May 15, 2023

Logistics/Expectations

Regular things

- 9am meetings Monday-Thurs
- Research journal each day

Goals/other things

- Chalk board talk explaining representation theory of wreath products
- Have a running annotated bibliography
- Potentially talk with Prof Pamela Harris at some point during the summer
- Work on developing good patterns for research and mathematical writing
- Have a list of grad schools to apply to and a solid draft of personal statement & resume. Also think
- Do fun math! But more specifically use parking functions as a set on which wreath products can act to help develop a sense of how to define local in the context of wreath products

Math

- **Representation theory of wreath products**

  Read more of Ceccherini-Silberstein, Scarabotti, and Tolli Representation Theory and Harmonic Analysis of Wreath Products to understand the irreducible representations of H wr G (Clifford theory, Gelfand pairs, understand equation on page 95).

  Some stray questions at the moment:

  - How does the strategy that I used for $G \wr_2$ work out to completion? (ie how do you decompose $\mathrm{Ind}_{GG}^{G \text{ wr } \mathbb{Z}_2} \rho \otimes \rho$ where ρ is an irreducible representation of G)
  - How does the representation theory change for $G \wr S_n$ if instead of the normal intermediate set of one element subsets, there is a different intermediary set X (like 2 element subsets)?

- **Parking Functions and related variations**

  Get to know these and their notation. Some questions:

  - How do you characterize the set of functions that are parking functions?

- How do different variations for the item describing the car's preference work? When does order of the cars matter?
- Make more solid the notation and the connection to representation theory

Keep an eye out for permutation statistic like things in this space so that we can work on characterizing those as local or not. (Current examples: how many cars get to park, how many cars are parked in their preferred spot, what is the sum (variance, range, mean etc) of the number of spots cars had to be displaced.

Could come up with more examples of these statistics before starting, and then later after starting. Also, one way to get to know parking functions and how an action would affect their value would be to think about if statistics that I come up with in brainstorming are affected by the action of the subgroup $1 \wr S_n$ (ie, does the order of the cars matter?)

- **Develop an understanding of how to charactarize spaces in a way that is finer than local**

  Connections: induced representations, multiplicity of trivial representation, the basis that you choose for your irreducible representation.

  To Prof O: Can we write out a slightly better set of notes for this bullet point as a starting point for thinking about these ideas at some point?

  One example to work through would be the looking at k element subsets acted on by S_n since apparently each increment of k adds an additional 1 to the multiplicity of the defining representation

- **Develop a notion of local for wreath products or just the smaller case of sets on which wreath products act**

  Starting points: eigenspaces for the graph given by a notion of adjacency.

  It would be good to think through how this would work for S_n acting on 1 element subsets. I wonder if you get the same notion of localness if you say that two one element subsets are adjacent if they differ by 1

- **Reading**
  - Hannah's Thesis
  - Ian's Thesis
  - papers from Prof O

One stray question

Do you have a good way of having a template in overleaf which lets you have a consistent set of commands that you have defined and usepackages? (I kept trying to use a couple of commands that I had from writing papers earlier, and then remembering that those commands were not in this document)