# ECE380 Digital Logic

Optimized Implementation of
Logic Functions:
Multilevel Synthesis and Analysis

Multilevel NAND & NOR circuits

- Two-level circuits consisting of AND and OR gates can easily be converted to networks that can be realized only NAND and NOR gates
  – A two-level AND-OR (SOP) circuit can be realized (directly) as a two-level NAND-NAND circuit
  – A two-level OR-AND (POS) circuit can be realized (directly) as a two-level NOR-NOR circuit
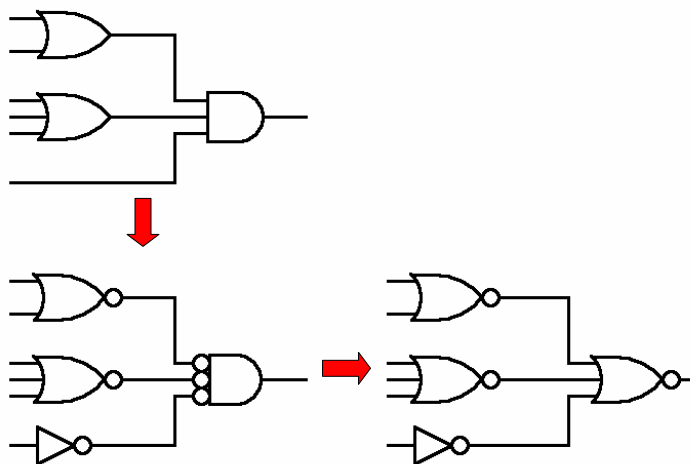- The same approach can be used for multilevel networks
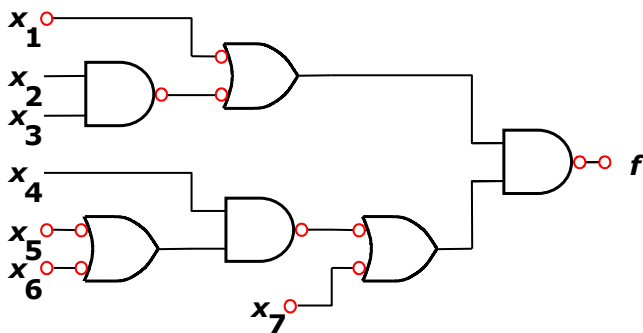
AND-OR to NAND-NAND example
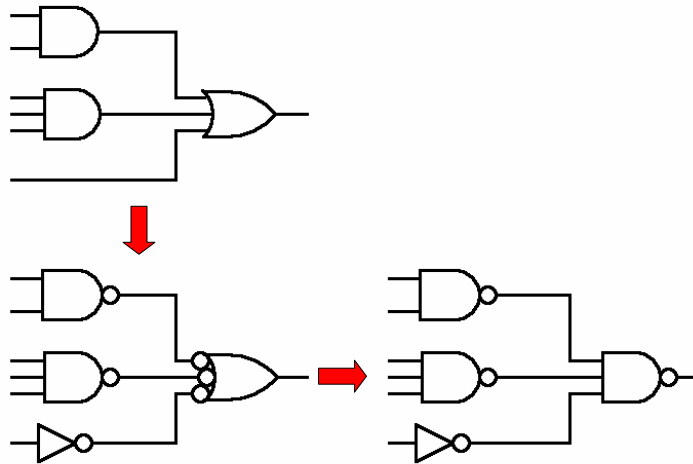
OR-AND to NOR-NOR example

Multilevel example
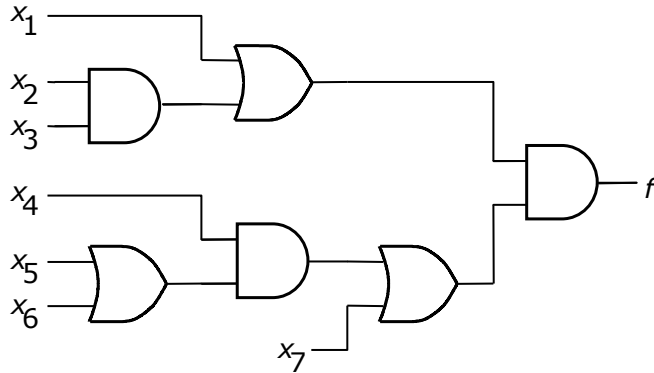
Circuit with AND and OR gates

Multilevel example (NAND)

Inversions needed to convert to NANDs

Note the inversion bubbles added to the output of every AND gate and to the input of every OR gate. Also, the inversion bubbles are always placed in pairs.

Multilevel example (NAND)
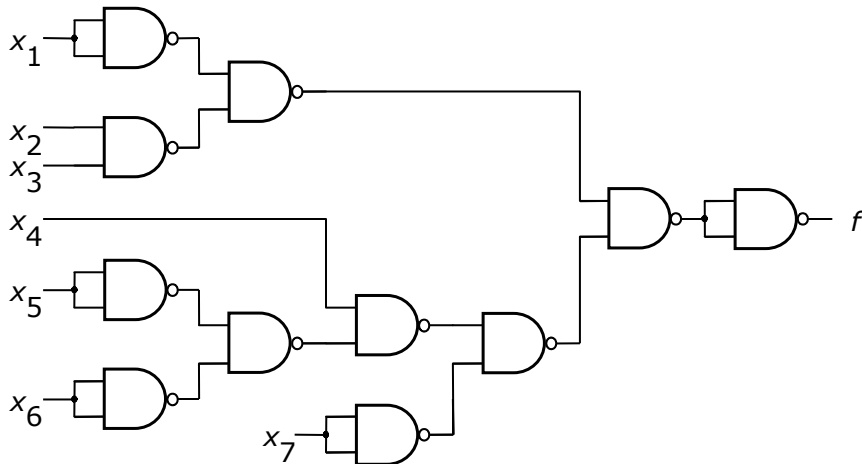

Conversion to a NAND-gate circuit

Multilevel example (NOR)

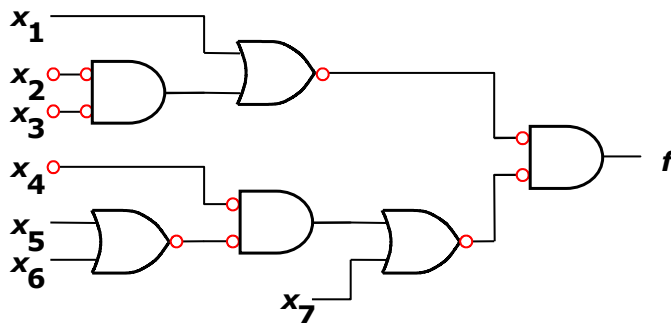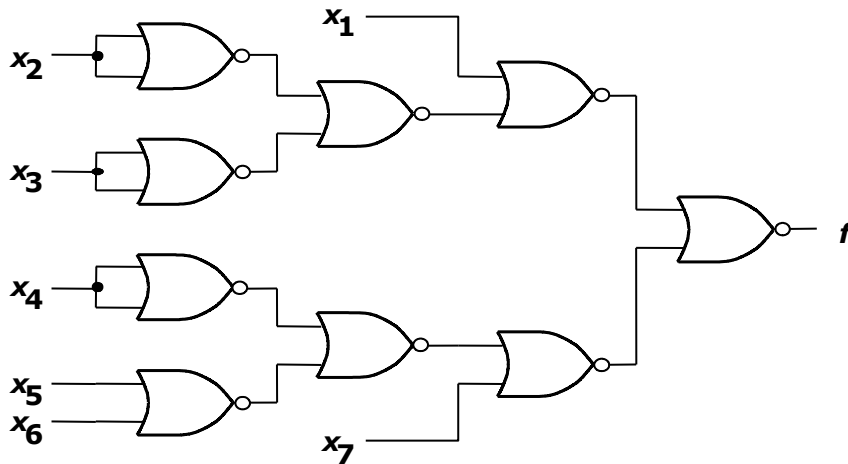## Inversions needed to convert to NORs

Note the inversion bubbles added to the output of every OR gate and to the input of every AND gate. Also, the inversion bubbles are always placed in pairs.
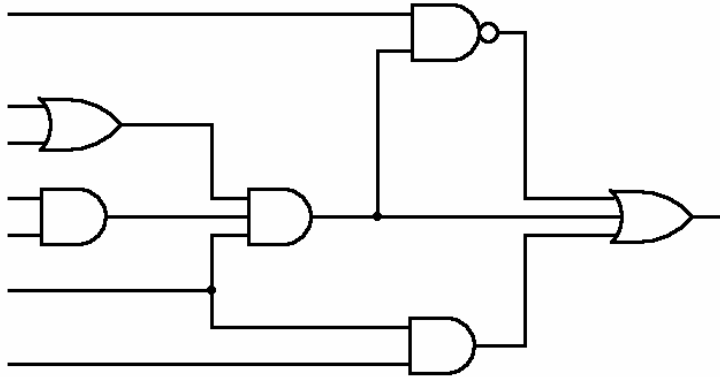
Multilevel example (NOR)

Conversion to a NOR-gate circuit

Multilevel conversion process

- The basic topology (wiring) of a circuit does not change substantially when converting from AND and OR gates to either NAND or NOR gates
- It may be necessary to insert additional gates (to serve as NOT gates) that implement inversions not absorbed as a part of other gates in the circuit
- The resulting circuit may not be minimum (i.e. such as a minimum 2-level NAND-NAND or NOR-NOR network)

Multilevel conversion example

- What is the NAND only equivalent of the following circuit?

Analysis of multilevel circuits

- In the previous examples, we synthesized multilevel circuits
- How can we easily determine a function that a given multilevel circuit implements?
  - For two-level circuits we simply wrote out the SOP or POS form equation from the circuit
  - This is easy to visualize for two-level circuits
  - This is more difficult for multilevel circuits because it is difficult to write an expression for the circuit by inspection
- We can write an expression for a function by analyzing it at intermediate points in the circuit
  - Write expressions for each of these subfunctions
  - Combine the subfunctions together into a single function

Multilevel circuit analysis

Label the output of every gate as a subfunction
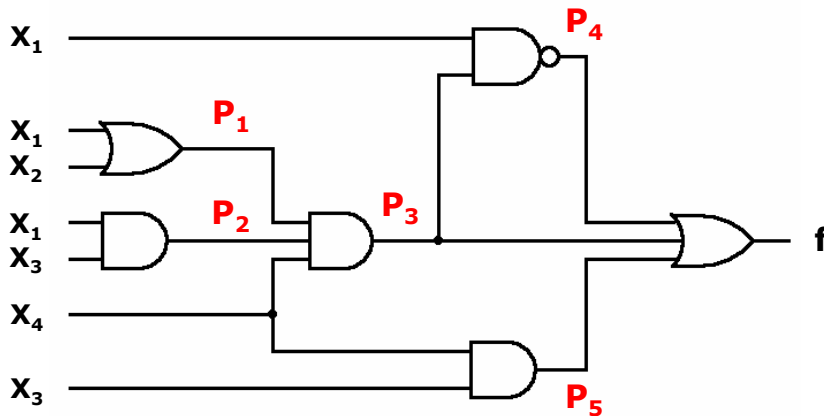
Multilevel circuit analysis

- Combine (and simplify if possible) these functions
  - $P_1 = x_1 + x_2$
  - $P_2 = x_1 x_3$
  - $P_3 = P_1 P_2 x_4 = (x_1 + x_2) x_1 x_3 x_4 = x_1 x_3 x_4 + x_2 x_1 x_3 x_4$
    $\quad\quad\quad = x_1 x_3 x_4$
  - $P_4 = (x_1 P_3)' = (x_1 x_1 x_3 x_4)' = (x_1 x_3 x_4)' = x_1' + x_3' + x_4'$
  - $P_5 = x_3 x_4$
  - $f = P_3 + P_4 + P_5 = (x_1 x_3 x_4) + (x_1 x_3 x_4)' + (x_3 x_4) = 1$