CSE301 Preparation Questions (From last year)

1. The weather of a year can be of either two types: good or bad. Suppose that any year's weather conditions depend on past years only through the previous year's condition. It has been observed that a good year is equally likely to be followed by either a good or a bad year, and that a bad year is twice as likely to be followed by a bad year as by a good. It is also observed that the average number of storms in a good year and bad year are 1 and 3 respectively. Suppose that this year 2014—call it year 0—is a good year. Now, answer the following:
   a. Define a Markov chain with an appropriate definition of states and state transition probability matrix to represent the above scenario.
   b. Compute the expected number of storms in year <u>2016</u>.

2. Each morning an individual leaves his house and goes for a run. He is equally likely to leave either from his front or back door. Upon leaving the house, he chooses a pair of running shoes (or goes running barefoot if there are no shoes at the door from which he departed). On his return he is equally likely to enter, and leave his running shoes, either by the front or back door. Assuming that he owns a total of k pairs of running shoes, construct a Markov chain $\{X_n, n >= 0\}$ where X_n is the <u>number of shoes at the front door</u> at the beginning of n-th day. Now,
   a. Draw a state transition diagram with associated transition probabilities.
   b. Show that the proportion of time the person runs barefooted is $\frac{1}{k+1}$.

3. Every time that a team wins a game, it wins its next game with probability 0.8; every time it loses a game, it wins its next game with probability 0.3. If the team wins a game, then it has dinner together with probability 0.7, whereas if the team loses then it has dinner together with probability 0.2. Now, answer the following:
   a. Define a Markov chain with appropriate states and state transitional probability matrix to represent the above scenario.
   b. Compute what proportion of games result in a team dinner.

4. An LRU (Least Recently Used) is a list of items where the items are ordered as they are requested. When an item is requested, it is placed at the beginning of the list (indexed as position 0) keeping the relative positions of others same. That is, if item C is requested from the list [B, C, A, D], it becomes [C, B, A, D]. Let X_n be the position of a *particular* item at the beginning of n-th operation. Assume that the list contains k items and all items are equally likely to be requested (irrespective their current positions and others). Now, answer the following:
   a. Argue that $\{X_n, n>=0\}$ is a Markov chain. Define appropriate states and state transition probabilities of the chain.
   b. What proportion of time the particular item remains at the <u>end</u> of the list?