PHYS1500J RC3

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1. Newton's laws of motion

1.1 Something that may appear in your quiz...

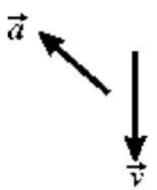
Interaction	Particles Involved	Relative Strength	Range
Gravitational always attractive holds planets in their orbits around Sun	any massive particle	$\sim 10^{-38}$ 最小	infinite
Electromagnetic attractive/repulsive fundamental in optics, chemistry, biology; source of friction	electric charge	$\sim 10^{-2}$	infinite
Weak necessary for buildup of heavy nuclei; responsible for radioactive decay (beta decay)	quarks, leptons	$\sim 10^{-6}$	short $\sim 10^{-18}$ m (0.1% of the diameter of the proton)
Strong holds protons and neutrons together in the nucleus	hadrons (protons, neutrons, mesons)	1	short $\sim 10^{-15}$ m (diameter of a medium sized nucleus)

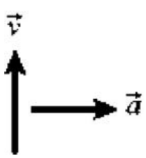
Materials	Coefficient of Static Friction, μ_s	Coefficient of Kinetic Friction, μ_k
Steel on steel	0.74	0.57
Aluminum on steel	0.61	0.47
Copper on steel	0.53	0.36
Brass on steel	0.51	0.44
Zinc on cast iron	0.85	0.21
Copper on cast iron	1.05	0.29
Glass on glass	0.94	0.40
Copper on glass	0.68	0.53
Teflon on Teflon	0.04	0.04
Teflon on steel	0.04	0.04
Rubber on concrete (dry)	1.0	0.8
Rubber on concrete (wet)	0.30	0.25

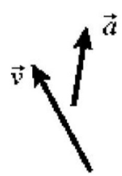
Sun's gravitational force on the earth	3.5×10^{22} N
Thrust of a space shuttle during launch	3.1×10^{7} N
Weight of a large blue whale	1.9×10^{6} N
Maximum pulling force of a locomotive	8.9×10^{5} N
Weight of a 250-lb linebacker	1.1×10^{3} N
Weight of a medium apple	1 N
Weight of smallest insect eggs	2×10^{-6} N
Electric attraction between the proton and the electron in a hydrogen atom	8.2×10^{-8} N
Weight of a very small bacterium	1×10^{-18} N
Weight of a hydrogen atom	1.6×10^{-26} N
Weight of an electron	8.9×10^{-30} N
Gravitational attraction between the proton and the electron in a hydrogen atom	3.6×10^{-47} N

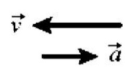
Quiz 2.

Shown below are the velocity and acceleration vectors for a person in several different types of motion. In which case is the person slowing down and turning to his/her right?









Quiz 3.

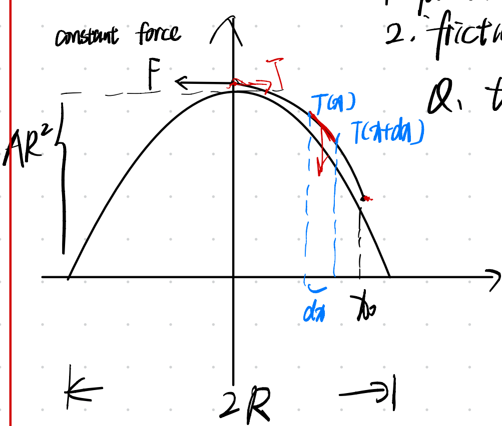
You are seated in a bus, moving on a straight road, and see that a box placed on the frictionless bus floor slides backwards. From this observation, you can conclude that

- [ ] You cannot conclude anything about the the velocity of the bus.
- [ ] The velocity of the bus is forward.
- [ ] The velocity of the bus is backward.
- [ ] The bus is slowing down down.
- [ ] The bus is speeding up.

1.2 Three Laws

Law of inertia	$\vec{F}_{net} = 0 \Rightarrow \vec{v}$ won't change
Law of acceleration	$\vec{F} = m\vec{a}$
Action-reaction law	$\lvert F' \rvert = \lvert F \rvert$ ☆☆

1.3 Exercise



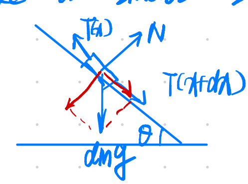
1. for the massive rope, length density $= \lambda$, m
2. frictionless. 3 at rest.

Q. to solve the tension force on the rope at x_0

$y = -A(x+R)(x-R)$

$y' = -2Ax \qquad \tan\theta = 2Ax$

take a small segment of rope dm.



$$\begin{cases} \dfrac{\lambda dx}{\cos\theta} = dm \quad \left(\text{notice that } \dfrac{dm}{dl} = \lambda\right) \\ dmg\sin\theta + T(x+dx) = T(x) \end{cases}$$

$$T(x+dx) - T(x) = -g\sin\theta \frac{\lambda dx}{\cos\theta} = -g \cdot 2Ax\,\lambda dx$$

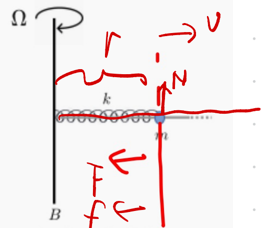
$$\frac{T(x+dx)-T(x)}{dx} = -2gA\lambda x$$

$\downarrow$

$T'(x) \Rightarrow T(x) = F - gA\lambda x^2$

$T(x_0) = F - gA\lambda x_0^2$

(*7 points*) [*Note.* ~~There is no gravitational force in this question.~~]
A linear spring with spring constant k and unstretched/uncompressed length is x_0 is coiled around a light long horizontal rod (see the figure). The rod is fixed to a vertical axle B, which spins always with the same angular speed $\Omega = \sqrt{k/m}$. One end of the spring is fixed at the axle. At the other end of the spring there is bead with mass m that can slide along the rod, but the rod is rough so that the bead moves with friction.



What should the value of the coefficient of kinetic friction μ be, so that the bead slides away from the axle with constant speed v_0? Comment briefly on whether the scenario described in this problem is sustainable?

(g)

$\dot{\varphi} = \sqrt{\frac{k}{m}}$ $\quad r = x_0 + v_0 t \quad \dot{r} = v_0 \quad \ddot{r} = 0$

① $kv_0 t + \mu mg = ma$

★★ $\bar{a} = (\ddot{r} - r\dot{\varphi}^2)\hat{n}_r + (r\ddot{\varphi} + 2\dot{r}\dot{\varphi})\hat{n}_\varphi$

$= -(x_0 + v_0 t)\frac{k}{m}\hat{n}_r + 2v_0\sqrt{\frac{k}{m}}\,\hat{n}_\varphi$

$\downarrow$

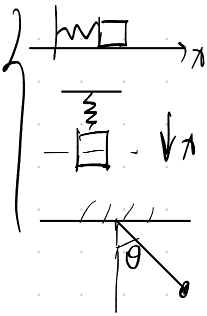
radial component of a $\Leftrightarrow$ radial force

★ $(x_0 + v_0 t)\frac{k}{m} = \frac{kv_0 t + \mu mg}{m}$

$\mu = \frac{x_0 k}{mg}$

2. Harmonic Oscillator and Mechanical Resonance

2.1 Simple Harmonic Oscillator



$ma_x = F_x \Rightarrow \ddot{x} + \frac{k}{m}x = 0$

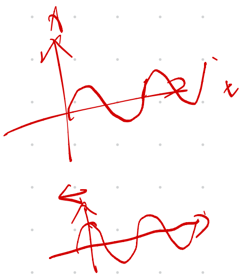
$ma_x = -kx \Rightarrow \ddot{x} + \frac{k}{m}x = 0$

$ma_\theta = F_\theta \Rightarrow ml\ddot{\theta} \approx -mg\theta \Rightarrow \ddot{\theta} + \frac{g}{l}\theta = 0$

In all three cases, the equation of motion is of the same form

$$\boxed{\ddot{x} + \omega_0^2 x = 0}$$

[with $\omega_0^2 = k/m$ for the mass–spring systems and $\omega_0^2 = g/l$ for the simple pendulum]



Any particle (system) with the equation of motion of the above form is called a **simple harmonic oscillator** (SHO)

note: x-t . v_x-t a_x-t 图.

向左移动$\frac{1}{4}$周期 向左移动$\frac{1}{4}$周期

2.2 Damped Oscillator

$\ddot{x} + \frac{b}{m}\dot{x} + \omega_0^2 x = 0$ $\Delta = \left(\frac{b}{m}\right)^2 - 4\omega_0^2$

underdamped regime $\Delta < 0$

$$x(t) = Ae^{-\frac{b}{2m}t}\cos\left(\sqrt{\omega_0^2 - \left(\frac{b}{2m}\right)^2}\, t + \phi_0\right)$$

overdamped regime $\Delta > 0$

$$x(t) = C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t} = $$
$$= C_1 e^{-\left(\frac{b}{2m} - \sqrt{\left(\frac{b}{2m}\right)^2 - \omega_0^2}\right)t} + C_2 e^{-\left(\frac{b}{2m} + \sqrt{\left(\frac{b}{2m}\right)^2 - \omega_0^2}\right)t}$$

critical damping $\Delta = 0$

(the equilibrium position is visited at most once)

$$x(t) = D_1 e^{-\frac{b}{2m}t} + D_2 t e^{-\frac{b}{2m}t}$$

2.3 Forced (or Driven) Oscillation

$$\ddot{x} + \frac{b}{m}\dot{x} + \frac{k}{m}x = \frac{F_0}{m}\cos\omega_{dr}t$$

For steady-state: $x_s(t) = A\cos(\omega_{dr}t + \phi)$

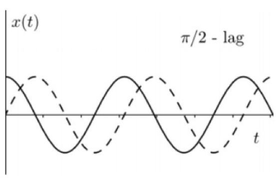
$$A(\omega_{dr}) = \frac{F_0}{m\sqrt{(\omega_0^2 - \omega_{dr}^2)^2 + \left(\frac{b\omega_{dr}}{m}\right)^2}}$$

$$\tan\phi = \frac{b\omega_{dr}}{m(\omega_{dr}^2 - \omega_0^2)}$$

Resonance Frequency: $\omega_{res} = \sqrt{\omega_0^2 - b^2/2m^2}$

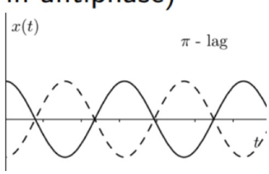
Note:
(1) resonance frequency is lower than the natural frequency if we have drags,
(2) the response of the system is not in phase with the drive.

- If $\omega_{dr} \to \omega_0$ (close to resonance[3]), then $\phi \to -\pi/2$.



The response ($x_s(t)$) lags the drive ($F_{dr}(t)$) by $1/4$ of the cycle.

- If $\omega_{dr} \to \infty$ (high frequencies), then $\phi \to -\pi$ the response lags the drive by $1/2$ of the cycle (displacement and drive are in antiphase)



The response ($x_s(t)$) lags the drive ($F_{dr}(t)$) by $1/2$ of the cycle.

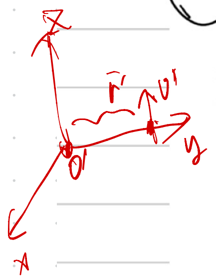
3. Dynamics in Non-Inertial Frames of Reference

$\boxed{\text{When talking about non-inertial force, don't forget "} - \text{" sign}}$

非惯性参考系下的加速度

$$m\bar{a}' = \bar{F} - m\bar{a}_{O'} - m\frac{d\bar{\omega}}{dt}\times\bar{r}' - 2m(\bar{\omega}\times\bar{v}') - m\bar{\omega}\times(\bar{\omega}\times\bar{r}')$$

d'Alembert force　　Euler　　Coriolis　　centrifugal force



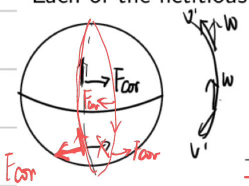
real force (of material origin)

pseudo-forces (also called *fictitious forces* or *forces of inertia*); these are kinematic corrections (which have units of [N]) that **must** be included due to the fact that we describe dynamics in a non-inertial FoR (*force* = *mass* × *acceleration* is valid only in inertial FoRs). These "forces" **never** appear in inertial FoRs!

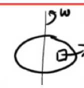
Each of the fictitious forces has its own name

北顺

南逆



$-m\bar{a}_{O'}$	d'Alembert (or drift) "force"	非惯性系的平动加速度
$-m\frac{d\bar{\omega}}{dt}\times\bar{r}'$	Euler "force"	系统角加速度，
$-2m(\bar{\omega}\times\bar{v}')$	Coriolis "force"	"北顺南逆"
$-m\bar{\omega}\times(\bar{\omega}\times\bar{r}')$	centrifugal "force"	

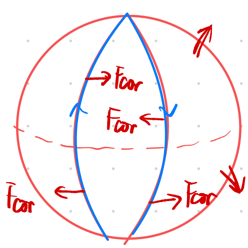


$v'=0$　　$\frac{d\omega}{dt}=0$

3.1 Coriolis Force & Geography

Northern Hemisphere: Rightward

(Clockwise Tendency)



Southern Hemisphere: Leftward

Counter-Clockwise Tendency

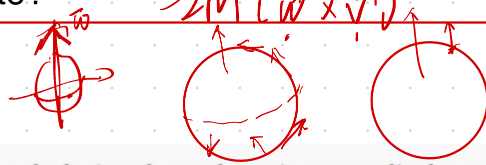
Coriolis force in nature: prevailing westerlies, cyclones and anticyclones

3.3 Exercises

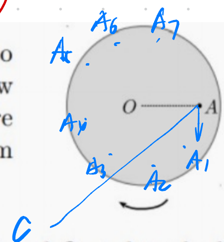
Can you say that there is a centrifugal force and a centripetal force acting upon a particle at the same time? Why not?

Will the oscillation plane of a Foucault pendulum, that is placed on the equator, rotate?

$-2m(\vec{\omega} \times \vec{v}')$

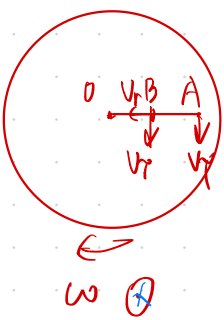


(*5 points*) A platform rotates clockwise about the axis perpendicular to the page and through the platform's center O, as shown in the top-view figure. The angular speed of rotation is constant. Suppose that you are standing at point A, throwing a ball towards the center of the platform O, along the radial direction AO.



(a) An observer, standing on the platform at O, will see the object being deflected to the left or to the right with respect to the radial direction AO (as he looks from O)?

(b) How will an immobile observer outside of the platform explain this deflection?



① non-inertial. Coriolis force.

$-2m(\bar{\omega} \times \bar{v}') \Rightarrow$ direction "↓"

⊗ ←

② inertial FoR: consider φ or ω to keep O, B, A in a line,

$$\frac{V_{\tau,B}}{r_{OB}} = \frac{V_{\tau,A}}{r_{OA}}$$

but actually, $r_{OB} < r_{OA}$, $V_{\tau,B} = V_{\tau,A}$

ω_B is greater than ω_A

tips: compare to a river in Southern Hemisphere.

北顺南逆 北右南左

More exercises to follow...

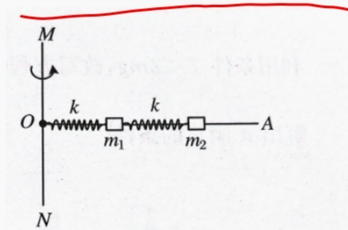
1\.

A ship of mass M sails with constant velocity, while its engine provides a constant propelling force of magnitude F_{prop}. There is a linear drag force acting upon the ship, $F_{drag} = -\beta v$, where $\beta>0$ is constant. At the instant t=0 s, the magnitude of the ship's propelling force suddenly drops to 25% of the original value F_{prop}. Find

- the speed of the ship before the reduction of the propelling force, and
- the dependence of ship's speed on time after the propelling force has been reduced.

2\.

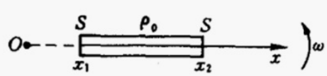
A smooth rod OA, can be rotated horizontally around point O, there are two identical springs(k), and two identical objects(m), if the system around the MN axis does stable uniform rotation, find the range of angular velocity of rotation.



3\.

Consider swinging a test tube filled with liquid with a string (which you probably should not imitate). The two ends of the test tube are at distance x_1 and x_2 from your hand. Area of the tube is S, density of liquid is ρ. Ignore earth gravity. Find:

1. Force exerted on the test tube by the string.
2. Liquid pressure at distance x from your hand.



Thanks!