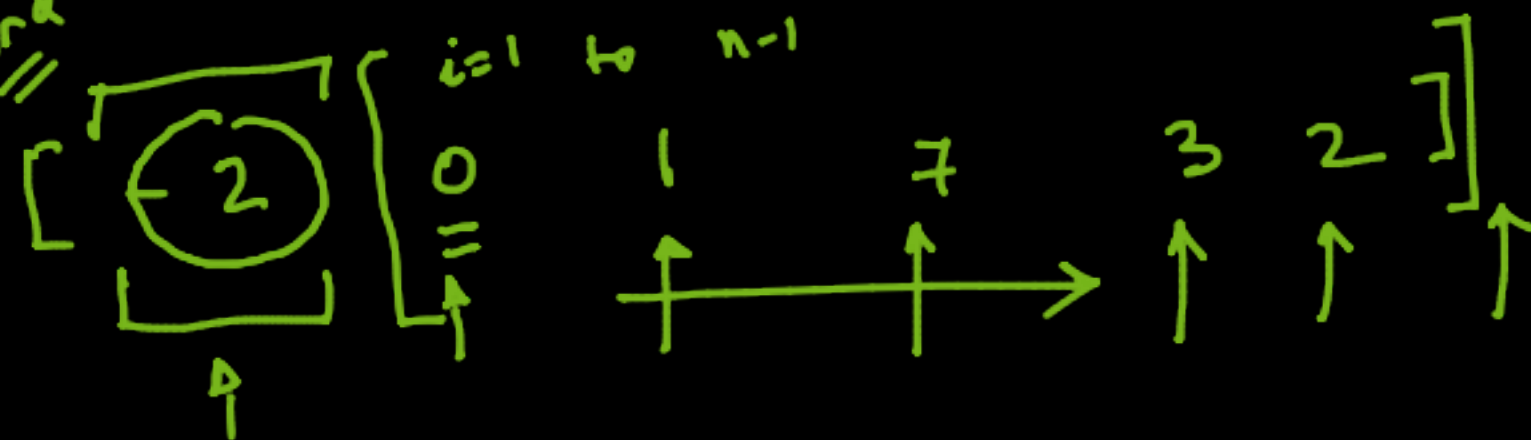
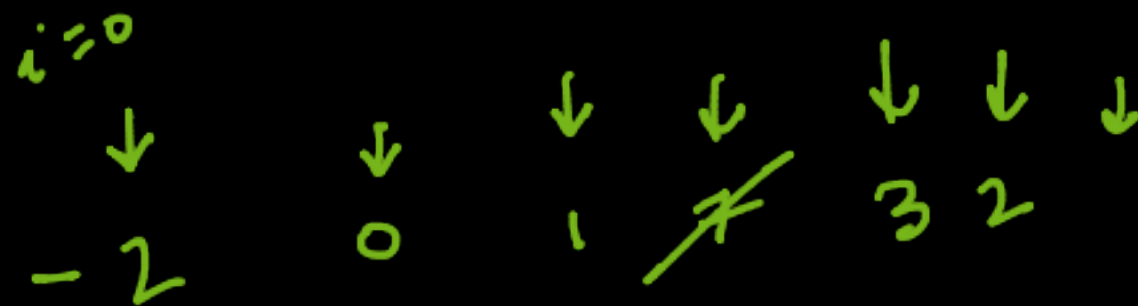




$A = []$   $\rightarrow$  empty  
 $\text{maxValue} = A[0]$   
 $\rightarrow$  Int out of Bound



$\text{maxValue} = \underline{\underline{-2}}$   
 $\uparrow$   
 $\rightarrow \text{INT-MIN}$   
 $\rightarrow \text{INT-MAX}$



$\text{maxValue} = \underline{\underline{-1}}$   
 $\cancel{-2}$   $\cancel{0}$   $\cancel{1}$   $\cancel{7}$

Bubble Sort n=5

idx	0	1	2	3	4
arr[]	5	4	3	2	1

1<sup>st</sup> pass

$i=0 =$

$n-i-1 =$

$5-0-1 = 4 =$

[ 5 4 3 2 1 ]

4 5 3 2 1

4 3 5 2 1

4 3 2 5 1

[ 4 3 2 1 ] [ 5 ]

2<sup>nd</sup> pass

$i=1 =$

$[n-i-1]$

$5-1-1 = 3$

[ 4 3 2 1 ] [ 5 ]

3 4 2 1

3 2 4 1

[ 3 2 1 ] [ 4 5 ]

3<sup>rd</sup> pass

$i=2 =$

$n-i-1$

$5-2-1 = 2$

[ 3 2 1 ] [ 4 5 ]

2 3 1

$n-i-1$

$5-2-1 = 2$

[ 2 1 ] [ 3 4 5 ]

$(n-i-1)$

4<sup>th</sup> pass

$i=3 =$

$5-3-1 = 1$

[ 2 1 ] [ 3 4 5 ]

[ 1 ] [ 2 3 4 5 ]

[ 1 2 3 4 5 ]

stop

$(n-1)$  passes
 
$$\left\{ \begin{array}{l} i=0 \\ i=1 \\ i=2 \\ \vdots \\ i=n-2 \end{array} \right.$$

$$\begin{array}{l} n-i-1 = n-1 \\ \quad \quad = n-2 \\ \quad \quad = n-3 \\ \quad \quad \vdots \\ \quad \quad = 1 \end{array}$$

$$\left. \begin{array}{l} \text{break (Best)} \\ \text{(worst)} \end{array} \right\}$$

$$= [1 + 2 + 3 + \dots + n-3 + n-2 + n-1] + n$$

$$= (1 + 2 + 3 + \dots + n) - n$$

$$= \frac{n(n+1)}{2} - n$$

worst

$$= \boxed{\frac{n(n-1)}{2}}$$

$O \rightarrow$  worst  $\Theta \rightarrow$  avg.  
 $n \rightarrow$  best

```

for (i=0; i < n-1; i++)
    int count = 0
    for (j=0; j < n-1-i; j++) {
        if (a[j] > a[j+1]) {
            swap(a[j], a[j+1])
            count++
        }
        if (count == 0) break
    }
    }
    
```

1 2 3 4 5  
 (n-1) steps

$\Rightarrow$  sorted  $\Theta \left( \frac{n + n^2}{2} \right) \Rightarrow \Theta(n^2)$

Best [1 2 3 4 5]  $\Rightarrow$

for (int i=0; i < n-1; i++) {

[ 1 time  
 (n-1)

for (int j=0; j < n-i-1; j++) {  
 $\begin{array}{c} \longleftrightarrow \\ \longleftrightarrow \end{array} \Rightarrow \frac{n(n-1)}{2}$   
 } break =

$n=5, n-1=4$

for (i=0; i < 4; i++)

$i=0, i=1, i=2, i=3$  stop

1 2 3 4 5  
 (n-1) steps (Best)  
 $\frac{n(n-1)}{2}$  steps (Worst)  
 5 4 3 2 1

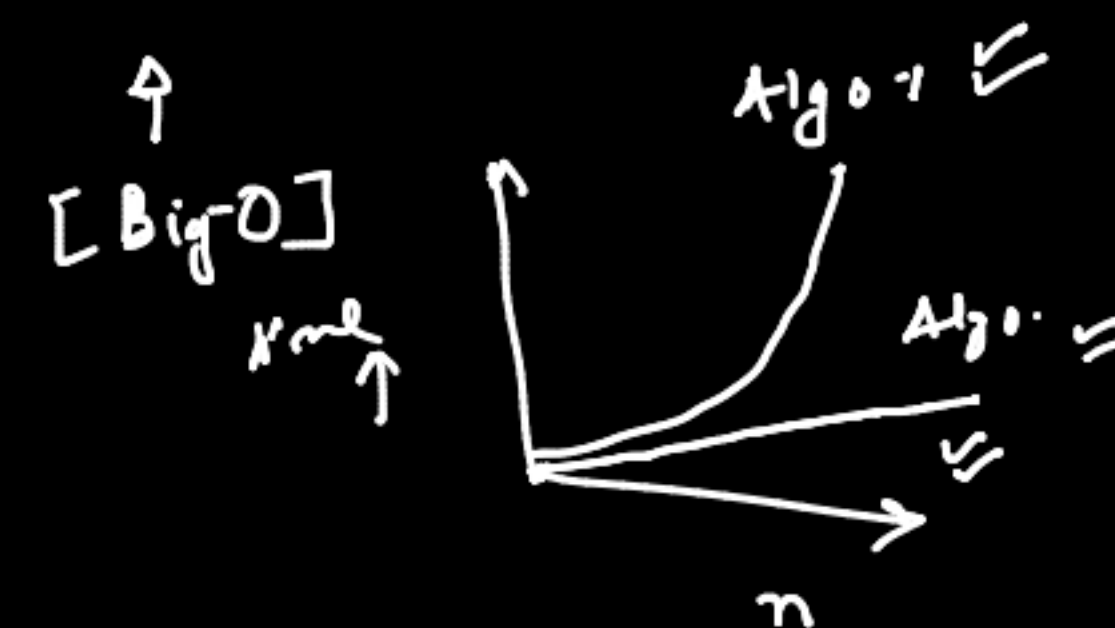
i/p  $\Rightarrow n$   
 1  $\rightarrow 1^2$  sec  
 2  $\rightarrow 2^2$  sec

$\Rightarrow \left[ \frac{n(n-1)}{2} \right] = \frac{n^2 - n}{2}$

$\Rightarrow \boxed{O(n^2)} \rightarrow$  worst  
 $\boxed{O(n)} \rightarrow$  best

worst =  $O(n^2)$  quadratic

Best =



Best case  
 (n-1) steps  
 $\Theta(n)$

$$x = 5 \quad y = 10$$

1.  $\Rightarrow$   $\left[ \begin{array}{l} \text{int temp} = x \\ x = y \\ y = \text{temp} \end{array} \right]$  swap

2.  $\Rightarrow$  builtin  
swap(x, y)

3.  $\Rightarrow x = (x + y) - (y = x)$

$$= 15 - 5$$

$$\boxed{x = 10} \quad \boxed{y = 5}$$

$$x = 5;$$

(4)  $\boxed{x = x \wedge y} \rightarrow \textcircled{1}$

$$\boxed{y = \textcircled{2} \wedge y = (x \wedge y) \wedge y} \rightarrow \textcircled{2}$$

$$x = x \wedge y = (x \wedge y) \wedge x$$

$$\boxed{x = y}$$

swap

$$\left[ \begin{array}{l} \Rightarrow x = 1 \\ \text{swap}(x, x) \end{array} \right]$$

$$\left[ \begin{array}{l} x = x + y = 5 + 10 = 15 \\ y = x - y = 15 - 5 = 10 \\ x = x - y = 15 - 10 = 5 \end{array} \right]$$

$$\text{temp} = x$$

$$x = x$$

$$x = \text{temp}$$

$$\text{temp} = 1$$

$$x = 1$$

$$\boxed{x = 1} \Rightarrow$$

$$\begin{array}{l} x = x \wedge x = 0 \\ x = \textcircled{2} \wedge x = \textcircled{2} \\ \boxed{x = \textcircled{2} \wedge x = 0} \end{array}$$

Integers  
are

$$x = 5 \quad y = 10$$

Slower

$$x = x \wedge y \quad \text{---} \textcircled{1}$$

$$y = \textcircled{2} = x \wedge y = (x \wedge y) \wedge y = x \wedge 0 = \textcircled{2}$$

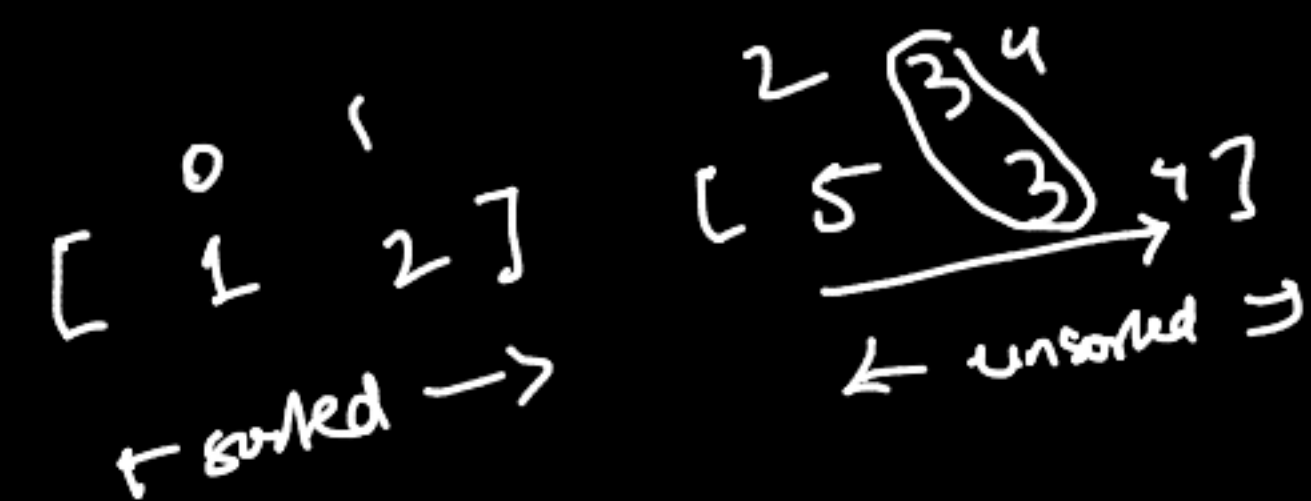
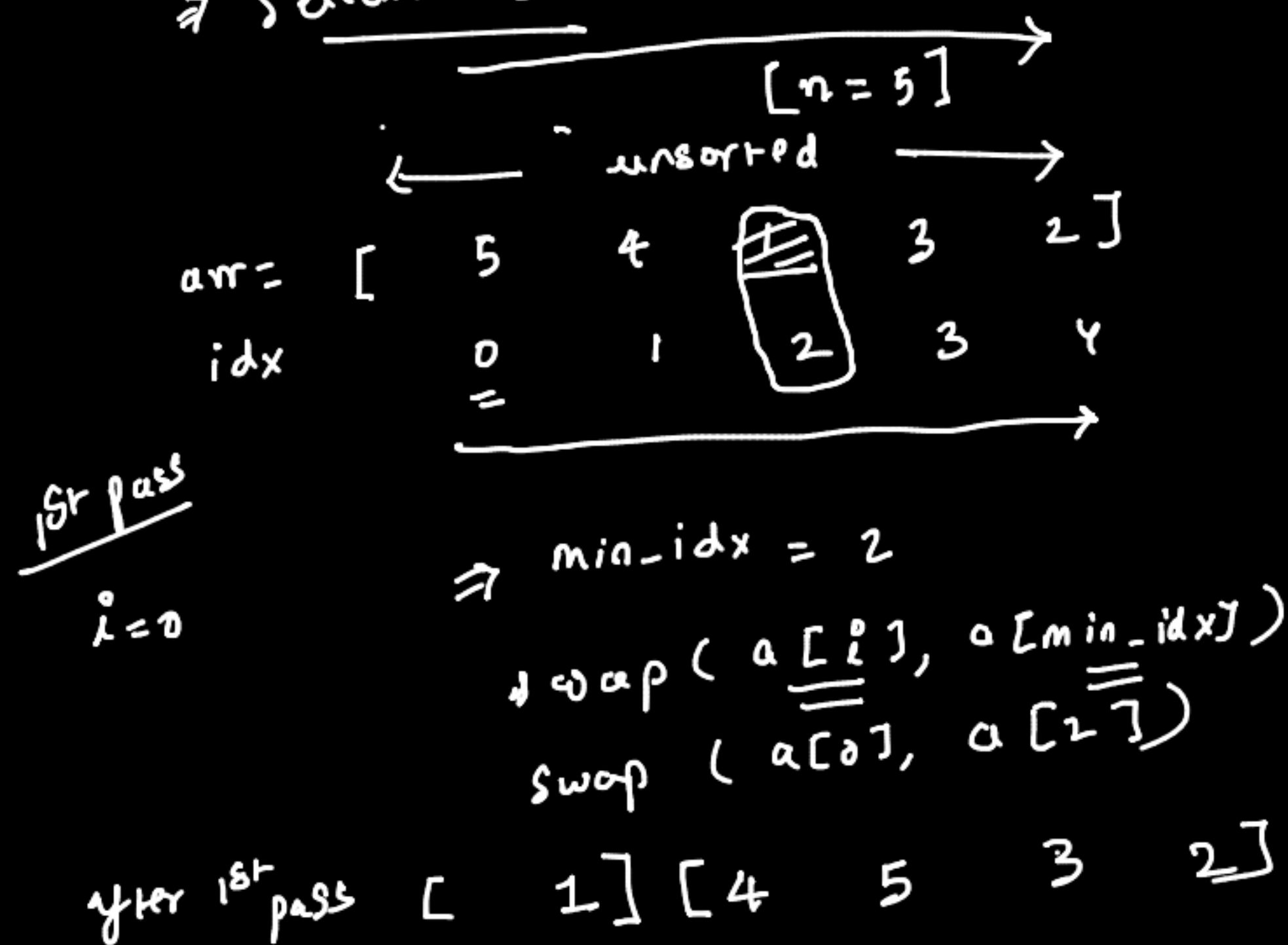
$$x = x \wedge y = (x \wedge y) \wedge x = y \wedge 0 = y$$

$$\left[ \begin{array}{l} x \wedge x = 0 \quad \leftarrow \\ x \wedge 0 = x \quad \leftarrow \end{array} \right]$$

swap (builtin)



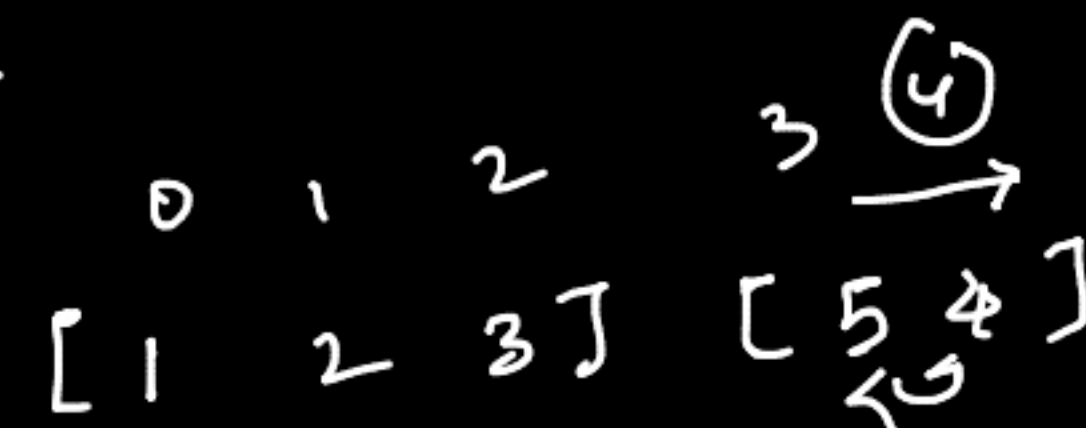
## Selection Sort



3rd pass

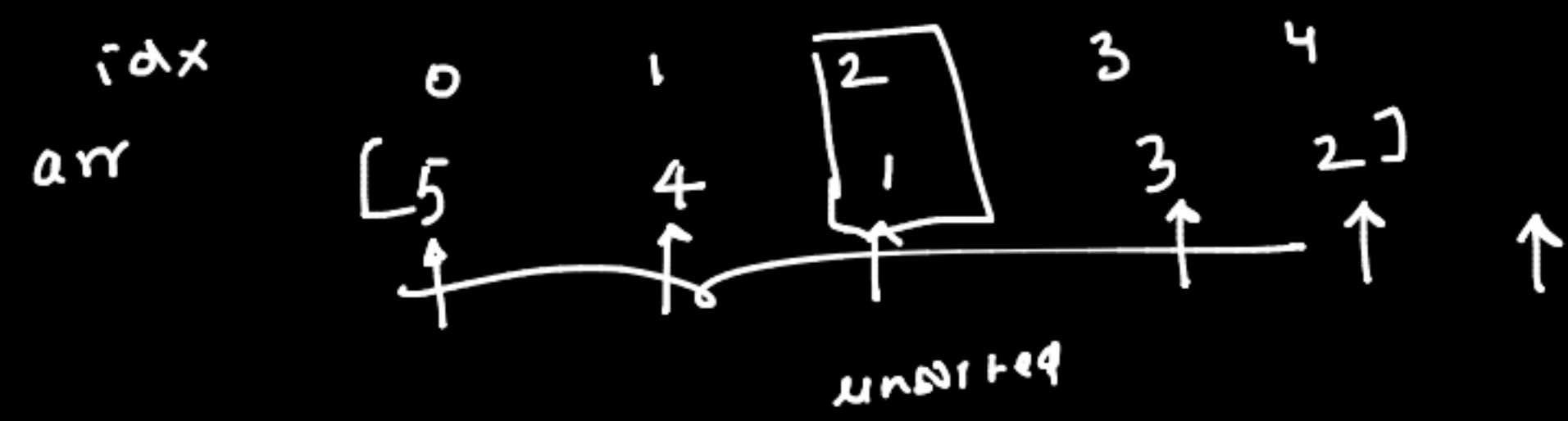
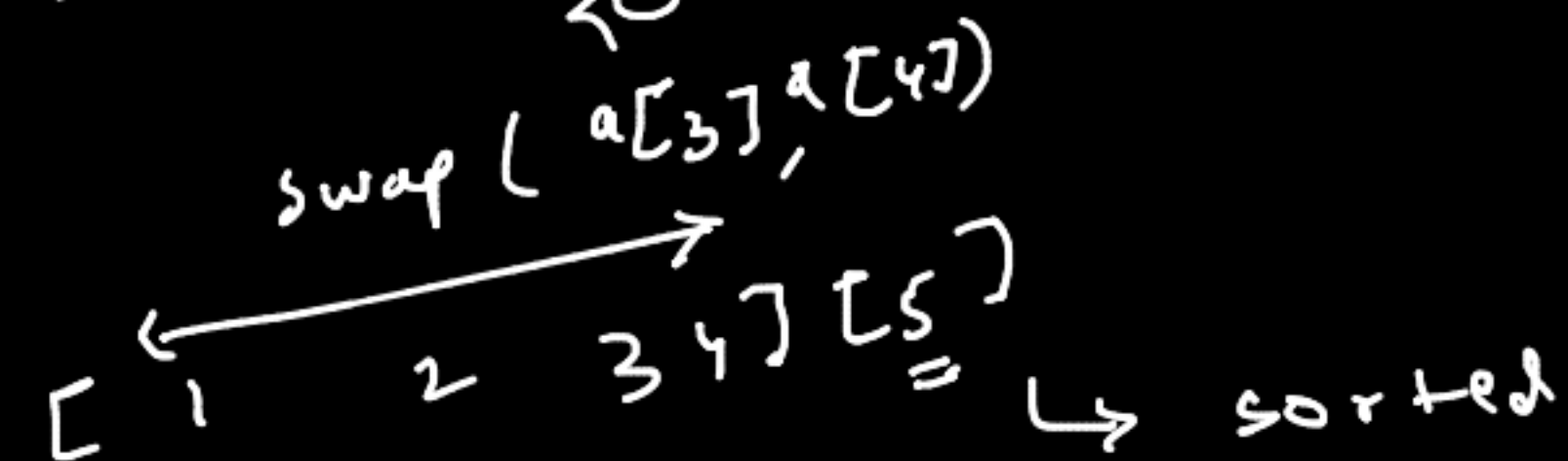
$i=2$

$\text{swap}(a[2], a[3])$



4th pass

$i=3$



$i=0$

$\text{min\_idx} = 2$

5 < 5

1st pass

[ 1 ] [ 4 5 3 2 ]

$\text{swap}(a[0], a[2])$

4 < 5 ✓

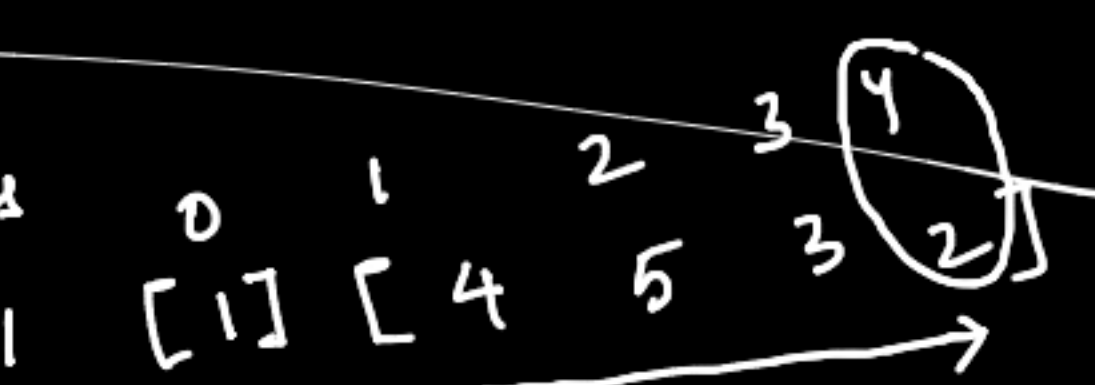
2 < 4 ✓

3 < 1 ✗

2 < 1 ✗

2nd pass

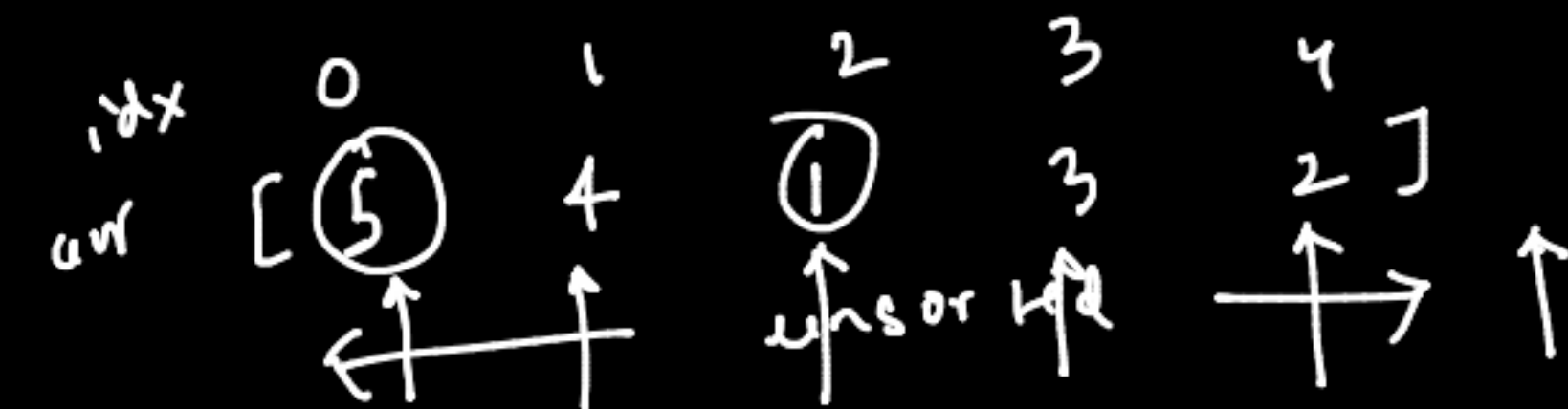
$\Rightarrow i=1$



$\text{swap}(a[1], a[4])$

after 2nd

$\rightarrow [ 1 2 ] [ 5 3 4 ]$



1st pass

$i=0$

$\text{min\_idx} = i = 2$

$\text{swap}(a[i], a[\text{min\_idx}])$   
 $\text{swap}(a[0], a[2])$

5 < 5 ✗

4 < 5 ✓

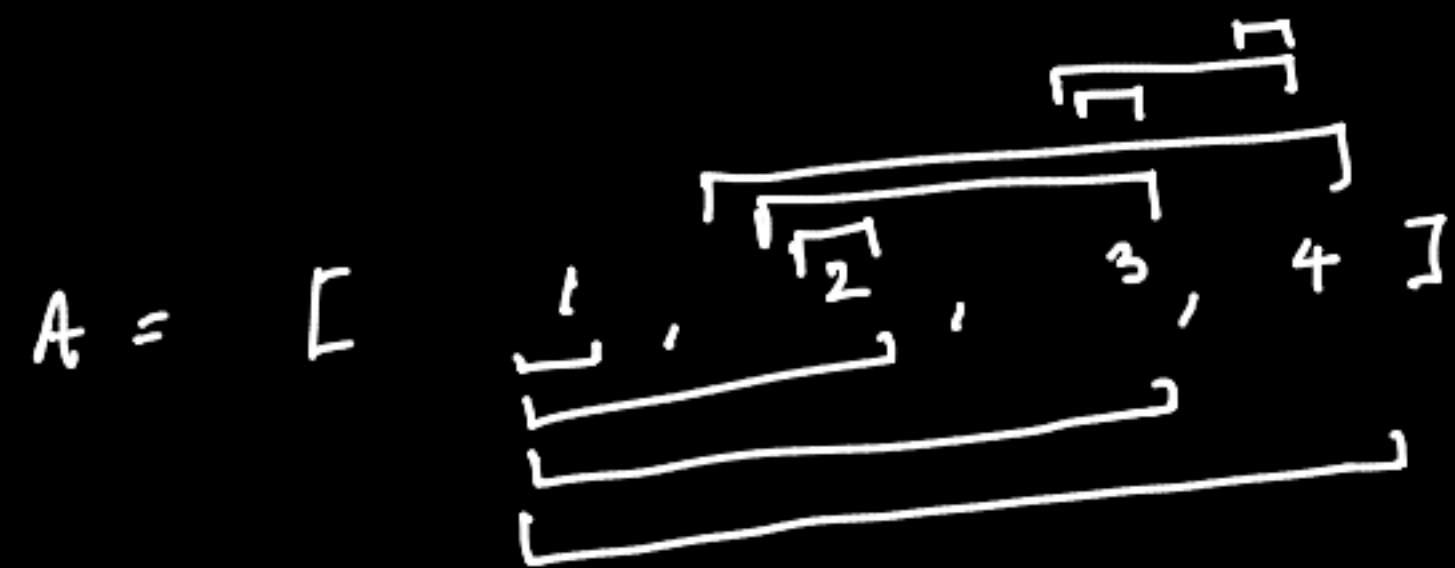
1 < 4 ✓

3 < 1 ✗

2 < 1 ✗

after 1st pass

[ 1 4 5 3 2 ]



⇒ sub-array  
 ⇒ (contiguous)

→ sub array is  
 a subset

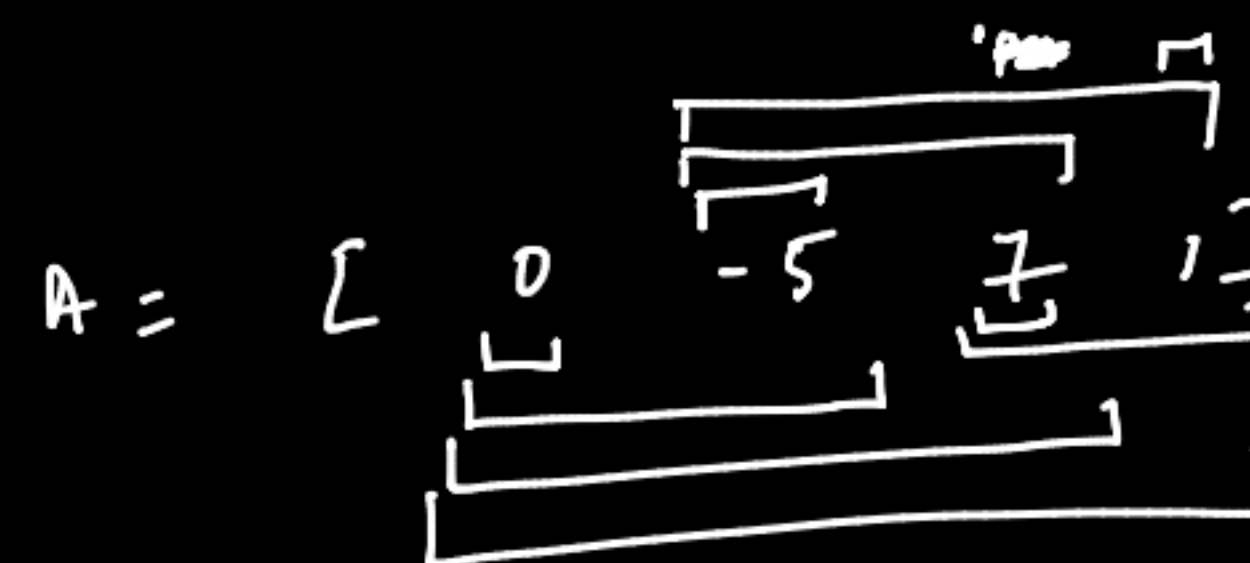
\* sub-seq (subset)

$A = [1, 2, 3, 4]$

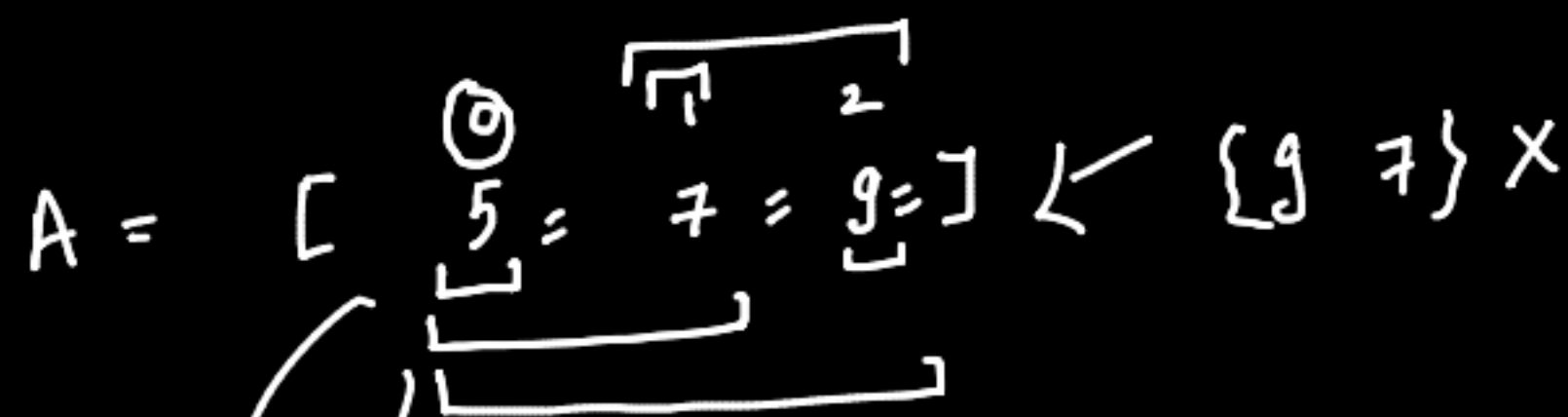
$\{1, 3\} \rightarrow$  subseq

↳ order is  
 main

$\{3, 1\} \times$



✓ order maintained ✓  
 subseq (✓) contiguous ✓ sub array =



$\{5\}$

$\{5, 7\}$  ✓

$\{5, 7, 9\}$  ✓

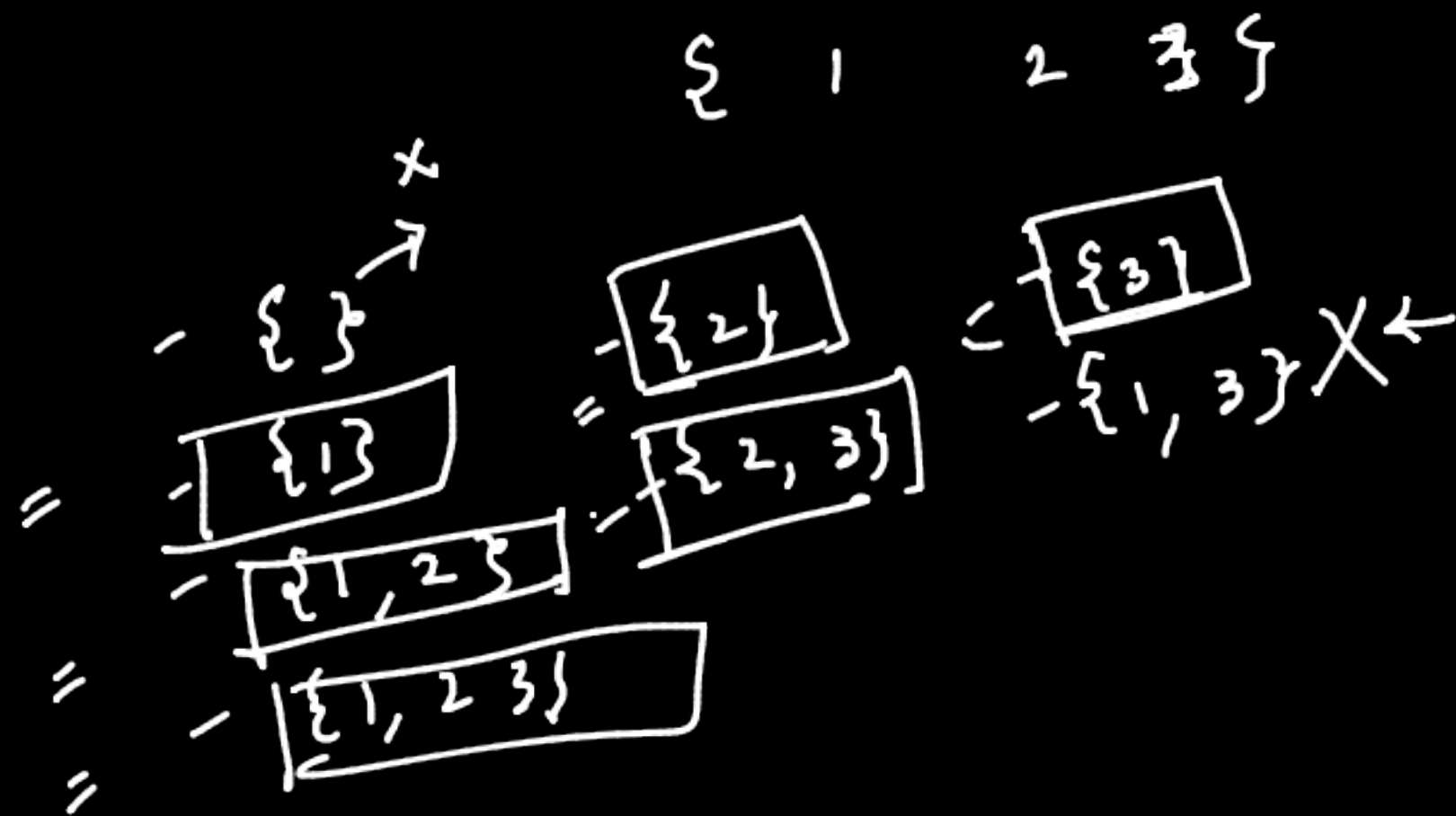
$\{7, 5, 9\}$  ✓

$\{9\}$

✓ order maintained  
 ✓ x contiguous / consecutive

$\neq$  subseq / subset

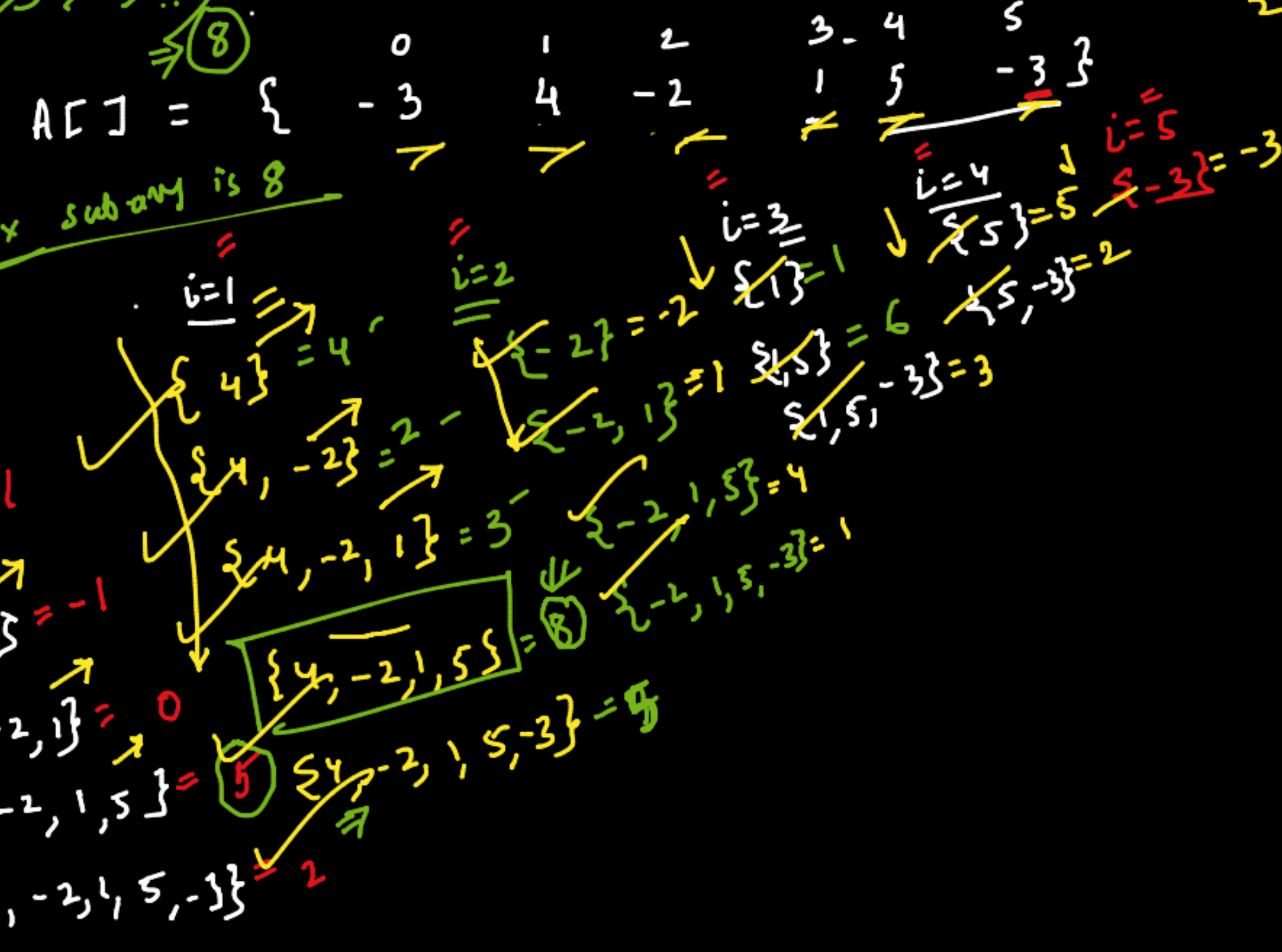
$$n=3 \Rightarrow 2^3=8$$





max sum = (sum of max sub array)

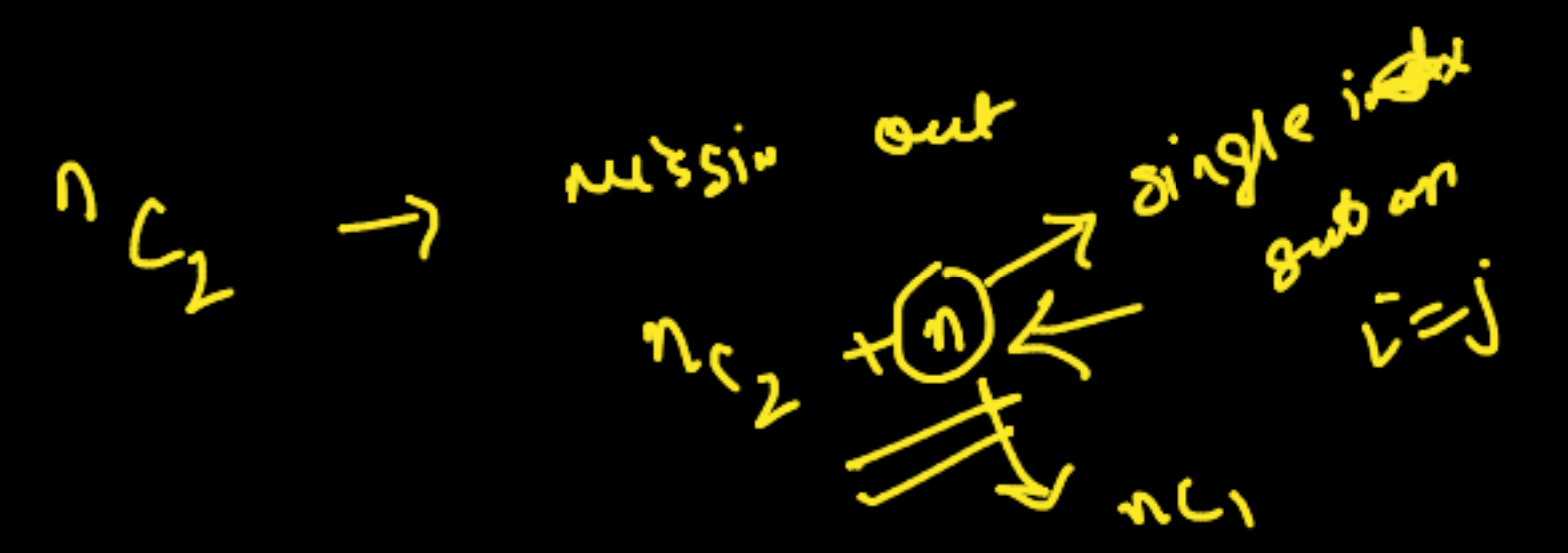
max sub array is 8



$$n=6 \Rightarrow \frac{6(6-1)}{2} = 15$$

$$\frac{n(n+1)}{2} = \frac{6(6+1)}{2} = 21$$

$$\frac{n(n+1)}{2} = \frac{6(6+1)}{2} = 21$$



$$\frac{n(n-1)}{2} + n = \frac{n(n-1+n)}{2} = \frac{n(n)}{2} = \frac{n^2}{2}$$

Array  $\rightarrow N$  how many subarray will it contain?

$$nC_2 + nC_1 = \frac{n(n-1)}{2} + n = \frac{n(n+1)}{2}$$

Every sub-array can be uniquely identified by its start index and end index

$n \rightarrow n$  indices  $[0 \text{ to } n-1]$

$$nC_2 = \frac{n!}{2! \cdot (n-2)!}$$

$(i, i) \Rightarrow$

$$\Rightarrow \frac{n \times (n-1) \times (n-2)!}{2! \cdot (n-2)!} = \frac{n(n-1)}{2}$$

total # of subarray