M374M
Homework #2

Logan.

Section 1.2: p.40 print [p.83 ut ebook] (#1[1], 3[2], 4[3], 9).

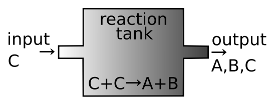
[1]This problem illustrates an alternative way to identify scales. Assume A, ω and λ are positive constants.

[2]Assume m, x are functions of t. The notation m' means dm/dt. As time t increases, show that the size x of an organism does not grow without bound, but instead approaches some maximum value.

[3]Find dimensions of constants m, a, k, V. For small restoring force scenario, find time and length scales from m, a, V and non-dimensionalize equation. Under what condition on k would we expect restoring force term to be negligible?

Mini-project.

A model for the mass concentration of a chemical `C` in a tank reactor is



$$\dot{c} = \frac{\eta}{V}(c_{\text{in}} - c) - \gamma c^2, \quad c(0) = 0. \tag{1}$$

Here $c(t)$ is the average concentration of `C` in the reactor and its output, c_{in} is the concentration at the input, V is the volume of the reactor, η is the volume flow rate and γ is a constant that describes the reaction rate. When γ is "small" we expect the nonlinear term to be negligible, so that (1) can be approximated by

$$\dot{c} = \frac{\eta}{V}(c_{\text{in}} - c), \quad c(0) = 0. \tag{2}$$

Here we derive a condition to determine what it means for γ to be "small" in this sense. Relevant physical dimensions are $[c] = ML^{-3}$, $[\eta] = L^3T^{-1}$ and $[\gamma] = L^3M^{-1}T^{-1}$. Below we use the units of kilograms, meters and seconds.

(a) In the case when γ is "small", or actually zero, the dominant effects in (1) are due to η, V and c_{in}. Use these quantities to find a characteristic time scale t_c and concentration scale c_c. Rewrite (1) in dimensionless form.

(b) Using the result from (a), identify a condition on γ under which the nonlinear term is expected to be negligible. In which of the two operating scenarios $(c_{\text{in}}, \eta, V, \gamma) = (10^{-2}, 10^{-2}, 10, 10^{-1})$ or $(c_{\text{in}}, \eta, V, \gamma) = (10^{-2}, 10^{-1}, 1, 10^{-1})$ is this condition reasonably met? That is, in which scenario can γ be regarded as "small"?

(c) Download the Matlab program file `program2.m` and function file `rtank.m` from the course webpage. These files numerically solve the reaction model; read the program file for instructions on how to run.

(d) For the first operating scenario in (b), simulate the systems in (1) and (2) for $t \in [0, 10V/\eta]$ and superimpose plots of c versus t for the two systems. Repeat for the second scenario. Do the simulations agree with the analysis in (b)? Specifically, does the nonlinear term seem to have a negligible or small effect in one of the scenarios as predicted?

Turn in: responses to (a), (b) and (d).