PROBLEM 1. [15 POINTS] Give the best possible Big-Oh characterization for each of the following running time estimates, where $n$ is the size of the input:

a) $2 \log n + 100000$;

b) $n^2 + 2^n$;

c) $(2n+1) + (2n-1) + \cdots + 5 + 3 + 1$;

d) $2^{20} + 3^{10}$,

e) $1 + n^2 + 2^n + n!$

a) $O(\log(n))$

b) $O(2^n)$

c) $O(n^2)$

d) $O(1)$

e) $O(n!)$

Please, explain your answers.

PROBLEM 2. [20 POINTS] Which of the following functions

$$\cancel{3^n}, \quad \cancel{2n+3}, \quad \cancel{n^2+n}, \quad \cancel{\log n^2}, \quad \cancel{\sqrt[3]{n}}, \quad \cancel{\log 2^n}$$

belong to

- $O(n)$;
- $\Theta(n)$;
- $\Omega(n)$?

- $3^n$
  - this function grows exponetially
  - this function does **NOT** belong to any of the linear functions because it grows faster than them
- $2n+3$
  - this function belongs to $O(n)$ because it grows linearly with $n$
  - it belongs to $\Theta(n)$ because it has the same growth rate
  - it belongs to $\Omega(n)$ because it is bounded below
- $n^2+n$
  - this belongs to $O(n^2)$ **NOT** $O(n)$
  - this does **NOT** belong to $\Theta(n)$ because it belongs to $\Theta(n^2)$
  - this belongs to $\Omega(n)$ because it bounds from below, a tighter bound would be $\Omega(n^2)$
- $\log(n^2)$ or $2\log(n)$
  - this belongs to $O(n)$ but a tighter bound would $O(3\log(n))$
  - this does **NOT** belong to $\Theta(n)$ because $\Theta(\log(n))$ is its average
  - this does **NOT** belong to $\Omega(n)$ because its out of lower bounds
- $\sqrt[3]{n}$ or $n^{1/3}$
  - this belongs to $O(n)$ but a tighter bound is $O(2\cdot n^{1/3})$
  - this does **NOT** belong to $\Theta(n)$ because it belongs to $\Theta(\sqrt[3]{n})$
  - this does **NOT** belong to $\Omega(n)$ because its out of lower bounds
- $\log(2^n)$ or $n\log(2)$
  - this belongs to $O(n)$
  - this belongs to $\Theta(n)$
  - this belongs to $\Omega(n)$