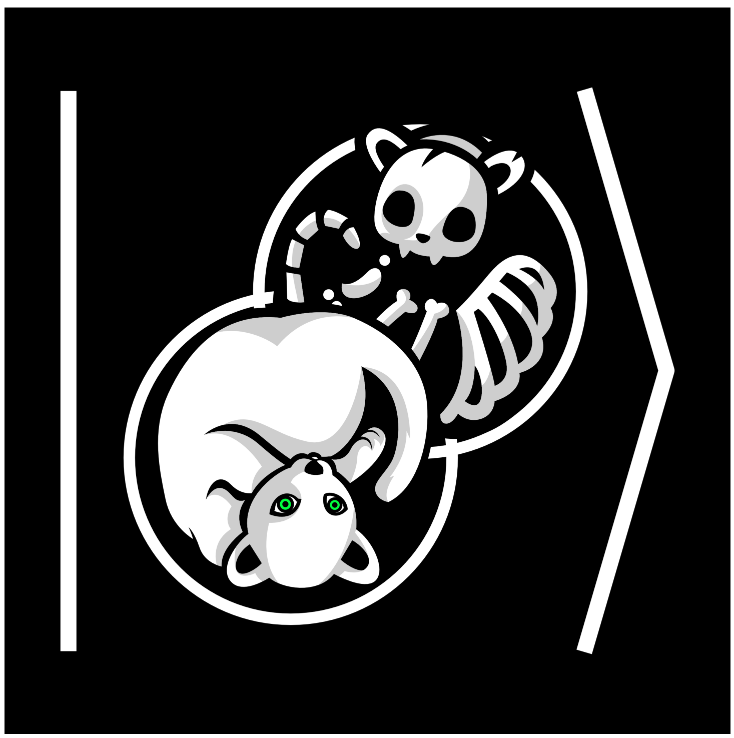
Entangled Kittens

Quantum Computing Tutorials



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A series of blog posts and video scripts
about quantum computing

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Preliminary Maths

Chapter 1

Complex Numbers

Complex numbers are essential in the mathematical formulation of quantum mechanics. In this chapter, I will quickly go over the basics of complex numbers and state some of their properties.

1.1 Imaginary Numbers

The imaginary unit, i, is a number defined such that

$$i^2 = -1$$

Some Properties of i:

$$i^3 = (i^2) \cdot i = (-1) \cdot i = -i$$

$$i^4 = (i^2)^2 = (-1)^2 = 1 = i^0$$

$$i \cdot (-i) = 1$$

$$\frac{1}{i} = -i$$

We call the product of a real number and i an "imaginary number".

1.2 Complex Numbers

The sum of a real number and an imaginary number is called a "complex number".

$$z = a + ib, \; z \in \mathbb{C}, \; a, b \in \mathbb{R}$$

a is called the real part of z

$$\mathrm{Re}(z) = a$$

and b is called the imaginary part of z

$$\mathrm{Im}(z) = b$$

1.2.1 The Complex Plane

We can represent complex numbers as points on a two-dimensional plane called the complex plane. The horizontal axis is called the real axis and the vertical axis is called the imaginary axis.

The point on the plane with Cartesian coordinates (a, b) represents the complex number $z = a + ib$.

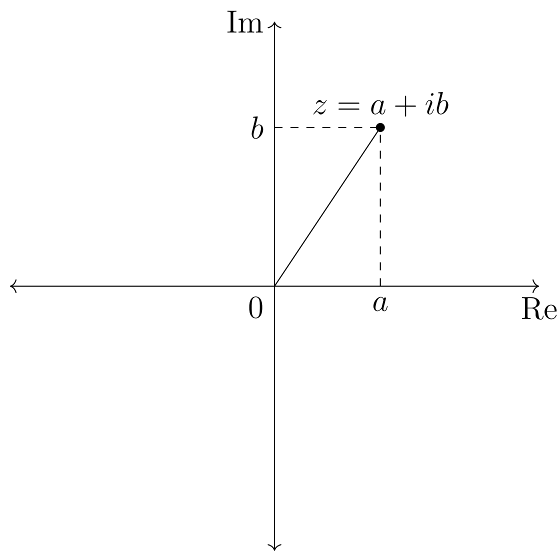

Figure 1.1: The complex plane

1.2.2 Algebra of Complex Numbers

Addition

To add complex numbers, add their real and imaginary parts respectively to get the sum

$$(a + ib) + (c + id) = (a + c) + i(b + d)$$

Multiplication

To multiply complex numbers, FOIL them

$$(a + ib)(c + id) = ac + a(id) + (ib)c + (ib)(id) = (ac - bd) + i(ad + bc)$$

Absolute Value

The distance from the origin to the complex number z in the complex plane is called the magnitude or the absolute value of z.

$$|z| = \sqrt{a^2 + b^2}$$

Complex Conjugate

Mirroring z about the real axis on the complex plane gives us its complex conjugate, z^*. This corresponds to flipping the sign of the imaginary part of z.

$$z^* = a - ib$$

Multiplying z with its complex conjugate gives us the square of the absolute value:

$$zz^* = (a + ib)(a - ib) = a^2 - (ib)^2 = a^2 + b^2 = |z|^2$$

As a rule of thumb, if asked to calculate the complex conjugate of an expression, just replace every i with $(-i)$.

1.3 Polar Form of Complex Numbers

The polar coordinates (r, ϕ) the complex plane correspond to Cartesian coordinates $(r\cos\phi, r\sin\phi)$, which correspond to the complex number

$$z = r(\cos\phi + i\sin\phi)$$

This is the polar form of a complex number. Comparing this to the Cartesian form, $z = a + ib$, we have:

$$r = |z| = a^2 + b^2$$

$$\phi = \tan^{-1}\left(\frac{b}{a}\right)$$

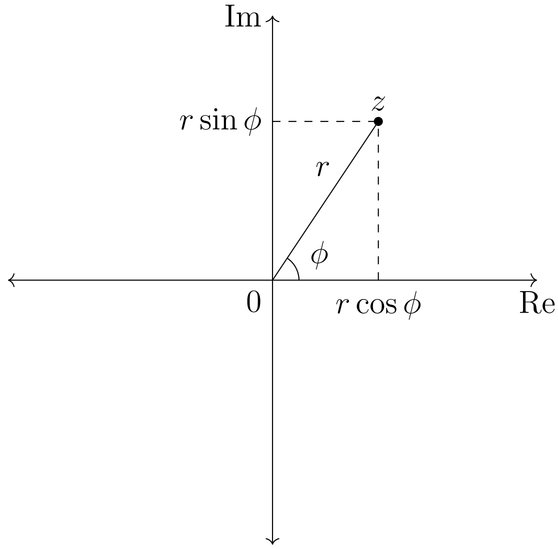

Figure 1.2: Polar form of a complex number

1.3.1 Euler's Formula

Complex numbers with $|z| = 1$ can be written $z = \cos \phi + i \sin \phi$. Another way of writing these numbers is

$$e^{i\phi} = \cos \phi + i \sin \phi$$

This is Euler's formula. The details of why this is true involves writing the power series expansions of $e^x, \sin x$, and $\cos x$, which I won't cover here.

In quantum computing literature, you'll see numbers of the form $e^{i\phi}$ referred to as "phases". Sometimes you'll see the complex number itself referred to as the phase, other times the angle ϕ will be called the phase. In either case remember that the complex number $e^{i\phi}$ will be present somewhere in that context.

1.3.2 Multiplication

Multiplication in polar form is much simpler thanks to Euler's formula. Given two complex numbers $z_1 = r_1 e^{i\phi_1}$ and $z_2 = r_2 e^{i\phi_2}$,

$$z_1 z_2 = (r_1 r_2) e^{i(\phi_1 + \phi_2)} = (r_1 r_2)[\cos(\phi_1 + \phi_2) + i \sin(\phi_1 + \phi_2)]$$

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