List of Preference List Funciton ideas

Function	Description
Parkability	The number of cars that can park
Defect number	The number of cars that cannot park (cars - parkability)
Displacement	The sum of the displacements for each car
Inclusive displacement	cars that don't park count as displaced to the end
Infinite displacement	cars that don't park count as if they had parked in an infinite parking lot
Exclusive displacement	cars that don't park are completely ignored
0-displaced	How many cars got their preferred spot?
d-displaced	How many cars were displaced exactly d spots? (must choose type)
m-moment of displacement	Must choose type of displacement (like Ian's thesis)
Partial Permutation	Extensions of descents, inversions, longest increasing subsequence, etc

Some more distributions that we have to work with (preference-list-statistic style):

- Which cars weren't able to park? (their preferences? their indices? First car not able to park?)
- Which spots are unoccupied? (First? Last?)

Could take inspiration from any permutation statistics on partial permutations, since the ultimate arrangement of the cars is a partial permutation. So given a preference list, you can determine a partial permutation (this function is NOT a homomorphism)

Could take inspiration from Diaconis, Hicks, and consider questions like for an arbitrary element of k-defects, what is the first preference? I wonder if there is a nice way of re-framing this question in terms of values of the parkability function on cosets of a particular subgroup (and if that is helpful at all)

Other notation/choices

Notation/language

- A preference list refers to the list of preferences.
  Notation: $PR_{n,m}$ for n cars and m spots. PR_n just means n cars and n spots
- A preference list with one element subset preferences $\pi \in PR_{n,m}$ is an element $\pi = (\pi_1, \pi_2, \cdots \pi_n)$ where each $\pi_i \in \mathbb{Z}$, $1 \leq \pi_i \leq m$
- A parking function is a preference list where all of the cars can park.
  Notation: PF_n (Diaconis, Hicks)

- Given a preference list π, $filling(\pi)$ indicates where each car ended up parking. $parking(\pi)$ indicates what car ended up parking in each parking spot. These are permutations in the case of parking functions and partial permutations in the more general case. In teh case of permutations, they are inverses of each other. (from Armstrong, Garsia, Haglund, Roades, and Sagan)

More choices that are still kind of open

- **Preference Type** For the above, the preference list statistics explored were for 1-element subsets, but there could also be preference lists for other preference types. (k-element subsets, 1-element subsets which will only go an additional k spots before giving up, posets, etc). I think that the next most interesting kind of preference to investigate might be 1-element subsets which will give up after k spots
- What group should we be working with? Options:
  - C_m^n
  - $C_m^n \wr S_n$
  - S_m^n
  - $S_m^n \wr S_n$

  Thoughts: I think that for 1 element subsets, maybe working with C_m^n rather than S_m^n would be interesting. I would be curious to keep wreath products in mind because of the way in which sometimes order matters, and sometimes order doesn't matter. It would be interesting if some notion of local on a wreath product could capture that information