Implementing Quantum Teleportation on a Real Quantum Computer using Qiskit
Physics 707 Term Paper

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(Dated: August 4, 2021)

The quantum teleportation protocol allows two parties to transfer a qubit state from one qubit to another by sharing only classical information and a pair of qubits in an entangled state. In this paper, I will show how to emulate the teleportation protocol to run on IBM's free to use quantum computers using their open source software development kit – Qiskit. Lastly, I will use quantum state tomography to measure the fidelity of the teleportation protocol on the *ibmq_5_yorktown* system which is one of the IBM Quantum Canary processors.

I. THE QUANTUM TELEPORTATION PROTOCOL

Alice and Bob each have one of two qubits that are in the Bell state $|\Psi\rangle_{AB} = \frac{1}{\sqrt{2}}(|0\rangle_A |0\rangle_B + |1\rangle_A |1\rangle_B)$. Alice also has another qubit in the state $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$, the teleportation protocol allows her to transfer or *teleport* her state $|\psi\rangle$ to Bob's qubit by making a measurement on both of her qubits. [1, 2] Doing so does not violate the no-cloning theorem as the state of Alice's qubit is collapses after measurement. The protocol is described using the quantum circuit in Figure 1.1.

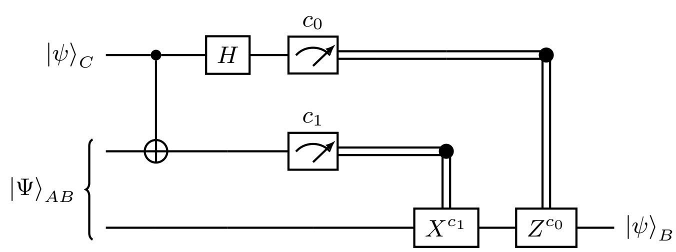


FIG. 1.1: The circuit describing the quantum teleportation protocol.

The initial state of the system is

$$\begin{aligned} |\psi_0\rangle &= |\psi\rangle_C \otimes |\Psi\rangle_{AB} \\ &= \frac{\alpha}{\sqrt{2}}(|0\rangle_C |0\rangle_A |0\rangle_B + |0\rangle_C |1\rangle_A |1\rangle_B) \\ &+ \frac{\beta}{\sqrt{2}}(|1\rangle_C |0\rangle_A |0\rangle_B + |1\rangle_C |1\rangle_A |1\rangle_B) \end{aligned} \tag{1.1}$$

After Alice applies a CNOT the state evolves to:

$$\begin{aligned} |\psi_1\rangle &= \mathrm{CNOT}_{C,A} |\psi_0\rangle \\ &= \frac{\alpha}{\sqrt{2}}(|0\rangle_C |0\rangle_A |0\rangle_B + |0\rangle_C |1\rangle_A |1\rangle_B) \\ &+ \frac{\beta}{\sqrt{2}}(|1\rangle_C |1\rangle_A |0\rangle_B + |1\rangle_C |0\rangle_A |1\rangle_B) \end{aligned} \tag{1.2}$$

And then after the Hadamard gate:

$$\begin{aligned} |\psi_2\rangle &= H_C |\psi_1\rangle \\ &= \frac{1}{2}\begin{bmatrix} |0\rangle_C |0\rangle_A (\alpha|0\rangle_B + \beta|1\rangle_B) \\ + |0\rangle_C |1\rangle_A (\alpha|1\rangle_B + \beta|0\rangle_B) \\ + |1\rangle_C |0\rangle_A (\alpha|0\rangle_B - \beta|1\rangle_B) \\ + |1\rangle_C |1\rangle_A (\alpha|1\rangle_B - \beta|0\rangle_B) \end{bmatrix} \\ &= \frac{1}{2}\begin{bmatrix} |0\rangle_C |0\rangle_A \otimes |\psi\rangle_B \\ + |0\rangle_C |1\rangle_A \otimes X|\psi\rangle_B \\ + |1\rangle_C |0\rangle_A \otimes Z|\psi\rangle_B \\ + |1\rangle_C |1\rangle_A \otimes XZ|\psi\rangle_B \end{bmatrix} \end{aligned} \tag{1.3}$$

After applying these gates, Alice measures qubit C to get a bit c_0 and measures qubit A to get a bit c_1. These measurements project Alice's qubits into one of the four 2-qubit standard basis states and simultaneously project's Bob's qubit into a state that is closely related to $|\psi\rangle$, as per Equation 1.3. Alice sends the measured bits to Bob via a classical communication channel, which tells Bob the corrective gates he needs to apply to transform his qubit into the state $|\psi\rangle$, given in Table 1.1.

c_0	c_1	Corrective Gate
0	0	I
0	1	X
1	0	Z
1	1	ZX

TABLE 1.1: The corrective gates Bob needs to apply based on the bits he receives from Alice.

II. TELEPORTATION ON IBM'S QUANTUM COMPUTERS

As of right now, IBM's quantum computers don't support classically controlled gates [2] like the corrective gates that Bob needs to apply in the teleportation protocol. As a result, we can't directly implement the protocol on an IBM quantum computer because teleportation requires the classical communication channel. However, it is possible to emulate the teleportation protocol via a quantum circuit that does not have any classically controlled gates and is equivalent to the one in Figure 1.1.

A. Principle of Deferred Measurement

The principle of deferred measurement states that measurements made in the middle of a quantum circuit can always be shifted to the end of the circuit, where any classically controlled gates dependent on those measurements are replaced by conditional quantum gates. [1]

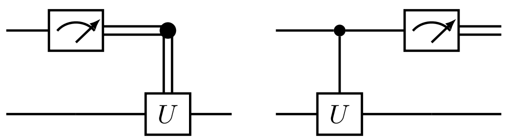


FIG. 2.1: The two circuits are equivalent as per the deferred measurement principle.

Making use of this principle, we can construct the following quantum circuit that will emulate the teleportation protocol.

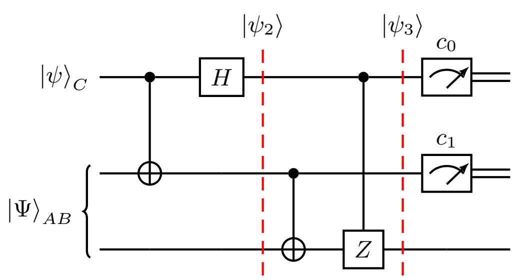


FIG. 2.2: The quantum circuit that will emulate the teleportation protocol on the IBM machine.

From Equation 1.3, it's clear that after applying the CNOT and controlled-Z gates, the system will be in the state

$$\begin{aligned}|\psi_3\rangle &= \mathrm{CZ}_{C,B}\,\mathrm{CNOT}_{A,B}\,|\psi_2\rangle \\ &= \frac{|0\rangle_A|0\rangle_B + |0\rangle_A|1\rangle_B + |1\rangle_A|0\rangle_B + |1\rangle_A|1\rangle_B}{2} \otimes |\psi\rangle_C\end{aligned} \tag{2.1}$$

And so, regardless of the measurement outcome, qubit B will be in the state $|\psi\rangle$.

III. MEASURING THE TELEPORTATION FIDELITY OF IBM'S QUANTUM COMPUTER

Unlike the implementation on simulators, we won't have access to the state vector of Bob's qubit at the end of running the protocol when we run it on an actual quantum computer. We can however, take multiple measurements on different runs of the protocol attempting to reconstruct Bob's state.

A. Quantum State Tomography

"Quantum state tomography is the attempt to discover the quantum-mechanical state of a physical system, or more precisely, of a finite set of systems prepared by the same process." [3]

The state of a single qubit can be characterized with a 2×2 density matrix

$$\rho = \frac{1}{2}(I + \bar{x}X + \bar{y}Y + \bar{z}Z) \tag{3.1}$$

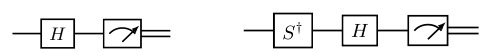
where X, Y and Z are the Pauli matrices and

$$\begin{aligned}\bar{x} &= \mathrm{tr}(X\rho), \\ \bar{y} &= \mathrm{tr}(Y\rho), \\ \bar{z} &= \mathrm{tr}(Z\rho),\end{aligned} \tag{3.2}$$

are the average values of the observables X, Y and Z respectively [1]. $\vec{r} = (\bar{x}, \bar{y}, \bar{z})$ is called the Bloch vector of ρ. $\|\vec{r}\| \leq 1$, with equality holding only for pure states [1].

We can find an approximation of the Bloch vector for a state ρ by instantiating $3N$ qubits in the state ρ, measuring the observables X, Y and Z on N instances each, and then taking the mean value of the observations. We can use this procedure to tomographically reconstruct the state of Bob's qubit after the teleportation protocol is finished.

IBM's hardware only allows the measurement of Z, with measurement outcome 0 denoting the eigenvalue $+1$ and 1 denoting the eigenvalue -1. The following circuits can be used to measure X and Y.



(a) X measurement (b) Y measurement

FIG. 3.1: Circuits to measure X and Y for a qubit.

$$S^\dagger = \begin{pmatrix} 1 & 0 \\ 0 & -i \end{pmatrix} \tag{3.3}$$

For a particular observable, say X, we can approximate the corresponding Bloch vector coordinate as

$$\bar{x} = \frac{N_0 - N_1}{N}, \tag{3.4}$$

where N_0 is the number of times we note the measurement outcome 0 and N_1 the number of times we note the measurement outcome 1.

B. Fidelity Calculation

Through the process described in the previous subsection, we obtain a density matrix ρ that approximates the state of Bob's qubit at the end of the teleportation protocol. If Alice had initialized her qubit to the pure state $|\psi\rangle$ at the beginning of the protocol, the fidelity between the $|\psi\rangle$ and ρ is [1]

$$F(|\psi\rangle, \rho) = \sqrt{\langle\psi|\, \rho\, |\psi\rangle}. \tag{3.5}$$

C. Results

ACKNOWLEDGMENTS

I acknowledge the use of IBM Quantum services for this work. The views expressed are those of the author, and do not reflect the official policy or position of IBM or the IBM Quantum team.

[1] M. A. Nielsen and I. L. Chuang, *Quantum Computation and Quantum Information* (Cambridge University Press, 2010).

[2] A. Asfaw, A. Corcoles, L. Bello, Y. Ben-Haim, M. Bozzo-Rey, S. Bravyi, N. Bronn, L. Capelluto, A. C. Vazquez, J. Ceroni, R. Chen, A. Frisch, J. Gambetta, S. Garion, L. Gil, S. D. L. P. Gonzalez, F. Harkins, T. Imamichi, H. Kang, A. h. Karamlou, R. Loredo, D. McKay, A. Mezzacapo, Z. Minev, R. Movassagh, G. Nannicini, P. Nation, A. Phan, M. Pistoia, A. Rattew, J. Schaefer, J. Shabani, J. Smolin, J. Stenger, K. Temme, M. Tod, S. Wood, and J. Wootton., Learn quantum computation using qiskit (2020).

[3] R. Schmied, Quantum state tomography of a single qubit: comparison of methods, Journal of Modern Optics **63**, 1744 (2016).

[4] IBM, Ibm quantum (2021).