M374M
Homework #4

Logan.

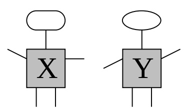
Sections 2.1, 2.2: p.87 print [p.158 ut ebook] (#3,5[1]). p.93 print [p.170 ut ebook] (#1a,1f,5[2]).

[1]Instead of the linearization, find all equilibria; sketch the direction of solution curves in a small region around each equilibrium in the phase plane.

[2]Assume α, β and γ are arbitrary positive constants; account for gains and losses in x and y with appropriate signs in the model equations.

Mini-project.

A simple model for the relationship dynamics between two people X and Y is



$$\dot{x} = ax + by, \quad x(0) = x_0$$
$$\dot{y} = cx + dy, \quad y(0) = y_0. \tag{1}$$

Here $x(t)$ is the intensity of X's feelings (for Y), and $y(t)$ is the intensity of Y's feelings (for X), where positive values indicate love, and negative values indicate hate. The constants a, b, c and d can take any value and characterize their personalities: $a > 0$ and $b > 0$ means X is eager and responsive, $a < 0$ and $b < 0$ means X is cautious and manipulative; similarly for Y. Here we perform a qualitative analysis to understand the ultimate fate of a relationship depending on the personality types.

(a) Consider the case of $a = 0$, $b > 0$, and $c < 0$, $d = 0$: so X is responsive and Y is manipulative, and neither is eager or cautious. In words, X warms up when Y is warm, cools down when Y is cool, and has no self-amplifying or self-suppressing tendencies. Y behaves analogously, but cools down when X is warm, and warms up when X is cool. Show that, if $(x_0, y_0) \neq (0, 0)$, then the relationship evolves as a never-ending cycle of love and hate. Illustrate the behavior of the system on a phase diagram.

(b) Consider the case of $a = d < 0$ and $b = c > 0$, so that X and Y are both cautious and responsive with identical characteristics. Show that if $|a| > b$ (more cautious than responsive), then the relationship always fizzles out to mutual apathy. On the other hand, if $|a| < b$ (more responsive than cautious), then the relationship is explosive: it will generally end up in extreme mutual love or mutual hatred depending on the initial feelings. What set of initial feelings lead to mutual love? What about mutual hatred? For each case, illustrate the behavior on a phase diagram.

(c) Download the Matlab program file `program4.m` and function file `lovehate.m` from the course webpage. Use the program to numerically simulate the model equation in (1) and produce solution curves for various (x_0, y_0). Although the results do not depend on specific magnitudes, use values of a, b, c and d from the set $\{-2, -1, 0, 1, 2\}$ to illustrate the different cases.

Turn in: responses to (a)–(c).