PHYS1500J RC1

Zhao Guojiao zgj_1.17@sjtu.edu.cn

Some Reminders

- Please **box** your answer in your homework. It will help your TAs find your answers a lot.
- Both the Mastering Physics homework and Problem Set homework will count to your final grade. Be careful. Details determine everything if you wanna get A/A+.
- Priority: MK's lecture -> MK's homework ->RC.
- It's relatively easy but still takes time to learn.
- My RC will be featured with **key concepts (usually confusing)+ exercise (helpful for your exam maybe)**

Scientific Notation and Unit

Scientific Notation

$$A = a \cdot 10^k \qquad \text{where } |a| \in [1, 10)$$

The Prefix of Units

Factor	Name	Symbol	Factor	Name	Symbol
10^{24}	yotta	Y	10^{-1}	deci	d
10^{21}	zetta	Z	10^{-2}	centi	c
10^{18}	exa	E	10^{-3}	milli	m
10^{15}	peta	P	10^{-6}	micro	μ
10^{12}	tera	T	10^{-9}	nano	n
10^{9}	giga	G	10^{-12}	pico	p
10^{6}	mega	M	10^{-15}	femto	f
10^{3}	kilo	k	10^{-18}	atto	a
10^{2}	hecto	h	10^{-21}	zepto	z
10^{1}	deka	da	10^{-24}	yocto	y

SI Units & Unit Conversions

SI Base Units		
Mass	kilogram	*kg*
Length	meter	*m*
Time	second	*s*
Amount of Substance	mole	*mol*
Electric Current	ampere	*A*
Thermodynamic Temperature	kelvin	*K*
Luminous Intensity	candela	*cd*

Unit conversion

$T \propto M^a L^b g^c$

$s = (kg)^a (m)^b (m/s^2)^c$

1. Denote the target unit in the form of SI.
2. Denote the known unit in the form of SI.
3. List equations to make sure that the exponent corresponding to the same SI unit the identical.
4. Solve the system of equations.

$\Rightarrow \begin{cases} a=0 \\ b+c=0 \\ -2c=1 \end{cases} \Rightarrow \begin{cases} a=0 \\ b=\frac{1}{2} \\ c=-\frac{1}{2} \end{cases}$

$T \propto L^{\frac{1}{2}} g^{-\frac{1}{2}}$ $\quad T = 2\pi\sqrt{\frac{L}{g}}$

> **Exercise** A simple pendulum consists of a light inextensible string AB with length L , with the end A fixed, and a point mass M attached to B. The pendulum oscillates with a small amplitude, and the period of oscillation is T. It is suggested that T is proportional to the product of powers of M , L and g, where g is the acceleration due to gravity. Use dimensional analysis to find this relationship.

Significant Figures

Calculation Rules (skip)

（1）和差运算：运算结果有效数字的末位和参与运算的数据中末位数最高的对齐 小数点后位数.

（2）乘除运算：运算结果有效数字的位数和参与运算的数据中有效数字位数最少的相同. 有效数字

（3）乘方开方：运算结果有效数字的位数和底数的有效数字位数相同.

（4）三角函数：让角度变化一个最小分度值，结果差异产生在哪一位，运算结果就保留到哪一位.

Counting Numbers of Significant Figures

Basic Rules

1. Non-zero digits are always significant.
2. Any zeros between two significant digits are significant.
3. Trailing zeros after a decimal point are significant.
   - Example: 2.300 has 4 significant figures (2, 3, and two zeros).
4. Leading zeros before the first non-zero digit are not significant.
   - Example: 0.0025 has only 2 significant figures (2 and 5).
5. Trailing zeros in a whole number without a decimal point are generally not significant (unless otherwise indicated).
   - Example: 1500 has 2 significant figures (1 and 5); but if written as **1500.**, it has 4 significant figures.
6. When using scientific notation, all digits in the coefficient are significant.
   - Example: 4.00 × 10^2 has 3 significant figures.

Back-of-the-Envelope Calculation

> **Exercise**
>
> How many senior high school Chinese teachers are there in Shanghai?
>
> Example
>
> 1. According to the population survey, there are about 15 million people living in Shanghai with household registration.
> 2. Assuming the population follows the uniform distribution with age, then for teenagers aged from 16 to 18, the population is about $1.5 \times 10^7 \times \frac{3}{80} = 5.6 \times 10^5$.
> 3. However, not every teenager in this age group studies in senior high school. About half of the students drop out of school or go to vocational school after junior high school. Thus, there are about $5.6 \times 10^5 \times \frac{1}{2} = 2.8 \times 10^5$ senior high school students. In fact, the actual number is about 1.5×10^5, while the error is accepetable.
> 4. Suppose every Chinese teacher teaches two classes, each class with 40 students, then there are $\frac{2.8 \times 10^5}{2 \times 40} = 3.5 \times 10^3$ senior high school Chinese teachers.

Vectors and Basic Vector Operations

Scalar

- time, mass, length, volume, density of matter, electric charge, potential energy, pressure, kinetic/potential energy...
- defined by a single number

- Property:

$+$ or $-$	only compatible units allowed as arguments
$*$ or $/$	may involve quantities with different units
sin(...), ln(...), exp(...), ...	only dimensionless arguments allowed

Vector

- velocity, force, linear momentum, angular velocity, angular momentum, electric/magnetic field, electric current density...
- defined by both a magnitude and a direction
- Property:

1. The dot product of two non-zero vectors is zero **iff** the vectors are perpendicular,

$$\overline{u} \circ \overline{w} = 0 \Leftrightarrow \overline{u} \perp \overline{w}$$

2. The cross product is anticommutative,

$$\overline{u} \times \overline{w} = -\overline{w} \times \overline{u}$$

3. The cross product is a zero vector **iff** the two non-zero vectors are parallel (or antiparallel)

$\overline{u}=0$ $\overline{w}=0$

$$\overline{u} \times \overline{w} = 0 \Leftrightarrow \overline{u} \parallel \overline{w}$$

> **Exercise**
> Consider two vectors $\mathbf{u} = 3\hat{n}_x + 4\hat{n}_y$ and $\mathbf{w} = 6\hat{n}_x + 16\hat{n}_y$. Find the $\boxed{\text{components}}$ of the vector $\mathbf{w}$ that are, respectively, parallel and perpendicular to the vector $\mathbf{u}$.
>
> vector

Cartesian Coordinate System

$\vec{u}\cdot\vec{w} = |\vec{u}||\vec{w}|\cos\langle\vec{u},\vec{w}\rangle$

$\vec{w}_{pa} = |\vec{w}|\cos\langle\vec{u},\vec{w}\rangle \frac{\vec{u}}{|\vec{u}|} = \frac{\vec{u}\cdot\vec{w}}{|\vec{u}|^2}\vec{u}$

$= \frac{18+64}{5^2}(3\hat{n}_x+4\hat{n}_y)$

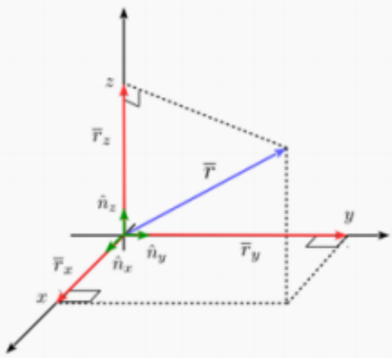
$\vec{w}_{per} = \vec{w} - \vec{w}_{pa}$

Definition

Defining fixed **unit vectors** (unit = of unit length) $\hat{n}_x$, $\hat{n}_y$, and $\hat{n}_z$ along the axes x, y, z, the position vector (or any other vector) can be represented as

$$\overline{r} = x\hat{n}_x + y\hat{n}_y + z\hat{n}_z.$$

The numbers x, y, z are called the Cartesian coordinates of the position vector.



Property

- mutually perpendicular
  $\hat{n}_x \circ \hat{n}_y = \hat{n}_y \circ \hat{n}_z = \hat{n}_z \circ \hat{n}_x = 0$
- unit length $|\hat{n}_x| = |\hat{n}_y| = |\hat{n}_z| = 1$, or equivalently
  $\hat{n}_x \circ \hat{n}_x = \hat{n}_y \circ \hat{n}_y = \hat{n}_z \circ \hat{n}_z = 1$
- For a right-handed system (note the cyclic permutation)

$$\begin{aligned} \hat{n}_x \times \hat{n}_y &= \hat{n}_z \\ \hat{n}_y \times \hat{n}_z &= \hat{n}_x \\ \hat{n}_z \times \hat{n}_x &= \hat{n}_y \end{aligned}$$

Basic Operation in Cartesian

Scalar Product

For $\overline{u} = (u_x, u_y, u_z)$ and $\overline{w} = (w_x, w_y, w_z)$, the dot product

$$\boxed{\overline{u} \circ \overline{w} = u_x w_x + u_y w_y + u_z w_z}$$

Vector Product

$$\begin{aligned} \overline{u} \times \overline{w} &= (u_y w_z - u_z w_y)\hat{n}_x + (u_z w_x - u_x w_z)\hat{n}_y + (u_x w_y - u_y w_x)\hat{n}_z \\ &= \begin{vmatrix} \hat{n}_x & \hat{n}_y & \hat{n}_z \\ u_x & u_y & u_z \\ w_x & w_y & w_z \end{vmatrix}. \end{aligned}$$

Differentiation

$$\begin{aligned} \boxed{\frac{d\overline{u}}{dt}} &= \frac{d}{dt}(u_x(t)\hat{n}_x + u_y(t)\hat{n}_y + u_z(t)\hat{n}_z) \\ &= \boxed{\dot{u}_x(t)\hat{n}_x + \dot{u}_y(t)\hat{n}_y + \dot{u}_z(t)\hat{n}_z} \end{aligned}$$

Integration

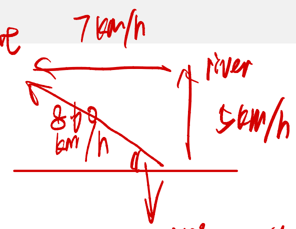
$$\int_{t_0}^{t_1} \overline{u} \mathrm{d}t = \left(\int_{t_0}^{t_1} u_x(t) \mathrm{d}t \right) \hat{n}_x + \left(\int_{t_0}^{t_1} u_y(t) \mathrm{d}t \right) \hat{n}_y + \left(\int_{t_0}^{t_1} u_z(t) \mathrm{d}t \right) \hat{n}_z$$

Exam Tip: Point out Frame of Reference!

> **Exercise** A river flows from south to north at 5 km/h. On this river, a boat is heading east to west, perpendicular to the current at 7 km/h. As viewed by an eagle hovering at rest over the shore, how fast and in what direction is this boat traveling?

① ② ☆ take ... as FOR

= velocity
magnitude
direction



35.54° north of west.

Reference

1. Tang Zenghao, 2024SU VP150 RC.
2. Mateusz Krzyzosiak, 2024SU VP150 Slides.