Excercise 3

Exercise 3 [10 points] (target due date: August 31)

This project is intended to give you some simple, concrete experience with the formal definitions of O and Θ. $c = 1000$

Let $f(n) = cn^3$. For each of the functions g given below, formally prove, directly from the definition, that $f \in \Theta(g)$. Each of your 6 separate proofs must clearly state N and c as in the definition of $O()$ and must clearly show the algebra proving your inequalities.

Make sure that you have some reason for choosing N and c, rather than just guessing and bluffing that the corresponding inequality is true (I'll be more skeptical of your inequalities if you haven't picked N and c sensibly).

a. $g(n) = 1000n^3$

b. $g(n) = n^3 + 1000n^2$

c. $g(n) = n^3 - 1000n^2$ (assume $n \geq 1000$ for this to be a legitimate function for our purposes)

a1) $n^3 \in O(1000n^3)$

for all $n \geq \overset{N}{\boxed{1}}$, $n^3 \leq \overset{c}{\boxed{1}}(1000n^3)$

$1n^3 \leq \boxed{1}(1000n^3)$

$\underline{-1n^3}$ $\quad 1\left(\dfrac{-1n^3}{1000}\right)$

0

$0 \leq 1000$

* Any number greater than the constant value of 1 for c, will always be greater for the inequality to be true. If c is negative then the inequality is false and is also proof by negation.

a2) $1000n^3 \in O(n^3)$

for all $n \geq \overset{N}{\boxed{1}}$, $1000n^3 \leq \overset{c}{\boxed{1001}}\, n^3$

$\dfrac{-1000n^3}{0} \leq \dfrac{-1000n^3}{1n^3}$

$-1(-1n^3) = 0$

$\dfrac{n^3}{n^2} = \dfrac{0}{n^2}$

$n = 0$

* Since the constant value for the left side of the equality is greater this time, c must be greater than or equal to g in this case for the inequality to be true.

b1) $n^3 \in O(n^3+1000n^2)$

$n \geq \overset{N}{\boxed{0}}$, $n^3 \leq \overset{C}{\boxed{1}}(n^3+1000n^2)$

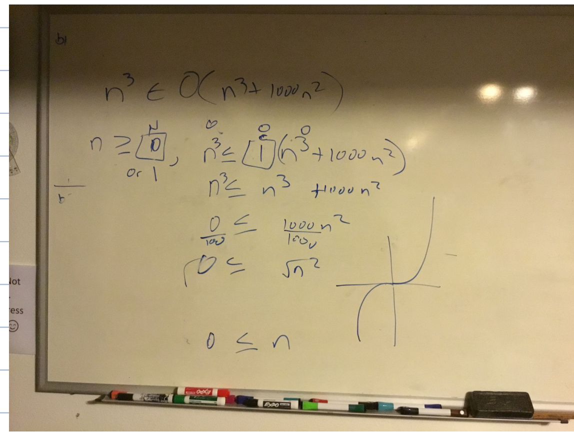
or 1

$n^3 \leq n^3+1000n^2$

$\frac{0}{1000} \leq \frac{1000n^2}{1000}$

$\sqrt{0} \leq \sqrt{n^2}$

$0 \leq n$



b2) $n^3+1000n^2 \in O(n^3)$

for all $n \geq \overset{N}{\boxed{0}}$, $n^3+1000n^2 \leq \overset{C}{\boxed{1000}}\, n^3$

$-1000n^3 \qquad -1000n^3$

$-999n^3+1000n^2 \leq 0$

$-n^2(999n-1000) \leq 0$

$((-n^2(999n-1000))(-1) \leq 0(-1)$

$n^2(999n-1000) \geq 0$

$n = \frac{1000}{999}$ or $n=0$

or

$n \approx 1.001001001$

c1) $n^3 \in O(n^3-1000n^2)$

For all $n \cancel{\geq} \overset{N}{\boxed{2000}}$ $\leq$, $n^3 \overset{\geq}{\cancel{\leq}} \overset{C}{\boxed{2}}(n^3-1000n^2)$

$n^3 \not\leq n^3-1000n^2$

$n^3 \leq 2n^3-2000n^2$

$-2n^3+2000n^2 \qquad -2n^3+2000n^2$

$n^3-2n^3+2000n^2 \leq 0$

$(-n^3+2000n^2) \leq 0$

$(-1)(-n^2(n-2000)) \leq 0(-1)$

$n^2(n-2000) \geq 0$

$n=0$ or $n=2000$

$n > 2000$

$$\boxed{\text{so... } n \le 2000 \,,\; n^3 \ge 2(n^3 - 1000n^2)}$$

c_2)

$n^3 - 1000n^2 \in O(n^3)$

For all $n \ge \boxed{0}$, $n^3 - 1000n^2 \le \boxed{1} n^3$

$$n^3 - 1000n^2 \le n^3$$
$$-n^3 \qquad -n^3$$

* If $c = 2$ then:

$n = -1000$

$-1000 < n < 0$

-or-

$n = 0$, $n > 0$

$$\frac{-1000n^2}{1000} \le \frac{0}{1000}$$

$$(-1)(-n^2) \le 0(-1)$$

$$n^2 \ge 0$$

$$n \ge 0$$