PHYS1500J Mid RC Part 1

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Some Reminders

- About the exam: "As last year, the exam is closed book and there is a formula sheet attached to the paper — I said that in class. A dictionary is ok — we have it as a rule in the syllabus."
- About parametric form and implicit form: "Both are fine, unless the question asks for a specific form."
- About differential equations: "The equations we have come across so far are not really differential equations, just some simple integration after separation of variables. I believe we had something like that last year indeed."

Scientific Notation and Unit

Scientific Notation

$$A = a \cdot 10^k \qquad \text{where } |a| \in [1, 10)$$

The Prefix of Units

Factor	Name	Symbol	Factor	Name	Symbol
10^{24}	yotta	Y	10^{-1}	deci	d
10^{21}	zetta	Z	10^{-2}	centi	c
10^{18}	exa	E	10^{-3}	milli	m
10^{15}	peta	P	10^{-6}	micro	μ
10^{12}	tera	T	10^{-9}	nano	n
10^{9}	giga	G	10^{-12}	pico	p
10^{6}	mega	M	10^{-15}	femto	f
10^{3}	kilo	k	10^{-18}	atto	a
10^{2}	hecto	h	10^{-21}	zepto	z
10^{1}	deka	da	10^{-24}	yocto	y

SI Units & Unit Conversions

SI Base Units		
Mass	kilogram	*kg*
Length	meter	*m*
Time	second	*s*
Amount of Substance	mole	*mol*
Electric Current	ampere	*A*
Thermodynamic Temperature	kelvin	*K*
Luminous Intensity	candela	*cd*

Dimensional Analysis Method:

1. Write down the target parameters' units in SI form.
2. Write down the known parameters' units in SI form.
3. Assume the exponent of the parameters.
4. Solve the system of equations to make sure the exponent of the same SI unit is identical.

Significant Figures

How to Count Numbers of Significant Figures

1. Non-zero digits are always significant.
2. Any zeros between two significant digits are significant.
3. Trailing zeros after a decimal point are significant.
   - Example: 2.300 has 4 significant figures (2, 3, and two zeros).
4. Leading zeros before the first non-zero digit are not significant.
   - Example: 0.0025 has only 2 significant figures (2 and 5).
5. Trailing zeros in a whole number without a decimal point are generally not significant (unless otherwise indicated).
   - Example: 1500 has 2 significant figures (1 and 5); but if written as **1500.** , it has 4 significant figures.
6. When using scientific notation, all digits in the coefficient are significant.
   - Example: 4.00 × 10² has 3 significant figures.

More Examples

Example

Measured Data	Number of Significant Figures
1098000	4
0.001098	4
1.098×10^6	4
1.09800×10^6	6
100.	3

Vectors and Cartesian Coordinate System

Scalar

- time, mass, length, volume, density of matter, electric charge, potential energy, pressure, kinetic/potential energy...
- defined by a single number
- Properties:

Ts + 1m

$+$ or $-$	only compatible units allowed as arguments
$*$ or $/$	may involve quantities with different units
sin(...), ln(...), exp(...), ...	only dimensionless arguments allowed

Vector

$3\hat{n}_x + 4\hat{n}_y$

- velocity, force, linear momentum, angular velocity, angular momentum, electric/magnetic field, electric current density...
- defined by both a magnitude and a direction
- Properties:

1. The dot product of two non-zero vectors is zero **iff** the vectors are perpendicular,

$$\overline{u} \circ \overline{w} = 0 \Leftrightarrow \overline{u} \perp \overline{w}$$

2. The cross product is anticommutative,

$$\overline{u} \times \overline{w} = -\overline{w} \times \overline{u}$$

3. The cross product is a zero vector **iff** the two non-zero vectors are parallel (or antiparallel)

$$\overline{u} \times \overline{w} = 0 \Leftrightarrow \overline{u} \parallel \overline{w}$$

  - Non-associative law of cross product: $$\vec{A} \times (\vec{B} \times \vec{C}) \neq (\vec{A} \times \vec{B}) \times \vec{C}$$ $$\vec{A} \times (\vec{B} \times \vec{C}) \neq -(\vec{A} \times \vec{B}) \times \vec{C}$$
  - "Back-Cab" Rule: (x_A, y_A, z_A)

$$\boxed{\mathbf{A}\times(\mathbf{B}\times\mathbf{C})=\mathbf{B}(\mathbf{A}\cdot\mathbf{C})-\mathbf{C}(\mathbf{A}\cdot\mathbf{B})}$$

Basic Operation in Cartesian

Basic Operation in Cartesian

Scalar Product

For $\overline{u}=(u_x,u_y,u_z)$ and $\overline{w}=(w_x,w_y,w_z)$, the dot product

$$\boxed{\overline{u}\circ\overline{w}=u_xw_x+u_yw_y+u_zw_z}$$

Vector Product

$$\begin{aligned}\overline{u}\times\overline{w} &= (u_yw_z-u_zw_y)\hat{n}_x+(u_zw_x-u_xw_z)\hat{n}_y+(u_xw_y-u_yw_x)\hat{n}_z\\ &= \begin{vmatrix}\hat{n}_x & \hat{n}_y & \hat{n}_z\\ u_x & u_y & u_z\\ w_x & w_y & w_z\end{vmatrix}.\end{aligned}$$

Differentiation

$$\begin{aligned}\boxed{\frac{d\overline{u}}{dt}} &= \frac{d}{dt}\left(u_x(t)\hat{n}_x+u_y(t)\hat{n}_y+u_z(t)\hat{n}_z\right)\\ &= \boxed{\dot{u}_x(t)\hat{n}_x+\dot{u}_y(t)\hat{n}_y+\dot{u}_z(t)\hat{n}_z}\end{aligned}$$

Integration

$$\int_{t_0}^{t_1}\overline{u}dt=\left(\int_{t_0}^{t_1}u_x(t)dt\right)\hat{n}_x+\left(\int_{t_0}^{t_1}u_y(t)dt\right)\hat{n}_y+\left(\int_{t_0}^{t_1}u_z(t)dt\right)\hat{n}_z$$