Exercise 7 [4 points] (target due date: Monday, August 31)

Suppose you have a function defined by

$$T(n) = \log_2(1) + \log_2(2) + \cdots + \log_2(n).$$

Your job on this Exerise is to apply the previous discussion to find a well-known function $g(n)$ such that $T(n) \in \Theta(g(n))$—and prove it, of course, by bounding $T(n)$ above and below by functions that you can integrate, both of which are in $\Theta(g(n))$.

To receive credit for this Exercise, you must draw sketches showing exactly what functions you are using to bound T by, both above and below, and work out the Θ categories for the two integrals, which need to be the same to give the Θ category for T.

$T(n) = \log_2(1) + \log_2(2) + \cdots + \log_2(n)$.

claim: $T(n) \in \Theta(n \log_2 n)$

Proof: Consider the integral being bounded by above

$$T(n) \le \int_{x=1}^{n} \log_2(n)\, dx$$

because each term in $T(n)$ has value at most $\log_2(n)$. The term inside the integral comes out as it doesnt depend upon x.

Therefore $T(n) \le \log_2(n) \int_{x=1}^{n} dx = (n-1)\log_2 n \in \Theta(n \log_2 n)$

because constants dont matter in Theta analysis.

To bound below, note $T(n) \le \int_{x=1}^{n} \log_2(n)\, dx$.

We notice that $T(n) = \sum_{x=1}^{n} \log_2(x)$

$T(n) \ge (n \log_2 n - n) - (1 \log_2 1 - 1) \in \Theta(n \log_2 n)$ because we only care about the fastest growing terms

This proves that $T(n) \in \Theta(n \log_2 n)$