M374M
Homework #3

Logan.

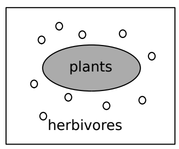
Section 1.3: p.53 print [p.103 ut ebook] (#1a[1],1n[1]). p.72 print [p.135 ut ebook] (#8a[2],9,13).

[1]This is a review question on solution methods for ODEs.

[2]Assume the parameter h can take any value: negative, zero or positive.

Mini-project.

A simple model for the population of plants in a plant-herbivore ecosystem is



$$\frac{dp}{dt} = rp\left(1 - \frac{p}{k}\right) - \frac{aqp}{1+bp}, \quad p(0) = p_0. \tag{1}$$

Here $p(t)$ is the number of plants, q is the constant number of herbivores, r and k are constants that describe the growth rate of the plants, and a and b are constants that describe the consumption rate of the plants by the herbivores. Here we perform a qualitative analysis to understand the behavior of solutions of the above model. To quantify the population sizes we introduce the dimensions $[p]$ = Plant and $[q]$ = Herbivore, and as usual $[t]$ = Time. All constants are assumed positive.

(a) Find the dimensions of r, k, a and b. Using the scales $t_c = 1/r$ and $p_c = k$, show that the dimensionless version of (1) takes the form

$$\frac{du}{d\tau} = u(1-u) - \frac{hu}{1+cu}, \quad u(0) = u_0 \tag{2}$$

where $u = p/p_c$ and $\tau = t/t_c$, and h and c are constants which you should identify.

(b) Assuming c is fixed, specifically $c > 1$, find (or characterize) all equilibrium solutions of (2) and determine their stability in terms of the parameter $h > 0$. Illustrate the results on a bifurcation diagram.

(c) Download the Matlab program file `program3.m` and function file `plant.m` from the course webpage. Use the program to numerically simulate the model equation in (2). Using $c = 4$ and a few different values of h, produce portraits of solutions for various u_0. Do the simulations agree with the analysis in (b)? Specifically, do solutions grow, decay, or remain constant according to the stability results in the bifurcation diagram?

(d) If $c = 4$ and the current plant population is at 25% of the carrying capacity so that $u(0) = 1/4$, for what range of the parameter h if any would the plant population survive as $\tau \to \infty$? For what range of h if any would the plant population die out as $\tau \to \infty$?

Turn in: responses to (a)–(d).