M374M
Homework #1

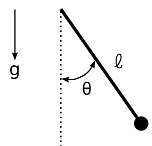
Logan.

Section 1.1: p.8 print [p.28 ut ebook] (#4[1]), p.27 print [p.59 ut ebook] (#5, 9[2], 13, 14[3]).

[1]Find reduced form of $g(t, r, \rho, E, P) = 0$ for $P \neq 0$. In special case when $P = 0$, deduce that $r = C(Et^2/\rho)^{1/5}$ as before.
[2]For fixed (T, r, C), show that Y must be proportional to V. The dimensions of concentration are M/L^3.
[3]Assuming a law $F(E, T, k) = 0$, derive an equivalent reduced law and show that $E = ckT$ for some constant c.

Mini-project.

A model for an ideal pendulum released from rest is



$$\left.\begin{array}{ll} \ell\ddot{\theta} + g\sin\theta = 0, & t \geq 0 \\ \dot{\theta} = 0, & t = 0 \\ \theta = \theta_0, & t = 0. \end{array}\right\} \tag{1}$$

Here θ is the pendulum angle, ℓ is the pendulum length, g is the gravitational acceleration constant, t is time, and over-dots denote time derivatives. A dimensional analysis of (1) reveals that the general solution $\theta = f(t, g, \ell, \theta_0)$ can be written in the form

$$\theta = \widetilde{f}(t\sqrt{g/\ell}, \theta_0) \tag{2}$$

for some function $\widetilde{f}$. Here we investigate various consequences of (2). Below we use meters and seconds as our units for length and time.

(a) Download the Matlab program file `program1.m` and function file `pendulum.m` from the course webpage. These files numerically solve the pendulum system in the first-order form $\dot{u}_1 = u_2$, $\dot{u}_2 = -(g/\ell)\sin u_1$, where $u_1 = \theta$, $u_2 = \dot{\theta}$. Read the program file for instructions on how to run.

(b) For the initial condition $\theta_0 = 3\pi/8$ and interval $t \in [0, 2.5]$, superimpose plots of θ versus t for different values of g and ℓ, say $(g, \ell) = (10, 0.25), (10, 0.5), (10, 1)$. Based on the general solution $\theta = f(t, g, \ell, \theta_0)$, briefly explain why different values of g and ℓ produce different curves.

(c) Repeat part (b) but now plot θ versus $t\sqrt{g/\ell}$. Based on the form of the solution in (2), briefly explain why different values of g and ℓ produce the same curve (or portion thereof). If θ_0 were changed in addition to g and ℓ, would we still get this same curve?

(d) Let T be the period of the pendulum motion for given θ_0, g and ℓ. Using the form of the solution in (2), show $T = \sqrt{\ell/g}\, h(\theta_0)$ for some function h. Does this agree with the observations in (b)? Specifically, for fixed θ_0 and g, does T increase by a factor of two when ℓ is increased by a factor of four?

Turn in: responses to (b), (c) and (d).

For all problems: must show correct supporting work to receive credit.