Exercise 8

$$\underbrace{\underbrace{3157}_{x_1}\underbrace{6248}_{x_2}}_{x} * \underbrace{\underbrace{1342}_{y_1}\underbrace{7964}_{y_2}}_{y}$$

a b c d

$x_{11} = 3157$, $x_{12} = 6248$, $x_{21} = 9715$, $x_{22} = 4208$

$y_{11} = 1342$, $y_{12} = 7964$, $y_{21} = 57403$, $y_{22} = 3786$

S_1

$a \cdot c$

S'_3: $p_1 = (a+b)(c+d) = (x_{11}+x_{12})(y_{11}+y_{12}) = (3157+6248)(1342+7964) = 87522930$

S'_1: $p_2 = a \cdot c = (3157)(1342) = 4236694$ $(100000000) = 4236694'0000'0000$

S'_2: $p_3 = a \cdot c = (6248)(7964) = 49759072$ ($x_{12} \cdot y_{12}$)

$$\begin{array}{r} 335271640000 \\ + \quad 49759072 \\ \hline 4240047216990 72 \end{array}$$

S'_4: $p_4 = s'_3 - s'_2 - s'_1 = p_1 - p_3 - p_2 = 33527164$ (10000)

$s'_1 \cdot (b^n) + s'_4 \cdot (b^{n/2}) + s'_2$ where $n = 8$, $b = 10$

$a \cdot c$

$x_1 \cdot y_1 = S_1 (10^8) + S_4 (10^4) + S_2 = \boxed{4240047216990 72}$

S_2

$b \cdot d$

$x_2 \cdot y_2 = (\underbrace{9715}_{a}\ \underbrace{4208}_{b})(\underbrace{5740}_{c}\ \underbrace{3786}_{d})$

$S_1 \rightarrow ac = (9715)(5740) = 55764100$

$S_2 \rightarrow bd = (4208)(3781) = 15931488$

$S_3 \rightarrow = (13923)(9526) = 132630498$

$S_4 \rightarrow = S_3 - S_2 - S_1 = 60934010$

$S_1 (b^n) + S_4 (b^{n/2}) + S_2$ where $b = 10$, $n = 8$

$= \boxed{5577019365631488}$

S_3

$S_3 = (a+b) \cdot (c+d) = (\overbrace{12873045}^{x}\overbrace{6}^{y})(70831750)$

a	b	c	d
x_1	x_2	y_1	y_2
1287	30456	7083	1750

$S_1 = ac = (1287)(7083) = 9115821$

$S_2 = bd = (30456)(1750) = 53298000$

$S_3 = (a+b)(c+d) = (31743)(8833) = 280385919$

$S_4 = S_3 - S_2 - S_1 = 217972098$

$S_1(b^n) + S_4(b^{n/2}) + S_2$ where $b=10$, $n=8$

$= 9118203476778000$

$S_4 = S_3 - S_2 - S_1 = 3117193903474 40$

$S_1(b^n) + S_4(b^{n/2}) + S_2$ where $b=10$, $n=16$

$= S_1(10^{16}) + S_2(10^8) + S_2$

$= 424004752570866461176336503 1488$