Aufgabe 1

$$A_1 = \begin{pmatrix} 2 & 2 & -3 \\ 0 & -1 & 3 \\ 0 & 0 & 0 \end{pmatrix} \in \mathbb{Q}^{4x3},\ b_1 = \begin{pmatrix} -8 \\ 5 \\ -4 \\ 0 \end{pmatrix} \in \mathbb{Q}^{4x1}.$$

$$\begin{aligned} 2x_3 = -4 &\iff x_3 = -0,5(-4) = 2 \\ x_2 + 3x_3 = 5 &\iff -x_2 + 3 \cdot 2 = 5 \\ &\iff -x_2 + 6 = 5. \\ &\iff x_2 = 1. \end{aligned}$$

$$\begin{aligned} 2x_1 + 2x_2 - 3x_3 = -8 & \\ &\iff 2x_1 + 2 \cdot 1 - 3 \cdot 2 = -8 \\ &\iff 2x_1 + 2 - 6 = -8 \\ &\iff 2x_1 = -1 \\ &\iff x_1 = -2 \end{aligned}$$

$$Sol(A_1, b_1) = \{ \begin{pmatrix} -2 \\ 1 \\ 2 \end{pmatrix} \}.$$

$$A_2 = \begin{pmatrix} 2 & 2 & -2 & 2 \\ 0 & -1 & 2 & -2 \\ 0 & 0 & 0 & -2 \end{pmatrix} \in \mathbb{F}_5^{3\times 4},\ b_2 = \begin{pmatrix} -1 \\ 1 \\ -2 \end{pmatrix} \in \mathbb{F}_5^{3\times 1}.$$

$$\begin{aligned} -2x_4 = -2 &\iff x_4 = 1 \\ -x_2 + 2x_3 - 2x_4 = 1 &\iff -x_2 + 2a - 2 = 1 \\ &\iff x_2 = 2a - 3 = 2a + 2. \end{aligned}$$

$$\begin{aligned} 2x_1 + 2x_2 - 2x_3 + 2x_4 = -1 & \\ &\iff 2x_1 + 2(2x + 2) - 2a + 2 = -1 \\ &\iff 2x_1 = 2a - 2 \\ &\iff x_1 = -a - 1. \end{aligned}$$

$$Sol(A_2, b_2) = \{ \begin{pmatrix} -a - 1 \\ 2a + 2 \\ a \\ 1 \end{pmatrix} | a \in \mathbb{F}_5 \}.$$

$$A_3 = \begin{pmatrix} 1 & 1 & 2 \\ 0 & 2 & -5 \\ 0 & 0 & 3 \end{pmatrix} \in \mathbb{F}_3^{3x3},\ b_3 = \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix} \in \mathbb{F}_3^{3x1}.$$

$$\left(\begin{array}{ccc|c} 1 & 1 & -1 & -1 \\ 0 & -1 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{array}\right).$$

$0x_1 + 0x_2 + 0x_3 = 1$

$\iff 0 + 0 + 0 = 1.$

$\iff 0 = 1...$

$Sol(A_3, b_3) = \{\}.$