M374M
Homework #7

Logan.

Section 3.1: p.165 print [p.289 ut ebook] (#4[1], 9[2], 7[3], 20[4]).

[1]Given $f(y, \epsilon)$ and $y(\epsilon) = y_0 + \epsilon y_1 + \cdots$ let $F(\epsilon) = f(y(\epsilon), \epsilon)$. Find Taylor expansion for $F(\epsilon)$ up to $O(\epsilon^2)$ term.

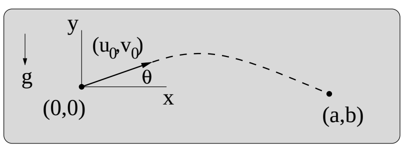
[2]Find approximation of each root up to $O(\epsilon^2)$ term.

[3]Show standard series in powers of ϵ does not work; then consider a generalized series in powers of $\epsilon^{1/3}$. It is helpful to introduce $\delta = \epsilon^{1/3}$ or $\epsilon = \delta^3$ and work with δ instead of ϵ. Find approximation of each root up to $O(\delta) = O(\epsilon^{1/3})$ term.

[4]Find approximation of solution up to $O(\epsilon)$ term.

Mini-project.

A basic problem in ballistics is to determine how to aim a given weapon in order for a bullet to strike a given target. Many factors are involved, such as gravity, wind, Coriolis, and bullet shape and spin effects, and all are important in long-range shots. Considering only gravity and air resistance, a simple model for the near-horizontal motion of a bullet is



$$\begin{aligned} m\ddot{x} &= -\alpha\dot{x}^2, & \dot{x}(0) &= u_0, & x(0) &= 0 \\ m\ddot{y} &= -\alpha\dot{x}\dot{y} - mg, & \dot{y}(0) &= v_0, & y(0) &= 0. \end{aligned} \tag{1}$$

Here $(x(t), y(t))$ is the bullet position, t is time, m is the bullet mass, g is gravitational acceleration, α is a resistance coefficient, and (u_0, v_0) is the bullet firing velocity; we assume $u_0^2 + v_0^2 = c^2$, where $c > 0$ is a constant that depends on the weapon. The problem is to determine the aiming angle $\theta = \arctan(v_0/u_0)$ required for the bullet path to intersect a fixed target at (a, b), where $a > 0$ and b is arbitrary.

(a) After dividing by m, consider the resulting system with small parameter $\epsilon = \alpha/m \geq 0$. In the case of $\epsilon = 0$, solve the system for $x(t)$ and $y(t)$ and show that the targeting problem has zero, one or two solutions for θ depending on a, b, c and g. If $c = 500\text{m/s}$ and $g = 10\text{m/s}^2$, what is the lowest aiming angle θ that can be used to strike a target at $(a, b) = (500, 2)$ meters? How much time is required for the impact? If s is the speed at impact, what is the ratio s/c?

(b) For the case of small $\epsilon > 0$, find a perturbation approximation for $x(t)$ and $y(t)$ up to $O(\epsilon)$ terms. Using this approximation, show that the targeting problem has zero, one or two solutions for θ depending on a, b, c, g and ϵ. If $c = 500\text{m/s}$ and $g = 10\text{m/s}^2$ as before, and $\epsilon = 0.0005\text{m}^{-1}$, what is the lowest aiming angle θ that can be used to strike the target at $(a, b) = (500, 2)$ meters? What is the corresponding impact time and speed ratio s/c now?

(c) Download the Matlab program file `program7.m` and function file `ballistic.m` from the course webpage. Use the program to numerically simulate the model equations in (1) and confirm your results in (a) and (b). Specifically, for the given values of the parameters and aiming angles, does the bullet path intersect (approximately) the target at the times and speeds as predicted?

Turn in: responses to (a)–(c).