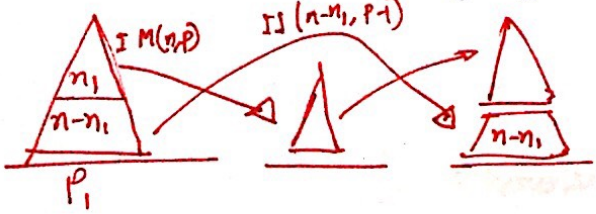
TOH

CTOH

DTOH

MPTOH (n, p, s, d, P)

Presumed Optimal Solution (POS)



MPTOH (n, p, s, d, P)

If $n=1$ then

$s \to d$

else $n_1 = f(n, p)$

MPTOH (n_1, p, s, i, P)

MPTOH $(n-n_1, p-1, s, d, P)$

MPTOH (n_1, p, i, d, P)

endif

Property:

P1: In POS strategy every disk requires 2^k moves to reach destination.

$$M(n,p) = \min_{0 \le n_1 < n} \{ 2M(n,p) + M(n-n_1, p-1) \}$$ → min number of moves

$N(k,p)$ – max # of disks each of which requires 2^k moves to reach destination.

P2. $N(k,p) = N(k-1,p) + N(k,p-1) = \binom{p-3+k}{k}$

$$\sum_{i=0}^{k_{max}-1} N(i,p) < n \le \sum_{i=0}^{k_{max}} N(i,p) = \bar{N}(k_{max}, p)$$

$$N_a(k_{max}, p) = n - \bar{N}(k_{max}-1, p)$$

$$\bar{N}(k_{max}-2, p) \le n_1 \le \bar{N}(k_{max}-2, p) + \min\{N(k_{max}-1, p), N_a(k_{max}, p)\}$$

$$\bar{N}(k_{max}-1, p-1) \le n - n_1 \le \bar{N}(k_{max}-1, p-1) + \min\{N(k_{max}, p-1), N_a(k_{max}, p)\}$$

N(K,P)

K\P	3	4	5	6	7	8
0	1	1	1	1	1	1
1	1	2	3	4	5	6
2	1	3	6	10	15	21
3	1	4	10	20	35	56
4	1	5	15	35	70	126
5	1	6	21	56	126	252
6	1	7	28	84	210	462

$(277, 7) \longrightarrow k_{max} = 6$

$N_a = 277 - 252 = 25$

$\downarrow$

$126 \le n_1 \le 126 + \min\{126, 25\}$

$126 \le 277 - n_1 \le 126 + \min\{84, 25\}$

Recurrent Problems

1. TOH - DTOH . CTOH . MTOH

2. Lines in the plane

3. Josephus problem

$$L_n = \begin{cases} 1, & n=0 \\ L_{n-1} + ((n-1)+1) & \text{otherwise} \end{cases}$$

$Z_n =$

$ZZ_n =$

$$P_n = \begin{cases} 1 \\ P_{n-1} + L_{n-1} \end{cases}$$

$Z_n = L_{2n} - 2n$

0.7

$1 + 1.7 + \dots + 42.7$

$1 + 2 + 4 + \dots + 2^{n-1}$

$J(2n) = 2J(n) - 1$

$J(2n+1) = 2J(n) + 1$

2. Sums

$1 + 2 + \dots + n$

$a_1 + a_2 + \dots + a_n$

$\sum_{k \text{ prime}} a_k = a_2 + a_3 + a_5 + a_7 + a_{11}$

~~sip~~ epilepsis

$1 + \dots + n$

$1 + 2 + \dots + (n-1) + n$

1 2 4 8 16 32

1 1 1 1 1 1

$n = 2^m + l$

$J(n) = 2l + 1$

$$\int_a^b \sin x\,dx = \int_a^b \sin y\,dy$$

$$K + \sum_{k=0}^{n} a_k = a_0 + a_1 + \dots + a_n$$

1820 Fourier

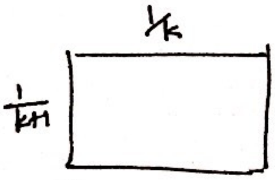
$$\sum_{k=0}^{n} a_k = \sum_{a \le k \le n} a_k = \sum_{a \le k \le n} a_k = a_0 + a_1 + \dots + a_n$$

integral method:

$$2x^2y + 5xy = f(x)$$

$$P(x)\, 2x^2y' + P(x)\, 5xy = f(x)\, P(x)$$

$$(P(x)y)'$$



$$\sum_{k=1}^{\infty} \frac{1}{k(k+1)} = \sum \left(\frac{1}{k} - \frac{1}{k+1}\right)$$

$$= 1 - \tfrac{1}{2} + \tfrac{1}{2} - \tfrac{1}{3} + \tfrac{1}{3} - \tfrac{1}{4} + \dots$$

$$= 1$$

$\boxed{\text{Chapter-2}}$