AI II HW #1. Ben Knight

problem 1.

$P(X=1) = 0.4$

$P(X=0) = 0.4$

$P(X=-1) = 0.2$.

- we have acess to a program A that generates a number in $[0,1]$ uniformly @ random. how can you use A to draw random samples of X.

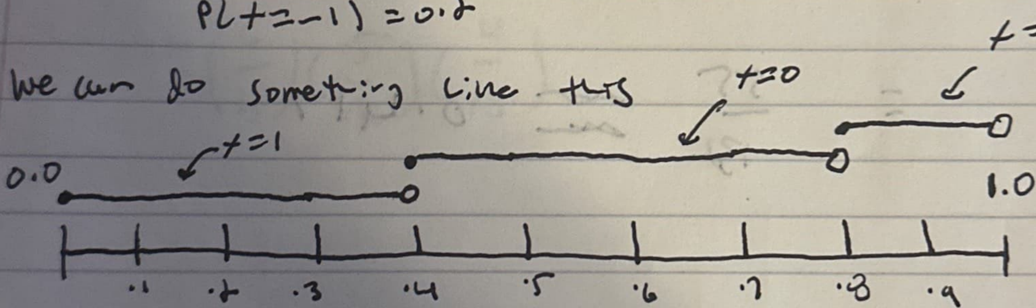
- we can simply map the output of A to the values that X can take on, so because

$P(X=1) = 0.4$

$P(X=0) = 0.4$

$P(X=-1) = 0.2$

we can do something like this



- we simply map the values of X to some interval over $[0,1]$ that coorsponds to the probability that X will take on that value.

Ben Knight

2.) A urn has 3 red balls, 4 blue balls, & 5 green balls. Alice draws a ball from the urn, & then Bob draws a ball. What is Prob Bob drew a green ball?

- Assumming no replacement, thus we have

total num balls = $3+4+5 = 12$

so

P(Bob draws green) = P(Alice draws green & BoB draws green) + P(Alice doesn't draw green & BoB does draw green)

$$= \frac{5}{12} \cdot \frac{4}{11} + \frac{7}{12} \cdot \frac{5}{11}$$

$$= \frac{20}{132} + \frac{35}{132}$$

$$= \frac{55}{132} \approx 0.417$$

3.) calculate $P(F = \text{empty} \mid S = \text{false})$

$$\Downarrow$$

$$P(\bar{f} \mid \bar{s})$$

we can right this prob calculation as

$$P(\bar{f} \mid \bar{s}) = \cancel{P(\bar{f})} \frac{P(\bar{f}, \bar{s}, G, B, T)}{P(\bar{s}, G, B, T)}$$

– we can get rid of denominator & represent it with α, we can normalize later to remove α

$$= \alpha\, P(\bar{f}, \bar{s}, G, B, T)$$

$$= \alpha \cdot P(\bar{f}) \cdot P(B) \cdot P(G \mid \bar{f}, B) \cdot P(T \mid B) \cdot P(\bar{s} \mid \bar{f}, T)$$

$$= \alpha \sum_{T} \sum_{B} \sum_{G} P(\bar{f}) \cdot P(B) \cdot P(\bar{G} \mid \bar{f}, B) \cdot P(T \mid B) \cdot P(\bar{s} \mid \bar{f}, T)$$

ignore we will normalize later.

$$\begin{aligned}
= & P(\bar{f})\, P(b) \cdot P(g \mid \bar{f}, b)\, P(t \mid b) \cdot P(\bar{s} \mid \bar{f}, t) \\
+ & P(\bar{f})\, P(b) \cdot P(\bar{g} \mid \bar{f}, b) \cdot P(t \mid b)\, P(\bar{s} \mid \bar{f}, t) \\
+ & P(\bar{f})\, P(\bar{b})\, P(g \mid \bar{f}, \bar{b}) \cdot P(t \mid \bar{b})\, P(\bar{s} \mid \bar{f}, t) \\
+ & P(\bar{f})\, P(\bar{b})\, P(\bar{g} \mid \bar{f}, \bar{b}) \cdot P(t \mid \bar{b})\, P(\bar{s} \mid \bar{f}, t) \\
+ & P(\bar{f})\, P(b)\, P(g \mid \bar{f}, b) \cdot P(\bar{t} \mid b)\, P(\bar{s} \mid \bar{f}, \bar{t}) \\
+ & P(\bar{f})\, P(b)\, P(\bar{g} \mid \bar{f}, b) \cdot P(\bar{t} \mid b)\, P(\bar{s} \mid \bar{f}\ \bar{t} \\
+ & P(\bar{f})\, P(\bar{b})\, P(g \mid \bar{f}, \bar{b}) \cdot P(\bar{t} \mid \bar{b})\, P(\bar{s} \mid \bar{f}\ \bar{t} \\
+ & P(\bar{f})\, P(\bar{b})\, P(\bar{g} \mid \bar{f}, \bar{b} \cdot P(\bar{t} \mid \bar{b})\, P(\bar{s} \mid \bar{f}, \bar{t}
\end{aligned}$$

$=$

$\downarrow$

$\downarrow$

$$= (0.05)(0.98)(0.03)(0.97)(0.92)$$
$$+ (0.05)(0.98)(0.97)(0.97)(0.92)$$
$$+ (0.05)(0.02)(0.01)(0.02)(0.92)$$
$$+ (0.05)(0.02)(0.99)(0.02)(0.92)$$
$$+ (0.05)(0.98)(0.03)(0.03)(0.99)$$
$$+ (0.05)(0.98)(0.97)(0.03)(0.99)$$
$$+ (0.05)(0.02)(0.01)(0.98)(0.99)$$
$$+ (0.05)(0.02)(0.99)(0.98)(0.99)$$

$$= \boxed{0.04449010231}$$

- we now need to calculate $P(f \mid \bar{s})$ so we can normalize,

$P(f \mid \bar{s}) =$

$$P(f)\,P(b)\,P(g \mid f,b)\,P(t \mid b) \cdot P(\bar{s} \mid f,t)$$
$$+ P(f)\,P(b)\,P(\bar{g} \mid f,b)\,P(t \mid b)\,P(\bar{s} \mid f,t)$$
$$+ P(f)\,P(\bar{b})\,P(g \mid f,\bar{b})\,P(t \mid \bar{b})\,P(\bar{s} \mid f,t)$$
$$+ P(f)\,P(\bar{b})\,P(\bar{g} \mid f,\bar{b})\,P(t \mid \bar{b})\,P(\bar{s} \mid f,t)$$
$$+ P(f)\,P(b)\,P(g \mid f,b)\,P(\bar{t} \mid b)\,P(\bar{s} \mid f,\bar{t})$$
$$+ P(f)\,P(b)\,P(\bar{g} \mid f,b)\,P(\bar{t} \mid b)\,P(\bar{s} \mid f,\bar{t})$$
$$+ P(f)\,P(\bar{b})\,P(g \mid f,\bar{b})\,P(\bar{t} \mid \bar{b})\,P(\bar{s} \mid f,\bar{t})$$
$$P(f)\,P(\bar{b})\,P(\bar{g} \mid f,\bar{b})\,P(\bar{t} \mid \bar{b})\,P(\bar{s} \mid f,\bar{t})$$

$$= (0.95)(0.98)(0.96)(0.97)(0.01)$$
$$(0.95)(0.98)(0.04)(0.97)(0.01)$$
$$(0.95)(0.02)(0.90)(0.02)(0.01)$$
$$(0.95)(0.02)(0.10)(0.02)(0.01)$$
$$(0.95)(0.98)(0.90)(0.03)(1.00)$$
$$(0.95)(0.98)(0.04)(0.03)(1.00)$$
$$(0.95)(0.02)(0.90)(0.98)(1.00)$$
$$(0.95)(0.02)(0.10)(0.98)(1.00)$$

$$= \boxed{0.0555845}$$

Normalize $\searrow$

$$P(\bar{f} \mid \bar{s}) = \frac{0.04449010231}{0.04449010231 + 0.0555845} = 0.447$$

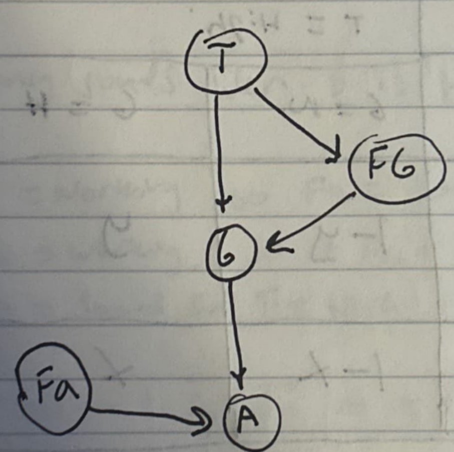
$$\boxed{\text{so } P(\bar{f} \mid \bar{s}) = 0.447}$$

4.)

A (alarm sounds)
FA (alarm is faulty)
FG (Gauge is faulty)
G (gauge reading)
T (actual core temp).

4a.) Gauge is more likely to Fail when core temp is to High, thus T has influence over FG

we get



4b.) $G \in \{\text{normal}, \text{High}\} = \{N, H\}$

$T \in \{\text{normal}, \text{High}\} = \{N, H\}$

correct temp when G working is x

correct temp when G is not working is y

Construct CPT.

N=normal
H=High

	T=normal		T=High	
	Gauge=N	G=High	G=N	G=H
FG=Faulty	y	1-y	1-y	y
FG=Working so not Faulty	x	1-x	1-x	x

False for FG

4c.) Alarm works correctly unless it is faulty, in which case it never sounds, give CPT associated with A.

	G = normal		G = High	
	A = Sound	A = Not Sound	A = Sound	A = no Sound
FA = Faulty	0	1	0	1
~FA = not Faulty	0	1	1	0

no Sound = False.

↑ False for FA

- note alarm only sounds when G is high, via problem description.

4d.) alarm = working, so Fa = not Faulty
gauge = working, so F_G = not Faulty
alarm = Sound so T = High

Find $P(T = high \mid \overline{fa}, \overline{fg}, a, g) = P(t \mid \overline{fa}, \overline{fg}, a, g)$

note

$$P(t \mid \overline{fa}, \overline{fg}, a, g) = \frac{P(t, \overline{fa}, \overline{fg}, a, g)}{P(\overline{fa}, \overline{fg}, a, g)}$$

- lets expand & cancel terms

$P(a \mid g, \overline{fa})$

$$= \frac{P(t) \cdot \cancel{P(\overline{fa})} \cdot P(\overline{fg} \mid t) \cdot P(g \mid \overline{fg}, t) \cdot \cancel{P(a \mid \overline{fa}, \overline{fg})}}{\cancel{P(\overline{fa})} \cdot P(\overline{fg}) \cdot P(g \mid \overline{fg}) \cdot \cancel{P(a \mid g, \overline{fa})}}$$

$$= \frac{P(t) \cdot P(\overline{fg} \mid t) \cdot P(g \mid \overline{fg}, t)}{P(\overline{fg}) \cdot P(g \mid \overline{fg})}$$

t

$$P(t|\bar{f}_a,\bar{f}_g,a,g) = \frac{P(t)\cdot P(\bar{f}_g|t)\cdot P(g|\bar{f}_g,t)}{\underbrace{P(\bar{f}_g)\cdot P(g|\bar{f}_g)}}$$

we need to get into terms of t as the top probabilities have a t term.

Quick note

~~Bayes rule~~ $P(A\cap B) = P(A)\cdot P(B|A) = P(B)\cdot P(A|B)$

thus we can say that

$$P(\bar{f}_g)\cdot P(g|\bar{f}_g) = P(\bar{f}_g, g) = P(\bar{f}_g\cap g)$$

~~so now~~ So now we have

$$P(t|\bar{f}_a,\bar{f}_g,a,g) = \frac{P(t)\cdot P(\bar{f}_g|t)\cdot P(g|\bar{f}_g,t)}{P(\bar{f}_g,g)}$$

Law of total prob

$$P(\bar{f}_g,g) = P(\bar{f}_g,g,t) + P(\bar{f}_g,g,\bar{t})$$

So we can say

$$P(t|\bar{f}_a,\bar{f}_g,a,g) = \frac{P(t)\cdot P(\bar{f}_g|t)\cdot P(g|\bar{f}_g,t)}{P(\bar{f}_g,g,t) + P(\bar{f}_g,g,t)}$$

$$= \frac{P(t).P(\bar{fg}\mid t).P(g\mid \bar{fg},t)}{P(\bar{fg},g,t) + P(\bar{fg},g,t)}$$

we can expand with help of our Bayes net.

$$= \frac{P(t).P(\bar{fg}\mid t).\overbrace{P(g\mid \bar{fg},t)}^{x \text{ from CPT}}}{P(t).P(fg\mid t).\underbrace{P(g\mid fg,t)}_{x \text{ from CPT}} + P(\bar{t}).P(\bar{fg}\mid \bar{t}).\underbrace{P(g\mid \bar{fg},\bar{t})}_{(1-x) \text{ from CPT}}}$$

$$P(t\mid \bar{fa},\bar{fg},a,g) = \frac{P(t).P(\bar{fg}\mid t)\,x}{P(t).P(\bar{fg}\mid t).x + P(\bar{t}).P(\bar{fg}\mid \bar{t})(1-x)}$$

5a.)

$$P(c|r,w,s) = \alpha \cdot P(c,r,w,s)$$
$$= \alpha \cdot P(c) \cdot P(r|c) \cdot P(s|c) \cdot P(w|r,s)$$
$$= (\alpha)(0.5)(0.8)(0.10)(0.99)$$
$$= (\alpha)(0.0396)$$

$$P(\bar{c}|r,w,s) = \alpha \cdot P(\bar{c},r,w,s)$$
$$= \alpha \cdot P(\bar{c}) \cdot P(r|\bar{c}) \cdot P(s|\bar{c}) \cdot P(w|r,s)$$
$$= \alpha(0.5)(0.2)(0.50)(0.99)$$
$$= \alpha(0.0495)$$

normalized

$$\boxed{\begin{aligned} P(c|r,w,s) &= 0.44 \\ P(\bar{c}|r,w,s) &= 0.56 \end{aligned}}$$

$$P(c \mid \bar{r}, w, s) = \alpha\, P(c, \bar{r}, w, s)$$
$$= \alpha\, P(c) P(\bar{r} \mid c) P(s \mid c) P(w \mid \bar{r}, s)$$
$$= (\alpha)(0.5)(0.2)(0.10)(0.90)$$
$$= \alpha\,(0.004)$$

$$P(\bar{c} \mid \bar{r}, w, s) = \alpha\, P(\bar{c} \mid \bar{r}, w, s)$$
$$= \alpha\, P(\bar{c}) P(\bar{r} \mid \bar{c}) P(s \mid \bar{c}) P(w \mid \bar{r}, s)$$
$$= \alpha (0.5)(0.8)(0.5)(0.90)$$
$$= \alpha (0.18)$$

normalized

$$\boxed{\begin{aligned} P(c \mid r, w, s) &= 0.5 \\ P(\bar{c} \mid r, w, s) &= 0.95 \end{aligned}}$$

$$P(r|c,w,s) = \alpha P(r,c,w,s)$$
$$= \alpha P(c) P(r|c) P(s|c) P(w|r,s)$$
$$= \alpha (0.5)(0.80)(0.10)(0.99)$$
$$= \alpha (0.0396)$$

$$P(\bar{r}|c,w,s) = \alpha P(\bar{r},c,w,s)$$
$$= \alpha P(c) P(\bar{r}|c) P(s|c) P(w|\bar{r},s)$$
$$= \alpha (0.5)(0.2)(0.10)(0.90)$$
$$= \alpha (0.009)$$

normalized

$$\boxed{\begin{aligned} P(r|c,w,s) &= 0.81 \\ P(\bar{r}|c,w,s) &= 0.19 \end{aligned}}$$

$$P(r|\bar{c},w,s) = \alpha P(r,\bar{c},w,s)$$
$$= \alpha P(\bar{c}) . P(r|\bar{c}) P(s|\bar{c}) P(w|r,s)$$
$$= \alpha (0.5)(0.2)(0.5)(0.99)$$
$$= \alpha (0.0495)$$

$$P(\bar{r}|\bar{c},w,s) = \alpha P(\bar{r},\bar{c},w,s)$$
$$= \alpha P(\bar{c}) . P(\bar{r}|\bar{c}) P(s|\bar{c}) . P(w|s,\bar{r})$$
$$= (0.5)(0.8)(0.5)(0.90)$$
$$= (\alpha)(0.18)$$

normalized

$$\boxed{\begin{aligned} P(r|\bar{c},w,s) &= 0.21569 = 0.22 \\ P(\bar{r}|\bar{c},w,s) &= 0.784 = 0.78 \end{aligned}}$$