PHYS1500J-Mid2-RC
Part3 Potential Energy and Conservation Laws
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1. Some Easy Concepts

$$\boxed{\text{net force} = \text{conservative force} + \text{non-conservative force}}$$

$$\delta W = \underbrace{\delta W_{\text{cons}}}_{-dU} + \delta W_{\text{non-cons}} \overset{work-k.e.\ thm}{=} dK.$$

Hence $-dU + \delta W_{\text{non-cons}} = dK$ and, equivalently,

$$\boxed{\delta W_{\text{non-cons}} = d(K+U) = dE} \quad \text{or} \quad \boxed{\Delta W_{\text{non-cons}} = \Delta K + \Delta U = \Delta E}$$

Conclusion. Work done by non-conservative forces changes the mechanical energy of the system.

Experiments show that $\Delta W_{\text{non-cons}} = -\Delta U_{\text{int}}$, where U_{int} is the *internal energy* (all forms of energy other than the kinetic energy and the potential energy due to external forces)

Hence, the **law of conservation of the total energy**

$$\boxed{\Delta K + \Delta U + \Delta U_{\text{int}} = 0}$$

Interpretation. Energy can be transformed between its different forms but the net change is always zero.

Conservation of Mechanical Energy (only elastic force present)

When only the elastic force does work, the sum $E = U_{el} + K$, called the *total mechanical energy* of the system is constant (that is, it is *conserved*).

Comment. Explicitly, $\frac{1}{2}kx^2 + \frac{1}{2}mv^2 = \text{const}$.

Conservation of Mechanical Energy (only gravitational forces present)

When only the force of gravity does work, the sum

$$E = U_{grav} + K,$$

called the *total mechanical energy* of the system is constant (that is, it is *conserved*).

Comments

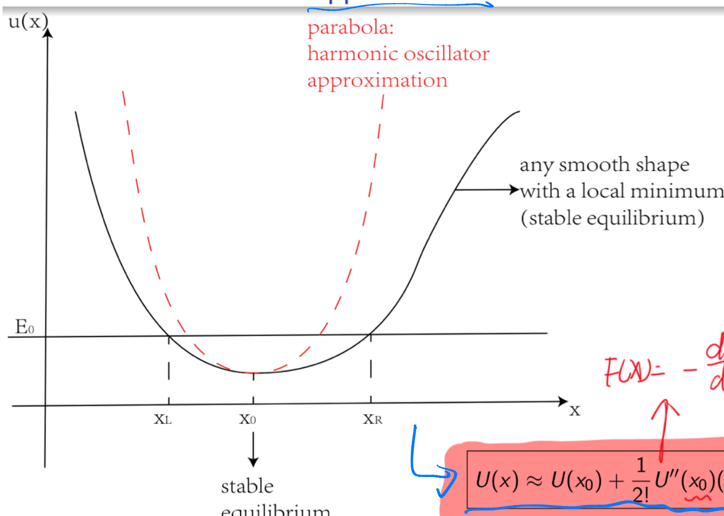
1. Explicitly, $mgy + \frac{1}{2}mv^2 = \text{const}$.
2. U_{grav} is determined up to an additive constant, $U_{grav} = mgy + C$. Only the difference ΔU_{grav} is measurable (physical), hence the constant plays no role (we can choose the reference level for U_{grav} at will.)

1.1 Conclusions

a. Conservative force doing work: transfer of energy between potential energy and kinetic energy
b. Non-conservative force doing work: change total mechanical energy (friction, air drag, pull force…)
c. Net forces doing work: change kinetic energy (kinetic theorem)

1.2 Potential Well

Motion in a Potential Well. Harmonic Approximation



Tips: Pay attention to the range of x, pay attention to the preconditions of this harmonic approximation

★ $|x-x_0| \to$ "small"

$|x-x_0| \ll \ldots$

$$F(x) = -\frac{du}{dx} = -k(x-x_0)$$

$$\boxed{U(x) \approx U(x_0) + \frac{1}{2!}U''(x_0)(x-x_0)^2.}$$

In nature: Taylor Expansion

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{n}(x_0)}{n!}(x-x_0)^n$$

$f^{n}(x_0)$: n-th derivative of $f(x)$ at $x = x_0$.

$$\approx \sum_{n=0}^{N} \frac{f^{n}(x_0)}{n!}(x-x_0)^n$$

usually N takes 2, but it depends,
try to be flexible during exam.

$$U(x) \approx U(x_0) + \frac{1}{2!}U''(x_0)(x-x_0)^2$$

for a spring, $U(x) = \frac{1}{2}kx^2$ (take $x_0 = 0$, $U(x_0) = 0$), so $k = \ddot{U}(x_0)$

Conclusion. A particle moving in a potential well of *any smooth shape,* not too far away from a stable equilibrium, moves as if it was in harmonic motion with the natural frequency

$$\omega_0 = \sqrt{\frac{k}{m}} = \sqrt{\frac{U''(x_0)}{m}}$$

前提: linear force

Consequently, $x(t) = A\cos(\omega_0 t + \phi)$.

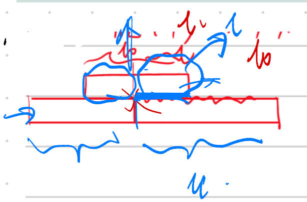
1.3 Some Exam Tips

1. $\vec{F} = \left(-\frac{\partial U}{\partial x}, -\frac{\partial U}{\partial y}, -\frac{\partial U}{\partial z}\right) = -\nabla U$.

各个方向上的偏导

F $\quad$ ∇U

2.



$f \propto l$.

$\mu mg \frac{l}{l_0}$

$Q = \int \mu mg \frac{l}{l_0}\, dl$.

$f = \mu mg\, x$.

$f = \mu mg \frac{l}{l_0}$

$Q = f x$.

N

2. Exercises

1. Two uniform, soft, and flexible ropes A and B have equal mass. They are laid flat on a horizontal ground, with rope A being longer than rope B. The midpoints of both ropes are slowly lifted until the entire ropes are off the ground. The final height reached by the midpoints of the two ropes is h_A and h_B, respectively. The work done by gravity during this process is W_A and W_B, respectively. Which of the following statements is correct?

A. If $h_A = h_B$, then it must be true that $W_A = W_B$

B. If $h_A > h_B$, then it is possible that $W_A < W_B$

C. If $h_A < h_B$, then it is possible that $W_A = W_B$

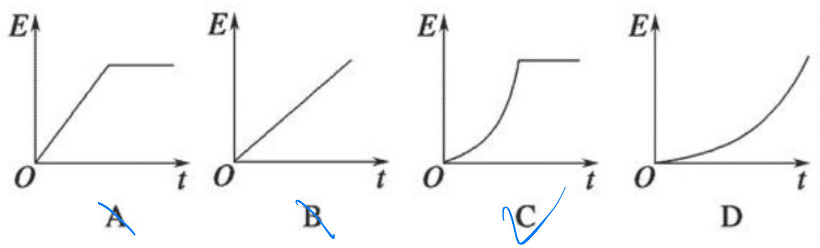
D. If $h_A > h_B$, then it must be true that $W_A > W_B$

A. m_A h_A' m_B h_B' $h_A' < h_B'$ $W_A < W_B$

B. A B

CD.. -

An object initially at rest on the ground is lifted vertically upward by a constant force. The force is removed when the object reaches a certain height. Ignoring air resistance, during the entire lifting process, the relationship between the **mechanical energy** of the object and **time** is represented by which of the following graphs?



a $v \propto t$ $h \propto t^2$ $\Delta E = Fh \propto Ft^2$

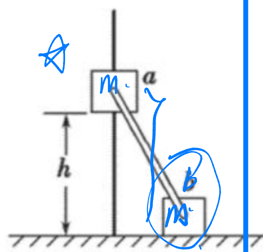
As shown in the figure, blocks a and b both have mass m. Block a is placed on a fixed vertical rod at a height h above the smooth horizontal ground. Block b is on the ground. The two blocks are connected by a light, rigid rod via hinges. The system starts from rest and begins to move due to gravity. Friction is neglected. Blocks a and b can be considered as point masses. Gravitational acceleration is g. Which of the following statements are correct:

A. Before block a hits the ground, the light rod is always doing ~~positive~~ work on block b.

B. The speed of block a when it hits the ground is $\sqrt{2gh}$.

C. During the fall of block a, its acceleration is always less than or equal to g.

D. Just before block a hits the ground, when its mechanical energy is at a minimum, the normal force exerted by the ground on block b is equal to mg.



A. $V_b \uparrow \downarrow$ W f → -

(B) b

$V_{b,x} = 0 = V_{a,x}$

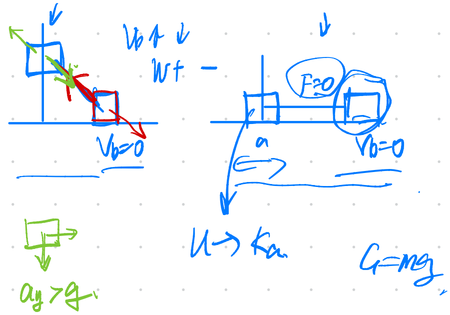
C. $a_y > g$

D. E_a min ⇒ E_b max

$mgh = \frac{1}{2}mv^2$

E_b max

k_b

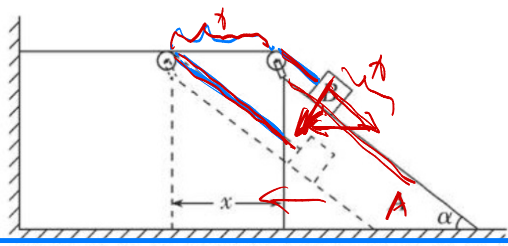


As shown in the figure, block A is on an inclined plane with angle α. One end of the string is fixed on the wall, the string passes over the top of the incline and connects via a pulley to block B. Block A is initially held stationary on the incline, and block B is resting on a horizontal frictionless table. The pulley on the right side is on the horizontal surface, and the string on the right side is parallel to the incline. The mass of block A and B is m. After removing the fixture holding A in place, blocks A and B start moving in a straight line, and all friction is neglected. Gravitational acceleration is g. Find:

(1) When A is fixed and not moving, the magnitude of the normal force N that A exerts on B; $mg\cos\alpha$

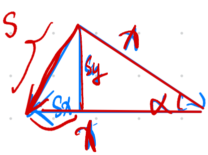
(2) When A has moved a distance x, the displacement s of B;

(3) When A has moved a distance x, the speed v_a of A.



V_A V_B

(2)



$\boxed{S = \sqrt{2(1-\cos\alpha)} \cdot x}$

$\begin{cases} S_x = x(1-\cos\alpha) \\ S_y = x\sin\alpha \end{cases}$

(3)

S ⇔ x

B A

$\begin{cases} \frac{ds}{dt} = V_B \quad \frac{dx}{dt} = V_A \\ mgS_y = \frac{1}{2}mV_A^2 + \frac{1}{2}mV_B^2 \end{cases}$

$\rightarrow V_B = \sqrt{2(1-\cos\alpha)} \cdot V_A$

$V_A = \sqrt{\frac{2gx\sin\alpha}{3-2\cos\alpha}}$

Some Mid1 Problems Given by Prof.MK Might Help You with Mid2…

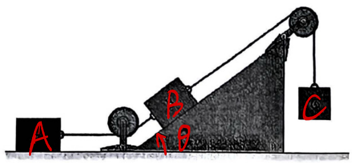
Problem 1. (*15 points*) Blocks A, B, and C are placed as in the figure below and connected by ropes of negligible mass. Both A and B have the same weight (of magnitude w), and the coefficient of kinetic friction between each block and the ramp is μ. The ramp consists of a horizontal section and a section inclined at an angle θ. Block C descends with constant velocity.

(a) Draw two separate free-body diagrams showing the forces acting on A and B.

(b) Find the tension in the rope connecting blocks A and B.

(c) What is the weight of block C?

(d) If the rope connecting A and B were cut, what would be the acceleration of C?

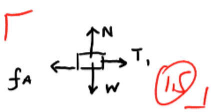


The acceleration due to gravity g is given.

P1

total 15

(a)



1.5

1.5

(b) The blocks move at constant speed, so there is no net force on A; then the tension in the cord connecting A and B is equal to the frictional force f_A. Hence

$$\boxed{T_1 = f_A = \mu w}$$ 3

(c) 1.5 The weight of C is equal to the tension in the rope connecting B and C. From the FBD for B

along ramp: $0 = T_2 - T_1 - f_B - w\sin\alpha$; $f_B = \mu N_B$ 1.5

$\perp$ to ramp: $0 = N - w\cos\alpha$ 1.5

Hence $T_2 = \underbrace{(\mu w)}_{T_1} + \mu w\cos\alpha + w\sin\alpha = W_c$ 1.5

$$\boxed{W_c = w(\mu + \mu\cos\alpha + \sin\alpha)}$$ 1

(d) If we cut the rope, T_2 will have a smaller value than that found in (c). Then the weight of C will not be balanced by T_2 and block C will be descending with acceleration (and block B will be moving up the ramp with the same acceleration). 2

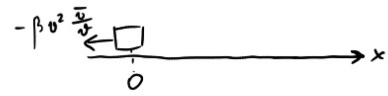
Problem 3. (*14 points*) A spacecraft of mass m, moves at constant speed v_0 in deep space along a straight line with its engines turned off. Suddenly, it enters a huge region filled with cosmic dust. The surrounding dust gives rise to a drag force of magnitude βv^2, where β is a constant and v is the instantaneous speed of the spacecraft.

How far into the dust region does the spacecraft travel, before its speed decreases to a half of the initial value? How would this distance change, if the initial speed was $2v_0$?

P3

total 14

After entering the cloud, the net force in the direction of motion equals the drag force



Eqn. of motion (2nd law)

$$m a_x = -\beta v_x^2 \iff m \frac{dv_x}{dt} = -\beta v_x^2 \quad (*)$$

2

(a) Use the chain rule $\frac{dv_x}{dt} = \frac{dv_x}{dx} \cdot \underbrace{\frac{dx}{dt}}_{v_x} = \frac{dv_x}{dx} v_x$. Hence

$$v_x \frac{dv_x}{dx} = -\frac{\beta}{m} v_x^2$$

4

Separate the variables and integrate

$$\int_{v_0}^{\frac{1}{2}v_0} \frac{dv_x}{v_x} = -\int_0^{s_1} \frac{\beta}{m}\, dx$$

3

$$\ln \frac{\frac{1}{2}v_0}{v_0} = -\frac{\beta}{m} s_1 \Rightarrow \boxed{s_1 = \frac{m}{\beta} \ln 2}$$

1,5 1

(b) The answer will not change, since

1

$$\ln \frac{v_0}{2v_0} = \ln \frac{\frac{1}{2}v_0}{v_0} \Rightarrow \boxed{s_1 = \frac{m}{\beta} \ln 2}$$

1,5

Good Luck with Your Mid2 Exam!

(X)

(✓)