PHYS1500J-RC2

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direction { a, v, x

→ +

← v↑

|a|↑

a < 0

Quizzes

Q3

Question

Consider an object moving along a straight line. If the object moves towards a point and its instantaneous speed increases, the instantaneous acceleration of this object must be positive.

- True
- False

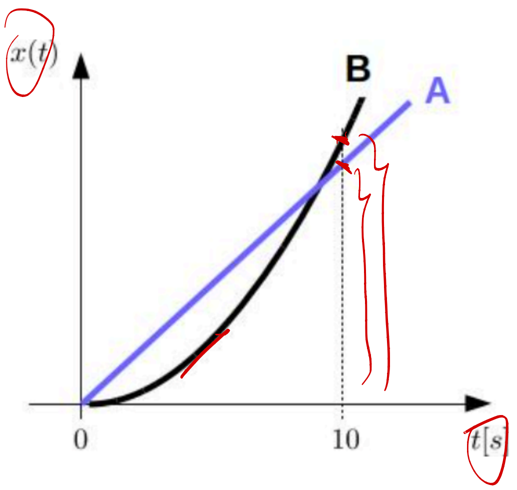
↗ a

"The body accelerates" ~~==~~ increasing speed ?

Q4

Two cars A and B start from the same point at *t*=0 and race along a straight section of a road. Their positions, as functions of time, are shown in the graph (the blue line is straight, and the black one is smooth).

Mark all the statements that are true.



x-t

v-t

slope: v

- [ ] There is an instant of time, between *t* = 0 [s] and *t* = 10 [s], at which car A accelerates. X v same
- [ ] There is an instant of time, between *t* = 0 [s] and *t* = 10 [s], at which car B slows down. X
- [ ] At some instant between 0 [s] and 10 [s], both cars have the same instantaneous speed.
- [ ] At t=10 [s] car A is closer to the starting point than car B.
- [ ] At t=10 [s] car A moves at a greater speed than car B. X

Q5 If $\overline{a}$ and $\overline{b}$ are non-zero vectors for which $\overline{a} \times \overline{b} = 0$ then it must follow that

[More than one answer may be correct; mark all that apply.]

- ☐ Both the vectors point in the ~~same~~ direction.
- ☐ The orthogonal projection of vector $\overline{b}$ onto vector $\overline{a}$ is ~~zero~~.
- ☐ The magnitude of the orthogonal projection of $\overline{a}$ onto $\overline{b}$ is equal to the magnitude of $\overline{a}$.
- ☐ $\overline{a} \circ \overline{b} \neq 0$
- ☐ The vector $(\overline{b} \times \overline{a})$ is perpendicular to the vector $(\overline{a} - 3\overline{b})$.

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Kinematics in 1D

Some Concepts

- Average Velocity & Instantaneous Velocity

$$v_{av} = \frac{x\left(t_0 + \Delta t\right) - x\left(t_0\right)}{\Delta t}$$

$$v\left(t_0\right) = \lim_{\Delta t \to 0} \frac{x\left(t_0 + \Delta t\right) - x\left(t_0\right)}{\Delta t} = \dot{x}\left(t_0\right)$$

- Average Speed & Average Velocity

$$\text{average speed} = \frac{\text{distance traveled}}{\text{time interval}}$$

$$\text{average velocity} = \frac{\text{displacement}}{\text{time interval}}$$

- Average Acceleration & Instantaneous Acceleration

$$a_{av} = \frac{v\left(t_0 + \Delta t\right) - v\left(t_0\right)}{\Delta t}$$

$$a_x(t) = \frac{dv(t)}{dt} = \dot{v}(t) = \ddot{x}(t)$$

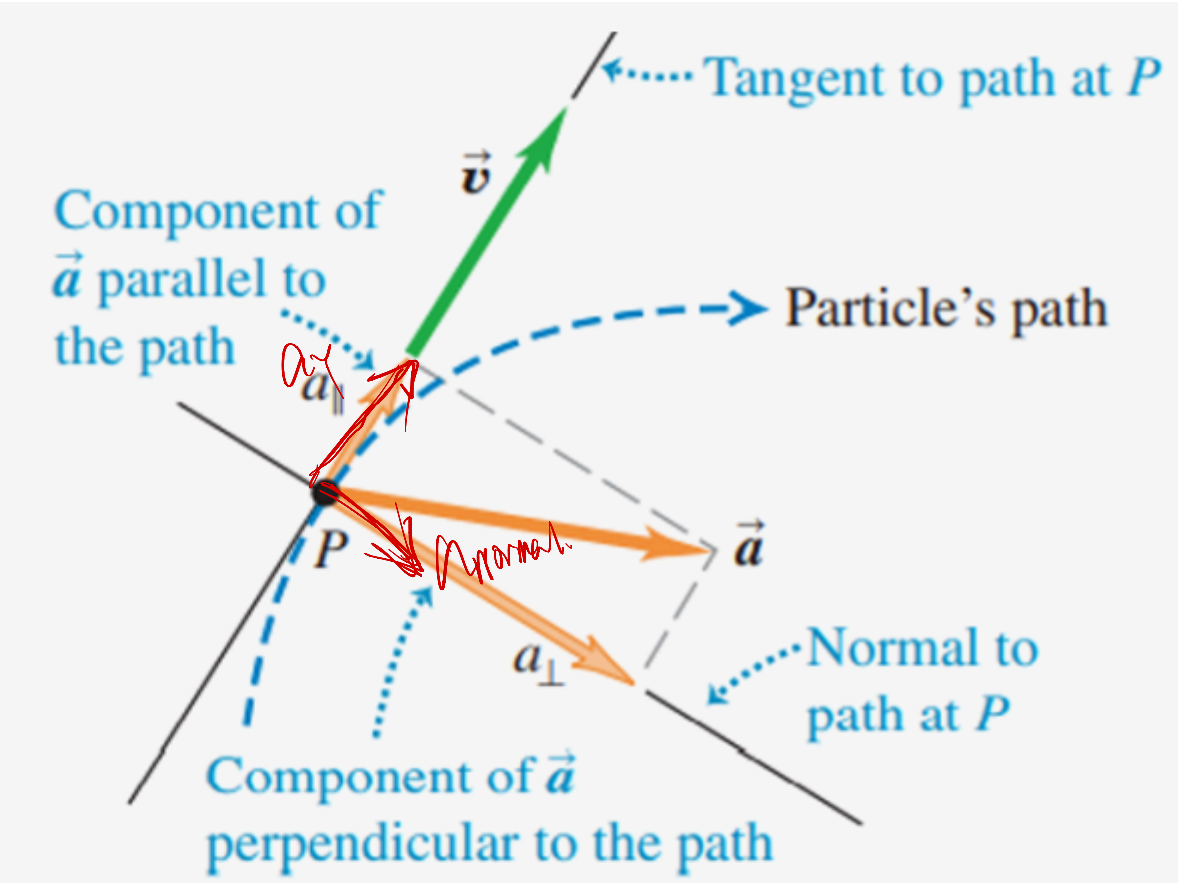
- The instantaneous speed is the magnitude of the instantaneous velocity.

Kinematics in 2D & 3D

3 perspectives in 2D & 3D motion

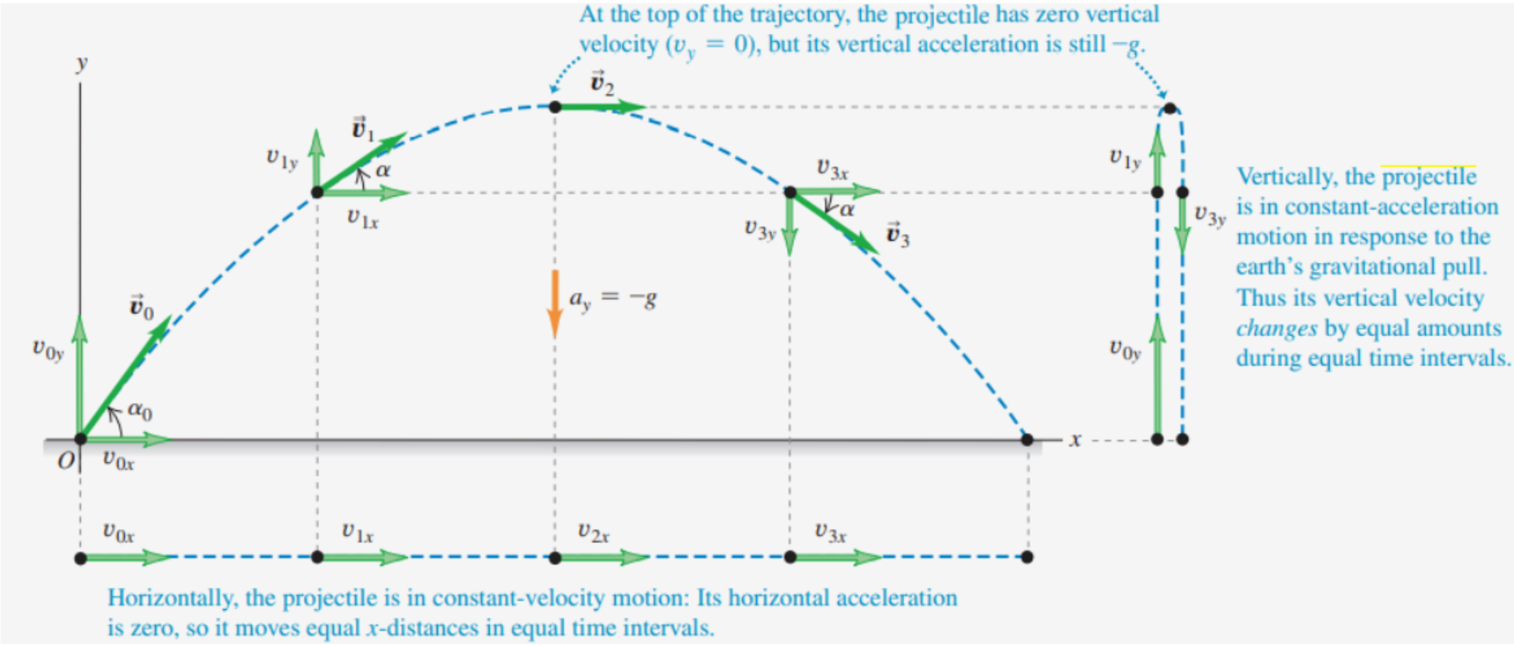
1. x,y,z components Cartesian coordinates ——> Cartesian coordinates
2. radial & transverse components ——> polar coordinates & cylinder coordinates
3. tangential & normal components ~~natural coordinates~~ ——> natural coordinates

Components of Acceleration



- Tangential Component: only changing the magnitude of the velocity.
- Normal Component: only changing the direction of the velocity.

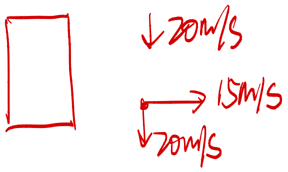
Projectile Motion



- Velocity: $v_x(t) = v_0 \cos\alpha, v_y(t) = v_0 \sin\alpha - gt$
- Displacement: $x(t) = v_0 t \cos\alpha, y(t) = v_0 t \sin\alpha - \frac{1}{2}gt^2$
- Maximum height: $t_h = \frac{v_0 \sin\alpha}{g}, y(t_h) = \frac{v_0^2 \sin^2\alpha}{2g} = h_{\max}$
- Range: $x_R = \frac{v_0^2 \sin 2\alpha}{g}$ (**maximum** range for $\alpha = \pi/4$)

A 124kg balloon carrying a 22kg basket is descending with a constant downward velocity of 20.0m/s. A 1.0kg stone is thrown from the basket with an initial velocity of 15.0m/s perpendicular to the path of the descending balloon, as measured relative to a person at rest in the basket. That person sees the stone hit the ground 5.00s after it was thrown. Assume that the balloon continues its downward descent with the same constant speed of 20.0m/s. Take g = 10m/(s^2).

1. How high is the balloon when the rock is thrown?
2. How high is the balloon when the rock hits the ground?
3. At the instant the rock hits the ground, how far is it from the basket?
4. Just before the rock hits the ground, find its horizontal and vertical velocity components as measured by an observer at rest on the ground.



1. $h_0 = V_0 t + \frac{1}{2}gt^2 = 225\,m$

2. $h' = h_0 - Vt = 225 - 5\times 20 = 125\,m$

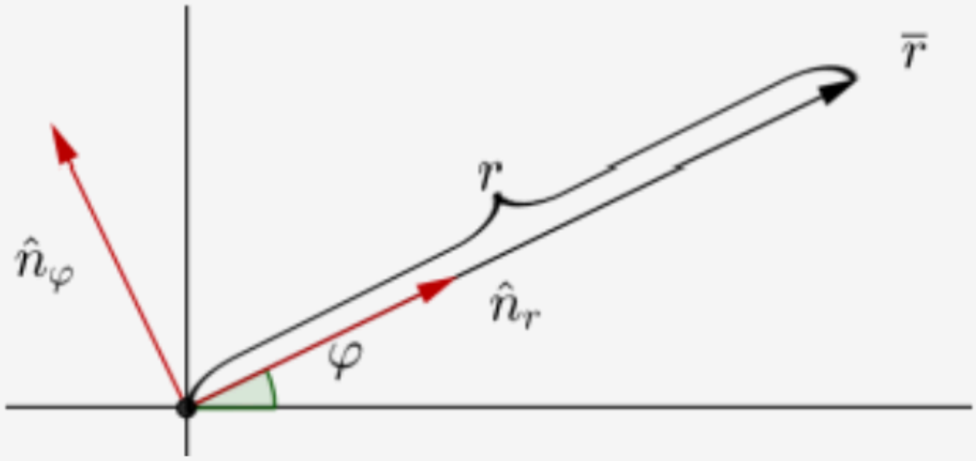
3. $S = 15 \times 5 = 75\,m$

4. take ... as FOR

$V_{\perp} = V_0 + gt = 70\,m/s$ downward.

$V_{\parallel} = 15\,m/s$ horizontal.

Polar System



(r,φ) in polar coordinate is equivalent as (rcosφ, rsinφ) in rectangular coordinates.

$\left\{ \begin{matrix} r \\ \varphi \end{matrix} \right. \to t$

- $\dot{\hat{n}}_r = \dot{\varphi}\hat{n}_\varphi$
- $\dot{\hat{n}}_\varphi = -\dot{\varphi}\hat{n}_r$
- $\vec{v} = \dot{r}\hat{n}_r + r\dot{\hat{n}}_r = \dot{r}\hat{n}_r + r\dot{\varphi}\hat{n}_\varphi$
- $\vec{a} = \dot{\vec{v}} = (\cancel{\ddot{r}} - r\dot{\varphi}^2)\hat{n}_r + (r\ddot{\varphi} + 2\cancel{\dot{r}}\dot{\varphi})\hat{n}_\varphi$

$\ddot{r} = \dot{r} = 0$

$\to -r\omega^2 \hat{n}_r + r\ddot{\varphi}\hat{n}_\varphi$

0

只有切向加速度

- Then we can derive expression for **uniform circular motion**:

$$\vec{a} = -r\dot{\varphi}^2\hat{n}_r = -r\omega^2\hat{n}_r$$

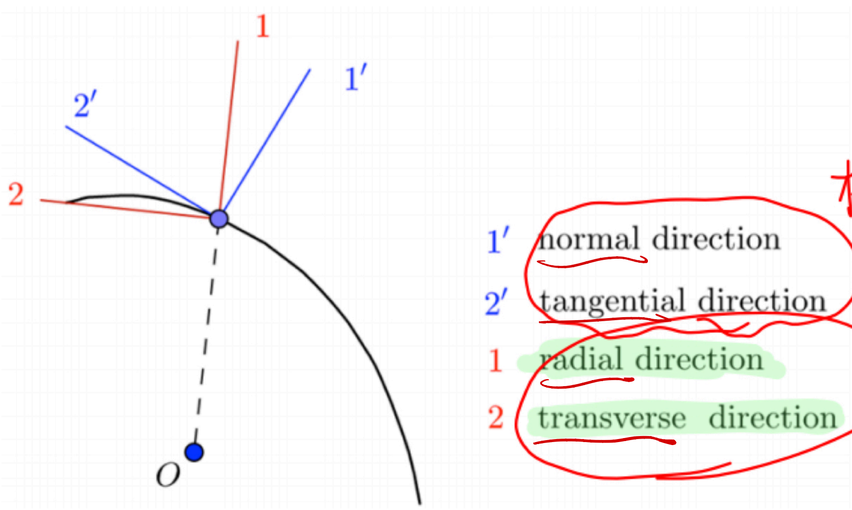
$$\vec{v} = r\omega\hat{n}_\varphi$$

- What if φ is a function of t? --> introduce angular acceleration $\epsilon(t)$

$$\vec{a} = -r\omega^2\hat{n}_r + r\epsilon(t)\hat{n}_\varphi$$

Radial & Transverse vs. Normal & Tangential

In general, radial ≠ normal, transverse ≠ tangential! (Though, it holds for uniform circular motion).

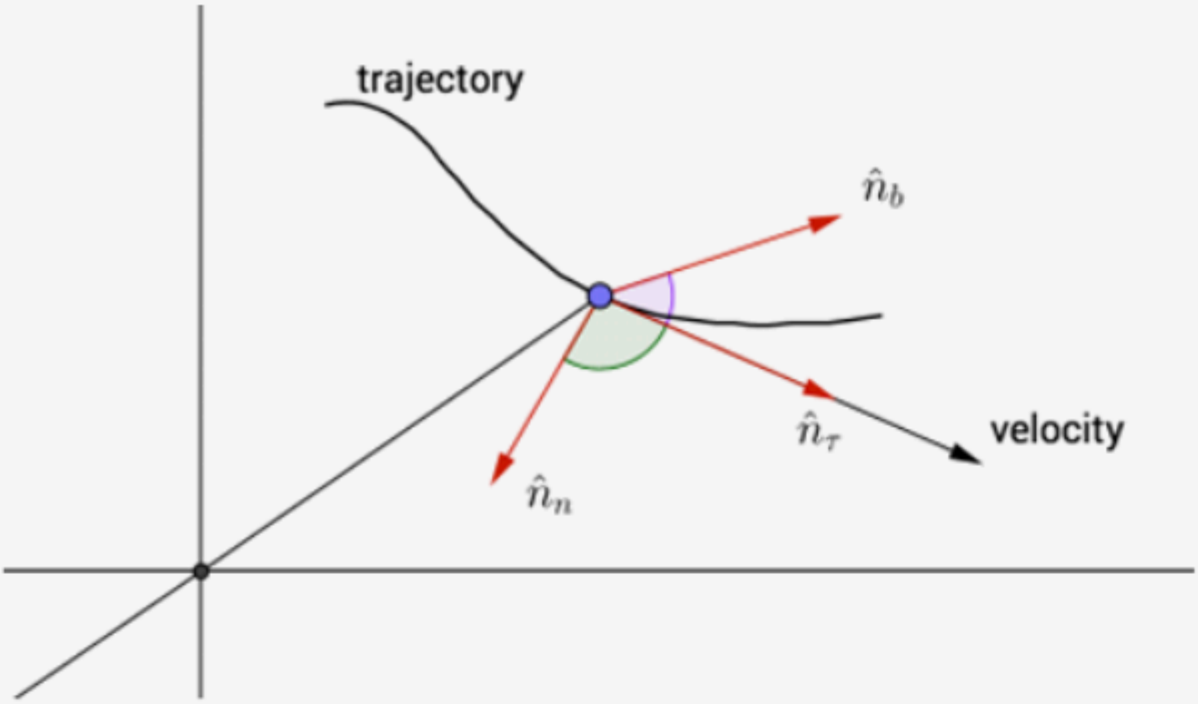


相较瞬时速度方向

相较原点方向

- Radial: tangent to the position vector.
  Transverse: normal to the position vector.
- Normal: normal to the trajectory.
  Tangential: tangent to the trajectory.

Natural Coordinates

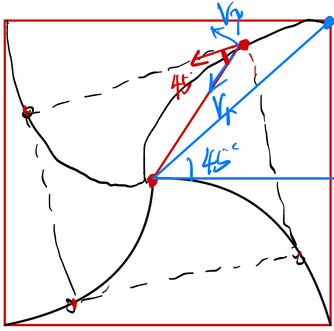


- $\hat{n}_\tau = \frac{\vec{v}}{v} = \frac{\dot{\vec{r}}}{|\dot{\vec{r}}|}$
- $\hat{n}_n = \frac{\frac{d\hat{n}_\tau}{dt}}{\left|\frac{d\hat{n}_\tau}{dt}\right|}$
- $\hat{n}_b = \hat{n}_\tau \times \hat{n}_n$

Four spiders are initially placed at the four corners of a square with side length L. The spiders crawl counter-clockwise at the same speed v and each spider crawls directly toward the next spider at all times. They approach the center of the square along spiral paths. Find

1. polar coordinates of a spider at any instant of time, assuming the origin is at the center of the square.
2. the time after which all spiders meet.
3. the trajectory of a spider $\boxed{\text{in polar coordinates}}$

$r \quad \varphi \qquad r = f(\varphi) \qquad \varphi = f(r)$



center - symmetric

(r, φ)

$v_r = v_T = \frac{v}{\sqrt{2}}$

(a) $r = r_0 - v_r t = \frac{L}{\sqrt{2}} - \frac{v}{\sqrt{2}} t$

$\Rightarrow \dot{\varphi} = \frac{v_T}{r} = \frac{\frac{1}{\sqrt{2}} v}{\frac{1}{\sqrt{2}}(L - vt)} = \frac{v}{L - vt}$

$\frac{d\varphi}{dt} = \frac{v}{L - vt} \Rightarrow \int_{\frac{\pi}{4}}^{\varphi} d\varphi = \int_0^t \frac{v}{L - vt} dt$

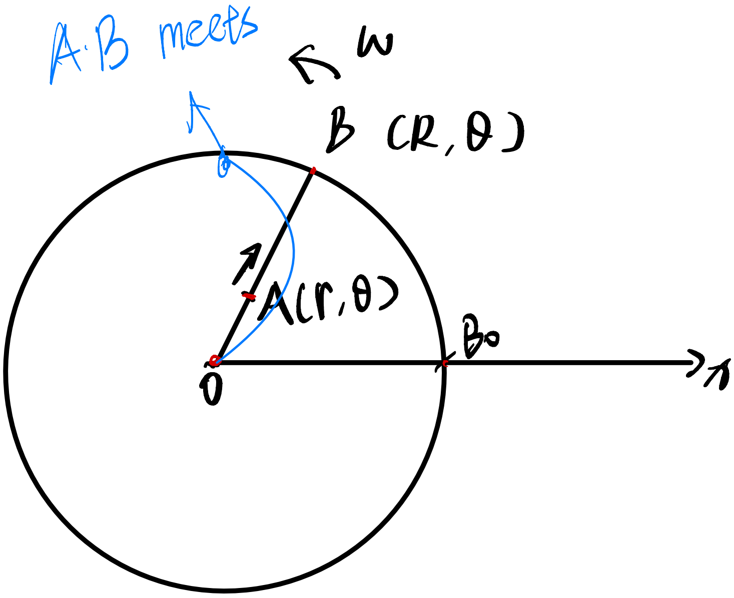
$\varphi = \ln \frac{L}{L - vt} + \frac{\pi}{4}$

(b) $t_{meet} = \frac{r_0}{v_r} = \frac{L}{v}$

(c) $\begin{cases} \varphi = \ln \frac{L}{L - vt} + \frac{\pi}{4} \\ r = \frac{1}{\sqrt{2}}(L - vt) \end{cases}$

$\Rightarrow \varphi = \ln \frac{L}{\sqrt{2} r} + \frac{\pi}{4}$ $\qquad r \in [0, \frac{\sqrt{2}}{2} L]$

$r = \frac{L}{\sqrt{2} e^{\varphi - \frac{\pi}{4}}}$ $\qquad \varphi \in [\frac{\pi}{4}, +\infty)$



$V_A = V_B$

A·B have same speed;

A always directs towards B
(O·A·B are on the same line);

B has a uniform circular motion;

A starts from O,

B starts from Bo.

Q: i). trajectory of A?

ii). What's θ when A meets B?

$\dot r \leftrightarrow \dot\theta$

$\theta = \varphi$ $\quad \frac{dr}{dt} \Leftrightarrow \frac{d\theta}{dt}$

$V = r\omega$

$r \leftrightarrow \theta$

$\frac{dr}{d\theta}$

i) $\vec v_A = \dot r \hat n_r + r\dot{\hat n}_r = \underline{\dot r \hat n_r + r\dot\varphi \hat n_\varphi}$

$|\vec v| = R\dot\varphi = \sqrt{\dot r^2 + (r\dot\varphi)^2} \Rightarrow \dot r = \sqrt{R^2-r^2}\,\dot\varphi$

$|\vec v_B|$

$$\frac{dr}{dt} = \sqrt{R^2-r^2}\,\frac{d\varphi}{dt}$$

$$\frac{dr}{d\varphi} = \sqrt{R^2-r^2}$$

$$\int_0^r \frac{dr}{\sqrt{R^2-r^2}} = \int_0^\varphi d\varphi$$

$$\arcsin\frac{r}{R}\Big|_0^r = \varphi \Rightarrow r = R\sin\varphi$$

$\varphi \in [0, \frac{\pi}{2}]$

ii) $\theta = \frac{\pi}{2}$