

Exercise 5

$$T(n) = 7T(n/2)$$

$$T(2^m) = 7[7T(\frac{2^m}{2^1})]$$

$$= 7[7T(2^{m-1})]$$

$$= 7^2 T(2^{m-1}) + \cancel{7 \cdot 0} \rightarrow 0$$

$$= 7^2 [7T(2^{m-2})]$$

$$= 7^3 (2^{m-2})$$

\vdots

$$= T(2^m) \in \Theta(7^m)$$

$$= \Theta(7^{\log_2(n)})$$

$$\boxed{= \Theta(n^{\log_2(7)})}$$

Exercise 6

$$T(n) = 3T(n/2) + n$$

$$T(2^m) = 3T(\frac{2^m}{2^1}) + 2^m$$

$$= 3[3T(2^{m-2}) + 2^{m-1}] + 2^m$$

$$= 3^2 T(2^{m-2}) + 3^1 \cdot 2^{m-1} + 3^0 \cdot 2^m$$

$$= 3^2 [3T(2^{m-3}) + 3^1 \cdot 2^{m-2} + 3^0 \cdot 2^{m-1}] + 3^0 \cdot 2^m$$

$$= 3^3 T(2^{m-3}) + (3 \cdot 3)^2 \cdot 2^{m-2} + 3^1 \cdot 2^{m-1} + 3 \cdot 2^m$$

$$T(2^m) \in \Theta(3^m)$$

$$= \Theta(3^{\log_2(n)})$$

$$\boxed{= \Theta(n^{\log_2(3)})}$$