

Case 1: $T(n) = \Theta(n^k)$
 Case 2: $T(n) = \Theta(n^2 \log n)$
 Case 3: $T(n) = \Theta(n)$

Master Theorem Worksheet

$$1) T(n) = 3T(n/2) + n^2 \quad 5) T(n) = 4T(n/2) + \log n$$

$$a=3, b=2 \rightarrow \log_2 3 \approx 1.585 \quad a=4, b=2 \rightarrow \log_2 4 = 2$$

$$n^2 > n^{1.585}$$

$$\log n < n^2$$

$$\rightarrow \text{Regularity: } 3(n/2)^2 \quad T(n) = \Theta(n^2)$$

$$T(n) = \Theta(n^2)$$

$$6) T(n) = T(n-1) + n$$

$$2) T(n) = 7T(n/2) + n^2$$

Doesn't fit theorem

$$a=7, b=2 \rightarrow \log_2 7 \approx 2.807$$

$$n^2 < n^{2.807}$$

$$T(n) = \Theta(n^{\log_2 7})$$

$$a=4, b=2 \rightarrow \log_2 4 = 2$$

$$3) T(n) = 4T(n/2) + n^2$$

$$n^2 \log n, n^2$$

$$a=4, b=2 \rightarrow \log_2 4 = 2$$

$$T(n) = \Theta(n^2 (\log n)^2)$$

$$n^2 = n^2$$

$$8) T(n) = 5T(n/2) + n^2 \log n$$

$$T(n) = \Theta(n^2 \log n)$$

$$a=5, b=2 \rightarrow \log_2 5 \approx 2.322$$

$$4) T(n) = 3T(n/4) + n \log n$$

$$n^2 \log n < n^{2.322}$$

$$a=3, b=4 \rightarrow \log_4 3 \approx 0.792$$

$$T(n) = \Theta(n^{\log_2 5})$$

$$n \log n > n^{0.792}$$

$$T(n) = \Theta(n \log n)$$

$$9) T(n) = 3T(n/3) + n^{1/\log_3 3}$$

$$a=3, b=3 \rightarrow \log_3 3 = 1$$

$$n^{1/\log_3 3} < n$$

$$T(n) = \Theta(n \log \log n)$$

$$10) T(n/4) + c$$

$$a=2, b=4 \rightarrow \log_4 2 = 1/2$$

$$1 < n^{0.5}$$

$$T(n) = \Theta(n^{1/2})$$

$$16) 2T(n/4) + \sqrt{n}$$

$$a=2, b=4 \rightarrow \log_4 2 = 0.5$$

$$\sqrt{n} = n^{0.5}$$

$$T(n) = \Theta(\sqrt{n} \log n)$$

$$11) T(n/4) + \lg n$$

$$a=1, b=4 \rightarrow \log_4 1 = 0$$

$$\lg n > 0$$

$$T(n) = \Theta((\lg n)^2)$$

$$17) 2T(n/4) + n^{0.51}$$

$$a=2, b=4 \rightarrow \log_4 2 = 0.5$$

$$n^{0.51} > n^{0.5}$$

$$T(n) = \Theta(n^{0.51})$$

$$12) T(n) = T(n/2) + T(n/4) + n^2$$

Doesn't fit theorem

$$18) T(n) = 16T(n/4) + n!$$

$$a=2, b=4 \rightarrow \log_4 2 = 0.5$$

$$n! > n^2$$

$$T(n) = \Theta(n!)$$

$$\lg n < n^{0.5}$$

$$19) T(n) = 3T(n/2) + n$$

$$T(n) = \Theta(n^{1/2})$$

$$a=3, b=2 \rightarrow \log_2 3 \approx 1.585$$

$$20) T(n) = 4T(n/2) + cn$$

$$T(n) = \Theta(n(\lg n)^2)$$

$$n \lg n > n$$

$$T(n) = \Theta(n^{\log_2 3})$$

$$a=4, b=2 \rightarrow \log_2 4 = 2$$

$$cn < n^2$$

$$15) T(n) = 8T((n-\sqrt{n})/4) + n^2$$

$$T(n) = \Theta(n^2)$$

Doesn't fit theorem

$$21) T(n) = 3T(n/3) + n/2$$

$$a=3, b=3 \rightarrow \log_3 3 = 1$$

$$n/2 < n$$

$$22) T(n) = 4T(n/2) + \frac{n}{\log n}$$

$$a=4, b=2 \rightarrow \log_2 4 = 2$$

$$\frac{n}{\log n} < n^2$$

$$T(n) = \Theta(n^2)$$

$$23) T(n) = 7T(n/3) + n^2$$

$$a=7, b=3 \rightarrow \log_3 7 \approx 1.77$$

$$n^2 > n^{1.77}$$

$$T(n) = \Theta(n^2)$$

$$24) T(n) = 8T(n/3) + 2^n$$

$$a=8, b=3 \rightarrow \log_3 8 \approx 1.89$$

$$2^n < n^{1.89}$$

$$T(n) = \Theta(n^{\log_3 8})$$

$$25) T(n) = 16T(n/4) + n$$

$$a=16, b=4 \rightarrow \log_4 16 = 2$$

$$n < n^2$$

$$T(n) = \Theta(n^2)$$