Exercise 10

1) $8 \cdot 15 \,\%\, 17 = 1$

2) $17 - 8 = 9$

3) Find the number in Z_{17} you can multiply by 11 to give 1.

$18 = 1$ $\qquad 17 - 18 = 1$

$\frac{11}{}$ $\;\; 18 \xrightarrow{+17} 35 \xrightarrow{+17} 52 \xrightarrow{+17} 69 \quad 86 \quad 103 \quad 120 \quad 137 \quad \frac{154}{17} = 9.05$

$9.05 - 1 = 0.05$

$18 + 17 = 35 \,\%\, 17 = 1$ $\qquad 0.05 \cdot 17 = 1$

4) No

$3^4 = 81 \,\%\, 17 = 13 \Rightarrow x^2 = 13 \Rightarrow 3.605\ldots$

$3^5 = 243 \,\%\, 17 = 5 \Rightarrow x^2 = 5 \Rightarrow 2.236\ldots$

$3^6 = 729 \,\%\, 17 = 15 \Rightarrow x^2 = 15 \Rightarrow 3.872$

$3^7 = 2186 \,\%\, 17 = 11 \Rightarrow x^2 = 11 \Rightarrow 3.316\ldots$

$3^8 = 6561 \,\%\, 17 = 16 \Rightarrow x^2 = 16 \Rightarrow 4$

No...

$9^2 \,\%\, 17$ $\quad 10^2 \,\%\, 17$

$13 \qquad 15$

5)

- It will turn out that there is a very useful fact about the powers of any given value in Z_n. To explore this, fill in this table of the various powers of $\alpha = 3$, in Z_{17}. Be sure to do this efficiently by noting that $\alpha^k = \alpha^{k-1} \cdot \alpha$ for $k \geq 1$—do not compute α^k in Z_{17} using your calculator and then reduce the result mod 17 (this approach will, later, quickly overwhelm your calculator, even for relatively small values of n):

k	$3^{0}=1$	$3^{1}=3$	$3^{2}=9$	$3^{3}=27$	$3^{4}=81$	$3^{5}=243$	3^{6} 729	3^{7} 2187	3^{8} 6561	3^{9} 19683	3^{10} 59049
3^k	1	3	9	10	13	5	15	11	16	14	8

k	3^{11} 177147	3^{12} 531441	3^{13} 1594323	3^{14} 4782969	3^{15} 14348907	3^{16} 43046721	3^{17} 129140163	3^{18} 387420489	3^{19} 1162261467	3^{20} 3486784401
3^k	7	4	12	2	6	1	3	9	10	13