1 Organized by Date

5/15/2023 Introduction day with questions

5/16/2023 Defining displacement, argument for order invariance of some properties

5/22/2023 Trick for checking if a preference list is a parking function, trick for checking the defect of a preference list, trick for calculating displacement of parking function, generating a random parking function. Stanley activities [3], Pamela Harris video

5/23/2023 Representation theory of wreath products

5/24/2023 Reading paper which enumerated defect d parking functions [1]. Decomposing preference lists as module graded by defect as an S_m module. Uses intermediate of tabloid spaces.

5/25/2023 Connecting preference lists to tabloid spaces/induced representations of cosets of young subgroups

5/30/2023 Notes on Tabloid spaces and Kostka numbers. Questions based on thinking of things in terms of tabloid spaces. Some incorrect computations for the S_m module decomposition.

6/02/2023 *code*

6/05/2023 Connecting the ideas of shuffle and the A function from [2] to break points, prime parking functions, and concatenating.

6/07/2023 Calculate $N_d(\lambda)$ for partitions of the form $1^{n-k}k$. Relating preference lists to permutations with circular parking. Visualization descriptions for $n = m = 3$. Clear outline of what groups can act on preference lists. Investigating maps to permutations/partial permutations can behave nicely with the group action. List of useful ideas/theory to learn more/keep in mind (two sided cosets, double cosets, mackey theory, gelfand pairs). Looking at number of circular preference lists which result in particular permutations for $n = m = 3$ with harmonic analysis on S_n in mind

6/12/2023 Looking at number of parking functions which result in particular permutations for $n = m = 4$ with harmonic analysis on S_n in mind as well as relation to circular preference lists. Writing out ideas in terms of double cosets. Writing down some useful maps and notes. Functions constant on cosets of the diagonal subgroup can be decomposed using only characters whose indices add to $0(modm)$. Handwritten notes on decomposition of some functions of interest for $n = m = 3$. Notes on the structure of the space as relates to orbits of S_m initially written to count (something that is easy to count). Code to compute coefficients. Example decompositions for $n = m = 4$ with patterns describing relation of $n \neq m$ to $n = m$

6/14/2023 Symmetric group S_n acting on characters χ_α of C_n^m. Developing adapted basis from coded examples. Notes on size of double cosets. Writing change of basis for $n = m = 3$ by hand

6/15/2023 Working out $N_d(\lambda)$ for a 2 part partition. Decomposition of $[n]^m$ as an S_m module.

6/19/2023 Reading Stanley and Yin

6/20/2023 Sampling from defect d preference lists, generating function for last break point, preliminaries on p-prime parking functions.

6/22/2023 Fourier transform of a sine wave, summary of convolution/multiplication for random walks

6/29/2023 Reading about quicksort. Working on counting p prime parking functions, Abel's identity, triangle of values. Starting to compare to [1]. Desmos plot of proportion of defect d-preference list. Desmos comparing two different sampling methods. Chaos notes from thesis reading

7/06/2023 Random Walks. Parallel result to Ian's thesis/Hultman. Averaging over orbits of S_m using character coefficients. The dimensions of functions constant on cosets of D and orbits of S_m is counted by the Catalan numbers (using relation to parking functions). Tensor product of sin. Using prime parking functions as building blocks for computation of averaged statistics by hand

7/07/2023 Combinatorial proof counting the number of parking functions for $n > m$, relating a simple expression to an expression iterating over partitions. Note that a similar argument can be used to relate the module structure of parking functions to that of prime parking functions. Some relevant maps from parking functions to other objects

7/10/2023 Writing code for decomposing functions into modified bases.

7/12/2023 **On indicator functions**, their basic properties, useful results that can follow, conjectures based on observations

7/13/2023 Relation between character decomposition of defect and displacement for circular parking procedures

7/24/2023 Understanding sampling from prime parking functions

8/2023 A combinatorial proof relating the number of prime parking functions to the first break point for parking functions. Counting p-prime parking functions with inclusion exclusion based on method 2 (correct edition with only one incomplete piece of logic). Crossing point for efficiency of the two different methods for sampling from p-prime parking functions

2 Organized by Topic

Relating to Sampling

5/24/2023 Reading paper which enumerated defect d parking functions [1].

5/22/2023 Trick for checking if a preference list is a parking function, trick for checking the defect of a preference list, trick for calculating displacement of parking function, generating a random parking function. Stanley activities [3], Pamela Harris video

6/05/2023 Connecting the ideas of shuffle and the A function from [2] to break points, prime parking functions, and concatenating.

6/20/2023 **Sampling from defect d preference lists**, generating function for last break point, preliminaries on p-prime parking functions.

6/29/2023 Working on counting p prime parking functions, Abel's identity, triangle of values. Starting to compare to [1]. Desmos plot of proportion of defect d-preference list. Desmos comparing two different sampling methods.

7/24/2023 Understanding sampling from prime parking functions

8/2023 A combinatorial proof relating the number of prime parking functions to the first break point for parking functions. Counting p-prime parking functions with inclusion exclusion based on method 2 (correct edition with only one incomplete piece of logic). Crossing point for efficiency of the two different methods for sampling from p-prime parking functions

Relating to harmonic analysis

6/12/2023 Looking at number of parking functions which result in particular permutations for $n = m = 4$ with harmonic analysis on S_n in mind as well as relation to circular preference lists. Writing out ideas in terms of double cosets. Writing down some useful maps and notes. Functions constant on cosets of the diagonal subgroup can be decomposed using only characters whose indices add to $0(modm)$. Handwritten notes on decomposition of some functions of interest for $n = m = 3$. Notes on the structure of the space as relates to orbits of S_m initially written to count (something that is easy to count). Code to compute coefficients. Example decompositions for $n = m = 4$ with patterns describing relation of $n \neq m$ to $n = m$

6/14/2023 Symmetric group S_n acting on characters χ_α of C_n^m. Developing adapted basis from coded examples. Notes on size of double cosets. Writing change of basis for $n = m = 3$ by hand

7/10/2023 Writing code for decomposing functions into modified bases.

7/12/2023 **On indicator functions**, their basic properties, useful results that can follow, conjectures based on observations

7/13/2023 Relation between character decomposition of defect and displacement for circular parking procedures

Random Walks specifically

6/22/2023 Fourier transform of a sine wave, summary of convolution/multiplication for random walks

7/06/2023 Random Walks. Parallel result to Ian's thesis/Hultman. Averaging over orbits of S_m using character coefficients. The dimensions of functions constant on cosets of D and orbits of S_m is counted by the Catalan numbers (using relation to parking functions). Tensor product of sin. Using prime parking functions as building blocks

Relating to Module Decomposition

5/24/2023 Decomposing preference lists as module graded by defect as an S_m module. Uses intermediate of tabloid spaces.

5/25/2023 Connecting preference lists to tabloid spaces/induced representations of cosets of young subgroups

5/30/2023 Notes on Tabloid spaces and Kostka numbers. Questions based on thinking of things in terms of tabloid spaces. Some incorrect computations for the S_m module decomposition.

6/15/2023 Working out $N_d(\lambda)$ for a 2 part partition. Decomposition of $[n]^m$ as an S_m module.

7/07/2023 Combinatorial proof counting the number of parking functions for $n > m$, relating a simple expression to an expression iterating over partitions. Note that a similar argument can be used to relate the module structure of parking functions to that of prime parking functions. Some relevant maps from parking functions to other objects

Ideas/theory to lean on

5/15/2023 Introduction day with questions

5/23/2023 Representation theory of wreath products

5/30/2023 Notes on Tabloid spaces and Kostka numbers.

6/07/2023 List of useful ideas/theory to learn more/keep in mind (two sided cosets, double cosets, mackey theory, gelfand pairs)

References

[1] Peter J. Cameron, Daniel Johannsen, Thomas Prellberg, and Pascal Schweitzer. Counting defective parking functions, 2008.

[2] Persi Diaconis and Angela Hicks. Probabilizing parking functions. *Advances in Applied Mathematics*, 89:125–155, 2017.

[3] Richard P. Stanley and Sergey Fomin. *Enumerative Combinatorics*, volume 2 of *Cambridge Studies in Advanced Mathematics*. Cambridge University Press, 1999.