CSE 301 Assignment

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Question   Is it possible to obtain Z_n regions with n bent lines when the angle at each zig is 30° ?

Answer   Yes, it is possible.

Proof   We proof it by induction. Assume z_n is the number of regions we get from n bent lines where the angle at each zig is 30°. Our hypothesis is, $z_n = Z_n$. The base case trivially holds. For $n = 1$, $Z_n = 2$ for $0° < \theta < 180°$, where θ is the angle of the zig. So, $z_1 = Z_1$.
Let's assume our hypothesis holds for $n = m$ i.e. $z_m = Z_m$. We place another bent line with 30° angle at zig. The only condition under which $z_{m+1}! = Z_{m+1}$ is when any of the two arms of the new bent line doesn't intersect with some arm from the existing $2m$ arms. This is possible only when any of the two arms of the new bent line is parallel with an exisiting arm. But there are inifinite ways to place the new bent line. So it is always possible to place the new bent line in a way so that it doesn't become parallel with any of the existing arms. Hence our hypothesis holds for $n = m + 1$ and the proof completes.