat{N})\hat{N} - \hat{I}$$



$$\hat{R} = 2(\hat{I} \cdot \hat{N})\hat{N} - \hat{I}$$

Transmission Vector – Snell's Law

$$\hat{I} = -\sin\theta_i \hat{M} + \cos\theta_i \hat{N}$$

$$\Downarrow$$

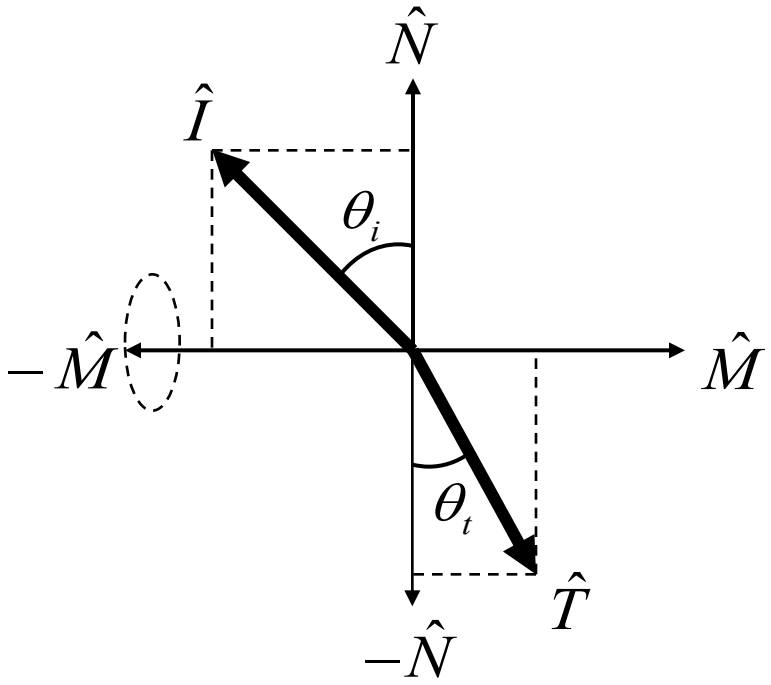
$$\hat{M} = \frac{\cos\theta_i \hat{N} - \hat{I}}{\sin\theta_i}$$

$$\hat{T} = \sin\theta_t \hat{M} - \cos\theta_t \hat{N}$$

$$\Downarrow$$

$$\hat{T} = \sin\theta_t \frac{\cos\theta_i \hat{N} - \hat{I}}{\sin\theta_i} - \cos\theta_t \hat{N} =$$

$$\frac{\eta_i(\cos\theta_i \hat{N} - \hat{I})}{\eta_t} - \cos\theta_t \hat{N}$$



$$\frac{\eta_i}{\eta_t} = \frac{\sin\theta_t}{\sin\theta_i}$$

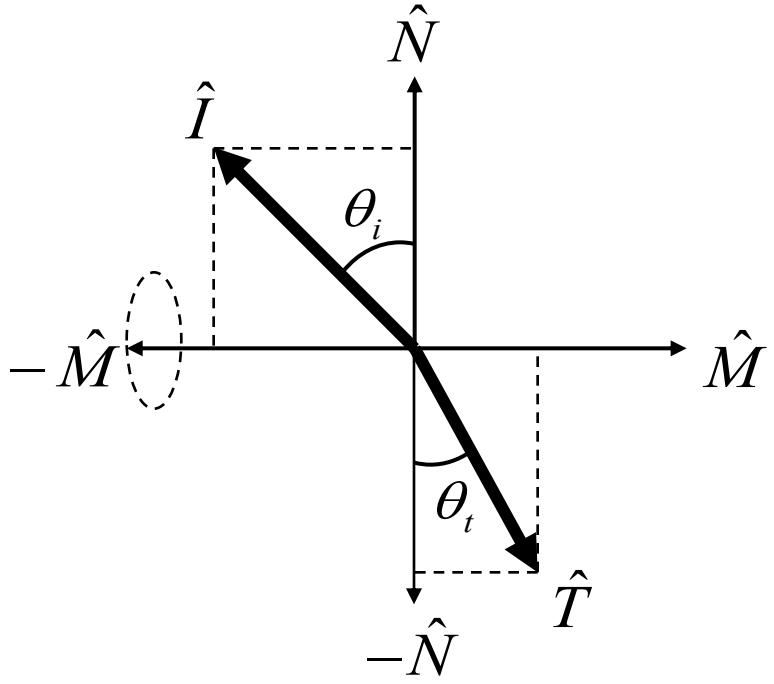
Transmission Vector – Snell's Law

$$\hat{T} = -\frac{\eta_i}{\eta_t}\hat{I} + \left(\frac{\eta_i}{\eta_t}\cos\theta_i - \cos\theta_t\right)\hat{N}$$

$$\eta_i \sin\theta_i = \eta_t \sin\theta_t \qquad \sin^2\theta + \cos^2\theta = 1$$

$$\cos^2\theta_t = 1 - \frac{\eta_i^2}{\eta_t^2}\left(1 - \cos^2\theta_i\right)$$

$$\cos\theta_i = \hat{I}\cdot\hat{N}$$

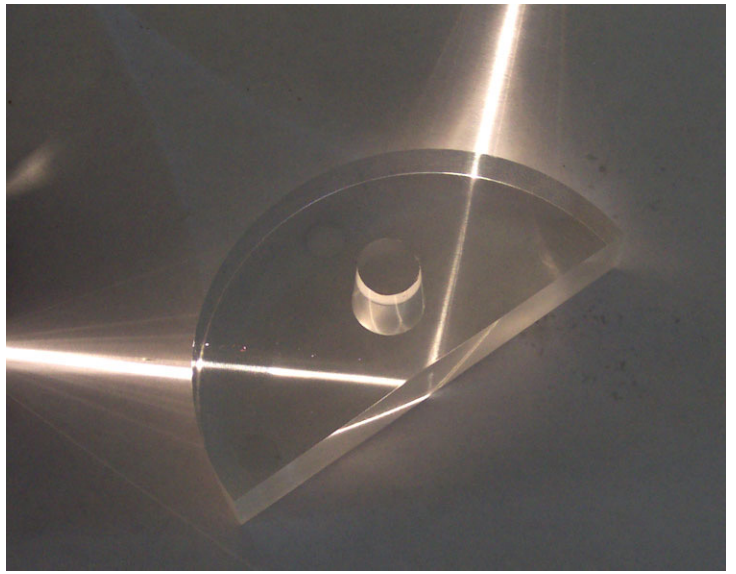


Is this an issue?

$$\hat{T} = -\frac{\eta_i}{\eta_t}\hat{I} + \left(\frac{\eta_i}{\eta_t}\hat{I}\cdot\hat{N} - \sqrt{1 - \frac{\eta_i^2}{\eta_t^2}\left(1 - \left(\hat{I}\cdot\hat{N}\right)^2\right)}\right)\hat{N}$$

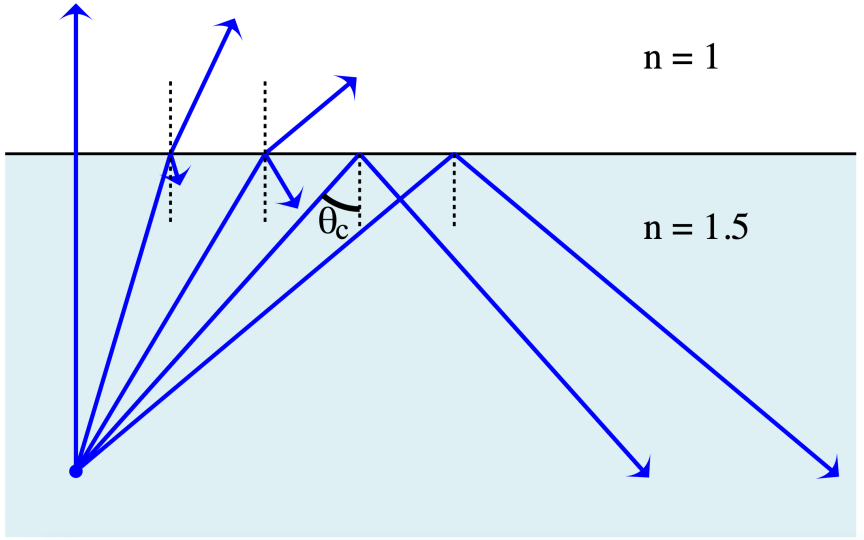
Total Internal Reflection

- Detection: $1-\frac{\eta_i^2}{\eta_t^2}\left(1-\left(\hat{I}\cdot\hat{N}\right)^2\right)<0$

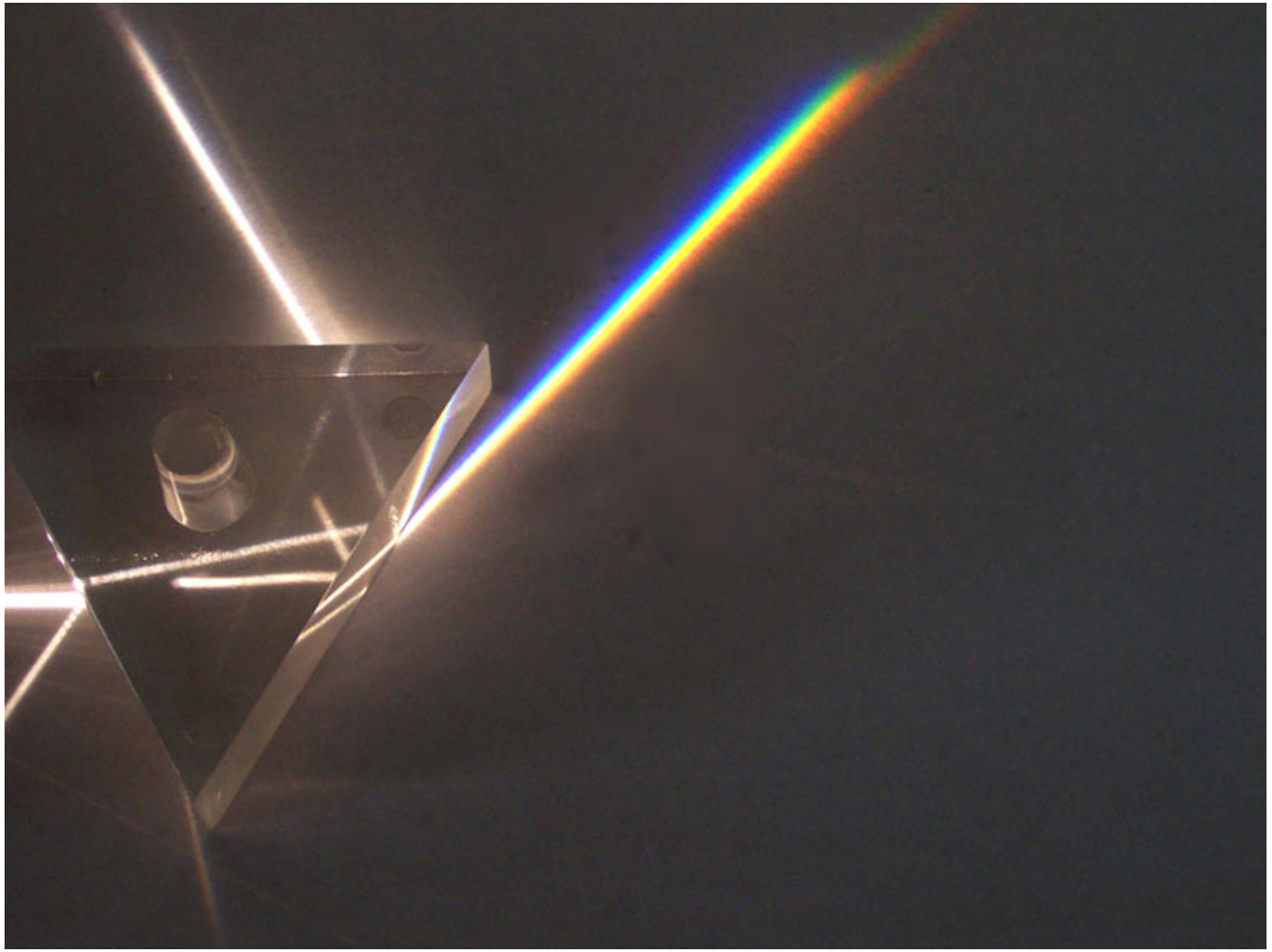


- Conditions
  - $\eta_i > \eta_t$
  - $\theta_i > \theta_c$ (critical angle)

- All light goes to reflection

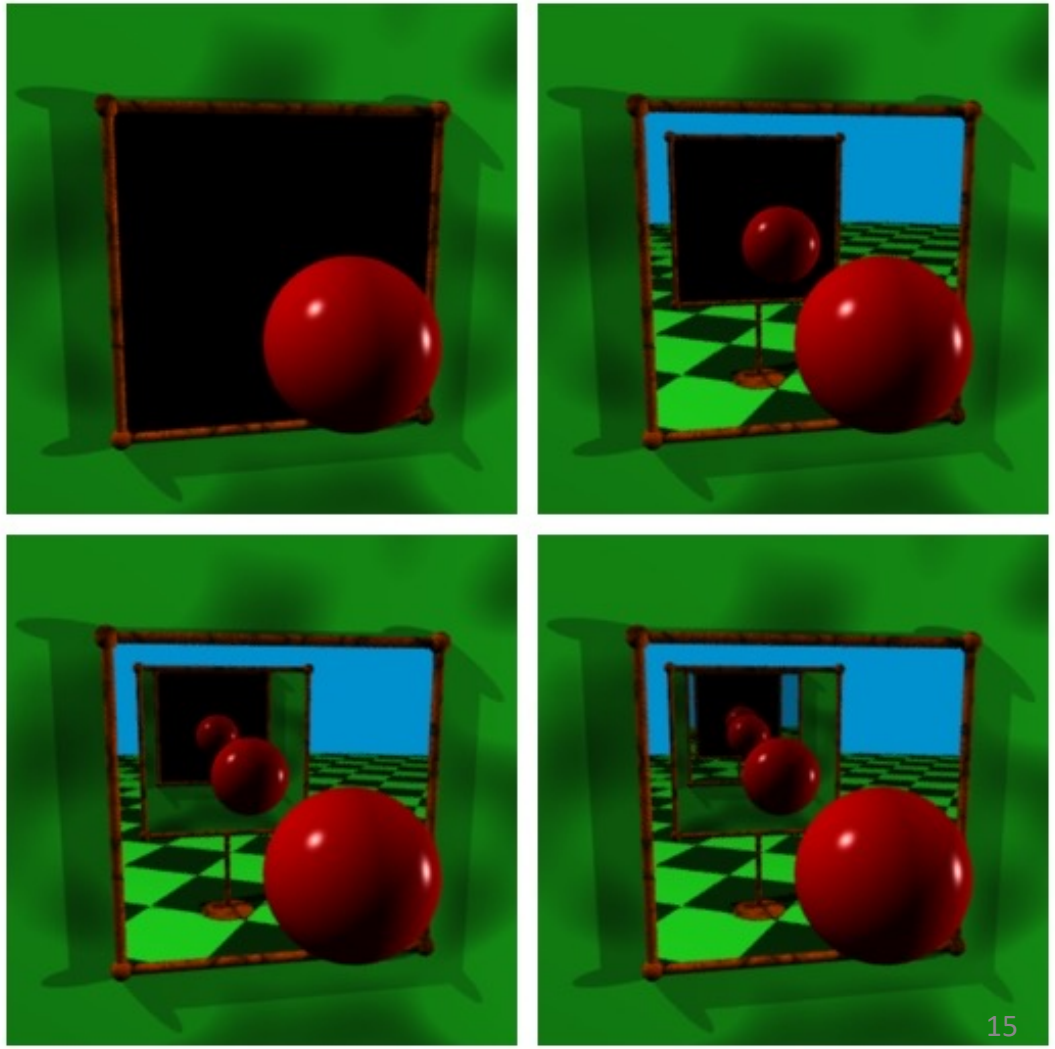


Refraction is Wavelength Dependent

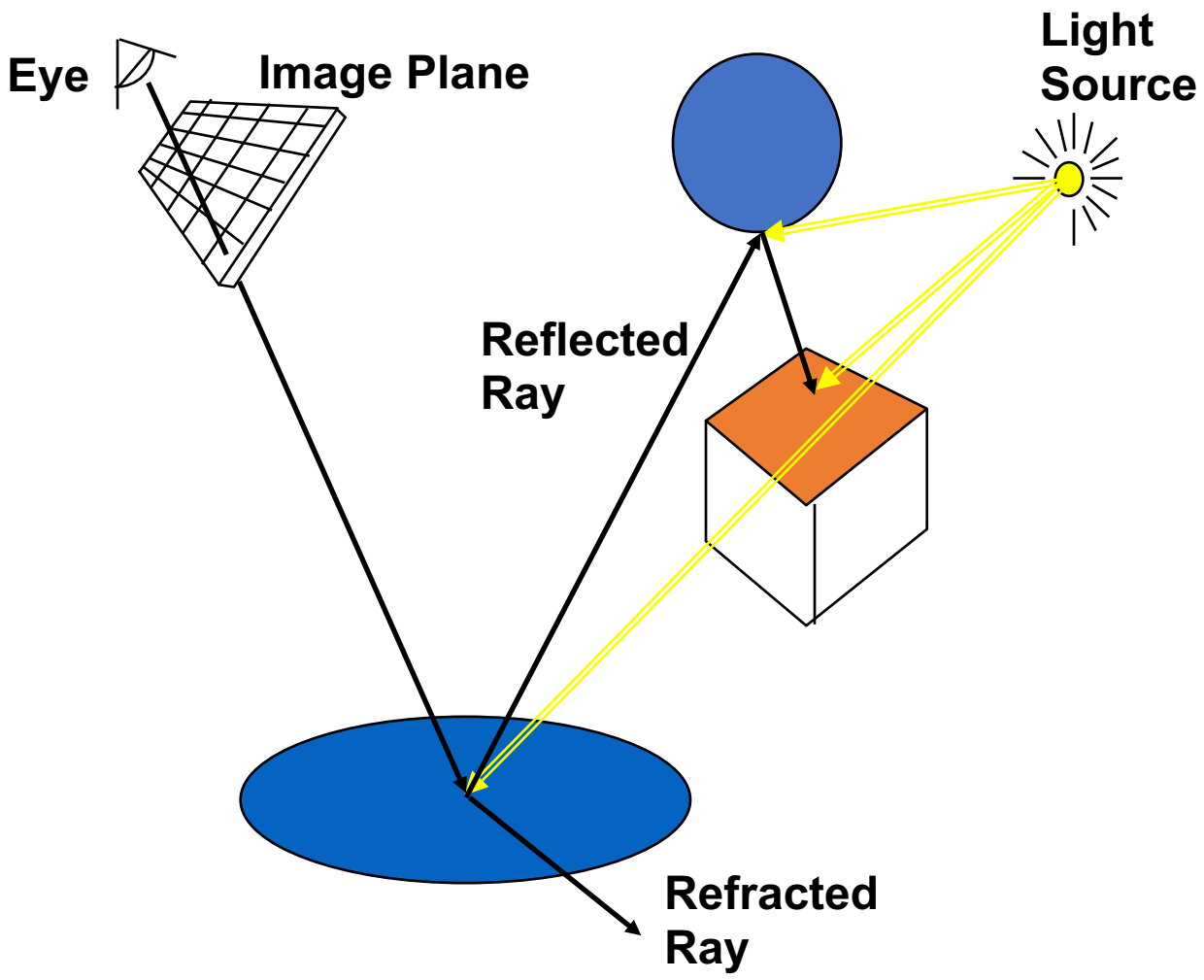


Termination in Ray-Tracing

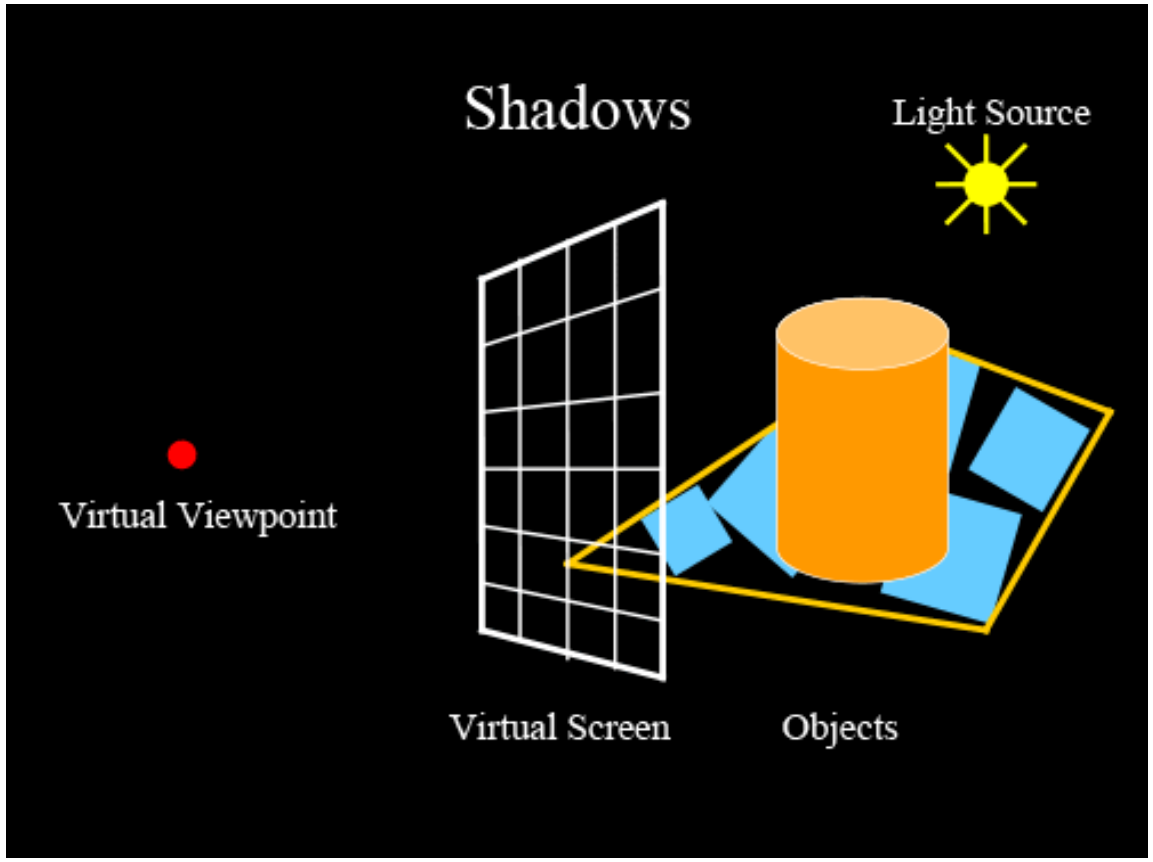
- Possible termination criteria:
  - No intersection
  - Contribution of secondary ray attenuated below a threshold
  - Maximal depth



Simulating Shadows



Shadows



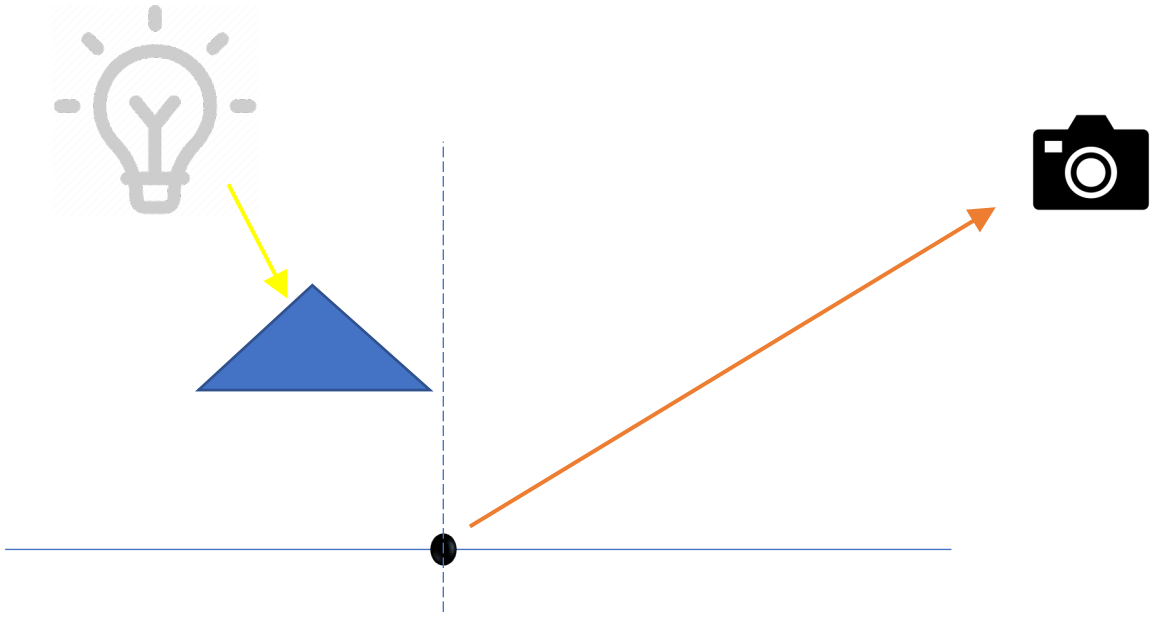


image credit http://studiomaven.org/Context__Overview_of_Ray-Traced_Rendering.html

Simulating Shadows

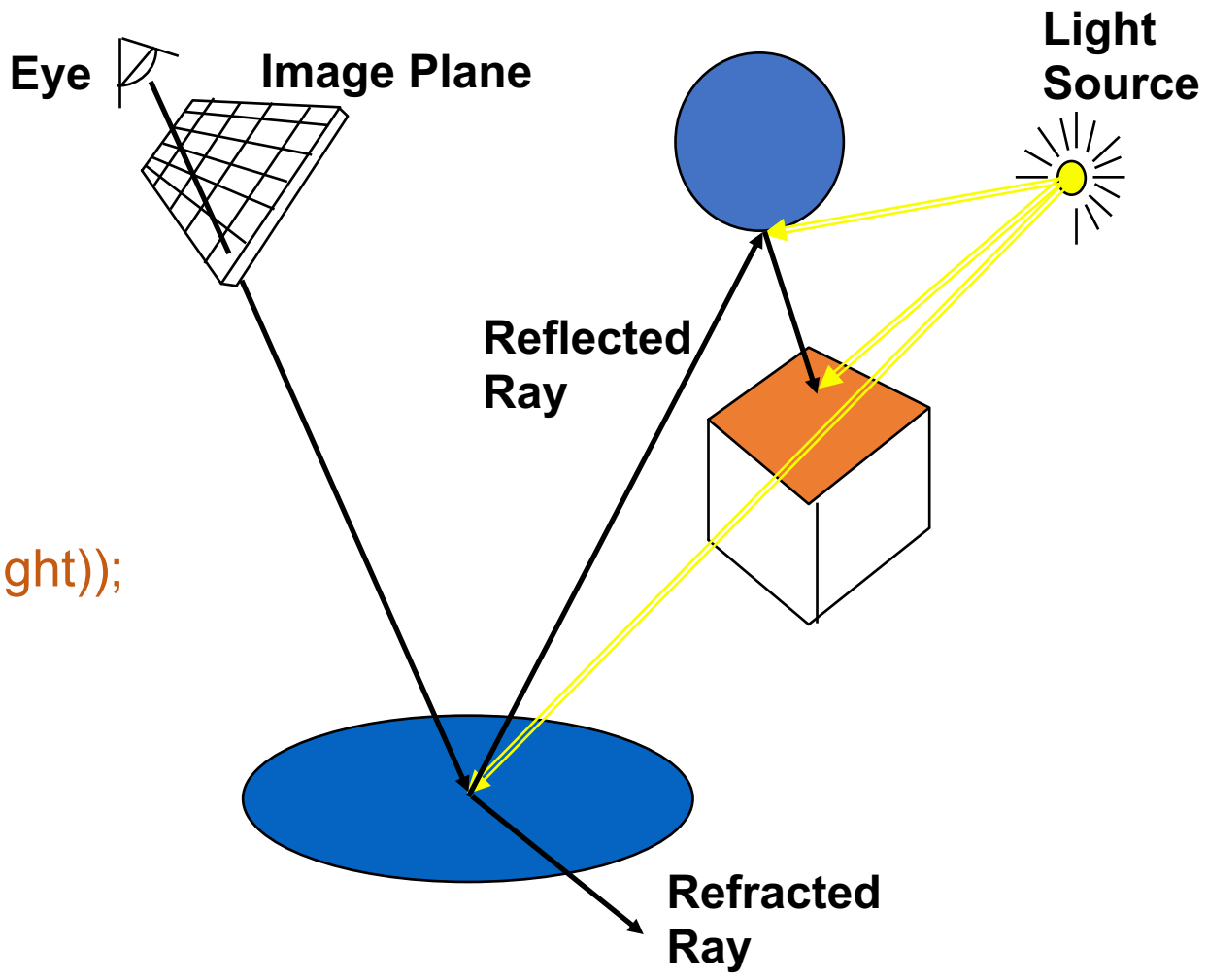
- Trace ray from each ray-object intersection point to light source(s)
  - If no line-of-sight ⇒ point is shadowed

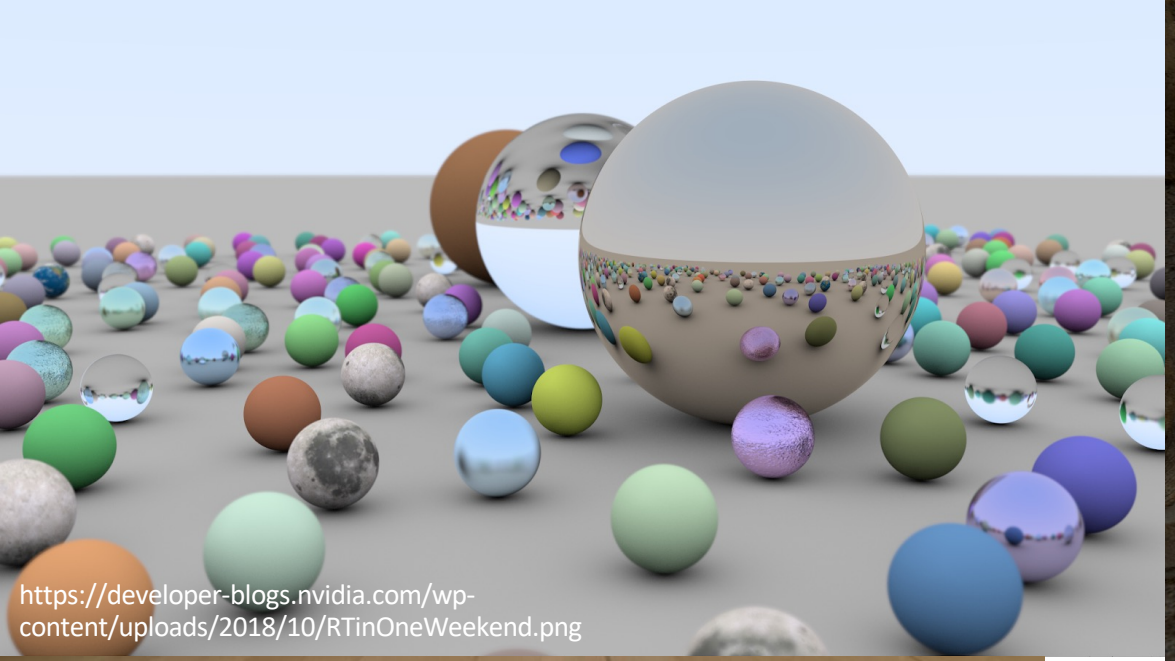
- A shadow computation routine:

```
shadow = RayTrace(LightRay(p,obj,light));

return Shade(shadow, ReflectColor,
RefractColor, p, obj);
```

- to be included in the final shading

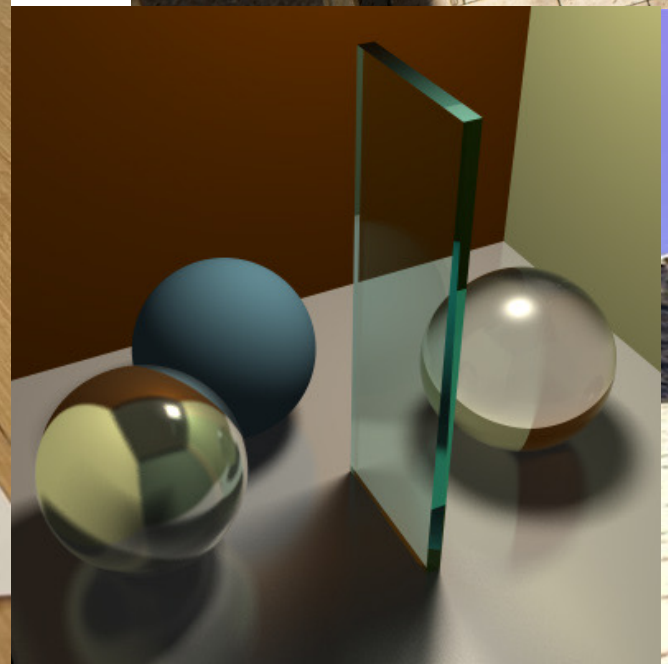




https://developer-blogs.nvidia.com/wp-content/uploads/2018/10/RTinOneWeekend.png

https://docs.unity3d.com/Packages/com.unity.render-pipelines.high-definition@12.0/manual/images/RayTracedShadows3.png

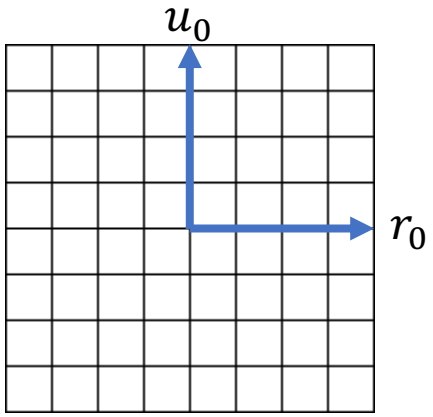
https://www.realtimerendering.com/blog/wp-content/uploads/2019/12/small_whitted_photo.jpg

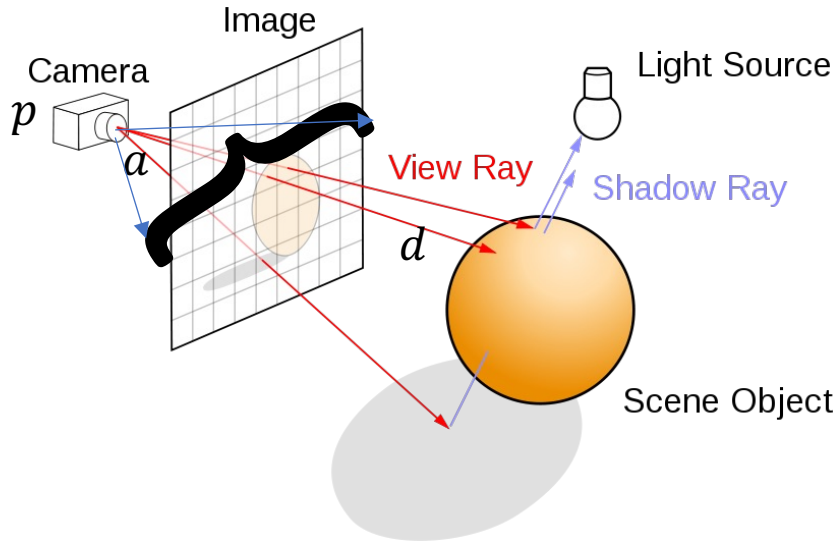


Camera Parameters

- p - camera location
- d - camera direction
- a - field of view angle (in the X axis)

$$r_0 = d \times y$$
$$u_0 = r_0 \times d$$



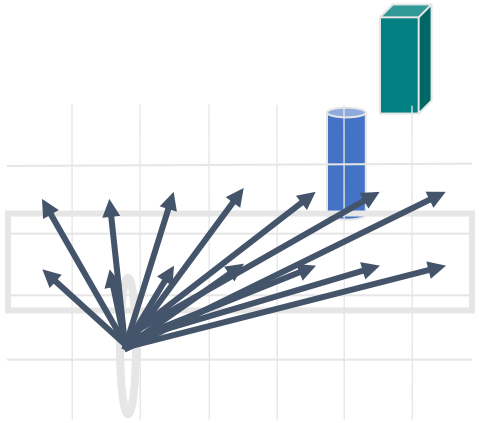


https://stackoverflow.com/questions/10012219/how-to-implement-depth-of-field-in-ray-tracer

- Assumption: The “up” vector of the camera has a positive Y-value

Shooting the Rays

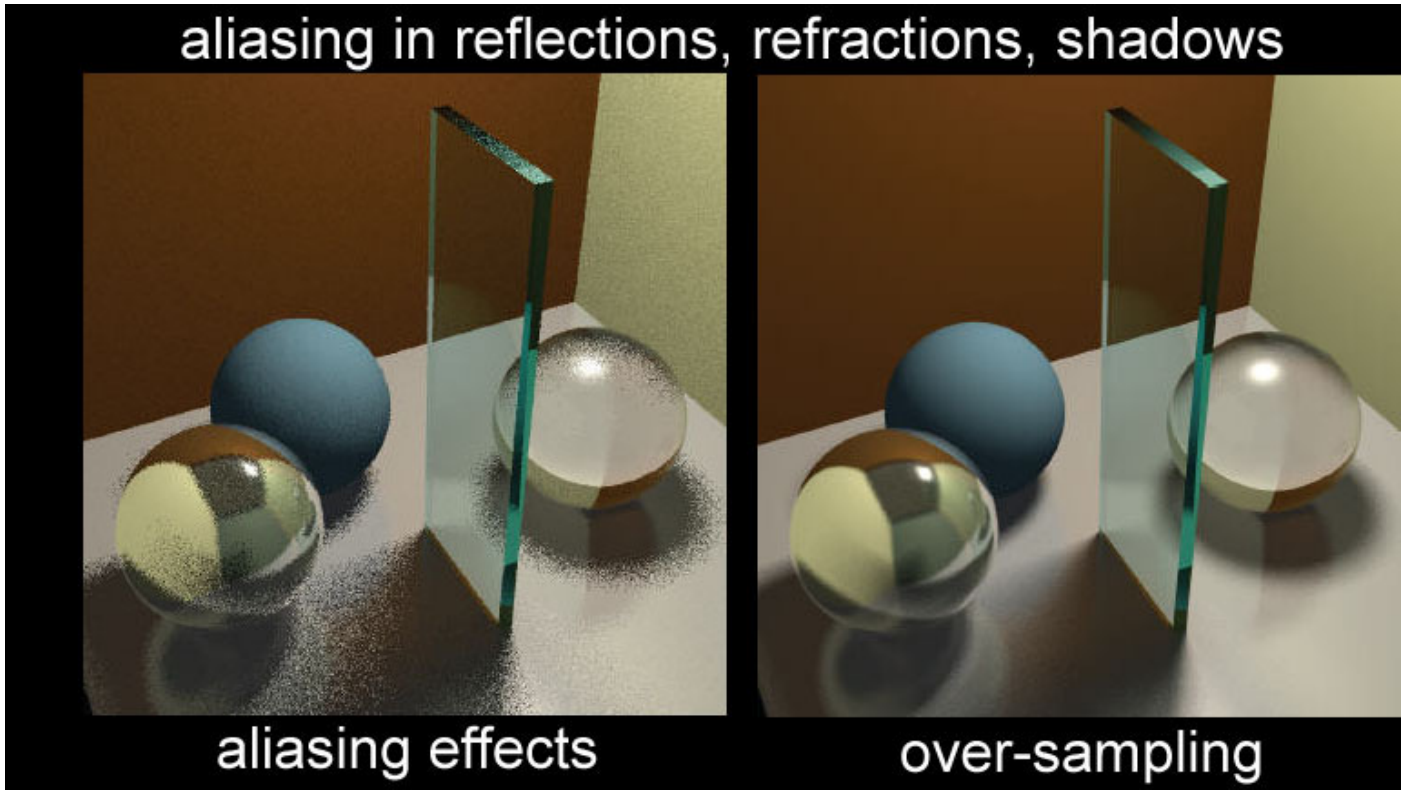
- One ray through center of each pixel

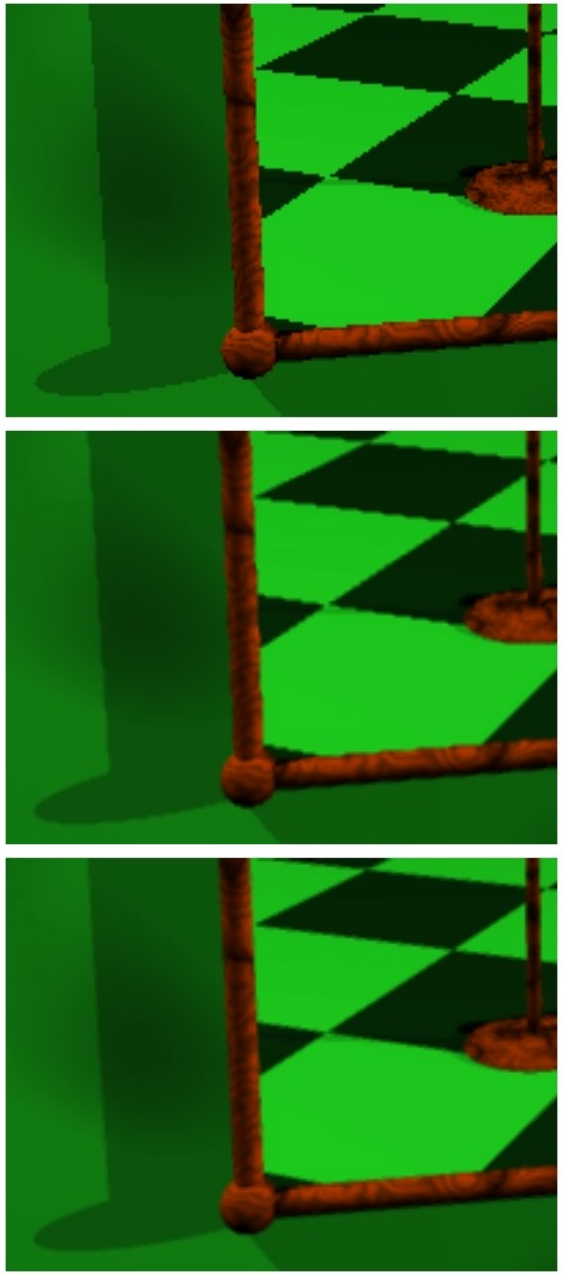


- More than one ray per pixel?

Supersampling

- Trace multiple primary rays per pixel and average their results



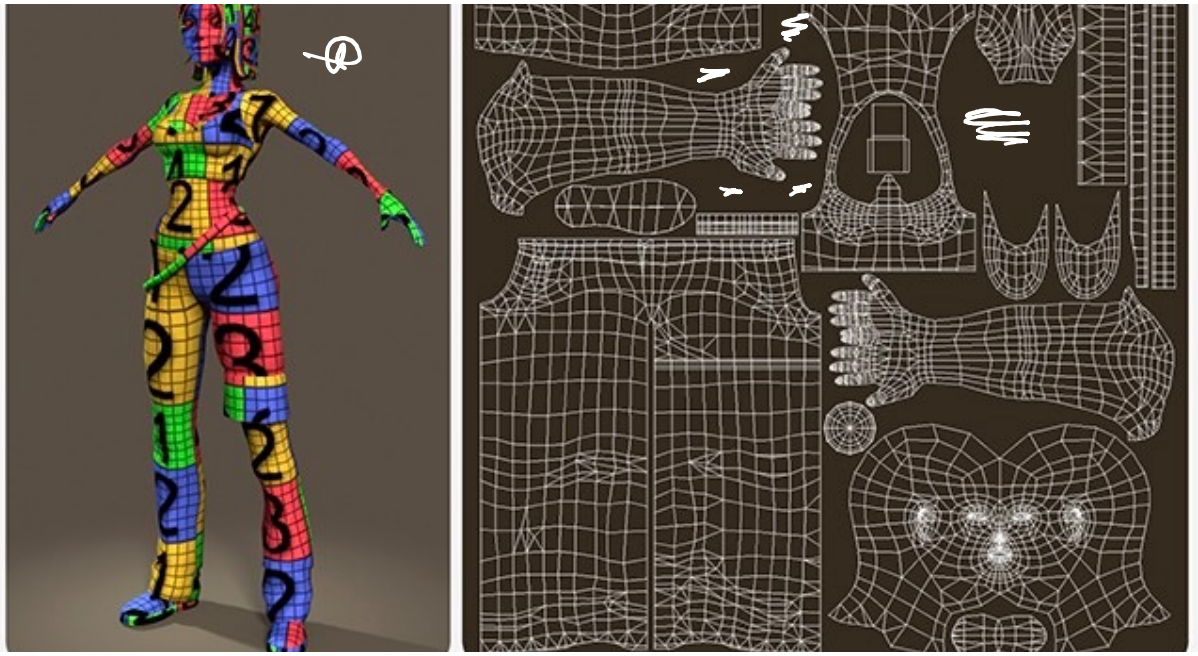


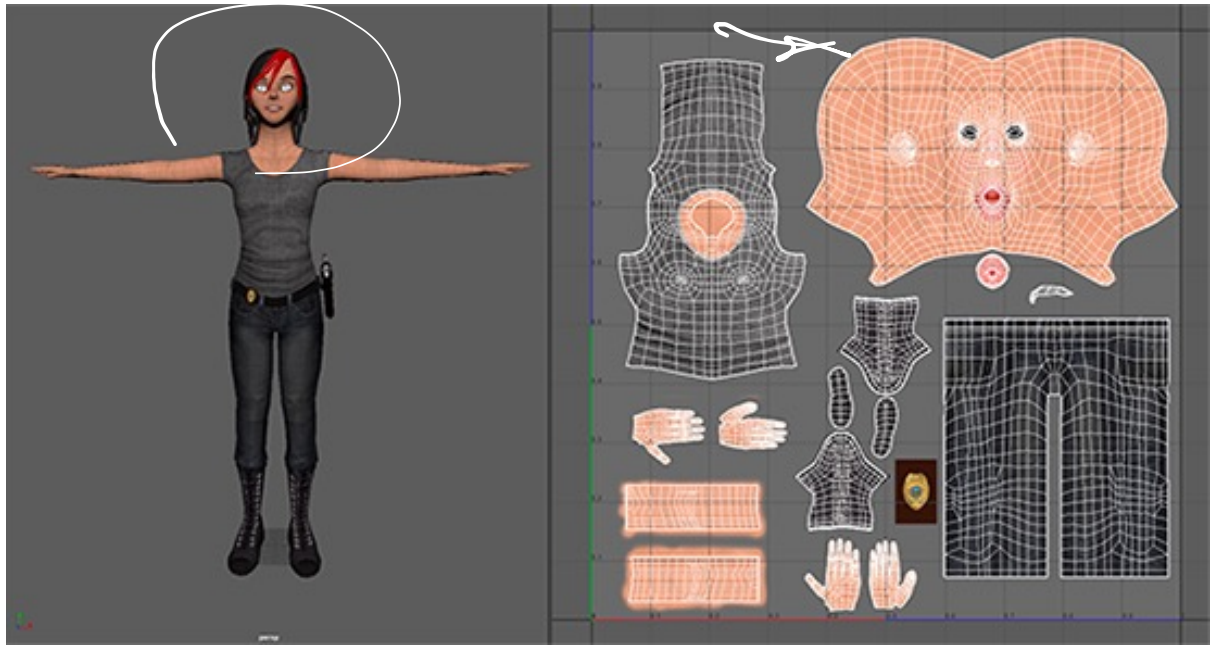
Advanced Phenomena

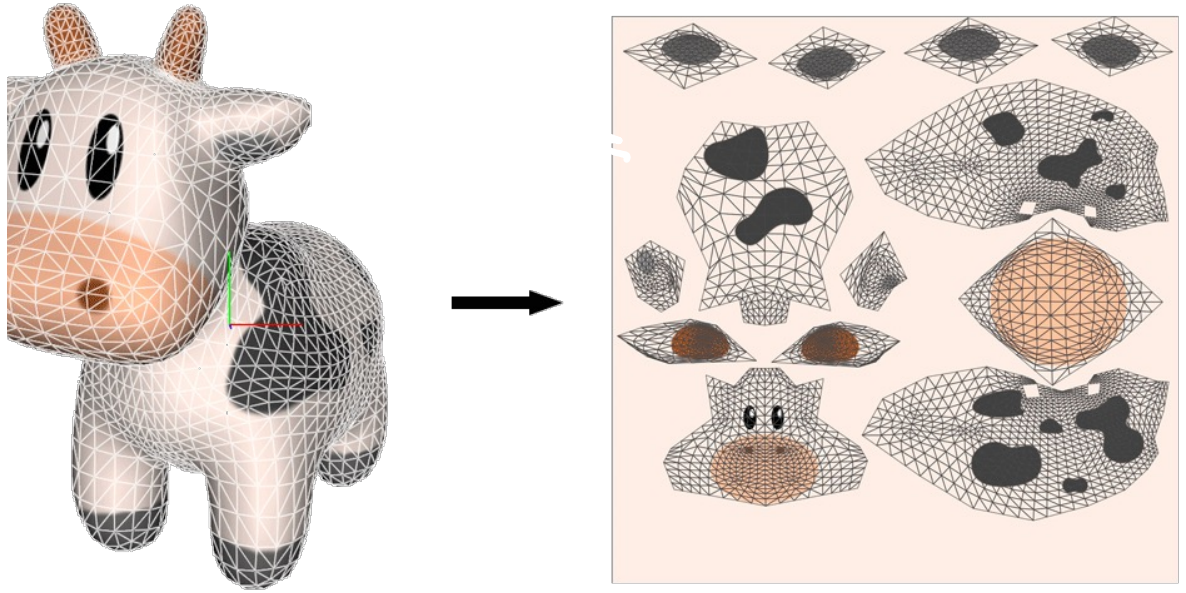
- Ray tracers cannot simulate:
  - Soft Shadows
  - Fog
  - Frequency-dependent light
  - Snell's law is different for different wave-lengths
  - Indirect light

How to deal with texture?

- Modulates diffuse color
- Or: the geometry
- Or: The normals

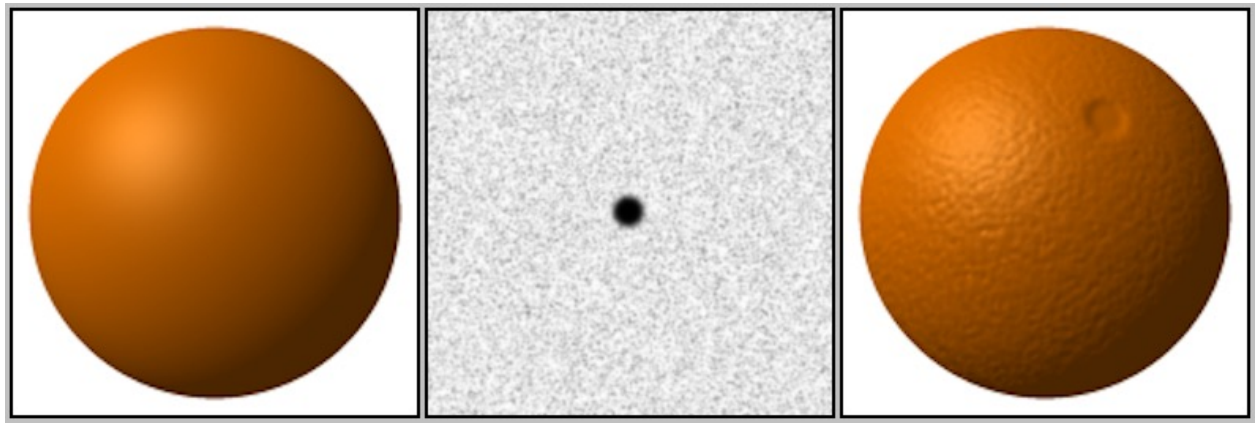






Bump Mapping

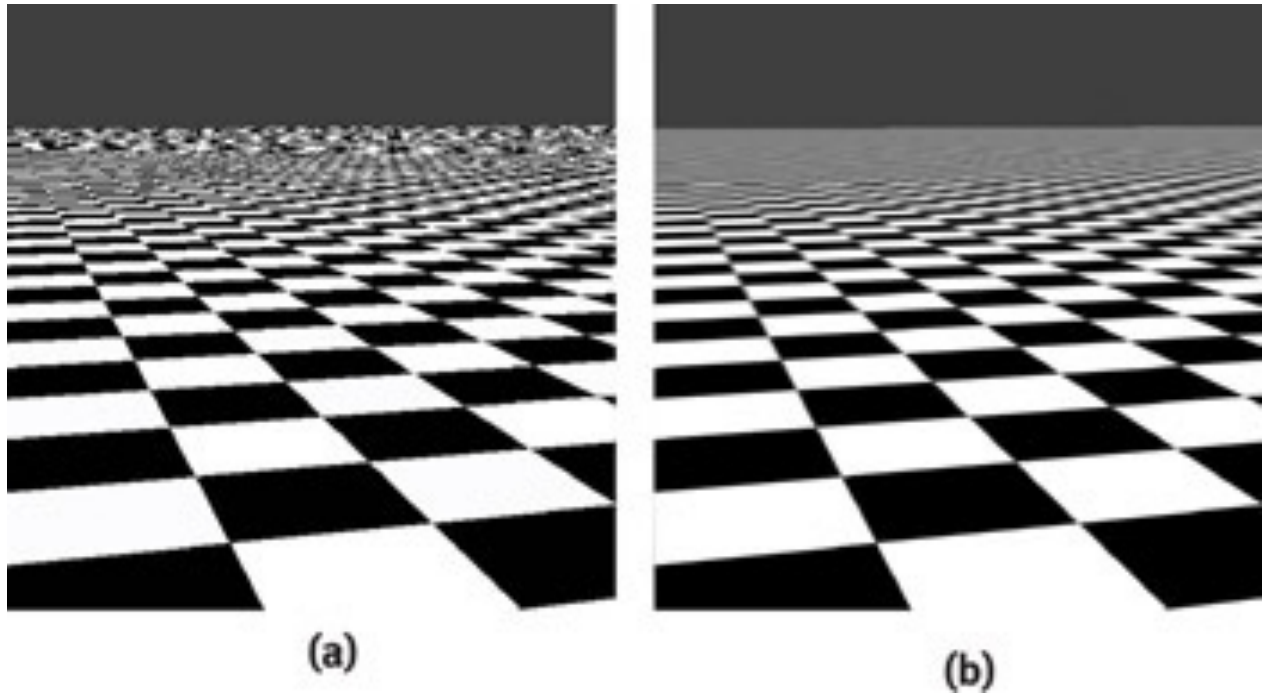
- Texture is normal perturbing, to “cheat” shading



https://www.sidefx.com/docs/houdini/shade/normalmaps.html

Problems

- How to generate maps?
- Finite resolution
- Artifacts
- More later!



http://www.cemyuksel.com/courses/conferences/siggraph2017-rethinking_texture_mapping/rethinking_texture_mapping_course_notes.pdf