

1. Представьте двойной интеграл $\iint_D f(x, y) dx dy$ в виде повторного интеграла

а) с внешним интегрированием по x ;

б) с внешним интегрированием по y ,

если область D ограничена указанными линиями.

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| 1.1. $D: x+2y=0, \quad x+6y-4=0, \quad y=0$ | 1.19. $D: x=-1, \quad x=-2, \quad y=0, \quad y=x^2$ |
| 1.2. $D: x^2=2y, \quad 5x+2y-6=0$ | 1.20. $D: y=0, \quad x^2=-y, \quad x=\sqrt{1-y^2}$ |
| 1.3. $D: y=\sqrt{8-x^2}, \quad y=x, \quad y=0$ | 1.21. $D: y=0, \quad y=1, \quad y=x, \quad x=-\sqrt{2-y^2}$ |
| 1.4. $D: y=0, \quad x=0, \quad y=1, \quad y=\ln x$ | 1.22. $D: x=0, \quad y=1, \quad y=4, \quad y=-x$ |
| 1.5. $D: x^2+y^2=4, \quad x^2+(y+2)^2=4, \quad y=0 (x \leq 0)$ | 1.23. $D: x^2+y^2=9, \quad x^2+(y-3)^2=9, \quad x=0 (x \leq 0)$ |
| 1.6. $D: y=\sqrt{2-x^2}, \quad y=x^2$ | 1.24. $D: x=0, \quad x=-2, \quad y=x^2+4, \quad y=0$ |
| 1.7. $D: y=x^2-2, \quad y=x$ | 1.25. $D: x=0, \quad y=0, \quad y=1, \quad (x-3)^2+y^2=1$ |
| 1.8. $D: y=1, \quad x=0, \quad y=3, \quad y=x$ | 1.26. $D: x=\sqrt{9-y^2}, \quad y=x, \quad y=0$ |
| 1.9. $D: y=x^2, \quad y=-x^2+2, \quad y=0 (x \geq 0)$ | 1.27. $D: x+2y-6=0, \quad x=y, \quad y=0$ |
| 1.10. $D: y=x, \quad x=0, \quad y=\sqrt{9-x^2}$ | 1.28. $D: 3x+y=3, \quad y=-x, \quad y=3$ |
| 1.11. $D: y^2=2-x, \quad y=x$ | 1.29. $D: x=0, \quad y=1, \quad y=-1, \quad y=\log_{1/2}(x)$ |
| 1.12. $D: x=\sqrt{2-y^2}, \quad x=y^2, \quad y=0$ | 1.30. $D: y=0, \quad x=0, \quad y=1, \quad x=\sqrt{4-y^2}$ |
| 1.13. $D: x=0, \quad y=\cos x, \quad y=\sin x$ | 1.31. $D: y=0, \quad x+2y-12=0, \quad y=\lg x$ |
| 1.14. $D: y=1, \quad x=0, \quad y=3, \quad y=-x$ | 1.32. $D: x^2=2-y, \quad x+y=0$ |
| 1.15. $D: y=0, \quad y=x, \quad y=-\sqrt{2-x^2}$ | 1.33. $D: y=3-x^2, \quad y=-2x$ |
| 1.16. $D: y=0, \quad x=\sqrt{y}, \quad y=\sqrt{6-x^2}$ | 1.34. $D: y^2=2x, \quad x^2=2y, \quad x=1$ |
| 1.17. $D: y=-x, \quad y^2=x+3$ | 1.35. $D: y=\sqrt{4-x^2}, \quad y=\sqrt{3x}, \quad x=0$ |
| 1.18. $D: x=1, \quad y=0, \quad x=0, \quad y=\sqrt{4-x^2}$ | |

2. Вычислите площадь плоской области D , ограниченной заданными линиями

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| 2.1. а) $D: y^2=4x, \quad x+y=3, \quad y=0$ б) $D: \begin{cases} x^2+4x+y^2=0, & x^2+6x+y^2=0, \\ y=0, & y=x \end{cases}$ | 2.19. а) $D: y^2=4x, \quad x=\frac{8}{y^2+4}$ б) $D: \begin{cases} x^2+2x+y^2=0, & x^2+10x+y^2=0, \\ y=0, & y=\sqrt{3} \cdot x \end{cases}$ |
| 2.2. а) $D: y=6x^2, \quad x+y=2, \quad x=0$ б) $D: \begin{cases} x^2-6y+y^2=0, & x^2-10y+y^2=0, \\ x=0, & y=-x \end{cases}$ | 2.20. а) $D: y=4-x^2, \quad y=x^2-2x$ б) $D: \begin{cases} x^2-2y+y^2=0, & x^2-4y+y^2=0, \\ y=-\frac{x}{\sqrt{3}}, & y=-\sqrt{3} \cdot x \end{cases}$ |
| 2.3. а) $D: y^2=x+2, \quad x=2$ | 2.21. а) $D: x=y^2+1, \quad x+y=3$ |

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| 6) $D: \begin{cases} x^2 + 2y + y^2 = 0, & x^2 + 4y + y^2 = 0, \\ x = 0, & y = \sqrt{3} \cdot x \end{cases}$ | 6) $D: \begin{cases} x^2 - 2x + y^2 = 0, & x^2 - 6x + y^2 = 0, \\ y = -\frac{x}{\sqrt{3}}, & y = -\sqrt{3} \cdot x \end{cases}$ |
| 2.4. a) $D: x = -2y^2, \quad x = 1 - 3y^2, \quad x = 0, \quad y = 0$ 6) $D: \begin{cases} x^2 - 4y + y^2 = 0, & x^2 - 6y + y^2 = 0, \\ x = 0, & y = -\sqrt{3} \cdot x \end{cases}$ | 2.22. a) $D: x^2 = 3y, \quad y^2 = 3x$ 6) $D: \begin{cases} x^2 + 2x + y^2 = 0, & x^2 + 8x + y^2 = 0, \\ y = \frac{x}{\sqrt{3}}, & y = \sqrt{3} \cdot x \end{cases}$ |
| 2.5. a) $D: x = \sqrt{49 - y^2}, \quad x = 7 - \sqrt{49 - y^2}$ 6) $D: \begin{cases} x^2 + 2y + y^2 = 0, & x^2 + 6y + y^2 = 0, \\ x = 0, & y = -x/\sqrt{3} \end{cases}$ | 2.23. a) $D: x = \cos y, \quad x = y + 1, \quad x = 0$ 6) $D: \begin{cases} x^2 - 2x + y^2 = 0, & x^2 - 4x + y^2 = 0, \\ y = 0, & y = -x/\sqrt{3} \end{cases}$ |
| 2.6. a) $D: y = x^2 + 1, \quad x + y = 3$ 6) $D: \begin{cases} x^2 - 2y + y^2 = 0, & x^2 - 10y + y^2 = 0, \\ y = -\sqrt{3} \cdot x, & y = -x/\sqrt{3} \end{cases}$ | 2.24. a) $D: x = 4 - y^2, \quad x - y + 2 = 0$ 6) $D: \begin{cases} x^2 + 2x + y^2 = 0, & x^2 + 6x + y^2 = 0, \\ y = 0, & y = x/\sqrt{3} \end{cases}$ |
| 2.7. a) $D: y^2 = 4x, \quad x + y = 3, \quad y = 0$ 6) $D: \begin{cases} x^2 + 4y + y^2 = 0, & x^2 + 10y + y^2 = 0, \\ y = -\sqrt{3} \cdot x, & y = -x/\sqrt{3} \end{cases}$ | 2.25. a) $D: x = y^2, \quad x = \sqrt{2 - y^2}$ 6) $D: \begin{cases} x^2 - 2x + y^2 = 0, & x^2 - 6x + y^2 = 0, \\ y = 0, & y = -x \end{cases}$ |
| 2.8. a) $D: y = \cos x, \quad y = x + 1, \quad y = 0$ 6) $D: \begin{cases} x^2 - 2y + y^2 = 0, & x^2 - 4y + y^2 = 0, \\ x = 0, & y = -x \end{cases}$ | 2.26. a) $D: \frac{x^2}{4} + \frac{y^2}{1} = 1, \quad y = \frac{x}{2}, \quad y = 0$ 6) $D: \begin{cases} x^2 + 2x + y^2 = 0, & x^2 + 4x + y^2 = 0, \\ y = 0, & y = \sqrt{3} \cdot x \end{cases}$ |
| 2.9. a) $D: x = \sqrt{4 - y^2}, \quad y = \sqrt{3x}, \quad x = 0$ 6) $D: \begin{cases} x^2 + 6y + y^2 = 0, & x^2 + 8y + y^2 = 0, \\ x = 0, & y = x \end{cases}$ | 2.27. a) $D: y = -\sqrt{64 - x^2}, \quad y = \sqrt{64 - x^2} - 8$ 6) $D: \begin{cases} x^2 - 4x + y^2 = 0, & x^2 - 8x + y^2 = 0, \\ y = 0, & y = -\sqrt{3} \cdot x \end{cases}$ |
| 2.10. a) $D: y = x^2 + 2, \quad x = y, \quad x = 2, \quad x = 0$ 6) $D: \begin{cases} x^2 - 4y + y^2 = 0, & x^2 - 12y + y^2 = 0, \\ x = 0, & y = -x \end{cases}$ | 2.28. a) $D: y = x^2, \quad y = \frac{3}{4}x^2 + 1$ 6) $D: \begin{cases} x^2 + 4x + y^2 = 0, & x^2 + 8x + y^2 = 0, \\ y = -x/\sqrt{3}, & y = -\sqrt{3} \cdot x \end{cases}$ |
| 2.11. a) $D: y = 4x^2, \quad 9y = x^2, \quad y = 2$ 6) $D: \begin{cases} x^2 - 4y + y^2 = 0, & x^2 - 8y + y^2 = 0, \\ x = 0, & y = \sqrt{3} \cdot x \end{cases}$ | 2.29. a) $D: x = y^2, \quad y^2 = 4 - x$ 6) $D: \begin{cases} x^2 - 4x + y^2 = 0, & x^2 - 6x + y^2 = 0, \\ y = -x/\sqrt{3}, & y = -\sqrt{3} \cdot x \end{cases}$ |
| 2.12. a) $D: y = \sqrt{49 - x^2}, \quad y = 7 - \sqrt{49 - x^2}$ 6) $D: \begin{cases} x^2 - 2y + y^2 = 0, & x^2 - 10y + y^2 = 0, \\ x = 0, & y = -x/\sqrt{3} \end{cases}$ | 2.30. a) $D: xy = 1, \quad x^2 = y, \quad y = 2$ 6) $D: \begin{cases} x^2 + 6x + y^2 = 0, & x^2 + 10x + y^2 = 0, \\ y = x/\sqrt{3}, & y = \sqrt{3} \cdot x \end{cases}$ |
| 2.13. a) $D: x = y^2, \quad x = \frac{3}{4}y^2 + 1, \quad y = 0$ 6) $D: \begin{cases} x^2 + 4y + y^2 = 0, & x^2 + 8y + y^2 = 0, \\ x = 0, & y = x/\sqrt{3} \end{cases}$ | 2.31. a) $D: y = \frac{8}{x^2 + 4}, \quad x^2 = 4y$ 6) $D: \begin{cases} x^2 - 4x + y^2 = 0, & x^2 - 8x + y^2 = 0, \\ y = x/\sqrt{3}, & y = 0 \end{cases}$ |
| 2.14. a) $D: y = \sqrt{2 - x^2}, \quad y = x^2$ | 2.32. a) $D: x = \sqrt{27 - y^2}, \quad x = 3\sqrt{3} - \sqrt{27 - y^2}$ |

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| б) $D: \begin{cases} x^2 - 6y + y^2 = 0, & x^2 - 8y + y^2 = 0, \\ y = -\sqrt{3} \cdot x, & y = -x/\sqrt{3} \end{cases}$ | б) $D: \begin{cases} x^2 - 2x + y^2 = 0, & x^2 - 4x + y^2 = 0, \\ y = -x, & y = 0 \end{cases}$ |
| 2.15. а) $D: x = -\sqrt{49 - y^2}, \quad x = \sqrt{49 - y^2} - 7$ б) $D: \begin{cases} x^2 + 8y + y^2 = 0, & x^2 + 10y + y^2 = 0, \\ y = -\sqrt{3} \cdot x, & y = -x/\sqrt{3} \end{cases}$ | 2.33. а) $D: y = x^2 + 4x, \quad y = x + 4$ б) $D: \begin{cases} x^2 + 4x + y^2 = 0, & x^2 + 8x + y^2 = 0, \\ y = -x, & y = 0 \end{cases}$ |
| 2.16. а) $D: 2y = \sqrt{x}, \quad x + y = 5, \quad x = 0$ б) $D: \begin{cases} x^2 - 10y + y^2 = 0, & x^2 - 12y + y^2 = 0, \\ y = -\sqrt{3} \cdot x, & y = -x \end{cases}$ | 2.34. а) $D: y = 2^x, \quad y = 2x - x^2, \quad x = 2, \quad x = 0$ б) $D: \begin{cases} x^2 - 2x + y^2 = 0, & x^2 - 12x + y^2 = 0, \\ y = -x, & y = -x/\sqrt{3} \end{cases}$ |
| 2.17. а) $D: x = -\sqrt{27 - y^2}, \quad x = \sqrt{27 - y^2} - 3\sqrt{3}$ б) $D: \begin{cases} x^2 + 2y + y^2 = 0, & x^2 + 12y + y^2 = 0, \\ y = x/\sqrt{3}, & y = x \end{cases}$ | 2.35. а) $D: y^2 = 4x, \quad x + y = 3, \quad y = 0$ б) $D: \begin{cases} x^2 + 8x + y^2 = 0, & x^2 + 12x + y^2 = 0, \\ y = -x, & y = -\sqrt{3} \cdot x \end{cases}$ |
| 2.18. а) $D: y = -2x^2 + 2, \quad y = -6$ б) $D: \begin{cases} x^2 - 4y + y^2 = 0, & x^2 - 12y + y^2 = 0, \\ y = -\sqrt{3} \cdot x, & y = -x/\sqrt{3} \end{cases}$ | 2.36. а) $D: y^2 = 4 - x, \quad y = x + 2, \quad y = 2, \quad y = -2$ б) $D: \begin{cases} x^2 - 4x + y^2 = 0, & x^2 - 12x + y^2 = 0, \\ y = -x/\sqrt{3}, & y = -\sqrt{3} \cdot x \end{cases}$ |

3. Вычислите массу неоднородной пластины D , ограниченной заданными линиями, если поверхностная плотность в каждой её точке $\mu = \mu(x, y)$

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| 3.1. $D: y^2 = x, \quad x = 3, \quad \mu(x, y) = x$ | 3.19. $D: y = x^2, \quad y = 4, \quad \mu(x, y) = 2x + 5y + 10$ |
| 3.2. $D: \begin{cases} x^2 + y^2 = 4, & x^2 + y^2 = 100, & x = 0, & y = 0, \\ (x \leq 0, & y \leq 0), & \mu(x, y) = -\frac{7y + x}{x^2 + y^2} \end{cases}$ | 3.20. $D: \begin{cases} x^2 + y^2 = 1, & x^2 + y^2 = 49, & x = 0, & y = 0, \\ (x \geq 0, & y \geq 0), & \mu(x, y) = \frac{5x + 2y}{x^2 + y^2} \end{cases}$ |
| 3.3. $D: y = 0, \quad x = 0, \quad 2x + 3y = 6, \quad \mu(x, y) = y^2 / 2$ | 3.21. $D: y = 0, \quad x = 0, \quad x + y = 1, \quad \mu(x, y) = x^2$ |
| 3.4. $D: \begin{cases} x^2 + y^2 = 1, & x^2 + y^2 = 49, & x = 0, & y = 0, \\ (x \geq 0, & y \geq 0), & \mu(x, y) = \frac{5x + 2y}{x^2 + y^2} \end{cases}$ | 3.22. $D: \begin{cases} x^2 + y^2 = 9, & x^2 + y^2 = 36, & x = 0, & y = 0, \\ (x \leq 0, & y \geq 0), & \mu(x, y) = \frac{3y - 4x}{x^2 + y^2} \end{cases}$ |
| 3.5. $D: y = 0, \quad x = 0, \quad y = 4, \quad x = \sqrt{25 - y^2}, \quad \mu(x, y) = x$ | 3.23. $D: y = \pi/2, \quad y = \pi, \quad x = 1, \quad x = 2, \quad \mu(x, y) = y \sin(xy)$ |
| 3.6. $D: x^2 + y^2 = 1, \quad \mu(x, y) = 2 - x - y$ | 3.24. $D: y = 0, \quad x = 0, \quad x + y = 2, \quad \mu(x, y) = x^2 + y^2$ |
| 3.7. $D: x^2 + y^2 = 4y, \quad \mu(x, y) = 4 - y$ | 3.25. $D: y = 0, \quad x = 0, \quad x + y = 1, \quad \mu(x, y) = 2x^2 + y^2$ |
| 3.8. $D: x = y, \quad y = -x, \quad y = 1, \quad \mu(x, y) = \sqrt{1 - y}$ | 3.26. $D: y \leq 0, \quad x \geq 0, \quad x^2 + y^2 = 4, \quad \mu(x, y) = 4 - x^2$ |
| 3.9. $D: y = 2x, \quad x = 0, \quad x + y = 2, \quad \mu(x, y) = 2 - x - y$ | 3.27. $D: y = x^2, \quad y = 2, \quad \mu(x, y) = 2 - y$ |
| 3.10. $D: x = 1, \quad x = y^2, \quad \mu(x, y) = 4 - x - y$ | 3.28. $D: y = 0, \quad x = 0, \quad x + y = 1, \quad \mu(x, y) = x^2 + y^2$ |
| 3.11. $D: y = 0, \quad x^2 = 1 - y, \quad \mu(x, y) = 3 - x - y$ | 3.29. $D: y = x^2 + 1, \quad x + y = 3, \quad \mu(x, y) = 4x + 5y + 2$ |
| 3.12. $D: y = x^2, \quad x = y^2, \quad \mu(x, y) = 3x + 2y + 6$ | 3.30. $D: y = x^2 + 1, \quad x + y = 1, \quad \mu(x, y) = 2x + 5y + 8$ |
| 3.13. $D: x^2 + y^2 = 4x, \quad \mu(x, y) = 4 - x$ | 3.31. $D: y = 1, \quad x = 0, \quad x = y, \quad \mu(x, y) = x^2 + 2y^2$ |
| 3.14. $D: y = 0, \quad y = 2x, \quad x + y = 6, \quad \mu(x, y) = x^2$ | 3.32. $D: y = 0, \quad x = 0, \quad x + y = 1, \quad \mu(x, y) = x^2 + y^2$ |

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| 3.15. $D: y^2 = 1 - x, x = 0, \mu(x, y) = 2 - x - y$ | 3.33. $D: y = x, y = x^2, \mu(x, y) = 2x + 3y$ |
| 3.16. $D: y = \sqrt{x}, x = y, \mu(x, y) = 2 - x - y$ | 3.34. $D: x = 0, x + 2y + 2 = 0, x + y = 1, \mu(x, y) = x^2$ |
| 3.17. $D: y = 0, x = 1, x = y,$ $\mu(x, y) = x^2 + 2y^2 + 10$ | |
| 3.18. $D: y = 0, x = 0, x + 2y = 1,$ $\mu(x, y) = 2 - x^2 - y^2$ | |

4. Вычислите объём тела V , ограниченного поверхностями

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| 4.1. $y = x^2, x = y^2, z = 3x + 2y + 6, z = 0$ | 4.19. $y = x^2, y + z = 2, y = 0, z = 0, x = 0$ |
| 4.2. $z = 20 - x^2, z = -8x, y = x^2 + z^2, y = 0$ | 4.20. $x = \sqrt{z}, x = 1/z, z = 16, y = 4x^2 + z^2, y = 0$ |
| 4.3. $y^2 = 1 - x, x + y + z = 2, x = 0, z = 0$ | 4.21. $y = 3\sqrt{x}, y = 3x, x + y + z = 10, z = 0$ |
| 4.4. $y = 5 - z^2, y = -4z, x = 4 + y^2 + z^2, x = 0$ | 4.22. $x = 10 + z^2 + 2y^2, z = y, z = 1, y = 0, x = 0$ |
| 4.5. $z = 3x^2 + 2y^2 + 1, y = x^2 - 1, y = 1, z = 0$ | 4.23. $z = x^2, x + y = 6, y = 2x, y = 0, z = 0$ |
| 4.6. $y = x + 3, x = 4, z = \sqrt{25 - x^2}, z = 0, y = 0$ | 4.24. $z = 10 + x^2 + 2y^2, x = y, x = 1, y = 0, z = 0$ |
| 4.7. $2x + 3y - 12 = 0, 2z = y^2, x = 0, y = 0, z = 0$ | 4.25. $y = 3\sqrt{z}, y = 3/z, x = 3 + y^2 + 2z^2, x = 0, z = 5$ |
| 4.8. $x = 4z^2 + 3y^2, y = z, y = 3z, z = 2, x = 0$ | 4.26. $z = 4 - x^2, x + y^2 = 4, x = 0, z = 0$ |
| 4.9. $x = 1 - z^2, x + y + z = 3, x = 0, y = 0$ | 4.27. $z = 2x^2 + y^2, x + y = 4, x = 0, y = 0, z = 0$ |
| 4.10. $y = x^2, x - 2z + 2 = 0, x + z - 7 = 0, x = 0, y = 0$ | 4.28. $y = 1 - x^2, x + y + z = 3, x = 0, y = 0, z = 0$ |
| 4.11. $y = x^3, y = 1, x = 0, z = 2 - x^2 - y^2, z = 0$ | 4.29. $y = \sqrt{x}, y = x, x + y + z = 2, z = 0$ |
| 4.12. $x + z = 2, z^2 = 4 - y, y = 0, x = 0, z = 0$ | 4.30. $z = x, y = 4, x = \sqrt{25 - y^2}, x = 0, z = 0$ |
| 4.13. $y = 7 - z^2, z = x, z = -x, y = 0, z = 2$ | 4.31. $z = 2x^2 + y^2, y = x, y = 3x, x = 2, z = 0, (z \geq 0)$ |
| 4.14. $y = 2x, x + y + z = 2, x = 2, y = 0, z = 0, x = 0$ | 4.32. $z = 2x^2 + 3y^2, y = x^2, y = x, z = 0, (z \geq 0)$ |
| 4.15. $y = x^2, y = 4, z = 2x + 5y + 10, z = 0$ | 4.33. $z = x^2, x - 2y + 2 = 0, x + y - 7 = 0, x = 0, z = 0$ |
| 4.16. $z = 2x^2 + y^2, z = 0, x + y = 1, x = 0, y = 0$ | 4.34. $z = 2 - x^2 - y^2, x + 2y = 1, x = 0, y = 0, z = 0$ |
| 4.17. $x = y^2, x = 1, x + y + z = 4, z = 0$ | 4.35. $z = x^2 + y^2, x + y = 1, x = 0, y = 0, z = 0$ |
| 4.18. $x^2 = 1 - y, y = 0, x + y + z = 3, z = 0$ | |

5. Вычислите объём тела V ,

а) ограниченного поверхностями, используя переход к цилиндрическим координатам;

б) ограниченного неравенствами, используя переход к сферическим координатам

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| 5.1. а) $\begin{cases} 9(x^2 + y^2) = z^2, x^2 + y^2 = 4, x = 0, y = 0, \\ z = 0, (x \geq 0, y \geq 0, z \geq 0) \end{cases}$ б) $\begin{cases} 1 \leq x^2 + y^2 + z^2 \leq 49, \\ -\sqrt{\frac{x^2 + y^2}{35}} \leq z \leq \sqrt{\frac{x^2 + y^2}{3}} \end{cases}$ | 5.19. а) $\begin{cases} x^2 + y^2 = 16z^2/49, x^2 + y^2 = 4z/7, \\ x = 0, y = 0, (x \geq 0, y \geq 0) \end{cases}$ б) $\begin{cases} 25 \leq x^2 + y^2 + z^2 \leq 100, \\ 3\sqrt{x^2 + y^2} \leq z \end{cases}$ |
| 5.2. а) $x^2 + y^2 + z^2 = 4, x^2 + y^2 = 1, (x^2 + y^2 \leq 1)$ б) $\begin{cases} 4 \leq x^2 + y^2 + z^2 \leq 81, \\ -x \leq y \leq 0 \end{cases}$ | 5.20. а) $\begin{cases} x^2 + y^2 + z^2 = 16, x^2 + y^2 = z^2, \\ x = 0, y = 0, (x \geq 0, y \geq 0, z \leq 0) \end{cases}$ |

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| | б) $\begin{cases} 16 \leq x^2 + y^2 + z^2 \leq 121, \\ 0 \leq y \leq \sqrt{3} \cdot x \end{cases}$ |
| 5.3. а) $\begin{cases} x^2 + y^2 = 1, \ x^2 + y^2 = z, \ x = 0, \ y = 0, \\ z = 0 \ (x \geq 0, \ y \geq 0) \end{cases}$ б) $\begin{cases} 4 \leq x^2 + y^2 + z^2 \leq 64, \\ -\sqrt{\frac{x^2 + y^2}{3}} \leq z \end{cases}$ | 5.21. а) $\begin{cases} 36(x^2 + y^2) = z^2, \ x^2 + y^2 = 9, \\ x = 0, \ z = 0, \ (x \leq 0, \ z \leq 0) \end{cases}$ б) $\begin{cases} 16 \leq x^2 + y^2 + z^2 \leq 64, \\ -\sqrt{\frac{x^2 + y^2}{63}} \leq z \leq \sqrt{\frac{x^2 + y^2}{3}} \end{cases}$ |
| 5.4. а) $\begin{cases} x^2 + y^2 = z^2 / 49, \ x^2 + y^2 = z / 7, \\ x = 0, \ y = 0, \ (x \geq 0, \ y \geq 0) \end{cases}$ б) $\begin{cases} 1 \leq x^2 + y^2 + z^2 \leq 36, \\ x \leq y \leq \sqrt{3} \cdot x \end{cases}$ | 5.22. а) $\begin{cases} x^2 + y^2 + z^2 = 16, \ x^2 + y^2 = 4, \\ (x^2 + y^2 \leq 4) \end{cases}$ б) $\begin{cases} 49 \leq x^2 + y^2 + z^2 \leq 100, \\ y \geq 0, \ y \geq \sqrt{3} \cdot x \end{cases}$ |
| 5.5. а) $\begin{cases} x^2 + y^2 + z^2 = 4, \ x^2 + y^2 = 4z^2, \\ x = 0, \ y = 0, \ (x \geq 0, \ y \geq 0, \ z \geq 0) \end{cases}$ б) $\begin{cases} 1 \leq x^2 + y^2 + z^2 \leq 49, \\ -\sqrt{\frac{x^2 + y^2}{3}} \leq z \leq 0 \end{cases}$ | 5.23. а) $\begin{cases} x^2 + y^2 = 4, \ x^2 + y^2 = 8z, \ x = 0, \ y = 0, \\ z = 0 \ (x \leq 0, \ y \geq 0) \end{cases}$ б) $\begin{cases} 25 \leq x^2 + y^2 + z^2 \leq 100, \\ -3\sqrt{x^2 + y^2} \leq z \leq -\sqrt{\frac{x^2 + y^2}{99}} \end{cases}$ |
| 5.6. а) $\begin{cases} 16(x^2 + y^2) = z^2, \ x^2 + y^2 = 1, \ x = 0, \ y = 0, \\ z = 0, \ (x \geq 0, \ y \geq 0, \ z \geq 0) \end{cases}$ б) $\begin{cases} 4 \leq x^2 + y^2 + z^2 \leq 81, \\ -x \leq y \leq -\sqrt{3} \cdot x \end{cases}$ | 5.24. а) $\begin{cases} x^2 + y^2 = 4z^2 / 25, \ x^2 + y^2 = 2z / 5, \\ x = 0, \ y = 0, \ (x \geq 0, \ y \leq 0) \end{cases}$ б) $\begin{cases} 16 \leq x^2 + y^2 + z^2 \leq 121, \\ \sqrt{3} \cdot x \leq y \leq 0 \end{cases}$ |
| 5.7. а) $x^2 + y^2 + z^2 = 16, \ x^2 + y^2 = 4, \ (x^2 + y^2 \leq 4)$ б) $\begin{cases} 4 \leq x^2 + y^2 + z^2 \leq 64, \\ 0 \leq z \leq \sqrt{\frac{x^2 + y^2}{15}} \end{cases}$ | 5.25. а) $\begin{cases} x^2 + y^2 + z^2 = 4, \ x^2 + y^2 = 3z^2, \\ x = 0, \ y = 0, \ (x \geq 0, \ y \leq 0, \ z \geq 0) \end{cases}$ б) $\begin{cases} 16 \leq x^2 + y^2 + z^2 \leq 81, \\ -\sqrt{\frac{x^2 + y^2}{63}} \leq z \end{cases}$ |
| 5.8. а) $\begin{cases} x^2 + y^2 = 4, \ x^2 + y^2 = 4z, \ x = 0, \ y = 0, \\ z = 0 \ (x \geq 0, \ y \geq 0) \end{cases}$ б) $\begin{cases} 1 \leq x^2 + y^2 + z^2 \leq 36, \\ \sqrt{3} \cdot x \leq y \leq -\sqrt{3} \cdot x \end{cases}$ | 5.26. а) $\begin{cases} 25(x^2 + y^2) = z^2, \ x^2 + y^2 = 4, \ y = 0, \\ x = 0, \ z = 0, \ (x \leq 0, \ z \leq 0, \ y \geq 0) \end{cases}$ б) $\begin{cases} 49 \leq x^2 + y^2 + z^2 \leq 100, \\ y \leq \sqrt{3} \cdot x, \ y \leq \frac{x}{\sqrt{3}} \end{cases}$ |
| 5.9. а) $\begin{cases} x^2 + y^2 = z^2 / 81, \ x^2 + y^2 = z / 9, \\ x = 0, \ y = 0, \ (x \geq 0, \ y \geq 0) \end{cases}$ б) $\begin{cases} 1 \leq x^2 + y^2 + z^2 \leq 49, \\ z \leq 3\sqrt{x^2 + y^2} \end{cases}$ | 5.27. а) $\begin{cases} x^2 + y^2 + z^2 = 9, \ x^2 + y^2 = 4, \ y = 0, \\ (x^2 + y^2 \leq 4, \ y \leq 0) \end{cases}$ б) $\begin{cases} 25 \leq x^2 + y^2 + z^2 \leq 100, \\ 0 \leq z \leq \sqrt{\frac{x^2 + y^2}{99}} \end{cases}$ |
| 5.10. а) $\begin{cases} x^2 + y^2 + z^2 = 64, \ x^2 + y^2 = 8z^2, \\ x = 0, \ y = 0, \ (x \geq 0, \ y \geq 0, \ z \geq 0) \end{cases}$ | 5.28. а) $\begin{cases} x^2 + y^2 = 1, \ x^2 + y^2 = 6z, \ x = 0, \ y = 0, \\ z = 0 \ (x \geq 0, \ y \leq 0) \end{cases}$ |

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| $\text{б)} \begin{cases} 4 \leq x^2 + y^2 + z^2 \leq 81, \\ y \leq \sqrt{3} \cdot x, \quad y \leq 0 \end{cases}$ | $\text{б)} \begin{cases} 16 \leq x^2 + y^2 + z^2 \leq 121, \\ -\sqrt{3} \cdot x \leq y \leq -\frac{x}{\sqrt{3}} \end{cases}$ |
| $5.11. \text{ а)} \begin{cases} x^2 + y^2 = z^2, x^2 + y^2 = 81, x = 0, y = 0, \\ z = 0, (x \geq 0, y \geq 0, z \geq 0) \end{cases}$ $\text{б)} \begin{cases} 4 \leq x^2 + y^2 + z^2 \leq 64, \\ -\sqrt{\frac{x^2 + y^2}{3}} \leq z \leq \sqrt{\frac{x^2 + y^2}{15}} \end{cases}$ | $5.29. \text{ а)} \begin{cases} x^2 + y^2 = z^2 / 25, x^2 + y^2 = z / 5, \\ x = 0, y = 0, (x \leq 0, y \leq 0) \end{cases}$ $\text{б)} \begin{cases} 16 \leq x^2 + y^2 + z^2 \leq 81, \\ \sqrt{\frac{x^2 + y^2}{63}} \leq z \leq \sqrt{\frac{x^2 + y^2}{3}} \end{cases}$ |
| $5.12. \text{ а)} \begin{cases} x^2 + y^2 + z^2 = 49, x^2 + y^2 = 4, z = 0, \\ (x^2 + y^2 \leq 4, \quad z \leq 0) \end{cases}$ $\text{б)} \begin{cases} 1 \leq x^2 + y^2 + z^2 \leq 36, \\ y \leq \sqrt{3} \cdot x, \quad y \leq -\sqrt{3} \cdot x \end{cases}$ | $5.30. \text{ а)} \begin{cases} x^2 + y^2 + z^2 = 4, x^2 + y^2 = 8z^2, \\ x = 0, y = 0, (x \leq 0, y \geq 0, z \geq 0) \end{cases}$ $\text{б)} \begin{cases} 49 \leq x^2 + y^2 + z^2 \leq 100, \\ y \leq 0, \quad y \leq -\frac{x}{\sqrt{3}} \end{cases}$ |
| $5.13. \text{ а)} \begin{cases} x^2 + y^2 = 25, x^2 + y^2 = 5z, x = 0, y = 0, \\ z = 0 (x \geq 0, y \geq 0) \end{cases}$ $\text{б)} \begin{cases} 1 \leq x^2 + y^2 + z^2 \leq 49, \\ -\sqrt{\frac{x^2 + y^2}{3}} \leq z \leq -\sqrt{\frac{x^2 + y^2}{35}} \end{cases}$ | $5.31. \text{ а)} \begin{cases} 4(x^2 + y^2) = z^2, x^2 + y^2 = 25, \\ y = 0, z = 0, (y \geq 0, z \geq 0) \end{cases}$ $\text{б)} \begin{cases} 25 \leq x^2 + y^2 + z^2 \leq 100, \\ \sqrt{\frac{x^2 + y^2}{99}} \leq z \leq 3\sqrt{x^2 + y^2} \end{cases}$ |
| $5.14. \text{ а)} \begin{cases} x^2 + y^2 = 4z^2 / 49, x^2 + y^2 = 2z / 7, \\ x = 0, y = 0, (x \geq 0, y \geq 0) \end{cases}$ $\text{б)} \begin{cases} 4 \leq x^2 + y^2 + z^2 \leq 81, \\ y \geq \sqrt{3} \cdot x, \quad y \geq 0 \end{cases}$ | $5.32. \text{ а)} \begin{cases} x^2 + y^2 + z^2 = 8, x^2 + y^2 = 4, y = 0, \\ (x^2 + y^2 \leq 4, \quad y \geq 0) \end{cases}$ $\text{б)} \begin{cases} 16 \leq x^2 + y^2 + z^2 \leq 121, \\ -\sqrt{3} \cdot x \geq y \geq -\frac{x}{\sqrt{3}} \end{cases}$ |
| $5.15. \text{ а)} \begin{cases} x^2 + y^2 + z^2 = 16, x^2 + y^2 = 9z^2, \\ x = 0, y = 0, (x \geq 0, y \geq 0, z \geq 0) \end{cases}$ $\text{б)} \begin{cases} 4 \leq x^2 + y^2 + z^2 \leq 64, \\ -\sqrt{\frac{x^2 + y^2}{3}} \leq z \leq -\sqrt{\frac{x^2 + y^2}{15}} \end{cases}$ | $5.33. \text{ а)} \begin{cases} x^2 + y^2 = 5, x^2 + y^2 = 2z, x = 0, y = 0, \\ z = 0 (x \leq 0, y \geq 0) \end{cases}$ $\text{б)} \begin{cases} 16 \leq x^2 + y^2 + z^2 \leq 64, \\ 0 \leq z \leq \sqrt{\frac{x^2 + y^2}{3}} \end{cases}$ |
| $5.16. \text{ а)} \begin{cases} 9(x^2 + y^2) = z^2, x^2 + y^2 = 4, \\ y = 0, z = 0, (y \geq 0, z \geq 0) \end{cases}$ $\text{б)} \begin{cases} 1 \leq x^2 + y^2 + z^2 \leq 36, \\ y \geq -\frac{x}{\sqrt{3}}, \quad y \geq -\sqrt{3} \cdot x \end{cases}$ | $5.34. \text{ а)} \begin{cases} x^2 + y^2 = z^2 / 64, x^2 + y^2 = z / 8, \\ x = 0, y = 0, (x \geq 0, y \leq 0) \end{cases}$ $\text{б)} \begin{cases} 49 \leq x^2 + y^2 + z^2 \leq 100, \\ -x \leq y \leq \sqrt{3} \cdot x \end{cases}$ |
| $5.17. \text{ а)} \begin{cases} x^2 + y^2 + z^2 = 4, x^2 + y^2 = 1, x = 0, \\ (x^2 + y^2 \leq 1, \quad x \leq 0) \end{cases}$ $\text{б)} \begin{cases} 1 \leq x^2 + y^2 + z^2 \leq 49, \\ \sqrt{\frac{x^2 + y^2}{35}} \leq z \leq \sqrt{\frac{x^2 + y^2}{3}} \end{cases}$ | $5.35. \text{ а)} \begin{cases} x^2 + y^2 + z^2 = 49, x^2 + y^2 = 6z^2, \\ x = 0, y = 0, (x \leq 0, y \geq 0, z \leq 0) \end{cases}$ $\text{б)} \begin{cases} 25 \leq x^2 + y^2 + z^2 \leq 100, \\ \sqrt{\frac{x^2 + y^2}{99}} \leq z \end{cases}$ |

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| 5.18. а) $\begin{cases} x^2 + y^2 = 1, & x^2 + y^2 = 2z, & x = 0, & y = 0, \\ z = 0 & (x \geq 0, & y \geq 0) \end{cases}$ б) $\begin{cases} 4 \leq x^2 + y^2 + z^2 \leq 81, \\ \frac{x}{\sqrt{3}} \leq y \leq \sqrt{3} \cdot x \end{cases}$ | 5.36. а) б) $\begin{cases} 16 \leq x^2 + y^2 + z^2 \leq 121, \\ 0 \leq y \leq -\frac{x}{\sqrt{3}} \end{cases}$ |
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6. Вычислите массу неоднородного тела V , ограниченного поверхностями, если плотность в каждой её точке $\mu = \mu(x, y, z)$

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| 6.1. $V : z = \sqrt{36 - x^2 - y^2}, \quad 9z = x^2 + y^2,$ $\mu(x, y) = \frac{4(x^2 + y^2)}{3}$ | 6.19. $V : z = \sqrt{25 - x^2 - y^2}, \quad z = \sqrt{(x^2 + y^2)/99},$ $\mu(x, y) = \frac{15(x^2 + y^2)}{7}$ |
| 6.2. $V : z = 9\sqrt{x^2 + y^2}, \quad z = 22 - x^2 - y^2,$ $\mu(x, y) = 9 z $ | 6.20. $V : z = \sqrt{100 - x^2 - y^2}, \quad z = 6, \quad x^2 + y^2 = 51,$ $(x^2 + y^2 \leq 51), \quad \mu(x, y) = 4 x $ |
| 6.3. $V : z = \sqrt{16 - x^2 - y^2}, \quad z = \sqrt{(x^2 + y^2)/15},$ $\mu(x, y) = \frac{4(x^2 + y^2)}{3}$ | 6.21. $V : z = \sqrt{16 - x^2 - y^2}, \quad 6z = x^2 + y^2,$ $\mu(x, y) = 7(x^2 + y^2)$ |
| 6.4. $V : z = \sqrt{36 - x^2 - y^2}, \quad z = 2, \quad x^2 + y^2 = 27,$ $(x^2 + y^2 \leq 27), \quad \mu(x, y) = 6z$ | 6.22. $V : z = \frac{21}{2}\sqrt{x^2 + y^2}, \quad z = \frac{23}{2} - x^2 - y^2,$ $\mu(x, y) = 3 z $ |
| 6.5. $V : z = \sqrt{4/9 - x^2 - y^2}, \quad z = x^2 + y^2,$ $\mu(x, y) = \frac{5(x^2 + y^2)}{2}$ | 6.23. $V : z = \sqrt{9 - x^2 - y^2}, \quad z = \sqrt{(x^2 + y^2)/80},$ $\mu(x, y) = 7(x^2 + y^2)$ |
| 6.6. $V : z = 12\sqrt{x^2 + y^2}, \quad z = 28 - x^2 - y^2,$ $\mu(x, y) = 3 x $ | 6.24. $V : z = \sqrt{81 - x^2 - y^2}, \quad z = 5, \quad x^2 + y^2 = 45,$ $(x^2 + y^2 \leq 45), \quad \mu(x, y) = 3 y $ |
| 6.7. $V : z = \sqrt{9 - x^2 - y^2}, \quad z = \sqrt{(x^2 + y^2)/8},$ $\mu(x, y) = \frac{5(x^2 + y^2)}{2}$ | 6.25. $V : z = \sqrt{1 - x^2 - y^2}, \quad 3z/2 = x^2 + y^2,$ $\mu(x, y) = \frac{7(x^2 + y^2)}{5}$ |
| 6.8. $V : z = \sqrt{25 - x^2 - y^2}, \quad z = 1, \quad x^2 + y^2 = 21,$ $(x^2 + y^2 \leq 21), \quad \mu(x, y) = 6 z $ | 6.26. $V : z = 6\sqrt{x^2 + y^2}, \quad z = 16 - x^2 - y^2,$ $\mu(x, y) = 15z$ |
| 6.9. $V : z = \sqrt{64 - x^2 - y^2}, \quad 12z = x^2 + y^2,$ $\mu(x, y) = \frac{3(x^2 + y^2)}{4}$ | 6.27. $V : z = \sqrt{36 - x^2 - y^2}, \quad z = \sqrt{(x^2 + y^2)/63},$ $\mu(x, y) = \frac{7(x^2 + y^2)}{5}$ |
| 6.10. $V : z = \frac{9}{2}\sqrt{x^2 + y^2}, \quad z = \frac{11}{2} - x^2 - y^2,$ $\mu(x, y) = 7 y $ | 6.28. $V : z = \sqrt{64 - x^2 - y^2}, \quad z = 4, \quad x^2 + y^2 = 39,$ $(x^2 + y^2 \leq 39), \quad \mu(x, y) = 2 x $ |
| 6.11. $V : z = \sqrt{49 - x^2 - y^2}, \quad z = \sqrt{(x^2 + y^2)/48},$ $\mu(x, y) = \frac{3(x^2 + y^2)}{4}$ | 6.29. $V : z = \sqrt{144 - x^2 - y^2}, \quad 18z = x^2 + y^2,$ $\mu(x, y) = 2(x^2 + y^2)$ |

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| 6.12. $V: z = \sqrt{49 - x^2 - y^2}, z = 3, x^2 + y^2 = 33,$ $(x^2 + y^2 \leq 33), \mu(x,y)=5z$ | 6.30. $V: z = \frac{3}{2}\sqrt{x^2 + y^2}, z = \frac{5}{2} - x^2 - y^2,$ $\mu(x,y)=13 z $ |
| 6.13. $V: z = \sqrt{9 - x^2 - y^2}, 9z/2 = x^2 + y^2,$ $\mu(x,y)=15(x^2 + y^2)$ | 6.31. $V: z = \sqrt{9 - x^2 - y^2}, z = \sqrt{(x^2 + y^2)/35},$ $\mu(x,y)=2(x^2 + y^2)$ |
| 6.14. $V: z = \frac{15}{2}\sqrt{x^2 + y^2}, z = \frac{17}{2} - x^2 - y^2,$ $\mu(x,y)=10 z $ | 6.32. $V: z = \sqrt{121 - x^2 - y^2}, z = 6, x^2 + y^2 = 21,$ $(x^2 + y^2 \leq 21), \mu(x,y)= x $ |
| 6.15. $V: z = \sqrt{4 - x^2 - y^2}, z = \sqrt{(x^2 + y^2)/255},$ $\mu(x,y)=15(x^2 + y^2)$ | 6.33. $V: z = \sqrt{9 - x^2 - y^2}, 8z = x^2 + y^2,$ $\mu(x,y)=\frac{2(x^2 + y^2)}{9}$ |
| 6.16. $V: z = \sqrt{64 - x^2 - y^2}, z = 1, x^2 + y^2 = 60,$ $(x^2 + y^2 \leq 60), \mu(x,y)=12z$ | 6.34. $V: z = 18\sqrt{x^2 + y^2}, z = 63 - x^2 - y^2,$ $\mu(x,y)= y $ |
| 6.17. $V: z = \sqrt{\frac{16}{9} - x^2 - y^2}, 2z = x^2 + y^2,$ $\mu(x,y)=\frac{15(x^2 + y^2)}{7}$ | 6.35. $V: z = \sqrt{49 - x^2 - y^2}, z = \sqrt{(x^2 + y^2)/195},$ $\mu(x,y)=\frac{2(x^2 + y^2)}{9}$ |
| 6.18. $V: z = 3\sqrt{x^2 + y^2}, z = 10 - x^2 - y^2,$ $\mu(x,y)=6 y $ | |

7. а) Найдите градиенты скалярных полей $u(x, y, z)$, $v(x, y, z)$ и угол между ними в точке M ;

б) Выясните, является ли векторное поле \vec{a} потенциальным, соленоидальным, гармоническим?

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| 7.1. а) $v = \frac{x^3}{2} + 6y^3 + 3\sqrt{6}z^3, u = \frac{yz^2}{x^2}, M\left(\sqrt{2}; \frac{1}{\sqrt{2}}; \frac{1}{\sqrt{3}}\right)$ б) $\vec{a}(M) = 2x\vec{i} + 3y\vec{j} + 2z\vec{k}$ |
| 7.2. а) $v = \frac{4\sqrt{6}}{x} - \frac{\sqrt{6}}{9y} + \frac{3}{z}, u = x^2yz^3, M\left(2; \frac{1}{3}; \sqrt{\frac{3}{2}}\right)$ б) $\vec{a}(M) = (yz - 2x)\vec{i} + (xz + yz)\vec{j} + xy\vec{k}$ |
| 7.3. а) $v = 9\sqrt{2}x^3 - \frac{y^3}{2\sqrt{2}} - \frac{4z^3}{\sqrt{3}}, u = \frac{z^3}{xy^2}, M\left(\frac{1}{3}; 2; \sqrt{\frac{3}{2}}\right)$ б) $\vec{a}(M) = z\vec{i} + x\vec{j} + y\vec{k}$ |
| 7.4. а) $v = \frac{3}{x} + \frac{4}{y} - \frac{1}{\sqrt{6}z}, u = \frac{z}{x^3y^2}, M\left(1; 2; \frac{1}{\sqrt{6}}\right)$ б) $\vec{a}(M) = x^2y\vec{i} - 2xy^2\vec{j} + 2xyz\vec{k}$ |
| 7.5. а) $v = \frac{x^3}{2} + 6y^3 + 3\sqrt{6}z^3, u = \frac{x^2}{yz^2}, M\left(\sqrt{2}; \frac{1}{\sqrt{2}}; \frac{1}{\sqrt{3}}\right)$ б) $\vec{a}(M) = x^2\vec{i} + y^2\vec{j} + z^2\vec{k}$ |

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| <p>7.6. a) $v = 3\sqrt{2}x^2 - \frac{y^2}{\sqrt{2}} - 3\sqrt{2}z^2, \quad u = \frac{z^2}{xy^2}, \quad M\left(\frac{1}{3}; 2; \sqrt{\frac{2}{3}}\right)$</p> <p>б) $\vec{a}(M) = (2x - yz)\vec{i} + (xz - 2y)\vec{j} + 2xyz\vec{k}$</p> |
| <p>7.7. a) $v = 6\sqrt{6}x^3 - 6\sqrt{6}y^3 + 2z^3, \quad u = \frac{xz^2}{y}, \quad M\left(\frac{1}{\sqrt{6}}; \frac{1}{\sqrt{6}}; 1\right)$</p> <p>б) $\vec{a}(M) = 5\vec{i} + 6\vec{j} + 8\vec{k}$</p> |
| <p>7.8. a) $v = \frac{\sqrt{6}}{2x} - \frac{\sqrt{6}}{2y} + \frac{2}{3z}, \quad u = \frac{yz^2}{x}z^3, \quad M\left(\frac{1}{\sqrt{2}}; \frac{1}{\sqrt{2}}; \frac{1}{\sqrt{3}}\right)$</p> <p>б) $\vec{a}(M) = 6xy\vec{i} + (3x^2 - 2y)\vec{j} + z\vec{k}$</p> |
| <p>7.9. a) $v = 3\sqrt{2}x^2 - \frac{y^2}{\sqrt{2}} - 3\sqrt{2}z^2, \quad u = \frac{xy^2}{z^2}, \quad M\left(\frac{1}{3}; 2; \sqrt{\frac{3}{2}}\right)$</p> <p>б) $\vec{a}(M) = yz\vec{i} + xz\vec{j} + yx\vec{k}$</p> |
| <p>7.10. a) $v = \frac{3}{x} + \frac{4}{y} - \frac{1}{\sqrt{6}z}, \quad u = \frac{x^3y^2}{z}, \quad M\left(1; 2; \frac{1}{\sqrt{6}}\right)$</p> <p>б) $\vec{a}(M) = (x^2 - z^2)\vec{i} - 3xy\vec{j} + (y^2 + z^2)\vec{k}$</p> |
| <p>7.11. a) $v = -\frac{4\sqrt{2}}{x} + \frac{\sqrt{2}}{9y} + \frac{1}{\sqrt{3}z}, \quad u = \frac{1}{x^2yz}, \quad M\left(2; \frac{1}{3}; \frac{1}{\sqrt{6}}\right)$</p> <p>б) $\vec{a}(M) = x\vec{i} + 2z\vec{j} + 2y\vec{k}$</p> |
| <p>7.12. a) $v = \frac{6}{x} + \frac{2}{y} - \frac{3\sqrt{3}}{2\sqrt{2}z}, \quad u = \frac{x^2}{y^2z^3}, \quad M\left(\sqrt{2}; \sqrt{2}; \frac{\sqrt{3}}{2}\right)$</p> <p>б) $\vec{a}(M) = 3x^2\vec{i} + 4(x - y)\vec{j} + (x - z)\vec{k}$</p> |
| <p>7.13. a) $v = x^2 + 9y^2 + 6z^2, \quad u = xyz, \quad M\left(1; \frac{1}{3}; \frac{1}{\sqrt{6}}\right)$</p> <p>б) $\vec{a}(M) = 5\vec{i} - 8z\vec{j} + 8y\vec{k}$</p> |
| <p>7.14. a) $v = \frac{2}{x} + \frac{3}{2y} - \frac{\sqrt{6}}{4z}, \quad u = \frac{y^3}{x^2z}, \quad M\left(\sqrt{\frac{2}{3}}; \sqrt{\frac{3}{2}}; \frac{1}{2}\right)$</p> <p>б) $\vec{a}(M) = (y - z)\vec{i} + 3xzy\vec{j} + (z - x)\vec{k}$</p> |
| <p>7.15. a) $v = \sqrt{2}x^2 - \frac{3y^2}{\sqrt{2}} - 6\sqrt{2}z^2, \quad u = xy^2z, \quad M\left(1; \frac{2}{3}; \frac{1}{\sqrt{6}}\right)$</p> <p>б) $\vec{a}(M) = z^3y^3\vec{i} + x^3\vec{j} + xy\vec{k}$</p> |
| <p>7.16. a) $v = -\frac{\sqrt{6}}{2x} + \frac{\sqrt{6}}{2y} - \frac{2}{3z}, \quad u = \frac{x}{yx^2}, \quad M\left(\frac{1}{\sqrt{2}}; \frac{1}{\sqrt{2}}; \frac{1}{\sqrt{3}}\right)$</p> <p>б) $\vec{a}(M) = (2x - 3y)\vec{i} + 2xy\vec{j} - z^2\vec{k}$</p> |
| <p>7.17. a) $v = \frac{6}{x} + \frac{2}{y} - \frac{3\sqrt{3}}{2\sqrt{2}z}, \quad u = \frac{y^2z^3}{x^2}, \quad M\left(\sqrt{2}; \sqrt{2}; \frac{\sqrt{3}}{2}\right)$</p> <p>б) $\vec{a}(M) = x^2\vec{i} + 4y\vec{j} + 2y^2\vec{k}$</p> |

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| <p>7.18. a) $v = \frac{1}{\sqrt{2}x} - \frac{2\sqrt{2}}{y} - \frac{3\sqrt{3}}{2z}, \quad u = \frac{y^2 z^3}{x}, \quad M\left(\frac{1}{\sqrt{2}}; \sqrt{2}; \frac{\sqrt{3}}{2}\right)$</p> <p>б) $\vec{a}(M) = 2x\vec{i} - 3y\vec{j} + z\vec{k}$</p> |
| <p>7.19. a) $v = 6\sqrt{6}x^3 - 6\sqrt{6}y^3 + 2z^3, \quad u = \frac{y}{xz^2}, \quad M\left(\frac{1}{\sqrt{6}}; \frac{1}{\sqrt{6}}; 1\right)$</p> <p>б) $\vec{a}(M) = 10z\vec{i} + 16\vec{j} + 10x\vec{k}$</p> |
| <p>7.20. a) $v = x^2 - y^2 - 3z^2, \quad u = \frac{yz^2}{x}, \quad M\left(\frac{1}{\sqrt{2}}; \frac{1}{\sqrt{2}}; \frac{1}{\sqrt{3}}\right)$</p> <p>б) $\vec{a}(M) = (x+y)\vec{i} - 2xz\vec{j} - 3(z+y)\vec{k}$</p> |
| <p>7.21. a) $v = \frac{3x^2}{\sqrt{2}} - \frac{y^2}{\sqrt{2}} + \sqrt{2}z^2, \quad u = \frac{z^2}{x^2 y^2}, \quad M\left(\frac{2}{3}; 2; \sqrt{\frac{2}{3}}\right)$</p> <p>б) $\vec{a}(M) = 5yz^5\vec{i} + 6z^6x\vec{j} + 7y^7x^7\vec{k}$</p> |
| <p>7.22. a) $v = \frac{x^3}{\sqrt{2}} - \frac{y^3}{\sqrt{2}} - \frac{8z^3}{\sqrt{3}}, \quad u = \frac{x^2}{y^2 z^3}, \quad M\left(\sqrt{2}; \sqrt{2}; \frac{\sqrt{3}}{2}\right)$</p> <p>б) $\vec{a}(M) = yz\vec{i} + (x-y)\vec{j} + z^2\vec{k}$</p> |
| <p>7.23. a) $v = \frac{3}{2}x^2 + 3y^2 - 2z^2, \quad u = x^2 yz^3, \quad M\left(2; \frac{1}{3}; \sqrt{\frac{3}{2}}\right)$</p> <p>б) $\vec{a}(M) = 5x\vec{i} + 5y\vec{j} + 8z\vec{k}$</p> |
| <p>7.24. a) $v = 9\sqrt{2}x^3 - \frac{y^3}{2\sqrt{2}} - \frac{4z^3}{\sqrt{3}}, \quad u = \frac{xy^2}{z^3}, \quad M\left(\frac{1}{3}; 2; \sqrt{\frac{3}{2}}\right)$</p> <p>б) $\vec{a}(M) = (5x^2 - 5z^2)\vec{i} - 15xy\vec{j} + (5y^2 + 5z^2)\vec{k}$</p> |
| <p>7.25. a) $v = \sqrt{2}x^2 - \frac{3y^2}{\sqrt{2}} - 6\sqrt{2}z^2, \quad u = \frac{1}{xy^2 z}, \quad M\left(1; \frac{2}{3}; \frac{1}{\sqrt{6}}\right)$</p> <p>б) $\vec{a}(M) = 8yz\vec{i} + 8xz\vec{j} + 8xy\vec{k}$</p> |
| <p>7.26. a) $v = x^2 + 9y^2 + 6z^2, \quad u = \frac{1}{xyz}, \quad M\left(1; \frac{1}{3}; \frac{1}{\sqrt{6}}\right)$</p> <p>б) $\vec{a}(M) = xy(3x-4y)\vec{i} + 3x^2(x-4y)\vec{j} + 3z^2\vec{k}$</p> |
| <p>7.27. a) $v = \frac{1}{\sqrt{2}x} - \frac{2\sqrt{2}}{y} - \frac{3\sqrt{3}}{2z}, \quad u = \frac{x}{y^2 z^3}, \quad M\left(\frac{1}{\sqrt{2}}; \sqrt{2}; \frac{\sqrt{3}}{2}\right)$</p> <p>б) $\vec{a}(M) = 8y\vec{i} + 8x\vec{j} + 8y\vec{k}$</p> |
| <p>7.28. a) $v = -\frac{4\sqrt{2}}{x} + \frac{\sqrt{2}}{9y} + \frac{1}{\sqrt{3}x}, \quad u = x^2 yz, \quad M\left(2; \frac{1}{3}; \frac{1}{\sqrt{6}}\right)$</p> <p>б) $\vec{a}(M) = 3x^2 y\vec{i} - 2xy^2\vec{j} + 2xyz\vec{k}$</p> |
| <p>7.29. a) $v = \frac{x^3}{\sqrt{2}} - \frac{y^3}{\sqrt{2}} - \frac{8z^3}{\sqrt{3}}, \quad u = \frac{y^2 z^3}{x^2}, \quad M\left(\sqrt{2}; \sqrt{2}; \frac{\sqrt{3}}{2}\right)$</p> <p>б) $\vec{a}(M) = \sqrt{z}\vec{i} + y^3\vec{j} + \frac{1}{2\sqrt{z}}\vec{k}$</p> |

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| 7.30. а) $v = -\frac{3x^3}{\sqrt{2}} + \frac{2\sqrt{2}y^3}{3} + 8\sqrt{3}z^3, \quad u = \frac{x^2z}{y^3}, \quad M\left(\sqrt{\frac{2}{3}}; \sqrt{\frac{3}{2}}; \frac{1}{2}\right)$ б) $\vec{a}(M) = (6x - 9y)\vec{i} + 6xy\vec{j} - 3z^2\vec{k}$ |
| 7.31. а) $v = x^2 - y^2 - 3z^2, \quad u = \frac{x}{yz^2}, \quad M\left(\frac{1}{\sqrt{2}}; \frac{1}{\sqrt{2}}; \frac{1}{\sqrt{3}}\right)$ б) $\vec{a}(M) = -5yz\vec{i} - 5xz\vec{j} - 5xy\vec{k}$ |
| 7.32. а) $v = \sqrt{3}x^4 + \frac{y^2}{\sqrt{3}} - 2\sqrt{3}z^3, \quad u = \frac{1}{x^2yz^3}, \quad M\left(1; -\frac{2}{3}; \frac{1}{\sqrt{6}}\right)$ б) $\vec{a}(M) = (y + x)\vec{i} + (z - y)\vec{j} + 2(x + z)\vec{k}$ |
| 7.33. а) $v = \frac{x}{\sqrt{2}} - \frac{2y}{\sqrt{2}} + \frac{\sqrt{3}z^3}{2}, \quad u = \frac{x^3z^2}{z}, \quad M\left(\sqrt{2}; \frac{1}{\sqrt{2}}; \sqrt{2}\right)$ б) $\vec{a}(M) = 3(x - z)\vec{i} + (x^2 - y^2)\vec{j} + 3z\vec{k}$ |
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8. Найдите работу силы при перемещении вдоль а) линии L от точки A к точке B ;
б) контура Γ

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| 8.1. а) $\vec{F} = (x^2 + 3y^2)\vec{i} + (2x^2 - y^2)\vec{j}, \quad L: y = \begin{cases} x, & 0 \leq x \leq 1, \\ 2 - x, & 1 < x \leq 2, \end{cases} \quad M(2;0), N(0;0)$ б) $\vec{F} = -x^2y^3\vec{i} + \vec{j} + z\vec{k}, \quad \Gamma: x = 4\cos t, y = 4\sin t, z = 3$ |
| 8.2. а) $\vec{F} = 4y\vec{i} - 5x\vec{j}, \quad L: x^2 + y^2 = 8, \quad (y \leq 0) \quad M(\sqrt{2};0), N(-\sqrt{2};0)$ б) $\vec{F} = x^2\vec{i} + y\vec{j} - z\vec{k}, \quad \Gamma: x = \cos t, y = \frac{\sqrt{2}}{2}\sin t, z = \frac{\sqrt{2}}{2}\cos t$ |
| 8.3. а) $\vec{F} = -y\vec{i} + 6x^2\vec{j}, \quad L = MN, \quad M(-2;1), N(3;-2)$ б) $\vec{F} = 2y\vec{i} - 3x\vec{j} + x\vec{k}, \quad \Gamma: x = 2\cos t, y = 2\sin t, z = 2 - 2\cos t - 2\sin t$ |
| 8.4. а) $\vec{F} = (x - y)\vec{i} + (2x + y)\vec{j}, \quad L: y = 2\sqrt{x} \quad M(0;0), N(1;2)$ б) $\vec{F} = y\vec{i} - x\vec{j} + z\vec{k}, \quad \Gamma: x = \cos t, y = \sin t, z = 2$ |
| 8.5. а) $\vec{F} = xy\vec{i} + 7y\vec{j}, \quad L: x^2 + y^2 = 16, \quad (x \geq 0) \quad M(0;-4), N(0;4)$ б) $\vec{F} = 3y\vec{i} - 5x\vec{j} + x\vec{k}, \quad \Gamma: x = 3\cos t, y = 3\sin t, z = 3 - 3\cos t - 3\sin t$ |
| 8.6. а) $\vec{F} = (x^2 + 2y)\vec{i} + (y^2 + 2x)\vec{j}, \quad L = MN, \quad M(-4;0), N(0;2)$ б) $\vec{F} = 6z\vec{i} - x\vec{j} + xy\vec{k}, \quad \Gamma: x = 3\cos t, y = 3\sin t, z = 4$ |
| 8.7. а) $\vec{F} = (x + 7y)\vec{i} + (y - 3x)\vec{j}, \quad L: y = \begin{cases} x, & 0 \leq x \leq 1, \\ 2 - x, & 1 < x \leq 2, \end{cases} \quad M(2;0), N(0;0)$ б) $\vec{F} = y\vec{i} - x\vec{j} + z^2\vec{k}, \quad \Gamma: x = \frac{\sqrt{2}}{2}\cos t, y = \frac{\sqrt{2}}{2}\cos t, z = \frac{\sqrt{2}}{2}\sin t$ |
| 8.8. а) $\vec{F} = (x + y\sqrt{x^2 + y^2})\vec{i} + (y - x\sqrt{x^2 + y^2})\vec{j}, \quad L: x^2 + y^2 = 81, \quad M(9;0), N(0;-9)$ б) $\vec{F} = (y - z)\vec{i} + (z - x)\vec{j} + (x - y)\vec{k}, \quad \Gamma: x = \cos t, y = \sin t, z = 2(1 - \cos t)$ |

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| 8.9. a) $\vec{F} = 3x\vec{i} - 2y^2\vec{j}$, $L = MN$, $M(4;2)$, $N(2;-4)$ б) $\vec{F} = 2z\vec{i} - x\vec{j} + y\vec{k}$, $\Gamma: x = 2\cos t$, $y = 2\sin t$, $z = 1$ |
| 8.10. a) $\vec{F} = x\vec{i} + (x^2 - 4y)\vec{j}$, $L: y = 2x^2$ $M(0;0)$, $N(1;2)$ б) $\vec{F} = x\vec{i} + z^2\vec{j} + y\vec{k}$, $\Gamma: x = \cos t$, $y = 2\sin t$, $z = 2\cos t - 2\sin t - 1$ |
| 8.11. a) $\vec{F} = x^2y\vec{i} - xy^2\vec{j}$, $L: x^2 + y^2 = 1$, $(x \leq 0, y \geq 0)$ $M(0;1)$, $N(-1;0)$ б) $\vec{F} = z\vec{i} + y^2\vec{j} - x\vec{k}$, $\tilde{A}: x = \sqrt{2}\cos t$, $y = 2\cos t$, $z = \sqrt{2}\sin t$ |
| 8.12. a) $\vec{F} = (2 - y)\vec{i} + y - x^2)\vec{j}$, $L = MN$, $M(-3;-1)$, $N(3;1)$ б) $\vec{F} = x\vec{i} - 0.5 \cdot z^2\vec{j} + y\vec{k}$, $\Gamma: x = \frac{\cos t}{2}$, $y = \frac{\sin t}{3}$, $z = \cos t - \frac{\sin t}{3} - \frac{1}{4}$ |
| 8.13. a) $\vec{F} = (y^2 - 2x)\vec{i} + (y^2 + x^2)\vec{j}$, $L: y = \begin{cases} x, & 0 \leq x \leq 2, \\ 4 - x, & 2 < x \leq 4, \end{cases}$ $M(4;0)$, $N(0;0)$ б) $\vec{F} = -z\vec{i} - x\vec{j} + xz\vec{k}$, $\Gamma: x = 5\cos t$, $y = 5\sin t$, $z = 4$ |
| 8.14. a) $\vec{F} = (2xy - y)\vec{i} + (x - y^2)\vec{j}$, $L: x^2 + y^2 = 4$, $(x \leq 0, y \leq 0)$ $M(0;-2)$, $N(-2;0)$ б) $\vec{F} = xz\vec{i} + x\vec{j} + z^2\vec{k}$, $\Gamma: x = \cos t$, $y = \sin t$, $z = \sin t$ |
| 8.15. a) $\vec{F} = (x + 5y)\vec{i} - 7y\vec{j}$, $L = MN$, $M(0;-1)$, $N(5;0)$ б) $\vec{F} = (y - z)\vec{i} + (z - x)\vec{j} + (x - y)\vec{k}$, $\Gamma: x = 4\cos t$, $y = 4\sin t$, $z = 1 - \cos t$ |
| 8.16. a) $\vec{F} = (2y - x^2)\vec{i} + (3x - 4)\vec{j}$, $L: y = 4x^2$ $M(0;0)$, $N(1;4)$ б) $\vec{F} = -x^2y^3\vec{i} + 2\vec{j} + xz\vec{k}$, $\Gamma: x = \sqrt{2}\cos t$, $y = \sqrt{2}\sin t$, $z = 1$ |
| 8.17. a) $\vec{F} = (x^2 - y)\vec{i} + y\vec{j}$, $L: x^2 + y^2 = 9$, $(x \geq 0)$ $M(0;3)$, $N(0;-3)$ б) $\vec{F} = xy\vec{i} + x\vec{j} + y^2\vec{k}$, $\Gamma: x = \cos t$, $y = \sin t$, $z = \sin t$ |
| 8.18. a) $\vec{F} = y^2/5\vec{i} + (4 - y^2)\vec{j}$, $L = MN$, $M(6;-1)$, $N(0;1)$ б) $\vec{F} = (y - z)\vec{i} + (z - x)\vec{j} + (x - y)\vec{k}$, $\Gamma: x = 3\cos t$, $y = 3\sin t$, $z = 2(1 - \cos t)$ |
| 8.19. a) $\vec{F} = (x - y)\vec{i} + (2x + y)\vec{j}$, $L: y = \begin{cases} 2x, & 0 \leq x \leq 1, \\ 3 - x, & 1 < x \leq 3, \end{cases}$ $M(0;0)$, $N(3;0)$ б) $\vec{F} = 7z\vec{i} - x\vec{j} + yz\vec{k}$, $\Gamma: x = 6\cos t$, $y = 6\sin t$, $z = 1/2$ |
| 8.20. a) $\vec{F} = (x + y)\vec{i} + (x - 2y)\vec{j}$, $L: x^2 + y^2/9 = 1$, $(x \geq 0, y \geq 0)$ $M(1;0)$, $N(0;3)$ б) $\vec{F} = x\vec{i} - z^2\vec{j} + 4y\vec{k}$, $\tilde{A}: x = 2\cos t$, $y = 3\sin t$, $z = 4\cos t - 3\sin t - 3$ |
| 8.21. a) $\vec{F} = (5 - x)\vec{i} - (3y - 4)\vec{j}$, $L: y = -4\sqrt{x}$ $M(0;0)$, $N(1;-4)$ б) $\vec{F} = -2z\vec{i} - x\vec{j} + x^2\vec{k}$, $\Gamma: x = \frac{\cos t}{3}$, $y = \frac{\sin t}{3}$, $z = 8$ |
| 8.22. a) $\vec{F} = (y - 5x)\vec{i} + (x + 4y)\vec{j}$, $L = MN$, $M(-1;3)$, $N(0;5)$ б) $\vec{F} = x\vec{i} - 2z^2\vec{j} + y\vec{k}$, $\Gamma: x = 3\cos t$, $y = 4\sin t$, $z = 6\cos t - 4\sin t + 1$ |
| 8.23. a) $\vec{F} = x\vec{i} + (x^2 - 4y)\vec{j}$, $L: y = -2x^2$ $M(0;0)$, $N(1;-2)$ б) $\vec{F} = z\vec{i} + x\vec{j} + y\vec{k}$, $\Gamma: x = 2\cos t$, $y = 2\sin t$, $z = 0$ |
| 8.24. a) $\vec{F} = (y - x\sqrt{x^2 + y^2})\vec{i} + (x + y\sqrt{x^2 + y^2})\vec{j}$, $L: x^2 + y^2 = 25$, $M(-5;0)$, $N(0;5)$ б) $\vec{F} = x\vec{i} + 5z^2\vec{j} + 3y\vec{k}$, $\Gamma: x = \cos t$, $y = 3\sin t$, $z = 2\cos t - 3\sin t - 2$ |
| 8.25. a) $\vec{F} = -y\vec{i} + 2x^2\vec{j}$, $L = MN$, $M(7;-5)$, $N(6;-1)$ |

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| б) $\bar{F} = x^2 y^3 \bar{i} + 3 \bar{j} + y \bar{k}, \quad \Gamma: x = \cos t, y = \sin t, z = 5$ |
| 8.26. а) $\bar{F} = -x \bar{i} + (x - y) \bar{j}, \quad L: y = \begin{cases} 2x, & 0 \leq x \leq 1, \\ 3 - x, & 1 < x \leq 3, \end{cases} \quad M(0;0), N(3;0)$ б) $\bar{F} = 4y \bar{i} - 3x \bar{j} + x \bar{k}, \quad \Gamma: x = 4 \cos t, y = 4 \sin t, z = 4 - 4 \cos t - 4 \sin t$ |
| 8.27. а) $\bar{F} = (x^2 - y) \bar{i} + (x + 2y^2) \bar{j}, \quad L: x^2/4 + y^2/9 = 1, \quad (x \leq 0) \quad M(0;-3), N(0;3)$ б) $\bar{F} = (y - z) \bar{i} - (x - z) \bar{j} + (x - y) \bar{k}, \quad \Gamma: x = 2 \cos t, y = 2 \sin t, z = 3 - 3 \cos t$ |
| 8.28. а) $\bar{F} = (2y + x) \bar{i} - xy^2 \bar{j}, \quad L = MN, \quad M(4;-2), N(0;0)$ б) $\bar{F} = 2y \bar{i} - z \bar{j} + x \bar{k}, \quad \Gamma: x = \cos t, y = \sin t, z = 4 - \cos t - \sin t$ |
| 8.29. а) $\bar{F} = (xy - y) \bar{i} + x^2/2 \bar{j}, \quad L: y = -2\sqrt{x} \quad M(0;0), N(1;-2)$ б) $\bar{F} = x \bar{i} + y^2 \bar{j} - z \bar{k}, \quad \Gamma: x = \sqrt{2} \cos t, y = 2 \sin t, z = \sqrt{2} \cos t$ |
| 8.30. а) $\bar{F} = y \bar{i} + 3x \bar{j}, \quad L: 2x^2 + y^2 = 1, \quad (y \geq 0) \quad M(1/\sqrt{2};0), N(-1/\sqrt{2};0)$ б) $\bar{F} = x \bar{i} - 3z^2 \bar{j} + y \bar{k}, \quad \Gamma: x = \cos t, y = 4 \sin t, z = 3 + 2 \cos t - 4 \sin t$ |
| 8.31. а) $\bar{F} = (3y - x^2) \bar{i} - 2x^2 \bar{j}, \quad L = MN, \quad M(-6;0), N(0;5)$ б) $\bar{F} = -y \bar{i} - 2z \bar{j} + xy \bar{k}, \quad \Gamma: x = \sqrt{3} \cos t, y = \sqrt{3} \sin t, z = 3$ |
| 8.32. а) $\bar{F} = (2x - 1) \bar{i} + (2 + y) \bar{j}, \quad L: y = \begin{cases} 3x, & 0 \leq x \leq 1, \\ 4 - x, & 1 < x \leq 4, \end{cases} \quad M(0;0), N(4;0)$ б) $\bar{F} = y/3 \bar{i} - 3x \bar{j} + x \bar{k}, \quad \Gamma: x = 2 \cos t, y = 2 \sin t, z = 1 - 2 \cos t - 2 \sin t$ |
| 8.33. а) $\bar{F} = (x^2 + y^2)(\bar{i} + 2 \bar{j}), \quad L: x^2/25 + y^2 = 1, \quad (x \geq 0, y \leq 0) \quad M(5;0), N(0;-1)$ б) $\bar{F} = 2x \bar{i} + 6z \bar{j} + 2y \bar{k}, \quad \Gamma: x = \sqrt{5} \cos t, y = \sqrt{5} \sin t, z = 6$ |
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9. Вычислите интеграл от функции комплексного переменного по данной кривой

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| 9.1. $\int_{AB} \bar{z}^2 dz, \quad AB = \{y = x^2, z_A = 2 + 4i, z_B = 1 + i\}$ |
| 9.2. $\int_{AB} (z + 1)e^z dz, \quad AB = \{ z = 1, \operatorname{Re} z \leq 0, z_A = -i, z_B = i\}$ |
| 9.3. $\int_{AB} \operatorname{Im} z^3 dz, \quad AB = \{z_A = -3 + 5i, z_B = 2 + 2i\}$ |
| 9.4. $\int_{AB} (z^2 + 7\bar{z} + 1) dz, \quad AB = \{z_A = 1 - 2i, z_B = 4 - 6i\}$ |
| 9.5. $\int_{ABC} z dz, \quad ABC = \{z_A = -1, z_B = -1 + i, z_C = 1 + 2i\}$ |
| 9.6. $\int_{AB} (12z^2 - 4z + 1) dz, \quad AB = \{ z = 3, z_A = -3i, z_B = 3i\}$ |
| 9.7. $\int_{AB} (\bar{z}^2 + \operatorname{Re} z) dz, \quad AB = \{y = 2x^2, z_A = -1 + 2i, z_B = 2 + 8i\}$ |
| 9.8. $\int_{ABC} z^3 e^{z^4} dz, \quad ABC = \{z_A = i, z_B = 1, z_C = 1 + i\}$ |

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| 9.9. | $\int_{AB} \operatorname{Re} \frac{\bar{z}}{z} dz, \quad AB = \{ z = 2, \operatorname{Im} z \geq 0, z_A = 2, z_B = -2\}$ |
| 9.10. | $\int_{ABC} (z^2 + \cos z) dz, \quad ABC = \{z_A = 3, z_B = 4 + i, z_C = 3 + 2i\}$ |
| 9.11. | $\int_{AB} \frac{\bar{z}}{z+1} dz, \quad AB = \{ z = 4, \operatorname{Re} z \leq 0, z_A = -4i, z_B = 4i\}$ |
| 9.12. | $\int_{ABC} (\operatorname{ch} z + \cos iz) dz, \quad ABC = \{z_A = 2i, z_B = -2 + 3i, z_C = i\}$ |
| 9.13. | $\int_{AB} (z ^2 \cdot \bar{z}) dz, \quad AB = \{z_A = 3 - 7i, z_B = -3 + 2i\}$ |
| 9.14. | $\int_{AB} (\operatorname{ch} z + z) dz, \quad AB = \{ z + 2 - i = 1, \operatorname{Im}(z - i) \leq 0, z_A = -3 + i, z_B = 1 + i\}$ |
| 9.15. | $\int_{AB} \operatorname{Re} z^3 dz, \quad AB = \{z_A = -3 + i, z_B = 6 - 2i\}$ |
| 9.16. | $\int_{AB} (3z^2 + 2\bar{z}) dz, \quad AB = \{ z = 5, \operatorname{Re} z \leq 0, z_A = 5i, z_B = -5i\}$ |
| 9.17. | $\int_{ABC} (z^2 + 1) dz, \quad ABC = \{z_A = -3 + i, z_B = 1, z_C = 2 - 3i\}$ |
| 9.18. | $\int_{AB} z \operatorname{Re} z^2 dz, \quad AB = \{z_A = 4 + 5i, z_B = -3 + 4i\}$ |
| 9.19. | $\int_{AB} (2\bar{z} + 3z^2 + 4) dz, \quad ABC = \{ z = 2, \operatorname{Im}(z) \leq 0, z_A = -2, z_B = 2\}$ |
| 9.20. | $\int_{AB} (\sin iz + z) dz, \quad AB = \{ z = 5, z_A = -4 + 3i, z_B = 3 + 4i\}$ |
| 9.21. | $\int_{AB} e^{ z ^2} \operatorname{Im} z dz, \quad AB = \{z_A = 1 + i, z_B = -1 + 3i\}$ |
| 9.22. | $\int_{AB} (2z + 1) \bar{z} dz, \quad AB = \{y = 3x^2 - 2, z_A = -2i, z_B = 1 + i\}$ |
| 9.23. | $\int_{AB} (\cos iz + 3z^2) dz, \quad AB = \{z_A = 2 - 4i, z_B = 3 + 5i\}$ |
| 9.24. | $\int_{AB} (2z + 1) \bar{z} dz, \quad AB = \{ z = 4, \operatorname{Im}(z) \geq 0, z_A = 4, z_B = -4\}$ |
| 9.25. | $\int_{AB} (z \cdot z) dz, \quad AB = \{ z = 7, \operatorname{Re} z \leq 0, z_A = -7i, z_B = 7i\}$ |
| 9.26. | $\int_{ABC} (\sin z + z^3) dz, \quad ABC = \{z_A = 0, z_B = 4 - i, z_C = 1 - i\}$ |
| 9.27. | $\int_{AB} z \operatorname{Im} z^2 dz, \quad AB = \{y = 2x^2, z_A = -2 + 8i, z_B = 1 + 2i\}$ |
| 9.28. | $\int_{AB} (\operatorname{Re} z^2 + z) dz, \quad AB = \{z_A = 1 + 2i, z_B = -2 + 3i\}$ |
| 9.29. | $\int_{ABC} (z^9 + 1) dz, \quad ABC = \{z_A = 5 - 2i, z_B = i, z_C = 1 + 2i\}$ |
| 9.30. | $\oint_l (z + 2i), \quad l^+ = \{z: z = 3\}$ |

10. Найдите все лорановские разложения данной функции $f(z)$ по степеням z

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| 10.1. $f(z) = \frac{z-1}{z^3+4z^2-5z}$ | 10.16. $f(z) = \frac{2z-4}{z^4-4z^3-21z^2}$ |
| 10.2. $f(z) = \frac{8z+4}{-9z^3+3z^2+6z}$ | 10.17. $f(z) = \frac{8z^2-28z+50}{2z^3-15z^2+25z}$ |
| 10.3. $f(z) = \frac{2z+1}{z^4+z^3-2z^2}$ | 10.18. $f(z) = \frac{13z+7}{6z^2-z^3+7z}$ |
| 10.4. $f(z) = \frac{5z-4}{5z^2-2z-2z^3}$ | 10.19. $f(z) = \frac{-7z+5}{z^3+6z^2+5z}$ |
| 10.5. $f(z) = \frac{10z^2-16z+2}{3z^3-7z^2+2z}$ | 10.20. $f(z) = \frac{9z^2+34z-15}{z^3+2z^2-3z}$ |
| 10.6. $f(z) = \frac{17z^2+10z+5}{2z^3+z-3z^2}$ | 10.21. $f(z) = \frac{12-z}{z^3-7z^2+12z}$ |
| 10.7. $f(z) = \frac{2-8z}{4z^2-3z^3-z}$ | 10.22. $f(z) = \frac{15z-36}{9z^2-2z^3-9z}$ |
| 10.8. $f(z) = \frac{10-6z}{3z^2-z^3-2z}$ | 10.23. $f(z) = \frac{28z+48}{24z-z^3-2z^2}$ |
| 10.9. $f(z) = \frac{-7z^2+7z-3}{7z^3-2z^4-3z^2}$ | 10.24. $f(z) = \frac{14z-48}{z^3+6z^2-16z}$ |
| 10.10. $f(z) = \frac{4z^2+13z-4}{-3z^3-5z^2+2z}$ | 10.25. $f(z) = \frac{13z-2}{3z^3+5z^2-2z}$ |
| 10.11. $f(z) = \frac{-5z^2+11z-5}{3z^2-2z^3-z}$ | 10.26. $f(z) = \frac{85z-36}{2z^3+11z^2-6z}$ |
| 10.12. $f(z) = \frac{-37z+12}{-4z^3+13z^2-3z}$ | 10.27. $f(z) = \frac{3z-2}{3z^3-4z^3+z}$ |
| 10.13. $f(z) = \frac{2z+7}{4z^3-2z^2-2z}$ | 10.28. $f(z) = \frac{2z^2-6z+15}{3z-z^3-2z^2}$ |
| 10.14. $f(z) = \frac{z-1}{z^3+4z^2-5z}$ | 10.29. $f(z) = \frac{5-4z}{2z^3-5z^2+2z}$ |
| 10.15. $f(z) = \frac{12z-39}{4z^3+26z^2+30z}$ | 10.30. $f(z) = \frac{z^2-4z+5}{4z^3+21z^2-z^4}$ |

11. Найдите разложение функции $f(z)$ в ряд Лорана в окрестности точки z_0

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| 11.1. $f(z) = z^2 \cos \frac{\pi}{z-1}, \quad z_0 = 1$ | 11.16. $f(z) = z \cos \frac{6z}{z-3i}, \quad z_0 = 3i$ |
| 11.2. $f(z) = z^2 e^{\frac{z}{z+1}}, \quad z_0 = -1$ | 11.17. $f(z) = (z+1) \sin \frac{\pi(z+1)}{z}, \quad z_0 = 0$ |
| 11.3. $f(z) = \sin \frac{2z}{z+i}, \quad z_0 = -i$ | 11.18. $f(z) = z^2 \sin \frac{2z}{z+i}, \quad z_0 = -i$ |
| 11.4. $f(z) = \cos \frac{2z+i}{2z-i}, \quad z_0 = \frac{i}{2}$ | 11.19. $f(z) = e^{\frac{z^2-4z}{(z-2i)^2}}, \quad z_0 = 2i$ |

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| 11.5. $f(z) = z \cos \frac{z+1}{z-1}, \quad z_0 = 1$ | 11.20. $f(z) = z^2 e^{\frac{1}{z+i}}, \quad z_0 = -i$ |
| 11.6. $f(z) = z \cos \frac{\pi(z-2)}{z}, \quad z_0 = 0$ | 11.21. $f(z) = \sin \frac{2z+i}{2z-i}, \quad z_0 = \frac{i}{2}$ |
| 11.7. $f(z) = \sin \frac{z^2-8z}{(z-4)^2}, \quad z_0 = 4$ | 11.22. $f(z) = e^{\frac{z+5}{(z-i)^2}}, \quad z_0 = i$ |
| 11.8. $f(z) = z^2 \cos \frac{z}{z+2}, \quad z_0 = -2$ | 11.23. $f(z) = z^2 \cos \frac{z^2-2z}{(z-1)^2}, \quad z_0 = 1$ |
| 11.9. $f(z) = e^{\frac{z^2-3z}{(z+2)^2}}, \quad z_0 = -2$ | 11.24. $f(z) = z \sin \frac{\pi(z-1)}{z+2}, \quad z_0 = -2$ |
| 11.10. $f(z) = \cos \frac{z^2-8z}{(z-4)^2}, \quad z_0 = 4$ | 11.25. $f(z) = z^2 e^{\frac{z}{z+1}}, \quad z_0 = -1$ |
| 11.11. $f(z) = z^2 e^{\frac{\pi}{(z-2)^2}}, \quad z_0 = 2$ | 11.26. $f(z) = z^2 \cos \frac{\pi(z+1)}{z}, \quad z_0 = 0$ |
| 11.12. $f(z) = z^2 \cos \frac{\pi(z+1)}{z}, \quad z_0 = 0$ | 11.27. $f(z) = z^2 \cos \frac{z+2}{z-4i}, \quad z_0 = 4i$ |
| 11.13. $f(z) = z \cos \frac{z-3}{z+3}, \quad z_0 = -3$ | 11.28. $f(z) = (z+i)^2 e^{\frac{z}{z-3i}}, \quad z_0 = 3i$ |
| 11.14. $f(z) = \cos \frac{z}{z+2i}, \quad z_0 = -2i$ | 11.29. $f(z) = e^{\frac{2z^2-z}{(z+5i)^2}}, \quad z_0 = -5i$ |
| 11.15. $f(z) = \cos \frac{\pi z}{z+1}, \quad z_0 = -1$ | 11.30. $f(z) = \sin \frac{i-3z}{2z-7i}, \quad z_0 = 3,5i$ |

12. Определить тип особой точки $z = 0$ для данной функции $f(z)$

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| 12.1. $f(z) = \frac{e^{2z} - 1 - 2z}{\operatorname{sh} 6z - 6z - 36z^3}$ | 12.16. $f(z) = \frac{\operatorname{ch} 3z - 1}{\operatorname{sh} 6z - 6z - 36z^3}$ |
| 12.2. $f(z) = z^4 \operatorname{ch} \left(\frac{5}{z^3} \right)$ | 12.17. $f(z) = \frac{e^{z^3} - 1}{\operatorname{ch} 2z - 1 - 2z^2}$ |
| 12.3. $f(z) = \frac{\cos 2z - 1 + 2z^2}{e^z - 1}$ | 12.18. $f(z) = z^2 e^{3/z^5}$ |
| 12.4. $f(z) = \frac{\cos 3z - 1}{\operatorname{sh} z - z - z^3/6}$ | 12.19. $f(z) = \frac{\operatorname{sh} z^3 - z^3}{e^z - 1 - z}$ |
| 12.5. $f(z) = \frac{\operatorname{ch} 4z - 1 - 8z^2}{e^z - 1 - z}$ | 12.20. $f(z) = \frac{\cos z^2 - 1}{\operatorname{sh} z - z - z^3/6}$ |
| 12.6. $f(z) = z^2 \sin \left(\frac{2}{z^3} \right)$ | 12.21. $f(z) = \frac{e^{3z} - 1 - 3z}{\cos 2z - 1 + 2z^2}$ |
| 12.7. $f(z) = \frac{\operatorname{sh} 3z - 3z}{\operatorname{ch} z - 1 - z^3/2}$ | 12.22. $f(z) = z^2 \cos \left(\frac{4}{z^3} \right)$ |

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| 12.8. $f(z) = \frac{e^{8z} - 1}{\operatorname{sh} 4z - 4z}$ | 12.23. $f(z) = \frac{\operatorname{sh} z^3 - z^3}{\cos z^2 - 1 + z^4/2}$ |
| 12.9. $f(z) = \frac{\operatorname{sh} z^2 - z^2}{\cos 2z - 1 + 2z^2}$ | 12.24. $f(z) = \frac{\operatorname{ch} 6z - 1 - 18z^2}{\operatorname{sh} 6z - 6z}$ |
| 12.10. $f(z) = \frac{\cos z^2 - 1}{z^5}$ | 12.25. $f(z) = z^5 \operatorname{sh}\left(\frac{2}{z^2}\right)$ |
| 12.11. $f(z) = \frac{\operatorname{ch} 8z - 1 - 32z^2}{e^{z^2} - 1}$ | 12.26. $f(z) = \frac{e^{z^2} - 1 - z^2}{\sin 3z - 3z}$ |
| 12.12. $f(z) = \frac{e^{4z} - 1 - 4z}{\cos 6z - 1}$ | 12.27. $f(z) = z^3 \sin\left(\frac{2}{z^4}\right)$ |
| 12.13. $f(z) = z^3 \operatorname{ch}\left(\frac{5}{z}\right)$ | 12.28. $f(z) = \frac{\cos 2z - 1 + 2z^2}{e^{z^2} - 1}$ |
| 12.14. $f(z) = \frac{\cos 9z - 1}{\sin z - z + z^3/6}$ | 12.29. $f(z) = \frac{\cos 9x - 1}{\operatorname{ch} z - 1 - z^3/2}$ |
| 12.15. $f(z) = \frac{\operatorname{sh}(z/2) - z/2}{4z^4}$ | 12.30. $f(z) = \frac{3\operatorname{sh} z^3 - 3z^3}{e^{4z} - 1 - 4z}$ |

13. Вычислите интеграл

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| 13.1. $\oint_{ z-3 =1} \frac{\sin 4z + 3}{z^2(z - \pi)} dz$ | 13.16. $\oint_{ z-1 =2} \frac{z(z + \pi)}{\sin 2z} dz$ |
| 13.2. $\oint_{ z =1} \frac{e^{iz} + 3}{\sin 3zi} dz$ | 13.17. $\oint_{ z-3/2 =1} \frac{7z(z + \pi)}{\sin 3z(z - \pi)} dz$ |
| 13.3. $\oint_{ z-1 =3/2} \frac{\ln(z+5)}{\sin z} dz$ | 13.18. $\oint_{ z-\pi =1} \frac{3(z^2 + \pi)^2}{i \sin z} dz$ |
| 13.4. $\oint_{ z+1 =1/2} \frac{\operatorname{tg} z + 4}{4z^2 + \pi z} dz$ | 13.19. $\oint_{ z-\pi =2} \frac{\cos^2 z}{2z \sin z} dz$ |
| 13.5. $\oint_{ z+1 =2} \frac{\sin^2 z - 7}{z^2 + 2\pi z} dz$ | 13.20. $\oint_{ z-1 =2} \frac{z^2 + 3}{(z^2 + 4) \sin \frac{z}{3}} dz$ |
| 13.6. $\oint_{ z =\pi/2} \frac{z^2 + z + 5}{\sin z(\pi + z)} dz$ | 13.21. $\oint_{ z =2} \frac{z^2 + \sin z + 6}{z^2 + \pi z} dz$ |
| 13.7. $\oint_{ z-1/2 =1} \frac{e^z + 6}{z(z-1)} dz$ | 13.22. $\oint_{ z =2} \frac{\sin^2 z}{3z \cos z} dz$ |
| 13.8. $\oint_{ z-2 =3} \frac{\cos^2 + 5}{z^2 - \pi^2} dz$ | 13.23. $\oint_{ z-3/2 =2} \frac{\sin 2z}{z(z - \pi) \left(z + \frac{\pi}{3}\right)} dz$ |

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| 13.9. $\oint_{ z-6 =1} \frac{\sin^3 z + 4}{z^2 - 4\pi^2} dz$ | 13.24. $\oint_{ z-1/2 =1} \frac{5iz(z-i)}{\sin \pi z} dz$ |
| 13.10. $\oint_{ z+3/2 =1} \frac{\cos^2 z + 9}{2z^2 + \pi z} dz$ | 13.25. $\oint_{ z-3/2 =2} \frac{4z}{\sin z} dz$ |
| 13.11. $\oint_{ z =1/4} \left(\frac{\ln(e+z)}{z \sin(z+\pi/4)} + 2z \right) dz$ | 13.26. $\oint_{ z-3/2 =2} \frac{z(\sin z + 3)}{\sin z} dz$ |
| 13.12. $\oint_{ z =1} \frac{z^3 - 2i}{\sin 2z(z-\pi)} dz$ | 13.27. $\oint_{ z =1} \frac{8 + \sin z}{z(z+2i)} dz$ |
| 13.13. $\oint_{ z =1/2} \frac{z+2}{z(z^2+1)} dz$ | 13.28. $\oint_{ z-1-i =5/4} \frac{7}{z^2(z-1)} dz$ |
| 13.14. $\oint_{ z-i =3/2} \frac{z-3}{z(z^2+4)} dz$ | 13.29. $\oint_{ z-1/4 =1/3} \frac{z(z+3)^2}{\sin 2\pi z} dz$ |
| 13.15. $\oint_{ z-3 =1/2} \frac{e^z - 1}{\sin z} dz$ | 13.30. $\oint_{ z-1 =3} \frac{ze^{2z}}{\sin z} dz$ |

14. Вычислите интеграл, используя теорию функций комплексного переменного

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| 14.1. $\int_{-\infty}^{+\infty} \frac{dx}{(x^2+2)(x^2+3)^2}$ | 14.16. $\int_0^{2\pi} \frac{dx}{10 - 3\sqrt{7} \sin x}$ |
| 14.2. $\int_0^{2\pi} \frac{dx}{\sqrt{21} \sin x + 5}$ | 14.17. $\int_{-\infty}^{+\infty} \frac{(x+2)dx}{(x^2+6)^3}$ |
| 14.3. $\int_{-\infty}^{+\infty} \frac{(x^2+1)dx}{(x^2+x+1)^2}$ | 14.18. $\int_0^{2\pi} \frac{dx}{\sqrt{5} \sin x + 3}$ |
| 14.4. $\int_0^{2\pi} \frac{dx}{3 \sin x + 5}$ | 14.19. $\int_{-\infty}^{+\infty} \frac{dx}{(x^2+4)(x^2+1)^2}$ |
| 14.5. $\int_{-\infty}^{+\infty} \frac{(x^2+4)dx}{(x^2+9)^2}$ | 14.20. $\int_0^{2\pi} \frac{dx}{\sqrt{35} \sin x - 6}$ |
| 14.6. $\int_0^{2\pi} \frac{dx}{4 - 2\sqrt{3} \sin x}$ | 14.21. $\int_{-\infty}^{+\infty} \frac{dx}{x^4 + 10x^2 + 9}$ |
| 14.7. $\int_{-\infty}^{+\infty} \frac{(x^2-x+2)dx}{x^4 + 10x^2 + 9}$ | 14.22. $\int_0^{2\pi} \frac{dx}{5 - 4 \sin x}$ |
| 14.8. $\int_0^{2\pi} \frac{dx}{3\sqrt{7} \sin x + 8}$ | 14.23. $\int_{-\infty}^{+\infty} \frac{x^2 dx}{(x^2+11)^2}$ |
| 14.9. $\int_{-\infty}^{+\infty} \frac{dx}{(x^2+9)(x^2+4)^2}$ | 14.24. $\int_0^{2\pi} \frac{dx}{\sqrt{3} \sin x - 2}$ |

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| 14.10. $\int_0^{2\pi} \frac{dx}{9 - 4\sqrt{5} \sin x}$ | 14.25. $\int_{-\infty}^{+\infty} \frac{dx}{(x^2 - 10x + 29)^2}$ |
| 14.11. $\int_{-\infty}^{+\infty} \frac{(x^2 + 1)dx}{(x^2 + 4x + 13)^2}$ | 14.26. $\int_0^{2\pi} \frac{dx}{7 + 4\sqrt{3} \sin x}$ |
| 14.12. $\int_0^{2\pi} \frac{dx}{4 \sin x + 5}$ | 14.27. $\int_{-\infty}^{+\infty} \frac{(x^2 + 5)dx}{x^4 + 5x^2 + 6}$ |
| 14.13. $\int_{-\infty}^{+\infty} \frac{dx}{x^4 + 7x^2 + 12}$ | 14.28. $\int_0^{2\pi} \frac{dx}{8 - 3\sqrt{7} \sin x}$ |
| 14.14. $\int_0^{2\pi} \frac{dx}{9 + 4\sqrt{5} \sin x}$ | 14.29. $\int_{-\infty}^{+\infty} \frac{dx}{(x^2 + 10)^2 (x^2 + 2)^2}$ |
| 14.15. $\int_{-\infty}^{+\infty} \frac{x^2 dx}{(x^2 + 3)^2}$ | 14.30. $\int_0^{2\pi} \frac{dx}{2\sqrt{2} \sin x + 3}$ |