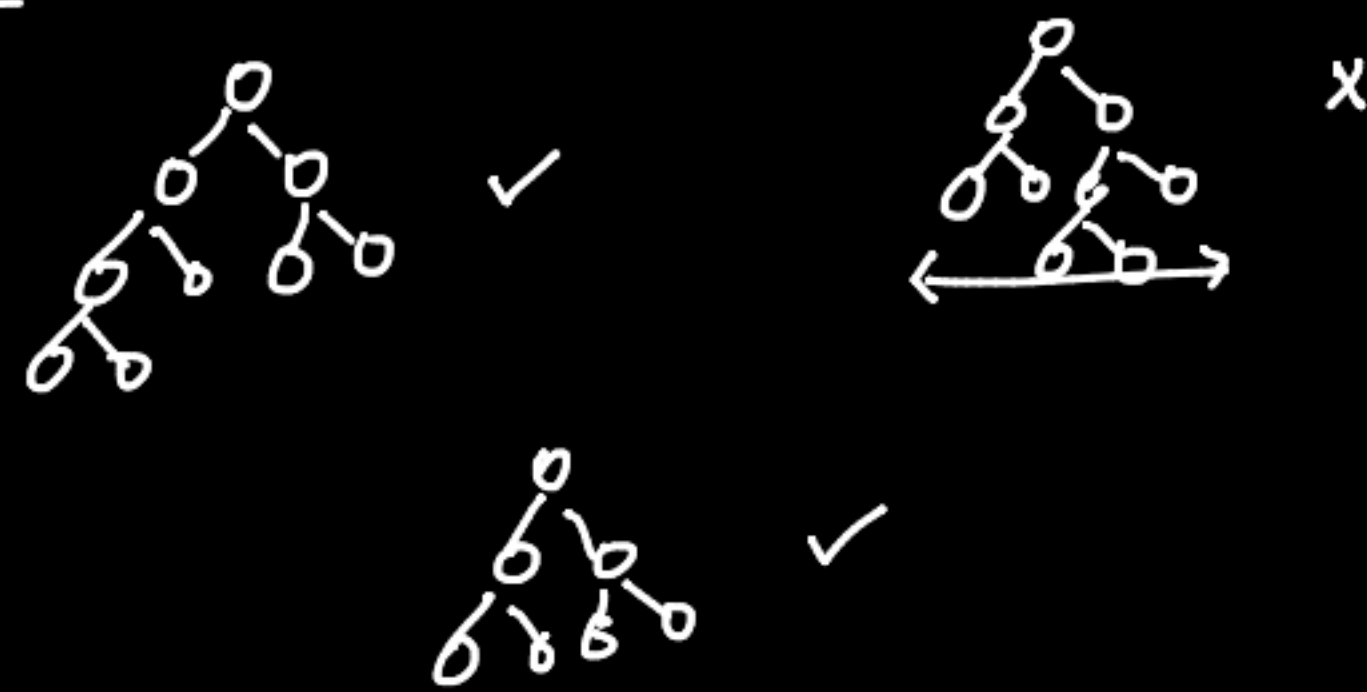
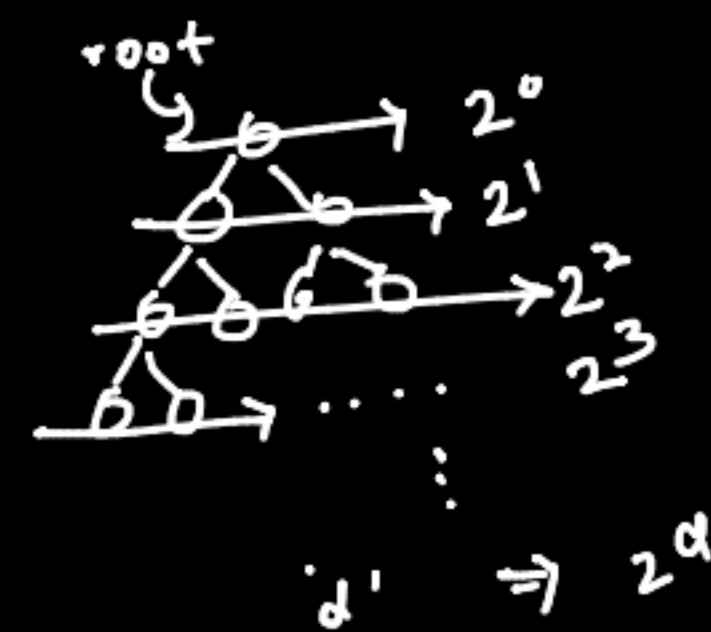


CBT

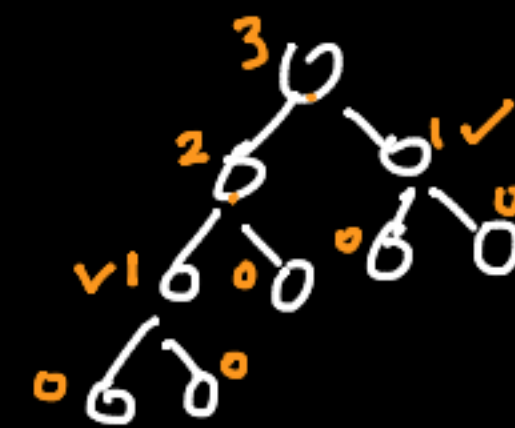


'n'

$$l(CBT_n) = \log_2 n$$



'n'
'l'



$$\begin{aligned} \checkmark 2 &\Rightarrow l=1 \\ \checkmark 5 &\Rightarrow l=0 \\ \checkmark 1 &\Rightarrow l=3 \checkmark \\ \checkmark 1 &\Rightarrow l=2 \end{aligned}$$

$$\Rightarrow \left\lfloor \frac{n}{2^{l+1}} \right\rfloor$$

$$\frac{9}{2^{3+1}} = \left\lfloor \frac{9}{8} \right\rfloor = 1 \quad n=9$$

$$l=1 \Rightarrow \left\lfloor \frac{9}{2^{1+1}} \right\rfloor$$

$$\Rightarrow \left\lfloor \frac{9}{4} \right\rfloor \Rightarrow 2$$

$$\frac{9}{2^{3+1}} = \frac{9}{16} = \left\lfloor 0.5625 \right\rfloor = 0$$

$$\left\lfloor \frac{n}{2^{l+1}} \right\rfloor$$

$$\left\lfloor \frac{n}{2^{l+1}} \right\rfloor$$

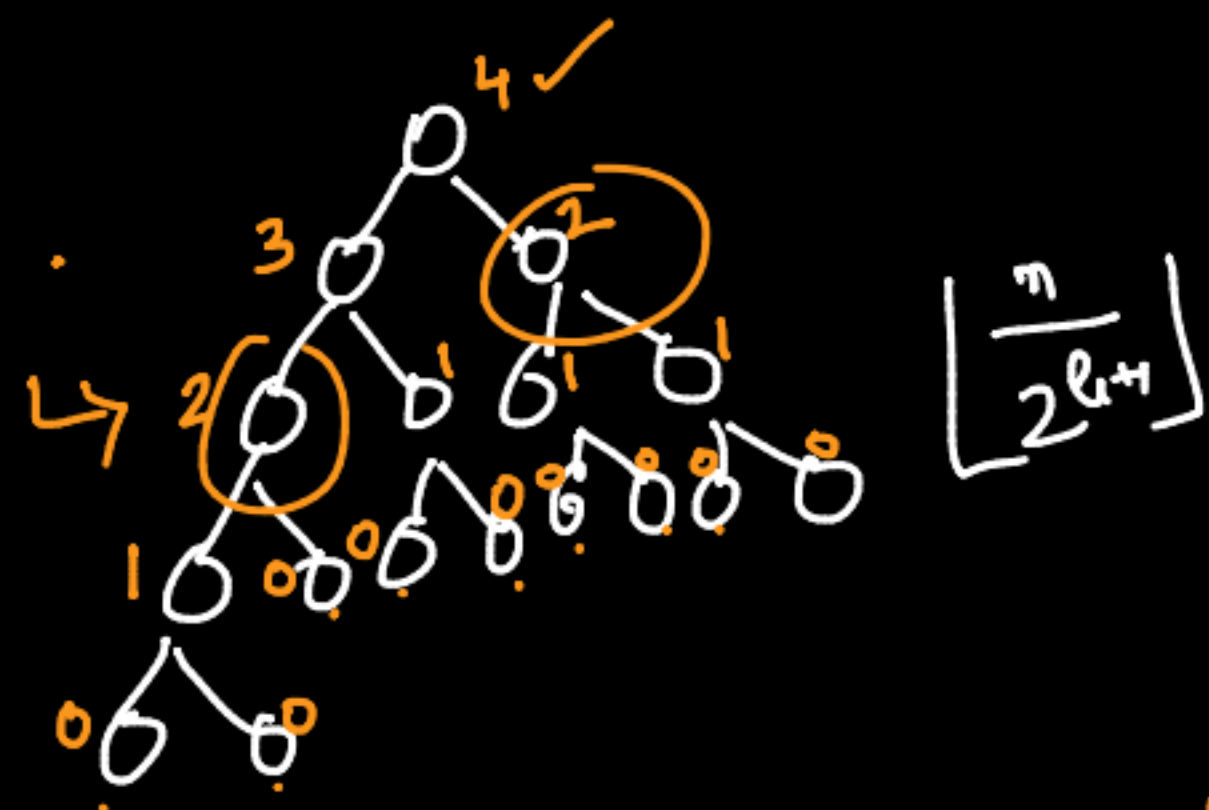
$$\frac{9}{2^{0+1}} = \frac{9}{2} = \left\lfloor 4.5 \right\rfloor = 4 \Rightarrow \frac{n}{2^{l+1}}$$

$$\frac{9}{2^{2+1}} = \frac{9}{8} = \left\lfloor 1.125 \right\rfloor = 1$$

$$\frac{9}{2^{1+1}} = \frac{9}{4} = \left\lfloor 2.25 \right\rfloor = 2$$

$$n=17$$

$$\left\lceil \frac{n}{2^{l+1}} \right\rceil$$



$$\left\lceil \frac{n}{2^{l+1}} \right\rceil$$

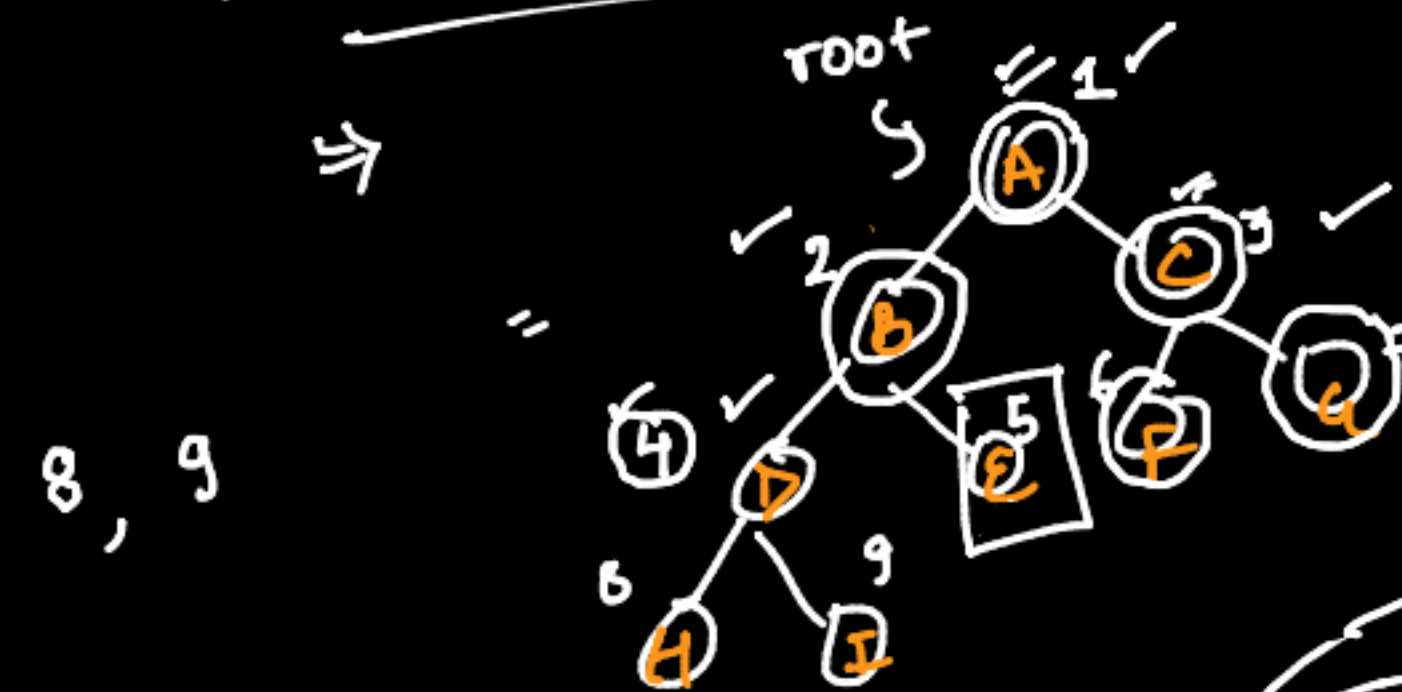
$$l=0 \Rightarrow \frac{17}{2^1} = \frac{17}{2} = \lceil 8.5 \rceil = 9 \checkmark$$

$$l=2 = \frac{17}{2^{2+1}} = \frac{17}{8} = \lceil 2.125 \rceil = 3 \checkmark$$

$$= 2 \frac{17}{2^{4+1}} = \frac{17}{32} = \lceil 0.53125 \rceil = 1 \checkmark$$

$$2 \frac{n}{2^{l+1}}$$

✓ representation of CBT



8, 9

Array

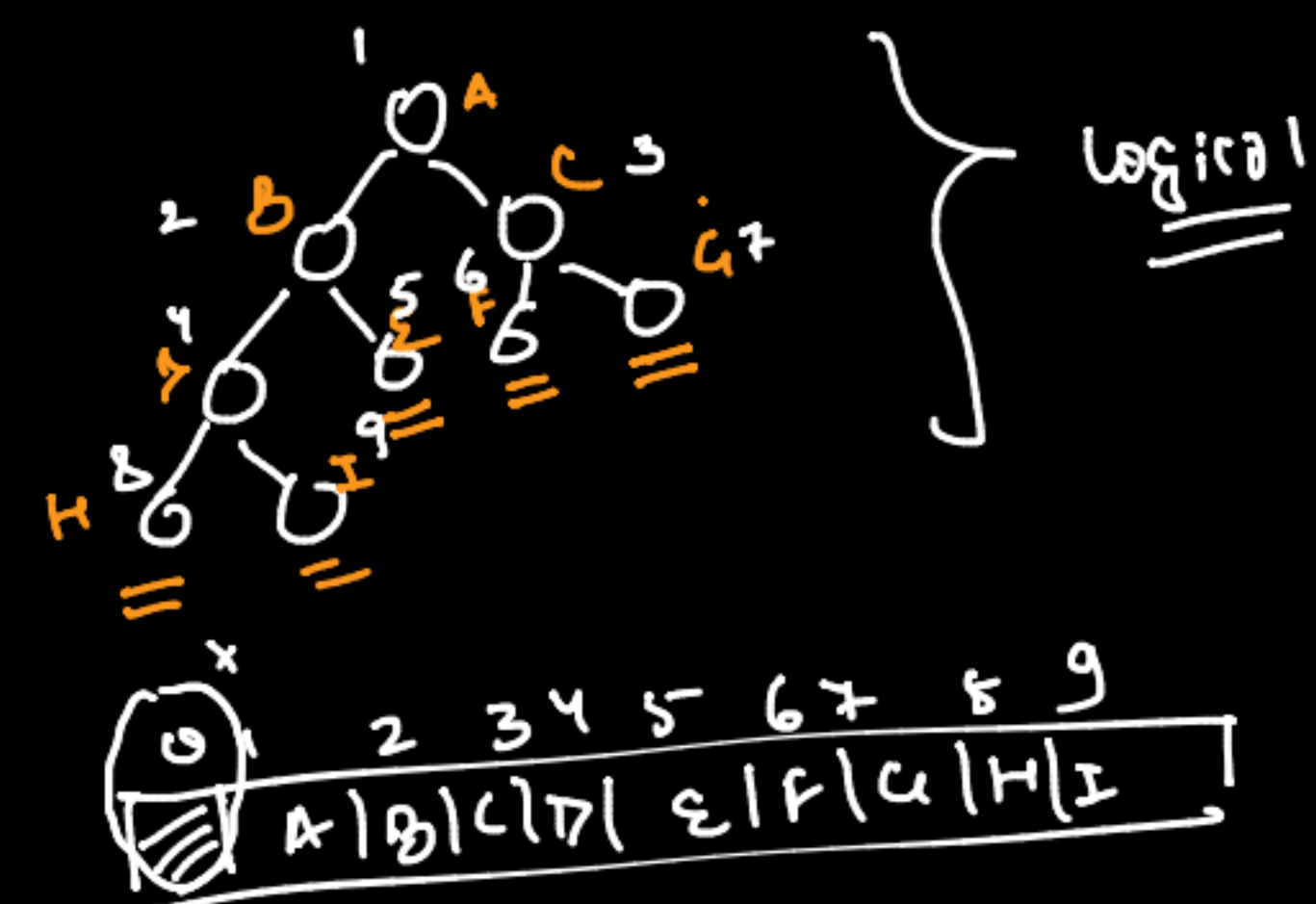


for any node i in $idx \rightarrow 2i$
 $\rightarrow right \rightarrow (2i+1)$

for any node i in $idx \Rightarrow \lfloor i/2 \rfloor$

$$4/2 = 2$$

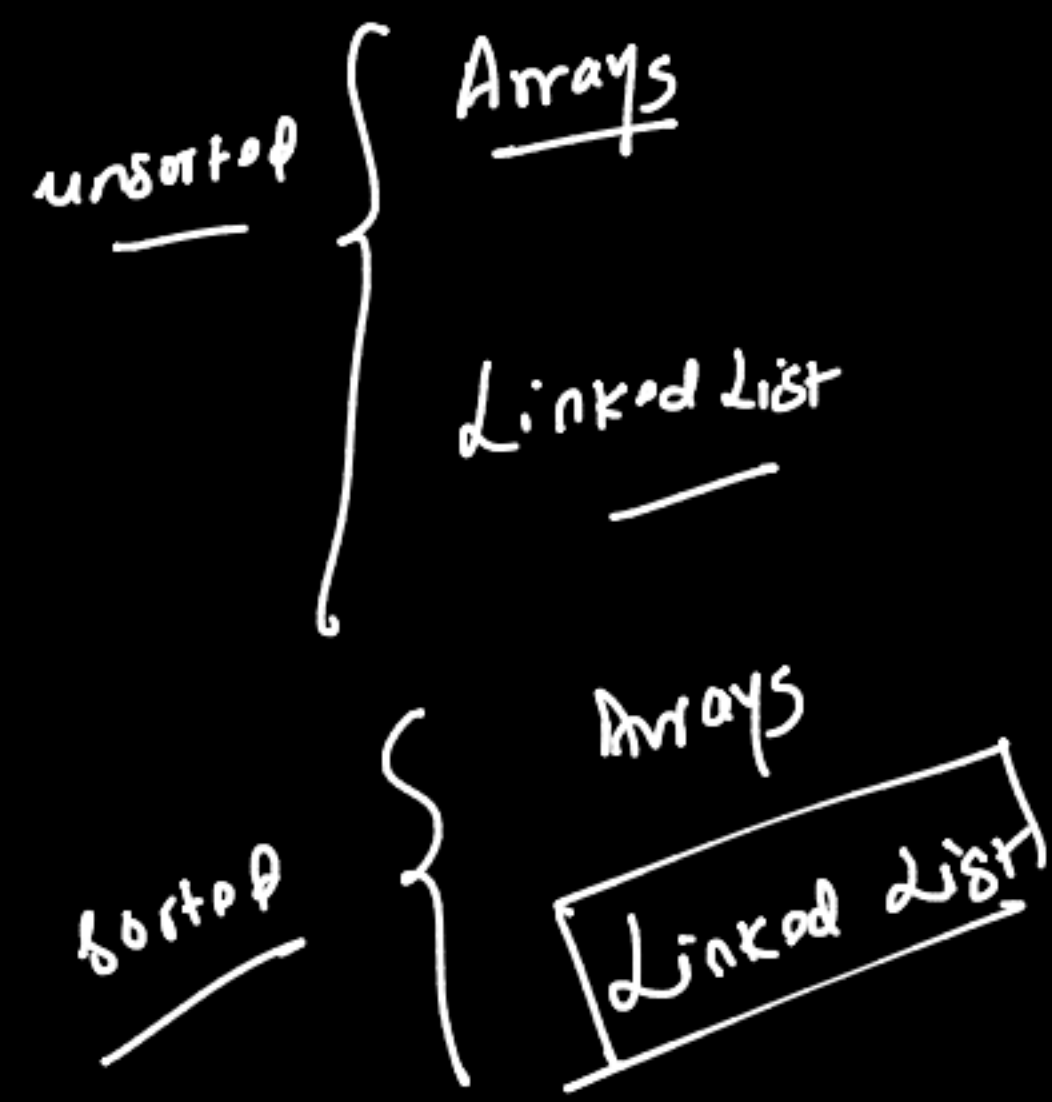
non-leaf
 $\lfloor n/2 \rfloor + 1$



logical

$$\text{starting pt. of leaf} = \lfloor n/2 \rfloor + 1 = \lfloor 9/2 \rfloor + 1 = 4 + 1 = 5$$

Heaps / Priority Queue



unbalanced { BST

balanced { BST

Heap / PQ insertion $O(\log n)$

min heap
max heap

<u>Insertion</u>	<u>find min</u>	<u>delete min</u>
$O(1)$	$O(n)$	$O(n)$
$O(1)$	$O(n)$	$O(n)$
$O(n)$	$O(1)$	$O(1)$
$O(n)$	$O(1)$	$O(n)$
$O(n)$	$O(\log n)$	$O(\log n)$

delete min/max
 $O(\log n)$

find min/max
 $O(1)$

Heap (min heap)

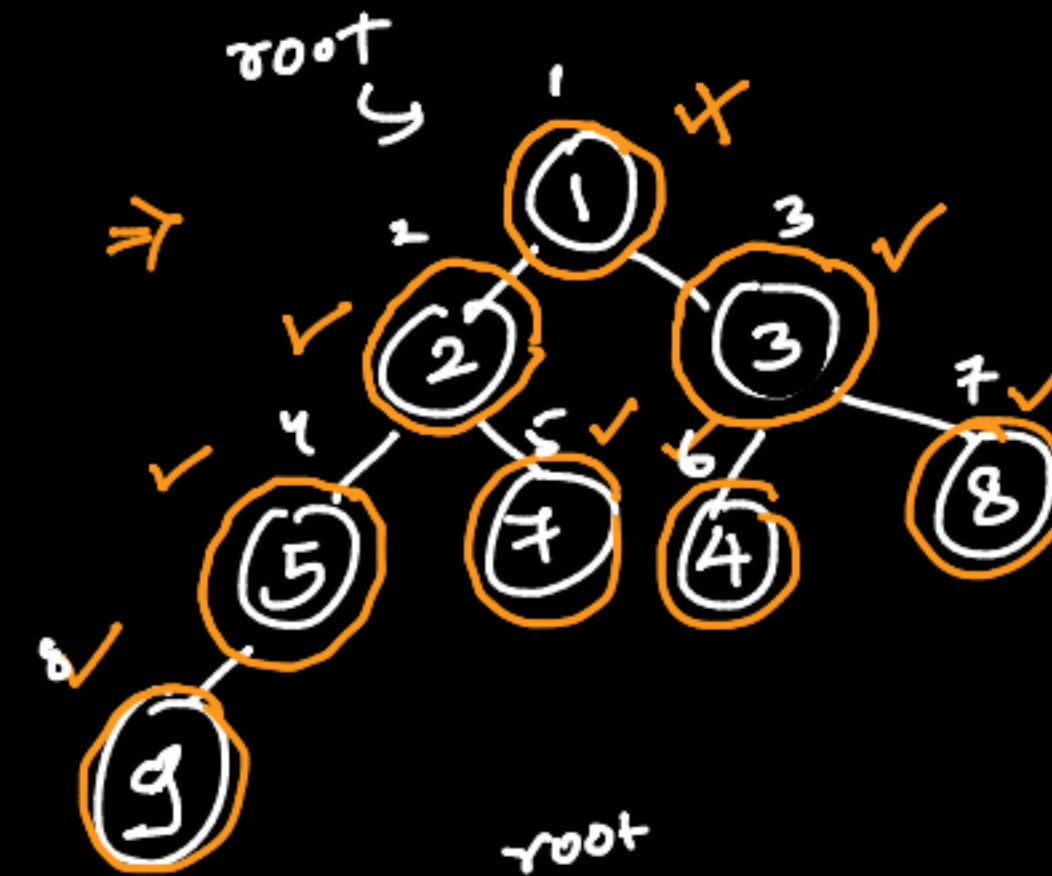
↳ CBT

↳ that satisfies heap property

⇒ (Min heap) for each node its
 value should \leq its
 child nodes

(Max heap) for each node its
 value should \geq its
 child nodes

Min heap

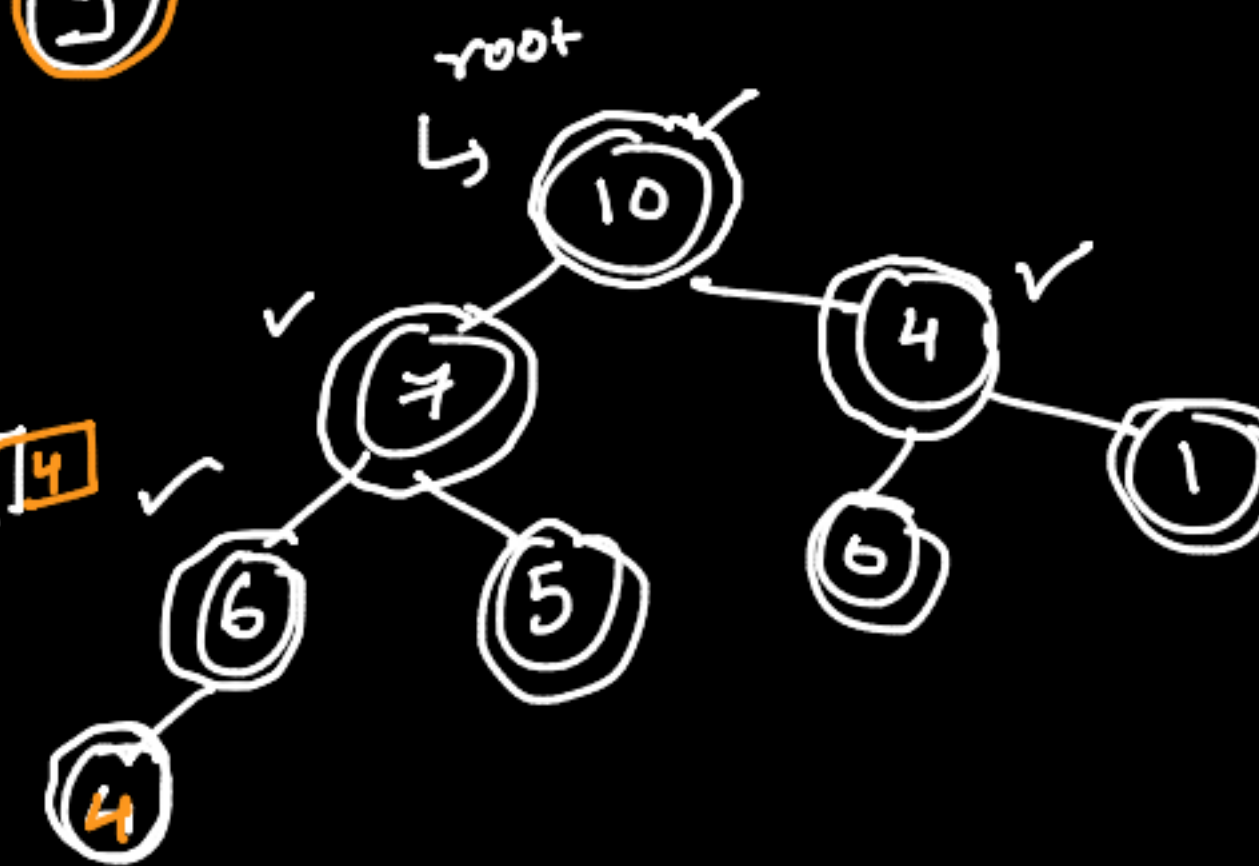


CBT ? ✓

0 1 2 3 4 5 6 7 8
1 2 3 5 7 4 8 9

Max heap

0 1 2 3 4 5 6 7 8
10 7 4 6 5 0 4

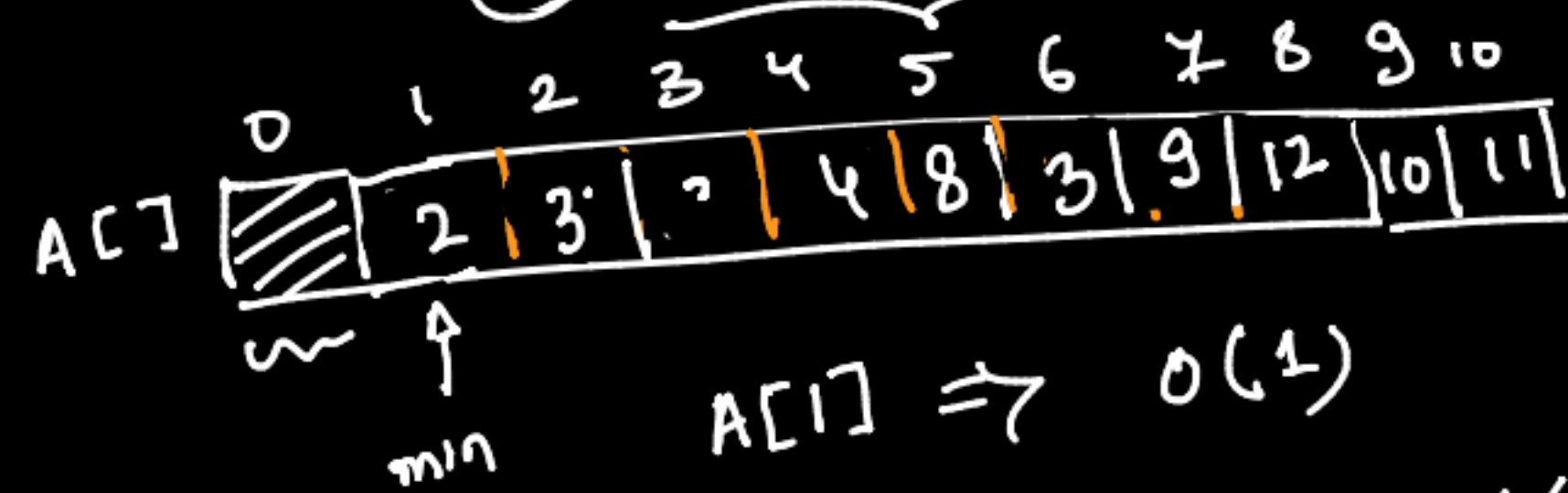
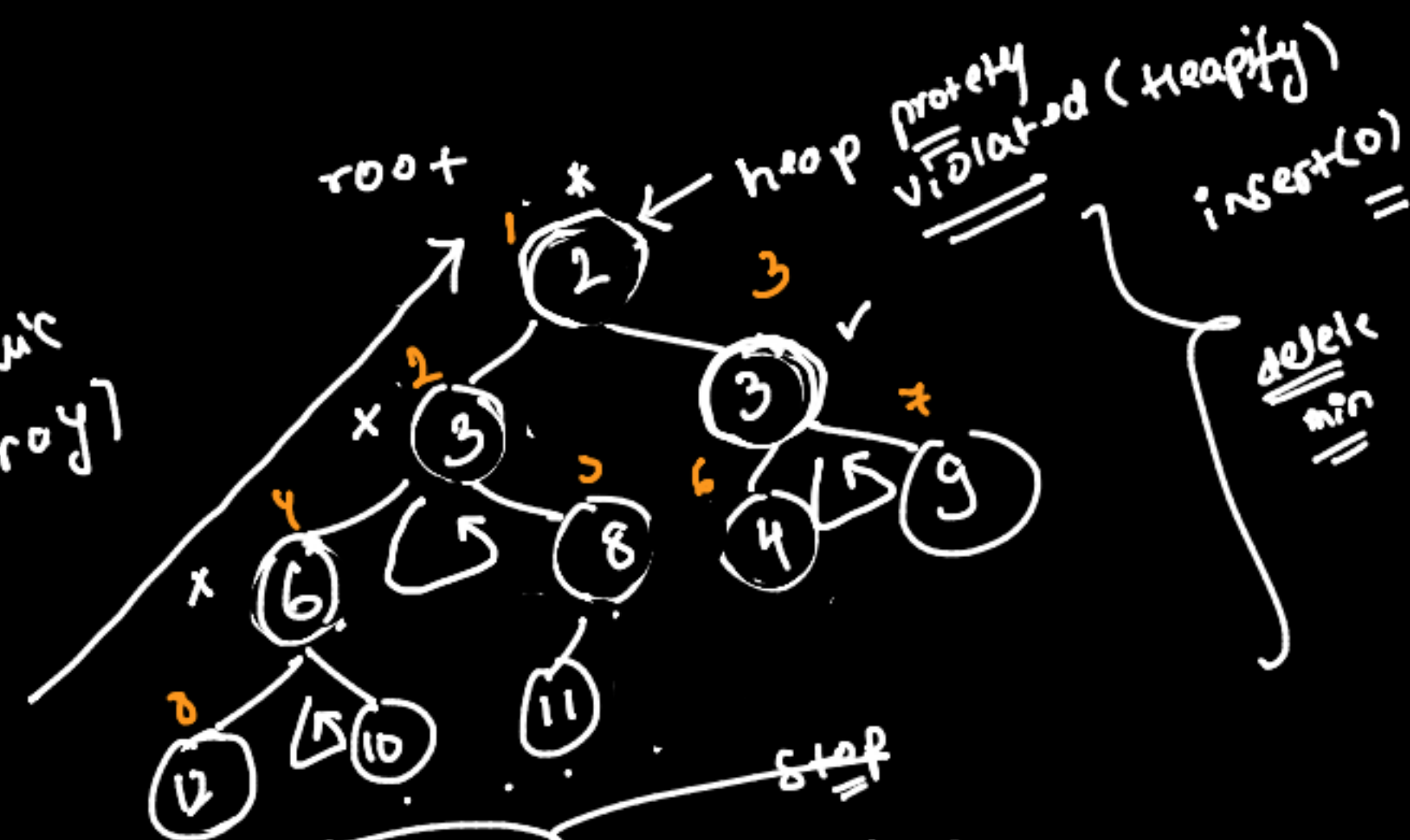


CBT ? ✓

Min-Heap

vector < dynamic array >

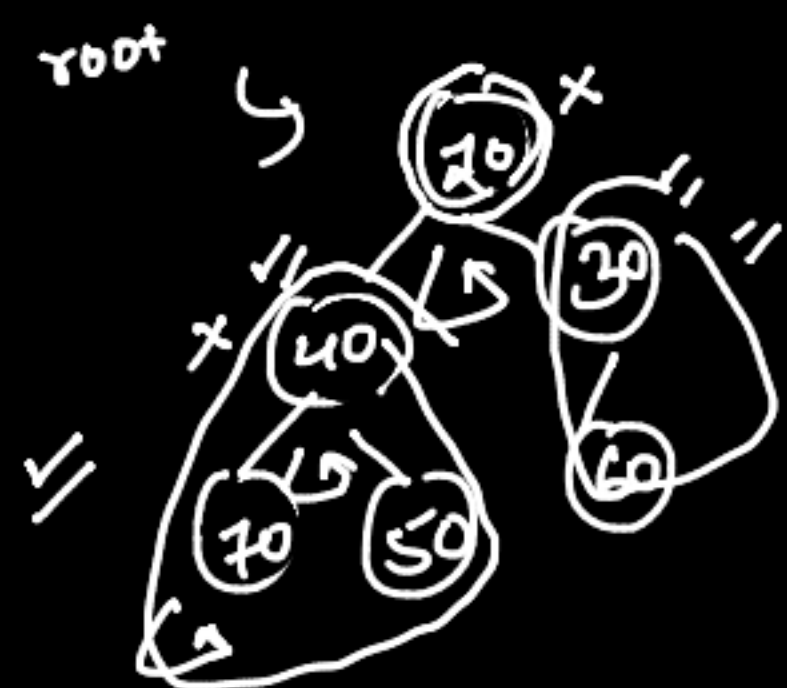
$O(\log N)$



$A[1] \Rightarrow O(1)$

$\propto O(4) \Rightarrow O(\log N)$

heapify



$\uparrow \log N$
 \downarrow
 $\propto O(4)$

