May 22, 2023

Typed conclusions from the past couple of days

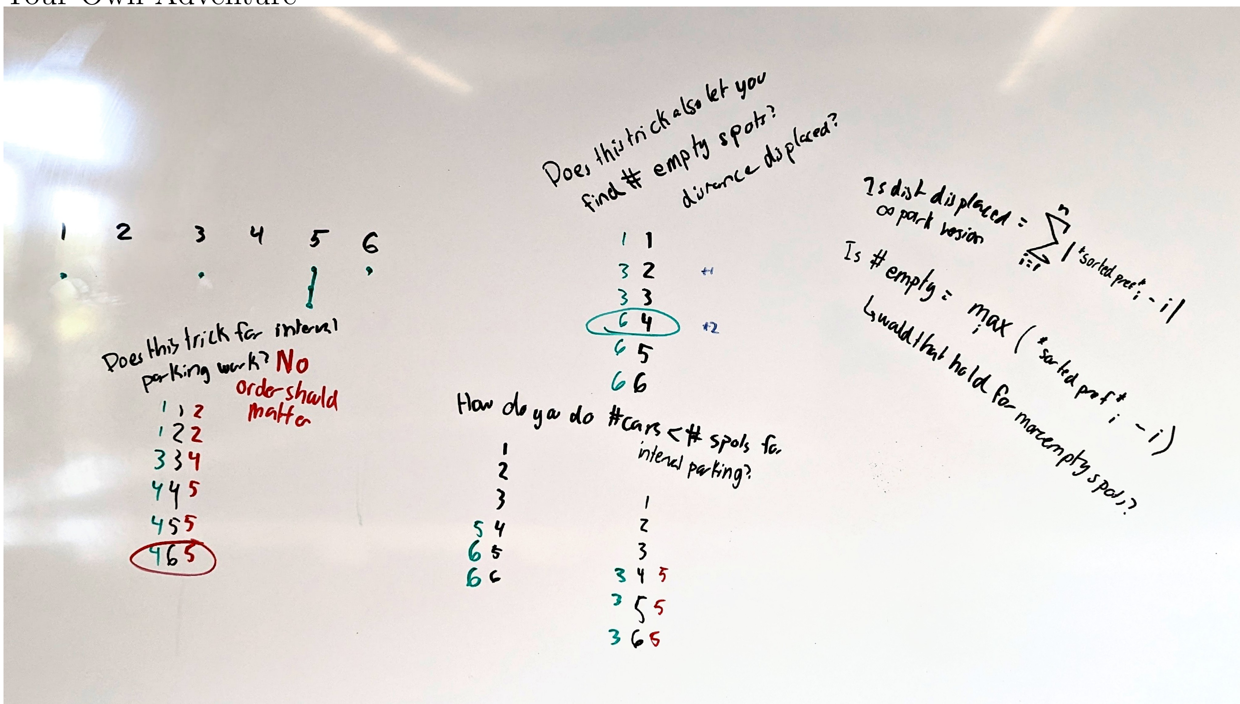
- Order doesn't matter for parkability and total displacement (inclusive) if every car would have the same set of back up spots upon missing their spot
- To check the defect of a preference list, there is a simple proceedure. First, order the preferences (reordering doesn't change parkability). Find the maximum difference $\pi_i - i$. This is the defect. If the function is a parking function, then $\sum_{i=1}^{n} i - \pi_i$ is the total displacement, since it would be the displacement for the cars parking in this order, and total displacement is order invariant.
- To generate a random parking function, take an element of C_n^{n+1}. If we let the diagonal subgroup be the subgroup generated by the all 1s element, then there must be exactly 1 parking function in each coset

Future directions/things I didn't get done/questions

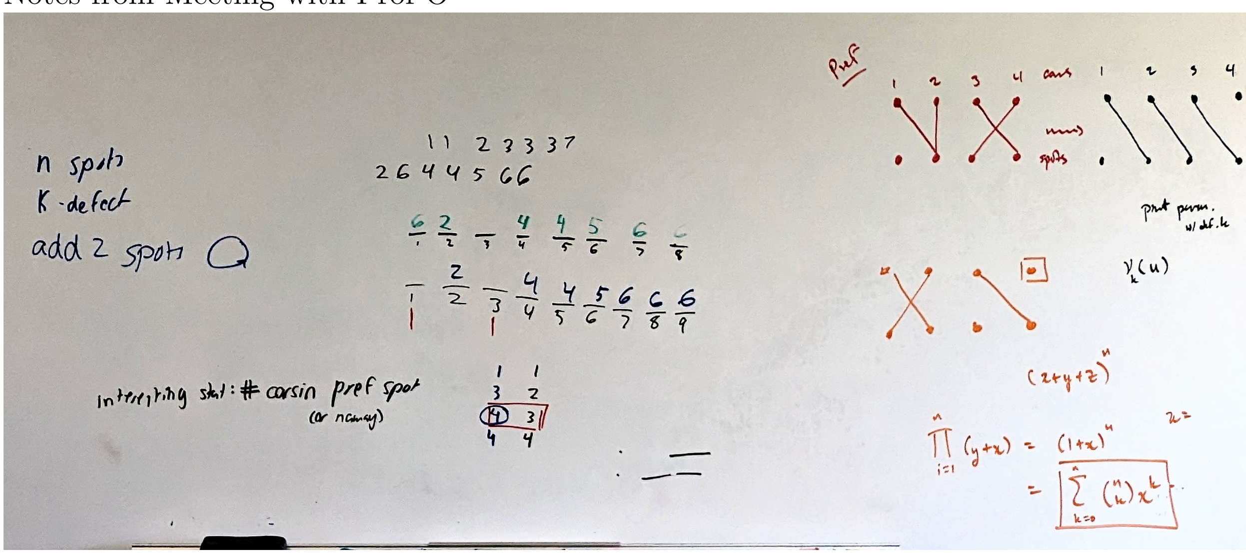
- Write down solutions to Stanley's excercises + extensions so far. (I thought I was going to work on this, but it was more of a reading/thinking/notation day than a writing up problems day) Continue thinking especially regarding potential extensions to non-parking functions
- How do you sample a 1-defect parking function at random? How do you count the number of 1-defect parking functions? (or d-defect, but you get the point)
- Irreducible representations of $C_n \wr S_m$
- Last 2 irreducible representations of $(\mathbb{Z}_2 \wr \mathbb{Z}_2) \wr \mathbb{Z}_2$
- From the Rhoades 2012 paper understand section 1.1, equation 1.1 and 1.2. This is the character for S_n resulting from the representation which results from viewing parking functions as an S_n module. It could be fun to see if along the way to understanding how they came up with this character and decomposition into irreducibles, maybe there could be some way of extending this. Can this be done for parking functions with defects? Relation to wreath products?
- Read more!

Photos from board work (chronological)

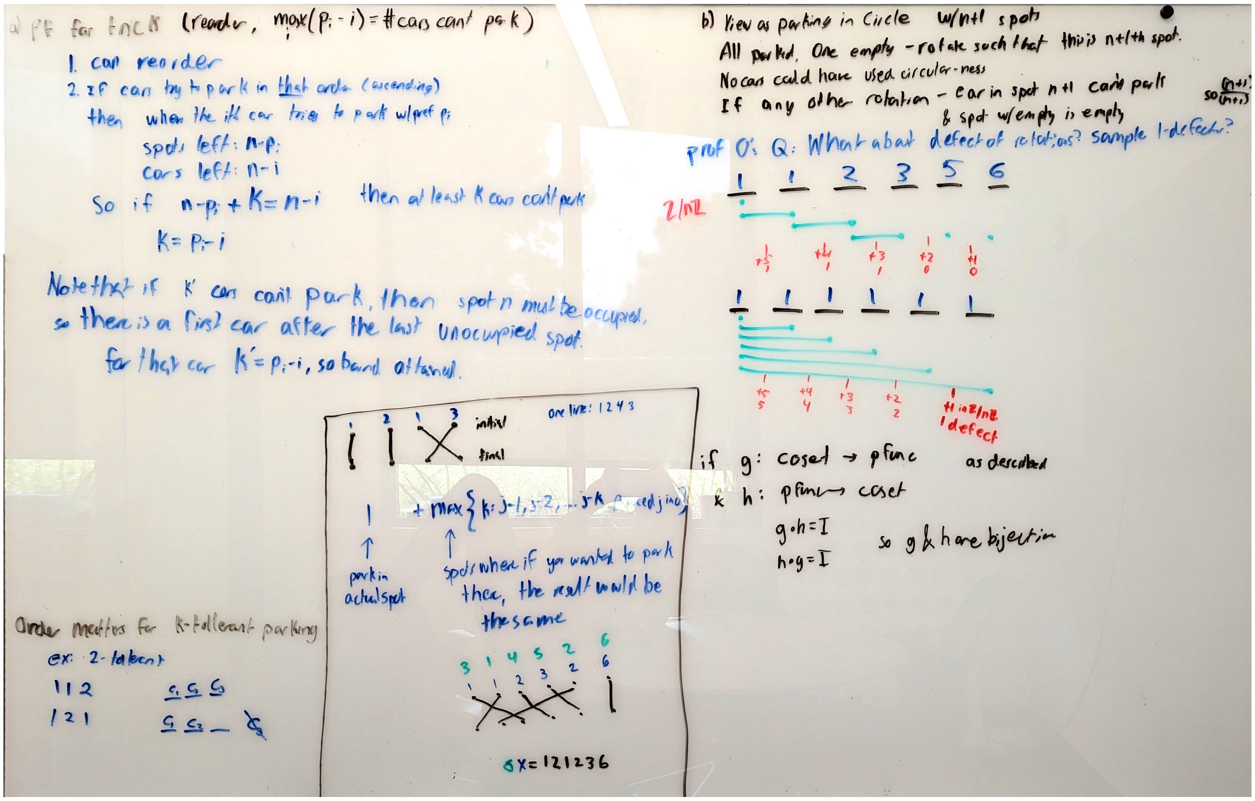
Whiteboard notes/investigation from watching Pamela Harris's "Parking Functions: Choose Your Own Adventure"



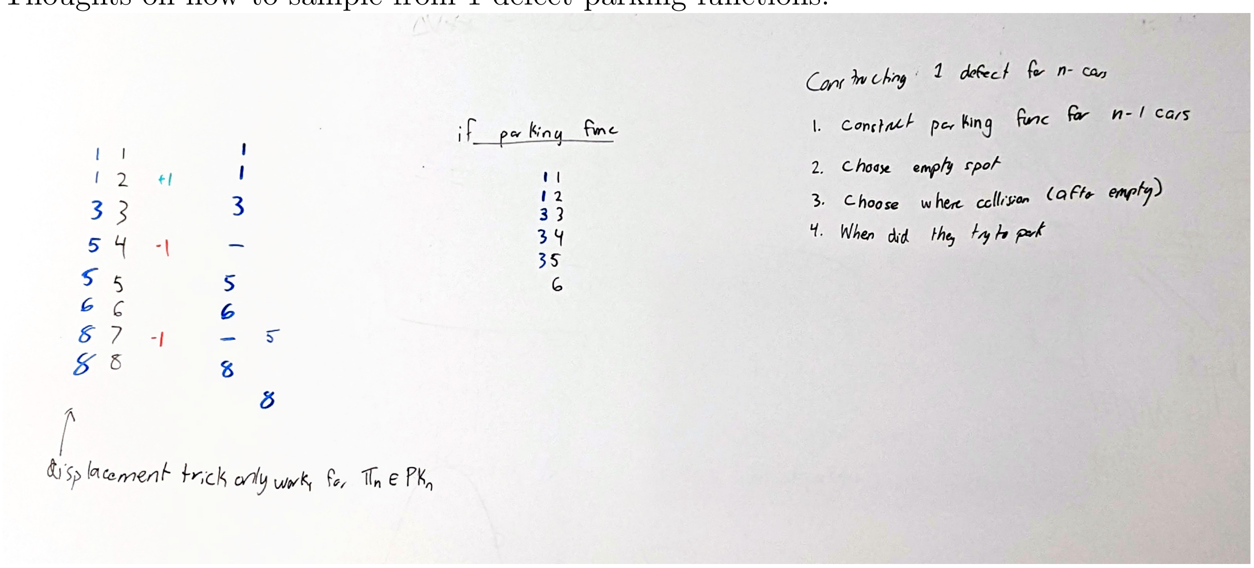
Notes from Meeting with Prof O



Working on problems by Stanley in makerspace



Determining that the difference of the sorted preference list and the index can NOT directly indicate the total displacement in general. However, for a parking function, it does. Thoughts on how to sample from 1-defect parking functions.



At the end of the day after working on laptop troubles (my data should be recoverable!) I wanted to think about wreath products a little bit. The wreath product that is most applicable in this case is probably $C_n \wr S_n$ so I was working on better understanding the representation theory of this wreath product by determining the conjugacy classes and irreducible representations for an example $\mathbb{Z}_2 \wr S_3$. I think that the two induced representations $\mathrm{Ind}_{\mathbb{Z}_2^3}^{\mathbb{Z}_2 \wr S_3} \chi_{000}$ and $\mathrm{Ind}_{\mathbb{Z}_2^3}^{\mathbb{Z}_2 \wr S_3} \chi_{000}$ will be simultaneously diagonalized by the same matrix, so each give 3 irreducible representations of dimension 1,1,2. For $\mathrm{Ind}_{\mathbb{Z}_2^3}^{\mathbb{Z}_2 \wr S_3} \chi_{001}$ and $\mathrm{Ind}_{\mathbb{Z}_2^3}^{\mathbb{Z}_2 \wr S_3} \chi_{011}$, each of these are reducible representations. I *think* that each of these representations could be decomposed into 2 distinct representations, corresponding in some way to the partition $= (2, 1)$. I need to work out the details of this, but this hunch is coming from the way that this played out for $G \wr \mathbb{Z}_2$. I think that I might need to also mess around with $\mathbb{Z}_2 \wr S_4$ particularly thinking about the V_4 subgroup (maybe I won't need to do this, but I think that it would probably be helpful to refine some of the distinctions if that doesn't happen by working through the details).