PHYSC1500J - RC5

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For some basic concepts before the topic of this RC, if you find lecture hard to understand, you may refer to the link below

https://zhuanlan.zhihu.com/p/681163731?share_code=nzJN6vMnsOrl&utm_psn=1928969363987342466

1. Torque

1.1 Concepts

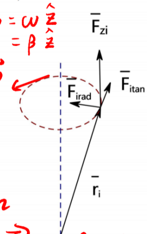
Definition

$$\overline{\tau} = \overline{r} \times \overline{F}$$

Rotation with Fixed Axis

The net force can be decomposed into three components: radial, tangential and along the z-axis $\bar{F}_i = \bar{F}_{i,rad} + \bar{F}_{i,tan} + \bar{F}_{i,z}$

$\vec{\omega} = \omega \hat{z}$
$\vec{\beta} = \beta \hat{z}$



$$\bar{F}_i = \bar{F}_i = \underbrace{\bar{F}_{i,rad}}_{\text{radial}} + \underbrace{\bar{F}_{i,tan}}_{\text{tangential}} + \underbrace{\bar{F}_{i,z}}_{\text{along the z-axis}}$$

the only component that contributes to rotation

if the motion is in a plane $\Rightarrow$ ω, β along z axis

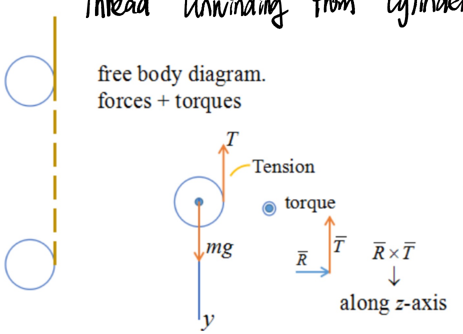
Formula

Formula:

$$\tau_z = I_z \varepsilon_z \quad \Rightarrow \quad F = ma$$

1.2 Two important examples

Thread Unwinding from Cylinder



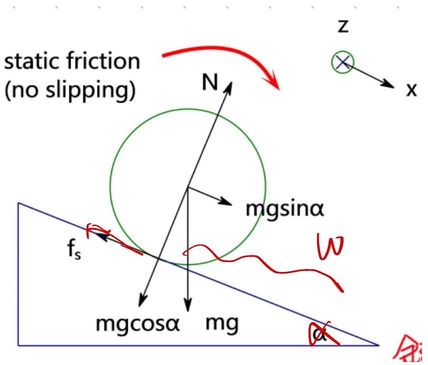
1. $mg - T = ma_{cm,y}$
2. $TR = I_{cm}\varepsilon_z = \frac{1}{2}mR^2\varepsilon_z$
3. $a_{cm,y} = R\varepsilon_z$ (no slipping; follows from $v_{cm,y} = \omega_z R$)

$\Rightarrow$

$$a_{cm,y} = \frac{2}{3}g$$

$$T = \frac{1}{3}mg$$

Sphere Rolling Down an Incline (no slipping)



1. Explain the direction of static friction
2. Is static friction doing work?

1. imagine small angle α
想象齿轮
the reason why sphere rotate ⊗ this way

f_s → G

2. translational kinetic energy ↓
rotational kinetic energy ↑
E_k f_s

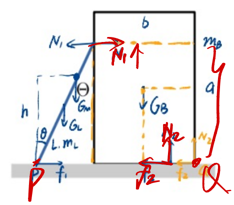
$$\begin{cases} ma_{cm,x} = mg\sin\alpha - f_s \\ f_s R = I_{cm}\varepsilon_z = \frac{2}{5}mR^2\varepsilon_z \\ a_{cm,x} = \varepsilon_z R \end{cases}$$

1.3 Exercise

Problem 3. (*15 points*) A man of mass m_M climbs up an unsecured ladder of length L and mass m_L. It is resting on the side of a large empty box of height a and width b, which has mass m_B. There is no friction between the ladder and the box.

(a) If the ladder makes an angle of θ degrees to the vertical side of the box, find the coefficient of friction between the box and the ground needed to prevent the box from sliding away from the ladder when the man stands at a height h above the ground.

(b) Assuming the friction is sufficient to prevent the box from sliding, find the maximum height the man can climb before the box begins to tip over.



The acceleration due to gravity g is known.

(a) for Box
$$\begin{cases} N_2 = G_B \\ N_1 = f_2 \end{cases}$$
$f_2 \leq \mu_s \cdot N_2$

for ladder: $N_1 L\cos\theta = G_L \cdot \frac{L}{2}\sin\theta + G_M h\tan\theta$

$$\Rightarrow \mu_s \geq \frac{m_L L\sin\theta + 2m_M h\tan\theta}{2m_B L\cos\theta}$$

(b) for Box $N_1 L\cos\theta \leq G_B \cdot \frac{1}{2}b$ (axis Q)

for ladder: $N_1 L\cos\theta = G_L \cdot \frac{L}{2}\sin\theta + G_M \cdot h\tan\theta$

$$\Rightarrow h \leq \frac{m_B b - m_L L\sin\theta}{2m_M\tan\theta}$$

2. Work and Conservation of Angular Momentum

2.1 Some Concepts

It's always a good idea to compare with concepts in translational motion dynamics ☺️

Definition and Formula

work

$$W_{12} = \int_{\theta_1}^{\theta_2} \tau_z \mathrm{d}\theta$$

$W = \int F dx$

power

$$P = \tau_z \omega_z$$

$P = F v$

Work-Kinetic Energy Theorem

$$W = K_2 - K_1,$$

where $K = \frac{1}{2} I \omega^2$

if there's only rotation

What if translation + rotation?

$K = \frac{1}{2} M_c V_c^2 + \frac{1}{2} I_c \omega^2$

linear momentum

$\boxed{L = r \times p}$

SI unit: $kg \cdot \frac{m^2}{s}$

$\Leftrightarrow I_{scalar} \vec{\omega} = \vec{L}_m$ (angular momentum)

$m\vec{V} = \vec{P}_m$ (linear momentum)

Definition and Formula

When the net external torque on a system is zero, then the total angular momentum of the system is conserved.

$$L_z = I_z \omega_z = const$$

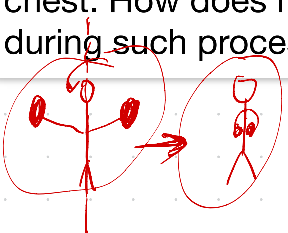
$\Rightarrow \vec{P} = m\vec{V}_{cm} = const$ if no external F.

in general, $\vec{L}$ not parallel to $\vec{\omega}$ except for case rotation is about a symmetry axis (or the object is planar)

A small question to help you understand :
A man holding dumbbell in hands with both arms raised, and he is rotating along his central axis. Then he pulls arms in towards chest. How does his angular velocity and kinetic energy change during such process?

$\omega \uparrow$

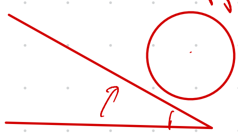
$\frac{1}{2} I \omega^2$



$I \downarrow$

$E_m \to E_k$

Another thinking question which may have appeared in lecture:
A rotating cylinder climbing up along an inclined plane



t=0
$w \neq 0$ at initial state

sliding → rolling

v_{cm} v_s

向上的
糙斜面滚筒，先滑动再滚动，停下来所需总时间一样

$t_h = \frac{R\omega_0}{2g\sin\alpha}$

但是停止滑动所需时间不一样

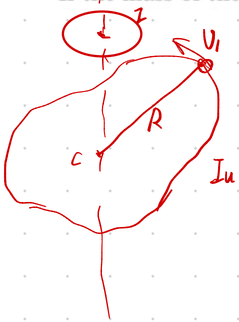
Inclined plane + rotating cylinder climbing + calculate work
1. Negative work done by kinetic friction → heat.
2. relative displacement = Vr - Vcm

2.2 Exercises

(5 points) A certain object with unknown shape is rotating about a fixed vertical axis passing through its center of mass. A scientist measures the part of the object farthest from the object's center of mass to be moving with speed v_1 at radius R_1. To measure the moment of inertia a scientist drops a stationary disk with $I = \frac{1}{2}M_d R_d^2$ on top of the unknown object. The disk is dropped such that its axis of symmetry (perpendicular to the disk's plane) is collinear with the axis about which the unknown object is rotating. The two objects hit and stick together and rotate as one object following the collision. With the addition of the disk the angular speed is found to decrease to 50% of its original value.

no use.

If the mass of the unknown object is equal to $\frac{3}{5}M_d$ and $R_d = R_1$, what is its moment of inertia?



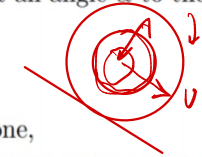
$\omega_0 = \frac{v_1}{R}$.

initial state: $L_i = I_u \cdot \omega_0$

final state: $L_f = (I_u + I)\frac{1}{2}\omega_0$

$\Rightarrow L_i = L_f \Rightarrow I_u = I = \frac{1}{2}M_d R_d^2$

(5 points) A uniform hollow cylinder has mass M, inner radius r and outer radius R. Another uniform cylinder with mass m and radius r is placed inside the hollow cylinder. Initially, the filled cylinder is held at rest at the top of a plane inclined at an angle α to the horizontal. It is then let to roll down the plane without slipping.

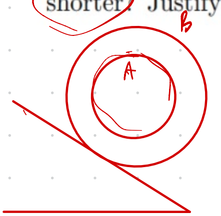


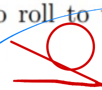
We repeat the experiment in two versions

(a) the inner cylinder is permanently glued to the hollow one,

(b) the inner cylinder is free to rotate without friction inside the hollow cylinder.

In which case is the time needed for the cylinder to roll to the bottom of the incline plane shorter? Justify your answer without calculations.





$K_t \propto K_r$

$\frac{1}{2}mv^2 \quad \frac{1}{2}I\omega^2$

$v \propto \omega$

$\Delta U_g = \Delta K = \Delta K_t + \Delta K_r$

large I ?

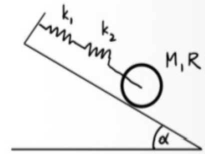
$\Rightarrow I_a = I_A + I_B \qquad I_a > I_b \qquad \Delta K_r = \frac{1}{2}I\omega^2$

$I_b = I_B$

(a) requires more E to reach the same translational speed.

$\Rightarrow$ (b)

(*15 points*) A uniform ball with mass M and radius R is placed on a long plane, inclined at the angle α to the horizontal. A yoke passing through the center of the ball is connected to two springs, obeying Hooke's law with spring constants k_1 and k_2, connected in series. The other end of the spring is connected to a fixed support at the top of the incline. The ball is released when the springs are neither stretched nor compressed. There is no slipping of the ball on the inclined plane at any time. The acceleration due to gravity g is known.



(a) How far down the plane does the ball travel until it stops instantaneously? What is the maximum speed of the center of mass of the ball?

(b) What is the direction of the force of friction between the plane and the ball at any instant of time?

(c) Derive an equation for the acceleration of the center of mass of the ball in terms of k_1, k_2, M, R, g, α, and the distance of the center of mass of the ball x, measured from its initial position.

(a) $\begin{cases} Mgh = \frac{1}{2} k L_1^2 \\ h = L_1 \sin\alpha \end{cases}$

$\Rightarrow L_1 = 2Mg \frac{k_1+k_2}{k_1 k_2} \sin\alpha$

$E_{tot} = \frac{1}{2}MV^2 + \frac{1}{2}\cdot\frac{2}{5}MR^2\cdot\left(\frac{V}{R}\right)^2 + Mg(L_1 - x)\sin\alpha + \frac{1}{2}kx^2$

$= MgL_1\sin\alpha$ (at initial state)

$\Rightarrow \frac{7}{10}MV^2 = -\frac{1}{2}k\left(x - \frac{1}{k}Mg\sin\alpha\right)^2 + \frac{1}{2k}M^2g^2\sin^2\alpha$

$\Rightarrow V_{max} = \sqrt{\frac{5M(k_1+k_2)}{7k_1k_2}}\, g\sin\alpha$ when $x = \frac{1}{2}L_1$

(b) according to (a):

$0 < x < \frac{1}{2}L_1$: $V\uparrow$ $\omega\uparrow$ $\Rightarrow \beta > 0 \Rightarrow f$ upwards along inclined plane

$\frac{1}{2}L_1 < x < L_1$: $V\downarrow$ $\omega\downarrow$ $\Rightarrow \beta < 0 \Rightarrow f$ downwards ~

(c) $0 < x < \frac{1}{2}L$

$\begin{cases} Ma = Mg\sin\alpha - kx - f_s \\ f_s R = I\beta = \frac{2}{5}MR^2\frac{a}{R} \end{cases} \Rightarrow a = \frac{5g\sin\alpha}{7} - \frac{5}{7M}\frac{k_1k_2}{k_1+k_2}x$

$\frac{1}{2}L < x < L$:

$\begin{cases} Ma = \cdots + f_s \\ f_s R = -I\beta \end{cases}$

3. Equilibrium and Elasticity

3.1 Equilibrium

Conditions for Equilibrium $\begin{cases} 1. \; F^{ext} = 0 \\ 2. \; \tau^{ext} = 0 \end{cases}$

3.2 Strain, Stress, and Elastic Modulus

Just remember some definitions

Some definition:

$$\text{stress} = \frac{\text{force}}{\text{area}}$$

$$\text{strain} = \frac{\Delta l \text{ (deformation)}}{l_0}$$

$$\text{elastic modulus} = \frac{\text{stress}}{\text{strain}}$$

$$\text{tensile stress} = \frac{F_\perp}{A}$$

$$\text{tensile strain} = \frac{\Delta l}{l_0}$$

Young's modulus is tensile stress divided by tensile strain, and is given by

$$Y = \frac{\frac{F_\perp}{A}}{\frac{\Delta l}{l_0}}$$

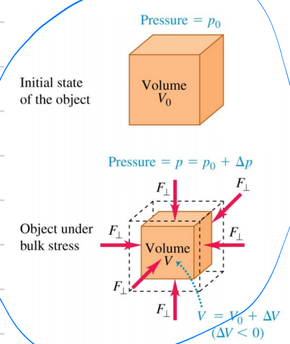
bulk stress and strain

pressure $P = \frac{F_\perp}{A}$

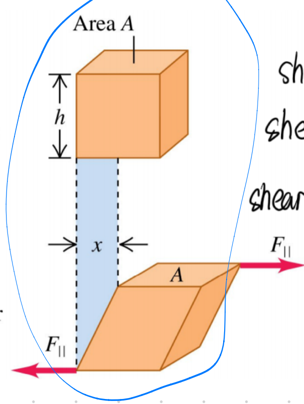
bulk stress : ΔP

bulk strain : $\frac{\Delta V}{V_0}$

bulk modulus $B = -\frac{\Delta P}{\frac{\Delta V}{V_0}}$ $(\Delta V < 0)$



shear stress & strain



shear stress $\frac{F_{\|}}{A}$

shear strain $\frac{x}{h}$

shear modulus $S = \dfrac{\frac{F_{\|}}{A}}{\frac{x}{h}}$