A Generalized Theory of Musical Contour: Its Application to Melodic and Rhythmic Analysis of Non-Tonal Music and its Perceptual and Pedagogical Implications

By

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CURRICULUM VITAE

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ABSTRACT

This dissertation proposes the thesis that abstract theories of pitch- and set-class structure do not reflect listeners' aural perception of sounding music as effectively as theories modelling the articulation of these underlying structures on the musical surface. This position is supported by a review of pertinent music-theoretical and music-psychological research. Based upon the data collected by various music-psychologists, published elsewhere but compared and critiqued here, this study concludes that listeners generally use figural cues drawn from musical context -- for example, melodic shapes, changes of direction, relative durational patterns, and so on -- to retain and recognize musical ideas in short-term memory. These figural cues may be represented in precise notation and compared with one another by application and generalization of Robert Morris's contour theories. Morris's comparison matrix and contour equivalence relations are introduced here, followed by this author's generalization of the theory to duration space and development of similarity relations for melodic contours of relative pitch height and rhythmic contours of relative duration successions. The similarity relations for musical contours build upon previous work of David Lewin, Robert Morris, and John Rahn. While the efficacy of these theories for modelling perceivable patterns in musical contexts cannot be proven without further psychological testing, their applicability to musical analysis is demonstrated. Analyses drawn from the music of Bartok, Webern, Berg, and Varèse illustrate ways in which melodic and rhythmic contour relationships may be used to shape a formal scheme, to differentiate melody from accompaniment, to associate musical ideas that belong to different set classes, and to create unity through varied repetition.

The concluding chapter explores avenues for future work. A section on music-psychological experimentation offers a critical overview of research in this area and proposes ideas for future experimentation. Second, the implications of music-psychological research for the pedagogy of non-tonal music theory are considered and a model curriculum for non-tonal music theory proposed. The dissertation concludes by proposing a number of ways in which contour theory might be generalized to other domains and illustrates the application of one such generalization to the analysis of chord spacing in a piano work of Luigi Dallapiccola.

TABLE OF CONTENTS

LIST OF FIGURES

CHAPTER I

CHAPTER II

CHAPTER III

CHAPTER IV

Chapter One

Music Perception and Non-Tonal Music Theory[1]

> When somebody says, "Can you hear these things?" the answer is that it's not a matter of hearing. Of course, you can hear these different notes.... But it's not a matter of hearing; it's a matter of the way you think it through conceptually with your musical mind.....it's not a matter of whether you hear it, it's a matter of how you conceptualize it, how you conceive it.[2]
>
> - Milton Babbitt

1. Introduction

The question posed to Babbitt, "Can you hear these things?," is a common query asked of music theorists and composers of non-tonal music by students, performers and audiences seeking a strategy for conceptualizing such music as it is heard. This dissertation examines some relationships between compositional and analytical models of musical structure and the perceptual capabilities of listeners. An overview of music-theoretical literature shows a predominance of abstract theories of pitch-class and set-class structure over theories modelling the articulation of these underlying structures on the musical surface. The thesis of this dissertation is that theories of the latter type more closely model aural perception; thus, such a theory for musical contour is presented and generalized to model various facets of musical articulation. The relevance of this theory to melodic and rhythmic analysis is demonstrated in a variety of musical contexts, and its relation to the cognitive strategies of listeners is examined. The dissertation departs from much previous music-theoretical work in its examination of literature from the field of cognitive psychology. By using conclusions drawn from music-psychological experimentation as the point of departure for a theory of musical articulation, it attempts to draw the fields of music theory and music cognition into closer proximity. Further, it demonstrates that one possible outcome of the cross-fertilization between these fields may be improved pedagogical methods for teaching non-tonal music theory and for training students to hear structure in this music.

The dissertation's primary theoretical contribution lies in its two central chapters, which develop a theory for relating musical contours and apply the theory to analysis of "pitch height" (Chapter 2) and duration successions (Chapter 3) in various non-tonal compositions. These first three chapters also include brief summaries of pertinent experimental studies of non-tonal music perception. Chapter 1 provides background material for the developments of the central chapters, defines terms, gives an overview of previous work in music theory and experimental music psychology, and places these into historical context. Chapter 4 summarizes some perceptual strategies by which listeners appear to structure non-tonal music as it is heard, and suggests ways in which knowledge of these strategies might influence the pedagogy of non-tonal music theory. Finally, the dissertation concludes with a discussion of ways in which the theories developed here might be generalized to other facets of musical structure and suggests additional avenues for future experimentation.

2. Theories of Non-Tonal Musical Structure

To this point in its relatively brief history, the systematic study of structure in non-tonal music has undergone tremendous changes.[3] Prominent composers such as Paul Hindemith, Olivier Messiaen, and Arnold Schoenberg were among the first in the 1930's-40's to publish technical explanations of their compositional techniques,[4] explanations that included early attempts to classify non-tonal pitch collections by intervallic content, and formulations of such concepts as combinatoriality and pitch-class invariance under transposition, among others to be defined and discussed below.[5] The 1950's-60's saw an outpouring of non-tonal music theory, including a movement toward context-free generalization of various principles of structure in non-tonal music and toward exhaustive classification of pitch-class collection types. In the former category belong such contributions as Milton Babbitt's and Donald Martino's work on combinatoriality,[6] David Lewin's interval function and his discussion of the complement and hexachord theorems,[7] as well as Babbitt's and John Rothgeb's examination of invariance and combinatoriality in relation to interval cycles.[8] In the latter category belong the contributions of Howard Hanson, Martino, George Perle, and Allen Forte,[9] each of whom independently developed a system for exhaustive classification of all possible pitch-class collections.

At the end of the 1960's, an exchange in Perspectives of New Music took place between Edward T. Cone and Lewin which, in retrospect, reflected a juncture in non-tonal music theory.[10] Cone questioned the value of analyses that did not attempt to explain the functional rationale behind a composer's musical choices--to explain, for example, why a composer might choose one twelve-tone row form over another when theoretically all rows related under transposition, retrograde, inversion, or retrograde-inversion are of equal functional status. Lewin responded with the argument that Cone was confusing theory with analysis. In so doing, Lewin effectively separated contextually-bound musical analysis from context-free theories of structure, as well as from the value judgements of music criticism. In the decade that followed, both the analytical and theoretical facets of the field flourished. The latter category included Forte's influential book, The Structure of Atonal Music,[11] further generalizations by

Robert Morris and Daniel Starr of principles governing construction of twelve-tone rows (such as all-interval and multiple-order-function rows, and rows with the potential to produce specific types of invariance or combinatoriality),[12] and a series of articles by Eric Regener, Lewin, Morris, and John Rahn that developed measurements of similarity among pitch-class collections.[13]

During the 1970's and early 80's, a growing body of literature, beginning with early reviews of Forte's book, began to question the perceptual validity and analytical cogency of such accepted tenets of non-tonal music theory as inversional equivalence, unordered pitch collections, and octave equivalence.[14] Reviews by William Benjamin and Richmond Browne, in particular, criticized Forte's assumption of octave equivalence. Several writers criticized inversional equivalence as criterion for set-class membership, among them Hubert Howe, Regener, and Richmond Browne. Browne, in particular, objected on perceptual grounds, questioning whether "anyone can really discriminate between presented instances of inversional equivalents, unless the configuration of the presentation very strongly emphasizes the inversional relation."[15] Among Benjamin's primary concerns was ordering, both in time and in register; indeed, in his opinion "the very name 'unordered pitch-class set' bespeaks a cavalier attitude about the matter of order within sets in the pre-twelve-tone literature."[16] All these criticisms share a concern for the musical contexts that articulate pitch-class sets.

Until recently, both the theory and analysis of non-tonal music have primarily been concerned with models of underlying pitch-class structure, rather than with the realizations of those structures in musical time and space. In other words, non-tonal music theory has, like tonal theory, contained a reductive component. To continue the analogy, tonal theorists influenced by the work of Heinrich Schenker have used reductive techniques in tonal music that "remove" embellishing tones and harmonies to reveal deeper levels of structure. Because that underlying structure is sometimes revealed at the expense of such elements as phrase structure, motive, rhythm, register, and so on, a group of theorists has recently emerged whose primary interest is the examination of relationships between underlying tonal structure and formal and rhythmic design.[17] In a somewhat similar vein, theorists of non-tonal music have generally focused upon principles of underlying pitch-class relationships, sometimes at the expense of such elements as register, contour, rhythm, and order. However, articles published in recent years by such writers as William Benjamin, Christopher Hasty and Jonathan Bernard[18] have begun to address some of these issues. Others, such as Michael Friedmann and Richard Cohn have developed music theories that model certain aspects of musical articulation.[19] Two textbooks published in the last decade, Charles Wourinen's on twelve-tone composition and Rahn's on non-tonal music theory, reflect a similar concern for both musical surface and underlying pitch-class relationships.[20] In addition to their thorough discussion of principles governing pitch-class structure in non-tonal music, these authors address such issues as register, dynamics, octave placement, rhythm, chord spacing, and aural perception. This latter area is particularly crucial, since it is the surface design of a composition that listeners immediately perceive and remember; whereas repeated hearings and detailed analyses are often necessary to reveal the principles underlying its pitch structure. In the present decade, the trends toward continued generalization of music-theoretical principles,[21] incorporation of non-pitch elements into theories of musical structure, and sensitivity to issues of musical perception combine in recent publications of Lewin and Morris,[22] upon whose work much of this dissertation builds.

The field of non-tonal music theory has traditionally been divided into two areas of research. One of these is characterized by the work of Forte and his colleagues and students at Yale University, and focuses on relationships among unordered pitch-class sets. The other area, associated with Babbitt and those at Princeton University, focuses on relationships among ordered sets and theories of twelve-tone musical structure. Although a particular theoretical emphasis has become associated with each university, separation of the two areas of specialization has never been complete: Martha Hyde, for example, has specialized in twelve-tone theory, but has enriched this study by incorporating unordered pitch-class set analysis;[23] further, Hubert S. Howe and Regener, who are associated with the twelve-tone Princeton school, have made significant contributions to the theory of unordered pitch-class sets.[24] More recent contributions to the field have ignored what is ultimately an artificial distinction between theories of unordered and ordered set structure, and have taken as their point of departure the operations common to both systems. Starr's work on invariance and partially-ordered rows, based upon properties of operator cycles, contains implications for theoretical systems modelling either ordered or unordered set structure.[25] In Morris's recent book, examination of ordered and unordered pitch-class relationships and operator properties leads as a natural consequence to generalizations of twelve-tone and pitch-class set theories. David Lewin extends this attitude; the transformational graphs and networks of his recent book describe structural principles in music drawn not only from the non-tonal and twelve-tone repertories, but also from the tonal repertory. Since this overview attempts to provide some historical perspective on developments in the field, and since the distinction between theories of unordered pitch-class sets and ordered twelve-tone rows has until recently been prevalent, portions of this discussion will proceed from this traditional "segregated" perspective.

An appropriate place to begin an overview of non-tonal music theory is with the concept of equivalence class. Equivalence classes must meet two criteria, to use Rahn's words: "exhaustivity" and "exclusivity."[26] That is, a set of equivalence classes must exhaustively partition its domain into discrete units, or classes, with no elements remaining. Further, each class must be exclusive; no two classes may share an element. For example, many music theorists since the time of Babbitt's early articles have distinguished between pitches and pitch classes.[27] Pitches extend linearly through the entire range of audible sound, low to high, and form the domain from which pitch-class equivalences may be drawn. Twelve pitch classes divide up this domain exhaustively, with no shared elements among classes; thus pitch class "C" represents all C's regardless of octave placement, pitch class "A#" represents all A#'s, and so on. Theorists have further extended these classes to include enharmonic equivalence (under equal temperament); thus, for example, pitch class C represents not only all C's, but all B#'s and D^{bb}'s as well. In seeking a musically-neutral way to discuss pitch classes--that is, without expressing a bias toward one octave designation or enharmonic spelling over another--theorists have turned to numeric representation. In this way, the twelve pitch classes can be expressed by the integers 0 through 11. This use of numbers not only eliminates octave and enharmonic biases, but also shows transpositional and inversional relationships between pitch-class collections without suggesting that registral contour is also preserved, as staff notation might imply. To quote Babbitt:

> ...compositional transposition, traditionally, implies contour preservation, a consideration that is, literally, meaningless in defining transposition in a twelve-tone operation, since contour is a function of the registral specification of the elements, and registral choice is as undefined by the structure of a set as is duration, intensity, timbre, or any of the other attributes necessarily associated with a compositional

> representation of a set; as a result, a set cannot be stated in musical notation without the additional qualification that each pitch sign be taken to signify the total pitch class a member of which it denotes. Since such a qualification only too easily leads to but another confusion of systematic principle with compositional permissive...it is both safer and more efficient to represent a twelve-tone set in numerical notation...[28]

Further, numerical representation simplifies certain procedures--calculation of intervals between pitches or pitch-classes, for example--by enabling the analyst to use simple mathematical operations, in this case subtraction.

The pros and cons of pitch-class (pc) numeration via a fixed- or moveable-"do" standard have been discussed in some detail by Lewin.[29] A fixed-"do" system would assign 0 always to C (or its enharmonic equivalents); thus pc1 = C#, pc2 = D, and so on regardless of musical context. A moveable-"do" system would assign 0 variously to the first note of what is perceived to be the primary referential pitch collection in a given composition, with all other pcs determined by counting semitones above that pitch.[30] Lewin argues that the moveable-"do" standard, used by Babbitt and George Perle among others, confers a "tonic function" upon the referential pitch class chosen as 0. He notes that "this is a difficult pill to swallow if one claims to be investigating 'atonal' music, or music composed with 'twelve tones related only to one another' and not to any one tone of reference."[31] The fixed-"do" standard, introduced by Forte[32] and now widely used, addresses this issue directly by assigning a single standard numeric representation to each pitch class regardless of context. In music that contains pitch-class centricity, however, this in turn may conflict with musical intuition: "even though I may hear E-flat as a musically 'tonic' pitch-class for Webern's Piano Variations, I am bound by the system to conceptualize my musical tonic as a '3' rather than a '0.'"[33] Lewin's solution to this conundrum is to develop a theory that does not label pitch-classes with numbers, but rather describes relations among pitch classes and pitch-class collections in terms of intervals (to be defined below) and operations upon intervals. This idea resurfaces repeatedly in Lewin's later work, and forms a point of departure for his Generalized Intervals and Transformations.

The method of pitch numeration to be used here is that advocated first by Rahn and more recently by Morris,[34] a method that clearly distinguishes between pitches and pitch classes. This is a fixed-"do" system whereby in pitch space, adapting Morris's terminology,[35] middle C (C4) will be arbitrarily assigned 0 and chromatic pitches above C numbered consecutively with positive integers, and below C with negative integers. Ordinary arithmetic can be used to calculate intervals spanned between pitches, and transpositions or inversions of pitch collections. Pitch-class space is a modular reduction of pitch space. It is a cyclic, rather than a linear space; thus its elements, regardless of octave placement, are numbered with the integers 0 through 11, and modular arithmetic used to model this universe of only 12 pitch classes. The 12-hour clock provides a convenient analogy to mod 12 arithmetic: five hours after 11:00, for example, is 4:00 ($5 + 11 = 16 = 4 \bmod 12$). Generally, addition or subtraction can be performed normally in this system, then the result divided by 12; the remainder (or residue) after division is the mod 12 equivalent.

The concept of musical spaces originated with Schoenberg; it can also be found in the writings of Babbitt, as well as in the more recent work of Lewin and Morris. Schoenberg conceived of musical space as having two or more dimensions, in which "elements of a musical idea are partly incorporated in the horizontal plane as successive sounds, and partly in the vertical plane as simultaneous sounds;"[36] thus he saw a horizontal component ordered in time, and a vertical component ordered by register. In the same essay, Schoenberg asserts a perceptual law requiring absolute "unity of musical space":

> In this space...there is no absolute down, no right or left, forward or backward. Every musical configuration, every movement of tones has to be comprehended primarily as a mutual relation of sounds...appearing at different places and times. To the imaginative and creative faculty, relations in the material sphere are as independent from directions or planes as material objects are, in their sphere, to our perceptive faculties. Just as our mind always recognizes, for instance, a knife, a bottle or a watch, regardless of its position, and can reproduce it in the imagination in every possible position, even so a musical creator's mind can operate subconsciously with a row of tones, regardless of their direction, regardless of the way in which a mirror might show the mutual relations, which remain a given quality."[37]

In other words, Schoenberg suggests a kind of perceptual pitch-class space (no absolute up or down) of unordered pitch collections (no "right or left, forward or backward") in which configurations are recognized by their internal structure (as a "mutual relation of sounds") regardless of their location within a piece, or of their musical articulation as chord or melody, or of their appearance in inverted form. In 1958, Babbitt discussed his conception of musical space, existing in five dimensions. Babbitt argued that every musical event

> ...is located in a five-dimensional musical space determined by pitch-class, register, dynamic, duration, and timbre. These five components not only together define the single event, but, in the course of a work, the successive values of each component create an individually coherent structure, frequently in parallel with the corresponding structures created by each of the other components.[38]

In accordance with Babbitt's own compositional style, his conception of musical space contains two facets: not only is each musical event five-dimensional, but each dimension may also be perceived separately in its own musical space, with its own "individually coherent" structure. Thus, for example, the succession of durations may have its own inherent structure in a "duration-space" that is differentiated from, but possibly parallel to, the pitch-class structure. Both Schoenberg and Babbitt describe musical spaces that are highly colored by their compositional practices; Lewin, on the other hand, has developed a number of musical spaces that model various types of music and various facets of musical structure.[39] He describes, for example, four types of pitch spaces: the chromatic pitch space and its mod 12 reduction to pitch-class space discussed above, as well as a diatonic pitch space with an analogous mod 7 reduction.[40] He notes that additional linear spaces could be devised of scales other than the chromatic and diatonic scales, and modular reductions devised as well. Lewin also develops a kind of "Riemannian" harmonic space based on just intonation with its attendant modular reduction, and six temporal spaces consisting of successions of either time points or durations and their modular reductions. Robert Morris has generalized musical spaces to even more dimensions using his concept of contour space, to be discussed at length in Chapter 2. In brief, he asserts that models of musical contour, because they are sequential from low to high, can be generalized to any sequential musical dimension, such as soft to loud dynamics, early to late time points, dull to bright timbres, and so on.[41]

An interval is a "directed measurement, distance, or motion"[42] between two points in any musical space. Intervals measure the distance between ordered points in musical space, while interval classes measure the distance between unordered ones. More specifically, in pitch space an ordered interval may be measured by positive or negative integers, representing an upward or downward direction, and determined by subtracting the first pitch integer from the second.[43] In the case of simultaneous pitches, their order, and thus the direction of the interval, is not considered. In such cases, the interval from c to e' belongs in the same equivalence class as that from e' to c; these are grouped together into the interval class, ± 4. In pitch-class space, on the other hand, the ordered pc interval between two pitch classes is determined by subtraction mod 12 of the second pc from the first. The use of mod 12 arithmetic eliminates the possibility of a negative result; thus the maximum intervallic span possible is 11 half steps.[44] In an unordered system, the interval from c to e (4) would belong in the same interval class as the interval from e to c (8); by convention, each interval class in pc-space is represented by the smaller of the two numbers.[45] Lewin's generalized interval, int(s,t), represents the directed distance or motion from point s to point t in any musical space. Thus, in pitch or pitch-class space the generalized interval may be understood as described above, but int(s,t) in other musical spaces might model a duration between two time points, or a move from one position to another on the two-dimensional "game board" that represents one of Lewin's harmonic spaces.[46] His generalized interval system (GIS) builds upon this expanded understanding of intervals within various musical spaces, by constructing a system consisting of a space, a mathematical group that represents intervals, and a function (int) that maps compared elements within that space into the group of intervals.[47] In the discussion that follows, pitch-class space and pc intervals should be assumed unless otherwise specified.

The universe of all possible unordered collections of pitch classes, called pc sets, may be partitioned into equivalence classes by various criteria. One such criterion might base an equivalence class upon cardinality, or number of elements, placing all 2-note sets into a single class, all 3-note sets into a class, and so on. This would result in a symmetrical partitioning of complementary classes: cardinality classes 4 and 8 would contain the same number of pc sets, as would 3 and 9, and likewise for any two cardinalities summing to 12. In the first half of the 1960's, in individual publications, Howard Hanson, George Perle, and Forte divided pitch-class sets into equivalence classes based upon the criterion of identical interval-class content.[48] In Forte's listing, which (after its 1973 revision) has now become a standard reference, each set is given a hyphenated number: the first number specifies the set's cardinality, while the second identifies its place on a list that is ordered by the interval-class content of the sets. The complement relation is represented in this labelling system as well: 4-12's complement is labelled 8-12, 5-6's complement is 7-6, etc. Finally, the interval-class content of each set is included on this list in a concise form that Forte calls the "interval vector,"[49] a six-position array in which each of the positions represents each of six interval classes (ic1 through ic6). Each position of the vector is filled with an integer that represents the tallied number of times that particular interval class is spanned between elements of the given set.[50]

Among the critical responses to Forte's theory as set forth in 1964,[51] were two primary concerns: the first pertained to his use of the same term, "set," both for the class of equivalent sets and for the individual members of each class; the second objected to interval-class content as the equivalence criterion. Different terms have been adopted by several writers in response to the first objection. Rahn, for example, distinguishes between "set" and "set type." This dissertation will use "set" to indicate a particular collection, and "set class" (SC) to indicate the entire class of equivalent collections to which this particular set belongs.[52] The second objection hinged upon the following fact: sets with identical interval-class content are generally transpositionally and/or inversionally related, with some exceptions (particularly among hexachords). It is this latter relation that Forte's critics would have preferred as equivalence criterion, rather than identity of interval-class content, since it would distinguish between those exceptional sets. Forte did recognize that certain sets with identical ic content were not transpositionally or inversionally related, and to these he gave the designation "Z-related."[53] In his 1973 The Structure of Atonal Music, the equivalence criterion for set classification was changed from interval-class content to transpositional/inversional equivalence (hereafter, T_n/T_nI equivalence).[54] As mentioned previously, it is this 1973 list that has become a fairly standard system of set-class labelling in the music-theoretical literature, and the system that will be used here. It should be reiterated, however, that not all theorists would, like Forte, divide the 4,096 possible pitch-class sets into set classes by the T_n/T_nI equivalence criterion.[55] A number of theorists--most notably, Hubert Howe and Eric Regener--prefer to base set class equivalence upon the T_n relation only, resulting in 352 distinct classes of cardinalities 0 through 12.[56] Morris has summarized the arguments for and against T_n as sole basis for set-class equivalence, as well as those for and against T_n/T_nI equivalence. Further, he has proposed a number of additional ways of partitioning sets into discrete set classes; he discusses, for example, a system based upon $T_n/T_nI/M/MI$ equivalence.[57] This equivalence criterion, developed most fully by Starr in 1978,[58] produces 158 distinct set classes, 66 fewer than are generated by T_n/T_nI equivalence alone.

Once all possible unordered sets have been exhaustively and exclusively partitioned into equivalence classes, a single set from each class is sufficient to serve as representative for the other members of its class. This representative is known variously in the literature as the best normal order, prime form, or normal form; likewise, the algorithm by which this representative form is determined differs somewhat among various authors.[59] This form represents a set class that ordinarily contains 24 sets (12 related by transposition and 12 by inversion), although some set classes with symmetrical properties contain fewer than 24 members.[60] For purposes of comparison, sets will be represented here in a normal order defined as the ascending order of pcs with the smallest possible intervallic span between its first and last pcs.

Comparison of the normal orders' cyclic interval successions between adjacent pcs ($CINT_1$) shows the transpositional (T_n) or inversional (T_nI) relationship between sets.[61] Two sets that are transpositionally related have the same $CINT_1$; sets that are inversionally related retain the same $CINT_1$ but in reverse order, with rotation of its elements sometimes required.[62] T_n/T_nI equivalence between sets may also be seen in their pc representations: the elements of two transpositionally-related sets will differ by a single constant integer; the elements of two inversionally-related sets will sum to a single constant integer known, after Babbitt, as the "index number."[63]

Two sets whose union comprises a complete twelve-tone collection, or aggregate, are called literal complements. Thus, {1, 2, 7} and {0, 3, 4, 5, 6, 8, 9, 10, 11} are literal complements. The first of these sets belongs to SC(3-5) [0, 1, 6] and the second to SC(9-5) [0, 1, 2, 3, 4, 6, 7, 8, 9] . Examination of the sets' prime form representatives obscures the fact that there exists some member of SC(3-5) that is a literal complement to some member of SC(9-5). Two set classes such as these, which contain sets that are literal complements, are known as abstract complements. The term "complement" will hereafter refer to abstract complementation, unless otherwise specified. One further relationship exists between the set classes under discussion: the smaller of the two is a subset of the larger, since every pc of the set [0, 1, 6] is also an element of [0, 1, 2, 3, 4, 6, 7, 8, 9]. Given two sets, A and B, in which every element of A is also an element of B, set A is defined as a literal subset of B (or A is literally included in B). By extension, set A is defined as an abstract subset of B (or A is abstractly included in B) if some member of set-class A is literally included in some member of set-class B. Thus, {1, 2, 7} is abstractly included in {0, 3, 4, 5, 6, 8, 9, 10, 11}. The terms "subset" and "inclusion" will hereafter refer to abstract relationships, unless otherwise specified.

Comparison of the interval-class content of any two complementary sets reveals certain consistent properties. The interval-class vectors (icvectors) of the sets given above are [100011] for SC(3-5) and [766674] for SC(9-5). For the first five positions of these two icvectors the integers in corresponding positions differ consistently by 6, which is also the difference in cardinality between the two sets. In the sixth position the two differ by 3 (half of 6) due to the fact that interval-class 6 contains only one member, half as many as ics 1 to 5, which contain two. This relationship between ic contents of two complementary sets, termed the "complement theorem," was first formalized by Lewin in 1960,[64] and holds true for sets of all cardinalities and their complements. Hexachords and their complements are a special case, however. Since the cardinalities of two hexachords differ by 0, all hexachords and their complements have identical interval class contents. This special case of the complement theorem is called the "hexachord theorem." Since complement-related hexachords share identical icvectors, a hexachord and its complement either belong to the same set class or belong to a Z-related pair. Set-theoretical proofs of one or both theorems are offered in articles by Regener, Starr, Morris, and Howard Wilcox.[65]

The multiplicity of intervals in a given pitch-class set has a direct correlation to the number of invariant pcs that will be produced when that set is transposed. Rahn calls this relationship between a set's ic content and its invariance under transposition the "common-tone theorem for T_n." This theorem states that the "number of pc in common between any set A and any nontritone transposition of A, $T_m(A)$, equals the multiplicity of the intervals of transposition 'm' in the set A."[66] Both Forte and Rahn note that the number of common tones between a set and its transposition can be predicted from examination of its icvector. However, patterns of invariance may be predicted between a set and its transformation under any twelve-tone operator (not just

T_n), or between a set and any other set, by means of invariance matrices. These matrices were set forth by Bo Alphonse in 1974[67] and have been generalized by Morris to predict invariance between sets in p- or pc-space.[68]

For purposes of this set-theoretical overview, an example of the invariance matrices in pc-space will suffice to show some pertinent applications. Figure I-1A illustrates two matrices based upon SC(5-8) [0, 2, 3, 4, 6]. The first is a T-matrix, which finds all possible differences between pcs of a set compared, in this instance, with itself. Because the vertical and horizontal sets in this example are equal, the matrix is symmetrical about its main diagonal, extending from the upper left- to lower right-hand corner. Each value in the upper right-hand triangle is mirrored by its inverse (mod 12) in the lower left-hand triangle. As Figure I-1B shows, the diagonal immediately above and to the right of the main diagonal provides what Morris calls INT_1--that is, the ordered pc-interval succession between the set's consecutive pcs. The next diagonal above and to the right of that provides INT_2, the ordered pc-interval succession between pcs two positions apart; the next diagonal provides INT_3, measured between pcs three positions apart, and so on. Patterns of invariance under transposition may be predicted from the T-matrix as follows (Figure I-1A): the matrix for set A contains three 2's (correspondingly, T_2A produces three common tones), it contains three 10's (thus $T_{10}A$ also produces three tones in common), it contains no 5's (showing complete variance under T_5A), and two 6's (two common tones with T_6A).

The second matrix of Figure I-1A is an I-matrix, which finds all possible sums between pcs of a set compared, in this case, with itself.[69] The association of the operation addition with inversionally-related sets has been discussed above in connection with Babbitt's index number. Because it represents sums rather than

FIGURE I-1

T- AND I-MATRICES FOR SC(5-8)

A is an element of SC(5-8) = [0, 2, 3, 4, 6]

FIGURE I-1A: T- and I-Matrices

T-Matrix (subtraction)

	0	2	3	4	6
0	0	2	3	4	6
2	10	0	1	2	4
3	9	11	0	1	3
4	8	10	11	0	2
6	6	8	9	10	0

I-Matrix (addition)

	0	2	3	4	6
0	0	2	3	4	6
2	2	4	5	6	8
3	3	5	6	7	9
4	4	6	7	8	10
6	6	8	9	10	0

T_0A = [0, 2, 3, 4, 6]

$T_2A = \{\underline{2}, \underline{4}, 5, \underline{6}, 8\}$* (max)

$T_{10}A = \{10, \underline{0}, 1, \underline{2}, \underline{4}\}$ (max)

$T_5A = \{5, 7, 8, 9, 11\}$ (min)

$T_6A = \{\underline{6}, 8, 9, 10, \underline{0}\}$

T_0A = [0, 2, 3, 4, 6]

*Underlined pcs are common tones with T_0A.

$T_2IA = \{8, 10, 11, \underline{0}, \underline{2}\}$

$T_4IA = \{10, \underline{0}, 1, \underline{2}, \underline{4}\}$

$T_6IA = \{\underline{0}, \underline{2}, \underline{3}, \underline{4}, \underline{6}\}$ (max)

$T_{11}IA = \{5, 7, 8, 9, 11\}$ (min)

FIGURE I-1B: INT_n for T-Matrix

$INT_1(A)$ = < 2 1 1 2 >

```
 2 1 1 2
0 2 3 4 6
```

$INT_2(A)$ = < 3 2 3 >

```
 -3-> -3->
0  2  3  4  6
     -2->
```

$INT_3(A)$ = < 4 4 >

```
  --4-->
0  2  3  4  6
     --4-->
```

$INT_4(A)$ = < 6 >

```
0  2  3  4  6
  ----6---->
```

differences, the I-matrix does not contain a row of zeros down its main diagonal. This matrix has a symmetrical structure like the T-matrix, since in this figure a set is compared with itself. In the I-matrix, unlike the T-matrix, each value in the upper right-hand triangle is mirrored exactly in the lower left-hand triangle. As the figure shows, the I-matrix predicts invariant pcs under T_nI in a similar manner to the way the T-matrix does for T_n-related sets. This discussion has only given a brief overview of the structure and power of these matrices for predicting patterns of pc invariance. Matrices can be constructed to examine relationships not just between a set and itself, but also between a set and any other set under any operation. In addition, submatrices showing one integer at a time can be used to discover patterns of ordered segmental invariance between ordered sets. Finally, Morris uses matrix representation to display the results of his COM-function comparing musical contours.[70] His COM-matrix plays an important role in defining the equivalence and similarity relations of the following chapters. Although this matrix does not predict invariance under T_n or T_nI, its diagonals show contour relationships between adjacent and non-adjacent elements of musical contours just as T- and I-matrices do for pitches or pitch-classes.

This overview has, to this point, discussed music theorists' concern with categorization of all possible sets into equivalence classes, and their identification and formalization of certain relationships among equivalent and complementary sets. It has not yet, however, approached the issue of similarity among sets that belong to different set classes. Similarity measurements fulfill all the requirements for equivalence discussed previously, except that of transitivity. Two types of similarity measurements have been proposed by various writers: one compares sets based upon shared intervallic content, while the other compares shared literal subset structure. Forte, Morris, and Charles Lord[71] have developed such measurements based upon compared interval-class contents. Forte's similarity relations in this category apply only to sets of equal cardinality, and signify either maximum or minimum similarity based upon compared interval-class vectors.[72] Robert Morris and Charles Lord have proposed alternative measurements that compare the interval-class contents of sets of differing cardinalities, and which operate along a single incremental scale from "least similar" to "most similar." Both Rahn and Lewin have responded by generalizing the applicability of such a model and comparing it with other possible measurements of similarity, both interval-based and inclusion-based, drawing upon previous work of Regener and Lewin.[73] Forte has also posited similarity relations based upon inclusion; his Rp relation denotes maximum similarity with respect to pitch class when two sets of equal cardinality share a common subset one pc smaller than the cardinality of the sets compared.[74] Further, Forte's set complex as it appears in his 1973 formulation is an inclusion-based similarity relation that combines abstract inclusion and abstract complementation into specified relationships (K and Kh) to create a network of interrelated set classes.[75] These SCs may be displayed graphically in the form of a diagram that charts the "connectedness" of each SC to the others in a given composition. A connected structure exists when every SC represented on this diagram is in the specified relation to a single SC, called the "nexus set," or to a small family of SCs that includes a primary nexus and one or two secondary nexus sets.

David Lewin's recent work generalizes many of the concepts discussed above, so as to apply in various generalized interval systems (GISs) in various musical spaces, not simply the pitch-class space that has been the primary focus of the discussion thus far. His embedding function, which counts the number of sets in set class /X/ that are included in set Y, may in itself be considered a type of inclusion-based similarity measurement, but it also generalizes Forte's interval vector to other musical spaces:

Lewin illustrates with a temporal-space interval vector, for example.[76] His injection function, INJ(X,Y)(f), counts the number of elements s in set X such that f(s) is a member of Y, where f is some transformation.[77] The transformation f is not necessarily one of the twelve-tone operators (TTOs). Lewin describes, as a non-TTO example, a wedge-transformation called "wedging-to-E," based on but not equivalent to the T_8I cycle. This "is not an operation; it is neither 1-to-1 nor onto,"[78] since E and B-flat map onto themselves under the wedge-transformation. As Lewin demonstrates, the INJ function generalizes Regener's common-note function, Babbitt's hexachord theorem, and Forte's K/Kh relation.[79]

The primary difference between the theories of non-tonal musical structure discussed to this point and those pertaining to twelve-tone music is characterized by Babbitt as the difference between a "combinational" and a "permutational" system:

> Given a collection of available elements, the choice of a sub-collection of these...provides a norm that is distinguishable by content alone; such a system, and the traditional tonal system is such, is therefore combinational. But if the referential norm is the totality of elements, there is but one such norm in terms of content, and deviations from this norm cannot exist within the system. But if an ordering is imposed upon this totality, and taken as a norm, this norm is so distinguished, in the case of twelve pitch class elements, from the 12! - 1 other possible orderings, that is, other possible permutations.
>
> Any consideration of the operations of the system must proceed from an awareness of their permutational nature. As a simple example: transposition, excepting the identity transposition, in a combinational system results in the adjoining of pitches which are not present in the original collection, and thus establishes a new sub-collection; transposition of a [twelve-tone] set results only in a permutation of the elements.[80]

In other words, if content alone were the criteria for distinguishing between one pitch collection and another, then all twelve-tone collections would be indistinguishable since they all contain the entire universe of twelve pitch classes. It is only when order is asserted as the criterion for distinguishing one twelve-tone set from another that two different sets may be identified, characterized by the differing permutations of their twelve elements.

A twelve-tone set (or row) is defined as an ordered collection of the twelve pitch classes, each of which appears once in the row. The basic set (often called the "prime" or P row) may be transformed by one of several operations: either by transposition, inversion, retrograde, or retrograde inversion (T_nP, T_nIP, RT_nP, and RT_nIP). The first two of these are operations upon pitch classes and the last two upon order positions.[81] Rows related by these operations are members of the same row class. Most row classes have 48 members: the twelve transpositions of the row's prime form, its inversion, its retrograde, and its retrograde-inversion. Transpositionally-related rows, as shown in Figure I-2A,[82] are rows that have identical INT_1 successions (by extension, such rows will generate identical T-matrices).[83] Row inversion is illustrated in Figure I-2B. Inversion in pitch space can be effected simply by reversing the direction of each of the row's intervals (exchanging "+" for "-" and vice versa); this creates a "mirror" inversion whose contour is the precise inverse of the original row.[84] Such a row is shown as the first of the T_5IP realizations in Figure I-2B. In much twelve-tone music such contour relationships are not preserved between rows, however; thus this figure also shows a T_5IP realization with a contrasting wedge-shaped contour. If the directed interval successions of these two rows are converted to ordered pc intervals, the successions are identical as shown.

Figure I-2C compares INT_1 of each row transformation to that of the original. Comparison of the prime and inverted rows reveals that their interval successions are inverse-related.[85] Because of this correspondence, inversionally-related rows will be defined here as rows with inverse-related INT_1 successions (as before, this definition

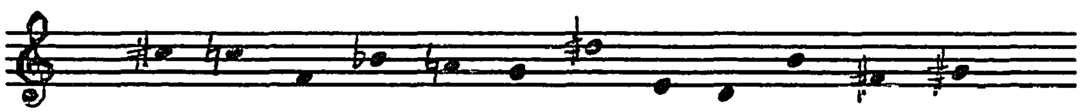
FIGURE I-2

INTERVALLIC RELATIONSHIPS AMONG ROWS BELONGING TO A SINGLE ROW CLASS

FIGURE I-2A:

T_1P row:	1	0	5	10	9	7	3	4	2	11	6	8
INT_1:	11	5	5	11	10	8	1	10	9	7	2	
T_3P row:	3	2	7	0	11	9	5	6	4	1	8	10
INT_1:	11	5	5	11	10	8	1	10	9	7	2	

FIGURE I-2B:



T_1P realized in pitch space	-1	-7	+5	-1	-2	+8	-11	-2	+9	-5	+2

T_5IP realized as "mirror" inversion

p-space INT_1:	+1	+7	-5	+1	+2	-8	+11	+2	-9	+5	-2
pc INT_1:	1	7	7	1	2	4	11	2	3	5	10

T_5IP realized without inverse-related contour

p-space INT_1:	+1	-5	+7	-11	+14	+4	-13	+14	-9	+5	-2
pc INT_1:	1	7	7	1	2	4	11	2	3	5	10

FIGURE I-2
(CONTINUED)

INTERVALLIC RELATIONSHIPS AMONG ROWS

FIGURE I-2C:

T_1P row:	1		0		5		10		9		7		3		4		2		11		6		8	
INT_1:		11		5		5		11		10		8		1		10		9		7		2		
T_5IP row:	5		6		1		8		9		11		3		2		4		7		0		10	
INT_1:		1		7		7		1		2		4		11		2		3		5		10		
RT_2P row:	9		7		0		3		5		4		8		10		11		6		1		2	
INT_1:		10		5		3		2		11		4		2		1		7		7		1		
RT_7IP row:	0		2		9		6		4		5		1		11		10		3		8		7	
INT_1:		2		7		9		10		1		8		10		11		5		5		11		

T_n-related rows have identical INT_1 successions.
T_nI-related rows have inverse-related INT_1 successsions.
RT_nI-related rows have retrograde-related INT_1 successions.
RT_n-related rows have inverse- and retrograde-related INT_1 successions.

may be extended to values across the entire T-matrix for each row).[86] Retrograde and retrograde-inversion rows are defined, respectively, as prime and inverted rows in reverse order. Figure I-2C compares the INT_1 successions of RT_2P and RT_7IP with T_1P. As the figure shows, RI-related rows contain identical pc interval successions, but in reverse order; while R-related rows have INT_1 successions that are inverse-related and in reverse order. Thus, if regarded as a transformation upon intervals rather than upon pitch, RI-related rows require only one transformation while R-related rows require two.

A comparison of the dyads formed harmonically between the pitch classes of the two inversionally-related row pairs, T_1P/T_5IP of Figure I-2C, reveals that pc 1 is twice paired harmonically with pc 5. A similar relationship exists between pcs 0 and 6, 4 and 2, 7 and 11, 8 and 10, while pitch classes 9 and 3 are paired with themselves. This relationship of preserved dyads is a reflection of the rows' index number.[87] The row-label subscripts sum (mod 12) to an index number of 6, as do the pc integers for each harmonic dyad between row pairs. Further, these preserved dyads will be maintained for any two inversionally-related rows whose subscripts add to 6. This mapping of pitch-class integers under the operation T_6I is summarized in cyclic notation as (0-6) (1-5) (2-4) (3) (7-11) (8-10) (9). Figure I-3 shows the generation of other selected cycles, as the successive application of a given operation to each member of the twelve-pc universe. As the figure shows, T_n cycles are of varying lengths, while T_nI cycles follow an invariant pattern: for odd values of n, there are six cycles of two elements each; for even values of n, there are five cycles of two elements each and two "singleton" cycles.

Among the earliest to discuss the implications of the T_n/T_nI cycles for musical analysis, Babbitt used these cycles to predict patterns of dyadic invariance and

FIGURE I-3

GENERATION OF SELECTED T_n AND T_nI OPERATOR CYCLES

T3:	0 -- 3	6 -- 9	T4:	0 -- 4	6 -- 10
	1 -- 4	7 -- 10		1 -- 5	7 -- 10
	2 -- 5	8 -- 11		2 -- 6	8 -- 0
	3 -- 6	9 -- 0		3 -- 7	9 -- 1
	4 -- 7	10 -- 1		4 -- 5	10 -- 2
	5 -- 8	11 -- 2		5 -- 9	11 -- 3

(0-3-6-9) (1-4-7-10) (2-5-8-11) (0-4-8) (1-5-9) (2-6-10) (3-7-11)

T4I:	0 -- 4	6 -- 10	T5I:	0 -- 5	6 -- 11
	1 -- 3	7 -- 9		1 -- 4	7 -- 10
	2 -- 2	8 -- 8		2 -- 3	8 -- 9
	3 -- 1	9 -- 7		3 -- 2	9 -- 8
	4 -- 0	10 -- 6		4 -- 1	10 -- 7
	5 -- 11	11 -- 5		5 -- 0	11 -- 6

(0-4) (1-3) (2) (5-11) (6-10) (7-9) (8) (0-5) (1-4) (2-3) (6-11) (7-10) (8-9) combinatoriality among rows in his articles from the early 1960's. In the second half of the decade, John Rothgeb used Babbitt's work as his point of departure for a masters' thesis and subsequent article that examined ordering relationships in the twelve-tone system.[88] Rothgeb gives a table of the T_n/T_nI cycles, and uses Babbitt's concept of "order inversions" (a tallied number representing the number of integer pairs from a given row that appear in reversed order in some transformation of that row) as a measure of similarity among forms of twelve-tone rows. In addition, he uses the cycles to draw generalizations about which operators will produce the maximum or minimum number of order inversions to a given row. In Starr's 1978 examination of the twelve-tone operator cycles, he includes not only T_n and T_nI cycles but also T_nM and T_nMI, and lists the complete cycles for these operators in a table.[89] Like Rothgeb, Starr notes that rows constructed by the symmetrical nesting of two-element cycles "have the property that the TTOs which generate them effectively retrograde them,"[90] but his is a generalization that encompasses all operations with two-element cycles whereas Rothgeb's formalization considered only the odd T_nI cycles. Starr provides a proof for what he terms the invariance theorem, which states that "if S comprises an F-cycle or a union of intact F-cycles, then S must be F-invariant."[91] Finally, Starr uses cycles to generate complement mappings, noting that sets consisting of "cross-sections" of the cycles (that is, one element from each cycle) for a given operator will map into their complement under that operation. Starr's work differs from that of previous writers since he uses the TTOs as his point of departure for a theory that describes general principles of structure among twelve-tone rows as well as unordered pitch-class sets.

This overview of theories pertaining to the twelve-tone system closes with a discussion of combinatoriality, which until recently has been considered a principle of twelve-tone composition only. Combinatoriality occurs when rows drawn from the same row class are combined contrapuntally in such a way that aggregates are formed by that convergence of rows before any single row has by itself stated all twelve tones. Arnold Schoenberg first described this procedure in his essay "Composition with Twelve Tones" (1941):

> Later, especially in the larger works, I changed my original idea...to fit the following conditions: the inversion a fifth below of the first six tones, the antecedent, should not produce a repetition of one of these six tones, but should bring forth the hitherto unusued six tones of the chromatic scale. Thus, the consequent of the basic set, the tones 7 to 12, comprises the tones of this inversion, but, of course, in a different order.[92]

Schoenberg describes hexachordal I-combinatoriality here, a pairing of a P- and I-row, such that the combination of the first hexachord of each produces an aggregate (by extension, so will the second hexachord). For this to be the case, the original row must consist of two hexachords that are inversionally-related, as he states.

Milton Babbitt was the first to publish detailed discussions of the principles underlying combinatoriality, presumably because his own compositions made extensive use of of these techniques. His 1958 "Some Aspects of Twelve-Tone Composition" (an article that Babbitt says is a condensation of his 1946 "The Function of Set Structure in the Twelve-Tone System"[93]) lists the six "all-combinatorial source sets." These six hexachords may be used to generate rows with the potential to form aggregates when paired with P-, I-, R-, and RI-related rows at the appropriate transposition levels. Babbitt also points out in this article the relationship between hexachordal interval-class content and the number of T_n levels at which P-combinatoriality may be achieved: any hexachord "missing" an interval, n, will map onto its complement under T_n and T_{12-n}. In addition to the six all-combinatorial hexachords, Babbitt touches upon the possibilities inherent in all-combinatorial source tetrachords and trichords. Tetrachordal combinatoriality results when three rows belonging to the same row class combine to create vertical aggregates, each formed of three tetrachords; trichordal combinatoriality requires four rows and results in aggregates formed of four trichords. Donald Martino develops these possibilities more fully in his 1961 article, "The Source Set and its Aggregate Formations,"[94] which includes not only the all-combinatorial hexachords but the all-combinatorial tetrachords and trichords as well. Finally, it is Babbitt's 1961 article, "Set Structure as a Compositional Determinant," that sets forth most clearly the requirements for each type of hexachordal combinatoriality.

Starr and Morris have proposed a more general theory of combinatoriality and the aggregate.[95] They note that previous writings have given the impression that combinatoriality is possible only with specially-constructed rows assembled from source sets, and that these theories have dealt only with partitions of rows into 2, 3, 4, or 6 equally-sized parts. This article corrects these impressions in favor of a more general theory that takes as its point of departure the combination matrix (CM), a two-dimensional array made up of rows and columns, which aligns specially-selected related twelve-tone rows to form a series of vertical aggregates.[96] The authors' four rules for forming two-row CMs effectively generalize the four types of hexachordal combinatoriality. The theory is general enough to include CMs of any number of rows, and draws heavily upon properties associated with the $T_n/T_nI/T_nM$ and T_nMI cycles in generating such matrices. The second part of the article focuses upon operations--such as merging, swapping of pcs, and overlaying--that may be performed upon CMs to enlarge and change the structure of CMs without destroying their aggregate-preserving properties. In his subsequent article, "Combinatoriality Without the Aggregate," Morris has continued the generalization of this theory of combinatoriality by positing that each of the rows and columns of a combination matrix need not represent the complete aggregate, but rather may represent a single set class.

As mentioned at the outset, many recent publications have begun to blur the distinction between theories of unordered pitch-class sets and ordered rows, primarily by focusing upon the operations and transformations that work upon these entities rather than upon the entities themselves. As Starr describes this perspective, "to approach various general aspects of twelve-tone or related types of music, I stress what I consider the operations that we apply to sets, rows, partitions, etc., rather than on those objects themselves, or...their classification."[97] While his "Sets, Invariance, and Partitions" uses operator cycles to discuss ordering properties arising from cross-sections and unions of cycles, it also develops theories for partially-ordered rows, and concludes by generating set-class lists for all possible unordered sets under the operations he has defined. Similarly, Chapter Four of Morris's Composition with Pitch Classes deals almost exclusively with operators, operator cycles, and operator groups. Throughout the book, Morris generalizes concepts across the ordered-vs.-unordered set boundaries: invariance matrices are constructed for pitch-class sets as well as for rows, combinatorial compositional designs are created with or without complete aggregates, operator cycles are used to generate invariant unordered sets as well as ordered segmental invariance, and so on. Finally, Lewin describes what he calls a "transformational attitude" in Generalized Musical Intervals:

> Instead of thinking: "i is the intervallic distance from s to t," we can think: "T_i is the unique transposition operation on this space that maps s into t." We can even shift our attention, if we wish, from the atomic "points" s and t to the one-element "Gestalts" X and Y, X being the set

> that contains the unique member s and Y being the set that contains the unique member t....[98]

After sketching a "mathematical dichotomy between intervals in a GIS and transposition-operations on a space," Lewin argues that "more significant than this dichotomy, I believe, is the generalizing power of the transformational attitude: It enables us to subsume the theory of GIS structure...into a broader theory of transformations."[99] His book concludes with a discussion of transformational networks that describe the structure of various compositions spanning the tonal and non-tonal repertories.

3. Music-Psychological Experimentation and Non-Tonal Music

The music-psychological literature is vast; overviews of the field are provided by Diana Deutsch, W. Jay Dowling and Dane L. Harwood, Robert Francès, and John A. Sloboda;[100] critical evaluations may be found in Mary Louise Serafine and Edwin Hantz.[101] Very little of this literature and very few experimental designs deal specifically with non-tonal musical materials, however. The summary that follows is therefore a fairly complete survey of experimental work in this area. Among the weaknesses of these publications is their general lack of music-theoretical input; that is, the lack of communication between the two fields. One of the studies on perception of twelve-tone rows, De Lannoy (1972), cites only one music theorist: Leonard Meyer.[102] None of the experiments on perception of row forms cite George Perle's work or any of Babbitt's articles, which date from the early 1960's; only one cites

Schoenberg's writings on twelve-tone composition.[103] Perhaps this trend is changing: recent publications by Carol Krumhansl and her colleagues cite Arthur Berger's and Pieter Van den Toorn's work on Stravinsky, as well as books on non-tonal theory and composition by Forte, Rahn, Messiaen, Walter Piston, and Wourinen,[104] while Diana Deutch's 1987 article, "The Tritone Paradox," cites Babbitt articles from 1960 and 1965, as well as Forte's The Structure of Atonal Music.[105] Still, this represents the exception rather than the rule. By the same token, only infrequently do music-theoretical books and articles cite results of music-psychological research. Morris's Composition with Pitch Classes, however, cites music-psychological publications by Burns and Ward, Deutsch, Meyer, Rasch and Plomp, and Shepard.[106] If citations in Krumhansl, Deutsch, and Morris are any indication, awareness of developments in both fields may become a growing trend contributing to an enrichment of both disciplines.

In searching for an experimental model to test perception of non-tonal music, psychologists have been drawn primarily to the compositional techniques of the twelve-tone school, specifically the use of ordered collections of the twelve pcs and transformations of those collections under the TTOs discussed above. The focus of these investigations has been upon the questions "can any melody comprised of twelve distinct pitch classes be retained in memory in the absence of a tonal hierarchy?" and "can the transformations retrograde, inversion, and retrograde-inversion be perceived by listeners?" Four of the studies to be discussed here--those by Francès, De Lannoy, Pedersen, and Krumhansl, Sandell, and Sergeant[107]--test perception of complete twelve-tone rows. Another, by Dowling, tests perception of the twelve-tone operators upon non-tonal segments of fewer than twelve tones.[108]

Dowling asked subjects to identify specific transformations upon five-note non-tonal melodies consisting of tones of equal duration. After hearing an initial melody, subjects were asked if a second melody was an exact retrograde (or inversion, or retrograde-inversion) of the first. Subjects were trained as to the meaning of these terms; they were warned not to accept same-contour imitations, but to prefer transformations that preserved exact interval sizes. In spite of the fact that much of his subject pool had little musical experience, recognition was somewhat above chance for all three conditions.[109] However, a high percentage of the errors made involved confusion between exact interval-for-interval transformations and same-contour imitations. Thus, there was no evidence that listeners were able to distinguish between transformations that preserved interval size and those that merely preserved contour, suggesting that perception of contour is more general than perception of pitch-class or interval. Further, Dowling presents the hypothesis that listeners remember non-tonal melodies in terms of their pitch structure (though he probably means pitch-class structure), rather than their interval structure.[110] He bases this conclusion on the ascending order of difficulty with which his subjects recognized these transformations: from least to most difficult, they were inversion, retrograde, and retrograde-inversion. The author asserts that the most difficult to perceive should be those transformations requiring two operations: the transformation requiring two operations if the row is retained as a pc construct would be retrograde-inversion; if retained as an intervallic construct, the two-operation transformation would be retrograde.[111] Dowling's experimental design does not distinguish between strategies used by trained musicians vs. untrained subjects, however. Further, it does not account for the possibility that listeners might use different strategies to recognize different transformations. It does not seem unlikely, for example, that listeners would use a pc-based strategy to recognize retrograde-related rows, but use an interval-based strategy to recognize I- and RI-related rows. For that matter, listeners might use a pc-based strategy to recognize RT_0 rows, since they contain an identical succession of pcs in reverse order, but an interval-based strategy for RT_n-related rows where n does not equal 0. Dowling's results must be regarded as inconclusive.

Early articles reporting on experiments testing perception of twelve-tone rows often contain polemical statements, and appear biased either for or against the comprehensibility of dodecaphonic music. Into the latter camp fall studies by Robert Francès and Paul Pedersen, while Christiaan De Lannoy's work falls into the former. Francès' subjects were asked to distinguish between two different row classes. Each trial could contain any one of the 48 transformations of two rows his subjects had "learned." These trials were divided into four categories:

> The first six trials were pure and simple expositions of the series in its straight version and its inversion and retrograde transformations, transposed and not transposed.... The next eight trials introduced rhythmic and melodic differentiation in keeping with the style of serial works, notably the substitution of octave equivalents.... In the next seven trials, chords of two or three tones were introduced to verify the persistence of serial unity in harmonic presentation.... Finally, the last seven trials were polyphonic.... Each of these categories included one item composed on the second series.[112]

His subject pool consisted entirely of trained musicians, half of which "had profound knowledge of serial technique."[113] Yet in most of the musical contexts, identification of the correct row class was made with no better than chance accuracy. Only in the first case (tones of equal duration with no octave displacement) was recognition slightly better than chance. However, the design of this experiment was highly biased against success at the recognition task.[114] First, the row classes between which subjects were asked to discriminate began with the same six pitch classes, five of which were presented in both cases with identical contours. Second, subjects were given little opportunity to learn the row before the trials began; they heard each series only twice, at a tempo so slow (one pitch per second) that retention of the entire series was made extremely difficult. Third, considering the reductionist methods employed by music-psychological experimenters for decades of tonal experimentation--testing perception of hierarchical tonal relationships using small diatonic subsets synthesized on sine waves, for example--Francès's introduction of varied contours, rhythmic complexity, and polyphonic trials shows a surprising disregard for scientific method, eliminates strict control over variables, and skews his results. Francès concludes his discussion of the experimental results by stating that "serial unity lies more on the conceptual than on the perceptual level; and that when thwarted by melodic motion, rhythm, and the harmonic grouping of tones, it remains very difficult to hear."[115]

In spite of the fact that this distinction between concept and percept seems uncharacteristic of the cognitive-psychological field, Francès's dichotomy forms the point of departure for De Lannoy's series of experiments and his defense of the twelve-tone system:

> In the trial of dodecaphony were often put forward arguments that made the aesthetic value of dodecaphonic music dependent on its perceptibility. Comparing the classical diatonic system to the dodecaphonic, the advocates of the former argue that the latter is no longer perceivable in larger perceptual structures, whereas always in the former "melodies" can be perceived as structural units.
>
> Therefore, according to the arguments of those inquisitors, the dodecaphonic system would be "conceptual", and not "perceptual" any more.[116]

De Lannoy notes the flaw in Francès's experimental design--that it requires subjects to distinguish among rows that begin with an identical sequence of tones. Further, he points out that elsewhere in Francès' book, in an experiment testing recognition of fugue themes in polyphonic tonal contexts, the author concludes that subjects generally identify these themes on the basis of the first few notes alone.[117] Thus Francès's decision to use rows that begin identically clearly shows a bias against success. De Lannoy is also troubled by Francès's lack of external criteria, suggesting that a parallel test should have been carried out upon transformations of tonal materials in order to compare the perceptibility of the two systems.[118] Based upon a group of experiments using three dissimilar rows and two Mozart melodies, De Lannoy concludes that both musicians and non-musicians can identify some rows and their transformations aurally, that success in this task is directly related to the distribution of intervals in those rows, and that listeners cannot always discriminate even between two Mozart melodies when they are transformed. These experiments have, in turn, been criticized for the author's treatment of data and for the vagueness of the instructions to his subjects,[119] and De Lannoy's work must be considered as biased in favor of the perceptibility of twelve-tone music as Francès' work is biased against it.

Paul Pedersen's prejudice is evident from his opening paragraph:

> It has often been noted that serialism does not seem to result in any perceptual order in the music where it forms the structural principle.... While attacking the perceptual validity of total serialism may, at this date, be equivalent to flogging a dead horse, there are perhaps many who still feel that the use of the techniques of "classical" (Schoenbergian) 12-tone theory will positively effect the perceptual coherence of a composition. The experiments presented here examine the effect of one of the basic tenets of 12-tone theory: that dealing with the question of octave equivalence on the perceptibility of 12-tone rows.[120]

Pedersen asked musically-experienced subjects to make a same-different judgement when presented with two twelve-tone rows that were either identical, or which had two adjacent pitches swapped in position. The first series of trials used a single all-interval row with pitches given in a single octave with equal durations. In 50% of these trials, two adjacent pitches of the row were swapped. Pederson reported the grand mean of correct judgements as 95.2% and had to admit that "...it is possible to learn a series of 12 different pitch classes with a fair degree of accuracy."[121] In the second series of trials, a second row was used, and each pitch class appeared in equal duration in one of five octaves, with octave placement randomly ordered. The grand mean for recognition was 54.3%. Pedersen noted that this result is "virtually at chance," and concluded that "random octave transposition of the members of a series of 12 different pitch classes destroys the perceptibility of the series."[122]

A more balanced perspective may be found in the recent work of Krumhansl, Sandell, and Sergeant. These authors have undertaken two types of experiments using rows drawn from two Schoenberg compositions. Only one of these experiment types addresses the issue of perceptibility of row classes under the twelve-tone operators; the other type investigates the question of whether non-tonal musical contexts generate a hierarchy of pitch-class relationships in the minds of listeners. Both experiment types were executed twice, first using pitches of equal duration and later using Schoenberg's musical contexts. In the first task, listeners heard a non-tonal melodic segment drawn from one of these rows followed by a "probe" tone, using each of the twelve pitch classes in random order as probe for each of twelve trials.[123] Subjects were asked to rate how well the probe tone fit with the preceding musical context in the "musical sense of the atonal idiom." In the second task, subjects were taught to distinguish between the two rows in prime form, and were trained as to the meaning of the R, I and

RI transformations. The trials included these transformations, and subjects were asked to determine whether each fragment sounded more like row 1 or row 2. In light of Dowling's conclusion that contour may influence such judgements, the equal-duration rows used "circular" electronic tones (also called "Shepard tones") generated to produce sounds with less clearly-defined octave placement.[124] In contrast to Francès's study, subjects had ample opportunity to learn the two rows, since listeners had already learned the prime form of the rows for the first task, and since "at the beginning of the present experiment they received as much training as was required to reach a fairly strict criterion of correctly labeling the two prime forms."[125]

Krumhansl, Sandell, and Sergeant found large individual differences in performance of these tasks, related for the most part to previous musical training. With respect to the tone-probe task, the experienced subgroup gave "low ratings for tones sounded more recently in the row contexts and high ratings for tones not yet sounded...[and] low ratings for tones fitting with local tonal implications."[126] Similar results were produced even in the musically-complex presentations drawn from Schoenberg compositions, which added rhythmic contexts and octave displacement of tones to the stimuli. The other subgroup had exactly the opposite results, giving high ratings for tones recently sounded and those fitting local tonal implications.[127] Results of the probe tone test in musically-complex contexts showed large individual differences between the two groups of listeners. The correlation of the results of this experiment with the musically-neutral tone-probe task showed that in the musically-complex contexts the responses by the experienced listeners largely replicated the previous results, while those of the less-experienced group varied considerably from their previous responses. The authors also compared data across various excerpts from the same composition; that is, they asked "whether the probe tone ratings for the different excerpts from the same piece were similar despite variations in rhythm and octave placements of the tones."[128] They found that Group 1 listeners produced ratings that were consistent across the different excerpts from a single piece, but that Group 2 listeners produced highly variable and inconsistent ratings. Finally, both the experienced and non-experienced group seemed to be influenced in some respects by local tonal implications suggested by the row's subsets:

> Group 1 listeners gave low ratings to tones consistent with tonal regions suggested by the context, and high ratings to tones that are inconsistent. This has the consequence that their probe tone ratings resembled keys very distantly related to the key region suggested by the contexts. In contrast, Group 2 listeners produced probe tone ratings that were generally consistent with the tonal implications of the context.[129]

The results of the second experiment type, testing R, I and RI transformations upon complete rows, produced a similar split between the two subject groups. Recognition of row transformations, even in Schoenberg's musical contexts, was above chance for all subjects. In the musically-neutral contexts, average performance in recognizing the R, I and RI forms ranged from 55% to 99%; further, the authors remark that overall performance among all subjects was remarkably accurate.[130] Performance was not so high for the experiment using Schoenberg's musical contexts, although in all cases average performance for row recognition was above chance. The authors conclude:

> ...these results suggest that the melodic and rhythmic variety of the excerpts made classification with respect to the underlying row difficult, but that some listeners possessed this ability after having extensive experience with the rows in previous experiments. It is likely that this experience is responsible for the difference in results between this experiment and the experiments reported by Francès (1972) and

> DeLannoy (1972), in which listeners were unable to identify musically complex variants of an underlying row.[131]

Finally, returning to Dowling's hypothesis that inexperienced listeners recognize transformations of non-tonal melodies as pc rather than interval transformations, it is interesting to compare his results with those of Krumhansl, Sandell, and Sergeant. These authors found that it was the experienced subgroup that most often correctly identified the I and R forms over the RI, while the less-experienced listeners identified the RI transformation most easily. They hypothesize that the inexperienced group may simply have used contour cues, since RI is the only transformation that preserves contour (though in reversed order). The experienced group may have used interval or pc information. Thus Dowling's conclusion that identification of transformations may have been made on the basis of contour alone should probably be extended to his conclusion about pitch vs. interval retention; his listeners may have structured the non-tonal melodies in memory as contours, retaining a pitch height succession rather than a pitch-class or interval succession.

Very few experimental studies have been published that focus upon perception of non-tonal materials in contexts other than the music of the twelve-tone school. Two of these are briefly summarized below. The first, by Krumhansl and Mark Schmuckler, tests whether the influence of each individual tonality is perceived in a polytonal context;[132] and the second, by Cheryl Bruner, tests perceived similarity among pitch-class sets.[133] Krumhansl and Schmuckler take as their point of departure the famous Petroushka chord from Stravinsky's ballet by the same name. The passage in question consists of two simultaneous parallel melodies of identical rhythm and contour, one of which expresses a C major triad and the other, an F# major triad--triads which, according to tonal theory, are drawn from maximally distantly-related keys. In the first of several experiments, using the probe-tone technique described above in Krumhansl, Sandell, and Sergeant's twelve-tone experiments, listeners' perception of the tonal hierarchy suggested by the individual keys was tested. When the keys were combined in a context-free experiment, some influence of both keys was felt. In the second and third experiments, however, excerpts from the Stravinsky composition were presented dichotically--that is, the C Major melodic line in one ear and the F# Major melodic line in the other--while the probe tone was sounded monaurally in either the right or left ear. Based upon these tests, the authors concluded that

> when presented dichotically the two components cannot be separated perceptually; this is attributed to the two voices having the same rhythmic and contour patterns and being sounded in the same pitch range.... The final experiment tested an alternative theoretical account, the octatonic collection. Probe-tone ratings following an octatonic scale did not account satisfactorily for the data for the musical passage, but the hierarchy of priorities proposed by Van den Toorn (1983) fit the data better than the major key profiles, especially for the experienced listeners.[134]

Thus, the experimenters concluded that the two triads, juxtaposed in a non-tonal musical context, lost their functional identity as major-key tonal entitites and were instead fused perceptually as a new type of sonority.

Bruner's experiments asked listeners to rate similarity among the twelve trichord types when articulated as chords, melodies, and arpeggios. Her results were compared to those obtained using Morris's SIM relation of 1981, and showed that listeners were more likely to base the similarity judgements upon such criteria as the number of common tones, the total number of semitones, and the degree of tonal association inherent in the set as presented, rather than upon total interval-class content. She concluded that

> the perception of similarity among contemporary pitch structures seems to be tied to the context in which the structures are presented. The results are consistent with compositional experience suggesting that the degree of perceived similarity between any two collections of pitches can be either diminished or enhanced by compositional choices having to do with the number of common tones, the spacing of chords, and the registral placement of pitches.[135]

Her results emphasize the need for a theory that describes musical articulation and which develops similarity measurements for such compositional features as registral contour and chord spacing. Her conclusion that listeners base similarity judgements upon registral placement, for example, instead of interval content is not so different from Dowling's conclusion that listeners recognize non-tonal melodic transformations based upon contour rather than interval structure. Such conclusions do not diminish the value of context-free theories that rate pcset similarity apart from their musical realizations, but they do point out a need for additional context-based theories.

Certainly other psychological approaches to the question of non-tonal music perception exist in addition to the experimental approaches detailed above. Leonard Meyer, for example, takes the tenets of Gestalt psychology as a point of departure for examination of the comprehensibility of non-tonal, and particularly serial, music. Meyer believes that "we perceive, understand, and respond to the world, including music, in terms of the patterns and models, concepts and classifications, which have been established in our traditions...."[136] After citing several musical examples, Meyer notes that these are

> ...instances of the more general laws of pattern-perception, discovered by the <u>Gestalt</u> psychologists, which tell us that regular, symmetrical, simple shapes will be more readily perceived, appear more stable, and be better remembered than those which are not. Thus, for instance, conjunct pitch sequences (the law of proximity), continuing

> timbres (the law of similarity), cyclic formal structures (the law of return)--all help to facilitate perception, learning and understanding. Though these normative modes of perceptual patterning need not necessarily be present in a particular kind of music, they must be taken into account even when...they are excluded from the syntactical norms of a style. For where they are not present (as they are not in most advanced music)...then the rate of other aspects of redundancy must be substantially increased.[137]

Contrary to Meyer's assertion, such patterns are not absent in non-tonal contexts. As Richmond Browne notes,

> When the tonal listener's expectation that he will be able to have a constant sense of ownership of the possible syntactic functions of the next pitch is no longer granted, the gestalts of sameness, continuation, shape, size, reversibility, conservation (belief that something is still there even though it is not now "visible"), interpolation, change of one parameter/non-change in another, etc., still obtain.[138]

The concept of Gestalt patterning seems most applicable in non-tonal musical analysis to the area of segmentation. Fred Lerdahl and Ray Jackendoff, whose theories are also indebted to the Gestalt psychologists, suggest ways in which the very principles cited by Meyer above might indeed be be adapted to non-tonal contexts:

> In order to flesh out a theory of atonal musical cognition, one would first have to develop well-formedness rules for possible segmentations. As suggested above, pitches could belong together if in some sense they were contiguous.... Then, out of these possible segmentations, preference rules would select sets actually heard by the experienced listener. Here parallels to the grouping principles of proximity..., similarity..., and parallelism...would be relevant.[139]

Meyer's assertion that regular, symmetrical shapes will be more readily perceived and appear more stable than those which are not implies that symmetrical scales, such as the whole tone or octatonic scales, are more stable than the asymmetrical diatonic scale. (Presumably, Meyer would not agree with this inference.) Further, this view conflicts with theories of Browne and the experimental findings of Helen Brown and David Butler,[140] who note that the comprehensibility of the diatonic scale is rooted in its asymmetry. Specifically, they note that its uneven but complete distribution of intervals assists the listener in identifying tonic; in particular, the rarer intervals--the single tritone and two half steps--present in the diatonic set provide "position finders" from which tonic can be determined.

4. Summation

In sum, over the course of the last several decades, theories of non-tonal music have become more and more refined. As non-tonal music continues to become accepted into the tradition of Western art music, so does our understanding of the general principles underlying the structure of this diverse body of music grow. Yet the role aural perception plays in conceiving this structure is still unclear, since no general theory of music perception has emerged and since few experimental psychologists have as yet addressed this area of research. This dissertation aims to develop a general theory that models certain aspects of musical perception in non-tonal contexts. It is a theory of musical articulation that describes the perceptible compositional vehicles by which underlying musical structure is articulated. When possible, this work draws upon previous research from both the music-theoretical and music-psychological literature in an effort to foster cross-fertilization between fields.

NOTES

[1]Non-tonal music refers to compositions that do not conform to principles of tonal structure and voice leading as codified in the theories of Heinrich Schenker. That is not to say that such music cannot contain pitch centricity, hierarchical structure, directed motion, triads, grammars, or other elements of the tonal system.

[2]Milton Babbitt, Words About Music, ed. Stephen Dembski and Joseph N. Straus (Madison: The University of Wisconsin Press, 1987), p. 23.

[3]Publications regarding non-tonal musical structure span about half a century, yet few sources provide any historical continuity to place each new contribution into context with reference to general developments in the field. The opening section of Janet Schmalfeldt's Berg's Wozzeck: Harmonic Language and Dramatic Design (New Haven: Yale University Press, 1983), entitled "Pitch-Class Set Theory: Historical Perspective," provides one account of early developments in the field. Beginning with the publications of J. M. Hauer from the 1920's in Vienna, she provides an historical account that culminates in the publication of Allen Forte's The Structure of Atonal Music (New Haven: Yale University Press, 1973). Her opening chapter also provides definitions for technical terms that are encountered in the theoretical literature and a succinct overview of Forte's theories. Schmalfeldt's account gives only cursory attention to twelve-tone theory, however, and does not attempt to discuss developments in the field after Forte. An excellent comparative overview of more recent work may be found in Robert Morris's review (Music Theory Spectrum 4 (1982), pp. 138-154) of John Rahn's Basic Atonal Theory (New York: Longman, 1980), although this overview proceeds topically rather than chronologically.

[4]Paul Hindemith, The Craft of Musical Composition (NY: Schott Music Corporation, 1937); Olivier Messiaen, The Technique of My Musical Language, trans. John Satterfield (Paris: A. Leduc, 1944; trans. 1956); and Arnold Schoenberg, "Composition with Twelve Tones" (1945), in Style and Idea, ed. Leonard Stein, trans. Leo Black (NY: St Martin's Press, 1975; reprint ed. with revisions, Berkeley: University of California Press, 1984), pp. 214-245.

[5]Because it is hoped that this dissertation will be read by those generally interested in the perception of non-tonal music, but whose primary training may not necessarily be in music theory, all technical terms used here will be defined in Part 2 of this chapter. Interested readers without specific training in this field may wish to read Part 2 first, then return to this introduction to place these music-theoretical concepts into historical context.

[6]Milton Babbitt, "Some Aspects of Twelve-Tone Composition," The Score and I.M.A. Magazine (1954), pp. 53-61; "Set Structure as a Compositional Determinant," Journal of Music Theory 5/1 (1961), pp. 72-94; and Donald Martino, "The Source Set and its Aggregate Formations," Journal of Music Theory 5/2 (1961), pp. 224-273.

[7]David Lewin, "Re: Intervallic Relations Between Two Collections of Notes," Journal of Music Theory 3/2 (1959), pp. 298-301, and "Re: The Intervallic Content of a Collection of Notes, Intervallic Relations Between a Collection of Notes and Its Complement: An Application to Schoenberg's Hexachordal Pieces," Journal of Music Theory 4/1 (1960), pp. 98-101.

[8]Milton Babbitt, "Twelve-Tone Invariants as Compositional Determinants," Musical Quarterly 46/2 (1960), pp. 246-259; reprinted in Problems of Modern Music, ed. Paul Henry Lang (NY: W.W. Norton & Co., 1962), pp. 108-121; and John Rothgeb, "Some Ordering Relationships in the Twelve-Tone System," Journal of Music Theory 11/2 (1967), pp. 176-197.

[9]Howard Hanson, Harmonic Materials of Modern Music: Resources of the Tempered Scale (NY: Appleton-Century-Crofts, Inc., 1960); George Perle, Serial Composition and Atonality: An Introduction to the Music of Schoenberg, Berg, and Webern (Berkeley: University of California Press, 1962); and Allen Forte, "A Theory of Set-Complexes for Music," Journal of Music Theory 8/2 (1964), pp. 136-183.

[10]Edward Cone, "Beyond Analysis," Perspectives of New Music 6/1 (1967), pp. 33-51; and David Lewin, "Behind the Beyond," Perspectives of New Music 7/2 (1969), pp. 59-69.

[11]Allen Forte, The Structure of Atonal Music , cited above.

[12]Robert Morris and Daniel Starr, "The Structure of the All-Interval Series," Journal of Music Theory 18/2 (1974), pp. 364-389; Robert Morris, "On the Generation of Multiple-Order-Function Twelve-Tone Rows," Journal of Music Theory 21/2 (1977), pp. 238-262; Daniel Starr and Robert Morris, "A General Theory of Combinatoriality and the Aggregate," Perspectives of New Music 16/1 & 2 (1977-78), pp. 3-35 (part 1) and 50-84 (part 2); and Daniel Starr, "Sets, Invariance, and Partitions," Journal of Music Theory 22/1 (1978), pp. 1-42.

[13]Chronologically, they are: Eric Regener, "On Allen Forte's Theory of Chords," Perspectives of New Music 13/1 (1974), pp. 191-212; David Lewin, "Forte's Interval Vector, My Interval Function, and Regener's Common-Note Function," Journal of Music Theory 21/2 (1977), pp. 194-237; Robert Morris, "A Similarity Index for Pitch-Class Sets," Perspectives of New Music 18/2 (1980), pp. 445-460; John Rahn, "Relating Sets," Perspectives of New Music 18/1 (1980), pp. 483-502; and David Lewin, "Response to a Response on Pcset Relatedness," Perspectives of New Music 18/2 (1980), pp. 498-502.

[14]See, for example, Richmond Browne's review of The Structure of Atonal Music in Journal of Music Theory 18/2 (1974), pp. 390-415 and Hubert S. Howe's in Proceedings of the American Society of University Composers 9-10 (1974-75), pp. 118-124. William Benjamin's "Ideas of Order in Motivic Music," (Music Theory Spectrum 1 (1979), pp. 23-34) is an expansion upon points made in his Forte review,

published in Perspectives of New Music 13/1 (1974), pp. 170-190. John Clough's 1965 article, "Pitch-Set Equivalence and Inclusion," (Journal of Music Theory 9/1, pp. 163-171), a response to Forte's earlier article,"A Theory of Set-Complexes," also raises important questions about musical perception, particularly the validity of an equivalence relation based upon interval content rather than upon transpositional and inversional equivalence. Hubert S. Howe also objected to interval-class content as equivalence criterion in classifying pitch collections ("Some Combinational Properties of Pitch Structures," (Perspectives of New Music 4/1 (1965), pp. 45-61). This was an issue that Forte evidently did reconsider; it was changed in The Structure of Atonal Music.

[15]Ibid., p. 404.

[16]Benjamin, "Ideas of Order in Motivic Music," p. 24.

[17]Among the first to examine this issue of structure and design was John Rothgeb in his article "Design as a Key to Structure in Tonal Music," Journal of Music Theory 15 (1971), pp. 230-253; reprinted in Readings in Schenker Analysis , ed. Maury Yeston (New Haven: Yale University Press, 1977), pp. 72-93. Other theorists currently working in this area include William Rothstein, David Beach, Dave Headlam, and Louise Trucks.

[18]William Benjamin, "Ideas of Order in Motivic Music," cited above; Jonathan Bernard, "Pitch/Register in the Music of Edgard Varèse," Music Theory Spectrum 3 (1981), pp. 54-73; Christopher Hasty, "Rhythm in Post-Tonal Music: Preliminary Questions of Duration and Meter," Journal of Music Theory 25/2 (1981), pp. 183-216; and "Phrase Formation in Post-Tonal Music," Journal of Music Theory 28/2 (1984), pp. 167-190.

[19]In his "Methodology for the Discussion of Contour: Its Application to Schoenberg's Music," Journal of Music Theory 29/2 (1985), pp. 223-248, Michael Friedmann develops a theory, to be discussed further in Chapter 2, that models aspects of musical contour. Chapter 3 of Richard Cohn's "Transpositional Combination in Twentieth-Century Music," (Ph.D. dissertation: University of Rochester, 1986) develops a number of format classes and surface transformations that model musical realizations of abstract pitch collections. Cohn's dissertation, incidently, contains an excellent overview of Howard Hanson's contributions to the field of non-tonal music theory, particularly his concept of "projection," which is a precursor of Cohn's generative theory of transpositional combination.

[20]Charles Wourinen, Simple Composition (NY: Longman, 1979) and John Rahn, Basic Atonal Theory , cited above.

[21]See, for example, Robert Morris's "Set Groups, Complementation, and Mappings Among Pitch-Class Sets," Journal of Music Theory 26/1 (1982) and "Combinatoriality Without the Aggregate," Perspectives of New Music 21/1 & 2 (1982-3), pp. 187-217.

[22]David Lewin, *Generalized Musical Intervals and Transformations* (New Haven: Yale University Press, 1987), and Robert Morris, *Composition with Pitch-Classes: A Theory of Compositional Design* (New Haven: Yale University Press, 1987).

[23]See Martha Hyde, "*Schoenberg's Twelve-Tone Harmony: The Suite Opus 29 and the Compositional Sketches* (Ann Arbor: UMI Research Press, 1982); "A Theory of Twelve-Tone Meter," *Music Theory Spectrum* 6 (1984), pp. 14-51; and "Musical Form and the Development of Schoenberg's Twelve-Tone Method," *Journal of Music Theory* 29/1 (1985), pp. 85-144.

[24]Howe, "Some Combinational Properties of Pitch Structures," and Regener, "On Allen Forte's Theory of Chords," cited above.

[25]Starr, "Sets, Invarance, and Partitions," passim.

[26]Rahn, *Basic Atonal Theory*, p. 74-75. Further, any equivalence relation must have the properties of reflexivity, symmetry, and transitivity. To quote Robert Morris (*Composition with Pitch Classes*, p. 341): "For elements a, b, and c, if R is an equivalence relation, then: an element a is in the R relation to itself; a is in the R relation to b and vice versa; and if a is in the R relation to b and b is in the same relation to c, then a is in the R relation to c."

[27]Babbitt uses the term "pitch class" in his 1958 article, "Who Cares if You Listen?" (*High Fidelity* 8/2 (Feb. 1958), pp. 38-40, 126-127; reprinted in *Contemporary Composers on Contemporary Music*, ed. Elliott Schwartz and Barney Childs (NY: Da Capo Press, 1978), but explains the term more fully in "Twelve-Tone Invariants as Compositional Determinants," reprint edition cited above, p. 110. Unfortunately, while many theorists assert a distinction between pitch and pitch class, this distinction is sometimes blurred in their publications. Thus, it is not uncommon that the term "pitch" appears when an author is describing a characteristic of pitch classes.

[28]Babbitt, "Twelve-Tone Invariants as Compositional Determinants," p. 110.

[29]David Lewin, "A Label-Free Development for 12-Pitch-Class Systems," *Journal of Music Theory* 21/1 (1977), pp. 29-48. See also his *Generalized Musical Intervals*, pp. 31-32.

[30] For example, the opening melody of Anton Webern's "Wie bin ich Froh!" from the *Drei Lieder* Op. 25/1, $F\sharp$-F-D-E-E^{b}, would be represented as $< 0, 11, 8, 10, 9 >$ in a moveable-do system but as $< 6, 5, 2, 4, 3 >$ by the fixed-do method. (Angle brackets (< >) denote ordered collections.)

[31]Lewin, "A Label-Free Development," p. 31.

[32]Forte,"A Theory of Set-Complexes," p. 139.

[33]Lewin, "A Label-Free Development, pp. 30-31.

[34]Rahn, Basic Atonal Theory, pp. 20-24, and Robert Morris, Composition with Pitch Classes, pp. 33-36 and 59-62.

[35]A more rigorous discussion of Morris's pitch and pitch-class spaces follows in Chapters 2 and 3 of this study. His pitch space does not necessarily specify chromatic semitones as implied here, but rather assumes the more general requirement that equal intervallic distances exist between consecutive pitches. Likewise, his pitch-class space is more general than that described here, since it is derived from a given pitch space by taking its pitches mod n, rather than specifically mod 12. In numbering pitch classes, Morris substitutes A and B for 10 and 11 to avoid the necessity of commas to distinguish between 1, 0 and 10. Other writers sometimes substitute "t" and "e" for 10 and 11 (see, for example, Andrew Mead's "Detail and the Array in Milton Babbitt's My Complements to Roger," in Music Theory Spectrum 5 (1983), p. 89-109).

[36]Schoenberg, "Composition with Twelve Tones," reprint edition cited above, p. 220.

[37]Ibid., p. 223.

[38]Babbitt, "Who Cares if You Listen?"; reprint edition cited above, p. 245.

[39]Lewin, Generalized Musical Intervals, pp. 16-24.

[40]John Clough has done some analysis in such a "diatonic pitch space," using a mod 7 system. See "Aspects of Diatonic Sets," Journal of Music Theory 23/1 (1979), pp. 45-61.

[41]This generalization is the topic of Chapter 7 of Morris's Composition with Pitch Classes, pp. 281-312.

[42]Lewin, Generalized Musical Intervals, p. 16. Although the present discussion deals primarily with intervals in pitch and pitch-class space, Lewin generalizes the concept through a variety of pitch and temporal spaces. The word "point" is therefore used in this definition and elsewhere to identify not only temporal positions, but also registral, harmonic, and other types of positions in various musical spaces.

[43]It should be noted that these definitions of interval and interval class differ considerably from the earlier formulations of Allen Forte, who did not always distinguish between pitch and pitch class in his definitions. These definitions are

modelled upon those that appear in Rahn's Basic Atonal Theory and Morris's Composition with Pitch Classes.

[44]To use Morris's words, "Since we take all arithmetic results mod-12 in pc-space, we always end up with an interval within the range of 0 to B. Thus, minus integers are always replaced by their positive counterparts." (Ibid., p. 62) Morris gives a chart of the correspondences between negative and positive integers mod 12 on pp. 62-63. It may help to visualize these as opposites across a clock face: -4 corresponds with 8, -5 with 7, -9 with 3, etc.

[45]For example, ic4 contains 4 and 8; ic5 contains 5 and 7. There are six pc interval classes, each of which has two members, except ic0 and ic6 which contain only one.

[46]Lewin, Generalized Musical Intervals, pp. 20-25.

[47]Ibid., p. 26. This discussion also gives the requirements of mathematical group structure for the interval group.

[48]Hanson, Harmonic Materials; Perle, Serial Composition and Atonality; and Forte, "A Theory of Set Complexes," cited above.

[49]Because this vector tallies the multiplicities of the six interval classes (not the 12 intervals), it will henceforth be called the interval-class vector or icvector. Morris's interval-class vector (ICV) differs in form, but not in principle, from Forte's (see, for example, Composition with Pitch Classes, p. 68-70). Morris's is a seven-place array that includes ic0 through ic6; inclusion of ic0 gives the cardinality of the set as the first digit of the ICV array. Morris also uses a twelve-position interval vector (IV), not to be confused with Forte's interval vector. Morris's IV(A,A) summarizes the intervals (not interval-classes) spanned between two statements of the set A, whereas the ICV(A) summarizes the interval classes within A itself (Ibid., p. 70).

[50] The sum of the numbers held in the vector's six positions--that is, the number of possible intervals spanned within any set--equals the sum of integers from 1 to (n - 1), with n representing the cardinality of the set. (Ibid., p. 68.) For Morris's seven-place ICV, the number of ics represented is the sum of integers from 1 to n.

[51]See Herbert S. Howe, "Some Combinational Properties," and John Clough, "Pitch-Set Equivalence," cited above.

[52]This distinction follows Morris's terminology. The Glossary of Composition with Pitch Classes, p. 347, provides a concise definition. Lewin uses "chord type" in some early work, but uses "set class" in his recent book. He notes that "set class" is a better term than "chord type" for the generalized sets, which may contain time points, durations, timbres, etc. (Generalized Musical Intervals p. 105).

[53]The Z-related sets (two 4- and 8-note sets, six 5- and 7-note sets, and 30 hexachords) share identical interval-class content but are not transpositionally or inversionally related. For example, the all-interval tetrachord, which contains one instance each of ic's 1 through 6, may be represented by [0, 1, 4, 6] or by [0, 1, 3, 7]. Comparison of their $CINT_1$ successions, < 1 3 2 6 > and < 1 2 4 5 >, reveals that they are not T_n/T_nI equivalent. Thus, using T_n/T_nI as the equivalence class basis, these two sets do not belong to the same class, despite their equivalent ic content. See notes 61 and 62 for a definition and further explanation of $CINT_1$ comparisons between sets.

[54]In this notation, T signifies the transposition operator, I the inversion, and the subscript "n" stands for the number of semitones by which the set is transposed.

[55]Robert Morris summarizes the various types of systems for determining set-class equivalence in his article, "Set Groups, Complementation and Mappings among Pitch-Class Sets" (cited above), and in Composition with Pitch Classes, pp. 78-81. This discussion is indebted to these summaries.

[56]See Howe's "Some Combinational Properties" and Regener's "On Allen Forte's Theory," cited above. The number of distinct set classes under each system is drawn from Morris, Composition with Pitch Classes, p. 80.

[57]See Morris, "Set Groups, Complementation and Mappings," passim, and Composition with Pitch Classes pp. 78-81. M and MI are multiplicative operations upon pcs. Multiplicative operations upon pcs are not so arbitrary as they might at first seem, since the more familiar T and I can also be defined as multiplicative operators, and since M_5 and M_7 produce consistent transformations upon interval-class content. Given a chromatic segment, for example, M_1 produces identity (equivalent to T_0), while M_{11} produces the set's inverse (equivalent to T_0I). The M_5 and M_7 operations (also known as M and MI, respectively) are sometimes called "circle-of-fifths" transformations, since they map the chromatic segment onto the circle of ascending fourths or fifths. Thus < 0 1 2 3 > under M becomes < 0 5 10 3 >, and under MI becomes < 0 7 2 9 >. While M_1 and M_{11} do not affect a set's interval-class content, M_5 and M_7 do: all interval-classes remain unchanged except for ics 1 and 5, which interchange (see Morris, Composition with Pitch Classes, pp. 79-80).

[58]Starr, "Sets, Invariance, and Partitions," cited above.

[59]See Forte, The Structure of Atonal Music, pp. 11-13; Rahn, Basic Atonal Theory, (as "normal form"), pp. 31-39; and Morris, Composition with Pitch Classes, pp. 83-84. The differences between algorithms is summarized in David H. Smyth's review of Rahn's Basic Atonal Theory, published in Perspectives of New Music 22/1 & 2, pp. 549-555. The normal form for five set classes differ between Forte's list and those generated by Rahn and Morris. These are for the set classes represented in Forte's nomenclature as 5-20, 6-29, 6-31, 7-20, and 8-26.

[60]For example, SC 3-12 [0, 4, 8] represents the symmetrically-structured augmented triad. This set class contains only 4 sets as members: {0, 4, 8}, {1, 5, 9}, {2, 4, 10}, and {3, 5, 11}. Transposition of any of these four sets by T_4 or T_8 reproduces the same pcs; similarly, inversion at T_0I, T_4I or T_8I reproduces the same pcs.

[61]This discussion of transpositional and inversional equivalence is indebted to Richard Chrisman's "Identification and Correlation of Pitch-Sets," Journal of Music Theory 15/1-2 (1971), pp. 58-83. Chrisman uses cyclic interval successions (Morris's $CINT_1$) to show the intervals not only between adjacent pcs but also "wrapping" back around cyclically to the first; thus the $CINT_1$ of [0, 1, 5] would be < 1 4 7 >. The numbers in a $CINT_1$ array always sum to 12.

[62]Because it is a cyclic array that includes the interval from the last pc back around to the first, the $CINT_1$ may require rotation to show inversional equivalence between sets. For example, $CINT_1$ of the set [0, 1, 4, 8] is < 1 3 4 4 >. Its inversion may be represented by the set [0, 4, 7, 8] with a $CINT_1$ of < 4 3 1 4 >, a rotation of < 4 4 3 1 >.

[63]Babbitt's clearest explanation of this concept may be found in Words about Music, pp. 33-42; however the concept may be found in some of his earliest articles, most notably "Twelve-Tone Invariants as Compositional Determinants."

[64]Lewin, "Re: The Intervallic Content of a Collection of Notes...," p. 99. "If P is a collection containing x notes with intervallic content p(i), and P' is the complement of P, with intervallic content p'(i), then p and p' are related by the formula: p'(i) = 12 - 2x + p(i) for every i." In this formulation p(i) may be interpreted as standing for a position in the icvector of the given set and p'(i) for the corresponding position in the icvector of its complement. The (12 - 2x) of the formula above may be conceptualized as ((12 - x) - x); that is, one subtracts from the "universe" of 12 the cardinality of the original set (x) to get the cardinality of its complement (12 - x), then subtracts this number x a second time to find the difference between cardinalities of the given set and its complement. Finally, that difference is added to the integer located in position p(i) of the set's icvector; the result is the integer that belongs in the corresponding position p'(i) of its complement.

[65]For proofs of the complement theorem, see the following articles (cited above): Regener, "On Allen Forte's Theory of Chords"; Starr, "Sets, Invariance and Partitions"; and Morris, "Set Groups, Complementation and Mappings." Howard J. Wilcox, in "Group Tables and the Generalized Hexachord Theorem,"; Perspectives of New Music 21/1 (1983), pp. 535-539, provides proofs of both theorems, as does Morris in Composition with Pitch Classes, pp. 74-77.

[66]Rahn, Basic Atonal Theory, p. 108. Forte notes this exception to the theorem as well. According to his discussion of the interval vector as predictor of invariance, "inverse-related values of t will always produce the same number of

invariants...[thus] the vector entry for ic6 must be multiplied by 2 to obtain the correct number of invariants for t = 6" (The Structure of Atonal Music, p. 31). Morris discusses an alternative to the icvector for predicting invariance, proposing that the invariance vector (to be discussed in more detail in the text following) be used. He also uses a twelve-place interval vector, IV(A,A) that summarizes the intervals spanned from set A to a copy of itself. Thus it accurately enumerates the number of common tones between A and T_nA for all n, and T_6A does not represent an exception to the rule (Composition with Pitch Classes, p. 72).

[67]Bo Alphonse, "The Invariance Matrix," (Ph.d. dissertation: Yale University, 1974). A similar approach is offered by Carlton Gamer and Paul Lansky, in "Fanfare for the Common Tone," Perspectives of New Music 14/2 (1976), pp. 229-235. For an excellent overview of Alphonse's work and an introduction to applications of the invariance matrix as used here, see Robert Morris's review of John Rahn's Basic Atonal Theory, in Music Theory Spectrum 4 (1982), pp. 138-154.

[68]This discussion of the invariance matrices' structure, including INT_1, INT_2, and other terms, is drawn from Morris, Composition with Pitch Classes, Chapters 2-3, passim.

[69]The $VTICS_A$ vector, formulated by Rahn, (Basic Atonal Theory, pp. 111-114) may be tabulated from the I-matrix and used to predict invariant pcs under T_nI. It is constructed and used in much the same way as is the interval-class vector for invariance under T_n.

[70]Morris, Composition with Pitch Classes, p. 28.

[71]Allen Forte, The Structure of Atonal Music; Robert Morris, "A Similarity Index for Pitch-Class Sets" (cited above); and Charles Lord, "Interval Similarity Relations in Atonal Set Analysis," Journal of Music Theory 25/1 (1981), pp. 91-111.

[72]Forte, The Structure of Atonal Music, p. 47-49. Maximally similar sets have identical integers in four of their six vector positions. In Forte's R_1 condition, the integers in the remaining two positions "swap," while in the R_2 condition there is no such requirement. Although Forte does not recognize the M/MI relation in his book, it should be noted that two sets in the R_1 relation that swap vector entries in positions 1 and 5 are M/MI-related as discussed above, since this transformation preserves all interval classes except 1 and 5, which exchange. Mimimum similarity with respect to interval class is denoted by R_0, and occurs between two sets that contain no identical corresponding vector entries.

[73]See John Rahn, "Relating Sets," David Lewin, "Forte's Interval Vector..." and Eric Regener, "On Allen Forte's Theory of Chords," all cited above. A brief summary of these approaches to similarity measurement is provided in Chapter 2 of this dissertation, in connection with the similarity relations to be developed there for musical contours.

[74]This relationship assumes abstract inclusion as distinct from literal inclusion, although Forte does not use these terms. He does note, however, that the Rp relation may be "weakly represented" (abstractly included) or "strongly represented" (literally included) in a musical composition.

[75]The set complex is first introduced in Forte (1964), "A Theory of Set-Complexes for Music," and reappears in a considerably revised form in The Structure of Atonal Music. The entire second half of the latter work is devoted to the set complex, pp. 93-177. The 1964 article differs from the 1973 reformulation in a number of ways: first, it defines the K and Kh relations (as well as an additional "Ks" relation) somewhat differently, based upon "interval-set inclusion"; second, the graphic display of the set complex differs from the K/Kh charts of 1973, and cannot show K/Kh interaction as effectively as the revised format does; finally, the concept of nexus set does not appear here, nor the idea of a "connected" or "unconnected" structure.

Forte's 1973 formulation is based upon abstract inclusion and complementation of pc sets. The K and Kh relations are defined most clearly by Morris. He uses a two-partition to divide (or partition) the aggregate into two complementary sets. Thus the inclusion and complementation relations are clearly visible. According to Morris's definition, "A pair of two-partitions (A|A') and (B|B') are in a (literal) K-relation if one position of one is included in either position of the other," and "The two-partitions (A|A') and (B|B') are in a (literal) Kh-relation if a member of one is included in both members of the other--(literally) included in one under T_0 and (abstractly) included in the other under a TTO not including M." (Composition with Pitch Classes, pp. 98-99.)

[76]Lewin, Generalized Musical Intervals, p. 105ff. To generalize the interval vector, EMB(SC_n,Y) would run through all two-note set classes (for n = 1 to 6) as values for X embedded in Y. By analogy, EMB(SC_n,Y) could run through all possible trichords embedded in Y, and so on. The embedding function is applied to a temporal space on pp. 112-120.

Lewin notes that choice of canonical operator is crucial to the embedding number. "Strictly speaking, we should write EMB(CANON,X,Y) to show that the embedding number varies with the canonical group as well as the sets X and Y" (Generalized Musical Intervals, p. 106). Thus, to run all the embedded trichord types through EMB(X,Y), n would range from 1 to 12 in a T_n/T_nI-equivalent system but would range from 1 to 19 in a system based upon T_n-equivalence only.

[77]More specifically, given sets X, Y, and function f, where f is the set of mappings (X_n, Y_m), such that X_n is an element of X and Y_m is an element of Y, and X and Y occur in a musical context, INJ(X,Y)(f) counts the number of (X_n, Y_m). ($0 \leq n \leq$ the cardinality of X, and $0 \leq m \leq$ the cardinality of Y.)

[78]Ibid., p. 125.

[79]Ibid. The INJ function is defined on p. 124. Lewin generalizes Regener on p. 144, Babbitt on p. 145, and Forte on p. 150.

[80]Babbitt, "Twelve-Tone Invariants," pp. 109-110.

[81]Other operations are possible as well: M and MI (defined above in connection with pitch class sets) are operations upon pcs; rotation is an operation upon order positions. Forty eight twelve-tone operators are cited in Appendix Two of Morris's Composition with Pitch-Classes: the twelve transpositions of a given set (T_0 - T_{11}), the twelve transpositions of its inversion (T_0I - $T_{11}I$), the twelve transpositions of its M-transform (T_0M - $T_{11}M$), and the twelve transpositions of its MI-transform (T_0MI - $T_{11}MI$).

[82]It should be noted that this figure and the following discussion assumes fixed-do approach to row labelling, chosen for the sake of consistency between twelve-tone and other non-tonal music theories. The reader should be aware, however, that a great deal of the twelve-tone theoretical literature uses the moveable-do approach, assigning 0 to the first note of the first prime row. See, for example, the articles of Milton Babbitt cited above and Perle's Serial Composition and Atonality.

[83]As Figure I-2A illustrates, transposition may be effected by mod 12 addition of some constant to each element of a given row; thus T_3P is derived by adding 2 to each element of T_1P. Because the INT_1 is identical for transpositionally-related rows, T_nP may also be constructed by beginning on pc n, then determining the remaining elements by adding to each successive pc (mod 12) the integer found in the corresponding position of INT_1.

[84]Transposition and inversion were defined somewhat differently in the previous discussion of set-class relationships. Transpositionally-related sets were defined as having identical $CINT_1$ successions, while inversionally-related sets were described as having retrograde-related $CINT_1$ successions, with rotation possibly required. This definition is only possible because sets in this system are reordered into a uniform ascending "normal order" before comparison. In ordered sets where such reordering is not possible, T_n preserves INT_1 while T_nI produces an inverse-related INT_1. As Figure I-2C shows, it is RI-related rows that produce retrograde-related INT_1 successions.

[85]Inverse-related pc intervals are those that sum to 12.

[86]Because of this inverse relationship between INT_1 successions, T_nIP may be found for any P-row by beginning on pc n, then determining the remaining elements by adding to each successive pc (mod 12) the inverse of the integer found in the corresponding position of the P-row's INT_1 succession. Babbitt's concept of index number, mentioned briefly above in connection with pc set inversion, may also be used to find a T_nIP row form. Note in Figure I-4C that the pc integers aligned vertically between the T_1P and T_5I row all add to 6. Given any two rows, T_nP and T_mIP, the

sum of (n + m) is the index number for these rows; all "harmonic" dyads formed between the two rows will sum to that index number. Thus, given T_nP, any T_mIP can be constructed by calculating for each integer of the P-row which integer in the I-row will sum to the index number (n + m).

87 See Babbitt's Words About Music, pp. 33-38, for an extended discussion of index numbers, with analytical examples drawn from the music of Webern and Dallapiccola. This discussion is a distillation of concepts originally introduced in his "Twelve-Tone Invariants as Compositional Determinants."

88 John Rothgeb, "Some Ordering Relationships in the Twelve-Tone System," Journal of Music Theory 11/2 (1967), pp. 176-197.

89 Starr, "Sets, Invariance, and Partitions," (cited above), pp. 12-13.

90 Ibid., p. 15.

91 Starr, p. 17. Rothgeb made a similar observation when he noted that the minimum number of order inversions under T_nI is produced when the row consists of the union of complete cycles.

92 Schoenberg, "Composition with Twelve Tones," (cited above) p. 225.

93 Babbitt, "Some Aspects of Twelve-Tone Composition," (cited above). His footnote 3 (p. 56) cites the earlier work but gives no publication information outside of the date. Presumably, this is an unpublished paper.

94 Martino, "The Source Set and its Aggregate Formations," (cited above).

95 See Starr and Morris, "A General Theory of Combinatoriality and the Aggregate," cited above.

96 This definition is drawn from Morris's later article, "Combinatoriality Without the Aggregate," p. 481.

97 Starr, "Sets, Invariance, and Partitions," p. 2.

98 Lewin, Generalized Musical Intervals, p. 157.

99 Ibid., p. 159.

100 Diana Deutsch, ed., The Psychology of Music (NY: Academic Press, 1982); W. Jay Dowling and Dane L. Harwood, Music Cognition (NY: Academic Press, 1986); Robert Francès, The Perception of Music, transl. W. Jay Dowling (Hillsdale, NJ: Lawrence Erlbaum Associates Publisher, in press); John A. Sloboda,

The Musical Mind: The Cognitive Psychology of Music (Oxford: Clarendon Press, 1985).

[101]Mary Louise Serafine, Music as Cognition: The Development of Thought in Sound (New York: Columbia University Press, 1988) and Edwin Hantz, "Studies in Musical Cognition: Comments from a Music Theorist," Music Perception 2/2 (1984), pp. 245-264.

[102]Christiaan De Lannoy, "Detection and Discrimination of Dodecaphonic Series." Interface 1 (1972), pp. 13-27.

[103]Two articles testing perception of twelve-tone rows--Pederson (1975) and Krumhansl, Sandell, and Sergeant (1987), to be discussed below--cite Reginald Smith Brindle, Serial Composition (London: Oxford University Press, 1966) as their primary source of twelve-tone theory. Pederson also cites S. Bauer-Mengelberg and M. Ferentz, "On Eleven-interval Twelve-tone Rows, Perspectives of New Music 3/2 (1965), pp. 93-103, and J. Kramer, "The Row as Structural Background and Audible Foreground: The First Movement of Webern's First Cantata," Journal of Music Theory 15 (1971), pp. 158-181. Krumhansl, Sandell, and Sergeant cite Schoenberg's "Composition with Twelve Tones" in Style and Idea, ed. Leonard Stein (London: Faber and Faber, 1941/1975) and Charles Wourinen, Simple Composition (New York: Longman, 1979).

[104]See the reference lists for Carol L. Krumhansl, Gregory J. Sandell, and Desmond C. Sergeant, "The Perception of Tone Hierarchies and Mirror Forms in Twelve-Tone Serial Music," Music Perception 5/1 (1987), pp. 31-78, and Carol L. Krumhansl and Mark A. Schmuckler, "The Petrouschka Chord: A Perceptual Investigation," Music Perception 4/2 (1986), pp. 153-184.

[105]Diana Deutsch, "The Tritone Paradox: Effects of Spectral Variables," Perception & Psychophysics 41/6 (1987), pp. 563-575.

[106]Morris, Composition with Pitch Classes, pp. 350-353.

[107]These studies are: Robert Francès, The Perception of Music, pp. 122-128; Christiaan De Lannoy, "Detection and Discrimination of Dodecaphonic Series"; Paul Pedersen, "The Perception of Octave Equivalence in Twelve-Tone Rows," Psychology of Music 3/2 (1975), pp. 3-8; and Krumhansl, Sandell, and Sergeant, "The Perception of Tone Hierarchies and Mirror Forms in Twelve-Tone Serial Music."

[108]W. Jay Dowling, "Recognition of Melodic Transformations: Inversion, Retrograde and Retrograde-Inversion," Perception & Psychophysics 12/5 (1972), pp. 417-421.

109 Future experimentation might replicate this study with a subject pool of trained musicians.

110 Dowling, p. 421.

111 In retrograde-related rows the direction of each interval is changed and their sequential order reversed, as discussed previously. Dowling's reasoning is nevertheless flawed because he fails to take T_n into consideration; thus, RI-related rows viewed as pitch constructs may have three operations, not two (RT_nIP is inverted, transposed, and reversed in order). This will be pursued more fully in Chapter 4.

112 Francès, pp. 123-124.

113 Ibid., p. 124.

114 This bias against success makes the above-chance results for recognition of row transformations in the equal-duration trials more significant.

115 Ibid., p. 126.

116 De Lannoy, p. 13.

117 Ibid., p. 14.

118 Ibid.

119 Krumhansl, Sandell, and Sergeant, p. 54.

120 Pederson, p. 3.

121 Ibid., p. 6.

122 Ibid., p. 6.

123 This technique was originally used by Krumhansl and Shephard in an experiment using diatonic scale patterns followed by probe tones. See Carol L. Krumhansl and Roger N. Shepard, "Quantification of the Hierarchy of Tonal Functions Within a Diatonic Context," Journal of Experimental Psychology: Human Perception and Performance 5 (1979), pp. 579-594.

124 Roger N. Shepard developed such a method that generates tones with no distinct octave placement--a musical "barber's pole" that continually ascends without

seeming to get higher. See Shepard, "Structural Representations of Musical Pitch," in The Psychology of Music, ed. Diana Deutsch (New York: Academic Press, 1982).

[125]Krumhansl, Sandell, and Sergeant, p. 54.

[126]Ibid., p. 31.

[127]Interestingly, upon replication of the first experiment with the same subjects (rating probe tones after hearing row excerpts in "musically-neutral" contexts), the experienced listeners gave the same responses 78% of the time for the Wind Quintet row, and 93% of the time for the Fourth String Quartet. Even the less experienced listeners gave the same response 61% and 72% of the time, respectively.

[128]Ibid., p. 68·

[129]Ibid., p. 48.

[130]Ibid., pp. 57.

[131]Ibid., p. 65.

[132]Carol L. Krumhansl and Mark A. Schmuckler, "The Petrouschka Chord: A Perceptual Investigation," cited above.

[133]Cheryl L. Bruner, "The Perception of Contemporary Pitch Structures," Music Perception 2/1 (1984), pp. 25-39.

[134]Krumhansl and Schmuckler, p. 153.

[135]Bruner, p. 38.

[136]Leonard Meyer, Music, The Arts, and Ideas: Patterns and Predictions in Twentieth-Century Culture (Chicago: University of Chicago Press, 1967), p. 273.

[137]Ibid., p. 289.

[138]Richmond Brown, "Review of The Structure of Atonal Music," p. 403.

[139]Fred Lerdahl and Ray Jackendoff, A Generative Theory of Tonal Music (Cambridge: The MIT Press, 1983), pp. 299-300.

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140 See Richmond Browne, "Tonal Implications of the Diatonic Set," and Helen Brown and David Butler, "Diatonic Trichords as Minimal Tone Cue-Cells," both in In Theory Only 5/6-7 (1981), pp. 3-21 and 39-55. In addition, two recent articles by Helen Brown touch upon this issue: "Tonal Hierarchies and Perceptual Context: An Experimental Study of Music Behavior," Psychomusicology 7/1 (1987), pp. 77-90; and "The Interplay of Set Content and Temporal Context in a Functional Theory of Tonality Perception," Music Perception 5/3 (1988), pp. 219-250.

Chapter Two

Musical Contour: Pitch Height Succession[1]

1. Psychological Studies of Contour Perception

Cognitive psychologists and music theorists have, for many years, understood that perception of pitch cannot simply be modelled along a single continuum from low to high.[2] Thus representational models for pitch perception have been developed to reflect a number of related dimensions,[3] among them the tendency of listeners familiar with Western tonal music to group octave-related pitches into equivalence classes. Nevertheless, in spite of this tendency, listeners are for the most part unable to recognize familiar melodies which have been distorted by octave displacement unless the melodic contour is retained. In a series of experiments, initiated by Diana Deutsch in 1972,[4] familiar melodies were distorted by assigning each pitch randomly to one of three octaves; thus pitch class was retained while contour and absolute interval size were altered. In most cases, Deutsch's subjects did not recognize the distorted melodies. She noted that recognition was no better under this condition than recognition of the same melody represented simply as a series of clicks that duplicated the tune's rhythm and eliminated pitch completely. More recently, Deutsch has posited a two-channel theory for processing pitch, whereby contexts consisting of single tones or chords related by harmonic inversion are processed along one channel that gives rise to the perception of octave equivalence, while contexts consisting of melodies or strings of intervals are processed along a second channel that does not recognize octave-related pitches as equivalent.[5]

Later experimenters who sought to replicate Deutsch's findings discovered that while pitch-class information alone was not sufficient for subjects to recognize familiar melodies, pitch-class plus contour information was.[6] Thus, octave-distorted tunes that retained the contour of the original were much more easily recognized than those that did not. Indeed, so important is the role of contour in the retention and recognition of well-known melodies that even the size of the interval between successive pitches may be altered. This alteration may simply involve a shift in interval quality between major and minor modes, or may involve a more marked change in interval size. Subjects will usually recognize the tune in spite of such intervallic changes if the contour remains unaltered.[7] Related experimentation has shown that listeners frequently confuse a fugue subject with its tonal answer--that is, they identify the two as identical on the basis of their equivalent contours and diatonic scale types, despite the fact that their interval contents differ. W. J. Dowling explains this tendency as follows:

> ...people remember the contours of melodies they hear without an exact notion of interval sizes between tones. Interval sizes can be reconstructed by comparing the contour to one of the overlearned scale systems the person knows--in our culture, major or minor. But if a novel melody just heard is repeated as a tonal imitation translated along the well-known scale, people are generally not able to detect the change in intervals.[8]

It should be noted that Dowling's statement applies to a subject pool containing a high proportion of musically-untrained subjects. In some of his experiments upon contour perception, the author directly compares the discriminatory abilities of musically-untrained subjects vs. trained musicians, finding that the musically-untrained are more likely to rely upon contour information alone while musicians are able to draw upon other types of information as well.[9] Nevertheless, it is primarily contour identity that associates fugue subjects with their tonal answers in listeners' ears and minds, whether musically trained or not.

Experimentation on the perception of non-tonal music is much less common, although some work on tonal music has had clear implications for the perception of non-tonal music. Judy Edworthy, for example, has tested listeners' recognition of alterations in transposed novel melodies of varying lengths. She discovered that subjects were significantly better at recognizing contour alterations for melodies of up to eleven notes, but better at recognizing interval alterations in fifteen-note melodies.[10] Her work poses the hypothesis that listeners require a substantial melodic length in order to recognize the tonal framework upon which the melodies are built. This conclusion would, however, be questioned by experimenters such as Helen Brown, David Butler, and Edwin Hantz, whose work focuses on minimal "cue-cells" of as few as three notes. Musically-trained listeners can identify a tonic key with little difficulty, depending on which scalar intervals the cue-cell contains and in what order.[11] According to Edworthy, once the listener has perceived the tonal framework, a melody's interval succession can be remembered, and is more resistant to forgetting. She concludes that

> ...the serially derived context enabling the listener to establish the key, scale, or tonal center of a transposed melody is important in interval perception but not in contour perception. The implication of this, in turn, is that a concept of a contour may be useful in the perception of music, particularly when the current tonal context is weak or confusing.[12]

Thus, accurate perception of contour does not depend on the listener's ability to establish a key. W. J. Dowling and D. S. Fugitani are among the few who have published results of experimentation using non-tonal musical materials. They discovered that listeners consistently confused the exact transposition of a novel non-tonal melody with a differing second non-tonal melody, if the latter retained the same contour. The authors conclude that listeners retain non-tonal melodies in memory solely in terms of contour.[13]

In non-tonal melodies of fewer than 15 tones, given the same or similar rhythmic pattern, listeners should be able to perceive equivalence or similarity among musical contours more easily than among pitch-class sets in melodic settings, since only the latter requires subjects to perceive intervallic information. Figure II-1, for example, illustrates several musical passages in which melodic patterns share contour identity but not set-class identity. The motives of Figure II-1A appear at various points in Bartok's Sonata for Two Pianos and Percussion, Mvt. II, and are aurally associated by identity of contour, rhythm, and metric placement, in spite of the fact that their set-class memberships differ. The melodies of Figure II-1B appear about six bars apart in the second movement of the Lyric Suite. Again, the listener will presumably associate these two on the basis of their identical contours and rhythmic similarity, in spite of the fact that their intervallic and pitch contents differ; the first melody is a member of SC 10-4, while the second belongs to SC 10-3. Finally, the melody of Figure II-1C, drawn from the second movement of Berg's Violin Concerto, may be divided into two parts as marked. The second unit is an intervallic expansion and rhythmic diminution of the first, but may be heard as a same-contour imitation of the first. As in the previous examples, each unit belongs to a different set class--the first to SC 4-27 and the second to SC 4-20.

FIGURE II-1

SAME-CONTOUR MELODIES

Figure II-1A:
Bartok, Sonata for Two Pianos and Percussion, II
Mm. 31, 33, 37

Figure II-1B:
Berg, Lyric Suite, II
Violin I, mm. 66-67 and 72-73

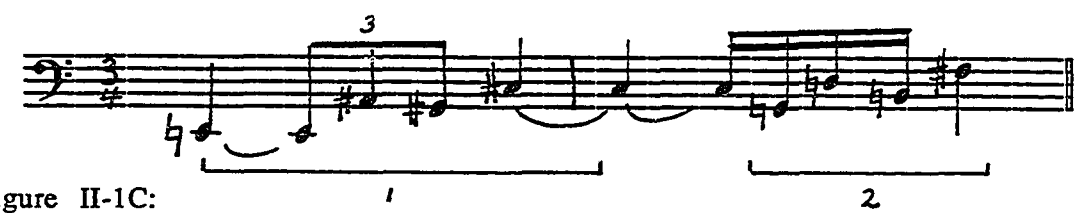


Figure II-1C:
Berg, Violin Concerto, II
Bassoon, mm. 35-36

2. Musical Spaces and Contour Space

For purposes of musical analysis and description, music theorists have found it useful to divide musical space into a number of interrelated spaces,[14] most commonly into pitch space (a linear space of pitches which extends from the lowest audible range to the highest) and pitch-class space (a cyclical space of twelve pitch classes that assumes octave equivalence and, because of its closed group structure under transposition (addition mod-12) enables equivalence classes not possible in pitch space).[15] Recently, a number of theorists have focused their attention upon the examination of another type of musical space, which has been called contour space.[16] In formulating this concept, music theorists recognize the fact that listeners may perceive similarity or equivalence among the contours of two phrases quite apart from accurately recognizing pitch-class or intervallic relationships between them, as noted above. In order to reflect this aspect of perception more precisely in musical analysis, new methods for comparing contours are necessary. The theories for contour similarity relations that follow take Robert Morris's definition of contour-space equivalence as their point of departure.

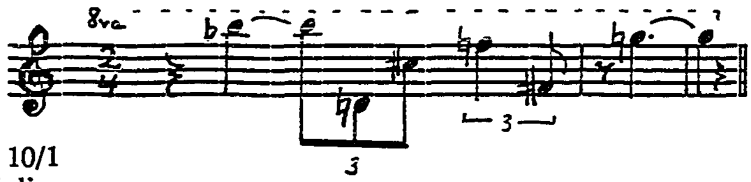
Morris defines contour space (c-space) as a type of musical space "consisting of elements arranged from low to high disregarding the exact intervals between the elements."[17] These elements are termed "c-pitches" ("cps") and are "numbered in order from low to high, beginning with 0 up to n-1," where n equals the cardinality of the segment, and where the "intervallic distance between the cps is ignored and left undefined."[18] The decision not to define the intervallic distance between c-pitches reflects a listener's ability to determine that one c-pitch is higher than, lower than, or equal to another, but not to quantify exactly how much higher or lower. In this respect, Morris's theory differs from that of Michael Friedmann. Many of the issues addressed in the latter part of Friedmann's article hinge upon the concept of contour intervals that measure the distance between c-pitches.[19] In this discussion, however, as in Morris, the intervallic distance between cps will remain undefined.

The definition of contour space adopted here also has crucial implications for evaluation of the research of music psychologists working in this area. Their definitions of contour deal exclusively with a simple determination of pitch direction between adjacencies. For example, Edworthy (following Dowling) defines contour as "the sequence of ups and downs in a melody independent of the precise interval sizes."[20] Dewitt and Chowder have a similar understanding of the term, defining contour as "the binary pattern of ups and downs of pitch direction...between any two adjacent pitches in the melody."[21] The primary difference between their definitions and the one to be used here is the postulation of a contour-space continuum extending from low to high. By numbering each contour pitch within this continuum, a precise measurement of its relative position to the others is possible, not only between adjacent cps but also between nonadjacencies. This possibility for comparison of relative pitch height is to become the basis upon which equivalence and similarity relations will be built.

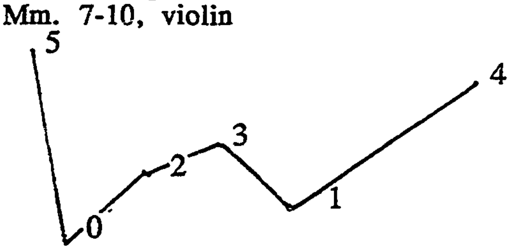
Because musical contours are ordered in time, a c-segment (or cseg) is defined as an ordered set of c-pitches in c-space.[22] Csegs will be labelled throughout by capital letters; the cps which make up csegs will be denoted by lower-case letters. Further, any ordered sub-grouping of a given cseg is defined as a c-subsegment (or csubseg). A csubseg may be comprised of either contiguous or non-contiguous c-pitches from the original cseg, as shown in Figure II-2. The contour diagrams used in

FIGURE II-2

C-SUBSEGMENTS



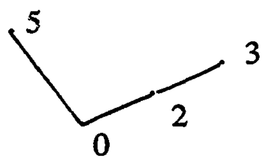
Webern, Op. 10/1
Mm. 7-10, violin



= <5 0 2 3 1 4>

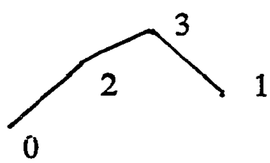
Selected csubsegs of cardinality 4:

A:



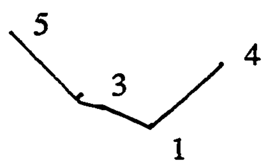
= <5 0 2 3> = <3 0 1 2>

B:



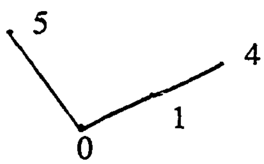
= <0 2 3 1>

C:



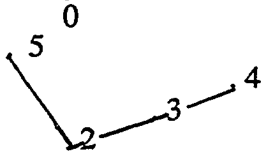
= <5 3 1 4> = <3 1 0 2>

D:



= <5 0 1 4> = <3 0 1 2>

E:



=<5 2 3 4> =<3 0 1 2>

this figure appear throughout as graphic representations of contour shape. There is experimental evidence that musicians and nonmusicians alike are able to draw such diagrams representing the shape of familiar melodies, although musicians are somewhat more accurate.[23] These diagrams make relationships among contours fairly easy to spot visually; thus, we see that csubsegs B and C are inversionally related, while A, D and E appear to be equivalent contours. More formal definitions of contour equivalence, the operation of inversion, and other relations among contours follow. Contours B and E illustrate segmentation of contour subsegments by register, or by the Gestalt concept of "proximity" in register,[24] with contour B isolating the four lowest c-pitches and contour E isolating the four highest. A number of experiments in auditory stream segregation support the perceptual validity of segmenting noncontiguous csubsegs such as contour E on the basis of register, thus cps in the higher register might form one perceptual "stream" and the lower-register cps a second one.[25] Noncontiguous csubsegs might also be associated by rhythm, timbre, or other musical factor.

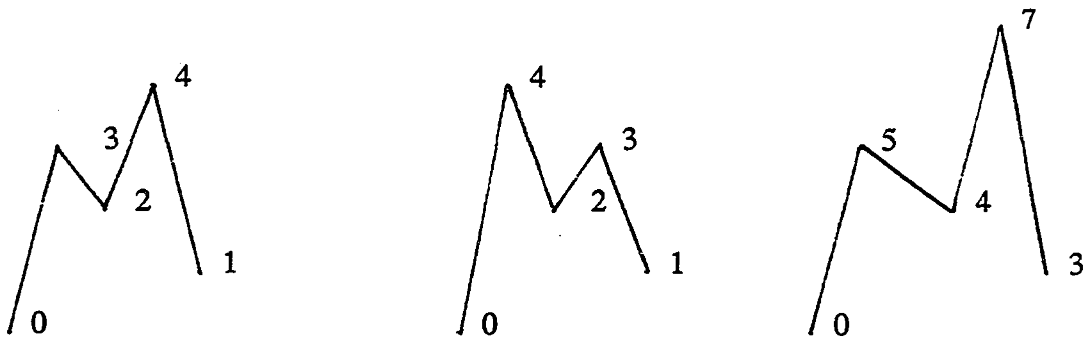
The uppermost contour in Figure II-2 is listed in "normal form," defined as an ordered array in which elements in a cseg of n distinct c-pitches are numbered from 0 to (n-1) and listed in temporal order. A csubseg's elements may retain the same numbers assigned to these cps in the original cseg, or may be "translated" to normal form. Translation is an operation through which a csubseg of n distinct c-pitches, not numbered in register from 0 to (n-1), is renumbered from 0 for the lowest c-pitch to (n-1) for the highest c-pitch in the csubseg, as illustrated in csubsegs A, C, D, and E of Figure II-2.[26]

Morris's comparison matrix (COM-matrix) will be used to compare contours in c-space, to define equivalence relations, and to develop similarity measurements for musical contours. The comparison matrix is a two-dimensional array that displays the results of the comparison function, COM(a,b), for any two c-pitches in c-space. If b is higher than a, the function returns "+1"; if b is the same as a, the function returns "0"; and if b is lower than a, COM(a,b) returns "-1."[27] The repeated instances of the integer "1" are omitted in the COM-matrix, as shown in Figure II-3. Each of these matrices has symmetrical properties in which the diagonal of zeros from the upper left-hand to lower right-hand corner (the "main" diagonal) forms an axis of symmetry. Each value in the upper right-hand triangle is mirrored on the other side of the main diagonal by its inverse. This symmetrical structure is a natural consequence of the fact that contour-pitch COM-matrices only compare a cseg with itself.

The comparison matrix provides a concise profile of a cseg's structure in much the same way as Friedmann's Contour Adjacency Series (CAS),[28] except that the COM-matrix furnishes a much more complete picture since it is not limited simply to relationships between adjacent contour pitches. Indeed, the CAS appears as a subset of the COM-matrix, as the first diagonal above and to the right of the main diagonal, as shown in Figure II-4A. Each of the diagonals to the right of the main diagonal is termed INT_n,[29] where n stands for the difference between order position numbers of the two cps compared; that is, INT_4 compares cps which are 4 positions apart. The term INT is used to be consistent with terminology for matrices in p- and pc-space, where integers appearing in each diagonal give information about a set's intervallic structure, including properties of invariance. Although intervals in pitch space have both magnitude and direction (e.g., the interval from d'' to e' is 10 semitones down, or "-10"), intervals in contour space have direction only. For this reason, the INT_n diagonals in c-space COM-matrices contain only pluses, minuses, or zeros, showing intervallic direction or equivalence. As Figure II-4B shows, INT_1 displays the results

FIGURE II-3

COMPARISON MATRICES



A = < 0 3 2 4 1 >   B = < 0 4 2 3 1 >   C = < 0 5 4 7 3 >*

	0	3	2	4	1
0	0	+	+	+	+
3	-	0	-	+	-
2	-	+	0	+	-
4	-	-	-	0	-
1	-	+	+	+	0

	0	4	2	3	1
0	0	+	+	+	+
4	-	0	-	-	-
2	-	+	0	+	-
3	-	+	-	0	-
1	-	+	+	+	0

	0	5	4	7	3
0	0	+	+	+	+
5	-	0	-	+	-
4	-	+	0	+	-
7	-	-	-	0	-
3	-	+	+	+	0

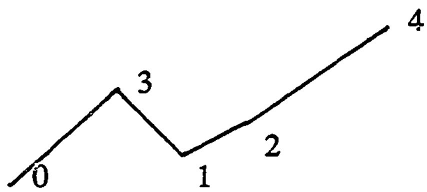
Csegs A and C are equivalent because they generate identical COM-matrices.

* Normal form of < 0 5 4 7 3 > = < 0 3 2 4 1 > by translation.

FIGURE II-4

STRUCTURE OF THE COM-MATRIX

FIGURE II-4A:



$$
\begin{array}{c|ccccc l}
 & 0 & 3 & 1 & 2 & 4 & \\
\hline
0 & 0 & + & + & + & + & \text{-- INT}_4 \\
3 & - & 0 & - & - & + & \text{-- INT}_3 \\
1 & - & + & 0 & + & + & \text{-- INT}_2 \\
2 & - & + & - & 0 & + & \text{-- INT}_1 \\
4 & - & - & - & - & 0 & \text{-- main diagonal}
\end{array}
$$

$INT_1 = <+ - + +>$ (= CAS) $INT_2 = <+ - +>$

$INT_3 = <+ +>$ $INT_4 = <+>$

FIGURE II-4B:

INT_1 -- <0 3 1 2 4>

\+ - + +

INT_2 -- <0 3 1 2 4>

\-

\+ +

INT_3 -- <0 3 1 2 4>

\+

\+

INT_4 -- <0 3 1 2 4>

\+

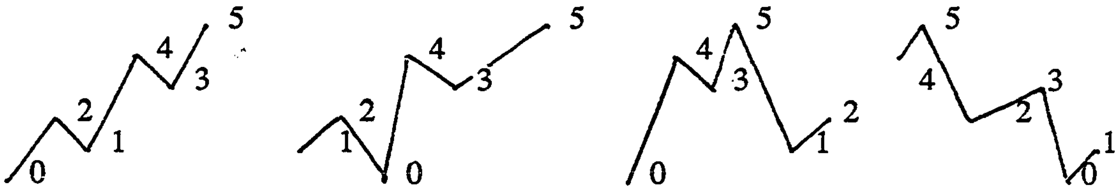
of the comparison function for each pair of adjacent cps in the cseg < 0 3 1 2 4 >: < + - + + > for the comparisons 0 to 3, 3 to 1, 1 to 2, and 2 to 4. INT$_2$ shows each comparison between a given c-pitch and a second cp twice removed from the first: < + - + > for 0 to 1, 3 to 2, and 1 to 4. Likewise, INT$_3$ displays each comparison between two cps three positions apart: < + + > for 0 to 2, and 3 to 4. Finally, INT$_4$ shows the comparison between two cps four positions apart. In this case, the predominance of pluses over minuses in each of the INTs illustrates the generally upward motion of this contour. The information provided by the COM-matrix gives a much more accurate profile of cseg structure than INT$_1$ alone, since c-pitches may be compared not only consecutively, but also non-consecutively with respect to relative height. By way of example, Figure II-5 contrasts several csegs which share an identical INT$_1$ but which differ a great deal with respect to their overall musical contours, a fact that is reflected in their respective comparison matrices.

Two contour equivalence classes based upon the comparison matrix have been proposed by Morris. The first of these is made up of all c-segments that share the same comparison matrix. Thus in Figure II-3 preceding, the first and third contours are equivalent csegs since they produce identical COM-matrices. Further, equivalent csegs may be reduced to the same normal order by the translation operation, as the figure shows. The design and results of experimentation on contour have been marred by the lack of a precise definition for equivalent contours. For example, Figure II-6A reproduces four of the non-tonal melodies used by Dowling and Fugitani in their 1971 experiment.[30] According to authors' definitions, melody A is an exact transposition of the target melody, B a "same-contour" imitation, and C a different contour comparison. Because their contours are defined only as the pattern of ups and downs between adjacent pitches, contour B is considered to be an exact-contour imitation of the target,

FIGURE II-5

COMPARISONS AMONG SELECTED CSEGS

WHERE INT_1 = < + - + - + >



A = < 0 2 1 4 3 5 >   B = < 1 2 0 4 3 5 >   C = < 0 4 3 5 1 2 >   D = < 4 5 2 3 0 1 >

	0	2	1	4	3	5
0	0	+	+	+	+	+
2	-	0	-	+	+	+
1	-	+	0	+	+	+
4	-	-	-	0	-	+
3	-	-	-	+	0	+
5	-	-	-	-	-	0

	1	2	0	4	3	5
1	0	+	-	+	+	+
2	-	0	-	+	+	+
0	+	+	0	+	+	+
4	-	-	-	0	-	+
3	-	-	-	+	0	+
5	-	-	-	-	-	0

	0	4	3	5	1	2
0	0	+	+	+	+	+
4	-	0	-	+	-	-
3	-	+	0	+	-	-
5	-	-	-	0	-	-
1	-	+	+	+	0	+
2	-	+	+	+	-	0

	4	5	2	3	0	1
4	0	+	-	-	-	-
5	-	0	-	-	-	-
2	+	+	0	+	-	-
3	+	+	-	0	-	-
0	+	+	+	+	0	+
1	+	+	+	+	-	0

Each contour has INT_1 = < + - + - + >.

As shown by contour graphs, contours A and B are most similar; A and D most dissimilar.

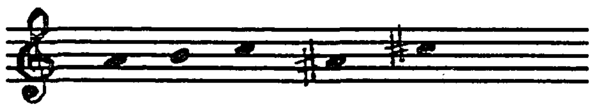
FIGURE II-6

MELODIES FROM DOWLING AND FUJITANI (1971)

FIGURE II-6A

< 0 2 3 1 4 > = < + + - + >

A:

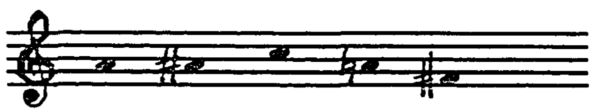


< 0 2 3 1 4 > = < + + - + >   exact transposition

B:

< 0 1 2 0 3 > = < + + - + >   "same-contour" imitation

C:



< 1 2 3 1 0 > = < + + - - >   different contour

FIGURE II-6B:

	0	2	3	1	4
0	0	+	+	+	+
2	-	0	+	-	+
3	-	-	0	-	+
1	-	+	+	0	+
4	-	-	-	-	0

A:

	0	2	3	1	4
0	0	+	+	+	+
2	-	0	+	-	+
3	-	-	0	-	+
1	-	+	+	0	+
4	-	-	-	-	0

B:

	0	1	2	0	3
0	0	+	+	0	+
1	-	0	+	-	+
2	-	-	0	-	+
0	0	+	+	0	+
3	-	-	-	-	0

C:

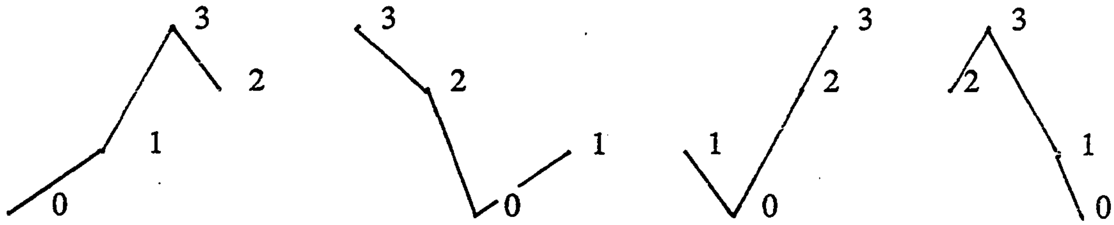
	1	2	3	1	0
1	0	+	+	0	-
2	-	0	+	-	-
3	-	-	0	-	-
1	0	+	+	0	-
0	+	+	+	+	0

since both share the pattern < + + - + >. Assuming the more precise measurement available in contour space, however, contours A and B contain different c-pitches, producing different normal orders and generating different matrices, as shown in Figure II-6B. The two contours are therefore nonequivalent. Certainly contours A and B will be perceived as similar by listeners, in part because of their INT_1 relationship. However, as mentioned above, not all INT_1-related contours contain such a high degree of similarity, as shown in Figure II-5. Thus a more precise definition of contour similarity, based upon contour equivalence as defined here, is needed.

Figure II-7 illustrates the second contour equivalence relation, the c-space segment class (or csegclass), an equivalance class made up of all csegs related by identity, translation, retrograde, inversion, and retrograde-inversion. Csegclasses (as distinct from csegs) are labelled throughout with underscored capital letters. Representatives of csegclass S, consisting of its prime form, < 0 1 3 2 >, together with its inversion, retrograde, and retrograde-inversion, and the COM-matrix for each, are given in this figure. As shown, the inversion of a cseg S comprised of n distinct cps is written IS, and may be found by subtracting each c-pitch from (n-1).[31] The retrograde of a cseg S (written RS) or its inversion (written RIS) may be found by listing the elements of the cseg S or IS in reverse order. The inversion, retrograde and retrograde-inversion of a contour S are also defined by Morris in terms of specific transformations of the COM-matrix for S.[32] The COM-matrix for IS, for example, merely exchanges each "+" from the S matrix for "-" in the IS matrix and likewise exchanges "-" for "+". The matrix for RIS is related in a somewhat more abstract manner, as though the S matrix had been "flipped" around the secondary diagonal (the diagonal proceeding from the lower left-hand corner to the upper right-hand corner). The COM-matrix for RS combines both the flip and the exchange features.

FIGURE II-7

C-SPACE SEGMENT CLASS < 0 1 3 2 >



P: < 0 1 3 2 >   I: < 3 2 0 1 >   RI: < 1 0 2 3 >   R: < 2 3 1 0 >

	0	1	3	2
0	0	+	+	+
1	-	0	+	+
3	-	-	0	-
2	-	-	+	0

	3	2	0	1
3	0	-	-	-
2	+	0	-	-
0	+	+	0	+
1	+	+	-	0

	1	0	2	3
1	0	-	+	+
0	+	0	+	+
2	-	-	0	+
3	-	-	-	0

	2	3	1	0
2	0	+	-	-
3	-	0	-	-
1	+	+	0	-
0	+	+	+	0

Inversion=

Exchange only

Retrograde =

Exchange & flip

Retrograde Inversion =

Flip only

Two csegs belonging to the same c-space segment class may be reduced to the same prime form, using an algorithm developed by Paul A. Laprade.[33] Simply expressed, the algorithm consists of three steps:

1) If necessary, translate the cseg so its content consists of integers from 0 to (n - 1),
2) If (n - 1) minus the last c-pitch is less than the first c-pitch, invert the cseg,
3) If the last c-pitch is less than the first c-pitch, retrograde the cseg.[34]

If for steps 2 and 3 the first and last cps are the same, compare the second and the second-to-last cps, and so on until the "tie" is broken. Figure II-8 illustrates the use of this algorithm for several csegs and shows that each is a member of the same csegclass. Appendix 1 gives a listing of Laprade's c-space segment classes for cardinalities 2 through 6 (larger csegs are excluded because of limitations of space). Contours in the following figures are occasionally labelled with csegclass names drawn from Laprade's table, with the first number of the label representing the cardinality of the csegclass, and the second number representing its ordinal position on the list. Thus, c5-12 represents the twelfth contour appearing on the list of five-note csegclasses.

The formulation of csegclasses reduces the possible number of csegs considerably, by assuming inversional, retrograde, and retrograde-inversional equivalence. There is, for example, only one possible cseg of cardinality 1: < 0 1 >. The other possibility, < 1 0 >, is an equivalent cseg by the operations retrograde and inversion. Similarly, only two possible csegclasses exist for trichords; all c-segments

FIGURE II-8

PRIME FORM ALGORITHM

Csegs:	<0 3 1 2>	<3 0 2 1>	<1 2 0 4>	<3 2 4 1>
1. **TRANSLATE**, if not consecutive integers 0 to (n - 1):	OK	OK	Translated = <1 2 0 3>	Translated = <2 1 3 0>
2. **INVERT**, if (n - 1) minus last cp < first cp	OK	Inverted = <0 3 1 2>	Inverted = <2 1 3 0>	OK
3. **RETROGRADE**, if last cp < first cp	OK	OK	Retrograde = <0 3 1 2>	Retrograde = <0 3 1 2>
PRIME FORM:	<0 3 1 2>	<0 3 1 2>	<0 3 1 2>	<0 3 1 2>

All four csegs belong to the same c-space segment class.

Operations:

To translate, renumber the cseg with consecutive integers from 0 to (n - 1), where n equals the cardinality of the cseg.

To invert, subtract each cp from (n - 1).

To retrograde, place the cps in reverse order.

will reduce either to < 0 1 2 > or to < 0 2 1 >. There are 8 possible csegclasses of cardinality 4, 32 of cardinality 5, and 161 of cardinality 6. A number of csegclasses have the property of RI-invariance--that is, the inverted form of the segment is the same as the retrograde form; similarly, the segment's RI-form is identical to its P-form. For example, the unidirectional ascending contour, c4-1 < 0 1 2 3 >, becomes < 3 2 1 0 > upon inversion or retrogression. Its RI-form is also < 0 1 2 3 >. Those csegclasses that are RI-invariant are marked with asterisks in Appendix 1.

To conclude the discussion of equivalence classes in contour space, it is important to address, once again, the question of aural perception. Is it possible to hear R-, I-, and RI- relations in contour space? What purpose is served by positing an equivalence class that includes these relations? The first of these questions may be answered by returning to the data of experimental psychology. In the first of two experiments designed to test recognition of melodic transformations such as inversion, retrograde, and retrograde-inversion, Dowling concludes

> ...that the melodic contour and the set of interval sizes in a melody are separable features or dimensions of the melodic pattern. Contour and interval size are handled in different and largely independent ways in cognitive processing. Recognition of exact repetitions and inversions seems to be mainly on the basis of contour.[35]

In the second of these experiments, Dowling extended this early study of inversionally-related diatonic melodies by using all three transformations upon short non-tonal melodies.[36] Subjects were asked to recognize R-, I-, and RI-transformations upon a number of melodies, and were warned not to accept transformations simply upon contour, but to prefer transformations that preserved exact interval sizes. In spite of this instruction, subjects tended to confuse exact-interval transformations with transformations upon contour that did not preserve interval size. Thus Dowling confirmed his previous conclusion that subjects made their recognition judgements based primarily upon contour information, and extended this result to retrograde and retrograde-inversion transformations. He summarized his results as follows:

> This study used a short-term recognition-memory paradigm and found that in the easier conditions all these transformations were recognized with better than chance accuracy. The ascending order of difficulty was: inversion, retrograde, retrograde-inversion. There was no evidence that listeners distinguish between transforms that preserve the exact interval content of the standard stimulus and those that merely preserve its contour.[37]

Although Dowling's conclusions raise a number of important questions,[38] the implications of his work for contour perception are clear: first, that transformations upon contour are perceived independently of transformations upon pitch or interval; and second, that no transformation is excluded from recognition, that subjects were able with better-than-chance accuracy to identify inversion, retrograde, and retrograde-inversion as transformations of an original non-tonal melody.[39] To return, finally, to the second question posed above, it is essential to posit R-, I-, and RI-equivalence to establish a finite collection of c-space segment classes of a manageable size. Once established, this collection may become a powerful analytical and compositional tool.

3. Similarity Relations for Musical Contours

A number of similarity measurements have been developed to describe relationships among unordered pitch-class sets. Generally, these fall into two categories: those based upon comparison of interval-class content, and those based upon inclusion relations. As David Lewin and John Rahn have noted,[40] the latter category can subsume the former, since the six interval classes and the six two-note set classes usually represent the same concept. The interval-class vector summarizes the number and types of two-note set classes included in a larger set class; thus similarity measurements based upon this vector also reflect an inclusion relation. The similarity relationships described by Allen Forte in The Structure of Atonal Music are of both types, but are limited to set classes of equal cardinality.[41] His R_0, R_1, and R_2 are relations based upon shared positions within the interval-class vector. His R_p, on the other hand, is an abstract pitch-inclusion relation between two set classes of equal cardinality, n, that share a common subset of cardinality (n - 1). Generalized and expanded versions of both interval-based and inclusion-based similarity measurements for unordered set classes of equal or differing cardinalities have appeared in the work of Eric Regener, Lewin, Morris, and Rahn.[42] The similarity measurements for musical contours proposed below are modelled, in part, upon their work.

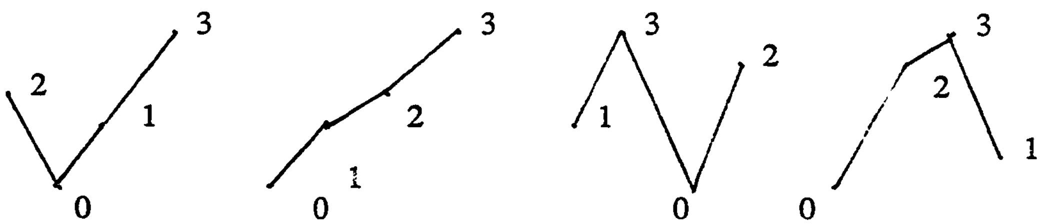
The similarity of two csegs or csegclasses in contour space may be measured either by comparing their structural profiles as summarized in the COM-matrix (an interval-based measurement),[43] or by examining their common csubseg structure (an inclusion-based measurement). The first of these will be called the contour similarity function (CSIM) and the second, the contour embedding function (CEMB). Both are designed to return a decimal number which approaches "1" as csegs become more similar. A function that returns the value "1" compares two equivalent csegs.[44]

The contour similarity function, CSIM(A, B), is illustrated in Figure II-9 and measures the degree of similarity between two csegs of the same cardinality. It compares specific positions in the upper right-hand triangle of the COM-matrix for cseg

FIGURE II-9

CSIM AS SIMILARITY MEASUREMENT

FOR CSEGS OF SAME CARDINALITY



A = <2 0 1 3>   B = <0 1 2 3>   C = <1 3 0 2>   D = <0 2 3 1>

$$\begin{array}{c|cccc} & 2 & 0 & 1 & 3 \\ \hline 2 & 0 & - & - & + \\ 0 & + & 0 & + & + \\ 1 & + & - & 0 & + \\ 3 & - & - & - & 0 \end{array} \qquad \begin{array}{c|cccc} & 0 & 1 & 2 & 3 \\ \hline 0 & 0 & + & + & + \\ 1 & - & 0 & + & + \\ 2 & - & - & 0 & + \\ 3 & - & - & - & 0 \end{array} \qquad \begin{array}{c|cccc} & 1 & 3 & 0 & 2 \\ \hline 1 & 0 & + & - & + \\ 3 & - & 0 & - & - \\ 0 & + & + & 0 & + \\ 2 & - & + & - & 0 \end{array} \qquad \begin{array}{c|cccc} & 0 & 2 & 3 & 1 \\ \hline 0 & 0 & + & + & + \\ 2 & - & 0 & + & - \\ 3 & - & - & 0 & - \\ 1 & - & + & + & 0 \end{array}$$

CSIM(A,B) = 4/6 = .67
CSIM(A,C) = 3/6 = .50
CSIM(A,D) = 2/6 = .33

A with the corresponding positions in the matrix of cseg B in order to total the number of similarities between them.[45] For each compared position of identical content, this total is incremented by 1. Such a similarity measurement, if it were simply to total the number of identical matrix positions, would not yield a uniform model of similarity among csegs of various cardinalities. That is, a similarity measurement of 3 between two three-note csegs would signify a much higher degree of similarity than would a similarity measurement of 3 between two seven-note csegs.[46] In order to create a more uniform measurement, the number of identical positions will be divided by the total number of positions compared;[47] thus CSIM(A,B) will return a decimal number which signifies greater similarity between csegs as this number approaches 1.

Although equivalent csegs will always return a value of "1," the values representing maximum and minimum similarity vary with the cardinality of the csegs. Figure II-10 summarizes the maxima and minima for csegs of cardinalities three through six. For example, the upper right-hand corners of the COM-matrices for two four-note csegs each contain six positions for comparison. Maximum similarity occurs when five of the six corresponding positions contain the same entry; thus CSIM(A,B) equals 5/6 or .83. When six of six positions contain the same entry the two segments are equivalent. Minimum similarity occurs when only one position is the same; thus CSIM(A,B) equals 1/6 or .17. If two segments have no positions in common, they are inversionally related. For two csegs of cardinality six, fifteen positions in each COM-matrix are compared; thus maximum similarity for two six-note csegs is 14/15 or .93, while minimum similarity is 1/15 or .07.

Reference back to the contour diagrams of Figure II-9 reveals that contours A and D are inversionally related. They are, in fact, RI-related and are members of the same csegclass, c4-4. CSIM(A,B) as yet accounts only for similarity between csegs,

FIGURE II-10

MAXIMA/MINIMA FOR CSIM(A,B) CARDINALITIES 3 TO 6

Cardinality:	3	4	5	6
Equivalence (Identity)	1	1	1	1
Maximum Similarity	.67	.83	.90	.93
Minimum Similarity	.33	.17	.10	.07
Inversionally-Related	0	0	0	0

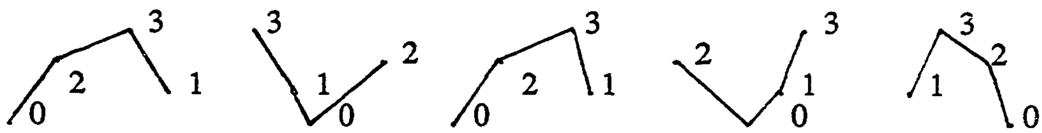
not csegclasses; thus, an extension of the similarity measurement is needed. The similarity function CSIM(A,B) compares the similarity between two csegclasses. CSIM(A,B) returns the largest decimal number, or 1, obtained by comparing the COM-matrix of one cseg representative of csegclass A with four cseg representatives (P, I, R and RI) of csegclass B. Therefore, CSIM(A, B) indicates the degree of highest possible similarity between two csegclasses.[48] If the two csegs are members of the same c-space segment class, CSIM(A,B) will return a value of "1". Maxima and minima for the CSIM function, as listed in Figure II-10, are the same whether comparing csegs or csegclasses. Figure II-11 offers two examples: if the csegs A < 0 2 3 1 > and B < 3 1 0 2 > are compared for similarity, CSIM(A,B) accurately reflects their total dissimilarity and inversional relationship with respect to contour (CSIM(A,B) = 0), but not the fact that these csegs belong to the same c-space segment class. CSIM(A, B), however, returns the value "1" since the two csegs are members of csegclass c4-4. In the second example of Figure II-11, csegs C and D are not members of the same csegclass; CSIM(C, D) returns the value .80.

One of the most intuitively satisfying ways of judging similarity in csegs of differing cardinalities is to count the number of times the smaller cseg is included in the larger.[49] Rahn has made a similar observation in his discussion of similarity relations among pitch-class sets. He illustrates his point by comparing two pairs of sets, each of which displays maximum similarity according to his generalization of the interval-based similarity index. He continues by showing that "set pairs yielding this constant maximum value for their sizes are yet of widely varying degress of 'intuitively' measured similarity, and that this 'intuitive' measure is related to embedding."[50] Rahn compares set A, [0, 1, 2] with two others: set B_1, [0, 1, 3, 4, 6, 8, 10], and set B_2, [0, 1, 2, 3, 4, 5, 6]. The function SIM(A,B_1) returns the value 18,[51] denoting

FIGURE II-11

CSIM FOR C-SPACE SEGMENT CLASSES

CSIM(A, B): A = <0 2 3 1> B = <3 1 0 2>



A PB IB RB RIB

	0	2	3	1
0	0	+	+	+
2	-	0	+	-
3	-	-	0	-
1	-	+	+	0

	3	1	0	2
3	0	-	-	-
1	+	0	-	+
0	+	+	0	+
2	+	-	-	0

	0	2	3	1
0	0	+	+	+
2	-	0	+	-
3	-	-	0	-
1	-	+	+	0

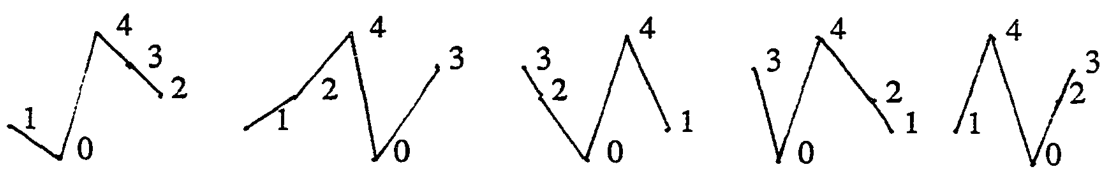
	2	0	1	3
2	0	-	-	+
0	+	0	+	+
1	+	-	0	+
3	-	-	-	0

	1	3	2	0
1	0	+	+	-
3	-	0	-	-
2	-	+	0	-
0	+	+	+	0

CSIM(A, PB) = 0/6 = 0
CSIM(A, IB) = 6/6 = 1
CSIM(A, RB) = 2/6 = .33
CSIM(A,RIB) = 4/6 = .67

CSIM(A, B) = 1

CSIM(C, D): C = <1 0 4 3 2> D = <1 2 4 0 3>



C PD ID RD RID

	1	0	4	3	2
1	0	-	+	+	+
0	+	0	+	+	+
4	-	-	0	-	-
3	-	-	+	0	-
2	-	-	+	+	0

	1	2	4	0	3
1	0	+	+	-	+
2	-	0	+	-	+
4	-	-	0	-	-
0	+	+	+	0	+
3	-	-	+	-	0

	3	2	0	4	1
3	0	-	-	+	-
2	+	0	-	+	-
0	+	+	0	+	+
4	-	-	-	0	-
1	+	+	-	+	0

	3	0	4	2	1
3	0	-	+	-	-
0	+	0	+	+	+
4	-	-	0	-	-
2	+	-	+	0	-
1	+	-	+	+	0

	1	4	0	2	3
1	0	+	-	+	+
4	-	0	-	-	-
0	+	+	0	+	+
2	-	+	-	0	+
3	-	+	-	-	0

CSIM(C, PD) = 6/10 = .60
CSIM(C, ID) = 4/10 = .40
CSIM(C, RD) = 8/10 = .80
CSIM(C, RID) = 2/10 = .20

CSIM(C, D) = .80

maximum similarity; likewise SIM(A,B_2) returns 18. In spite of that fact that the two pairs return an equivalent similarity value based upon interval-class inclusion, he concludes that it seems "utterly unreasonable" to assert that the latter pair (A,B_2) is not more similar than the former, since in the latter A is embedded five times in B while in the former A is embedded zero times.[52]

Inclusion relations among musical contours may be determined in one of two ways: either by examining the two COM-matrices to determine the number of times the smaller cseg's COM-matrix is embedded in the COM-matrix of the larger cseg, or by looking at all possible csubsegs within the larger cseg and determining by translation how many are equivalent to the smaller cseg. The contour embedding function (CEMB(A,B)) is designed to count the number of times cseg A is embedded in cseg B. This total then is divided by the number of csubsegs of the same cardinality as A possible, in order to return a value that approaches 1 for csegs of greater similarity. The formula for determining the number of m-sized subsets of an n-sized set is:[53]

$$\frac{n!}{m!\,(n-m)!}.$$

Because the order of c-pitches is crucial in determining csubsegs, all embedding functions count embedded csegs not csegclasses (thus, < 2 0 1 > differs from < 1 0 2 >).

Figure II-12 illustrates two rather dissimilar csegs of unequal cardinality: CEMB(A,B) = 2/20 = .10. Cseg c3-1 < 0 1 2 > is embedded only twice in cseg c6-96 < 4 5 2 3 6 1 >, as the contiguous csubseg < 2 3 6 > and the noncontiguous < 4 5 6 >. In Figure II-12B, the complete matrix of cseg < 0 1 2 > is found as a contiguous subset of the large cseg's matrix, while Figure II-12C shows the matrix of < 0 1 2 > embedded as a noncontiguous subset. The c-pitches associated with each

FIGURE II-12

CEMB(A,B)

A = < 0 1 2 > = c3-1
B = < 4 5 2 3 6 1 > = c6-96

Fig. II-12A - MATRIX OF c3-1:

$$\begin{array}{c|ccc} & 0 & 1 & 2 \\ \hline 0 & 0 & + & + \\ 1 & - & 0 & + \\ 2 & - & - & 0 \end{array}$$

<0 1 2>

Fig. II-12B - MATRIX OF c3-1 EMBEDDED AS CONTIGUOUS SUBSET OF c6-96:

$$\begin{array}{c|cccccc} & 4 & 5 & 2 & 3 & 6 & 1 \\ \hline 4 & 0 & + & - & - & + & - \\ 5 & - & 0 & - & - & + & - \\ 2 & + & + & 0 & + & + & - \\ 3 & + & + & - & 0 & + & - \\ 6 & - & - & - & - & 0 & - \\ 1 & + & + & + & + & + & 0 \end{array}$$

<2 3 6>=<0 1 2>

Fig. II-12C - MATRIX OF c3-1 EMBEDDED AS NON-CONTIGUOUS SUBSET OF c6-

$$\begin{array}{c|cccccc} & 4 & 5 & 2 & 3 & 6 & 1 \\ \hline 4 & 0 & + & - & - & + & - \\ 5 & - & 0 & - & - & + & - \\ 2 & + & + & 0 & + & + & - \\ 3 & + & + & - & 0 & + & - \\ 6 & - & - & - & - & 0 & - \\ 1 & + & + & + & + & + & 0 \end{array}$$

<4 5 6>=<0 1 2>

Fig.II-12D - UPPER RIGHT-HAND TRIANGLE:

$$\begin{array}{c|ccc} & 0 & 1 & 2 \\ \hline 0 & 0 & + & + \\ 1 & - & 0 & + \\ 2 & - & - & 0 \end{array}$$

$$\begin{array}{c|cccccc} & 4 & 5 & 2 & 3 & 6 & 1 \\ \hline 4 & \boxed{0} & + & - & - & + & - \\ 5 & - & 0 & - & - & + & - \\ 2 & \boxed{+} & \boxed{+} & 0 & + & + & - \\ 3 & + & + & - & 0 & + & - \\ 6 & \boxed{-} & \boxed{-} & - & - & \boxed{0} & - \\ 1 & + & + & + & + & + & 0 \end{array}$$

$$\begin{array}{c|cccccc} & 4 & 5 & 2 & 3 & 6 & 1 \\ \hline 4 & \boxed{0} & + & - & - & + & - \\ 5 & - & 0 & - & - & + & - \\ 2 & + & + & 0 & + & + & - \\ 3 & \boxed{+} & \boxed{+} & - & 0 & + & - \\ 6 & - & - & - & - & 0 & - \\ 1 & \boxed{+} & \boxed{+} & + & + & \boxed{+} & 0 \end{array}$$

position of the embedded matrix are members of the csubseg < 0 1 2 >. Note that in the noncontiguous instance, the entire structure of each embedded row and column must remain intact in order to reflect the csubseg relation accurately. It is for this reason that CEMB(A, B) must consider the structure of the entire embedded COM-matrix as a whole rather than the upper right-hand triangle alone. In Figure II-12D, the pluses of the upper right-hand triangle of the smaller cseg's matrix have been circled in non-adjacent positions of the larger cseg's matrix. If the rows and columns are not violated, the corresponding matrix entries for the main diagonal and lower left-hand triangle (indicated in the figure by squares) are incorrect for the embedded subset. Thus, the information given in the upper right-hand triangle is not sufficient to identify csubsegments.

Since the embedding function checks for non-contiguous subsets as well as contiguous ones, it accounts for such instances as a contour which we perceive as generally rising, even though it also includes some descents. In Figure II-13A, for example, the embedded csubseg < 0 1 2 > appears repeatedly, both as a non-contiguous and a contiguous subset of < 0 2 1 3 4 >, and its role in our perceiving this contour as an ascending line is clearly audible. As the comparison matrix and corresponding contour diagrams show, < 0 1 2 > is embedded seven times in the larger cseg. CEMB(A,B) can also be found by extracting all three-note csubsegs from the larger cseg, translating each to normal form, and counting the number of times < 0 1 2 > is found, as shown in Figure II-13B.

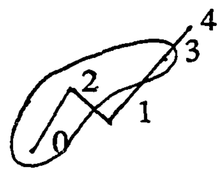
Although the CSIM and CEMB functions provide an adequate measure of similarity between most csegs (of equal or unequal cardinality), they are not alone sufficient to describe relationships between any two csegs. For example, our embedding function only describes relationships between csegs of differing

FIGURE II-13A

CEMB(A,B) - ADDITIONAL EXAMPLES

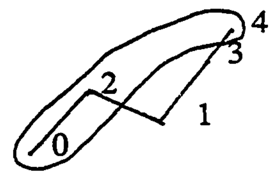
Matrix embedding: A = < 0 1 2 >   B = < 0 2 1 3 4 >

	0	1	2
0	0	+	+
1	-	0	+
2	-	-	0



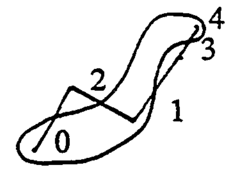
	0	2	1	3	4
0	0	+	+	+	+
2	-	0	-	+	+
1	-	+	0	+	+
3	-	-	-	0	+
4	-	-	-	-	0

csubsegs: < 0 2 3 >



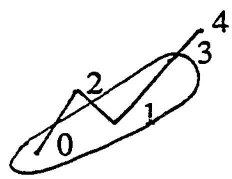
	0	2	1	3	4
0	0	+	+	+	+
2	-	0	-	+	+
1	-	+	0	+	+
3	-	-	-	0	+
4	-	-	-	-	0

< 0 2 4 >



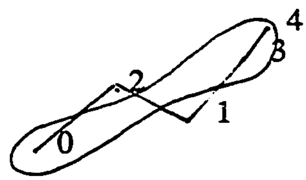
	0	2	1	3	4
0	0	+	+	+	+
2	-	0	-	+	+
1	-	+	0	+	+
3	-	-	-	0	+
4	-	-	-	-	0

< 0 1 4 >



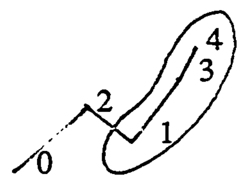
	0	2	1	3	4
0	0	+	+	+	+
2	-	0	-	+	+
1	-	+	0	+	+
3	-	-	-	0	+
4	-	-	-	-	0

csubsegs: < 0 1 3 >



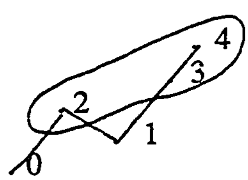
	0	2	1	3	4
0	0	+	+	+	+
2	-	0	-	+	+
1	-	+	0	+	+
3	-	-	-	0	+
4	-	-	-	-	0

< 0 3 4 >



	0	2	1	3	4
0	0	+	+	+	+
2	-	0	-	+	+
1	-	+	0	+	+
3	-	-	-	0	+
4	-	-	-	-	0

< 1 3 4 >



	0	2	1	3	4
0	0	+	+	+	+
2	-	0	-	+	+
1	-	+	0	+	+
3	-	-	-	0	+
4	-	-	-	-	0

csubseg: < 2 3 4 >

CEMB(A,B) - 7/10 = .70

FIGURE II-13B

EMBEDDED CSUBSEGS BY TRANSLATION

A=<0 1 2> B=<0 2 1 3 4>

Possible csubsegs are:
<0 2 1>=<0 2 1>
<0 2 3>=<0 1 2>*
<0 2 4>=<0 1 2>*
<0 1 3>=<0 1 2>*
<0 1 4>=<0 1 2>*
<0 3 4>=<0 1 2>*
<2 1 3>=<1 0 2>
<2 1 4>=<1 0 2>
<2 3 4>=<0 1 2>*
<1 3 4>=<0 1 2>*

* ~~Embedded < 0~~ 1 2 > identified by translation.

CEMB(A, B) = 7/10 = .70.

cardinalities. What of the situation in which two csegs of equal cardinality share one or more common csegs? Following Rahn's generalization of Lewin's embedding function,[54] two additional functions are proposed to count the csubsegs mutually embedded in csegs A and B. For both functions, csegs A and B may be of equal or unequal cardinality. CMEMB_n(X,A,B) counts the number of times the csegs, X (of cardinality n), are embedded in both csegs A and B. The variable "X" may successively represent more than one cseg-type during the course of the function, as shown in Figure II-14. Each cseg X must be embedded at least once in both A and B; then, all instances of X are counted in both A and in B. The total number of mutually-embedded csegs of cardinality n is divided by the number of n-cardinality csubsegs possible in both csegs to return a decimal number approaching 1 as the csegs A and B are more similar. Generally, CMEMB_n(X,A,B) returns a higher decimal number for embedded csegs of smaller cardinality since there are fewer cseg types, and therefore a higher probability of inclusion in both csegs A and B. In particular, CMEMB_2(X,A,B) provides a weak representation of cseg similarity, as shown in Figure II-14, since only two cseg representatives exist: < 0 1 > and < 1 0 >. Thus a refinement of the function, introduced below as the adjusted mutual-embedding (ACMEMB) function, is necessary.

The CMEMB_n(X,A,B) function does, however, accurately model the degree of similarity between contours such as those given in Figure II-15. Although they differ in cardinality, contours A and B of this figure share identical patterns of contour reversals. That is, their figural diagrams share the same direction and number of "corners," although the number of intervening c-pitches between corners in contour B is greater than in contour A. These two contours are maximally similar with respect to the contour embedding function CMEMB_n(X,A,B) for values of n ranging from 2 to

FIGURE II-14

CMEMB$_n$(X, A, B)

Cseg A = c5-26: < 1 0 4 3 2 >	**Cseg B = c5-24: < 2 0 1 4 3 >**
Csubsegs of A:	**Csubsegs of B:**
<10432>=<10432>	<20143>=<20143>
<1043>=<1032>	<2014>=<2013>
<1432>=<0321>	<2013>=<2013>
<1032>=<1032>	<2043>=<1032>
<1042>=<1032>	<0143>=<0132>
<0432>=<0321>	<2143>=<1032>

CMEMB$_4$(X, A, B) = 5/10= .50

<104>=<102>	<201>=<201>
<103>=<102>	<204>=<102>
<102>=<102>	<203>=<102>
<143>=<021>	<214>=<102>
<142>=<021>	<213>=<102>
<132>=<021>	<243>=<021>
<043>=<021>	<014>=<012>
<042>=<021>	<013>=<012>
<432>=<210>	<043>=<021>
<032>=<021>	<143>=<021>

CMEMB$_3$(X, A, B) = 16/20 = .80

<10>=<10>	<20>=<10>
<14>=<01>	<21>=<10>
<13>=<01>	<24>=<01>
<12>=<01>	<23>=<01>
<04>=<01>	<01>=<01>
<03>=<01>	<04>=<01>
<02>=<01>	<03>=<01>
<43>=<10>	<14>=<01>
<42>=<10>	<03>=<01>
<32>=<10>	<43>=<10>

CMEMB$_2$(X, A, B) = 20/20 = 1

Common csubsegs are underlined.

(m-1) where m represents the cardinality of the smaller cseg, which contains no extra cps between corners. Experimentation has demonstrated the perceptual salience of contour reversals. M. C. Dyson and A. J. Watkins, in their 1984 study,[55] noted that while previous experimentation had shown memory for corners of visual shapes to be important in recognition, the analogous situation for aural shapes had not previously been tested. Their work departs from most previous contour experimentation in that it breaks musical contours down into component parts. Their results "support the idea that corners of melodic contours act as features and are perceptually more salient than the intervening notes."[56] Thus the fact that contours A and B of Figure II-15 are alike with respect to their corners explains, in part, their aural similarity, and $\mathrm{CMEMB}_n(X,A,B)$ reflects this maximum degree of similarity by returning a value of "1."

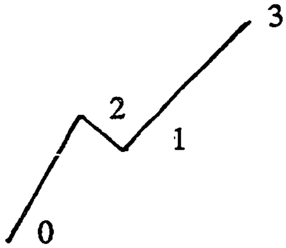
ACMEMB(A,B) counts the total number of significant mutually-embedded csegs of cardinality 2 through the cardinality of the smaller cseg, and adjusts this to a decimal value by dividing by the total number of possible subsets of A and B (excluding the null sets for each and the one-note csubsegs).[57] Figure II-16A shows the adjusted mutual embedding function for two csegs of the same cardinality, and II-16B for csegs of differing cardinalities.[58]

Finally, the embedding functions may be generalized abstractly for csegclasses in much the same manner as was the CSIM function. That is, CEMB($\underline{A}$, $\underline{B}$), $\mathrm{CMEMB}_n(X, \underline{A}, \underline{B})$ and ACMEMB($\underline{A}$, $\underline{B}$) will compare the csubseg content of cseg A with each of the four transforms of cseg B (PB, IB, RB and RIB) and return the highest of these values. Thus, if A and B are members of the same csegclass, each of these functions will return a value of "1."

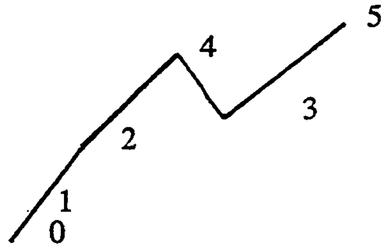
FIGURE II-15

MAXIMUM SIMILARITY $CMEMB_n(X,A,B)$

Contour A:



Contour B:



Contour A: <0213>

2-note csubsegs:

<02>=<01>
<01>=<01>
<03>=<01>
<21>=<10>
<23>=<01>
<13>=<01>

Contour B: <012435>

2-note csubsegs:

<01>=<01>	<15>=<01>
<02>=<01>	<24>=<01>
<04>=<01>	<23>=<01>
<03>=<01>	<25>=<01>
<05>=<01>	<43>=<10>
<12>=<01>	<45>=<01>
<14>=<01>	<35>=<01>
<13>=<01>	

$CMEMB_2(X,A,B)$ = 21/21 = 1

Contour A, 3-note csubsegs:

<021>=<021>
<023>=<012>
<013>=<012>
<213>=<102>

Contour B, 3-note csubsegs:

<012>=<012>	<124>=<012>
<014>=<012>	<123>=<012>
<013>=<012>	<125>=<012>
<015>=<012>	<143>=<021>
<024>=<012>	<145>=<012>
<023>=<012>	<135>=<012>
<025>=<012>	<243>=<021>
<043>=<021>	<245>=<012>
<045>=<012>	<235>=<012>
<035>=<012>	<435>=<102>

$CMEMB_3(X,A,B)$ = 24/24 = 1

FIGURE II-16

ACMEMB(A,B) FOR CSEGS OF EQUAL CARDINALITY

FIGURE II-16A: CSEGS OF EQUAL CARDINALITY

A = < 0123 > B = < 0213 >

Csubsegs of A:	<01>=<01>	Csubsegs of B:	<02>=<01>
	<02>=<01>		<01>=<01>
	<03>=<01>		<03>=<01>
	<12>=<01>		<23>=<01>
	<23>=<01>		<13>=<01>
	<13>=<01>		<21>=<10>
	<012>=<012>		<021>=<021>
	<013>=<012>		<023>=<012>
	<023>=<012>		<013>=<012>
	<123>=<012>		<213>=<102>
	<0123>=<0123>		<0213>=<0213>

17 csegs mutually embedded in both csegs; **ACMEMB(A, B) = 17/22 = .77**

FIGURE II-16B: CSEGS OF UNEQUAL CARDINALITY

C = < 02134 >

Csubsegs of C:	<0214>=<0213>	<021>=<021>	<02>=<01>
	<0234>=<0123>	<023>=<012>	<01>=<01>
	<0134>=<0123>	<024>=<012>	<03>=<01>
	<0213>=<0213>	<013>=<012>	<04>=<01>
	<2134>=<1023>	<014>=<012>	<23>=<01>
		<213>=<102>	<24>=<01>
		<214>=<102>	<13>=<01>
	<02134>=<02134>	<234>=<012>	<14>=<01>
		<134>=<012>	<34>=<01>
		<034>=<012>	<21>=<10>

29 csegs mutually embedded in csegs A and C; **ACMEMB(A, C)=29/37=.78**
33 csegs mutually embedded in csegs B and C; **ACMEMB(B, C)=33/37=.89**

4. Extensions of the Theory for Context-Dependent Analysis

Up to this point, relations among contours have been considered without reference to the musical contexts in which these contours appear. The application of contour theory to context-dependent analysis poses a number of problems, not the least of which is the segmentation of the music into meaningful units. Friedmann has discussed segmentation in some detail; his examples provide considerable insight into this difficult problem.[59] A second context-dependent issue with considerable theoretical ramifications is the common occurrence of repeated notes within a musical contour. Consecutive repeated notes pose no problem, since they may be treated as single contour pitches, as shown in Figure II-17A. Csegs containing nonconsecutive repeated c-pitches may be numbered in order from low to high with 0 representing the lowest pitch and (n - 1 - r) the highest; repetitions of a c-pitch will be represented by the same integer. Here the variable "n" stands for the cardinality of the cseg, while "r" equals the number of times any c-pitch is repeated. Thus, the contour of the melody in Figure II-17B is < 1 2 3 0 3 1 >. The cardinality of the cseg is 6, cp 1 is repeated once and cp 3 is repeated once; thus the cps are numbered from 0 to 3, since (n - 1 - r) equals (6 - 1 - 2) or 3. Translation of a cseg including repeated notes is defined as the renumbering of the cseg with integers ranging from 0 to (n - 1 - r). The inversion of a repeated-note cseg is calculated by subtracting each cp from (n - 1 - r). Previously stated definitions of R and RI still hold, as does the prime form algorithm, although "ties" may occur more frequently (if for steps 2 and 3 of the algorithm the first and last

FIGURE II-17A

REPEATED-NOTE SEGMENTS

CONSECUTIVE REPEATED NOTES

Webern, Op. 10/1
Mm. 10-11, harp

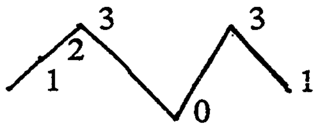
<0 5 3 2 4 1> NOT <0 5 3 3 2 4 1>

FIGURE II-17B

REPEATED-NOTE SEGMENTS

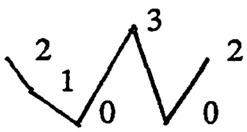
NON-CONSECUTIVE REPEATED NOTES

Webern, Op. 10/1
Mm. 3-6, clarinet

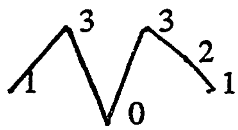


P = < 1 2 3 0 3 1 >

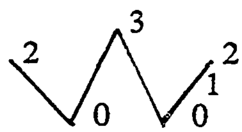
	1	2	3	0	3	1
1	0	+	+	-	+	0
2	-	0	+	-	+	-
3	-	-	0	-	0	-
0	+	+	+	0	+	+
3	-	-	0	-	0	-
1	0	+	+	-	+	0



IP = < 2 1 0 3 0 2 >*



RP = < 1 3 0 3 2 1>



RIP = < 2 0 3 0 1 2 >

* To invert, each cp is subtracted from (n - 1 - r), where n represents the cardinality of the cseg and r is the number of times a particular cp is repeated. In this instance, r = 2, since cp 1 is repeated once and cp 3 is repeated once.

cps are the same, the second and the second-to-last cps are compared, and so on until the "tie" is broken).

Since contours containing consecutive repeated notes may also generate matrices, the matrices may be used as a point of departure for identifying and comparing such contours. The matrices for repeated-note contours are identical to those previously discussed, except that they contain zeros in positions other than along the main diagonal. Comparison of the repeated-note matrices with those of csegclasses appearing on the list of prime forms reveals that each repeated-note contour is maximally similar to two other contours--those which contain pluses where the repeated-note contour has zeros, and those which contain minuses where the repeated-note contour has zeros. Thus, any repeated-note contour may be represented by a hyphenated composite label that identifies the two related nonrepeated-note contours, as shown in Figure II-18. The cardinality of the cseg appears to the left of the hyphen. To the right of the hyphen, separated by slashes, are the ordinal numbers of the two related csegclasses. The first ordinal number represents the csegclass label of some cseg whose COM-matrix is identical to that of the repeated-note cseg except that it contains a plus in the place of each 0 in the upper right-hand triangle. The second ordinal number represents the cseg which contains a minus in each of those positions. In Figure II-18A, csegclasses c5-2 and c5-4 differ from the repeated-note cseg in only one position each; the composite label is rc5-2/4 ("rc" stands for "repeated-note csegclass").[60] Two repeated notes will result in two zeros in the upper right-hand triangle, as shown in Figure II-18B, and so on. Because the COM-matrix for each of the csegclasses represented in the composite label differs from that of the repeated-note cseg precisely in those positions where an extra "0" appears , the COM-matrix of the repeated-note cseg (and therefore the normal form of the cseg itself) may be generated

FIGURE II-18

CSEGCLASS LABELS FOR REPEATED-NOTE CSEGS

FIGURE II-18A: CSEG WITH ONE REPETITION

A = < 0 1 2 3 2 >

$$\begin{array}{c|ccccc|}
 & 0 & 1 & 2 & 3 & 2 \\ \hline
0 & 0 & + & + & + & + \\
1 & - & 0 & + & + & + \\
2 & - & - & 0 & + & \boxed{0} \\
3 & - & - & - & 0 & - \\
2 & - & - & 0 & + & 0
\end{array}$$

Csegclass label = 5-?

Related Matrices:

B = c5-2: < 0 1 2 4 3 >

$$\begin{array}{c|ccccc|}
 & 0 & 1 & 2 & 4 & 3 \\ \hline
0 & 0 & + & + & + & + \\
1 & - & 0 & + & + & + \\
2 & - & - & 0 & + & \boxed{+} \\
4 & - & - & - & 0 & - \\
3 & - & - & - & + & 0
\end{array}$$

C = c5-4: < 0 1 3 4 2 >

$$\begin{array}{c|ccccc|}
 & 0 & 1 & 3 & 4 & 2 \\ \hline
0 & 0 & + & + & + & + \\
1 & - & 0 & + & + & + \\
3 & - & - & 0 & + & \boxed{-} \\
4 & - & - & - & 0 & - \\
2 & - & - & + & + & 0
\end{array}$$

Therefore: A = **rc5-2/4.**

CSIM(A,B) = CSIM(A,C) = CSIM(B,C).

FIGURE II-18

CSEGCLASS LABELS FOR REPEATED-NOTE CSEGS

FIGURE II-18B: CSEG WITH TWO REPETITIONS

D = < 1 2 3 0 3 1 >

$$
\begin{array}{c|cccccc|}
 & 1 & 2 & 3 & 0 & 3 & 1 \\
\hline
1 & 0 & + & + & - & + & \boxed{0} \\
2 & - & 0 & + & - & + & - \\
3 & - & - & 0 & - & \boxed{0} & - \\
0 & + & + & + & 0 & + & + \\
3 & - & - & 0 & - & 0 & - \\
1 & 0 & + & + & - & + & 0
\end{array}
$$

Csegclass label = 6-?

Related matrices:

E = c6-145: < 1 3 4 0 5 2 >

$$
\begin{array}{c|cccccc|}
 & 1 & 3 & 4 & 0 & 5 & 2 \\
\hline
1 & 0 & + & + & - & + & \boxed{+} \\
3 & - & 0 & + & - & + & - \\
4 & - & - & 0 & - & \boxed{+} & - \\
0 & + & + & + & 0 & + & + \\
5 & - & - & - & - & 0 & - \\
2 & - & + & + & - & + & 0
\end{array}
$$

F = c6-154: < 2 3 5 0 4 1 >

$$
\begin{array}{c|cccccc|}
 & 2 & 3 & 5 & 0 & 4 & 1 \\
\hline
2 & 0 & + & + & - & + & \boxed{-} \\
3 & - & 0 & + & - & + & - \\
5 & - & - & 0 & - & \boxed{-} & - \\
0 & + & + & + & 0 & + & + \\
4 & - & - & + & - & 0 & - \\
1 & + & + & + & - & + & 0
\end{array}
$$

Therefore: **D = rc6-145/154.**

CSIM(D,E) = CSIM(D,F) = CSIM(E,F).

from the composite label of the repeated-note cseg merely by comparing the matrices of the two csegclasses in its name and placing zeros in those positions that differ. Finally, the similarity and embedding functions[61] still hold for repeated-note csegs, as for nonrepeated-note csegs.

Unlike equivalence relations, similarity relations are in principle non-transitive. That is, a similarity measurement that obtains between csegs A and B, and also between B and C, will not necessarily obtain between A and C. However, the CSIM relationship among a repeated-note cseg and the two csegs represented in its composite label is transitive. Thus a cseg with one repetition, resulting in one extra zero-position in its matrix, forms a transitive-triple[62] with the two csegs of its composite label, as shown in Figure II-18A. This is due to the fact that the CSIM relation counts the number of identical matrix positions between csegs, and in the case of these so-related csegs all matrix positions will be identical except the positions that contained a zero in the repeated-note contour. Figure II-18B illustrates a contour with two repeated c-pitches. The two resulting zeros in the upper right-hand triangle may be replaced with < +, + >, < -, - >, < +, - >, or < -, + >. These four related matrices, along with the repeated-note matrix, represent a transitive-quintuple among the csegs shown in Figure II-19. Generally, a cseg containing n repeated notes forms a transitive tuple of cardinality $(2^n + 1)$, where "1" represents the repeated-note cseg and "2^n" represents the number of related csegs. Thus a cseg with three repeated notes forms a transitive 9-tuple with related csegs, a contour with four repeated notes forms a transitive 17-tuple, and so on.

FIGURE II-19

TRANSITIVE QUINTUPLE AMONG REPEATED-NOTE CSEGS

CSEG WITH TWO REPETITIONS:

< 1 2 3 0 3 1 > = rc6-145/154

	1	2	3	0	3	1
1	0	+	+	-	+	$\boxed{0}$
2	-	0	+	-	+	-
3	-	-	0	-	$\boxed{0}$	-
0	+	+	+	0	+	+
3	-	-	0	-	0	-
1	0	+	+	-	+	0

RELATED CSEGS:

< 1 3 4 0 5 2 > = c6-145

	1	3	4	0	5	2
1	0	+	+	-	+	$\boxed{+}$
3	-	0	+	-	+	-
4	-	-	0	-	$\boxed{+}$	-
0	+	+	+	0	+	+
5	-	-	-	-	0	-
2	-	+	+	-	+	0

< 2 3 5 0 4 1 > = c6-154

	2	3	5	0	4	1
2	0	+	+	-	+	$\boxed{-}$
3	-	0	+	-	+	-
5	-	-	0	-	$\boxed{-}$	-
0	+	+	+	0	+	+
4	-	-	+	-	0	-
1	+	+	+	-	+	0

< 2 3 4 0 5 1 > = c6-166

	2	3	4	0	5	1
2	0	+	+	-	+	$\boxed{-}$
3	-	0	+	-	+	-
4	-	-	0	-	$\boxed{+}$	-
0	+	+	+	0	+	+
5	-	-	-	-	0	-
1	+	+	+	-	+	0

< 1 3 5 0 4 2 > = c6-148

	1	3	5	0	4	2
1	0	+	+	-	+	$\boxed{+}$
3	-	0	+	-	+	-
5	-	-	0	-	$\boxed{-}$	-
0	+	+	+	0	+	+
4	-	-	+	-	0	-
2	-	+	+	-	+	0

CSIM(A,B) AMONG ANY TWO OF THE FIVE CSEGS RETURNS 26/30 = .87.

5. Analytical Applications

Because the perception of contour is more immediate than of pitch or pc, it seems likely that contour perception, on some level, plays a role in how listeners might differentiate among melodies, associate thematic relationships, determine formal schemes, and so on, as they attend to a non-tonal composition. Analytical applications of contour theory therefore model some aspects of the listener's musical perception that pitch-class analysis alone does not address. Michael Friedmann cites the classics of the 12-tone literature as being ripe for the application of contour analysis, since "the series of interval classes in time is invariant, and contour therefore becomes an important source of transformation and motion." Indeed, Arnold Schoenberg, in his essay on composition with twelve tones, repeatedly stresses the idea that "the possibilities of evolving the formal elements of music--melodies, themes, phrases, motives, figures, and chords--out of a basic set are unlimited."[63] By varying contour and rhythm but retaining the order of the series with its mirror (inversion) and retrograde forms, the composer draws varied thematic material from the basic set. Schoenberg makes clear in his examples that the "mirror" form need not necessarily mirror the contour of the original set. For example, he cites the Rondo movement of his Wind Quintet, Opus 27 (reproduced as Figure II-20) and notes that

> ...the principal theme of the Rondo of this Quintet, shows a new way of varying the repetitions of a theme.... While rhythm and phrasing significantly preserve the character of the theme so that it can easily be recognized, the tones and intervals are changed through a different use of BS [basic set] and mirror forms.[64]

FIGURE II-20

ARNOLD SCHOENBERG, <u>WIND QUINTET</u>, "RONDO"

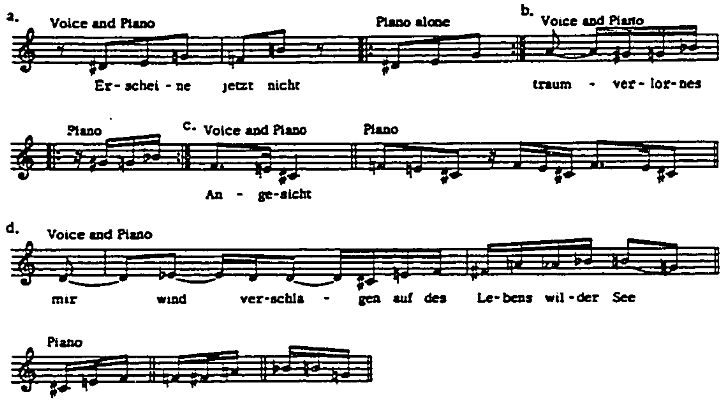
Here, Schoenberg combines the basic set with its pc-retrograde and retrograde inversion, using not the retrograde and retrograde-inversion contour, but instead a contour that is highly similar to that of the basic set, and a rhythm that is identical. With this and numerous other examples, he shows that the ordered set of pitch-classes and its` musical realization in contour space do not necessarily undergo the same transformations; thus a retrograde row need not employ a retrograde contour, etc.

In his analysis of the Four Orchestral Songs, Opus 22,[65] Schoenberg illustrates his discussion of motivic development with a series of examples that show musical ideas associated by their membership in the same c-space segment class, but which may or may not belong to the same set class. A few of his examples are reproduced here as Figure II-21. Schoenberg's example No. 18, drawn from the first of the songs, "Seraphita," shows recurrences of a three-note motive belonging for the most part to c3-1. Note that the sets C#-E-F and F-F#-A, given in the last line of No. 18, are pc-inversionally related but have identical contours. In No. 36, Schoenberg cites motivic statements in which the intervallic content, and hence the set class, has been altered but the contour retained. His Nos. 36 and 37 show a motivic connection with the second song, "Alle, welche dich suchen," where the recurring idea--marked with his "x's" in the first case--is comprised of different intervals, but the same contour. The composer summarizes his thoughts as follows:

> It can be pronounced a law of music that it is possible to recognize (i.e., to perceive) not only the regular rearrangements of musical shapes, but, given favorable conditions, irregular ones as well, provided only that enough will remain constant, once the intervals have been exchanged. The effect will therefore be of repeating or of making a variant....[66]

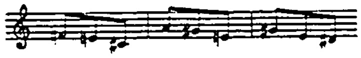
FIGURE II-21

EXAMPLES FROM SCHOENBERG'S ANALYSIS OF THE FOUR ORCHESTRAL SONGS, OPUS 22

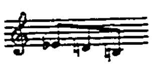


No. 18

No. 36a (Piano and Voice)



No. 36b (Piano alone)



No. 37a (Piano)

No. 37b

In the analytical applications that follow, two cases may be discerned: contour information either directly reinforces perception of set-class or row-class relations or it associates musical ideas that are not members of the same collection, thus enriching the fabric of association among musical ideas. An example of contour relations reinforcing other structural relationships may be found in the first movement of Alban Berg's Lyric Suite. The movement's primary row, P, appears most frequently at T_5, as < 5 4 0 9 7 2 8 1 3 6 10 11 >. It is an all-interval row, and pc-retrograde invariant. That is, all retrograde row forms are identical to an associated prime form at T_6, resulting in only 24 possible row forms rather than 48. This is due to the structure of the two hexachords as shown in Figure II-22; the second is the retrograde of the first, transposed by six semitones. This may also be seen as "nested" tritones; that is, the interval between the first and last pc is a tritone, likewise between the second and penultimate, and so on. Berg's initial musical realization of the row reinforces the retrograde pc-relationship between hexachords by use of retrograde-related contours. The first violin melody in measures 2-4 comprises a complete statement of T_5P, overlapping with the beginning of a T_{11}IP statement.[67] If the melody is segmented into two halves by the climactic high A-flat, the halves form contours < 1 0 3 2 5 4 6 > and its retrograde (excluding cp 6) < 4 5 2 3 0 1 >, thus making the retrograde relationship among pitch-classes in each hexachord of the row clear aurally.[68] In addition, the derived subsidiary row S,[69] illustrated here as T_{10}S < 10 3 8 1 6 11 4 9 2 7 0 5 >, displays invariance under the operation R(T_3I), since the first hexachord inverted and transposed by three semitones produces the pitch classes of the second in retrograde order. Again, the contour of Berg's first complete linear row presentation

FIGURE II-22

PROPERTIES OF ROW INVARIANCE REFLECTED IN MELODIC CONTOURS

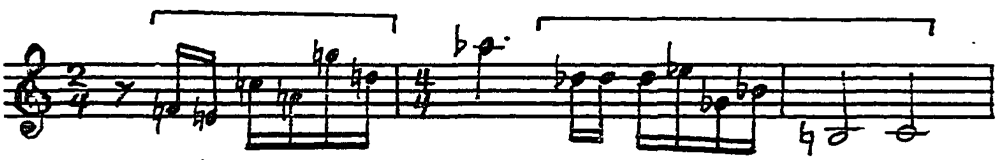
OF BERG, <u>LYRIC SUITE</u>

ROW P:

T_5P = < 5 4 0 9 7 2 8 1 3 6 10 11 >

$H_2P = R(T_6)H_1P$ Retrograde of H_1 = < 2 7 9 0 4 5 >
Transposed by T_6 = < 8 1 3 6 10 11 >

Row T_5P (overlapping with $T_{11}IP$)
Mm. 2-4 (violin I):



contour: < 1 0 3 2 5 4 6 > followed by < 4 5 2 3 0 1 > (retrograde related)

ROW S:

$T_{10}S$ = < 10 3 8 1 6 11 4 9 2 7 0 5 >

$H_2P = R(T_3I)H_1P$ Retrograde of H_1 = < 11 6 1 8 3 10 >
Transformed by T_3I = < 4 9 2 7 0 5 >

Row $T_{10}S$
Mm. 7-9 (cello):

contour: < 0 1 2 3 4 5 > followed by < 5 4 3 2 1 0 > (retrograde related) alludes to this relationship between hexachords, since the contour of the second hexachord is an exact retrograde (or inversion) of the first, as shown in Figure II-22.

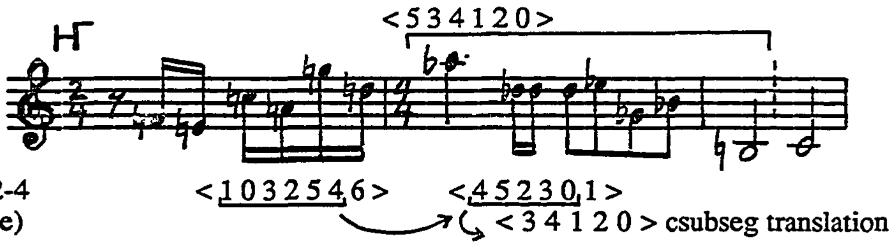
Berg also uses contour and rhythmic similarity to associate phrases drawn from different rows or row forms. Figure II-23 gives six prominant melodies from the Lyric Suite first movement; all but melody E are marked "Hauptstimme" in the score. Melodies A through D are interpretations of the P row, while melodies E and F are members of row-class S. Melodies A and B are associated both by rhythm and by contour. Although not equivalent, the contours of the ascending and descending portions of the two melodies are highly similar. By segmenting at the conclusion of the climactic high note and including only the first 12 pcs of each, the rows divide as 7 + 5. The two seven-cp segments, < 1 0 3 2 5 4 6 > and < 0 1 2 3 5 4 6 >, return a CSIM measurement of .90, while the two five-cp segments, < 3 4 1 2 0 > and < 4 3 2 1 0 >, return .80. Even more striking, however, is the relation of melody C to the opening melody. The rhythmic context of this melody is rather dissimilar to melody A. Yet the contour of each hexachord in C may be derived directly from A. The last six cps of A form contour c6-117, < 4 5 2 3 0 1 > (belonging to the same c-space segment class as melody A's first six cps), as do the first six cps of melody C. Further, the last six cps of melody C, contour < 0 2 1 4 3 5 >, may also be derived from melody A via a different segmentation. Melody A, as mentioned above, contains a complete statement of the twelve pitches of row P (T_5P), plus one pitch drawn from another row form. If this "extra" pitch is excluded from consideration, the contour of the last six cps is < 5 3 4 1 2 0 >, the retrograde of the second hexachord's contour in melody C. Thus despite their rhythmic differences, melodies A and C are closely associated, since

FIGURE II-23

CONTOUR RELATIONS AMONG SELECTED LINEAR ROW PRESENTATIONS

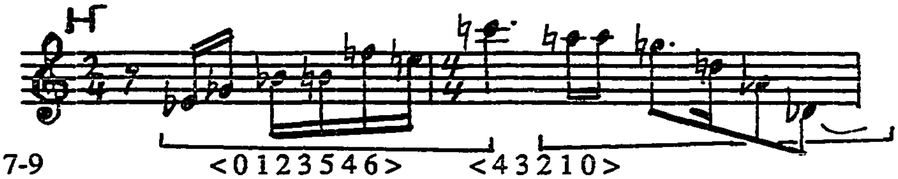
IN BERG, LYRIC SUITE

MELODY A:



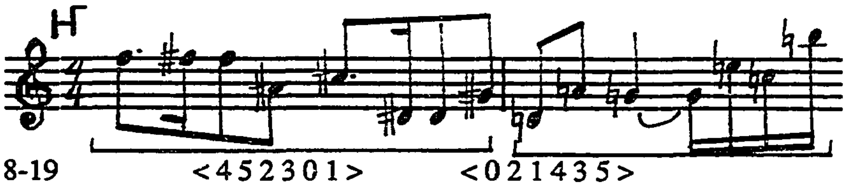
Vln. I, mm. 2-4
T_5P (plus one)

MELODY B:



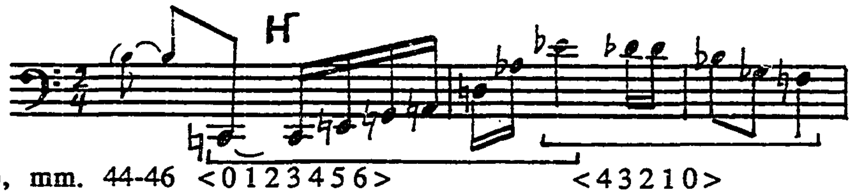
Vln. I, mm. 7-9
r_8T_5P

MELODY C:



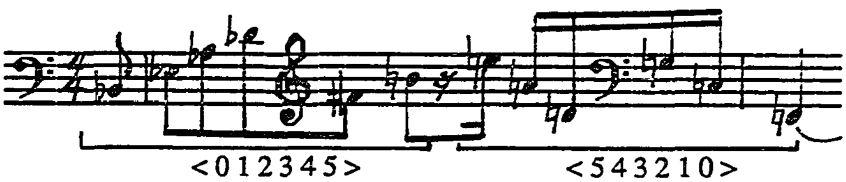
Vln. I, mm. 18-19
T_5IP

MELODY D:



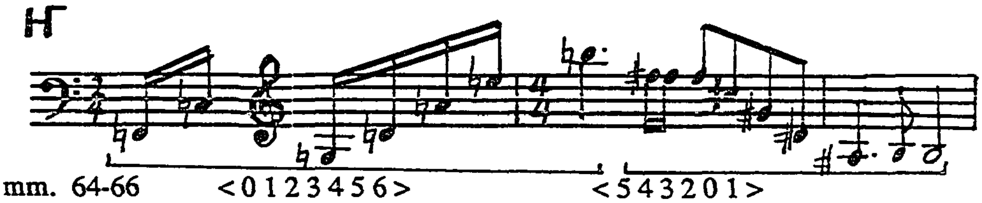
Vc. (canonic), mm. 44-46
$T_{11}IP$

MELODY E:



Vc., mm. 7-9
$T_{10}S$

MELODY F:



Vc. & vln., mm. 64-66
$RT_{10}S$

each hexachord of the latter melody belongs to a c-space segment class previously heard in the former.

Melodies D, E, and F contain contours belonging to the same c-space contour class. Contours c5-1, c6-1, and c7-1, all unidirectional, are heard prominently. The use of these distinctive c-space segments associates rows from two different row classes. D is a statement of the prime row, while both E and F are forms of the subsidiary row. The pcs of these latter two melodies are retrograde-related, while the contours that associate them are almost identical. Although drawn from a different row class, melody F resembles melody A in several ways. Its rhythm is strikingly similar to the rhythms of the first two melodies. Further, F consists of a complete twelve-tone row plus one note drawn from another row form, as did melody A. Segmentation of the final six cps, including the "extra" note, reveals yet another association with a previous melody. This latter contour, < 5 4 3 2 0 1 >, belongs to the same segment class as the first hexachord of melody B, and is related by inversion. The descending contour of the second hexachord in B, may also be linked aurally with the rising contour of F's first hexachord. Melody F appears in the coda of the movement and forms an obvious aural association with the opening melodies by virtue of instrumentation, rhythm, and contour. Yet its intervallic structure differs markedly from melody A, since A is an all-interval row and F, on the other hand, maximizes interval-class 5.

The preceding examples from Berg's Lyric Suite have shown instances in which contour relations either illuminated aspects of serial pitch structure, or associated musical ideas that differed with respect to row structure. Bela Bartok's "Diminished Fifth," from Mikrokosmos (Vol. IV),[70] provides a clear example of contour equivalence mirroring set-class equivalence. Figure II-24 gives a formal diagram of the

FIGURE II-24

FORMAL DIVISIONS IN BARTOK, "DIMINISHED FIFTH"

Figure II-24A: Formal diagram

		A			B	A'
	a	a'	b	a''	c	a'''
Mm:	1-5	6-11	12-19	20-25	26-34	35-44
Scale:	#1	#1	#2	#1	#3 ---- #2	#1

Octatonic scale types:

Octatonic #1

Octatonic #2

Octatonic #3

FIGURE II-24

FORMAL DIVISIONS IN BARTOK, "DIMINISHED FIFTH"

Figure II-24B: "Head motives" of each section

movement, along with the "head motive" of each section and its representation in c-pitches. A modified ternary form, the work's initial formal section also displays a ternary design. The A section extends from measures 1 to 25, B from 26 to 34, and A' from 35 to 44. The A section comprises four smaller divisions: a (mm. 1-5), a' (mm. 6-11), b (mm. 12-19), and a" (mm. 20-25). Every formal division is heralded by a change in octatonic scale and melodic contour, as shown in the figure, with the exception of the recurring "a" sections which share a single scale type and c-space segment class.

The initial phrase of the work, measures 1-3, contains the first instances of the contour segment class that is to become a primary formal determinant, recurring prominently at the onset of three other sections. The movement opens with statements of set class 4-10 simultaneously in the right and left hands, as shown in Figure II-24B. The repeated-note contour in the right hand, < 0 1 2 3 0 > or rc5-9/22, recurs in the left hand in retrograde order as < 0 3 2 1 0 >, thus reinforcing the set-class association. The left hand's retrograde statement is followed immediately by repetition of the < 0 1 2 3 0 > contour at the section's close. The a' section opens with a swapping of pitch material from left- to right-hand, and vice versa, preserving set-class and contour segment-class content. Segmentation by phrase markings reveals that while the right hand presents the < 0 1 2 3 0 > contour once again, the left hand presents a slightly longer repeated-note melody, which still forms set-class 4-10 but contour < 3 0 1 2 3 0 >. Interestingly, this contour contains two overlapping statements of rc5-9/22; the contour of the first five cps is the retrograde-inversion of the contour formed by the last five cps.

The a" section features a distinctive subset of the opening contour's retrograde: < 3 2 1 0 >.[71] Each of the four repetitions of this contour belongs to set class 4-10 except the first, which belongs to 4-13. These two set classes are fairly similar by most standard measurements; they are in both R_2 and R_p of Forte's similarity relations, and return a decimal value of .83 in Rahn's generalization (ak(A,B)) of Morris's similarity index.[72] The similarity of intervallic structure and identity of contour presentation between the two give the section a motivic cohesiveness in spite of the change in set class. The final section (a''' in measures 35 and following) represents a return to the pitch content of the work's opening. Although the right hand begins with the expected < 0 1 2 3 0 > contour, the left hand is a varied reprise and instead states the contour < 3 2 1 3 0 >, rc5-8/17, before its statement of < 0 1 2 3 0 >.

Contrast is created in the b and c sections, as distinct from the recurring a sections, partly by their differing contours. For example, the c section's opening pitch and rhythmic material is directly analogous to that of the a section, with statements of set class 4-10 in both right and left hands. The contour of these statements differs from the initial presentations of rc5-9/22, however. Here four-note csegs are featured, including c4-3, < 0 2 1 3 >, in the right hand and its retrograde (or inversion), < 3 1 2 0 >, in the left. In measures 30-31, the music shifts from octatonic #3 to #2, but the featured contour of the section remains invariant. Thus each of the short melodic phrases from m. 30-35 begins with c4-3, as < 3 1 2 0 >. The b section is the only one that features trichordal pitch collections rather than tetrachordal; thus the recurring set class in both hands is 3-7, a subset of 4-10. It also differs from most previous sections in that the initial contour used, rc5-12/18 or < 0 1 0 2 0 >, contains more than one repeated note. The contour of the left hand, < 2 1 2 0 > or c4-5/3, belongs to the same c-space segment class as the first four cps of the right, and is related by inversion. Further, contour < 2 1 2 0 > is maximally similar, as measured by the CSIM relation, to the < 3 1 2 0 > stated repeatedly in the left hand of the c section (CSIM = 5/6 = .83). This fact is reflected in their csegclass labels: c4-5/3 for < 2 1 2 0 > and c4-3 for < 3 1 2 0 >.

In summary, contour plays an important role in segmenting this brief work into distinct formal units. In addition, the use of retrograde- or inversionally-related contours between right and left hands at the beginning of each section reinforces the identity of set classes between hands. Given a highly unified octatonic pitch structure including repetition of only a small number of set classes, contour serves to associate the recurring a sections and to differentiate them from the contrasting b and c sections.

Finally, an analytical application in which set-class relations and contour relations provide differing, complementary musical associations is illustrated in Figure II-25, in the first of Anton Webern's Fünf Stücke für Orchester, Opus 10. The movement divides into four two- and four-bar phrases -- A (mm. 1-2), B (mm. 3-6), C (mm. 7-10), and D (mm. 10-11) -- plus a concluding 1-bar "codetta" of a single reiterated pitch. The two central phrases are joined in an antecedent-consequent relationship. Both consist of a broad solo line played in the upper register over a sustained celesta trill. Both melodies have substantial accompaniments: a series of chords beneath the antecedent phrase, and a thicker, more contrapuntal accompaniment to the consequent. Flanking this central portion on either side are opening and closing sections of sparser texture, consisting of solo lines without accompaniment. The first and last bars of the movement feature striking instances of Klangfarbenmelodie, while the second and penultimate bars consist of unaccompanied solo lines on distinctive, coloristic instruments. Thus the opening and closing sections frame the central portion in a symmetrical arrangement.

Each of the four principal melodies forms a melodic contour of cardinality six. Yet in each case the six cps are partitioned differently in terms of rhythm, register,

FIGURE II-25

PRIMARY MELODIC CONTOURS IN WEBERN, OP. 10/1

A		B	C	D	"codetta"
3 + 3		4 + 2	5 + 1	6 (+ 0)	1
<u>Klang-farben</u>	solo Glock.	solo clarinet "antecedent"	solo violin "consequent"	solo harp	<u>Klang-farben</u>
solo texture		chordal accompaniment	contrapuntal accompaniment	solo texture	

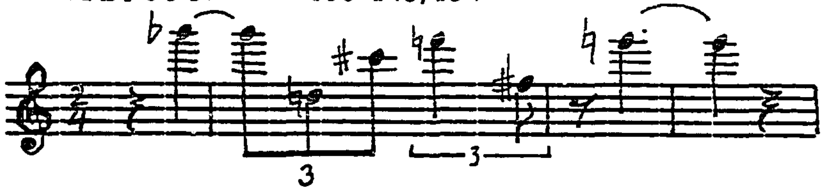
Contour A:

Mm. 1-2 Harp et. al. A=<0 1 0 4 3 2> rc6-29/133

Contour B:

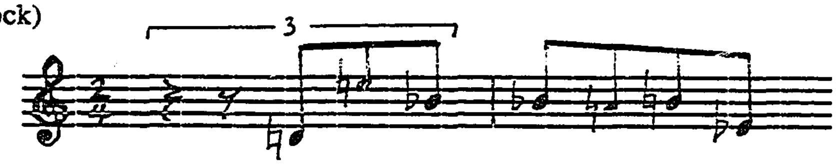
Mm. 3-6 Clarinet B=<1 2 3 0 3 1> rc6-145/154

Contour C:



Mm. 7-10 Violin (& Glock) C=<5 0 2 3 1 4> c6-104

Contour D:



Mm. 10-11 Harp D=<0 5 3 2 4 1> c6-104

and/or timbre: the first as 3 | 3, the second as 4 | 2, and the third as 5 | 1. The final melody is uninterrupted by rests and has no change in instrumentation; thus it forms a 6 | 0 partition. Comparison of set-class membership reveals that no pair of melodies belongs to the same set class. In fact, since two are repeated-note csegs, the cardinalities of the pitch-class sets differ; the first is a pentachord, the second a tetrachord, and the last two, hexachords. Although the two hexachords' do not belong to the same set class (C = 6-Z44, D = 6-Z6), they are members of the same c-space segment class, c6-104. Contour D immediately follows C musically, and is its contour inversion. Further, the ordering of cps in c6-104 produces a successive pattern of preserved adjacencies between the inversionally-related contours:[73]

C = < 5 0 2 3 1 4 >

D = < 0 5 3 2 4 1 >.

The relationship between successive contours is, for the most part, one of high dissimilarity: CSIM(A,B) and CSIM(B,C) equal .27 and CSIM(C,D) equals 0. On the other hand, connections between the opening melodies and the concluding one are much stronger (CSIM(A,D) = .53 and CSIM(B,D) = .60). Thus the third melody, at the highpoint of the movement, has the contour most dissimilar to those which precede and follow it, a contour which sets it apart from the others (CSIM(A,C) = .40, CSIM(B,C) =.27 and CSIM(C,D) = 0).

All four of the primary melodies are related by their csubseg structure. Each has c4-6 embedded at least once as four successive cps, often prominently positioned. Yet in no case do these successive pitches belong to the same set class, despite their membership in the same csegclass. For example, contour A ends with < 0 4 3 2 > (or, by translation, < 0 3 2 1 >), and is immediately followed by its retrograde in the first four cps of contour B, < 1 2 3 0 >. This segmentation into fours is aurally suggested by the isolation of these tetrachords by rests on either side. Like contour B, contours C and D begin with c4-6 as the first four cps. Contour C begins with < 5 0 2 3 >, which is the inversion of the original csubseg as stated in A (by translation < 5 0 2 3 > becomes < 3 0 1 2 >, and by inversion, < 0 3 2 1 >). Contour D's initial tetrachord is a return to < 0 3 2 1 > as it initially appeared. Finally, csegclass c4-6 appears embedded as noncontiguous csubsegs in contours A, C, and D as well. It occurs a total of three times in A and 5 times in D, and is in fact the only four-note csubseg these two contours share (CMEMB4(X, A, D) = 8/30 = .27). Contour C also contains five embedded statements of c4-6, but in the inverted form.

Secondary melodic material (of cardinality four or greater) is shown in Figure II-26 as contours E through H. In contour F, c4-6 appears again in exactly the same form as in contour C, the melody which it accompanies. Thus the contours of the violin and cello lines (mm. 7-8) form a heterophonic texture of overlapping statements of c4-6 in close temporal proximity, as shown in Figure II-27. Contour heterophony occurs only at this highpoint of the piece, where the contrapuntal texture is most complex. In every other case, the csegclass of the accompanying line is not an embedded csubseg of the melody it accompanies; thus the distinction between melody and accompaniment is clear.

Since only two possible csegclasses exist for c-segments of cardinality three, occasional instances of recurring three-note csubsegs may be of relatively trivial analytical importance. The distinctive repeated-note csubseg rc3-2/2, < 0 1 0 >, occurs with enough frequency throughout the movement to warrant discussion, however. This "neighbor note" motive opens the movement with its vivid Klangfarben scoring. Its inverted form is embedded repeatedly in contour B which follows, as the contiguous cps < 3 0 3 > and as the noncontiguous csubsegs < 1 2 1 > , < 1 3 1 > (twice), and

FIGURE II-26

SECONDARY MELODIC MATERIAL: WEBERN, OP. 10/1

Contour E:

Mm. 4-6
Flute & Cello

E=<2013> c4-4

Contour F:

Mm. 7-8
Cello

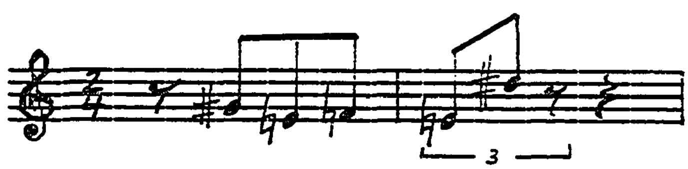
F=<3012> c4-6

Contour G:

Mm. 6-7
Trumpet

G=<0213> c4-3

Contour H:

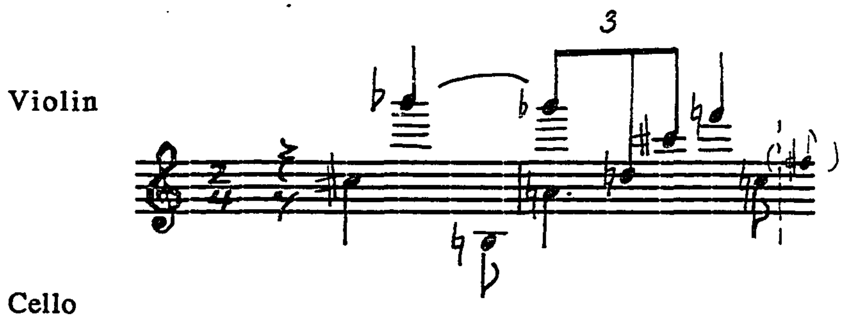


Mm. 8-9
Flute

H=<20103> c5-14/20

FIGURE II-27

CONTOUR HETEROPHONY: WEBERN, Op. 10/1, MM. 7-8



Both contours are c4-6, < 3 0 1 2 >.

< 1 0 1 >. Further, it occurs as the central three consecutive cps of contour H. Most striking, however, is its prolonged statement over the course of measures 3 through 10 -- first in the extended trill (which in itself contains repeated instances of < 0 1 0 >) and then in the continuation of this line in the trumpet/harp of m. 9 and celesta/cello of m. 10. This extended < 0 1 0 > clearly refers back to the opening gesture, even with respect to its instrumentation.

Finally, if music theorists model analytical theories to reflect aural perceptions, then a theory that describes relationships among musical contours is certainly overdue. The theory detailed above defines equivalence and similarity relations for contours in contour space. The analytical applications that follow briefly illustrate how specific contour relationships may be used to shape a formal scheme, to differentiate melody from accompaniment, to associate musical ideas which belong to different set classes, and to create unity through varied repetition.

NOTES

[1]Portions of this chapter appear in "Relating Musical Contours: Extensions of a Theory for Contour," Journal of Music Theory 31/2 (1987), pp. 225-267, which I wrote in collaboration with Paul A. Laprade. I am particularly indebted to Laprade for his development of a prime form algorithm and the computer-generated table of contour-space segment classes to be used in my analyses. A copy of Laprade's table is given in Appendix I.

[2]A bi-dimensional model for pitch, distinguishing pitch (or pitch height) from pitch class (called pitch quality or chroma) has existed in the psychological literature since the mid-nineteenth century. Christian Ruckmick ("A New Classification of Tonal Qualities," Psychological Review 36/2, 1929, p. 172), for example, cites a M. W. Drobisch article from 1846 ("Uber die mathematische Bestimmung der musikalischen") as the earliest attempt to depict pitch perception as a helical model. This model shows the close perceptual proximity of octaves as distinct from rising pitch height by the vertical alignment of octave-related pitches within each turn of the helix.

[3] In recent years, several psychologists have posited models for pitch perception on the basis of experimentation, among them Diana Deutsch, Carol Krumhansl and Roger N. Shepard. Shepard's multi-dimensional model for pitch is a double helix wrapped around a helical cylinder, where ascent represents pitch height with octave-related chroma aligned vertically while a downward projection of each pitch produces a circle of fifths model. Further, a vertical plane passing through the double helix model divides those tones which are diatonic to a given key from those which are not. See Shepard's "Structural Representations of Musical Pitch," in Diana Deutsch, ed., The Psychology of Music (NY: Academic Press, 1982, pp. 343-390) for an overview of representational models for pitch perception. Shepard notes elsewhere, however, that certain aspects of pitch perception differ markedly among listeners depending upon their musical backgrounds. In experiments undertaken jointly with Krumhansl in 1979, Shepard discovered that musical listeners perceived octave-related pitches as functionally equivalent, whereas subjects with less musical experience did not perceive such an equivalence. See his "Individual Differences in the Perception of Musical Pitch," in Documentary Report of the Ann Arbor Symposium (Reston, VA: Music Educators National Conference, 1981), pp. 152-174, for further details of this phenomenon. For purposes of this article, we will therefore assume experienced musical listeners in discussions relating to perceptual issues.

[4]Diana Deutsch, "Octave Generalization and Tune Recognition," Perception & Psychophysics 11/6 (1972), pp. 411-412.

[5]See her recent discussion of the two-channel theory in "Octave Equivalence and the Immediate Recall of Pitch Sequences," in Music Perception 2/1 (1984), pp. 40-51, co-authored with Richard C. Boulanger.

[6]W. J. Dowling and Ava W. Hollombe, "The Perception of Melodies Distorted By Splitting Into Several Octaves: Effects of Increasing Proximity and Melodic Contour," Perception & Psychophysics 21/1 (1977), pp. 60-64, and W. L. Idson and D. W. Massaro, "A Bidimensional Model of Pitch in the Recognition of Melodies," Perception & Psychophysics 24 (1978), pp. 551-565,

[7]See Diana Deutsch, "The Processing of Pitch Combinations," Chapter 9 in Deutsch (Ed.), The Psychology of Music (pp. 277-289) for an overview of experiments on recognition of melodies distorted by octave displacement or by alteration of interval size. See also B. White's "Recognition of Distored Melodies," American Journal of Psychology 73 (1960), pp. 100-107 and W. J. Dowling and D. S. Fujitani, "Contour, Interval, and Pitch Recognition in Memory for Melodies," The Journal of the Acoustical Society of America 49/2 (1971), pp. 524-531.

[8]W. J. Dowling, "Mental Structures Through Which Music is Perceived" in Documentary Report of the Ann Arbor Symposium (Reston, VA: Music Educator's National Conference, 1981), pp. 147.

[9]W. J. Dowling, "Scale and Contour: Two Components of a Theory of Memory for Melodies" in Psychological Review 85 (1978), pp. 341-354.

[10]Judy Edworthy, "Interval and Contour in Melody Processing," Music Perception 2/3 (1985), pp. 375-388.

[11]See, for example, Helen Brown and David Butler, "Diatonic Trichords as Minimal Tonal Cue-Cells," In Theory Only 5/6-7 (1981), pp. 39-55; Helen Brown, "Tonal Hierarchies and Perceptual Context: An Experimental Study of Music Behavior," Psychomusicology 7/1 (1987), pp. 77-90, and "The Interplay of Set Content and Temporal Context in a Functional Theory of Tonality Perception," Music Perception 5/3 (1988), pp. 219-250; and Edwin Hantz, "The Perception of Tonality: A Study in Perceptual Contexts," unpublished paper presented at the Annual Meeting of the Semiotic Society of America (Pensacola, FL: 1987).

[12]Ibid., p. 375.

[13]W. J. Dowling and D. S. Fugitani, "Contour, Interval, and Pitch Recognition in Memory for Melodies," Journal of the Acoustical Society of America 49 (1971), pp. 524-531.

[14]Robert Morris, in his Composition with Pitch Classes: A Theory of Compositional Design (New Haven: Yale University Press, 1987), develops five such spaces (see pp. 23-26). David Lewin's Generalized Musical Intervals and Transformations (New Haven: Yale University Press, 1987) posits six temporal and six pitch- and/or pc-related musical spaces (pp. 16-25).

[15]John Rahn, in his Basic Atonal Theory (New York: Longman, 1980) clearly and consistently distinguishes between pitch relationships and pitch-class relationships, effectively separating theoretical concepts which apply only to pitch space from those which operate in pitch-class space.

[16]In addition to Robert Morris's Composition with Pitch Classes, cited above, another resource is Michael Friedmann's "A Methodology for the Discussion of Contour: Its Application to Schoenberg's Music," Journal of Music Theory 29/2 (1985), pp. 223-248. Friedmann's work raises important issues regarding musical structure, analysis, and perception. His article posits a number of theoretical constructs for comparing and relating musical contours, including the contour adjacency series and related vector, the contour class with its associated vector, and the contour interval succession and array. Although these formulations differ from those posited here in a number of crucial aspects, his article influenced this work in its initial stages.

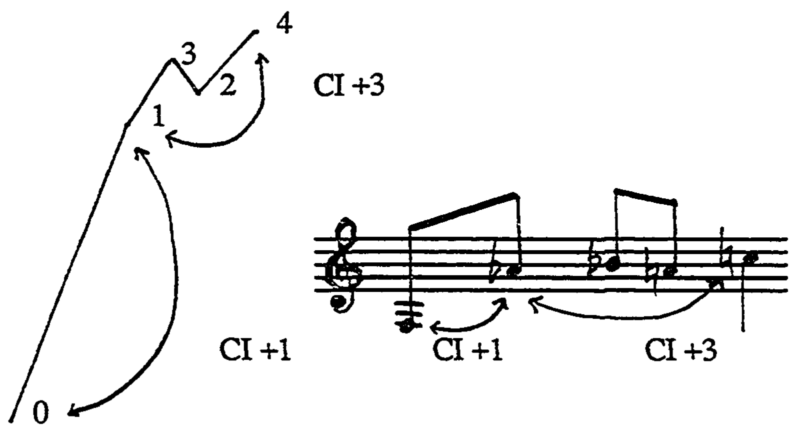
Discussion of musical contour is not without earlier precedents, however, particularly in the writings of music theorist-composers, such as Arnold Schoenberg (Fundamentals of Musical Composition, New York: St. Martin's Press, 1967, pp. 113-115, for example), Ernst Toch (The Shaping Forces in Music, New York: Criterion Music Corp., 1948, Chapter 5, "The Wave Line") and Robert Cogan, whose Sonic Design: The Nature of Sound and Music, written in conjunction with Pozzi Escot, (Englewood Cliffs, NJ: Prentice-Hall, Inc., 1976) makes extensive use of contour graphs in musical analysis. See also Cogan's New Images of Musical Sound (Cambridge: Harvard University Press, 1984).

Contour graphing has also been used widely among ethnomusicologists in their categorizing of melodic types. "Melodic Contour Typology," by Charles R. Adams (Ethnomusicology 20/2 (1976), pp. 179-215), is an excellent summary of ethnomusicological approaches to contour analysis including symbolic narration, computer encoding, word-list typology, and contour graphing. Adams posits a new theory of contour typology based on initial and end points, as well as high and low points. "The Structure of Melodic Movement: A New Method of Analysis," by Mieczyslaw Kolinski (in Studies in Ethnomusicology, vol. 2, pp. 96-120), categorizes a large number of contour diagrams via narrative description ("widening hanging complex," "rising overlapping complex," "wide-centered falling up-zigzags," etc.) and creates a methodology for diagramming melodic contours via hollow or filled dots denoting recurrent or non-recurrent movements. These dots are connected by lines, and placed between margins marked with letter names and semitone measurements. Both theories of contour analysis appear in Nicholas Cook's recent book, A Guide to Musical Analysis (London: J. M. Dent & Sons Ltd., 1987), pp. 196-208.

[17]Morris, Glossary, s.v. "c-space," p. 340.

[18]Morris, Definition 1.1, p. 26.

[19]Friedmann defines contour intervals (CIs) as "the distance between one element in a CC [Contour Class] and a later element as signified by the signs + or - and a number. For example, in CC < 0-1-3-2 >, the CI of 0 to 3 is +3, and the CI of 3 to 2 is -1" (p. 246). He readily acknowledges that the contour interval is "infinitely expandable or contractable in pitch space," and that "a larger CI contains a greater number of intervening pitches in the registral order of the musical unit . . . [and] is by no means necessarily a larger interval in pitch space" (p. 230). Although this concept is interesting, it seems somewhat counterintuitive from the perspective of a listener's perceptions, since a contour interval of +3 may be considerably smaller in pitch space than a CI of +1. For example, the cseg < 0 1 3 2 4 > may be realized as follows:



In this case, CI +3 (measured from contour pitches 1 to 4) is only a major third, while CI +1 is a minor tenth. Other musical realizations of this cseg may produce even larger differences in CI size. Further, Friedmann uses the contour interval, contour interval array, and associated vectors as an equivalence criterion (p. 231 and 234), and to compare similarities among contours in his analyses (p. 240 and following). Because intervals between contour pitches remain undefined in this formulation, the equivalence criteria and similarity relations posited here differ markedly from Friedmann's in concept.

[20]Judy Edworthy, "Melodic Contour and Musical Structure," in Musical Structure and Cognition , ed. Peter Howell, Ian Cross, and Robert West (London: Academic Press, Inc., 1985), pp. 169-188.

[21]Lucinda A. Dewitt and Robert G. Crowder, "Recognition of Melodies after Brief Delays," Music Perception 3/3 (1986), pp. 259-274.

[22]This is slightly different from Morris's definition, since all contours here are designated c-segments, not c-sets.

[23]See J. B. Davies and J. Jennings, "Reproduction of Familiar Melodies and the Perception of Tonal Sequences," Journal of the Acoustical Society of America 61 (1977), pp. 534-541.

[24]A number of discussions of the Gestalt principles for perception of musical groups are available in the literature, among them Diana Deutsch's "Grouping Mechanisms in Music, pages 99-134 of The Psychology of Music , cited above, and Fred Lerdahl and Ray Jackendoff's A Generative Theory of Tonal Music (Cambridge: The MIT Press, 1983), pp. 40-43 and 302-307. W. Jay Dowling and Dane L. Harwood discuss the Gestalt grouping mechanisms on pages 154-158 of their book, Music Cognition (New York: Academic Press, Inc., 1986).

[25]See, for example, Albert S. Bregman and Jeffrey Campbell, "Primary Auditory Stream Segregation and Perception of Order in Rapid Sequences of Tones," Journal of Experimental Psychology 89/2 (1971), pp. 244-249, and Albert S. Bregman, "Auditory Streaming: Competition Among Alternative Organizations," Perception & Psychophysics 23/5 (1978), pp. 391-398.

[26]Note that these definitions do not account as yet for repeated tones within a musical contour. This is a separate issue that will be addressed at a later point.

[27]Morris, Definition 1.2, p. 28.

[28]Friedmann, pp. 226-227.

[29]This term is adopted from Morris, Composition with Pitch Classes, p. 40. His Definition 2.2.3 states that "the function, INT_m , of a pseg X is defined as the ordered set Y where $Y_n = ip <X_n, X_{(n+m)}>$."

[30]Dowling and Harwood, Music Cognition, p. 134.

[31]Morris's Definition 1.4, p. 29, is rephrased slightly: the inversion of a cseg P, of cardinality n, is the cseg IP. Each IP_m equals $(n - 1) - P_m$ where the subscript m denotes order positions within the cseg P.

[32]Morris, Chapter 2, passim.

[33]See Note 1.

---

[34]More formally:

Let [cp(1)...cp(n)] be a cseg with cps numbered in time from 1 to n.
Let "n" equal the cardinality of the cseg;
Let "x" equal an ordinal position within the cseg, ranging from 1 to n (thus, "cp(x)" is a particular c-pitch, located "xth" from the left).

1) If necessary, translate the cseg to normal form,
2) If $(n - 1) - cp(n) < cp(1)$, then invert the cseg,
3) If $cp(n) < cp(1)$, then retrograde the cseg.

[35]W. Jay Dowling, "Recognition of Inversions of Melodies and Melodic Contours," Perception & Psychophysics 9/3B (1971), p. 349.

[36]W. Jay Dowling, "Recognition of Melodic Transformations: Inversion, Retrograde, and Retrograde-Inversion," Perception & Psychophysics 12/5 (1972), pp. 417-421.

[37]Ibid., p. 417.

[38]Among the issues Dowling's experiment was designed to address was the question of whether listeners perceive R-, I-, and RI-transformations as operations upon pitch or upon interval. He hypothesized that listeners using a pitch-transformation strategy would find retrograde-inversion the most difficult to identify, since two operations are necessary (each pitch must first be replaced with its inversion, and then the order of pitches retrograded). Listeners using an interval-transformation strategy would find the retrograde most difficulty to identify, since two operations upon intervals are necessary (the directions are reversed, and the order retrograded). Because the majority found the RI-transformation to be most difficult, Dowling concludes that all three transformations are processed along a "pitch-vector model." I suspect that his conclusion is flawed, and that listeners may use different strategies to recognize different operations. Even Dowling (p. 421) provides evidence for intervallic processing when he states that

> ...the inversions might be easier because among the three they allow for the element-by-element comparison of the comparison stimulus with memory of the standard in the same temporal order. Suppose the standard is encoded..."Down, Down, Up, Down." Then if S[ubject] hears a stimulus going "Up, Up, Down, Up," he need only compare the two, using the rule that for each "up" there should be a corresponding "down" in memory and vice versa.

He describes not a comparison of element-to-element, but of the direction of interval between elements. Thus his description of the strategy for recognizing inversions uses an interval model. More specifically, however, it uses a contour-space model since only the direction of each interval is described and not its magnitude.

[39]Recognition of the R-, I-, and RI-transformations upon melodies was discussed in some depth in Chapter 1. Although in Dowling's experiment recognition of contour transformations was only somewhat better than chance, the experiments descibed in the previous chapter show that the subjects' previous musical training, instructions to the subjects during the experiment, and construction of the musical stimuli have a great deal of influence upon these results. For some subject pools and in some musical contexts, recognition of the R-, I-, and RI-transformations upon 12-tone rows was extremely accurate, while in other subject pools and contexts recognition was far lower.

[40]David Lewin, "Forte's Interval Vector, My Interval Function, and Regener's Common-Note Function," Journal of Music Theory 21/2 (1977), p. 198, and John Rahn, "Relating Sets," Perspectives of New Music 18 (1979-80), p. 488.

[41]Allen Forte, The Structure of Atonal Music (New Haven: Yale University Press, 1973), pp. 46-60.

[42]See Eric Regener's "On Allen Forte's Theory of Chords," Perspectives of New Music 13/1 (1974), pp. 191-212; David Lewin's "Forte's Interval Vector, my Interval Function, and Regener's Common-Note Function," cited above; Robert Morris's "A Similarity Index for Pitch-Class Sets," Perspectives of New Music 18 (1979/80), pp. 445-460; and Rahn's "Relating Sets," in the same volume, pp. 483-498.

[43]As mentioned previously, the term "interval" is retained here, as is the INT_n diagonal of the COM-matrix, for its analogy with terminology in p- and pc-space. Intervals in contour space have no absolute measurement of magnitude, only of direction.

[44]This conforms to John Rahn's example in "Relating Sets," cited above.

[45]This similarity measurement is based upon compared positions in the upper triangles alone, since the entries in the lower left-hand triangle of the COM-matrices used here simply mirror (with inverse values) those in the upper right-hand triangle.

[46]Rahn, "Relating Sets", p. 490.

[47]This total number of comparisons between right triangles is sigma(n); which is represented by:

$$\sum_{S=1}^{(n-1)} (S)$$

(in other words, the summation of an arithmetic series from 1 to (n - 1), where n equals the cardinality of the cseg).

[48]Just as a distinction was made in the previous chapter between abstract and literal inclusion and complementation, so might CSIM(A,B) be considered an abstract similarity relation since it compares cseg A with members of csegclass B that are not literally represented in the music.

[49]This method of comparing csegs of unequal cardinality is chosen instead of an expansion and generalization of the CSIM measurement for two reasons. First, the embedding relation is easier to hear and therefore is intuitively more satisfying. Second, any generalization of CSIM to csegs of unequal cardinality would, in effect, create another type of embedding function, since it would involve comparing matrices of unequal size (thus embedding one matrix within another and systematically shifting the position of the embedded smaller matrix to make comparisons with each position of the larger matrix).

[50]Rahn, "Relating Sets," p. 490.

[51]Morris, "A Similarity Index," p. 446-447. This similarity index is determined by taking the absolute value of the difference between interval-class vector entries and adding. The index is "based on the total number of ics that are different: the less different the ics, the greater the similarity."

[52]Rahn, "Relating Sets," pp. 492.

[53]Rahn, Basic Atonal Theory, p. 122.

[54]Rahn, "Relating Sets," p. 492.

[55]M. C. Dyson and A. J. Watkins, "A Figural Approach to the Role of Melodic Contour in Melody Recognition," Perception & Psychophysics 35/5 (1984), pp. 477-488.

[56]Ibid., p. 483.

[57]More formally:

$$\mathrm{ACMEMB}(A,B) = \frac{\sum_{n=2}^{c} \mathrm{CMEMB}_n(X,A,B)}{2^{\#A} + 2^{\#B} - (\#A + \#B + 2)}$$

where c = cardinality of the smaller of the 2 csegs,

n = cardinality of X,
X = mutually embedded cseg, and
# stands for "cardinality of."

The numerator of this fraction loops through the $CMEMB_n(X, A, B)$ function successively for cardinalities 2 through the cardinality of the smaller cseg. The denominator divides this figure by the total number of csubsegs possible ($2^{\#A} + 2^{\#B}$) minus the one-note csubsegs (#A + #B) and minus the null set for each (2).

[58]A possible area of future investigation suggested by the ACMEMB function is an additional function that counts embedded csegclasses, rather than csegs. Although such a function no longer measures similarity (since order is crucial to perception of embedded subsegments), it does reveal some interesting properties of selected csegclasses. I am grateful to Greg Sandell for pointing out to me that c5-10 and c5-12 contain identical embedded csegclass contents, and form a kind of "Z-related" pair.

[59]Friedmann, pp. 234-236.

[60]In symmetrically-structured csegs of odd cardinality (i.e., < c b r x r b c > or < 1 3 2 0 2 3 1 >), the composite label will reflect the cseg's symmetry. For example, the COM-matrix for the repeated-note cseg < 1 0 2 0 1 > is shown below with the two matrices which determine its composite label:

$$
\begin{array}{c|ccccc}
 & 1 & 0 & 2 & 0 & 1 \\
\hline
1 & 0 & - & + & - & \boxed{0} \\
0 & + & 0 & + & \boxed{0} & + \\
2 & - & - & 0 & - & - \\
0 & + & 0 & + & 0 & + \\
1 & 0 & - & + & - & 0
\end{array}
\qquad
\begin{array}{c|ccccc}
 & 2 & 0 & 4 & 1 & 3 \\
\hline
2 & 0 & - & + & - & \boxed{+} \\
0 & + & 0 & + & \boxed{+} & + \\
4 & - & - & 0 & - & - \\
1 & + & - & + & 0 & + \\
3 & - & - & + & - & 0
\end{array}
\qquad
\begin{array}{c|ccccc}
 & 3 & 1 & 4 & 0 & 2 \\
\hline
3 & 0 & - & + & - & \boxed{-} \\
1 & + & 0 & + & \boxed{-} & + \\
4 & - & - & 0 & - & - \\
0 & + & + & + & 0 & + \\
2 & + & - & + & - & 0
\end{array}
$$

rc5-28/28 c5-28 also c5-28

In cases such as these, the related csegs that determine the composite label belong to the same c-space segment class. The composite label reflects this dual relationship by listing the csegclass's ordinal number twice.

[61]The maximum possible value for CSIM(A,B) between cseg A with repeated notes and cseg B without, is equal to $\frac{\text{sigma}(n) - r}{\text{sigma}(n)}$, where r is the total number of cp repetitions. Such a comparison cannot therefore return a value of "1."

[62]Allen Forte (The Structure of Atonal Music, page 53) notes, in reference to his similarity relations, that

...those cases in which transitivity holds are of special structural significance since they represent an extension of the particular relation beyond the scope of a single pair of sets to form a collection of interrelated sets. A collection of this kind will be referred to as a *transitive n-tuple*, or simply a transitive tuple.

[63] Arnold Schoenberg, "Composition with Twelve Tones," in Style and Idea: Selected Writings of Arnold Schoenberg, ed. Leonard Stein, trans. Leo Black (Berkeley: University of California Press, 1975), p. 226.

[64] Ibid., pp. 229-230.

[65] Arnold Schoenberg,"Analysis of the Four Orchestral Songs Opus 22," in Perspectives on Schoenberg and Stravinsky, ed. Benjamin Boretz and Edward T. Cone, trans. Claudio Spies (New York: W.W. Norton & Company Inc., 1972), pp. 25-45.

[66] Ibid., p. 41.

[67] More specifically, B and C both belong to another row form; B is an overlap, belonging both to T_5P and $T_{11}IP$.

[68] Because this latter contour (c6-117) is RI-invariant, the two halves of the melody may be considered inversionally-related or retrograde-related.

[69] This row has the same hexachordal pitch-class content as row P and is derived by an operation on order. Row S contains the pcs from the first hexachord of P, order positions 0, 2, 4 followed by 5, 3, 1 (in other words, alternate pitches proceding in one direction, then the other direction). The same algorithm may be used to generate the second hexachord of row S from the retrograde of P's second hexachord.

[70] A score to this work is available in Charles Burkhart's Anthology for Musical Analysis, Fourth Edition (New York: Holt, Rinehart and Winston, 1986), p. 482.

[71] Because c4-1 is an RI-invariant csegclass, < 3 2 1 0 > may also represent the opening contour's inversion. See Appendix I for a complete list of RI-invariant csegclasses; these are marked in the appendix by an asterisk.

[72] Rahn, "Relating Sets," p. 489-490.

[73] Such a pattern will always result between inversionally-related csegs in which adjacent cps add to an odd index number, in this case, 5. Other patterns of

invariance between inversionally-related contours may be predicted using the T_nI cycles. See Daniel Starr, "Sets, Invariance, and Partitions," Journal of Music Theory 22/1 (1978), p. 1-42, for a detailed examination of this subject.

Chapter Three

Rhythmic Perception: Duration Succession

1. Psychological Studies of Rhythmic Perception

Almost without exception, psychologists agree that listeners familiar with Western tonal music perceive musical rhythms in relation to equally-spaced, internally-generated beats whenever possible.[1] In support of the beat hypothesis, Dirk-Jan Povel has undertaken a series of experiments designed to test how well listeners can reproduce various temporal patterns, concluding that

> ...the perception of time, or more precisely the perception of temporal sequences, is determined in large part by an internal structure on which subjects try to map presented temporal sequences. Only when a temporal sequence completely fits this mental structure are subjects able to imitate the temporal sequence correctly. In all other cases severe distortions occur.... Our results clearly show that subjects are unable to store durations independently.[2]

Povel's "mental structure," as further developed in a subsequent study, is a beat-based model in which equally-spaced pulses of medium duration (generally 250 to 1,500 msec, corresponding to M.M. 240 to M.M. 40)[3] may be either subdivided or concatenated by the listener to structure a duration succession. With his colleague Peter J. Essens, Povel compares this hierarchical organization to that of clock time, in which minutes may be concatenated into hours or subdivided into seconds.[4]

Povel and Essens posit three possible types of perceptual clocks: first, an absolute clock, pulsing at a single fixed rate; second, a clock that pulses at a rate derived from the smallest time unit of a given rhythmic sequence; and third, a hierarchical beat-based clock as described above.[5] They discard the first two of these clocks on the basis of their experimental results. Subjects in these experiments were asked to listen to duration successions and to reproduce them by tapping. The authors conclude that the absolute clock is "unable to explain why a temporal pattern presented at a different tempo will be recognized as structurally identical," and further that "such a model would imply that all sequences having the same number of intervals will be equally well perceived and reproduced regardless of the durations of the intervals."[6] They continue their discussion by noting that

> The measuring problem would be simplified if the subject were able to select a time unit for this clock identical to the shortest interval in the sequence.... Such a representation is independent of the actual tempo of the pattern. The model predicts that all sequences having the same number of intervals will be equally well conceptualized, provided that all intervals in the sequence comprise integer multiples of the smallest interval. Hence both the sequence 200 200 400 [msec.] and 200 400 400 ought to be easy to conceptualize and consequently they should both be equally well reproduced. In fact, however, subjects reproduce the first sequence perfectly, but the second poorly.[7]

They argue that the first of these sequences is more easily reproduced because its first two durations may be heard as subdivisions of the third, which represents the beat unit of a hierarchical clock. Essens and Povel have undertaken further experimentation to determine expressly "whether subjects can use the smallest interval in a temporal pattern as a basic unit in representing other (longer) intervals in the same pattern."[8] According to this hypothesis, duration successions in which intervals relate as 3:1 or 4:1, exact multiples of the basic duration unit, should be reproduced more accurately than ratios of 2.5:1 or 3.5:1. In fact, their results

> do not support a distinction between the representation of patterns containing integer ratio intervals (e.g., 2:1, 3:1, and 4:1) and those containing noninteger-ratio intervals (e.g., 1.5:1, 2.5:1, and 3.5:1) in a nonmetrical context. From this result, we must conclude that the smallest interval is not used in specifying the time structure of the patterns.[9]

The implication of their work is that rhythmic theories based upon tallied multiples of a composition's smallest durational value do not model aural perception. That is not to say that such theories cannot reveal important aspects of a work's rhythmic structure,[10] particularly in compositions where serialized rhythm is directly linked to pitch structure[11] and in certain non-Western musics.[12]

Given the results of experimentation in cognitive processing, the rhythmic theories of Maury Yeston have music-psychological support. His hierarchical conception of musical time simplifies the notion of "meter," eliminating reliance upon accentual relationships in its definition. Borrowing from Yeston's formulation,[13] meter is defined here as a relationship between two different levels of equally-spaced pulses, requiring both a faster- and a slower-paced pulse in consonant relationship with each other. Rhythmic consonance refers to proper alignment among levels, as shown in Figure III-1, while rhythmic dissonance is represented by misalignment between levels resulting in syncopation. This definition also accounts for tempo changes, as Yeston notes:

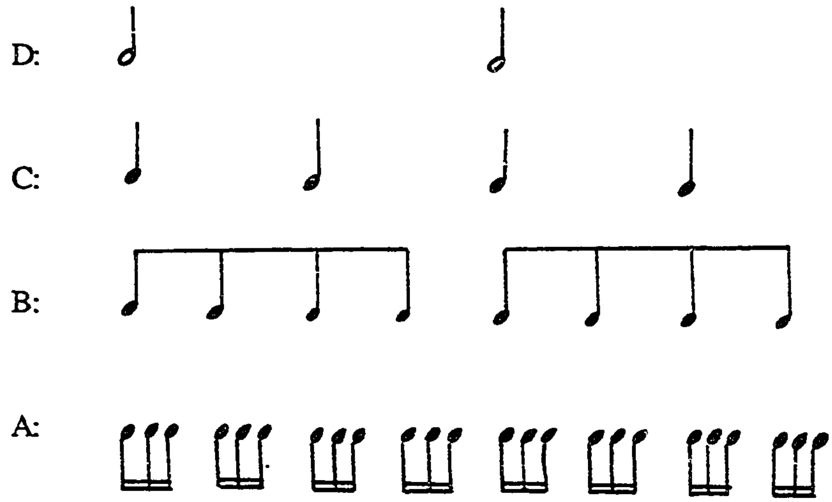
> It may appear at first blush that ritardando, accelerando, and tempo rubato instructions are the kinds of phenomena that...present problems for the theory, but this is not the case. If meter were misunderstood to be solely a foreground phenomenon, then a gradual slowing down of a pulse on the surface of a composition would have the effect of dissipating all semblance of regular motion. But since meter has been defined as a relationship between levels, this dissipation does not occur.[14]

A number of authors in the music-psychological community come close to Yeston's hierarchical definition in their understanding of rhythmic structure; among these are Essens and Povel, as well as Handel and Oshinsky, whose work is discussed below.

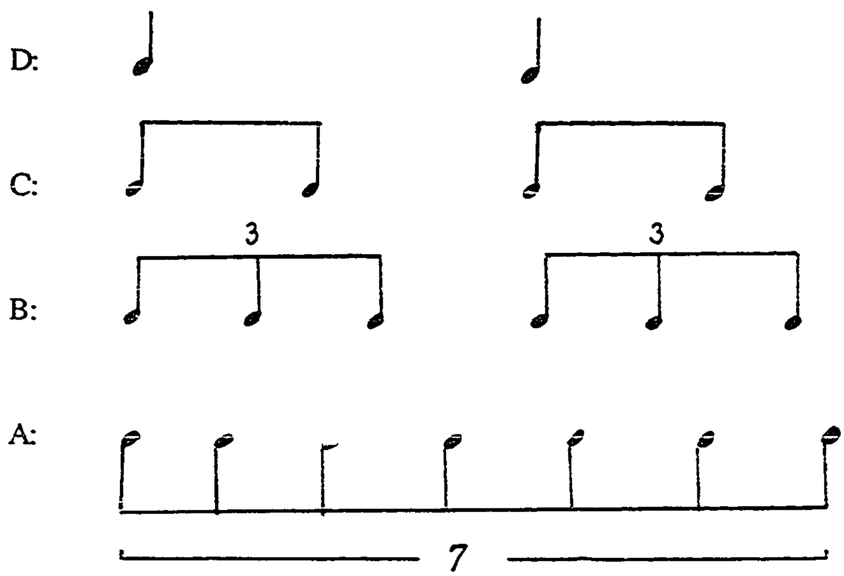
FIGURE III-1

YESTON'S RHYTHMIC CONSONANCE AND DISSONANCE BETWEEN LEVELS

Rhythmic consonance (Yeston's Example 4:1):



Rhythmic dissonance (Yeston's Example 4:2):



Essens and Povel define a metrical representation of a temporal pattern as "one based upon a metrical framework, meaning that the pattern is mapped on a frame formed of equal time intervals. For example, the interval sequence 2 2 3 1 2 1 1 4...may be perceived metrically as having a metrical framework with intervals of 4 time units."[15] Implicit in this definition is the concept of two distinct levels of pulses, one pulse-level measured by the time unit "1" and a second slower level moving at a rate of 4 time units. For Essens and Povel, a nonmetrical representation occurs when "the sequence cannot be divided into equal time intervals, such as the interval sequence 1 3 2 1 4 with a total duration of 11 time units."[16]

Stephen Handel and James Oshinsky, whose experiments focus on dissonant polyrhythms (3 against 4, 2 against 3, 2 against 5, and so on), rely like Yeston upon rhythmic levels in defining meter. They aptly note that difficulties arise in psychological research from "the lack of an appreciation that rhythm emerges from the interaction of the hierarchical levels of music. The simple rhythmic concepts of meter, beat, accent, and phrase are not found at any single level...but arise out of the interactions among levels."[17] These authors see the function of meter as providing "a lattice against which the foreground elements can be structured. Thus, in general, the meter should be the "slower" moving pulse train...."[18] Further, they reject reliance upon accents in formulating rhythmic theory:

> We think it critical that rhythmic theory leave behind notions based on simple accented/unaccented rhythmic units. Analyses of this sort always involve post hoc considerations based on the theorist's intuitions about the intentions of the composer. While an appeal to musical intentions may prove successful in a small number of instances, it will clearly limit the ultimate generalizability of any theory.[19]

By questioning the validity of rhythmic theories based upon accentuation, Handel and Oshinsky strike at the very heart of Grosvenor W. Cooper and Leonard B. Meyer's theories, as set forth in The Rhythmic Structure of Music. Cooper and Meyer define meter as "the measurement of the number of pulses between more or less regularly recurring accents," and state further that "in order for meter to exist, some of the pulses in a series must be accented."[20] They define rhythm by similar criteria, as "the way in which one or more unaccented beats are grouped in relation to an accented one."[21] Thus, to Cooper and Meyer meter implies measurement of--and rhythm, grouping of--accented and unaccented events. For purposes of this study, however, accentuation will not be considered in defining terminology. Rather, following Yeston, meter will be understood as a consonant relationship between two hierarchical levels of equally-spaced pulses, while rhythm will be understood simply as a succession of durations that may or may not be metrical, and thus may or may not contain a perceived beat.

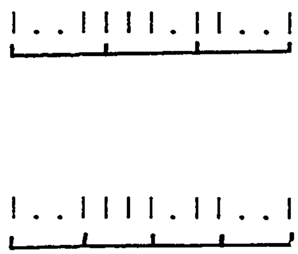
Duration successions that can be understood metrically are more accurately perceived and reproduced by subjects. Povel and Essens call such rhythms "clock-inducing patterns." Their work shows that duration successions with strong clock-inducing patterns are better reproduced than weaker clock-inducing patterns, and that assisting the listener to induce the clock, by supplying a second slower pulse train, assists in reproduction. Further, they show the importance of meter in shaping perception by presenting listeners with two rhythmic patterns that are identical in terms of duration succession, as shown in Figure III-2, but which can be interpreted in terms of two different meters. When a second pulse stream is introduced to structure the rhythms according to different meters, subjects are unable to recognize the two duration successions as identical.[22] Yeston makes similar observations in discussing his

FIGURE III-2

TWO METRIC INTERPRETATIONS OF IDENTICAL DURATION SUCCESSIONS

Povel and Essens, Figure 14:

Pattern = 3 1 1 1 2 1 3



Possible metrical interpretations (in musical notation):

Example 2:1, which shows various metric realizations of the "uninterpreted rhythmic structure" 1 2 1 2 1 2 (etc.).[23]

Duration successions that can not be interpreted in terms of a relatively simple beat-based hierarchy are usually perceived imprecisely and are distorted when reproduced.[24] Musicians and nonmusicians alike tend to alter relationships within such patterns toward metrical interpretations, usually with temporal intervals relating as 2:1.[25] Jeanne Bamberger and David Lewin have discussed this phenomenon of listeners' distortion of rhythmic patterns. They cite a rhythm in which a consistent beat unit is not present; rather, the rhythm consists of durations that are multiples of a small temporal unit.[26] Figure III-3a reproduces Bamberger's example in rhythmic notation, as a duration succession of 2, 3, 4, and 5 eighth notes, while Figure III-3b shows a common interpretation by listeners--one that imposes a meter featuring 2:1 ratios.[27]

Povel hypothesizes that listeners have at least two possible ways of understanding temporal sequences, and that the nature of the rhythm itself determines which method will be used. In the first two of his 1981 experiments, he discovered that

> The stimuli...did not fit a beat-based coding and were therefore internally represented as rather unstructured groups of tones. Such an unstructured representation is imprecise and is strongly influenced by a tendency to conceptualize duration differences in a 1:2 relation.[28]

Rhythms that do not "fit a beat-based coding," to use Povel's words, abound in non-western musics and in Western music of this century.[29] Examples in the latter category may be found in the serialized rhythms of Babbitt, the Gestalt-like rhythmic grouping of Boulez, and the rhythmic dissonance of Carter and Stockhausen. In subsequent studies, Povel determined that rhythmic patterns that do not invoke an

FIGURE III-3

NON-METRICAL DURATION SUCCESSION AND ITS INTERPRETATION

Figure III-3a:

Figure III-3b:

internal clock, those without perceived beats, are coded differently from those that do. He concludes that "This alternative coding, called 'figural coding' by Bamberger (1978), capitalizes on the perceptual grouping of events. In this latter group strategy, detailed information about the relative durations of intervals would seem to be left uncoded."[30] This hypothesis has been substantiated more recently by Jeffrey Summers, Simon Hawkins, and Helen Mayers, who also describe two schools of thought regarding temporal organization: first, Gestalt-like grouping and second, hierarchical beat-based models.[31] They note that the first is used to interpret non-metrical rhythms, while the second is used for metrical ones. To summarize, psychologists find that in rhythms where no beat unit is perceived, listeners are likely to distort the durational values in favor of a metrical interpretation or, where this is not possible, to interpret the rhythm as a "figural" pattern by employing such Gestalt principles as proximity. In the latter case, relative longs and shorts are retained, rather than precise durational relationships, and "clustering" of successive short durations may be perceived by virtue of their close temporal proximity. A more general view of this dichotomy might divide musical literature into that with perceived rhythmic patterning and that without patterning, rather than that with hierarchical beat structure and that without. Thus, music based on systematic durational patterning, including metrical music, will be perceived in terms of that cognitive pattern-based framework, whereas music without underlying pattern will be perceived as a Gestalt.

Many of the studies cited above carefully distinguish between musically-trained and untrained listeners. Summers, Hawkins, and Mayers, as well as Povel and Essens, believe that similarities in performance between the two groups of listeners interpreting metrical patterns suggest a universal cognitive structure, although they find some differences between groups. Povel notes that musically-trained subjects discover the beat more quickly than the untrained; further, they are more likely to try to impose a beat on patterns containing irregularly spaced durations. But he concludes that "cognitive structures are not specific to musically active subjects."[32] Jacqui Smith, on the other hand, has argued that skilled and unskilled subjects may have different psychological representations of rhythmic structures, and that beat-based representations are far from universal.[33] She found that musically-skilled subjects were able to make use of higher-order properties in a temporal sequence, using the beat as a framework, whereas nonmusicians' responses reflected an organization based on proximity. In other words,

> When musically skilled subjects were incorrect, they still maintained the global quality of the sequence and in particular maintained a steady beat. This suggests that they were able to make use of higher order dimensions in the sequence, using the beat as a temporal framework and simply remembering where to add or delete sounds.... Irregular interval sizes and a simplification of the number of different interval sizes characterized the errors of the unskilled subjects. Their coding of the sequences seems to involve grouping sounds on the basis of temporal proximity. Many non-musicians reported adopting a "counting" strategy, tallying the number of sounds that formed a group, and the number of such groups in a sequence.[34]

Each of the durational units in a beat-based hierarchy may also be subdivided, providing an additional level to the rhythmic hierarchy. It is unclear, however, exactly how the listener perceives this subdivision. Experiments in listener reproduction of temporal patterns from aural stimuli have shown that the cognitive structure for perception of rhythmic subdivisions is far from precise. Eric F. Clarke summarizes his perceptual model of this structure as follows:

> ...it is proposed that rather than internally representing rhythms by means of complex ratio information, a combination of two simple components is used. One is a system of metrical markers that are directly timed at one level but exist as abstract relationships at both higher and lower levels.... The second component is a system of untimed procedures, organised around these markers, specifying subdivisions in terms of equal and unequal time spans, the unequal subdivisions using a simple distinction between long and short.[35]

This categorization of unequal subdivisions simply into longs and shorts is an imprecise measurement of relative duration, and explains the common misapperception of the dotted eighth-sixteenth subdivision for a triplet's quarter-eighth subdivision, since both are heard as long-short.[36] In a second article, based upon a study of expressive rhythm in performances of Erik Satie's "Gnossienne No. 5," Clarke elaborates upon this theory. He suggests here that listeners can compare their perceived string of long-short durations to a "metrical matrix" to produce a more accurate perception of a given temporal pattern:

> The procedural specifications of individual rhythmic figures are extremely simple in construction. They collect together those durations that constitute a rhythmic group and specify the component durations in terms of equal or unequal divisions of the overall time span, long and short components being labelled in unequal divisions. For instance, the last beat of Bar 5 of Gnossienne no. 5 ( ) would simply be specified as four equal divisions and the first beat of Bar 6 ( ) as an unequal division into a long and short component. The position of the beat in relation to each group is also marked, and when this package of information is combined with the metrical matrix, a unique rhythmic specification results.[37]

Clarke's metrical matrix[38] may be modelled in terms of the hierarchical pulse trains discussed previously. The interaction between two levels specifies meter and beat, while a third level may be added to specify beat subdivisions. Although early researchers believed that listeners are able to perceive only duple subdivisions of the beat accurately, Povel's subsequent work has shown this to be untrue:

> It is indeed clear from research done so far that temporal sequences are not stored as a series of independent durations.... Also the idea, advocated by Fraisse (1946, 1956) that in the coding of temporal sequences the occurring intervals are subdivided into two classes, namely long and short intervals that roughly relate as 1:2, has been shown to be too simple...[39]

Povel discovered that the metrical contexts in which the various subdivisions appeared were crucial variables in how well the rhythms were perceived. In his 1981 experiments, for example, Povel was surprised that even musicians had more trouble reproducing 1:3 and 1:4 ratios than 1:2. In reevaluating the data, he realized that this result was directly linked to the rhythmic contexts in which these ratios were presented. When these contexts were revised, subjects accurately reproduced 1:3 and 1:4 ratios. He describes the revised contexts as follows:

> One characteristic of the correctly imitated sequences is that the shorter interval is repeated so many times that the total duration of repetitions is equal to the longer interval. This configuration enables the subject to define the shorter interval as a subdivision of the longer one while this longer interval is repeated continuously.[40]

That is, the duration succession 250 750 750 msecs. was poorly reproduced, while the succession 250 250 250 750 was accurately reproduced. Thus Clarke's metrical matrix (Povel's "temporal grid") may exist as a cognitive framework by which listeners may categorize an equal or unequal (long-short, short-long) subdivision more precisely in terms of either a duple or triple subdivision. Further experimentation might reveal whether the matrix model can be expanded to include other subdivisions.[41]

Clarke terms his theory a "two-component system," in which the perceived pattern of relative longs and shorts is overlaid upon a metrical matrix that represents beat structure and subdivision. He notes that this theory has certain similarities to

Dowling's two-component model of melodic contour overlaid upon a diatonic scale framework.[42] Dowling and Harwood note the similarity between theories as well, citing Caroline Monahan's suggestion that

> rhythmic subdivision patterns are laid on the beat framework in a way analogous to the way melodic pitch contours are laid on the scale framework.... Rhythmic subdivisions can thus be said to be encoded in rhythmic contours of relative, not absolute, temporal relationships. Rhythmic contours are like melodic contours in being able to stretch to fit different frameworks (as with change of tempo) and in being able to slide along a given framework (as in displacement of rhythmic accent).[43]

Thus "rhythmic contours" may be understood as analogous to melodic contours; they represent relative durations in much the same way that melodic contours represent relative pitch height, without a precise calibration of the intervals spanned.

2. Temporal Spaces and Duration Space

The extension of contour theory into temporal spaces necessitates a few preliminary comments, particularly regarding the relation between music theory and music perception. A theory is a model; it may be a pure abstraction, as are some mathematical or logical theories. A music theory models musical structure by formalizing some aspect or aspects of a given body of music; it defines equivalence classes among its elements or sets of elements, as well as transformations upon those elements or sets. According to the model defined here, contour theory in duration space is isomorphic with contour theory in contour space; it does indeed model rhythmic transformations occurring in the compositions cited and analyzed below. To assert that a music theory accurately models musical structure is not necessarily to say that it also models a listener's perception of that music; the two are separate issues. It has been one of the primary aims of this dissertation to address the relationship between the two areas--between the theoretical model proposed and the listener's perception of the music that the theory models, as documented in the music-psychological literature. It is not intended, however, that the fundamental distinction between the two be blurred. In generalizing contour theory to duration space, a number of important perceptual questions arise--questions that have not to date been addressed by music-psychological experimenters. Although these questions will be raised when appropriate during the discussion that follows, the reader is reminded that the perceptual applicability of the theory is a separate issue from the theory itself.

The pitch spaces discussed in the previous chapter have direct analogs in the temporal domain. Recent publications by Robert Morris and David Lewin have drawn upon this relationship in discussing temporal spaces. Morris, for example, has defined three types of temporal spaces, the structures of which are isomorphic with the three pitch spaces discussed previously: sequential time with contour space, measured time with pitch space, and modular time with pitch-class space. According to his definition, a segment of cardinality n in sequential time (s-time) "consists of n s-time points numbered in order from 0 to n-1 (corresponding to their temporal order)."[44] The interval between time points remains undefined, as is the case in contour space. Morris's measured time, the pitch-space analogy,

> at MM n, consists of m-time points (m-tps) arranged in temporal order. The duration between each m-time point is equal to the 'minimal duration' which is equal to 60/n seconds. The m-time points are labeled from 0, a mid point, to 'later' by increasingly positive integers and to 'earlier' by increasingly negative integers.[45]

Finally, modular time (mod-time), the temporal space corresponding to pitch-class space, is defined as "a set of time-points (tpc) derived from a m-time by taking its m-tps mod n. Tpcs are labeled successively with integers from 0 to n-1."[46]

Lewin's formulation contains six temporal spaces, the first two of which correspond to Morris's M-time and mod-time. Lewin terms his time points beats, and in his modular space beat classes, such that "beat-class 0 comprises all pulses...that occur at some bar-line; beat-class 1 comprises all the pulses...that occur one unit after some barline," etc.[47] In addition, he defines four types of temporal spaces based not on sequential time points, but upon durations. Two of these spaces, based upon "a family of durations, each duration measuring a temporal span in time units,"[48] differ only in the way intervallic spans are measured: in the first case, as quotients and in the second, as differences. Thus, "if s spans 4 time units and t spans 3 time units, then int(s,t) = 3/4" in the first case, while int(s,t) would equal 1, in the second.[49] The remaining two temporal spaces are reductions of these two systems "by a durational modulus M greater than 1."[50] Two durations belong to the same duration class if one is some integral power of M times the other.

An additional type of temporal space is proposed here: a duration space analogous to contour space, that models relative duration in much the same way as contour space models relative pitch height. Thus, just as contour space is represented analytically by an integer system that assigns 0 to the lowest element of the contour and (n-1) to the highest, so duration space will assign 0 to the shortest duration and (n-1) to the longest. The relationship between contour space and pitch space discussed by Dowling and Monahan above is analogous to that between duration space and measured time. That is, just as melodic contours may be overlaid upon various scale types in pitch space--from the major or minor scale in diatonic tonal contexts to the complete chromatic scale in non-tonal contexts--so may "rhythmic contours"[51] be overlaid upon various metric frameworks.

In generalizing contour theory from contour space to duration space, a number of important differences between the spaces must be acknowledged. While these differences do not affect the general applicability of the theory to duration space, they do have important implications for the perception of various transformations that occur in the music to be studied below. First, a fundamental perceptual difference between contour space and duration space is that while melodic contours are easily remembered, and same/different comparisons accurately made by most listeners regardless of context (tonal or non-tonal), rhythmic contours may not be recognized by listeners as identical if their underlying metrical structures differ (as was the case in Figure III-2 above). It is in contexts where no consistent beat unit or metrical structure can be perceived that rhythmic contours of relative shorts and longs best model the listener's perception.[52] At least one composer has confirmed this premise about rhythmic perception of non-tonal music; Gérard Grisey states that

> without a reference pulse we are no longer talking of rhythm but of durations. Each duration is perceived quantitatively by its relationship to preceding and successive durations. This is the case in the rhythmic writing of Messiaen and of the serialist school. In fact, a micro-pulse allows the performer or conductor to count and execute these durations, but it only exists as a way of working and has no perceptual reality. The more complex the durations...the more our appreciation of them is only relative (longer or shorter than...).[53]

Compositions chosen for analysis and discussion here are therefore works that do not strongly invoke a perceived beat or meter. Second, contour theory in c-space compares points in that space, while in duration space (d-space) it compares pairs of points. Since the elements of this latter space are durations, measured from the onset of one event to the onset of the next, each duration is dependent upon two points, rather than one, for its identity. Thus, while a point in contour space is immediately perceivable, a duration in d-space is not perceived until the second point of the pair defines its length. Further implications of this distinction for music perception are considered below. Third, segments in c-space or d-space may be divided into subsegments for comparison and analysis. Intuition suggests that listeners should be able to perceive not only contiguous subsegments of a melodic contour, but also certain non-contiguous ones. The subsegment made up of non-adjacent cps that are associated by extreme low or high register, or associated by a pattern of accentuation, for example, might easily be perceived as a contour subsegment, while other subsegments might be more difficult or impossible to isolate perceptually. The issue of perceived non-contiguous subsegments in duration space differs somewhat, since intuition suggests that it would be unlikely for listeners to group non-adjacent durations on the basis of their length alone (identifying all the longest durations as a distinct perceived subsegment, for example). However, that is not to say that non-contiguous subsegments in duration space cannot be perceived, since the interaction of melodic contour, accentuation, timbral change or any other number of musical features might cause the listener to associate non-adjacent durations. Finally, while the operations of inversion and retrograde inversion have a clear perceptual basis in pitch-space and contour-space as discussed in the previous chapter, application of these operations to duration successions in duration-space may be more difficult to perceive. Again, it should be stressed that the validity of the theory proposed here to model certain rhythmic transformations in non-tonal music should be considered separately from the perceptual applicability of that theory. In both contour space and duration space, the operations of identity, inversion, retrograde and retrograde inversion can be shown to model certain transformations occuring in non-tonal compositions. In contour space, experimentation has shown that these transformations can indeed be perceived; in duration space, however, some questions remain as to the perceptibility of the I and RI operations.

Qualifications aside, the theory proposed here does indeed model aspects of rhythmic structure in nonmetrical music, not only in the Western non-tonal repertory that is considered here, but in certain non-Western musics as well. For example, in their discussion of South Indian rhythmic talas, Kanthimathi Kumar and Jean Stackhouse describe the divisions of tala in Karnatic music.[54] They list the seven main talas in terms of their number and grouping of counts, then note that each of these seven talas has five forms. An adaptation of their table showing the five forms (Jatis) of the tala dhruva follows:[55]

Different Jatis of Dhruva	Count Distribution	Total Counts	Dseg
1. Tisra	3 + 2 + 3 + 3	11	< 1 0 1 1 >
2. Chaturasra	4 + 2 + 4 + 4	14	< 1 0 1 1 >
3. Khanda	5 + 2 + 5 + 5	17	< 1 0 1 1 >
4. Misra	7 + 2 + 7 + 7	23	< 1 0 1 1 >
5. Sankeerna	9 + 2 + 9 + 9	29	< 1 0 1 1 >

Note that each of the five variations of dhruva has an equivalent rhythmic contour, < 1 0 1 1 >. Each of the other six main talas' Jatis shares the same dseg. Thus the seven talas are equivalence classes according to the definition of dseg proposed here.

To return to the generalization of contour theory in the temporal domain, duration space (d-space) is defined here as a type of temporal space consisting of elements arranged from short to long. Elements in d-space are termed durations (durs), and are numbered in order from short to long, beginning with 0 up to (n - 1), where n equals the number of elements in the segment and where the precise, calibrated duration of each dur is ignored and left undefined.[56] A precise profile of the structure of a duration succession in d-space is provided by Morris's COM-matrix. As stated previously, the COM-matrix is a two-dimensional array that displays the results of the comparison function, COM(a,b). In this case, a and b represent any two durs in d-space. If b is longer than a, the function returns "+1"; if b is the same length as a, the function returns "0"; if b is shorter than a, COM(a,b) returns "-1." The repeated instances of the integer "1" are omitted in the COM-matrix. The symmetrical structure of the matrix and properties of its diagonals (INT_n), as discussed in the previous chapter, remain the same whether applied to contours in c-space or to rhythms in d-space.

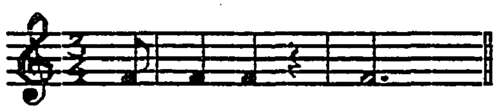
In accordance with the definitions established for contours in contour space, a d-segment (dseg) is defined as an ordered set of durs in d-space. Dsegs are ordered in s-time rather than in m-time, to use Morris's terms, since the distance between time points in the former is flexible. The equally-spaced time points of the latter are less appropriate to a space in which precise durations are not calibrated by a single time unit. Just as a cseg can be realized in pitch space in an indefinite number of ways, so can a dseg be realized in measured time by an indefinite number of rhythms. By way of example, Figure III-4 shows several realizations of dseg < 0 1 2 3 >.[57] In numbering durs from short to long, the determination must be made from the onset of one dur to the onset of the next, regardless of whether the pitch in question extends through the entire temporal interval spanned or is interrupted by a rest. Thus, dseg B of Figure III-4 still represents < 0 1 2 3 > even though it contains a rest. Music psychologists call this temporal span the inter-onset interval (IOI). As Eric Clarke notes,

FIGURE III-4

MULTIPLE REALIZATIONS OF DSEG < 0 1 2 3 > IN MEASURED TIME

Overlaid upon three possible "temporal grids":

A:


B:


C

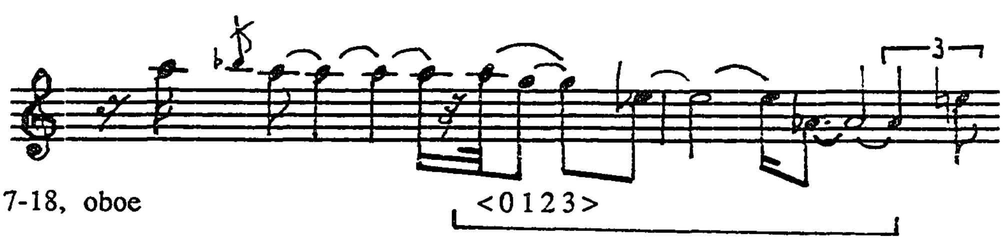

From Varèse, Octandre, Movement I:

D:


m. 2, oboe

E:


mm. 13-15, trumpet

F:


mm. 17-18, oboe

> this is the most significant measure as far as the rhythmic function of the note is concerned since the other possible measures (onset to offset or offset to onset) refer mainly to the articulation properties of the note.[58]

This requirement is one of the features mentioned above that distinguishes contour space from duration space, since c-space compares points in space while d-space compares pairs of points (onset to onset). This presents certain analytical difficulties in determining the duration of the final note of a succession, because there is no following onset by which to measure the length of that final duration. In the abstract and in the case of certain non-sustaining instruments, the theory might be formulated to exclude the duration of the final note, restricting its role so that it serves only to define the length of the penultimate duration. However, in musical contexts--particularly those involving performance by sustaining instruments, such as voice or wind instruments--the cut-off of the final note has some perceptual validity. It is for this reason that the examples following include the final note's duration. In examining music written for a non-sustaining instrument, the analyst might define duration successions differently, however, consistently omitting the duration of the final note. Finally, while a rest that is internal to a duration succession (Figure III-4B and F) generally adds to the duration of the note preceding, it may also be seen as an important criterion for segmenting the succession into d-subsegments.

A d-subsegment (dsubseg) is defined as any ordered subgrouping of a given dseg. Figure III-5 illustrates dsubsegs drawn from a prominent oboe melody in the first movement of Varèse's Octandre (mm. 8-11). Dsubsegs in duration space are assumed to be contiguous subgroupings, unlike csubsegs in contour space. As mentioned previously, in c-space the listener may group non-contiguous high cps aurally by their close proximity in pitch height, for example, whereas the temporal

FIGURE III-5

DSEGS AND DSUBSEGS IN DURATION SPACE

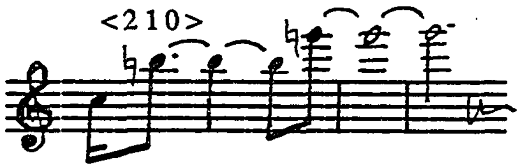
Varèse, Octandre
Mvt. I, Oboe, mm. 8-12

<2 1 0 3 5 4>

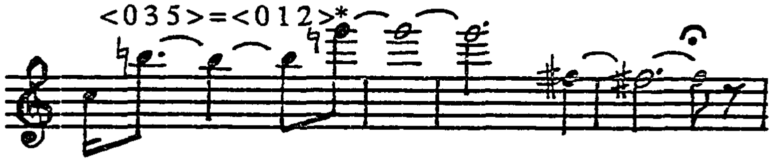
Dsubsegs:

A: 

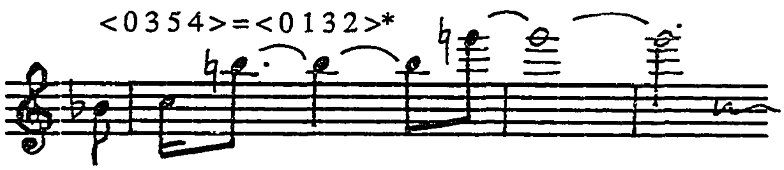
<2 1 0>

B: 

<0 3 5> = <0 1 2>*

C: 

<0 3 5 4> = <0 1 3 2>*

D: 

<1 0 3 5> = <1 0 2 3>*

* = by translation

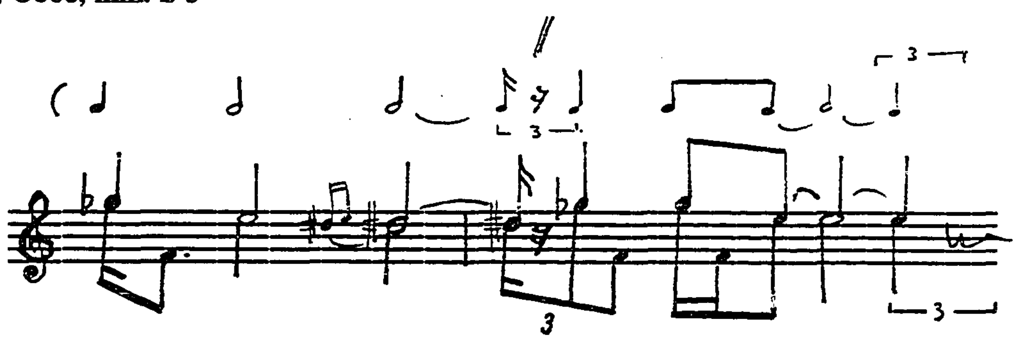
nature of d-space prevents the listener from grouping all long durs simply by virtue of their length. Only in the case where melodic contour or some other musical feature, such as accentuation or timbre change, interacts with perception of rhythmic contour might a case for noncontiguous dsubsegs be made. In compound melody, for example, registral pitch proximity might cause the listener to perceive the higher or lower voice (or both) as an independent duration stream. Figure III-6 shows such an instance. Here, the reiterated low f might be heard as a separate stream, resulting in a noncontiguous dsubseg heard in the upper voice (indicated by stems up). To determine durs in this dsubseg, the note values given in parentheses, which sum the rhythmic values of both streams, must be considered. Non-contiguous dsubsegs are clearly a special case; thus the term dsubseg will generally refer only to contiguous dsubsegs unless otherwise specified.

Two types of equivalence relations are posited for dsegs in duration space. First, equivalent dsegs are those that generate identical COM-matrices;[59] they may be translated to the same normal form following the algorithm detailed in Chapter Two. This definition asserts equivalence for any two duration successions related as those in Figure III-7a. Measured in terms of the smallest durational unit (the sixteenth note), succession A may be represented as < 3 1 4 > and B as < 6 2 8 >. Succession B is an augmentation of A in m-time, which may be shown numerically by multiplying A's durational values by 2. The two successions generate identical matrices, and in d-space are equivalent representatives of < 1 0 2 >. Dseg equivalence may also explain why rhythms such as those of Figure III-7b are so easily confused by students in early states of aural skills training, since the two are equivalent in duration space. Figure III-7c illustrates an additional instance of dseg equivalence in which the durations of the two successions in m-time are not related by any precise mathematical relationship. Yet

FIGURE III-6

NONCONTIGUOUS DSUBSEGS

Varèse, <u>Octandre</u>
Mvt. I, Oboe, mm. 2-3



stems down: < 0 1 2 3 > < 1 1 0 0 2 >

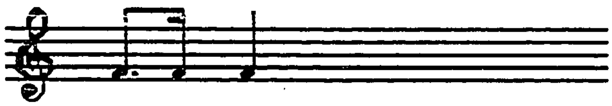
stems up:* < 0 1 2 > < 1 0 2 >

*noncontiguous dsubsegs

FIGURE III-7

DSEG EQUIVALENCE

Figure III-7a:

A: 

< 3 1 4 > in sixteenth-note durations = < 1 0 2 > in d-space

B: 

< 6 2 8 > in sixteenth-note durations = < 1 0 2 > in d-space

Matrices:

A:

$$\begin{array}{c|ccc} & 3 & 1 & 4 \\ \hline 3 & 0 & - & + \\ 1 & + & 0 & + \\ 4 & - & - & 0 \end{array}$$

B:

$$\begin{array}{c|ccc} & 6 & 2 & 8 \\ \hline 6 & 0 & - & + \\ 2 & + & 0 & + \\ 8 & - & - & 0 \end{array}$$

Figure III-7b:

< 1 0 2 > in d-space

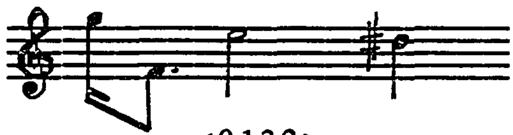
< 1 0 2 > in d-space

FIGURE III-7

DSEG EQUIVALENCE

Figure III-7c:

C.



<0 1 3 2>

Varèse, <u>Octandre</u>
Oboe, m. 1

D:

<0 3 5 4>=<0 1 3 2>

Varèse, <u>Octandre</u>
Oboe, m. 9-12

Matrices:

C:

	0	1	3	2
0	0	+	+	+
1	-	0	+	+
3	-	-	0	-
2	-	-	+	0

D:

	0	3	5	4
0	0	+	+	+
3	-	0	+	+
5	-	-	0	-
4	-	-	+	0

succession D is a free augmentation of C, and in d-space the two are equivalent representations of < 0 1 3 2 >, producing identical matrices as shown. Succession D is one of the dsubsegs from Octandre previously cited in Figure III-5, and is numbered as in that figure; succession C is comprised of the first four notes of the movement. The two successions have clear aural associations--both are prominent solo oboe lines, and the melody of succession D represents a rhythmic expansion, or development, of the melody with which the movement opened.

The second equivalence relation, the duration-space segment class (dsegclass), is defined as an equivalence class made up of all dsegs related by identity, translation, retrograde, inversion, and retrograde inversion, analogous to the csegclass of contour space.[60] As in the previous chapter, the inversion of a dseg S of n distinct durs is written IS, and may be found by subtracting each dur from (n-1), where n represents the cardinality of the segment. In effect, this results in durations "swapping" positions within the segment. Given an odd value of (n-1), the longest and shortest durations swap positions, the next-to-longest and next-to-shortest swap positions, and so on. If (n-1) is even, the same holds true except that n/2 retains its position:

P:	< 0 2 1 3 >	P:	< 0 4 1 3 2 >	P:	< 3 1 2 4 6 5 0 >
I:	< 3 1 2 0 >	I:	< 4 0 3 1 2 >	I:	< 3 5 4 2 0 1 6 >.

This algorithm for finding the inversion of a rhythmic segment is precisely the one used by Milton Babbitt in composing his Three Compositions for Piano. The retrograde (RS) or retrograde inversion (RIS) of a duration succession may be found by listing the elements of S or IS in reverse order. Thus dsegs A and B of Figure III-5 preceding are retrograde-related, while C and D are RI-related. Because d-space is isomorphic with c-space, the csegclass labels of Appendix 1 may be adopted for use in analyzing rhythmic contours as well as melodic contours. Figure III-8 shows members of dsegclass d4-2, < 0 1 3 2 >, representing S and its R-, I-, and RI-transformations, and the corresponding COM-matrix for each. For each cseg, one possible m-time realization is given in musical notation.[61]

Music psychologists have not explored the question of whether listeners can perceive R-, I-, and RI-transformations upon duration successions. Retrograde rhythms have been used for centuries in conjunction with pitch retrogrades in musical composition; yet it is unclear how well listeners recognize the pitch transformation, much less the rhythmic retrograde. In the case of metric music, rhythmic retrogrades violate such expectations as long notes coinciding with "strong" beats or initiating measures, thus the new metric context of the retrograde succession makes this transformation difficult to recognize aurally. Yet, musical experience and intuition suggest that rhythmic retrogrades may be perceivable if their length is not excessive. Certainly the palindromic rhythms of Webern's <u>Piano Variations</u>, Opus 27, and <u>Symphonie</u>, Opus 21, can be heard for a short while, if not for the full extent of their length. The issue of rhythmic inversion is a more complex one. Few composers before this century attempted to "invert" duration successions, since there was no established procedure as to how inversion might operate in a temporal space. In some RI-invariant contours, such as the continuously-longer duration pattern of the rhythmic contour < 0 1 2 3 >, an inverted statement might seem easier to perceive. In such cases, however, the segment's inversion and its retrograde are equivalent; thus the listener may hear a relationship between the two segments because of their retrograde relation rather than their inversion relation. Finally, as noted in the previous chapter,

FIGURE III-8

DSEGCLASS EQUIVALENCE

P: <0 1 3 2> I: <3 2 0 1> RI: <1 0 2 3> R: <2 3 1 0>

	0	1	3	2
0	0	+	+	+
1	-	0	+	+
3	-	-	0	-
2	-	-	+	0

	3	2	0	1
3	0	-	-	-
2	+	0	-	-
0	+	+	0	+
1	+	+	-	0

	1	0	2	3
1	0	-	+	+
0	+	0	+	+
2	-	-	0	+
3	-	-	-	0

	2	3	1	0
2	0	+	-	-
3	-	0	-	-
1	+	+	0	-
0	+	+	+	0

W. Jay Dowling has asserted that perception of R-, I-, and RI-transformations upon melodies is based not on recognition of exact pitch or interval relationships, but primarily upon a more figural perception of the transformation upon melodic contour.[62] Perhaps such a case can also be made for transformations upon duration successions, perceived not in relation to absolute durations but to rhythmic contours of relative durations, particularly in contexts where that perception is not structured by beat or meter. Experimental testing into the perceptual validity of the d-space segment class remains for future research.

3. Similarity Relations for Rhythmic Contours

Two types of similarity relations between dsegs are proposed, the first based upon shared intervallic structure as shown in the COM-matrix, and the second upon shared dsubseg structure. These measurements are based on the similarity relations developed for contours in contour space, although they differ in a few respects. The first of these relations, DSIM(A,B), is illustrated in Figure III-9. The DSIM relation is restricted to comparisons between two csegs of equal cardinality. Like CSIM, this function compares specific positions in the upper right-hand triangles of the COM-matrices of two segments to total the number of similarities between them. The COM-matrix provides a useful tool for comparing every interval of duration to every other. For each compared position of identical content, the total is incremented by 1. Finally, in order to create a more uniform measurement across dsegs of various cardinalities,[63] the total is divided by the number of positions compared to return a number that

FIGURE III-9

DSIM(A,B)

A:

Varèse, <u>Octandre</u>
Oboe, m. 1 < 0 1 3 2 >

B:

Trumpet, m. 10 < 0 2 3 1 >

Matrices:

A:

$$\begin{array}{c|cccc} & 0 & 1 & 3 & 2 \\ \hline 0 & 0 & + & + & + \\ 1 & - & 0 & + & + \\ 3 & - & - & 0 & - \\ 2 & - & - & + & 0 \end{array}$$

B:

$$\begin{array}{c|cccc} & 0 & 2 & 3 & 1 \\ \hline 0 & 0 & + & + & + \\ 2 & - & 0 & + & - \\ 3 & - & - & 0 & - \\ 1 & - & + & + & 0 \end{array}$$

DSIM(A,B) = 5/6 = .83

Note: The duration succession < 0 2 3 1 > can also be realized in musical notation as:

This realization visually appears more similar to the rhythm of dseg A, and shows that .83 is an accurate representation of high similarity between the two dsegs.

FIGURE III-9
(CONTINUED)

DSIM(A,B)

FIGURE III-9B:

A:

Varèse, <u>Octandre</u>
Oboe, m. 8-11

<2 1 0 3 5 4>

B:

Varèse, <u>Octandre</u>
Oboe, m. 17-18

<5 0 2 3 4 1>

Matrices:

A:

	2	1	0	3	5	4
2	0	-	-	+	+	+
1	+	0	-	+	+	+
0	+	+	0	+	+	+
3	-	-	-	0	+	+
5	-	-	-	-	0	-
4	-	-	-	-	+	0

B:

	5	0	2	3	4	1
5	0	-	-	-	-	-
0	+	0	+	+	+	+
2	+	-	0	+	+	-
3	+	-	-	0	+	-
4	+	-	-	-	0	-
1	+	-	+	+	+	0

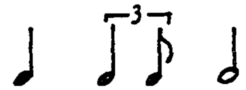
DSIM(A, B) = 9/15 = .60

signifies greater similarity as it approaches 1. Equivalent dsegs return a value of 1. Maxima and minima for segments of cardinalities three through six are discussed in Chapter Two and are presented in tabular form as Figure II-9. In Figure III-9A, the two matrices differ by only one position of the six; INT_1 and INT_3 are identical for both dsegs, only INT_2 differs. The two dsegs are maximally similar; DSIM(A,B) returns .83. Figure III-9B illustrates two larger and less similar dsegs. Here, only nine matrix positions are shared and none of the INT diagonals is identical between the two matrices. DSIM(A,B) in this case returns only .60. Finally, DSIM(A,B) may be generalized for dsegclasses in the same manner as was CSIM(A,B). That is, DSIM(A,B) returns the largest decimal number, or 1, obtained by comparing the COM-matrix of one dseg representative of dsegclass A with four dseg representatives (P, I, R, and RI) of dsegclass B. Therefore, DSIM(A,B) indicates the degree of highest possible similarity between two dsegclasses.

The second similarity measurement is the embedding function, of which there are several types. All are designed to count the number of times specific dsegs are embedded in the d-segments to be compared. The assumption here is that duration successions sharing a large number of equivalent dsubsegs will be perceived as more similar than duration successions that do not share such components. The most crucial difference between the embedding functions in d-space and their counterparts in c-space is the fact that the former consider only contiguous dsubsegs. Figure III-10 illustrates the three duration embedding functions directly analogous to the contour embedding functions discussed in the previous chapter. The first of these functions, DEMB(A,B), compares two dsegs of unequal cardinality, and is designed to count the number of times the smaller dseg, A, is embedded in the larger dseg, B. This total is then divided by the number of contiguous dsubsegs of the same cardinality as A possible in order to

FIGURE III-10

DURATION EMBEDDING FUNCTIONS

A =  = <2130> = d4-6

B =  = <2031> = d4-8

C =  = <35206417>

Contiguous dsubsegs of A, B, and C:

A = <2103>

<2103>

<210> = <210>
<103> = <102>

<21> = <10>
<10> = <10>
<03> = <01>

B = <2031>

<2031>

<203> = <102>
<031> = <021>

<20> = <10>
<03> = <01>
<31> = <10>

C = <35206417>

<35206417>

<3520641> = <3520641>
<5206417> = <4205316>

<352064> = <241053>
<520641> = <420531>
<206417> = <204315>

<35206> = <23104>
<52064> = <31042>
<20641> = <20431>
<06417> = <03214>

<3520> = <2310>
<5206> = <2103>
<2064> = <1032>
<0641> = <0321>
<6417> = <2103>

<352> = <120>
<520> = <210>
<206> = <102>
<064> = <021>
<641> = <210>
<417> = <102>

<35> = <01>
<52> = <01>
<20> = <10>
<06> = <01>
<64> = <10>
<41> = <10>
<17> = <01>

DEMB(A,C) = 2/5 = .40
DEMB(B,C) = 0

DMEMB3(A,C) = 3/8 = .38
DMEMB3(B,C) = 5/8 = .63

ADMEMB(A,C) = 16/24 = .67
ADMEMB(B,C) = 15/24 = .62
ADMEMB(A,B) = 8/12 = .67

return a value that approaches 1 for dsegs of greater similarity. The formula for determining the number of m-sized contiguous subsets of an n-sized set is (n + 1) - m. The figure shows two four-element dsegs, members of dsegclass d4-6 and d4-8, and an eight-element dseg to which they are compared. While dseg A is embedded in C two times, dseg B is not embedded in C at all. Thus DEMB(A,C) = 2/5 or .40 and DEMB(B,C) = 0. This measurement does not reflect the fact both dsegs B and C share a fairly large number of three-note subsegments, thus an extension of this similarity function is needed.

DMEMB$_n$(X,A,B) is designed to count the number of times the dsegs, X (of cardinality n), are embedded in both A and B. The total number of mutually-embedded dsegs of cardinality n is then divided by the number of n-cardinality dsubsegs possible in both dsegs to return a decimal number approaching 1 as dsegs A and B are more similar. As Figure III-10 shows, segments B and C share 5 instances of dsegs < 1 0 2 > and < 0 2 1 > out of the eight three-element dsubsegs possible between them (thus, DMEMB$_3$(X,B,C) = 5/8 = .63). Dsegs A and C, on the other hand, share only < 1 0 2 > three times. As mentioned in the previous chapter, this type of function returns a higher decimal number for embedded segments of smaller cardinalities since there are fewer segment types and therefore a higher probability of inclusion in both segments A and B. Thus, DMEMB$_2$(X,A,B) provides a rather weak representation of dseg similarity since only two dsubsegments, < 0 1 > and < 1 0 >, are possible. Further, the restriction of dsubsegments to contiguous occurrences also weakens the power of both these functions for predicting dseg similarity. A four-note segment, for example, can be embedded contiguously in a five-element dseg only twice; thus the function comparing these two dsegs can return only 1 (2/2), .50 (1/2), or 0 (0/2).

Because of the limitations cited above, ADMEMB(A,B) is designed as a finer measurement of similarity among d-segments. This function generalizes the $DMEMB_n(X,A,B)$ function over all cardinalities, n, 2 through the cardinality of the smaller of the two segments. It counts the total number of significant mutually-embedded dsegs and adjusts this to a decimal value by dividing by the sum of possible contiguous dsubsegs of both A and B, up to the cardinality of the smaller dseg.[64] As Figure III-10 shows, the dsegs A and B give almost identical similarity ratings in comparison with C when the mutual embedding function is generalized over various cardinalities. Finally, the theory can be extended for dsegclasses in the same way as the contour similarity functions were generalized. That is, DEMB(<u>A,B</u>), $DMEMB_n$(<u>X,A,B</u>) and ADMEMB(<u>A,B</u>) would compare the dsubseg content of dseg A with each of the four transforms of B (PB, IB, RB, and RIB) and return the highest of these values. It remains to be seen in experimental contexts, however, whether listeners can recognize these transformations upon duration successions.

To return briefly to the data on temporal perception discussed at the outset of this chapter, rhythms that can be understood in terms of a beat have been shown to be remembered and reproduced better by Western listeners than nonbeat-based rhythms. As previously noted in the discussion of experiments undertaken by Povel and Essens, listeners discern a beat most easily in duration successions where successive equally-spaced shorter durations may be heard as subdivisions of a longer duration. It is only in nonbeat-based rhythms--by extension, rhythms with fewer instances of repeated equal durations--that listeners perceive a rhythmic contour of relative shorts and longs without a precise notion of their proportional relationships. For this reason, the theory that models these rhythmic contours has not, up to this point, accounted for the instance of repeated equal durations. For the most part, however, the theory can be extended in duration space in the same way as it was in contour space. That is, a rhythm such as eighth-quarter-eighth-half would be modelled as < 0 1 0 2 >, as a "repeated-note contour" in duration space. Such a contour would generate a matrix containing zeros in positions other than along the main diagonal (as was shown in Figure II-17 and II-18). Further, a hyphenated composite dsegclass label could be assigned by comparing this repeated-note matrix with the matrices generated by the segments appearing on the list of prime forms. The primary difference between this generalization for duration space and that for contour space concerns the instance of consecutive equally-spaced durations. In c-space, two or more consecutive cps of the same pitch height were considered a single cp (as in Figure II-17A). In contrast, two or more consecutive equal durations in d-space must be considered independently. Consequently, a rhythm such as eighth-eighth-eighth-quarter will be modelled as < 0 0 0 1 >, a member of rd4-1/6.[65] This type of duration succession is more likely to be perceived in relation to a beat, however, and is not characteristic of the non-tonal works under consideration here, which tend to be ametrical.

4. Analytical Applications

Generalization of contour theory to the temporal domain enables analysts to address two aspects of musical structure--melodic contour relations and nonmetrical rhythmic structure--that are too often slighted in analyses of non-tonal compositions. The analysis of Edgard Varèse's Density 21.5 that follows incorporates both types of analysis, focusing upon recurring melodic and rhythmic contours that work in conjunction with pitch- and set-class structure to shape the work's formal design.

The rhythmic structure of Varèse's music has been the subject of some discussion in Jonathan Bernard's The Music of Edgard Varèse and the articles collected by Sherman Van Solkema in The New Worlds of Edgard Varèse: A Symposium,[66] although only the former source discusses Density 21.5 in any detail. Bernard devotes an entire chapter of his book to the questions of rhythmic and durational analysis, yet his discussion focuses almost entirely upon rhythm as a means of articulating pitch structure, rather than purely upon rhythmic and durational structure.[67] The chapter opens with a consideration of the following premise: that "some scholars and critics have said, explicitly or implicitly, that Varèse assigned the domains of rhythm, duration, timbre, and loudness a status equal to that of pitch."[68] He notes that there is no way to prove such a claim except to demonstrate its validity through analysis, and he questions how such an analysis might proceed:

> To consider pitch, register, rhythm, duration, and so forth all on an equal footing, there would have to be some sort of continuum that included all these domains: a scale of values held in common.
> As an example, consider the hypothetical assertion that dynamic levels in some given instance are as important as the pitches subjected to these fluctuations in loudness.... To make sense of this statement one would have to find a way of expressing the information conveyed by the pitches and that conveyed by the dynamics as components of a single analytical image.[69]

Substitution of "durations" for "dynamics" in the hypothetical assertion above produces exactly the type of analysis to be undertaken here. That is, a single theoretical system will be used to compare similarity and equivalence relations among contours (as registral pitch height successions), rhythms (as duration successions), and pitch structures (as pitch-class sets). Chapter four of this study will suggest ways in which the same theory might be used to model other facets of musical structure, such as dynamics and chord spacing. Varèse's music provides an ideal context in which to illustrate analytical applications of the rhythmic theory proposed here since, as Bernard notes, the composer's "penchant for rhythmic complexity seems to have been aimed at nearly complete and constant disruption of pulse, of any semblance of regularity in beat pattern.... It is difficult," he continues, "to find passages in Varèse where the beat, or even some simple subdivision or compound of it, is literally stressed for more than a couple of measures."[70] He describes here precisely the type of nonbeat-based context that listeners are most likely to perceive in terms of a rhythmic contour.

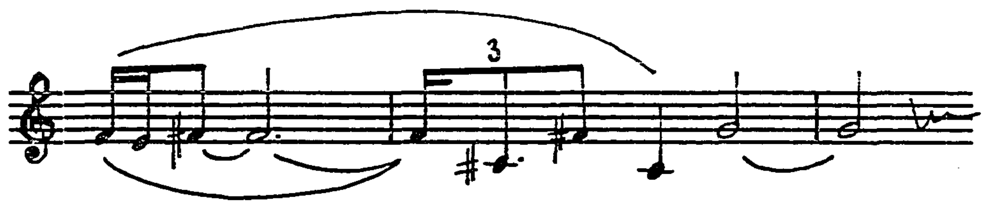
Density 21.5 is the most often analyzed of Varèse's compositions. Bernard, for example, cites four other analyses before providing his own.[71] James Tenney and Larry Polansky graphically compare their Gestalt-based segmentation with Jean-Jacques Nattiez's formal analysis.[72] In spite of the fact that the work's structure is as much founded upon recurring rhythmic contours as it is upon recurring pitches or set classes, none of these analyses adequately addresses the issue of rhythmic structure. Density 21.5 may be divided into three large sections, as illustrated in Figure III-11. Two of these sections are further subdivided: B containing two contrasting subsections marked B-I and B-II, and A' comprising a return of material from both the A and B sections. The boundary between subsections in each case is marked by a recurring fanfare-like motive labelled "x" in the diagram.

A recurring rhythmic figure, < 0 0 1 >, initiates most phrases of the A section, as shown in melodies A through F of Figure III-12. Segment D consists entirely of this duration succession. All of the remaining segments begin with either < 0 0 3 2 1 > or < 0 0 2 3 1 >. Although these two successions do not share many embedded subsegments, comparison of their matrices reveals a high degree of similarity; their content is identical in nine out of ten positions. Three of the six segments cited here (labelled B, C, and F) begin with the rhythmic contour < 0 0 2 3 1 >, yet their

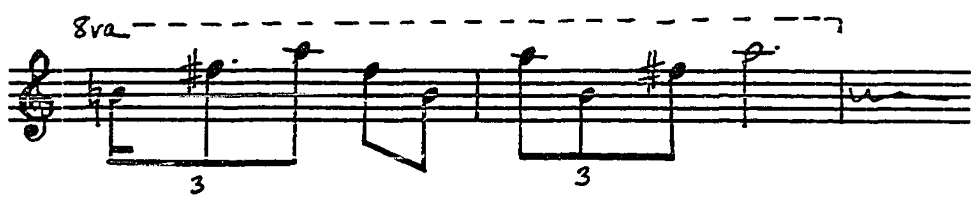
FIGURE III-11
FORMAL DESIGN OF VARESE, <u>DENSITY 21.5</u>

A (mm. 1-23)	B (mm. 24-40)				A' (mm. 41-61)		
	B-I	"x"	B-II	A'	B-II'	"x"	B-I'
	24-28	29-32	33-40	41-45	46-51	51-53	53-61

<u>Primary thematic material:</u>

A:


B-I:


B-II:


"x":


FIGURE III-12

PRIMARY RHYTHMIC CONTOURS IN DENSITY 21.5 A AND A' SECTIONS

A Section:

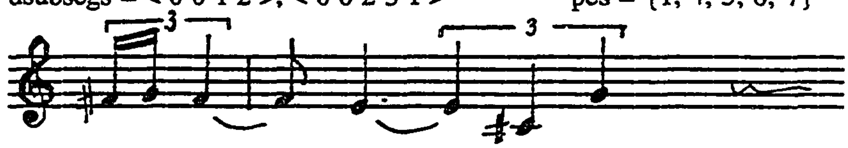
A: mm. 1-2

dseg = < 0 0 4 2 1 3 5 > cseg = < 2 1 3 0 3 0 4 > sc 5-4
dsubsegs = < 0 0 2 1 >, < 0 0 3 2 1 > pcs = {1, 4, 5, 6, 7}
by translation

B: mm. 3-4

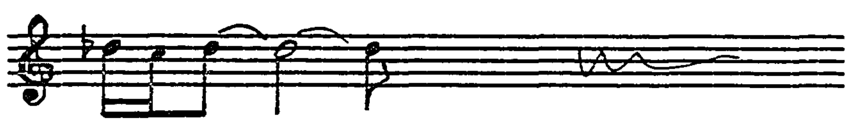
dseg = < 0 0 2 3 1 2 > cseg = < 2 1 3 4 0 4 > sc 5-4
dsubsegs = < 0 0 1 2 >, < 0 0 2 3 1 > pcs = {1, 4, 5, 6, 7}

C: mm. 4-5



dseg = < 0 0 2 3 1 1 > cseg = < 2 3 2 1 0 3 > sc 4-13
dsubsegs = < 0 0 1 2 >, < 0 0 2 3 1 > pcs = {1, 4, 6, 7}

D: m. 9



dseg = < 0 0 1 > cseg = < 1 0 1 > sc 2-1
pcs = {0, 1}

E: m. 15

dseg = < 0 0 3 2 1 > cseg = < 1 0 2 0 2 > sc 3-1
dsubseg = < 0 0 2 1 > pcs = {3, 4, 5}

F: mm. 21-22

dseg = < 0 0 2 4 1 3 4 5 > cseg = < 2 1 2 0 1 2 0 1 > sc 3-1
dsubsegs = < 0 0 1 2 >, < 0 0 2 3 1 > pcs = {9, 10, 11}

FIGURE III-12
(CONTINUED)

PRIMARY RHYTHMIC CONTOURS IN DENSITY 21.5 A AND A' SECTIONS

< 0 0 1>, < 0 0 2 1 > and < 0 0 1 2 > Embedded as Dsubsegs

A' Section:

G: mm. 41-42

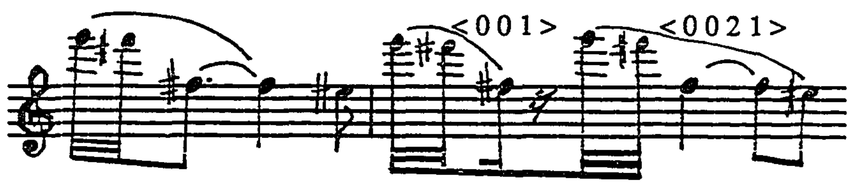
dseg = < 0 0 1 >   cseg = < 1 0 2 >   sc 3-1
pcs = {5, 6, 7}

H: mm. 42-43

dseg = < 0 0 3 3 3 1 1 2 2 2 4 >
cseg = < 2 1 3 1 2 3 2 1 3 0 4 >   sc 5-4
dsubseg = < 0 0 1 >   pcs = {2, 5, 6, 7, 8}
bracketed dsubsegs: both = < 0 0 1 1 1 >

"X" Sections (based on A's rhythmic contour):

I: mm. 29-30

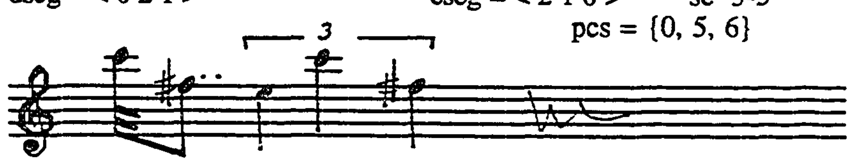


dseg = < 0 0 2 1 >   cseg = < 3 2 1 0 >   sc 3-1
pcs = {5, 6, 7}

J: mm. 51-52

dseg = < 0 2 1 >   cseg = < 2 1 0 >   sc 3-5
pcs = {0, 5, 6}

K: m. 52



dseg = < 0 2 1 1 1 >   cseg = < 2 1 0 2 1 >   sc 3-5
dsubseg = < 0 2 1 >   pcs = {0, 5, 6}

melodic contours and set-class structures differ. Melody C may be heard as a variation of B, since it immediately follows B musically and since its rhythmic contour differs only with respect to the final duration. Further, the pitches of melody C are a literal subset of those in melody B (which, incidently, are the same pitches as melody A--thus A, B, and C form a kind of "continuous variation"). The melodic contours of the first four-notes of B and C differ a great deal: in terms of adjacent pitches, < - + + > in B as opposed to < + - - > in C. Although melody F begins with the same rhythmic contour as segments B and C, it contains no common pitch classes with either segment. Further, segment F's contour differs markedly from the others. This melody contains only 3 distinct pitches, forming the repeated-note contour < 2 1 2 0 1 2 0 1 > and the chromatic set class 3-1. The remaining two segments (A and E) are very similar rhythmically, since the rhythmic contour of the latter, < 0 0 3 2 1 >, can be embedded contiguously in the former. The first three notes of each forms the 3-1 trichord, as did segment F. Neither the melodic contour nor pitch class content of E can be embedded literally in A, however.

The three-note figure with which the A' section begins, segment G of Figure III-12, marks a return to the duration succession, melodic contour, and 3-1 set class of the composition's opening, although the precise pitches differ by a semitone. The composer repeats and expands this motive in melody H, which follows G immediately in the score. Segment H is a member of set class 5-4, as were the opening two melodies of the work, this time stated a semitone higher. The rhythmic contour of this final reference to the A section's material differs most strongly from the contours of the rhythms that preceded it. This duration succession, < 0 0 3 3 3 1 1 2 2 4 >, contains four instances of repeated equal durations, and more closely resembles the repeated-duration successions that are featured in the B section than the rhythms of the A section.

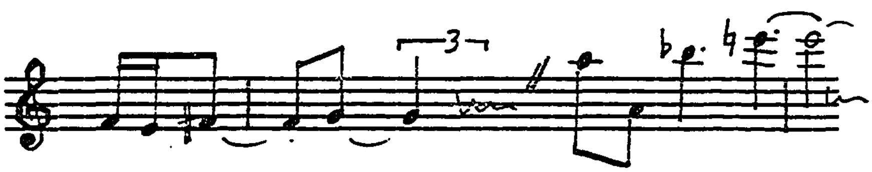
As mentioned previously, both the B and A' sections are bisected by the fanfare-like motive that has been labelled "x". This motive provides contrast to the material that surrounds it by virtue of its sudden change of register and dynamic, but the contour segments and duration successions used here are not new to the work. Segment I of Figure III-12 contains three short statements of "x"--the first and last forming dseg < 0 0 2 1 > (which was heard previously in segments A and E) and the central statement forming dseg < 0 0 1 >, the segment that has been heard repeatedly in the work as a kind of rhythmic "head motive" of every melody discussed thus far. In this case, however, the motive is initiated with equal-duration thirty-second notes rather than the sixteenths of most previous statements. The melodies of segments I, J, and the initial notes of K are the continuously-descending melodic contours < 2 1 0 > and < 3 2 1 0 >; these contours recur regularly throughout the piece as do their inversions, to be discussed in Figure III-14 following.

This analysis of melodic and rhythmic contours in Varèse's Density 21.5 closes with a discussion of two compositional techniques used in this work, the analysis and discussion of which are greatly enhanced by the precise language of contour theory. The first of these involves the composer's development of melodic material by registral expansion--that is, by varying the pitch-space realization of a reiterated c-segment. The second involves his use of contour equivalence spanning both the pitch-registral and temporal domains. Figure III-13 shows three instances of pitch-space expansion within recurring equivalent contours, a technique that plays an important role in linking the A and A' sections, as well as the B and B' sections. The first example of contour expansion occurs within the A section, measures 3-4 and 13-14. Here, the cseg

FIGURE III-13

CONTOUR EXPANSION IN <u>DENSITY 21.5</u>

A section:



cseg < 1 0 2 3 > expanded, mm. 3-4 and 13-14
dseg < 0 0 1 2 > expanded, mm. 3-4 and 13-14

A and A' sections:

mm. 9-10

mm. 46-48

cseg < 1 0 1 0 1 0 1 0 1 0 > expanded

B and B' sections:

mm. 22-24

mm. 56-58

cseg < 0 1 3 0 3 0 > expanded (second time adds one cp 1)

< 1 0 2 3 > is expanded from a total pitch compass of a minor third to one of a perfect twelfth. Likewise, the rhythm of mm. 13-14 represents a contour expansion of the first in sequential time--both are d-space statements of < 0 0 1 2 >. Second, measures 9-10 of the A section contain a long melody that for two measures oscillates up and down a semitone, creating the cseg < 1 0 1 0 1 0 1 0 1 0 >. This contour recurs in the A' section, measures 46-48, this time as a minor third oscillation. In the third instance, the melody beginning in the second measure of the B section (m. 22) forms cseg < 0 1 3 0 3 0 >. It returns in the B' section expanded in register by a semitone, and with one additional cp.

A striking feature of this work is the occurence of several segments that can be represented by the same succession of c-pitches as durations--that is, segments that are equivalent in c-space and d-space. Melody A of Figure III-14 is one such example. This melody is structured such that each successive pitch is both higher and longer than the one that preceded it; thus both facets of its structure can be represented by the succession < 0 1 2 3 >. Although melody B begins with a repeated duration, the rhythmic and melodic contours of the last three notes can be represented by the succession < 0 1 2 >. Similarly, melody C contains one repeated duration (this time at the end of the melody) but, disregarding this repetition, its initial contour may be represented in both the registral and temporal domains as < 0 1 2 3 >. A case might be made that the continuously-ascending melody functions as a cadential gesture in this work, since melody C serves this function immediately before the B-II section begins, and the ascending melody of mm. 44-45 (not shown in the figure) concludes the first subsection of the A' section. Further, the melody that concludes the entire composition (melody E of Figure III-14) is the longest continuously-rising contour of the work, composed of csegclass c7-1, cps < 0 1 2 3 4 5 6 >. Although its duration succession

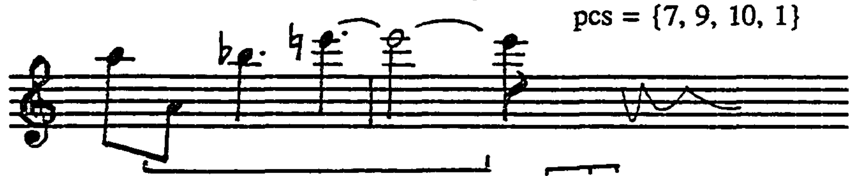
FIGURE III-14

A RECURRING MELODIC/RHYTHMIC CONTOUR, DENSITY 21.5

A: m. 6

dseg = < 0 1 2 3 > cseg = < 0 1 2 3 > sc 4-12
pcs = {7, 9, 10, 1}

B: mm. 13-14



dseg = < 0 0 1 2 > cseg = < 1 0 2 3 > sc 3-5
csubseg = < 0 1 2 > (twice)
pcs = {4, 9, 10}

C: mm. 31-32

dseg = < 0 1 2 3 3 > cseg = < 0 1 2 3 4 > sc 5-5
pcs = {4, 8, 9, 10, 11}

D: m. 55

dseg = < 0 1 2 > cseg = < 0 1 2 > sc 3-3
pcs = {4, 7, 9}

E: mm. 58-61

dsegs = < 1 0 1 2 >, < 0 1 2 > segmented by slurs
cseg (entire excerpt) = < 0 1 2 3 4 5 6 > sc 7-37
pcs = {10, 11, 1, 2, 3, 5, 6}

does not also represent d7-1, the melody's subsegments (as indicated by the composer's slurring) contain two instances of embedded d3-1's. Thus, the final melody of the work has been prepared aurally by similar cadential gestures, and its contour and rhythmic structure heard previously in other contexts.

Finally, generalization of contour theory into other domains enables the analyst to compare diverse facets of musical structure along a single sequential scale. The preceding analytical examples drawn from Varèse's Octandre and Density 21.5 have shown ways in which analysis of duration successions as rhythmic contours clarifies some aspects of one composer's musical language.

NOTES

[1]This is not to say that such a perceptual model is universal. Listeners native or acculturated to Arabic or Indian music hear rhythms as additive rather than divisive. Thus, one would expect that the pattern eighth-quarter-quarter would be as easily structured cognitively as the pattern eighth-eighth-quarter to these listeners, despite the fact that the former does not conform to a beat-based model. It seems clear that the psychological studies cited in this chapter are biased toward Western tradition by virtue of the musical backgrounds of the listeners who participate in these experiments. However, it is interesting to note that the duration-space classes to be posited below model certain aspects of Indian rhythmic practice, since some rhythmic talas considered to be variations of each other belong to the same d-space segment class (see page 155 and note 54 following).

[2]Dirk-Jan Povel, "Internal Representation of Simple Temporal Patterns," Journal of Experimental Psychology: Human Perception and Performance 7/1 (1981), pp. 16-17.

[3]For purposes of this discussion, it is assumed that the beat unit falls within this medium duration. As Stephen Handel and James Oshinsky note ("The Meter of Syncopated Auditory Polyrhythms," Perception & Psychophysics 30/1 (1981), "Rhythm cannot be abstracted from presentation rate or tempo. Rhythm does not remain invariant across changes in tempo; rather, the rhythm emerges at a specific tempo" (p. 9). More specifically,

> ...there are rates so slow that the elements of a pulse train are too separated temporally to remain coherent. The elements are phenomenologically "individual" and are not heard as a regularly recurring background. The essential periodicity or repetition of the rhythm is lost. Similarity, there are rates so fast that the elements of a pulse train are too close temporally to remain background. (p. 8)

[4]Dirk-Jan Povel and Peter J. Essens, "The Perception of Temporal Patterns," Music Perception 2/4 (1985), p. 414.

[5]Povel and Essens, p. 413-414.

[6]Ibid., p. 413.

[7]Ibid., p. 413-414. The duration successions cited by Povel and Essens here may be transcribed in musical notation as in note 1 above. The reader is reminded that these experimental results show a Western bias toward divisive rhythms. One might

predict that the same experiment conducted with subjects from non-Western musical traditions (particularly Indian or Arabic) would produce different results.

[8]Peter J. Essens and Dirk-Jan Povel, "Metrical and Nonmetrical Representations of Temporal Patterns," Perception and Psychophysics 37/1 (1985), p. 3.

[9]Ibid., p. 4.

[10]See, for example, Allen Forte's "Aspects of Rhythm in Webern's Atonal Music," Music Theory Spectrum 2 (1980), pp. 90-109. His proportional graph is designed in precisely this way:

> The integer value 1 is assigned to the smallest durational value in the work (movement). The largest value is the least common multiple of all the other values.... The result is a depiction of a precise calibration of component durations, so that any temporal span or pattern can be compared with any other. (p. 91)

[11]See Milton Babbitt, " Twelve-Tone Rhythmic Structure and the Electronic Medium," Perspectives of New Music 1/1 (1962), pp. 49-79; reprinted in Perspectives on Contemporary Music Theory , ed. Benjamin Boretz and Edward T. Cone (New York: W.W. Norton & Company, Inc., 1972), pp. 148-179.

[12]In "African Rhythm: A Reassessment," (Ethnomusicology 24/3, 1980, pp. 393-415), Robert Kauffman expands upon various current theories of rhythmic structure in African music. Among these is analytical use of the "density referent," a concept defined by Mantle Hood in The Ethnomusicologist (New York: McGraw Hill, 1971), that refers to the fastest regularly recurring event. Kauffman notes that the density referent

> ...can be used to study and understand temporal elements that would be rendered ambiguous by reference to more subjective concepts of beat. For example, a beat of MM60 can also be perceived as two beats at 120. Density referent, being faster than beat, is not subject to such ambiguities. (p. 396)

While density referent is a useful analytical tool, Kauffman suspects that it is not commonly used by African performers or listeners to structure rhythmic patterns perceptually. After observing a teacher instructing African drumming students, Kauffman notes that

> ...Ayitee did not ask his students to count out eight fast pulses. Instead he wanted them to respond to the gestalt of the two drum parts. This would seem to suggest that density referent is only one level of a larger metrical organization. (p. 396)

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[13]Maury Yeston, The Stratification of Musical Rhythm (New Haven: Yale University Press, 1976), p. 151-152.

[14]Ibid., p. 148.

[15]Essens and Povel, "Metrical and Nonmetrical Representations," p. 1.

[16]Ibid.

[17]Stephen Handel and James S. Oshinsky, "The Meter of Syncopated Auditory Polyrhythms," Perception & Psychophysics 30/1 (1981), p. 2.

[18]Ibid., p. 8.

[19]Ibid, p. 9.

[20]Grosvenor W. Cooper and Leonard B. Meyer, The Rhythmic Structure of Music (Chicago: The University of Chicago Press, 1960), p. 4.

[21]Ibid., p. 6.

[22]Povel and Essens, "The Perception of Temporal Patterns," p. 432.

[23]See Yeston, The Stratification of Musical Rhythm, Example 2:1 for two metric and two nonmetric interpretations of this duration succession.

[24]In addition to the works of Povel and Essens already cited, see Diana Deutsch's "Recognition of Durations Embedded in Temporal Patterns, Perception & Psychophysics 39/3 (1986), pp. 179-186 for further experimental confirmation of this statement.

[25]Povel, "Internal Representation," p. 9.

[26]David Lewin ["Some Investigations into Foreground Rhythmic and Metric Patterning," in Music Theory: Special Topics, ed. Richmond Browne (NY: Academic Press, 1981), pp. 101-137] cites Bamberger's lecture in the panel "Cognitive Approaches to Music Composition and Perception," at the First International Conference on Computer Music, at the Massachusetts Institute of Technology, October 29, 1976, as his point of departure for a study of rhythmic and metric patterning based upon a temporal interval vector.

[27]Lewin, p. 101.

[28]Povel, "Internal Representation," p. 16.

[29]Further, nonbeat-based patterns are found in the rhythms of speech. One type of 16th-century chanson (vers mesuré) contains melodic lines in which rhythms are approximated transcriptions of the rhythmic patterns of speech. Although the rhythms are limited by the constraints of Western notation (toward 1:2 or 1:3 ratios) this technique resulted in highly syncopated, rhythmically-complex textures.

[30]Povel and Essens, "The Perception of Temporal Patterns," p. 437.

[31]Jeffrey J. Summers, Simon R. Hawkins, and Helen Mayers, "Imitation and Production of Interval Ratios," Perception & Psychophysics 39/6 (1986), pp.437.

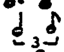
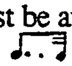
[32]Povel, "Internal Representation," p. 17.

[33]Jacqui Smith, "Reproduction and Representation of Musical Rhythms: The Effects of Musical Skill," in The Acquisition of Symbolic Skills, ed. Don Rogers and John A. Sloboda (NY: Plenum Press, 1983), pp. 273-282.

[34]Ibid., p. 276.

[35]Eric F. Clarke, "Structure and Expression in Rhythmic Performance," in Musical Structure and Cognition, ed. Peter Howell, Ian Cross, and Robert West (London: Academic Press, 1985), pp. 225-226.

[36]This type of error has also been discussed with reference to African drumming performance. In discussing applications of the "density referent" concept, Kauffman (cited above) notes that

> A performer of the pattern  must be aware of the density referent  in order to avoid the errors  or  , but he will ultimately respond to the larger gestalt of each beat or of the entire measure. Thus it would also seem that African musicians respond to some type of metrical organization, which may include various combinations of the density referent. (pp. 396-397)

The rhythms that Kauffman cites are all unequal beat divisions, which may be categoried as long-short, and are therefore easily confused.

[37]Eric F. Clarke, "Some Aspects of Rhythm and Expression in Performances of Erik Satie's 'Gnossienne No. 5,'" Music Perception 2/3 (1985), pp. 323-4.

[38]Povel posits a similar construct, a "temporal grid," in "A Theoretical Framework for Rhythm Perception," in Psychological Research 45 (1984), pp. 315-337.

[39]Ibid., p. 320.

[40]Povel, "Internal Representation," p. 13.

[41]It should be mentioned here that production of rhythmic patterns from notation and reproduction from aural stimuli are two very different tasks. Summers, Hawkins, and Mayers, cited above, have discovered that different strategies are used for the two tasks, and that a musician may be able to tap a nonmetrical rhythm accurately (probably by counting multiples of a small duration unit) but be unable to reproduce the same rhythm from aural stimuli with the same degree of accuracy. By extension, a performer may be able to perform septuplets, for example, but this beat subdivision may not be accurately perceived by his or her audience since division into sevenths may not be a part of the perceivable metrical matrix.

[42]Clarke mentions this analogy in both the articles cited above, "Some Aspects of Rhythm and Expression," pp. 324-5, and "Structure and Expression in Rhythmic Performance," p. 226.

[43]W. Jay Dowling and Dane L. Harwood, Music Cognition (NY: Academic Press, Inc., 1986), pp. 187-188. The authors cite, in particular, Chapter 5 of Monahan's dissertation, "Parallels between Pitch and Time: The Determinants of Musical Space" (Ph.D. dissertation: University of California, Los Angeles, 1984).

[44]Robert Morris, Composition with Pitch Classes, Definition 7.2, p. 299. The final chapter of Morris's book discusses isomorphisms between musical time and pitch spaces. He generalizes the concept of contour as well, as "a set of points in one sequential dimension ordered by any other sequential dimension" (p. 283). Thus, S_1-time ordered by S_2-time is a contour, where S_1-time is order in time and S_2-time is order in C-space.

[45]Ibid., Definition 7.3.

[46]Ibid., Definition 7.4, p. 301.

[47]David Lewin, Generalized Musical Intervals and Transformations, pp. 22-23.

[48]Ibid., pp. 23-24.

[49]Ibid.

[50]Ibid., p. 24.

[51]As mentioned in a previous note, Morris defines a contour (Def. 7.1.2) as "a set of points in one sequential dimension ordered by any other sequential dimension."

In this case, the first sequential dimension is duration, ranging from short to long, ordered by s-time corresponding to temporal order.

[52] As Povel noted in the experiments discussed previously, a beat emerges in duration successions where successive equally-spaced shorter durations may be heard as subdivisions of a longer duration. By extension, in beat-based rhythms, strings of equally-spaced durations are common (two eighths or four sixteenths, for example). However, in duration successions where no two durations are of equal length, a beat is difficult to hear unless supplied in an accompanying line. Musically-trained listeners may try to "impose" a beat to structure their listening, but if this strategy fails they too rely upon a perceived rhythmic contour of relative shorts and longs.

[53] Gérard Grisey, "Tempus ex Machina: A Composer's Reflections on Musical Time," Contemporary Music Review 2/1 (1987), pp. 240.

[54] Kanthimathi Kumar and Jean Stackhouse, Classical Music of South India: Karnatic Tradition in Western Notation (Stuyvesant, NY: Pendragon Press, 1988), pp. 21-23.

[55] Ibid., p. 23.

[56] Formulation of these definitions and those that follow are indebted to those for contour space in Morris, cited above. Note that this application of the COM-matrix differs somewhat from Morris's temporal applications of contour theory.

[57] Note in Figure III-4 that the fourth dur of dseg D is ornamented by d#-e grace notes. The grace notes will not be considered two separate (very short) durs, but rather as ornaments (like a trill) belonging to the d#, and lengthening its duration slightly.

[58] Eric F. Clarke, "Structure and Expression in Rhythmic Performance," p. 212.

[59] The COM-matrix forms the basis for equivalence and similarity relations both in c-space and d-space. It should be noted that the entire matrix is necessary to give a complete profile of a segment's structure. It would be meaningless to compare only duration adjacencies (INT_1), for example; rather, meaningful statements about equivalence or similarity can only be made when every pair of durations, contiguous or not, is compared to every other pair. This is not to say that these comparisons should be viewed as some sort of entity in themselves (as actual subsegments, for example) that appear in the score or are perceived upon hearing the dseg.

[60] As mentioned previously, the isomorphism between contour space and duration space provides a useful theoretical model for analysis of rhythmic transformations in some non-tonal music. This is not to assert that the operations R, I, and RI have the same perceptual validity in both spaces.

[61]These may be compared with the contours graphs of Figure II-6 for < 0 1 3 2 > realized in c-space.

[62]See Chapter Two, footnote 32.

[63]For further explanation, see discussion of CSIM in Chapter Two.

[64]The term "significant" here simply refers to dsubsegs of cardinality larger than 2. The number of contiguous subsegments in a segment of cardinality n, excluding the null and one-note subsegments, may be found using the following formula:

$$\frac{(n^2-n)}{2.}$$

[65]Here, "rd" signifies a repeated-duration dseg. Its membership in rd4-1/6 is demonstrated by the matrices below:

$$\begin{array}{c|cccc} & 0 & 0 & 0 & 1 \\ \hline 0 & 0 & 0 & 0 & + \\ 0 & 0 & 0 & 0 & + \\ 0 & 0 & 0 & 0 & + \\ 1 & - & - & - & 0 \end{array} \qquad \begin{array}{c|cccc} & 0 & 1 & 2 & 3 \\ \hline 0 & 0 & + & + & + \\ 1 & - & 0 & + & + \\ 2 & - & - & 0 & + \\ 3 & - & - & - & 0 \end{array} \qquad \begin{array}{c|cccc} & 2 & 1 & 0 & 3 \\ \hline 2 & 0 & - & - & + \\ 1 & + & 0 & - & + \\ 0 & + & + & 0 & + \\ 3 & - & - & - & 0 \end{array}$$

4-1/6 4-1 4-6

[66]Jonathan Bernard, The Music of Edgard Varèse (New Haven: Yale University Press, 1987) and Sherman Van Solkema, ed., The New Worlds of Edgard Varèse: A Symposium (NY: Institute for Studies in American Music, 1979), which includes Elliot Carter's "On Edgard Varèse," Robert Morgan's "Notes on Varèse's Rhythm," and Chou Wen-chung's "Ionisation: The Function of Timbre in Its Formal and Temporal Organization."

[67]The two primary subdivisions of Bernard's (cited above) fourth chapter, "Rhythm and Duration," are entitled "Pitch Stasis" and "The Role of Rhythm in 'Motivic' or 'Thematic' Reference."

[68]Ibid., p. 128.

[69]Ibid., p. 129.

[70]Ibid., p. 133.

[71]These analyses are Jean-Jacques Nattiez,"Varèse's Density 21.5: A Study in Semiological Analysis," trans. by Anna Barry, Music Analysis 1/3 (1982), pp. 243-340; Marc Wilkinson, "An Introduction to the Music of Edgar Varèse," The Score and I.M.A. Magazine 19 (1957), pp. 5-18; Martin Gümbel, "Versuch an Varèse Density 21.5," Zeitschrift für Musiktheorie 1/1 (1970), pp. 31-38; and James Tenney with Larry Polansky, "Temporal Gestalt Perception in Music," Journal of Music Theory 24/2 (1980), pp. 205-241. Bernard's analysis concludes The Music of Edgard Varèse, pp. 217 to 232.

[72]Tenny and Polansky, pp. 222-226.

Chapter Four

Perceptual Strategies to Pedagogical Strategies

1. Music-Psychological Research: Critical Overview and Avenues for Future Experimentation

An overview of experimental research in the perception of melodic and rhythmic contours, such as that provided here, would not be comlete without critical examination of some of the problems inherent in these studies and an enumeration of some perceptual questions still to be answered. To recapitulate, experimental data, principally from the work of W. Jay Dowling and Judy Edworthy,[1] supports the hypothesis that moderately-experienced listeners (without "absolute" pitch) retain brief tonal melodies in short-term memory primarily as contours related to an underlying diatonic framework, rather than as pitch-class or interval successions.[2] Without specific training and in the absence of a diatonic framework, however, listeners appear to remember non-tonal melodies in short-term memory solely in terms of contour. Several psychologists have shown that listeners use different strategies depending upon whether short-term or long-term memory is required, upon the length of the melody, and upon the amount of their previous musical training.[3] Experimenters agree that pitch or interval information is more salient in long-term memory, while contour information is more salient in short-term; contour information is immediately available after an initial hearing, but interval information, once acquired, is more resistant to forgetting. The primary problem with these experiments' conclusions is the imprecise way in which certain terms are defined. For example, the authors use the terms like "pitch perception" when they may be referring to pitch-class perception; further, their understanding of contour may be based solely on INT_1, as mentioned previously. There is some evidence that Edworthy confuses pitch- or pc-based strategies with interval-based strategies. She makes this statement about one of her experimental conditions, for example:

> ...it suggests that the intervallic information (pitch) is better processed after a few notes have been heard, rather than at the beginning. It seems eminently reasonable to suggest that, if the key of a melody is not known beforehand (which is the normal state of affairs) then a few notes are required to actually establish a tonality; up to this point, the contour relationships may be more salient.[4]

In a subsequent article, she concludes that "although immediately precise, contour information is easily lost whereas interval information once obtained is more resistant to forgetting."[5] This seems highly suspect, since directed intervals "contain" contour information; thus if intervals are retained, contour should not be lost. Again, the terminology is imprecise and these conclusions must remain inconclusive.

The strategies by which listeners recognize R-, I-, and RI-transformations of melodic segments are discussed primarily by Dowling, and Krumhansl, Sandell, and Sergeant.[6] The question of whether listeners use a contour-, pc-, or interval-based strategy to recognize row transformations still remains unresolved. As discussed in the first chapter, Dowling hypothesizes that listeners probably use a pitch-based strategy because listeners in his experiments identified the inverted transformation with the fewest errors and the retrograde-inversion with the most errors.[7] The author does not carefully distinguish between pitch and pitch class in his discussion, nor between strategies used by trained vs. untrained musicians. Further, he notes that his subjects often confused exact transformations with same-contour transformations. For this reason, his attempt to determine whether listeners use an interval- or pc-based strategy seems futile, since they may have relied entirely upon a contour-based strategy. Dowling bases his conclusion on the assumption that a two-operation transformation should be more difficult to perceive than a one-operation transformation, and notes that a pc-based strategy requires two operations for the RI-transformation and one for I- and R-transformations. In contrast, he points out that an interval-based strategy requires two for the R-transformation and one for the others.[8] Dowling's argument is flawed for several reasons. As mentioned previously, it neglects the role of contour as a strategy. Second, it fails to take transposition into account; his experiment uses identical pcs for the original segment and, in reverse order, for its retrograde. Except under T_0, the RI-transformation takes not two but three operations (R, T_n, and I), while the I- and R-transformations take two each (T_n and I, or T_n and R). An interval-based strategy, on the other hand, is unaffected by transposition: inversion and retrograde-inversion require only one transformation each, and retrograde two (Figure I-2C). Thus, his finding that I-transformations are most easily recognized points to a one-operation interval-based strategy. Further, his hypothesized pc-based strategy intuitively seems unlikely, since it implies that all listeners have some capacity for absolute pitch-class memory.

Because they were aware of Dowling's conclusions, Krumhansl, Sandell, and Sergeant attemped to minimize the influence of contour upon subjects' determinations in one experiment by using "Shepard tones"--circular electronic tones that have no clearly-perceived octave placement.[9] In another experiment, they used rhythms and contours drawn from Schoenberg's musical contexts, which might or might not preserve contour relationships under the various transformations. Thus in neither experiment could recognition of row transformations be based entirely upon contour cues, although the latter may have had some influence. No experimental condition was designed specifically to test whether subjects used contour-, pc-, or interval-based strategies to make their determinations. In both experiments Krumhansl, Sandell, and Sergeant found that their experienced subjects, like Dowling's, recognized I- and R-transformations over RI-, while inexperienced subjects recognized the RI-transformation most easily. Because RI- is the only transformation to preserve contour (albeit in reversed order) they hypothesize that musically-inexperienced listeners are more likely to use contour cues in recognizing transformations of twelve-tone rows, while experienced listeners use either pitch-class or interval information. In addition, for both groups of listeners, they found that the number of changes in contour direction correlated significantly with classification accuracy--that is, rows presented with more complex contours were more difficult to recognize.

Music psychologists have also addressed the issue of octave equivalence, which is assumed in both tonal and most non-tonal music theories. Diana Deutsch posits that listeners' strategies differ according to musical context; in contexts consisting of single tones or chords related by harmonic inversion, octave equivalence can be perceived, while in melodic contexts listeners do not recognize octave-related pitches as equivalent.[10] This has direct implications for contour perception, since octave displacement alters melodic contour. Experimentation using octave-displaced tones in twelve-tone rows has shown that such displacement negatively affects listeners' ability to recognize row transformations; however, Krumhansl, Sandell, and Sergeant have shown that for musically-experienced listeners it does not destroy perceptibility.[11]

Rhythmic experimentation, particularly that of Dirk-Jan Povel and Eric F. Clarke, has shown that subjects cannot remember precise durations independently; rather, they relate the durations in temporal sequences to an internally-generated, equally-spaced beat.[12] This relationship to the beat is not precisely measured, but perceived as equal or unequal beat subdivisions. Further, unequal subdivisions are remembered simply as patterns of longs and shorts rather than as precise ratios of beat subdivisions.[13] So strongly is perception of temporal sequences tied to this underlying beat framework that listeners usually do not recognize identical successions when presented in contrasting frameworks (Figure III-2). When an underlying beat is absent listeners perceive the durations imprecisely and distort such rhythms when asked to reproduce them. In the absence of a beat with which to structure their perception, listeners generally remember rhythms in terms of what Jeanne Bamberger calls "figural" patterns, as rhythmic contours of relative longs and shorts.[14] Listeners may also use Gestalt-like grouping mechanisms to perceive successive short durations as distinct subgroups.

There remain many questions still to be answered by music-psychological experimentation. Some questions have already been addressed by psychologists, but their answers remain inconclusive. Thus, new experiments need to be designed to test the effect of octave displacement on same/different comparisons, the effect of tonal perceptual mechanisms upon non-tonal music perception, similarity relations among set classes, and perception of row class membership, for example. Among the questions not adequately addressed by most experimenters is the issue of varying cognitive strategies among subjects: how do the strategies used by trained musicians differ from those used by inexperienced listeners? how do they differ between musicians with "absolute pitch"--or perhaps more accurately, absolute pitch class[15]--and those with "relative pitch"? are there any physiological differences associated with different strategies? might listeners with absolute pc have different neurophysiological responses?[16] This dissertation concludes with several proposals for experimentation that might be undertaken to address these and other issues.

All previous experiments on octave equivalence in non-tonal music have used complete twelve-tone rows. One possible experiment would test recognition of shorter segments under various transformations, including octave displacement. One of the problems with previous work has been the length of the stimuli--twelve-tone rows exceed the maximum length established by G. H. Miller for retention without "chunking": 7 ± 2.[17] Initially, the experiment might be limited to recognition of three-note segments, but follow-up experiments might use successively larger segments. Subjects should be divided into groups by the level of musical training (inexperienced, moderately-experienced, experienced), and subjects with absolute pc distinguished from those without. Before beginning the experiment, subjects should be trained as to the types of acceptable transformations--change of contour, order, or spacing, and octave displacement. An analogy might be made during the initial training period with the various possible arpeggiations and voicings of a single chord type. During the actual experiment, subjects would be presented with two trichords and asked to rate their similarity on a scale of 0 to 10, with 0 representing equivalence under the transformations cited above. At the conclusion of the experiment, subjects should be asked what strategies they devised to make their determinations. A second experiment might then be undertaken that also admits inversion as an acceptable transformation, with appropriate additional training supplied. Both experiments, as described, ask subjects to judge equivalence outside of any musical context; yet musicians know that chords with equivalent musical functions may not necessarily sound alike in isolation (further, chords that sound alike in isolation may not serve equivalent musical functions). Thus, a series of experiments might also be envisioned in which sets are presented for judgements of similarity or equivalence in musical contexts.

Experiments such as those just described, which allow subjects to identify equivalence as well as similarity, should have been a part of Cheryl Bruner's test of perceived similarity relations among trichords.[18] Perception of similarity hinges upon listeners's ability to perceive identity--segments that are more similar are those that are "closer" to identity. Further, the similarity measurement her experiment was designed to test is an incremental scale of this type, beginning with 0 as a representation of set-class identity.[19] In spite of Bruner's conclusion that "the perception of similarity among pitch-class sets is tied to context and manner of presentation,"[20] her initial experiment did not include a series of trials in which context and manner of presentation were held invariant. Thus, this experiment may not have measured perceived similarity among pitch-class sets at all; rather it may have measured perceived similarity among contours or spacing. At no time did she ask her subjects to describe the strategies by which they made their similarity judgements, nor did she check to see whether similarity measurements other than Morris's SIM relation better fit her data. Bruner states in her concluding section that, "the degree of perceived similarity between any two collections of pitches can be either diminished or enhanced by compositional choices having to do with the number of common tones, the spacing of chords, and the registral placement of pitches."[21] In a second experiment, Bruner presented each trichord as a chord with identical spacing; although this is a more satisfactory experiment, it is still flawed by its neglect of the equivalence relation. Further, perception of non-tonal sets as chords is much more difficult a task than perception of melodic segments; yet she chose to use chordal realizations.

Following the set-class equivalence experiment described above, an experiment testing similarity relations among sets should be undertaken to see whether the results replicate Bruner's findings. Similarity should be tested among trichords with identical contours and spacings before proceeding to a test in which these elements are varied. Subjects in such an experiment would be asked to rate the similarity of two presented trichords along an incremental scale from 0 to 10, where 0 represents equivalence. Finally, Bruner points out a number of variables, other than contour or spacing, that seemed to affect perceived similarity: number of common tones between pairs, total number of semitones, and degree of tonal associations the sets suggest.[22] A series of trials might also be devised that tests the influence of these factors--first among sets with invariant contours, then among more varied representations.

Another group of experiments, related to those described above, would take the contour segment class as their point-of-departure, rather than the pitch-class set. Thus, after appropriate training, subjects would be asked to judge whether two non-tonal melodies have identical contours, according to the definitions posited in Chapter 2 of the present work. After determining that subjects can make such determinations accurately, additional related experiments could be run. One experiment might test perception of contour segment class membership by testing perception R-, I-, and RI-related contours. An important variable in such experimentation is the length of the non-tonal melodies used.[23] In successive trials, the length of the stimulus ought to be gradually increased to determine the effect this has upon subjects' accuracy; and the results of increased length related to perception of twelve-tone rows. Another related experiment might be devised to test the perceptual validity of the similarity measurements posited here, following an experimental procedure similar to that suggested for perceived similarity among pitch-class sets. In each of these experiments, subjects' results should be divided according to musical experience. Further, it would be interesting to determine whether musicians possessing absolute pc are as capable of (or as likely to) remember global features such as contour, or whether their absolute pitch perception somehow interferes with contour perception. Analysis of the neurophysiological responses to this task by subjects with and without absolute pc might also prove enlightening.

In addition, a group of experiments designed to test perception of rhythmic contours should be performed. Few experiments have intentionally tested perception of non-metric rhythms, although some have included non-metric temporal sequences among their rhythmic patterns. An experiment analogous to the melodic contour experiment described above might begin by training subjects as to the criteria for rhythmic contour equivalence, then ask them to make same-different judgements for pairs of non-metric temporal sequences according to these criteria. A series of related experiments might test whether it is possible for listeners to recognize R-, I-, and RI-transformations of temporal sequences. Yet another group of experiments might test the validity of the similarity measurements for rhythmic contours posited in Chapter 3.

Music-psychological experimentation has much to offer those working in the field of non-tonal music theory. Increased communication between theorists and psychologists can influence the design of experiments so as to test relationships of musical and theoretical value. By considering results of music-psychological experimentation, theorists can improve theoretical models and pedagogical techniques to coincide with listeners' perceptual strategies. Indeed, cross-fertilization between disciplines can only benefit both fields of endeavor.

2. Implications for Theory Pedagogy

Knowledge of the cognitive strategies by which listeners structure their perception of non-tonal music may be used in designing a pedagogy of non-tonal music theory for undergraduate theory and analysis classes. Such a curriculum is proposed here, including a description of the course, student objectives, methodology and sequence of presentation, and discussion of students' application of theoretical skills to listening and analysis. The course systematically incorporates melodic contour perception into both its written and aural skills components,[24] and clearly distinguishes between c-space, p-space and pc-space concepts and between theories pertaining to ordered and unordered sets throughout. The approach used here is based upon the assumption that theoretical concepts reinforced by aural experience will be better applied and remembered by students; thus a coordinated sequence of sight singing, dictation, and keyboard exercises complements the written component of the course. While the discussion below focuses primarily upon methodological issues, a more detailed outline of the curriculum's sequence of presentation appears in Appendix III, to which the reader may wish to refer periodically. Finally, because no textbook of non-tonal music theory systematically incorporates progressive ear-training studies,[25] an overview of sight-singing and ear-training texts offering non-tonal exercises is given in Appendix IV.

The course proposed here is a one-semester undergraduate course in which students develop the basic concepts, terminology, and analytical skills necessary to discover and articulate aspects of structure in non-tonal Western music. Repertoire for analysis is drawn from compositions of the first half of this century, including music of

Babbitt, Bartok, Berg, Dallapiccola, Schoenberg, Stravinsky, and Webern. Coordinated dictation, sight singing and keyboard exercises reinforce theoretical concepts with aural experience. By the end of the semester, students should be able to: 1) analyze a passage of non-tonal music and discuss details of its structure, including aspects of its musical design (form, phrase structre, rhythmic articulation, contour relations, etc.) and its underlying pitch and/or pitch-class structure; 2) recognize, by eye and ear, all twelve trichord types; 3) identify, by eye and ear, transformations in contour-, pitch-, and pitch-class space upon a given segment or set; 4) identify set classes larger than trichords in analysis with the aid of a set-class list; 5) improvise at the keyboard using non-tonal materials; 6) take non-tonal melodies in dictation, and sing such melodies at sight; and 7) synthesize the concepts covered in class by presenting their original insights into the structure of non-tonal music in the form of analytical papers, model compositions, performances, and/or class presentations.

The curriculum takes as its point-of-departure the distinction between c-space, p-space, and pc-space, and deals first with ordered segments in contour- and pitch-space before introducing pitch-class space and unordered sets.[26] Thus it begins with easily-perceived relationships between contours, and between ordered sets in pitch space, before asking students to compare sets without regard to contour, order, or octave placement. Students initially learn to recognize equivalent csegs, aurally and in analysis, before the I-, R-, and RI-transformations are added. As much as possible, concepts are reinforced in aural skills training and illustrated in the compositions studied. Bartok's "Bulgarian Rhythm," (Mikrokosmos, Vol. IV, #115), for example, provides compositional examples of equivalent contours that contain different intervals, and clear examples of inversionally-related contours, while "Diminished Fifth" (Vol. IV, #101) contains R- and RI-related contours as well.

Ear-training exercises begin with c-space discrimination between equivalent and non-equivalent contours, eventually adding recognition of I-, R-, and RI-relationships as these are studied.[27] Both contour diagrams and c-space numeric notation may be used to identify and describe contour relationships. Contour diagrams may be used throughout the semester for modal, atonal, or twelve-tone melodies, not only to identify related contours but also as an initial step in dictation exercises. Paul Hindemith advocates this technique in his Elementary Training for Musicians:

> The students' abilities to perceive primarily the general outlines of an example will be promoted by dictating in fast tempo, and by frequent repetitions of a complete example. In dictating melodic lines it is advisable to let the students symbolize their course graphically in a continuous pencil line, before writing them down in notes. After the first vague impression has settled, main points of the example may be fixed...[28]

Teachers may wish to encourage contour graphing by means of worksheets prepared for this task. Figure IV-1 shows two possible contour grids for this purpose--the first for pc-centric melodies, in which students may hear scale-degree relationships, and the second for twelve-tone and other non-tonal melodies.[29]

In the proposed curriculum, students are first introduced to T_n and T_nI in pitch space, as contour-preserving operations upon ordered pitch intervals. Staff notation ought not to be abandoned entirely in favor of numeric pitch notation at this stage. In pitch space, students may benefit from the graphic layout that staff notation provides for visualizing the transposition and inversion operations, and from the visual analogy with contour graphs in contour space. By first using definitions based upon interval successions rather than pitch successions, the instructor closely models aural perception via "relative" as opposed to "absolute" pitch.[30] Figure IV-2A shows the ordered pitch-space segment < 7 8 11 13 > and the ordered pitch-interval succession between adjacent elements, < +1 +3 +2 >. Transpositionally-related segments are defined as those with identical INT_1 successions; for inversionally-related p-space segments, the direction of each interval is reversed as shown.[31] Error-detection and pattern recognition drills are effective initial p-space aural skills exercises. The former presents students with a notated melody, while they hear a same-contour melody with some pitch differences that they must locate and notate; the latter presents several INT_1 successions--which may represent melodies with identical contours--and asks students to determine which one belongs to the melody played.[32] The advantage of using same-contour materials is that students cannot rely upon easily-identified and remembered contour relationships to make judgements, but must listen to intervallic relationships.

Two other concepts may be introduced during the study of pitch-space before

FIGURE IV-5

CONTOUR GRIDS

Figure IV-1A: Bartok, "In Oriental Style"

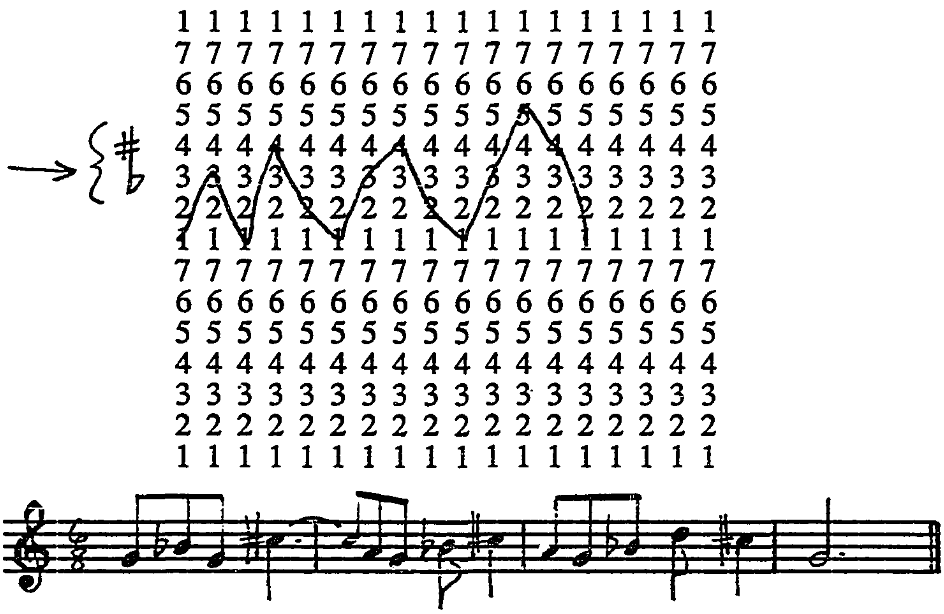


Figure IV-1B: Webern Cantata, No. 2/1

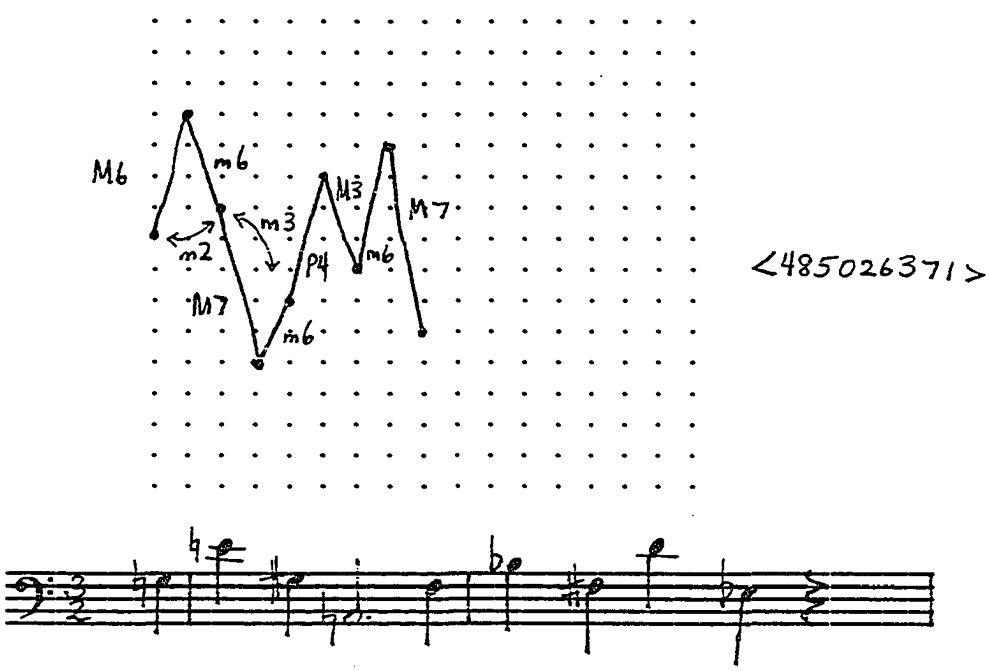


FIGURE IV-6

INTRODUCTION TO T_n/T_nI OPERATORS

Figure IV-2A: T_n/T_nI as Operations on Pitch-Space Segments

S $\quad$ T_nS $\quad$ T_nIS

$INT_1 = \langle +1\ +3\ +2 \rangle$ $\quad$ $INT_1 = \langle +1\ +3\ +2 \rangle$ $\quad$ $INT_1 = \langle -1\ -3\ -2 \rangle$

Figure IV-2B: T_n/T_nI as Operations on PC-Space Sets
Realized in Pitch Space in Normal Order

S $\quad$ T_nS $\quad$ T_nIS

$CINT_1 = \langle 1\ 3\ 2\ 6 \rangle$ $\quad$ $CINT_1 = \langle \underbrace{1\ 3\ 2}\ 6 \rangle$ $\quad$ $CINT_1 = \langle \underbrace{2\ 3\ 1}\ 6 \rangle$

retrograde-related

$\langle 2\ 3\ 1\ 6 \rangle$ is the retrograde of
$\langle 6\ 1\ 3\ 2 \rangle$, a rotation of $\langle 1\ 3\ 2\ 6 \rangle$

introducing pc-space: the segment class and the unordered set. Segments in c-space or p-space belong to the same segment class if they are related by inversion, retrograde, or retrograde-inversion. The latter two relationships may, as before, be defined in p-space in terms of operations upon INT_1 and related ear-training exercises be devised to teach their recognition. The concept of unordered sets in p-space may be introduced simply by noting that a chord can be represented by any of its arpeggiations.[33] Although all pitch sets are ordered in time or register in their musical contexts, they may be considered conceptually unordered for purposes of comparison in analysis. To facilitate comparison, elements in these sets are customarily placed in numerical order, then the INT_1 of this invariant order can be compared to reveal T_n or T_nI equivalence. Analysis of Bartok's "Diminished Fifth" illustrates application of the theory of unordered sets since each of the formal sections is characterized by a different octatonic scale. When considered as unordered pitch sets, these scales form equivalent sets.

Introduction to pitch-class space may best be accomplished by a comparison with pitch space. If unordered sets have been described as the various equivalent arpeggiations of a chord, the assertion of pc-space opens up many new arpeggiations, doublings, and spacings. Further, octave duplication of pitches may be disregarded, so that pitch sets of differing sizes may be compared as pc sets of equal size. After integer notation in each space has been compared, an analysis should be undertaken that incorporates both p- and pc-space observations. Bartok's "Isle of Bali" is ideal for such an analysis, since students may note on the one hand its inversional symmetry in p-space, and on the other, the identity of its octave-related repetitions and doublings. Further, the work clearly illustrates the usefulness of unordered sets as analytical tools, since the opening theme of the first four measures is repeated pc-for-pc in the second four measures in a different order, and in final four measures as chords.

In pitch-class space, as in pitch space, elements in unordered sets are placed in numerical order for purposes of comparison. Because pc-space is a cyclical space, pcs must be placed in an ascending cyclic order that "wraps" around from the last pc to the first. Chrisman's successive interval array (hereafter called, after Morris, $CINT_1$)[34] is used to compare pc sets for T_n/T_nI equivalence, as shown in Figure IV-2B. At this point, staff notation is less appropriate and integer notation preferred, since the latter does not imply any octave designation. Students should be aware that staff notation, if used, is not truly representative of pc-space, but instead shows only one possible p-space realization of a pc-space concept. The normal order used for comparison of unordered pc sets is the rotation of the set that has the largest interval at the right-hand end of its $CINT_1$ and smallest intervals to the left.[35] Depending on individual students' ability for abstraction, the rotations may be visualized in their heads, or on paper in integer or staff notation. Transpositionally-related pc sets have identical $CINT_1$ successions, inversionally-related sets have retrograde $CINT_1$ successions (with rotation required, as shown in the figure).[36]

Because students are already familiar with concept of segment classes, set classes will seem a logical extension of the theory. The instructor may wish to begin by introducing unordered pc intervals, or interval classes, as two-note set classes. The students can easily generate the entire list (2-1 through 2-6) together, and thus begin to incorporate SC identification drills into their aural skills exercises. Students should understand that they are hearing realizations of interval classes, or two-note SCs, as intervals in p-space. A review of the eleven pc intervals should be a part of the aural skills curriculum from the beginning of the semester, beginning with simple intervals in the piano's middle register, and progressing to widely-spaced compound intervals and extreme registers. Students should fluently recognize all eleven in ascending and descending melodic contexts and in harmonic contexts. Further, students can be prepared for non-tonal melodic dictation tasks by taking short chains of intervals in dictation; the student may notate the chain as a contour graph, followed by a p-space INT_1 representation, and finally, if the first pitch is given, staff notation.

Just as the class generated the dyadic SC list, so should they generate the list of trichords together. This exercise serves to reinforce the idea of set classes as representative of a collection of sets, graphically illustrates T_n/T_nI pitch-class invariance as limiting the number of sets in a given set class, and demonstrates that there are indeed only twelve possible types. The set classes should not be presented for ear training all at once, however, nor should they be presented in the order of Forte's set-class list. Rather, the set classes should be introduced as appropriate, alongside compositions in which they appear. For example, set classes that may be perceived as subsets of the pentatonic, whole tone, and octatonic scales should be taught first in conjunction with analyses of pentatonic, whole tone, and octatonic compositions. Figure IV-3A proposes an order of presentation following such a plan.[37] Set classes grouped together may be presented together in a single class period, with students' initial task simply to distinguish between them. As the list grows longer, students should be asked to identify sets across these artificial boundaries. The sets on the right of the figure are in "inverted normal order," which should be introduced separately after the students are thoroughly familiar with the form of the sets given to the left, since some of these sound quite different from the sets in normal order. The dotted lines represent dividing points for a helpful mnemonic device: for sets appearing above the top line, the second digit of Forte's set-class name can be derived by adding the pc

FIGURE IV-3

PEDAGOGICAL STRATEGIES FOR AURAL TRAINING

OF SET-CLASS RECOGNITION

Figure IV-3A:

Set-Class Name	Normal Order on C	Possible Scalar Superset	Inverted Normal Order on C	Suggested Composition
3-9	[0, 2, 7]	pentatonic		Bartok,
3-7	[0, 2, 5]	pentatonic	[0, 3, 5]	Five-Tone Scale
3-6	[0, 2, 4]	whole tone		Bartok,
3-8	[0, 2, 6]	whole tone	[0, 4, 6]	Whole-tone Scale
3-12	[0, 4, 8]	whole tone		
				Stravinsky,
3-11	[0, 3, 7]	diatonic/octatonic	[0, 4, 7]	Rite of Spring,
3-10	[0, 3, 6]	diatonic/octatonic		Intro. to Part II
3-3	[0, 1, 4]	chromatic/octatonic	[0, 3, 4]	Berg,
3-5	[0, 1, 6]	chromatic/octatonic	[0, 5, 6]	Altenberglieder #2
3-2	[0, 1, 3]	diatonic/octatonic	[0, 2, 3]	Webern,
3-4	[0, 1, 5]	chromatic	[0, 4, 5]	Op. 5/3
3-1	[0, 1, 2]	chromatic		Bartok, Minor Seconds, Major Sevenths

FIGURE IV-3

PEDAGOGICAL STRATEGIES FOR AURAL TRAINING

OF SET-CLASS RECOGNITION

(CONTINUED)

Figure IV-3B: Method of Presentation:

3-9:

[0, 2, 7]

ps-seg = < 0 1 >*	ps-seg = < 0 0 >	ps-seg = < 1 0 >
$CINT_1$ = < 2 5 5 >	$CINT_1$ = < 5 5 2 >	$CINT_1$ = < 5 2 5 >

3-8a:

[0, 2, 6]

ps-seg = < 0 1 >	ps-seg = < 0 1 >	ps-seg = < 1 0 >
$CINT_1$ = < 2 4 6 >	$CINT_1$ = < 4 6 2 >	$CINT_1$ = < 6 2 4 >

3-8b:

[0, 4, 6]

ps-seg = < 1 0 >	ps-seg = < 0 1 >	ps-seg = < 0 1 >
$CINT_1$ = < 4 2 6 >	$CINT_1$ = < 2 6 4 >	$CINT_1$ = < 6 4 2 >

*See Section 3 (particularly pp. 221 and following) for definition of segments in ps-space. Any trichord that in some rotation has a ps-seg of < 0 0 > is inversionally symmetrical.

integers of the set's normal order; for those appearing below the bottom line, subtraction of the middle from the final pc integer of normal order produces the set-class label.

In aural training for set class recognition, it is important to stress that students are hearing one possible pitch-set realization of the set class. While each of the twelve trichords is gradually entered into the students' aural recognition repertory, variations in contour and spacing should be kept to a minimum--that is, the instructor should limit the number of possible p-set realizations of each pc-set. One possible way to limit these realizations is to teach each trichord at first with an invariant rising contour, in "close" position (within an octave span), in each of its three rotations. That is, set class 3-9 would be introduced in ear-training exercises as illustrated in Figure IV-3B. Students should recognize the set in any of the twelve possible transpositions, either as stacked fourths, or as a perfect fourth with whole step above or below. The instructor, with the class, should invert inversionally symmetrical sets such as 3-9 to show that only twelve representations, as unordered pc sets, are possible. Sets that are not inversionally symmetrical must be taught both in their normal order and their inverted normal order, as Figure IV-3B shows for set class 3-8. Although inversion preserves interval-class content, it does change the characteristic sound, as demonstrated by the inversionally-related major and minor triads or diminished and dominant sevenths. The rotations of SC 3-8 illustrate another important point--that is, that tonal associations are weakened in some rotations: for example, the second statement of [0, 2, 6] suggests an incomplete third inversion dominant seventh, while the third statement of [0, 4, 6] suggests an incomplete second inversion half diminished seventh--yet the other rotations do not seem to suggest these sonorities quite so strongly . Tonal associations are not reliable "crutches" for learning any set class, and should not be encouraged.

In fact, they should be discouraged since they imply resolutions that in non-tonal musical contexts are unlikely to occur.

Generally, set-class identification drills should follow a progressive plan of increasing perceptual difficulty. One possible plan might progress as follows: first, pc sets are realized as pitch sets spanning less than an octave, with ascending contours, in each possible rotation; second, either the first or last pitch is displaced an octave (contour remains invariant); next, the first and second conditions are repeated with sets expressed as chords instead of melodies;[38] third, order is varied (thus contour changes) within an octave span; fourth, various orderings include octave displacement; finally, the third and fourth conditions are repeated as chords. Throughout, students may also find aural reinforcement in keyboard assignments, each of which focuses upon a specific trichord type. Students should be encouraged to experiment with different doublings and spacings, and to improvise melodies and accompaniments based upon specific trichord types.

At the completion of this unit that introduces contour-, pitch-, and pc-space, ordered segments, unordered sets, segment classes, set classes, and their analytical application in various short piano works, students will be encouraged in the first of three "application units" to synthesize the material presented thus far and apply it to composition, analysis, and performance. During this period, no new theoretical constructs will be introduced; rather, class periods will consist of analyses of larger works, such as excerpts from Stravinsky's Rite of Spring and Bartok's Music for Strings, Percussion, and Celesta. Class discussions will incorporate various issues, such as the role of timbre, the articulation of phrase structure and form, the role of pitch centricity, and so on. Each student will be responsible for a small project: either a model composition based upon the works studied, a short analysis paper, or an in-class presentation that relates analysis to performance of a non-tonal work chosen in consultation with the instructor. Each "application unit" ends with an exam, consisting of both a written and aural component.

The second unit of the curriculum proposed here builds upon the groundwork laid in the first. As Appendix III shows, this unit includes such concepts as the interval-class content of pc sets, pc invariance under T_n and T_nI, the Z-relation, complementary SCs, and similarity relations. Although the unit focuses primarily upon pc-space, the distinction between c-, p-, and pc space is maintained throughout. Thus pitch invariance is distinguished from pc invariance, for example. Pc invariance is described initially, as Chrisman does, in terms of characteristics of $CINT_1$ successions in pc-invariant sets,[39] thus building upon previous definitions of equivalence based upon $CINT_1$ identity, rotation, or retrogression. This is followed by application of pc-space T- and I-matrices for predicting invariance. Similarity relations among contours are introduced, using the COM-matrix, before ic-based (or T-matrix based) similarity measurements for unordered pc sets are discussed. The aural skills component completes the introduction of the remaining trichords--in conjunction with analysis of compositions by Schoenberg, Webern, and Berg that use these sonorities--and continues all previous types of exercises, such as interval chains, contour graphing, error detection, pattern recognition, interval and set-class identification, and dictation, using progressively longer and more challenging examples. The second unit closes, like the first, with an application unit that enables students to analyze more extended works, and to apply the theoretical concepts of this unit to composition, analysis, or performance (see suggested projects and compositions for analysis in Appendix III).

The course's final unit marks a return to the study of segment classes or, more specifically, row classes. Because Dallapiccola's "Die Sonne Kommt" (<u>Goethelieder</u>)

contains an exact pitch-space palindrome, analysis and comparison of this composition with the same composer's "Quartina" (Quaderno Musicale di Annalibera) at the beginning of the unit, reasserts the distinction between pitch- and pc-space operations and between row transformations that do or do not preserve segments in contour-space. Later, Webern's Piano Variations Op. 27/2 may be used to illustrate frozen register and inversional symmetry in p-space. Since each of the twelve-tone operators was defined in Unit I in terms of its effect upon the segment's INT_1 succession, these need only be reviewed at this time for twelve-tone rows in p- and pc-space. In addition to these interval-based definitions, students should also become familiar with the TTOs defined as pc-based operations, since these definitions will facilitate matrix construction and their understanding of the TTO cycles. Because students are already familiar with the construction and use of COM-, T-, and I-matrices, the introduction of the row matrix and discussion of its properties will have been prepared. Students may also be taught to scan the matrix to find patterns of segmental invariance, and later, combinatoriality. The T_n and T_nI cycles should be introduced in conjunction with compositions, such as the Webern Piano Variations and Symphony, in which their analytical significance can be seen. Students may also be shown how the cycles are used to construct rows with special properties, such as the row of Berg's Lyric Suite. Like the others, this unit closes with an application unit in which students are given the opportunity to examine various compositions in more depth and to demonstrate their mastery of the concepts introduced in this unit through composition, analytical papers and an examination.

In aural skills exercises associated with this unit, students will be asked to recognize T_n, T_nI, RT_n, and RT_nI relationships between rows by taking their INT_1 successions in dictation and comparing them. Students should continue all other ear-training drills, such as error detection, pattern recognition, and melodic dictation, now using complete twelve-tone rows. Once students have mastered the twelve trichords, selected tetrachords, or possibly the all-combinatorial hexachords,[40] might be the logical next step in set-class recognition tasks. Alternatively, larger sets could be viewed in terms of the component subsets included within them; set-classes that may partitioned trichordally might be featured initially in this type of aural skills exercise. Another possible approach to aural SC identification is suggested by Richard Cohn's theory of transpositional combination.[41] This generative theory models such perceptions as the 6-20 hexachord being "formed" by two statements of the augmented triad (3-12) a half-step apart, or the octatonic scale (8-28) formed by two diminished seventh chords (4-28) a half-step apart. According to Cohn's notation, the first would be represented as 3-12 * 1 = 6-20, and the second as 4-28 * 1 = 8-28. The "1" in these instances represents SC 2-1 (which is also ic 1)--3-12 is "projected" up by a semitone; thus {0, 4, 8} plus {1, 5, 9} equal {0, 1, 4, 5, 8, 9}.[42] With careful reading of Cohn's dissertation, Appendix 2, a list of sets might be compiled that can be taught aurally in terms of projected trichords. Another possible approach to ear-training of large sets might include recognition of complementary sets.

3. Further Generalization of Contour Theory

The theory of musical contours presented in the preceding chapters can be generalized to any pair of sequential dimensions. This section takes Morris's generalized definition as a point of departure for a theory of pitch-span analysis that models various chord spacings. The utility of this and other generalizations for musical analysis is illustrated in several short musical passages. According to Morris's definition, a contour is "a set of points in one sequential dimension ordered by any other sequential dimension," and "a sequential dimension of order n is a basic musical attribute whose points (or states) are listed in order corresponding to the number 0 to n-1."[43] He cites some sequential dimensions of music that might be combined in pairs to model various musical contours. Often one of the sequential dimensions is s-time; melodic contours model pitch classes ordered in s-time, for example. Figure IV-4 shows a few brief passages drawn from the first piece of Stockhausen's Klavierstücke, No. 2, in which one sequential dimension, loudness, is ordered either by register or by s-time. Comparison of these examples shows some interesting relationships between contours. The six-note chord in m. 20 has six dynamic indications, which may be seen as overlapping statements of < 2 1 0 2 > and < 0 2 1 3 >. Likewise, the last six dynamics of the sequence in Figure IV-4B, ordered in s-time, contain the same overlapping statements of < 2 0 1 2 > and < 0 2 1 3 >. This latter sequence also contains a subsegment < 0 1 2 1 > that is I-related to < 2 0 1 2 >. Further analysis along these lines seems a fruitful area for future research.

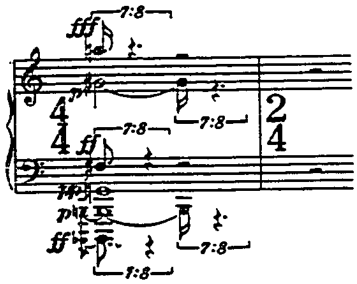
Morris illustrates this generalization of musical contours by using a sequential dimension of noise content ordered by loudness, location ordered by c-pitches, and envelope ordered by vowel color, among others.[44] As these dimensions suggest, contour-based methodology seems a promising vehicle for analysis of electronic music. Since, in many cases, no score exists for electronic compositions, one way to proceed

FIGURE IV-4

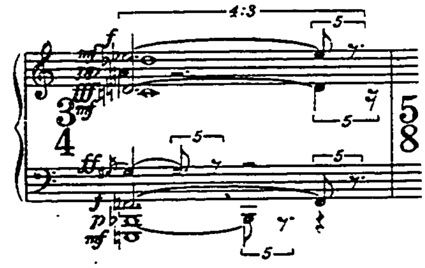
LOUDNESS AS A SEQUENTIAL DIMENSION

IN STOCKHAUSEN'S KLAVIERSTÜCKE, NO. 2 - I

Figure IV-4A: Loudness Ordered by Register



m. 20



m. 54

< 2 1 0 2 >

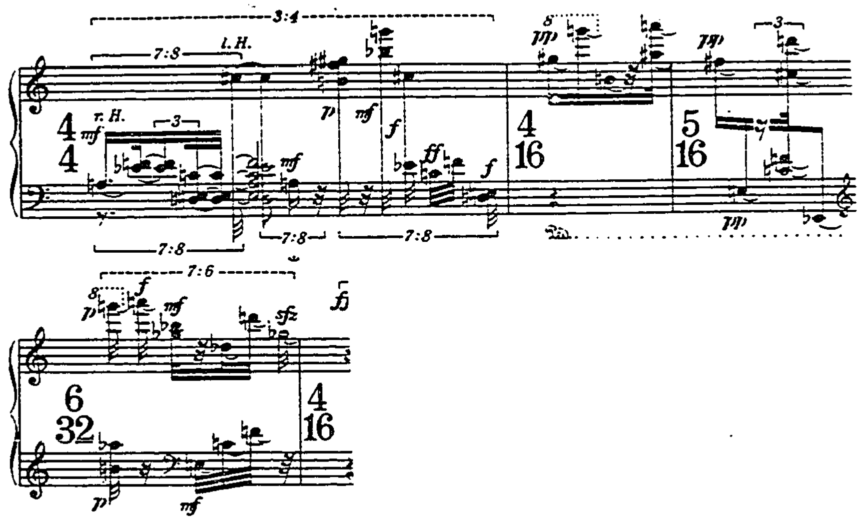
< 2 1 0 2 1 3 >

< 0 2 1 3 >

< 2 1 3 4 2 5 0 2 3 >

Figure IV-4B: Loudness Ordered by S-Time

Mm. 47-50



Sequence of dynamics: < mf p mf f ff f pp p f mf sfz >

< 0 1 2 1 > < 0 2 1 3 >

Contour: < 2 1 2 3 4 3 0 1 3 2 5 >

< 2 0 1 2 >

in analyzing such music is by transcribing various elements for comparison. Because rhythms in some electronic pieces are non-metric, analysts in the past may have relied solely upon graphic representation for analysis. However, discovery of related duration successions over spans of music can now be facilitated by using contour notation. Similarly, melodic contour notation may facilitate analysis of microtonal or other nonequally-tempered passages. Finally, if, like Morris, the analyst views envelope (short to long), noise content (pure to noisy), and timbre (dull to bright) as sequential dimensions, contour notation may be used to compare the colors of electronic sounds.[45]

Other extensions of contour theory may also be envisioned. For example, rhythmic contours differ from melodic contours in that their elements do not represent "points," but rather, relative spans between points. The concept of a musical space consisting of a series of spans can be generalized to other domains, and in any sequential dimension the spans between points may be compared using a theory analogous to that of duration space. As in duration space, segments would consist of relative spans numbered from 0 to (n-1) as they extend from narrow to wide, and would be ordered in s-time. One such space might measure the relative span between pitches, from narrow to wide. Another might measure the "distance" spanned between varying dynamics--pp to mp occupying a narrower span than pp to ff, for example. Others could measure the relative distance between varying timbres, envelopes, or any other sequential dimension.

To illustrate one such generalization, pitch-span space (ps-space) is defined here as a type of space consisting of elements arranged from narrow to wide. Unlike intervals, which also measure spans between pitches, ps-space spans are not calibrated by some equally-spaced means of measurement. That is, ps-space spans represent a relative span, rather than a precise measurement in semitones. Elements in ps-space are termed spans, and are numbered in order from narrow to wide, beginning with 0 up to (n - 1), where n equals the number of elements in the segment and where the calibrated intervallic distance between pitches is ignored and left undefined. Ps-space segments (ps-segs) are ordered sets of spans in ps-space, ordered in s-time.

Figure IV-5A shows six realizations of pitch-class set {1, 2, 4, 7}. The first three, labelled X in the figure, feature a pitch-space expansion; each successive chord encompasses a wider range. Yet all are equivalent spacings that can be represented as ps-seg < 0 1 2 >, a spacing that features successively-larger spans from bottom to top.[46] By way of contrast, the three chords labelled Y in the figure are p-space expansions of the same pitch-class set, this time realized as ps-seg < 1 2 0 >. Again, all are equivalent spacings. Figure IV-5B shows additional members of ps-space segment class < 1 2 0 >, giving one possible realization each of I-, R-, and RI-related spacings. Figure IV-5C illustrates that pitch-space inversion of Y--in this case, about a middle-C axis--produces a retrograde-related spacing. Identity of ps-seg type may explain the aural similarity of some chords that do not belong to the same set class. The ps-segment may provide an alternative to Michael Friedmann's contour interval (CI), discussed previously.[47] Figure IV-6 reproduces an example from Chapter 2 that illustrates one possible realization of cseg < 0 1 3 2 4 >. Friedmann's contour interval is illustrated here as a measurement between contour pitches; for example, cp 0 to 1 spans a CI of +1, and cp 1 to 4 spans a CI of +3. The realization given in Figure IV-6 shows that +1 may, in fact, be smaller than +3 according to this system. If, however, the spans between pitches are measured in ps-space, the resulting ps-seg < 3 1 0 2 > accurately represents the relative space spanned between cps.

FIGURE IV-5

PS-SPACE SEGMENTS

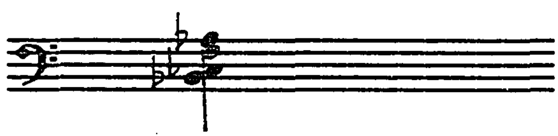
Figure IV-5A: Equivalent Spacings in Ps-Space

X: <0 1 2> <0 1 2> <0 1 2> Y: <1 2 0> <1 2 0> <1 2 0>

Figure IV-5B: Ps-space Segment Class

IY: <1 0 2> RY: <0 2 1> RIY: <2 0 1>

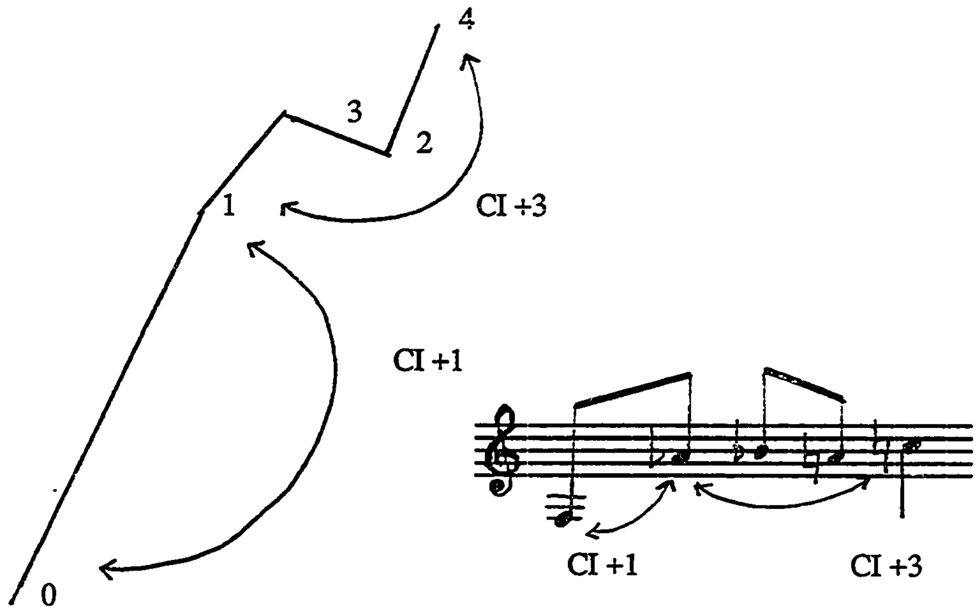
Figure IV-5C: P-Space Inversion of Y (about Middle C) Produces R-related Spacing



<0 2 1>

FIGURE IV-6

A PS-SPACE ALTERNATIVE TO FRIEDMANN'S "CONTOUR INTERVAL"



Ps-space: < 3 1 0 2 >

This section closes by demonstrating an analytical application of ps-space segments in Dallapiccola's "Accenti" (Quaderno Musicale di Annalibera, II). As Figure IV-7A shows, the work has a ternary form, A-B-A', in which the A' section likewise divides into a smaller a-b-a. Its texture consists primarily of 5-note chords, statements of set classes 5-26 [0, 2, 4, 5, 8] and 5-27 [0, 1, 3, 5, 8]. Only two chords break this pattern--that which closes the A section in measure 4, and that which closes the first division (the a section) of A' in measure 10. These cadential exceptions are six-note chords, and members of SC 6-31 [0, 1, 4, 5, 7, 9]. The spacing of the five-note chords, measured in ps-space and ordered from low to high, is remarkably consistent. Of 17 attack-point spacings for these pentachords,[48] all but three belong to either ps-segclass 4-4, 4-5, 4-6, or some repeated-note segclass that is a combination formed from two of those three. Of the three exceptions, two belong to ps 4-1; the other--the first chord of the piece--belongs to a repeated-note segment, ps 4-2/4, that includes one of the primary three ps segments. Ps 4-4 is the most common spacing, with five statements (mm. 3, 5, 6, 14, and 16), all appearing as < 2 0 1 3 >. The final statement--the last chord of the piece--differs in its set-class membership from the first three; most are members of SC 5-27, while the final statement is a member of 5-26. Identity of spacing may contribute to the chords' aural similarity in spite of their differing set-class membership. A comparison of the two statements of ps 4-6 (m. 8 and 9) reveals that they too are drawn from different set classes, likewise for the statements of ps 4-1 (mm. 5-6 and 11), and for ps 4-5/6 (mm. 10 and 15). Two instances of retrograde-related ps-segs occur in "Accenti." Ps 4-4/6 appears first in m. 2 as < 1 0 0 2 > and again in m. 12 as < 2 0 0 1 >. Similarly, ps 4-5/6 appears in m. 10 as < 1 1 0 2 >, but in m. 13 as < 2 0 1 1 >.

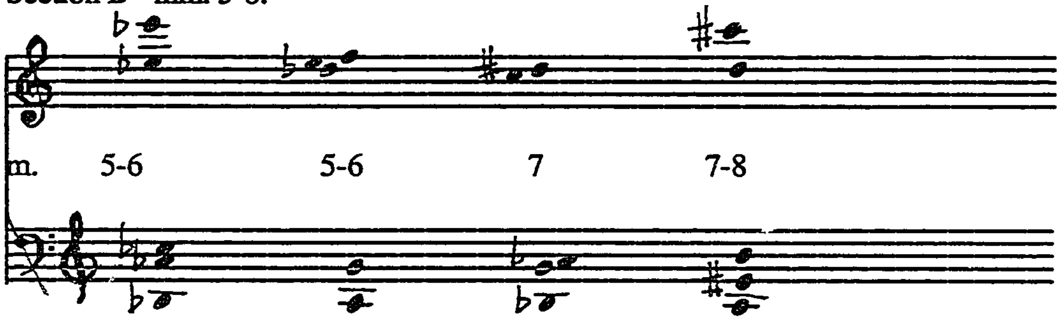
FIGURE IV-7

PS-SPACE SEGMENTS IN DALLAPICCOLA'S "ACCENTI"

Figure IV-7A: Form and Ps-space Segments
A section - mm. 1-4:

<1 0 1 2>	<1 0 0 2>	<2 0 1 3>	<2 0 1 0 1>
ps 4-2/4	ps 4-4/6	ps 4-4	ps 5-12/20

Section B - mm. 5-8:



<2 0 1 3>	<3 2 1 0>	<3 0 2 1>	<2 1 0 3>
ps 4-4	ps 4-1	ps 4-5	ps 4-6

Section A' (a-b-a) - mm. 9-16:
a (mm. 9-10) b (mm. 11-12) a (mm. 13-16)

< 2103>	<1102>	<30210>	<3210>	<2001>	<2011>	<2013>	<1102>	<2013>
ps 4-6	ps 4-5/6	ps 5-8/17	ps 4-1	ps 4-4/6	ps 4-5/6	ps 4-4	ps 4-5/6	ps 4-4

FIGURE IV-7

PS-SPACE SEGMENTS IN DALLAPICCOLA'S "ACCENTI"

(CONTINUED)

Figure IV-7B: COM-matrices and CSIM relations

A: ps 4-4

$$\begin{array}{c|cccc} & 2 & 0 & 1 & 3 \\ \hline 2 & 0 & - & - & + \\ 0 & + & 0 & \boxed{+} & + \\ 1 & + & - & 0 & + \\ 3 & - & - & - & 0 \end{array}$$

B: ps 4-6

$$\begin{array}{c|cccc} & 2 & 1 & 0 & 3 \\ \hline 2 & 0 & - & - & + \\ 1 & + & 0 & \boxed{-} & + \\ 0 & + & + & 0 & + \\ 3 & - & - & - & 0 \end{array}$$

C: ps 4-4/6

$$\begin{array}{c|cccc} & 1 & 0 & 0 & 2 \\ \hline 1 & 0 & - & - & + \\ 0 & + & 0 & \boxed{0} & + \\ 0 & + & 0 & 0 & + \\ 2 & - & - & - & 0 \end{array}$$

D: ps 4-5/6

$$\begin{array}{c|cccc} & 1 & 1 & 0 & 2 \\ \hline 1 & 0 & 0 & - & + \\ 1 & 0 & 0 & - & + \\ 0 & + & + & 0 & + \\ 2 & - & - & - & 0 \end{array}$$

E: ps 4-5

$$\begin{array}{c|cccc} & 3 & 0 & 2 & 1 \\ \hline 3 & 0 & - & - & - \\ 0 & + & 0 & + & + \\ 2 & + & - & 0 & - \\ 1 & + & - & + & 0 \end{array}$$

F: ps 4-1

$$\begin{array}{c|cccc} & 3 & 2 & 1 & 0 \\ \hline 3 & 0 & - & - & - \\ 2 & + & 0 & - & - \\ 1 & + & + & 0 & - \\ 0 & + & + & + & 0 \end{array}$$

CSIM(A, B), (A,C) or (B,C) = .83
CSIM (B,D) = .83
CSIM(D,F), (A,F) or (C,F) = .33
CSIM(D, E) = .33
CSIM(D, E) = .83

Figure IV-7B gives COM-matrices and similarity measurements for various ps-sets in "Accenti." Matrices A through C show the transitive triple existing for the similarity relation between ps 4-4 < 2 0 1 3 >, 4-6 < 2 1 0 3 >, and 4-4/6 < 1 0 0 2 >. CSIM returns .83, or maximum similarity, between any two of the three. Maximum similarity is also found between segments D (ps 4-5/6) and B (ps 4-6). As might be expected, ps 4-1 is the least similar to the others, CSIM equals .33 between segment F and A, C, or D, and .50 between F and B. What is more surprising is the low similarity between ps 4-5 and 4-5/6 as they appear in the work: CSIM(D,E) = .33. This is due to the fact that it is not < 3 0 2 1 > from which the repeated-note segment < 1 1 0 2 > is formed, but rather its retrograde < 1 2 0 3 >. Thus, maximum similarity exists between the two ps-segclasses--CSIM(D, E) = .83--but not between the two segments as they appear here.

4. Summation and Conclusions

This dissertation proposes the thesis that abstract theories of pitch- and set-class structure do not reflect listeners' aural perception of sounding music as effectively as theories modelling the articulation of these underlying structures on the musical surface. This position is supported by a review of pertinent music-psychological experimentation. Based upon the data collected by various psychological researchers, published elsewhere but compared and critiqued here, this study concludes that listeners generally use figural cues drawn from musical context -- for example, melodic shapes, changes of direction, relative durational patterns, and so on -- to retain and recognize musical ideas in short-term memory. These figural cues may be represented in precise notation and compared with one another by application and generalization of Robert Morris's contour theories. Morris's comparison matrix and contour equivalence relations are introduced here, followed by this author's generalization of the theory to duration space and development of similarity relations for contours of pitch height or duration successions. While the efficacy of these theories for modelling perceivable patterns in musical contexts cannot be proven without further psychological testing, their applicability to musical analysis is demonstrated. Analyses drawn from the music of Bartok, Webern, Berg, and Varèse illustrate ways in which melodic and rhythmic contour relationships may be used to shape a formal scheme, to differentiate melody from accompaniment, to associate musical ideas that belong to different set classes, and to create unity through varied repetition.

This concluding chapter explores avenues for future work. A section on music-psychological experimentation offers a critical overview of research in this area and proposes ideas for future experimentation: first, experiments that replicate previous work using the more precise definition of contour equivalence assumed here; and second, experiments designed to test and compare listeners' perceived ratings of similarity among melodic and rhythmic contours with the similarity measurements proposed here. Second, the implications of music-psychological research for the pedagogy of non-tonal music theory are considered and a model curriculum proposed. The curriculum incorporates contour theory from the beginning; further, it advocates an instructional pattern that procedes from mastery of theories based upon musical context to more abstract theories of pitch- and set-class structure. The dissertation concludes by proposing a number of ways in which contour theory might be generalized to other domains and illustrates the application of one such generalization to the analysis of chord spacing in a piano work of Luigi Dallapiccola.

NOTES

[1]Material for this discussion is drawn primarily from Judy Edworthy, "Towards a Contour-Pitch Continuum Theory of Memory for Melodies," in The Acquisition of Symbolic Skills, ed. Don Rogers and John Sloboda (NY: Plenum Press, 1983), pp. 263-271, and "Interval and Contour in Melody Processing," Music Perception (1985) 2/3, pp. 375-388; W. J. Dowling, "Scale and Contour: Two Components of a Theory of Memory for Melodies," Psychological Review 85 (1978), pp. 341-354; W. J. Dowling and D. S. Fujitani, "Contour, Interval, and Pitch Recognition in Memory for Melodies," Journal of the Acoustical Society of America 49 (1971), pp. 524-531; and W. J. Dowling and J. C. Bartlett, "The Importance of Interval Information in Long-Term Memory for Melodies," Psychomusicology 1 (1981) pp. 30-49.

[2]As mentioned at the outset of this dissertation, all experimental designs and conclusions summarized here show a Western bias. The term "experienced listener" refers to subjects who are experienced in Western tonal music.

[3]Judy Edworthy, in particular, hypothesizes that listeners retain short tonal melodies as contours but longer melodies as interval or pitch successions. See "Towards a Contour-Pitch Continuum" and "Interval and Contour" cited above.

[4]Edworthy, "Towards a Contour-Pitch Continuum," p. 270. She may not be referring to a contour-pitch continuum at all, but rather a contour-interval continuum.

[5]Edworthy, "Interval and Contour," p. 375. She notes that "Subjects were significantly better at detecting contour alterations for melodies of up to 11 notes but significantly better at detecting interval alterations in the 15-note melodies." (Ibid.) It would seem that contour alterations in these melodies would also affect their interval succession; thus her conclusion must be questioned.

[6]W. Jay Dowling, "Recognition of Melodic Transformations: Inversion, Retrograde and Retrograde-Inversion," Perception & Psychophysics 12/5 (1972), pp. 417-421; and Carol L. Krumhansl, Gregory J. Sandell, and Desmond C. Sergeant, "The Perception of Tone Hierarchies and Mirror Forms in Twelve-Tone Serial Music," Music Perception 5/1 (1987), pp. 31-78.

[7]Dowling, ibid. He asserts that listeners use a pitch-based strategy, though he is probably referring to a strategy based on memory of pcs rather than pitches. This discussion will therefore use the term "pc-based" rather than "pitch-based".

[8]Ibid., p. 421.

[9]This is noted in Chapter 1 as well. Roger N. Shepard is credited as having developed this technique; see Shepard, "Structural Representations of Musical Pitch," in The Psychology of Music, ed. Diana Deutsch (New York: Academic Press, 1982). One recent experiment by Diana Deutch and her colleagues has negative implications for listeners' ability to perceive contour, given a non-tonal segment composed of Shepard tones and containing tritones. See Diana Deutsch, William L. Kuyper, and Yuval Fisher, "The Tritone Paradox: Its Presence and Form of Distribution in a General Population," Music Perception 5/1 (1987), pp. 79-92. Among their findings was the following:

> The tritone paradox occurs when an ordered pair of tones is presented, with each tone consisting of a set of octave-related components, and the pitch classes of the tones separated by a half-octave. Such a patterns is heard as ascending in one key, but as descending in a different key. Further, the pattern in any one key is heard as ascending by some listeners but as descending by others (p. 79).

[10]This two-channel theory for octave equivalence is discussed in Diana Deutsch and Richard C. Boulanger, "Octave Equivalence and the Immediate Recall of Pitch Sequences," in Music Perception 2/1 (1984), pp. 40-51.

[11]See Paul Pederson, "The Perception of Octave Equivalence in Twelve-Tone Rows," Psychology of Music 3/2 (1975), pp. 3-8; and Krumhansl, Sandell, and Sergeant, cited above.

[12]This view is prevalent in Dirk-Jan Povel's publications. See, in particular, "Internal Representation of Simple Temporal Patterns," Journal of Experimental Psychology: Human Perception and Performance 7/1 (1981), pp. 3-18.

[13]Eric F. Clarke, "Structure and Expression in Rhythmic Performance," in Musical Structure and Cognition, ed. Peter Howell, Ian Cross, and Robert West (London: Academic Press, 1985), pp. 209-236.

[14]Jeanne Bamberger, "Intuitive and Formal Musical Knowing: Parables of Cognitive Dissonance," in The Arts, Cognition and Basic Skills, ed. by S. S. Madeja, (New Brunswick, NJ: Transaction Books, 1978).

[15]Subjects with absolute pitch frequently make errors in octave identification, although they correctly identify the pitch class. See, for example, Mark Klein, Michael Coles, and Emanuel Donchin, "People with Absolute Pitch Process Tones Without Producing a P300," Science 223 (1984), p. 1306-1307.

[16]This question has already been investigated to some extent; see Klein, Coles, and Donchin, "People with Absolute Pitch...," pp. 1306-1309. According to these authors,

> The P300 is a positive-going component of the event-related brain potential (ERP) that may be a manifestation of the processes of maintaining or updating working memory.... It is quite easily obtained in the "oddball" procedure, in which two discrete stimuli (one frequent and one rare) are presented in a Bernoulli sequence; the subject counts the rare stimulus, which elicits a large P300. A consideration of the variables that control the amplitude and latency of the P300 has led Donchin and his colleagues to suggest that the component is the manifestation of a subroutine invoked whenever there is a need to update the model of the environment in working memory. If AP [absolute pitch] subjects process acoustic stimuli without reliance on such schema, they should not emit a P300 in response to novel tones. We have confirmed this prediction. (p. 1306)

They also suggest that subjects with absolute pitch have "access to a set of internal 'standards' that allows them to fetch the name of a tone without comparing the representation of the tone they have just heard with a recently fetched representation of a standard. If so, those with AP may not need to maintain, or update, in their working memory the representations of infrequently occuring events" (Ibid.). Experimentation at the University of Rochester under the direction of Robert Frisina and Joseph Walton is currently testing the extent to which the P300 is elicited by subjects in timbre discrimination tests.

[17]G. H. Miller, "The Magic Number Seven, Plus or Minus Two," Psychological Review 63 (1956), pp. 81-97.

[18]Cheryl L. Bruner, "The Perception of Contemporary Pitch Structures," Music Perception 2/1 (1984), pp. 25-39.

[19]Ibid., p. 26. Her experiment attempts to test the perceptual validity of the SIM relation, presented in Robert Morris, "A Similarity Index for Pitch Class Sets," Perspectives of New Music 18 (1979-80), pp. 445-460.

[20]Bruner, p. 25.

[21]Ibid., p. 38.

[23]This is especially important in light of Judy Edworthy's somewhat

[23]This is especially important in light of Judy Edworthy's somewhat questionable conclusion that as melody length increases, interval information is retained and contour information is lost. (Edworthy, "Interval and Contour in Melody Processing," p. 375.)

[24]Because of the limited time in a one-semester framework and because all the music literature to be studied can be understood metrically, duration space and pitch-span space will not figure into this curriculum. These, and other generalizations of

contour theory, might be included in an upper-level course. The rhythmic aspect of aural skills training in this course will therefore focus upon such features of non-tonal metric music as asymmetrical meters, changing meters, less common divisions of the beat, metric modulation, syncopation, and rhythmic dissonance.

[25]John Rahn's Basic Atonal Theory (New York: Longman, Inc., 1980) offers a number of listening exercises, and in its writing style and approach shows great sensitivity to aural experience, but it has no systematic ear-training program.

[26]Concepts appropriate to analysis of twelve-tone music are therefore introduced from the beginning of the course, thus weakening the artificial boundary between theories of atonal and twelve-tone musical analysis.

[27]The concepts of contour recognition and graphing are not unfamiliar to music education specialists. A standard way of having a class of children internalize melodic contour is to have them move their hands and arms in the air to show contour as they hear or sing a tonal melody. They may also be asked to draw melodic contour.

[28]Paul Hindemith, Elementary Training for Musicians (New York: Associated Music Publishers, 1949), p. 182.

[29]For students who have been surrounded by tonal music all their lives, it is difficult not to hear modes in terms of altered major or minor scales, whether these modes are heard in isolation or in pitch-centric modal compositions. Likewise, certain set classes may suggest tonal sonorities to students (triad and seventh chord types, for example). Such set-class associations should be abandoned as soon as possible in aural skills training because, although trichords and tetrachords in isolation may evoke these associations, they rarely do in non-tonal musical contexts.

[30]As discussed previously, experimental results show that non-tonal melodies are initially stored in short-term memory in terms of their contours. For this reason, ear-training exercises initially focus upon perception of contour relationships. However, as melodies are learned and stored in long-term memory, common sense implies that for most listeners they are retained as interval successions rather than pc successions, since most listeners use an interval-based "relative pitch" strategy rather than an "absolute" pitch-class recognition strategy. Although this is the premise from which the following discussion of pedagogy proceeds, it has yet to be proven conclusively by experimentation.

[31]When sets are inverted upon the staff in pitch space, the axis of inversion is graphically illustrated. In this case, the axis is g (7); however, if the inverted set were transposed up an octave the axis would be c# (13). With pitch-space numeric notation, the axis of inversion can be found by summing the pitch numbers between sets (adding the first of each set, second of each set, etc.) and dividing by two. With the two sets given in Figure IV-3A, < 7 8 11 13 > and < 7 6 3 1 >, each pair sums to 14; thus the axis of inversion is 7. If the second set were transposed up an octave to < 19 18 15 13 >, each would sum to 26, with the axis at 13. If the pairs sum to an odd number,

the axis of inversion occurs between two pitches. As Morris points out (Composition with Pitch Classes, p. 147), axis of inversion is not a meaningful concept in pc space:

> If we define...a pc-space center of inversion where two entities related by T_nI have n/2 as their center, it turns out that for every n there are two answers for n/2, one greater or less than the other by 6.... From this we would have to posit two centers of inversion.... Moreover, by realizing our pcsets in different regions of p-space many other inversional centers can be produced...in that case, there is no inversional center at all.

[32] This is one of the approaches Bruce Benward uses in Workbook in Advanced Ear Training (Dubuque, Iowa: Wm. C. Brown Co., Publishers, 1969).

[33] Changes in arpeggiation in pitch-space involve changes in order only, while in pitch-class space it may also involve changes in octave placement and thus spacing.

[34] See Morris, Composition with Pitch Classes, pp. 40-41. Richard Chrisman's formulation appears in "Identification and Correlation of Pitch-Sets," Journal of Music Theory 15 (1971), pp. 58-83, and "Describing Structural Aspects of Pitch-Sets Using Successive-Interval Arrays," Journal of Music Theory 21/1 (1977), pp. 1-28. The latter article also discusses ways in which the successive-interval array may be used to predict pc-invariance under transposition or inversion.

[35] Chrisman, "Identification and Correlation," pp. 78-80. Eric Regener describes a similar interval notation, though he prefers the largest number appearing to the left; thus, "the normal form of an n-note chord...is simply that circular permutation having the largest value when considered as an n-digit number" (Eric Regener, "On Allen Forte's Theory of Chords," Perspectives of New Music 13/1 (1974), pp. 196-197).

[36] When students are comfortable with T_n and T_nI as transformations upon intervals, the instructor may lead the students to discover that these transformations can also be defined as operations upon pcs. This can be presented as an alternative "short cut," using addition and subtraction of pc integers (as defined in Chapter 1).

[37] That is not to say that these set classes can only be understood or recognized in terms of their scalar supersets (or that the supersets suggested are the only ones possible), but simply to assert that this is one way to go about organizing their presentation in aural skills training.

[38] Just as students beginning aural skills training find harmonic intervals much more difficult than melodic, so do students find harmonic realizations of set classes more difficult than melodic realizations.

[39] Chrisman, "Describing Structural Aspects," pp. 13-16.

[40]Michael Rogers suggests various sonorities that might be used for ear-training, as well as classroom techniques for drilling these patterns, in Teaching Approaches in Music Theory: An Overview of Pedagogical Philosophies (Carbondale: Southern Illinois University Press, 1984), pp. 140-142.

[41]Richard L. Cohn, "Inversional Symmetry and Transpositional Combination in Bartók," Music Theory Spectrum 10 (1988), pp. 19-42; also his "Transpositional Combination in Twentieth-Century Music" (Ph.d. dissertation: University of Rochester, 1986).

[42] Sets of larger cardinalities may be used to project other sets. SC 3-1 projected by SC 4-28 produces the chromatic scale, for example: {0, 1, 2} plus {3, 4, 5}, {6, 7, 8}, and {9, 10, 11}. The transposition operators, {0, 3, 6, 9} form SC 4-48.

[43]Robert Morris, Composition with Pitch Classes: A Theory of Compositional Design (New Haven: Yale University Press, 1987), p. 282-283.

[44]Ibid.

[45]Other approaches to timbre analysis in terms of sequential dimensions may be found in David L. Wessel, "Timbre Space as a Musical Control Structure," Computer Music Journal 3/2 (1979), pp. 45-52; Kaija Saariaho, "Timbre and Harmony: Interpolations of Timbral Structures," Contemporary Music Review 2/1 (1987), pp. 93-133; and Fred Lerdahl, "Timbral Hierarchies," in the same issue, pp. 135-160. Wessel describes a two-dimensional model with one axis representing "dull" to "bright," and the other "less bite" to "more bite." Saariaho uses a sound-noise axis. Lerdahl discusses a vibrato dimension and a "harmonicity" dimension (its prototype has entirely natural harmonics), which are combined in a two-dimensional array. Lerdahl even evokes the idea of a "timbral interval," as "the distance from one timbre to another, where distance is judged not by acoustic specification but by perceived difference (as psychologists know well, there is often a discrepancy between the two)." (p. 145)

[46]Robert Morris calls this type of spacing "inverse overtone spacing," one of six spacing types defined in Composition with Pitch Classes, pp. 54-55. He notes that spacing types are somewhat related to the idea of contour-classes in c-space. While his six spacing types are not equivalence classes, he observes that it is possible to define criteria for spacing-types that would form equivalence classes.

[47]The contour interval is defined and discussed on p. 230 ff. of Michael Friedmann's "A Methodology for the Discussion of Contour: Its Application to Schoenberg's Music," Journal of Music Theory 29/2 (1985).

[48]The chords shown in the figure are taken from their attack point; in some cases there is a change in pitch after the initial attack.

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APPENDIX I

C-SPACE SEGMENT-CLASSES OF CARDINALITIES 2 THROUGH 6

The following table of csegclasses, cardinalities 2 through 6, is a portion of the output of a computer program written by Paul A. Laprade in March 1986. The program, written in standard Pascal language, was implemented on a Digital PRO-350 using the Xenix Pascal compiler and editor.

The csegclasses are listed in prime form, grouped by cardinality, and numbered in ascending order by prime form considered as an integer value. An asterisk (*) following the csegclass name indicates the property of identity under retrograde-inversion. For referential purposes, the INT_1 of a csegclass is listed at the right of its csegclass representative.

C-space segment classes for cseg cardinality 2

Csegclass/RIinv.	Prime form	INT(1)
c 2-1*	< 0 1 >	< + >

C-space segment classes for cseg cardinality 3

Csegclass/RIinv.	Prime form	INT(1)
c 3-1*	< 0 1 2 >	< + + >
c 3-2	< 0 2 1 >	< + − >

C-space segment classes for cseg cardinality 4

Csegclass/RIinv.	Prime form	INT(1)
c 4-1*	< 0 1 2 3 >	< + + + >
c 4-2	< 0 1 3 2 >	< + + − >
c 4-3*	< 0 2 1 3 >	< + − + >
c 4-4	< 0 2 3 1 >	< + + − >
c 4-5	< 0 3 1 2 >	< + − + >
c 4-6	< 0 3 2 1 >	< + − − >
c 4-7*	< 1 0 3 2 >	< − + − >
c 4-8*	< 1 3 0 2 >	< + − + >

C-space segment classes for cseg cardinality 5

Csegclass/RIinv.	Prime form	INT(1)
c 5-1*	< 0 1 2 3 4 >	< + + + + >
c 5-2	< 0 1 2 4 3 >	< + + + − >
c 5-3	< 0 1 3 2 4 >	< + + − + >
c 5-4	< 0 1 3 4 2 >	< + + + − >
c 5-5	< 0 1 4 2 3 >	< + + − + >

APPENDIX I (Continued)

c 5-6	< 0 1 4 3 2 >	< + + − − >
c 5-7	< 0 2 1 4 3 >	< + − + − >
c 5-8	< 0 2 3 1 4 >	< + + − + >
c 5-9	< 0 2 3 4 1 >	< + + + − >
c 5-10	< 0 2 4 1 3 >	< + + − + >
c 5-11	< 0 2 4 3 1 >	< + + − − >
c 5-12	< 0 3 1 4 2 >	< + − + − >
c 5-13*	< 0 3 2 1 4 >	< + − − + >
c 5-14	< 0 3 2 4 1 >	< + − + − >
c 5-15	< 0 3 4 1 2 >	< + + − + >
c 5-16	< 0 3 4 2 1 >	< + + − − >
c 5-17	< 0 4 1 2 3 >	< + − + + >
c 5-18	< 0 4 1 3 2 >	< + − + − >
c 5-19	< 0 4 2 1 3 >	< + − − + >
c 5-20	< 0 4 2 3 1 >	< + − + − >
c 5-21	< 0 4 3 1 2 >	< + − − + >
c 5-22	< 0 4 3 2 1 >	< + − − − >
c 5-23*	< 1 0 2 4 3 >	< − + + − >
c 5-24	< 1 0 3 4 2 >	< − + + − >
c 5-25	< 1 0 4 2 3 >	< − + − + >
c 5-26	< 1 0 4 3 2 >	< − + − − >
c 5-27	< 1 2 4 0 3 >	< + + − + >
c 5-28	< 1 3 0 4 2 >	< + − + − >
c 5-29	< 1 3 4 0 2 >	< + + − + >
c 5-30	< 1 4 0 3 2 >	< + − + − >
c 5-31*	< 1 4 2 0 3 >	< + − − + >
c 5-32	< 1 4 3 0 2 <	< + − − + >

C-space segment classes for cseg cardinality 6

Csegclass/RIinv.	Prime form	INT(1)
c 6-1*	< 0 1 2 3 4 5 >	< + + + + + >
c 6-2	< 0 1 2 3 5 4 >	< + + + + − >
c 6-3	< 0 1 2 4 3 5 >	< + + + − + >
c 6-4	< 0 1 2 4 5 3 >	< + + + + − >
c 6-5	< 0 1 2 5 3 4 >	< + + + − + >
c 6-6	< 0 1 2 5 4 3 >	< + + + − − >
c 6-7*	< 0 1 3 2 4 5 >	< + + − + + >
c 6-8	< 0 1 3 2 5 4 >	< + + − + − >
c 6-9	< 0 1 3 4 2 5 >	< + + + − + >
c 6-10	< 0 1 3 4 5 2 >	< + + + + − >
c 6-11	< 0 1 3 5 2 4 >	< + + + − + >
c 6-12	< 0 1 3 5 4 2 >	< + + + − − >
c 6-13	< 0 1 4 2 3 5 >	< + + − + + >
c 6-14	< 0 1 4 2 5 3 >	< + + − + − >
c 6-15	< 0 1 4 3 2 5 >	< + + − − + >
c 6-16	< 0 1 4 3 5 2 >	< + + − + − >
c 6-17	< 0 1 4 5 2 3 >	< + + + − + >
c 6-18	< 0 1 4 5 3 2 >	< + + + − − >
c 6-19	< 0 1 5 2 3 4 >	< + + − + + >
c 6-20	< 0 1 5 2 4 3 >	< + + − + − >
c 6-21	< 0 1 5 3 2 4 >	< + + − − + >
c 6-22	< 0 1 5 3 4 2 >	< + + − + − >

APPENDIX I (Continued)

c 6-23	< 0 1 5 4 2 3 >	< + + − − + >
c 6-24	< 0 1 5 4 3 2 >	< + + − − − >
c 6-25	< 0 2 1 3 5 4 >	< + − + + − >
c 6-26*	< 0 2 1 4 3 5 >	< + − + − + >
c 6-27	< 0 2 1 4 5 3 >	< + − + + − >
c 6-28	< 0 2 1 5 3 4 >	< + − + − + >
c 6-29	< 0 2 1 5 4 3 >	< + − + − − >
c 6-30	< 0 2 3 1 5 4 >	< + + − + − >
c 6-31	< 0 2 3 4 1 5 >	< + + + − + >
c 6-32	< 0 2 3 4 5 1 >	< + + + + − >
c 6-33	< 0 2 3 5 1 4 >	< + + + − + >
c 6-34	< 0 2 3 5 4 1 >	< + + + − − >
c 6-35*	< 0 2 4 1 3 5 >	< + + − + + >
c 6-36	< 0 2 4 1 5 3 >	< + + − + − >
c 6-37	< 0 2 4 3 1 5 >	< + + − − + >
c 6-38	< 0 2 4 3 5 1 >	< + + − + − >
c 6-39	< 0 2 4 5 1 3 >	< + + + − + >
c 6-40	< 0 2 4 5 3 1 >	< + + + − − >
c 6-41	< 0 2 5 1 3 4 >	< + + − + + >
c 6-42	< 0 2 5 1 4 3 >	< + + − + − >
c 6-43	< 0 2 5 3 1 4 >	< + + − − + >
c 6-44	< 0 2 5 3 4 1 >	< + + − + − >
c 6-45	< 0 2 5 4 1 3 >	< + + − − + >
c 6-46	< 0 2 5 4 3 1 >	< + + − − − >
c 6-47	< 0 3 1 2 5 4 >	< + − + + − >
c 6-48*	< 0 3 1 4 2 5 >	< + − + − + >
c 6-49	< 0 3 1 4 5 2 >	< + − + + − >
c 6-50	< 0 3 1 5 2 4 >	< + − + − + >
c 6-51	< 0 3 1 5 4 2 >	< + − + − − >
c 6-52	< 0 3 2 1 5 4 >	< + − − + − >
c 6-53	< 0 3 2 4 1 5 >	< + − + − + >
c 6-54	< 0 3 2 4 5 1 >	< + − + + − >
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APPENDIX I (Continued)

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APPENDIX I (Continued)

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APPENDIX I (Continued)

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c 6-191*	< 2 5 1 4 0 3 >	< + − + − + >
c 6-192*	< 2 5 4 1 0 3 >	< + − − − + >

APPENDIX II
GLOSSARY OF TERMS[1]

COM-matrix (comparison matrix) - a two-dimensional array that displays the results of the comparison function, COM(a,b). For any two c-pitches in c-space, if b is higher than a, the function returns "+"; if b is the same as a, the function returns "0"; and if b is lower than a, COM(a,b) returns "-." For any two d-pitches in d-space, if b's duration is longer than a, the function returns "+"; if b is the same duration as a, the function returns "0"; and if b is shorter than a, COM(a,b) returns "-."

C-pitches (cps) - elements in c-space, numbered in order from low to high, beginning with 0 up to (n - 1), where n equals the number of elements.

C-segment (cseg) - an ordered set of c-pitches in c-space.

C-space (contour space) - a type of musical space consisting of elements arranged from low to high disregarding the exact intervals between elements.

C-space segment class (csegclass) - an equivalence class made up of all csegs related by identity, translation, retrograde, inversion, and retrograde-inversion.

C-subsegment (csubseg) - any ordered subgrouping of a given cseg. May be comprised of either contiguous or non-contiguous c-pitches from the original cseg.

D-segment (dseg) - an ordered set of durs in d-space.

D-space (duration space) - a temporal musical space consisting of elements arranged from short to long, where the precise, calibrated duration of each element is ignored and left undefined. D-space models relative duration in the same way that c-space models relative pitch height.

D-space segment class (dsegclass) - an equivalence class made up of all dsegs related by identity, translation, retrograde, inversion, and retrograde-inversion.

D-subsegment (dsubseg) - an ordered subgrouping of a given dseg. Dsubsegs in duration space are assumed to be contiguous subgroupings, unlike csubsegs in c-space.

Durs (durations) - elements in d-space, numbered in order from low to high, beginning with 0 up to (n - 1), where n equals the number of elements.

Equivalent segments - segments that generate identical COM-matrices are considered equivalent. Equivalent segments can be reduced to the same normal form.

INT_n - any of the diagonals to the right of the main diagonal (upper left-hand to lower right-hand corner) of the COM-matrix, in which n stands for the difference between order position numbers of the two elements compared; that is, INT_3 compares those elements which are 3 positions apart.

Inversion - the inversion of a segment S comprised of n distinct elements is written IS, and may be found by subtracting each element from (n - 1).

Meter - a relationship between two different levels of equally-spaced pulses, requiring both a faster- and a slower-paced pulse in consonant relationship (proper alignment) with each other.

Normal form - an ordered array in which members of a segment of n distinct elements are numbered from 0 to (n - 1) and listed in temporal order.

Prime form - a representative form for identification of segment classes, derived by the following algorithm: (1) if necessary, translate the segment so its content consists of integers from 0 to (n - 1); (2) if (n - 1) minus the last element is less than the first, invert the segment; (3) if the last element is less than the first, retrograde the segment.

Rhythm - a succession of durations that may or may not be metrical, and may or may not contain a perceived beat.

Translation - an operation through which a segment is renumbered from 0 for the lowest element to (n - 1) for the highest.

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[1]The reader is referred to the chapter texts for fuller explanation of terms and for bibliographic citations, where appropriate, linking these concepts and terms to those of other music theorists.

APPENDIX III

CURRICULUM DESIGN

COURSE DESCRIPTION:

In this course, students develop the basic concepts, terminology, and analytical skills necessary to understand and articulate aspects of structure in non-tonal Western music. Repertoire for analysis is drawn from "classic" compositions of the first half of this century, including music of Bartok, Stravinsky, Schoenberg, Webern, Berg, Dallapiccola, and Babbitt. Coordinated dictation, sight singing and keyboard exercises reinforce theoretical concepts with aural experience.

COURSE OBJECTIVES:

By the end of the semester, students should be able to:

1) Analyze a passage of non-tonal music and discuss details of its structure, including aspects of its musical design (form, phrase structre, rhythmic articulation, contour relations) and its underlying pitch and/or pitch-class structure;
2) Recognize, by eye and ear, all twelve trichord types;
3) Identify, by eye and ear, transformations in contour-, pitch-, and pitch-class space upon a given segment or set;
4) Identify set classes larger than trichords in analysis, with the aid of a set-class list;
5) Improvise at the keyboard using non-tonal materials;
6) Take non-tonal melodies in dictation, and sing such melodies at sight;
7) Synthesize the concepts covered in class by presenting their original insights into the structure of non-tonal music in the form of analytical papers, model compositions, and/or class presentations.

REQUIRED TEXTS:

Adler, Samuel. Sight Singing: Pitch, Interval, Rhythm. New York: W. W. Norton & Company, 1979.

Burkhart, Charles. Anthology for Musical Analysis. 4th edition. New York: Holt, Rinehart and Winston, 1986.

Other supplementary materials--including John Rahn's Basic Atonal Theory (New York: Longman, 1980), Murray J. Gould's Paths to Musical Thought (New York: Holt, Rinehart and Winston,1979), and various scores and recordings--are on reserve in the library or will be handed out in class.

SEQUENCE OF PRESENTATION:

Note: Ear training tasks listed near the beginning of the curriculum are to be continued throughout the term, gradually increasing the length and difficulty of materials used. They will not be repeated continuously on the outline due to space limitations. Sight singing assigments are drawn from the Adler and Gould texts or from class handouts. Keyboard assignments are not difficult technically, but are important aural reinforcement of concepts discussed in class.

Concepts	Pieces	Keyboard	Sight Singing & Ear Training
I. Pitch Spaces: C-space and P-space A. C-space segments - equivalence - I-relation - subsegments B. P-space segments - T_n/T_nI defined via INT_1	Bartok, "Bulgarian Rhythm," Mikrokosmos IV - #115	Given a cseg or pseg, improvise an extended non-tonal melody that is a continuous variation on that segment and its inversion.	"Model" non-tonal cseg given, followed by three others; students choose equivalent segment of the three. Students may draw & use contour graphs. Later, add I-, R-, and RI-related csegs for students to recognize. Gradually increase length & difficulty (prep. for 12-tone unit).
II. Segment Class Membership: $T_n/T_nI/RT_n/RT_nI$ equivalence A. RT_n/RT_nI defined in c-space and p-space (via INT_1)	Bartok, "Diminished Fifth" Mikrokosmos IV - #101		
B. Segclasses in various spaces compared (2 segs, same csegclass but different psegclass) C. Intervals as 2-element segment classes		Interval drills: chains of intervals, non-tonal sequences for continuation, improvisation based on models featuring specific intervals.	
III. Unordered Sets in P-space A. Psets as conceptually unordered; order occurs in musical context B. Ascending order for comparison			Error detection exercises: student identifies discrepancies between notation and melody played (identical contour); gradually increase length & difficulty

Concepts	Pieces	Keyboard	Sight Singing & Ear Training
C. Octatonic collections as unordered sets; relation to form (Dim 5th)			Interval drills - SS: Adler, "Preparatory Exercises" & "Strike & Sing" from various chapters. ET: students identify all pc intervals (0-11) in isolation; simple intervals progress to wide compound int's; later, extremes in register. Later, unordered ic's identified by SC label (2-1 to 2-6).
IV. Intro. to Pc-space: C-space, p-space, & pc-space compared A. Integer notation compared B. Intervals in pc-space C. Inversional symmetry in p-space only	Bartok, "Isle of Bali" Mikrokosmos IV - #109		
V. Unordered Sets in Pc-space A. Interval classes in pc-space as two-element SCs; generate list B. Larger SC's: trichords - normal order as in Chrisman - $CINT_1$ to define T_n/T_nI - generate list - SC labels	Bartok, "Five-Tone Scale" Mikrokosmos III - #78	Improvise WT and/or pentatonic tune, accompany with approp. trichords as studied (experiment with diff. chordal spacings)	Dictation: interval chains. Student notates as contour graph, then as p-space INT_1. SC identification: [0, 2, 5] and [0, 2, 7]. Drill in 3 rotations; gradually alter octave & contour. SS: Gould SC drills; Adler, P4 & P5 chapters; pentatonic tunes.

Concepts	Pieces	Keyboard	Sight Singing & Ear Training
			Pattern recogn: several INT_1's given; student identifies which was played. Gradually longer & more difficult.
C. SC membership in pc-space - norm is 24 - T_n/T_nI invariance	Bartok, "Whole-Tone Scale" Mikrokosmos V - #136		
D. Prime form as representative; SC's larger than trichords; SC lists			Add: [0, 2, 4], [0, 2, 6], [0, 4, 8] & SS associated Gould exercises; then add "inverted normal orders": [0, 3, 5], [0, 4, 6]

VI. APPLICATION OF THEORY TO COMPOSITION, ANALYSIS, AND PERFORMANCE:

Application units give the students a chance to examine larger works in more depth, to apply theories as composition tools or performance aids, and to discuss other issues of musical structure (form, rhythm, phrase structure, "stylistic" considerations). During each application unit students are required to complete a small project; class sessions are devoted to analysis and discussion of larger works.

Possible projects:
- Bartok-style composition project; piano piece for performance and explanation in class; student hands in both score and short analysis
- Analysis paper; Bartok piano piece chosen in consultation with instructor
- Mini lecture-recital in class; student chooses Bartok piano piece to perform and discuss structure (presentation and handout graded).

Class sessions: Analysis and discussion of larger works such as:

Bartok, Music for Strings, Percussion, & Celesta, I & II
Bartok, Sonata for Two Pianos & Percussion I
Stravinsky, Rite of Spring, Introduction to Part II
Stravinsky, Sonata for Two Pianos, I

Class discussion:

What musical elements determine phrase structure and form in the absence of tonal structure?
How is "sonata form" articulated in a non-tonal context?
Is there pitch-centricity in this music? How is it established?
etc.

Skills components:

Continue all previous types of ET exercises (error detection, pattern recognition, SC identification, contour graphing, etc.); add [0, 3, 7], [0, 3, 6], [0, 4, 7]. Add dictation of melodies based on chains of trichords studied. Sight singing associated Gould exercises, octatonic melodies, Adler chapters on M/m thirds.

Keyboard: improvise a simple Bartok-style piano work, following certain constraints imposed by teacher and using Mikrokosmos as model.

END OF UNIT I - EXAM I (written and ear training)

Concepts	Pieces	Keyboard	Sight Singing & Ear Training
VII. Interval-class content of unordered pcsets A. T-matrix B. ICvector C. T_n invariance - $CINT_1$ -T-matrix D. Z-relation	Schoenberg, "Farben," Op. 16/3	Auditor names any of seven trichords studied thus far; student plays in four different chordal realizations, altering T_n, T_nI and/or spacing.	Continue all previous: interval chains, contour graphing, error detection, interval ID, set class ID, pattern recognition. SS: Adds Adler chapter on M/m 6ths; review melodies using trichords studied thus far.
VIII. T_nI invariance -$CINT_1$ -I-matrix			

Concepts	Pieces	Keyboard	Sight Singing & Ear Training
IX. Complementary SCs in pc-space A. Literal vs. abstract complementation B. Complement and hexachord theorems	Berg, Altenberglieder #2 & #3	Auditor names trichord, student improvises extended tune that is a chain of T_n/T_nI statements of that trichord.	Add: [0, 1, 4] & [0, 1, 6] with associated Gould exercises and dictation drills. SS: adds Adler Ch. on M/m 2nds
X. Similarity Relations A. In c-space: - COM matrix - CSIM	Webern, Orchestra Piece Op. 10/1		Continue all previous; add [0, 3, 4] & [0, 5, 6]
B. In pc-space: IC-based - R_1, R_2, R_0 - SIM, ASIM C. In pc-space: Inclusion-based - literval vs. abstract - R_p	Webern, String Quartet Op. 5/3 & 4	Same drill as previous two; now using remaining trichords as studied to create chords or melodies.	Add [0, 1, 3] & [0, 1, 5] with associated Gould drills; SS adds Adler Ch. on M/m 7ths

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XI. APPLICATION OF THEORY TO COMPOSITION, ANALYSIS, AND PERFORMANCE:

Possible projects: - Webern-style string trio (score & parts) for performance and explanation in class; student hands in both score and short analysis

Projects (continued) - Analysis paper; short atonal work of Schoenberg, Webern, or Berg, chosen in consultation with instructor
- Mini lecture-recital in class; student chooses atonal piece to perform and discuss structure (presentation and handout graded).

Class sessions: Analysis and discussion of larger works such as:
Schoenberg, Piano Piece, Op. 11/1
Berg, Altenberglieder, Op. 4 (selected remaining movements)
Webern, Orchestra Pieces, Op. 10 (selected movements)

Skills components:

Ear training adds [0, 1, 2], [0, 2, 3] and [0, 4, 5]. Keyboard and sight singing combined and consist of Gould's "Sing and Play" exercises; forms review of all trichord types.

END UNIT II - EXAM II

Concepts	Pieces	Keyboard	Sight Singing & Ear Training
XII. Segment classes continued; intro. to 12-tone rows A. Ordered segments in c-space, p-space, pc-space (review) B. Row classes: - review of TTOs - operations defined by INT_1 in p- and pc-space C. Contour preservation vs. non-preservation (effect on INT_1)	Dallapiccola Goethelieder, "Die Sonne Kommt"		Previous ET drills now use complete rows: error detection, pattern recognition (from INT_1), etc.; row dictation in steps, including contour graphs, INT_1 in p-space, staff notation. Recognition of T_n-, T_nI-, RT_n- and RT_nI-related rows via INT_1
XIII. Row Matrix A. Properties of Matrix B. Row Labelling C. Segmental invariance read from matrices	Dallapiccola, Quaderno, "Quartina"	Realization of rows in p-space from matrix; played as melodies (w/rhythmic context) or chord series	
XIV. Row realization in p-space A. Frozen register B. Inversional symmetry C. T_nI cycles	Webern, Piano Variations Op. 27/2 Webern, Symphony Op. 21/1		Continue ET of all trichord types; now in melodic and harmonic presentation; varying contours and registral placement.

Concepts	Pieces	Keyboard	Sight Singing & Ear Training
XV. Combinatoriality	Schoenberg, Piano Piece Op. 33a Babbitt, 3 Compositions for Piano, I	Given row(s), students determine transformations requested by auditor without a matrix; play as a melody.	Continue all previous in preparation for final exam.

XVII. APPLICATION OF THEORY TO COMPOSITION, ANALYSIS, AND PERFORMANCE:

Possible projects:
- Twelve-tone composition for performance and explanation in class; any style, but must use combinatoriality; student hands in both score and short analysis
- Analysis paper; Webern Op. 27/I or other 12-tone work chosen in consultation with instructor
- Mini lecture-recital in class; student chooses 12-tone piece to perform and discuss structure (presentation and handout graded).

Class sessions: Analysis and discussion of larger works such as:
Schoenberg, Violin Concerto, I
Dallapiccola, Quaderno (selected movements)
Berg, Lyric Suite, I
Babbitt, Semi-Simple Variations

Skills components:

Ear training review; sight singing uses all intervals and includes rows as well as other non-tonal melodies. Final keyboard assignment asks students to compose a row and its I-combinatorial pair; student brings integer notation to audit and improvises short piano piece from that notation.

END UNIT III - EXAM III

APPENDIX IV

TEXBOOK SURVEY: NON-TONAL AURAL SKILLS

Non-Tonal Aural Skills Texts:

Adler, Samuel. Sight Singing: Pitch, Interval, Rhythm. NY: W. W. Norton & Company, 1979.

Benward, Bruce. Advanced Ear Training. Dubuque, Iowa: Wm. C. Brown Publishers, 1985.

________. Workbook in Advanced Ear Training. Dubuque, Iowa: Wm. C. Brown Co., Publishers, 1969.

Edlund, Lars. Modus Novus: Studies in Reading Atonal Melodies. London: J. &. W. Chester Ltd., 1963.

Gould, Murray J. Paths to Musical Thought: An Approach to Ear Training Through Sight Singing. NY: Holt, Rinehart and Winston, 1979.

Herder, Ronald. Tonal/Atonal: Progressive Ear Training, Singing and Dictation Studies in Diatonic, Chromatic, and Atonal Music. NY: Continuo Music Press, Inc., 1973.

Hindemith, Paul. Elementary Training for Musicians. NY: Associated Music Publishers, 1946.

Kliewer, Vernon. Music Reading: A Comprehensive Approach. Englewood Cliffs, NJ: Prentice-Hall, Inc., 1973.

Self, George. Aural Adventure: Lessons in Listening. London: Novello, 1969.

Stearns, Peter Pindar. 235 Modern Dictation Exercises. Bryn Mawr: Coburn Press, 1987.

Thomson, William. Advanced Music Reading. Tucson: Sonora Music, 1975.

Wittlich, Gary E. and Humphries, Lee. Ear Training: An Approach through Music Literature NY: Harcourt Brace Jovanovich, Inc., 1974.

Commentary:

Because there is no definitive text for aural skills training of non-tonal music, the teacher asked to design an undergraduate non-tonal music theory curriculum enters a field in which diverse approaches abound. This discussion gives an overview of the pedagogical approaches offered in various ear-training and sight-singing textbooks. Two primary methodologies for aural skills training can be discerned: one views tonality and atonality along a continuum, each using the same materials but in different ways; the other isolates itself from tonal practice and takes as its point-of-departure aural memory for collections of "typical" non-tonal sonorities. Texts in both categories, and some that do not fall neatly into either category, use an organization based upon intervals, most commonly arranged in ascending order, minor second to major seventh, though other orderings are possible.[1]

Ronald Herder's Tonal/Atonal falls most conspicuously into the first of these categories.[2] Herder's text "sets up a graded series of eye and ear experiences...in the form of progressive interval studies" based on "the idea of chromatic transformations of a tonal melody."[3] Melodies drawn from composers such as Beethoven, Brahms, Mozart, and Bach are progressively distorted by chromatic alteration until they become completely "atonal." These altered melodies appear together with melodies from Schoenberg, Berg, Carter, and Hindemith, for sight singing and dictation. Herder also advocates the use of imaginary diatonic reference notes in inner-hearing exercises to assist the student in singing difficult chromatic melodies.

Vernon Kliewer's and Samuel Adler's sight singing texts also adopt the pedagogical stance that views tonal and non-tonal music along a continuum.[4] Both are organized in progressive units--Kliewer's by successively longer sets with larger and more difficult intervals, and Adler's by successively larger intervals. Within each unit, tonal and non-tonal melodies appear side-by-side in both rhythmic and arhythmic settings; both books contain melodies composed specifically for the text within the constraints of each unit, as well as melodies from music literature. Of all the texts discussed thus far, Adler's relies most heavily upon interval perception. Indeed, his preface states:

> In this volume, I have focused on the one musical element common to all styles and creative periods, namely the interval. I contend that if you sing by intervals at all times, you will be able to read music that is tonal, modal, pantonal, nontonal, or a combination of all of these means of organization and polarization.[5]

The psychological experimentation summarized in this dissertation has shown, however, that subjects with even moderate musical experience are unlikely to use absolute interval information to understand or remember tonal melodies. Rather, they tend to use contour and scale-degree information; this is probably the case for pitch-centric modal music as well. Thus, Adler's argument for the general applicability of an interval-based pedagogy for all musical styles is not supported by experimental evidence. Such a pedagogy probably is, however, correct for teaching sight-singing of non-tonal melodies. The text is invaluable for its interval-based organization of progressively more-difficult non-tonal melodies, even if its premise is not supported in the music-psychological literature.

Lars Edlund's Modus Novus is also organized by interval, though his perspective differs quite a bit from Adler's, in particular.[6] First, he divorces himself from tonal practice, comparing twentieth-century compositional techniques to the triadic tonal practice that preceded it:

> Does the music so far composed in the 20th century contain any such logical structural principles which could serve as a basis for a method of training the ear?.... The answer to this, of course, is "No"! The study material presented in this book, however, has been built up on a number of tonal and melodic figures which in the author's opinion have played some part in avoiding the major/minor-tonal limitations in 20th century music.[7]

Although most units contain newly-composed tunes organized by interval, one of the author's main theses is "that great accuracy in singing individual intervals is not always a guarantee of accuracy in reading atonal melodies; this is because most students still feel the interval with a major/minor interpretation."[8] This strikes at the very heart of Adler's premise, and with some justification. Adler would probably be the first to agree, on the other hand, that each interval must be learned free of tonal associations if students are to benefit from an interval-based approach to sight-singing non-tonal music. Finally, William Thomson offers yet another contrasting opinion as to the efficacy of an interval-based pedagogy. He contends that intervals cannot be apprehended as "absolutes, exclusive of any background context.... attempting to imagine the two pitches of an interval exclusive of any context beyond themselves is like trying to imagine two points in vision without a spatial reference."[9] Thompson advocates use of contextual tonality frames--"either a frame that is made explicit by the patterns of the melody itself or...a frame that is imposed by the reader when the melody does not clearly project its own."[10] These frames posit local tonic pitches that may change every few bars, but which enable students to use scale degree information in sight singing non-tonal melodies.

The primary texts based upon teaching aural recognition of a finite group of non-tonal sonorities are those of Murray Gould and Bruce Benward.[11] Gould's is the only text that systematically teaches the twelve trichord types, using pc-notation and representing each trichord type by its normal order on zero. His theoretical explanations are fairly good, as far as they go, although he does not cite the work of Forte or others in shaping his discussion. He makes no clear distinction between pitch and pitch class, though he does between set and set type; he discusses inversional equivalence, total interval (not ic) content of each trichord as given in staff notation, and invariance of certain trichord types under T_n/T_nI to produce fewer than 24 representatives. He orders presentation of the trichords for aural training by a similar criterion to that advocated here, beginning with [0, 2, 7] and [0, 2, 5] and ending with [0, 1, 3] and [0, 1, 2]. Each trichord is introduced individually, and for each, nine different types of exercises are given--some are arhythmic melodies with small registral spans, others add rhythm and use octave displacement with wider leaps, some are to be sung with trichords played as chordal accompaniments on the piano to a free non-tonal melody sung by the student; each unit concludes with a small single-line composition constructed entirely as a trichordal chain.

Bruce Benward's ear-training texts use some "old-fashioned" terminology and theoretical concepts;[12] yet the methods by which he organizes his aural skills exercises are among the soundest pedagogically, since he pays careful attention to contour, order, and spacing in designing his exercises. His sensitivity to the importance of contour as an aural cue is shown in such tasks as those in which students are asked to distinguish between non-tonal melodies that have identical contours or chords that have identical spacing, but which belong to different set classes. In one type of exercise, the student is presented with a set of pitches, a series of chords, or a twelve-tone row divided into a series of trichords; then the student hears the pitches, chords, or trichords in a new order and is asked to notate the order heard. Benward's more recent text eliminates the Hindemith classifications. It retains many of the same types of error-detection and sonority-identification exercises, however. The author adds trichord and tetrachord identification exercises to this text, though he does not use Forte labels. Most are multiple choice identifications, in which students are asked to circle in a workbook the trichord or tetrachord just heard.

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[1]One textbook that falls into none of these categories deserves mention: Ear Training: An Approach through Music Literature (New York: Harcourt Brace Jovanovich, Inc., 1974) by Gary Wittlich and Lee Humphries. This textbook is a chronologically-arranged program in aural skills training that uses complete pieces from music literature, rather than single-line melodies from the literature or artificially-constructed exercises. It can therefore address issues such as instrumentation, texture, tempo changes, articulation, and so on, that are rarely treated in aural skills classes. The text contains four works that might be classified as modal or non-tonal: Debussy's Prelude to the Afternoon of a Faun, Stravinsky's Sérénade en La, and movements from Bartok's Contrasts and Dallapiccola's Quaderno. The major flaw in their approach is that it is not organized in progressive steps toward some goal, nor does it have enough exercises using non-tonal materials for students to develop any proficiency with this material. It would, however, complement the work students do in another sight singing or dictation text.

[2]Ronald Herder, Tonal/Atonal: Progressive Ear Training, Singing and Dictation Studies in Diatonic, Chromatic, and Atonal Music (New York: Continuo Music Press, 1973).

[3]Ibid., p. v.

[4]Vernon Kliewer, Music Reading: A Comprehensive Approach (Englewood Cliffs, NJ: Prentice-Hall, Inc., 1973) and Samuel Adler, Sight Singing: Pitch, Interval, Rhythm (New York: W. W. Norton & Company, 1979).

[5]Adler, p. x.

[6]Lars Edlund, Modus Novus: Studies in Reading Atonal Melodies (London: J. & W. Chester, Ltd., 1963). Edlund chooses an unusual ordering for his interval-based organization: seconds, fifths, thirds, tritones, sixths, sevenths.

[7]Ibid., p. 13.

[8]Ibid.

[9]William Thompson, Advanced Music Reading (Sonora Music, 1979), p. x.

[10]Ibid.

[11]Murray J. Gould, Paths to Musical Thought: An Approach to Ear Training Through Sight Singing (New York: Holt, Rinehart, and Winston, 1979), and Bruce Benward, Workbook in Advanced Ear Training (cited above) and Advanced Ear Training (Dubuque, Iowa: Wm. C. Brown Publishers, 1985).

12 Benward's 1969 book is, understandably, quite dated; it relies, for example, upon Hindemith-classifications for identification of twentieth-century chord types.