To Do/Ideas to think about

From Yesterday

- Answer questions about maps - also think about if/how the correspondence between PK_n and PC_{n+1} works when $n \neq m$
- Write code to figure out distance from uniform distribution for distribution of desired spots?
- Look at $N_d(\lambda)$ for 2 row partitions (maybe even just $n-2, 2$). Can you take advantage of the fact that $N_d(\lambda)$ counts the double cosets $S_m x S_m$ where x has type λ and defect d
- Read (continued) - papers from Prof O
- Read - regarding Mackey theory

From meeting with Prof O

- Play with bases for characters - is there an adapted basis which can collapse dimensions further? To find what basis might make sense, you could look at what characters always appear grouped for functions of interest.
- Think about indicator functions. One possible candidate would be $1_{I,D}$ which is 1 on preference lists which send the ith car d cars past it's preference. (ie specifying the displacement of the ith car

How does Symmetric group S_n act on characters χ_α of C_n^m

I didn't do a good job explaining why this was true in our meeting. I essentially started at the middle and things got somewhat muddled. So here it is in a more linear manner.

Let χ_a be the character associated with the group element a, so $\chi_a(g) = \omega^{\langle a,g\rangle}$, where $\omega = e^{2\pi i/n}$ and the inner product is defined as follows:

$$\langle a, g\rangle = \sum_{i=0}^{m} a_i g_i$$

I want to show that $\sigma \cdot \chi_a = \chi_{\sigma a}$. This can be shown by the following computation

$$\begin{aligned}
\sigma \cdot \chi_a(g) &= \chi_a(\sigma^{-1} \cdot g) && \text{definition of convolution} \\
&= \omega^{\langle a, \sigma^{-1} g\rangle} && \text{definition of } \chi_a \\
&= \omega^{\sum_{i=0}^{m} a_i (\sigma^{-1} \cdot g)_i} && \text{definition of inner product} \\
&= \omega^{\sum_{j=0}^{m} (\sigma \cdot a)_j g_j} && \text{reindexing summation} \\
&= \omega^{\langle \sigma \cdot a, g_j\rangle} && \text{definition of inner product} \\
&= \chi_{\sigma \cdot a}(g) && \text{definition of } \chi_{\sigma \cdot a}
\end{aligned}$$

The key point in this chain of equality is the reindexing of the summation.

I wrote some code to figure out what elements are never differentiated from eachother. It's not pretty code, but here's the output. The initial list gives the values of the character decomposition of the number of the function counting the number of cars circularly displaced by exactly i. The list afterwards gives the set of characters.

3 cars 3 spots

```
In [370]: by_value(group_characters(3,3), 3,3, hspace = 45)
6
[-9.+0.j  9.+0.j  0.+0.j]                        : 210, 120,
[-4.5-2.6j  0. +5.2j  4.5-2.6j]                  : 102, 012,
[-4.5+2.6j  0. -5.2j  4.5+2.6j]                  : 201, 021,
[ 4.5-2.6j -4.5-2.6j -0. +5.2j]                  : 222,
[ 4.5+2.6j -4.5+2.6j -0. -5.2j]                  : 111,
[54.+0.j 18.+0.j  9.+0.j]                        : 000,
```

4 cars 4 spots

```
    In [369]: by_value(group_characters(4,4), 4,4, hspace = 45)
25
[-64.+0.j  48.+0.j  16.+0.j   0.+0.j]         : 3100, 1300,
[-64.+0.j  80.+0.j -16.+0.j   0.+0.j]         : 2200,
[-48.-16.j  16.+32.j  32.-16.j   0. +0.j]     : 1030, 0130,
[-48.+16.j  16.-32.j  32.+16.j   0. +0.j]     : 3010, 0310,
[-32.+0.j  16.+0.j  16.+0.j   0.+0.j]         : 2020, 0220,
[-24.+0.j  24.+0.j -24.+0.j  24.+0.j]         : 2002, 0202, 0022,
[-20.-20.j -20.+20.j  20.+20.j  20.-20.j]     : 1003, 0103, 0013,
[-20.+20.j -20.-20.j  20.-20.j  20.+20.j]     : 3001, 0301, 0031,
[-12.-20.j  20.-12.j  12.+20.j -20.+12.j]     : 1111,
[-12.+20.j  20.+12.j  12.-20.j -20.-12.j]     : 3333,
[-8.+0.j  8.+0.j -8.+0.j  8.+0.j]             : 3212, 2312, 3122, 1322, 2132, 1232,
[-4.-4.j -4.+4.j  4.+4.j  4.-4.j]             : 2213, 2123, 1223,
[-4.+4.j -4.-4.j  4.-4.j  4.+4.j]             : 3221, 2321, 2231,
[ 4.-4.j  4.+4.j -4.+4.j -4.-4.j]             : 3311, 3131, 1331,
[ 4.+4.j  4.-4.j -4.-4.j -4.+4.j]             : 3113, 1313, 1133,
[ 8.-16.j -8.+16.j  8.-16.j -8.+16.j]         : 3302, 3032, 0332,
[ 8.+0.j -8.+0.j  8.+0.j -8.+0.j]             : 2222,
[ 8.+16.j -8.-16.j  8.+16.j -8.-16.j]         : 1102, 1012, 0112,
[ 12. -4.j  -4.-12.j -12. +4.j   4.+12.j]     : 3203, 2303, 3023, 0323, 2033, 0233,
[ 12. +4.j  -4.+12.j -12. -4.j   4.-12.j]     : 2101, 1201, 2011, 0211, 1021, 0121,
[ 16.-16.j -16. +0.j   0.+16.j   0. +0.j]     : 3230, 2330,
[ 16.+16.j -16. +0.j   0.-16.j   0. +0.j]     : 2110, 1210,
[ 32. +0.j -48.-32.j  16.+32.j   0. +0.j]     : 3320,
[ 32. +0.j -48.+32.j  16.-32.j   0. +0.j]     : 1120,
[640.+0.j 208.+0.j 112.+0.j  64.+0.j]         : 0000,
```

5 cars 5 spots

```
    In [368]: by_value(group_characters(5,5), 5,5, hspace = 75)
99
[-625. -0.j  341.8+0.j  202.3-0.j   80.9+0.j   -0. +0.j]                      : 41000, 14000,
[-625. -0.j  733.2-0.j  -77.3-0.j  -30.9-0.j   -0. +0.j]                      : 32000, 23000,
[-538.6-118.9j  169.1+237.8j  288.6-118.9j   80.9  -0.j     0.   -0.j ]       : 10400, 01400,
[-538.6+118.9j  169.1-237.8j  288.6+118.9j   80.9  +0.j     0.   +0.j ]       : 40100, 04100,
[-398.9 -73.5j  280.9+146.9j  148.9 -73.5j  -30.9  -0.j    -0.   -0.j ]       : 20300, 02300,
[-398.9 +73.5j  280.9-146.9j  148.9 +73.5j  -30.9  +0.j    -0.   +0.j ]       : 30200, 03200,
[-368.4-210.5j  -74.3+258.1j  264.8+115.4j  178. -163.j     0.   -0.j ]       : 10040, 01040, 00140,
[-368.4+210.5j  -74.3-258.1j  264.8-115.4j  178. +163.j    -0.   -0.j ]       : 40010, 04010, 00410,
[-256.6-31.5j  149.3+60.9j  -14.8-27.2j  122.  -2.1j   -0.  +0.j ]            : 20030, 02030, 00230,
[-256.6+31.5j  149.3-60.9j  -14.8+27.2j  122.  +2.1j   -0.  -0.j ]            : 30020, 03020, 00320,
[-208.9  -9.1j  302.3-278.4j  101.1+358.8j -194.5 -71.3j   -0.   -0.j ]       : 11120,
[-208.9  +9.1j  302.3+278.4j  101.1-358.8j -194.5 +71.3j   -0.   +0.j ]       : 44430,
[-175.8 -57.1j  108.7+149.6j    0. -184.9j -108.7+149.6j  175.8 -57.1j]       : 20003, 02003, 00203, 00023,
[-175.8 +57.1j  108.7-149.6j   -0. +184.9j -108.7-149.6j  175.8 +57.1j]       : 30002, 03002, 00302, 00032,
[-136.7-188.1j -221.2 +71.9j   -0. +232.5j  221.2 +71.9j  136.7-188.1j]       : 10004, 01004, 00104, 00014,
[-136.7+188.1j -221.2 -71.9j   -0. -232.5j  221.2 -71.9j  136.7+188.1j]       : 40001, 04001, 00401, 00041,
[ -45.2-62.2j   93.4+86.j  -101.1+14.7j   53. -38.5j   -0.  -0.j ]            : 43120, 34120, 41320, 14320, 31420, 13420,
[-45.2 -3.5j   9.5 -0.j   -0. +18.2j  35.7-14.7j   0.  +0.j ]                 : 33130, 31330, 13330,
[-45.2 +3.5j   9.5 +0.j   -0. -18.2j  35.7+14.7j   0.  +0.j ]                 : 42220, 24220, 22420,
[ -45.2+62.2j   93.4-86.j  -101.1-14.7j   53. +38.5j   -0.  +0.j ]            : 42130, 24130, 41230, 14230, 21430, 12430,
[-43.1-10.9j  28.5+34.1j  -3. -44.3j -23.7+37.6j  41.3-16.5j]                 : 33103, 31303, 13303, 33013, 30313, 03313, 31033, 13033, 30133, 03133, 10333, 01333,
[-43.1+10.9j  28.5-34.1j  -3. +44.3j -23.7-37.6j  41.3+16.5j]                 : 42202, 24202, 22402, 42022, 24022, 40222, 04222, 20422, 02422, 22042, 20242, 02242,
[-41.1-38.5j  22.7+25.1j -38.6+57.5j  57. -44.1j   0.  -0.j ]                 : 22240,
[-41.1+38.5j  22.7-25.1j -38.6-57.5j  57. +44.1j   0.  +0.j ]                 : 33310,
[-31.9-46.2j  34. -44.6j  53. +18.6j  -1.3+56.1j -53.8+16.1j]                 : 21101, 12101, 11201, 21011, 12011, 20111, 02111, 10211, 01211, 11021, 10121, 01121,
[-31.9+46.2j  34. +44.6j  53. -18.6j  -1.3-56.1j -53.8-16.1j]                 : 44304, 43404, 34404, 44034, 40434, 04434, 43044, 34044, 40344, 04344, 30444, 03444,
[-27.1 -2.9j -11.2+24.9j  20.2+18.3j  23.7-13.6j  -5.6-26.7j]                 : 44214, 42414, 24414, 44124, 41424, 14424, 42144, 24144, 41244, 14244, 21444, 12444,
[-27.1 +2.9j -11.2-24.9j  20.2-18.3j  23.7+13.6j  -5.6+26.7j]                 : 43111, 34111, 41311, 14311, 31411, 13411, 41131, 14131, 11431, 31141, 13141, 11341,
[-18.1-40.3j  -9. +43.2j  32.7-29.7j -43.9 +4.8j  38.3+22.j ]                 : 41113, 14113, 11413, 11143,
[-18.1 -5.9j  11.2+15.4j   0. -19.j  -11.2+15.4j  18.1 -5.9j]                 : 42103, 24103, 41203, 14203, 21403, 12403, 42013, 24013, 40213, 04213, 20413, 02413,
                                                                                41023, 14023, 40123, 04123, 10423, 01423, 21043, 12043, 20143, 02143, 10243, 01243,

[-18.1 +5.9j  11.2-15.4j   0. +19.j  -11.2-15.4j  18.1 +5.9j]                 : 43102, 34102, 41302, 14302, 31402, 13402, 43012, 34012, 40312, 04312, 30412, 03412,
                                                                                41032, 14032, 40132, 04132, 10432, 01432, 31042, 13042, 30142, 03142, 10342, 01342,

[-18.1+40.3j  -9. -43.2j  32.7+29.7j -43.9 -4.8j  38.3-22.j ]                 : 44412, 44142, 41442, 14442,
[-17.3-100.7j  65.5  -0.j   -0.  +76.9j -48.2 +23.8j  -0.   +0.j ]            : 21110, 12110, 11210,
[-17.3 -5.6j -18.4+20.3j  38.6-23.8j  -3.  +9.1j  -0.  +0.j ]                 : 32140, 23140, 31240, 13240, 21340, 12340,
[-17.3 +5.6j -18.4-20.3j  38.6+23.8j  -3.  -9.1j  -0.  -0.j ]                 : 43210, 34210, 42310, 24310, 32410, 23410,
[-17.3+100.7j  65.5  +0.j   -0.  -76.9j -48.2 -23.8j   0.   -0.j ]            : 44340, 43440, 34440,
[-14.6-23.2j -26.6 +6.7j  -1.8+27.4j  25.5+10.2j  17.6-21.1j]                 : 22204, 22024, 20224, 02224,
[-14.6+23.2j -26.6 -6.7j  -1.8-27.4j  25.5-10.2j  17.6+21.1j]                 : 33301, 33031, 30331, 03331,
[-10.4 -4.8j  11.2 -2.2j  -7.7 +8.4j   1.3-11.3j   5.6 +9.9j]                 : 32212, 23212, 22312, 32122, 23122, 31222, 13222, 21322, 12322, 22132, 21232, 12232,
[-10.4 +4.8j  11.2 +2.2j  -7.7 -8.4j   1.3+11.3j   5.6 -9.9j]                 : 43323, 34323, 33423, 43233, 34233, 42333, 24333, 32433, 23433, 33243, 32343, 23343,
[ -6.9 -9.5j -11.2 +3.6j  -0. +11.8j  11.2 +3.6j   6.9 -9.5j]                 : 32104, 23104, 31204, 13204, 21304, 12304, 32014, 23014, 30214, 03214, 20314, 02314,
                                                                                31024, 13024, 30124, 03124, 10324, 01324, 21034, 12034, 20134, 02134, 10234, 01234,

[-6.9-1.4j -3.5+6.1j  4.8+5.2j  6.4-2.9j -0.8-7.j ]                           : 33324, 33234, 32334, 23334,
[-6.9+1.4j -3.5-6.1j  4.8-5.2j  6.4+2.9j -0.8+7.j ]                           : 32221, 23221, 22321, 22231,
[ -6.9 +9.5j -11.2 -3.6j   0. -11.8j  11.2 -3.6j   6.9 +9.5j]                 : 43201, 34201, 42301, 24301, 32401, 23401, 43021, 34021, 40321, 04321, 30421, 03421,
                                                                                42031, 24031, 40231, 04231, 20431, 02431, 32041, 23041, 30241, 03241, 20341, 02341,

[ -5.6 -9.9j  -1.3+11.3j   7.7 -8.4j -11.2 +2.2j  10.4 +4.8j]                 : 22213, 22123, 21223, 12223,
[ -5.6 +9.9j  -1.3-11.3j   7.7 +8.4j -11.2 -2.2j  10.4 -4.8j]                 : 43332, 34332, 33432, 33342,
[ -2.1-18.3j  16.8 -7.7j  12.5+13.6j  -9. +16.1j -18.1 -3.6j]                 : 44331, 43431, 34431, 43341, 34341, 33441,
[ -2.1+18.3j  16.8 +7.7j  12.5-13.6j  -9. -16.1j -18.1 +3.6j]                 : 22114, 21214, 12214, 21124, 12124, 11224,
[  2.1-90.8j  51.6+74.7j -85.7-30.1j  87. -26.j  -55.1+72.2j]                 : 11102, 11012, 10112, 01112,
[  2.1+90.8j  51.6-74.7j -85.7+30.1j  87. +26.j  -55.1-72.2j]                 : 44403, 44043, 40443, 04443,
[  5.6-26.7j -23.7-13.6j -20.2+18.3j  11.2+24.9j  27.1 -2.9j]                 : 31114, 13114, 11314, 11134,
[  5.6+26.7j -23.7+13.6j -20.2-18.3j  11.2-24.9j  27.1 +2.9j]                 : 44421, 44241, 42441, 24441,
[  6.9-15.4j   3.5+16.5j -12.5-11.3j  16.8 +1.8j -14.6 +8.4j]                 : 44322, 43422, 34422, 44232, 42432, 24432, 43242, 34242, 42342, 24342, 32442, 23442,
[ 6.9-2.2j -4.3+5.9j -0. -7.3j  4.3+5.9j -6.9-2.2j]                           : 33202, 32302, 23302, 33022, 30322, 03322, 32032, 23032, 30232, 03232, 20332, 02332,
[ 6.9+2.2j -4.3-5.9j -0. +7.3j  4.3-5.9j -6.9+2.2j]                           : 32203, 23203, 22303, 32023, 23023, 30223, 03223, 20323, 02323, 22033, 20233, 02233,
[  6.9+15.4j   3.5-16.5j -12.5+11.3j  16.8 -1.8j -14.6 -8.4j]                 : 32113, 23113, 31213, 13213, 21313, 12313, 31123, 13123, 11323, 21133, 12133, 11233,
[  10.4-157.8j -146.9 -58.6j -101.1+121.6j   84.4+133.7j  153.3 -38.9j]       : 44444,
[10.4 -1.1j  4.3 +9.5j -7.7 +7.j  -9.  -5.2j  2.1-10.2j]                      : 33211, 32311, 23311, 33121, 31321, 13321, 32131, 23131, 31231, 13231, 21331, 12331,
[10.4 +1.1j  4.3 -9.5j -7.7 -7.j  -9.  +5.2j  2.1+10.2j]                      : 43224, 34224, 42324, 24324, 32424, 23424, 42234, 24234, 22434, 32244, 23244, 22344,
[  10.4+157.8j -146.9 +58.6j -101.1-121.6j   84.4-133.7j  153.3 +38.9j]       : 11111,
[  11.7-112.j    56.4 +97.5j -103.  -45.7j  110.2 -23.5j  -75.3 +83.8j]       : 44002, 40402, 04402, 40042, 04042, 00442,
[  11.7+112.j    56.4 -97.5j -103.  +45.7j  110.2 +23.5j  -75.3 -83.8j]       : 11003, 10103, 01103, 10013, 01013, 00113,
[ 14.6 -8.4j -16.8 -1.8j  12.5+11.3j  -3.5-16.5j  -6.9+15.4j]                 : 44223, 42423, 24423, 42243, 24243, 22443,
[ 14.6 +8.4j -16.8 +1.8j  12.5-11.3j  -3.5+16.5j  -6.9-15.4j]                 : 33112, 31312, 13312, 31132, 13132, 11332,
[ 17.3-23.8j -35.7+32.9j  38.6 +5.6j -20.2-14.7j  -0.  -0.j ]                 : 32230, 23230, 22330,
[ 17.3+23.8j -35.7-32.9j  38.6 -5.6j -20.2+14.7j   0.  -0.j ]                 : 33220, 32320, 23320,
[ 18.1-24.9j  29.3 +9.5j  -0. +30.8j -29.3 +9.5j -18.1-24.9j]                 : 44101, 41401, 14401, 44011, 40411, 04411, 41041, 14041, 40141, 04141, 10441, 01441,
[ 18.1 -3.6j   9. +16.1j -12.5+13.6j -16.8 -7.7j   2.1-18.3j]                 : 42211, 24211, 22411, 42121, 24121, 41221, 14221, 21421, 12421, 22141, 21241, 12241,
[ 18.1 +3.6j   9. -16.1j -12.5-13.6j -16.8 +7.7j   2.1+18.3j]                 : 43314, 34314, 33414, 43134, 34134, 41334, 14334, 31434, 13434, 33144, 31344, 13344,
[ 18.1+24.9j  29.3 -9.5j  -0. -30.8j -29.3 -9.5j -18.1+24.9j]                 : 41104, 14104, 11404, 41014, 14014, 40114, 04114, 10414, 01414, 11044, 10144, 01144,
[ 27.1-31.8j -40.6 +9.8j  38.6+16.j  -21.9-35.6j  -3.3+41.7j]                 : 33333,
[ 27.1-12.4j -29.3 -5.9j  20.2+22.j   -3.5-29.7j -14.6+26.j ]                 : 44313, 43413, 34413, 44133, 41433, 14433, 43143, 34143, 41343, 14343, 31443, 13443,
[ 27.1+12.4j -29.3 +5.9j  20.2-22.j   -3.5+29.7j -14.6-26.j ]                 : 42112, 24112, 41212, 14212, 21412, 12412, 41122, 14122, 11422, 21142, 12142, 11242,
[ 27.1+31.8j -40.6 -9.8j  38.6-16.j  -21.9+35.6j  -3.3-41.7j]                 : 22222,
[  45.2-14.7j   48.2+53.2j -101.1-62.2j    7.7+23.8j   -0.  -0.j ]            : 44110, 41410, 14410,
[  45.2+14.7j   48.2-53.2j -101.1+62.2j    7.7-23.8j   -0.  +0.j ]            : 41140, 14140, 11440,
[ 50.8-10.1j   6.1-51.5j -47. -21.7j -35.2+38.j   25.3+45.2j]                 : 33004, 30304, 03304, 30034, 03034, 00334,
[ 50.8+10.1j   6.1+51.5j -47. +21.7j -35.2-38.j   25.3-45.2j]                 : 22001, 20201, 02201, 20021, 02021, 00221,
[ 56.9-61.6j -82.2+16.3j  76.1+35.1j -41. -73.1j  -9.9+83.2j]                 : 43003, 34003, 40303, 04303, 30403, 03403, 40033, 04033, 00433, 30043, 03043, 00343,
[ 56.9+61.6j -82.2-16.3j  76.1-35.1j -41. +73.1j  -9.9-83.2j]                 : 21002, 12002, 20102, 02102, 10202, 01202, 20012, 02012, 00212, 10022, 01022, 00122,
[ 68.1-14.5j   7.2-69.2j -63.6-28.3j -46.5+51.8j  34.9+60.3j]                 : 42004, 24004, 40204, 04204, 20404, 02404, 40024, 04024, 00424, 20044, 02044, 00244,
[ 68.1+14.5j   7.2+69.2j -63.6+28.3j -46.5-51.8j  34.9-60.3j]                 : 31001, 13001, 30101, 03101, 10301, 01301, 30011, 03011, 00311, 10031, 01031, 00131,
[ 69.1 -58.8j -38.2 -65.7j -38.6+107.7j   7.7 +16.8j  -0.   +0.j ]            : 33040, 30340, 03340,
[ 69.1 +58.8j -38.2 +65.7j -38.6-107.7j   7.7 -16.8j  -0.   +0.j ]            : 22010, 20210, 02210,
[ 79.8-44.1j -42.3-53.2j -62.5+38.5j  25. +58.8j   0.  +0.j ]                 : 43030, 34030, 40330, 04330, 30430, 03430,
[ 79.8+44.1j -42.3+53.2j -62.5-38.5j  25. -58.8j  -0.  +0.j ]                 : 21020, 12020, 20120, 02120, 10220, 01220,
[  86.4-118.9j -100.7 +20.3j   62.5 +45.4j  -48.2 +53.2j   -0.   -0.j ]       : 42400, 24400,
[ 86.4-118.9j -44.8  +4.8j -77.3+146.9j  35.7 -32.9j   0.   +0.j ]            : 33400,
[  86.4+118.9j -100.7 -20.3j   62.5 -45.4j  -48.2 -53.2j   -0.   +0.j ]       : 31100, 13100,
[ 86.4+118.9j -44.8  -4.8j -77.3-146.9j  35.7 +32.9j   0.   -0.j ]            : 22100,
[107.7-71.3j -70.2-32.9j -62.5 +9.1j  25. +95.1j  -0.  -0.j ]                 : 42040, 24040, 40240, 04240, 20440, 02440,
[107.7+71.3j -70.2+32.9j -62.5 -9.1j  25. -95.1j  -0.  +0.j ]                 : 31010, 13010, 30110, 03110, 10310, 01310,
[ 180.9 -95.1j -261.8+106.3j  101.1-198.j   -20.2+186.7j   -0.   +0.j ]       : 44020, 40420, 04420,
[ 180.9 +95.1j -261.8-106.3j  101.1+198.j   -20.2-186.7j   -0.   -0.j ]       : 11030, 10130, 01130,
[ 226.1 -73.5j -380.2+364.4j  202.3-237.8j  -48.2 -53.2j    0.   +0.j ]       : 11300,
[ 226.1 -73.5j -324.3 -86.j    62.5+192.4j   35.7 -32.9j   -0.   -0.j ]       : 43300, 34300,
[ 226.1 +73.5j -380.2-364.4j  202.3+237.8j  -48.2 +53.2j    0.   -0.j ]       : 44200,
[ 226.1 +73.5j -324.3 +86.j    62.5-192.4j   35.7 +32.9j   -0.   -0.j ]       : 21200, 12200,
[9375.+0.j 2975.+0.j 1625.+0.j 1025.+0.j  625.+0.j]                           : 00000,
```

After a couple of wrong turns, I constructed a better basis which only requires one basis vector for each unique value. (specifically for $n = m = 3$)

```
In [95]: by_value(character_to_new_basis_3.dot(decompose(disp2, 3,3, False)), 3,3, labels = basis_labels_3)
5
-0.-5.2j             : 111,
-0.+5.2j             : 222,
4.5-2.6j             : S_2*012,
4.5+2.6j             : S_2*102,
9.+0.j               : 000,

In [96]: by_value(character_to_new_basis_3.dot(decompose(disp1, 3,3, False)), 3,3, labels = basis_labels_3)
6
-9.-5.2j             : S_2*102,
-9.+5.2j             : S_2*012,
-4.5-2.6j            : 222,
-4.5+2.6j            : 111,
9.+0.j               : S_3*012,
18.+0.j              : 000,

In [97]: by_value(character_to_new_basis_3.dot(decompose(disp0, 3,3, False)), 3,3, labels = basis_labels_3)
4
-9.-0.j              : S_3*012,
4.5-2.6j             : S_2*012, 222,
4.5+2.6j             : S_2*102, 111,
54.+0.j              : 000,

In [98]: by_value(character_to_new_basis_3.dot(decompose(total_displacement, 3,3, False)), 3,3, labels = basis_labels_3)
4
-4.5-7.8j            : 111,
-4.5+7.8j            : 222,
9.-0.j               : S_3*012,
36.+0.j              : 000,

In [99]: by_value(character_to_new_basis_3.dot(decompose(max_displacement, 3,3, False)), 3,3, labels = basis_labels_3)
6
-6.-5.2j             : 111,
-6.+5.2j             : 222,
3.-0.j               : S_3*012,
4.5-2.6j             : S_2*012,
4.5+2.6j             : S_2*102,
30.+0.j              : 000,
```

6/13 & 6/14

Choice for adaptive basis?

Should have $\chi_{123}+\chi_{312}$

$\chi_{1234}+\chi_{1324}+\chi_{2134}+\chi_{2314}+\chi_{3214}+\chi_{3124}$ etc

AKA respect subgroups of symmetric group when they act.

Double cosets of $S_m \times S_m$ where x has multiplicity type λ

$$|S_m x S_m| = \frac{|S_m||S_m|}{|S_\lambda|} = \frac{n!\,n!}{\prod_{p\in\lambda} p!} \quad \text{wreath product elements}$$

$$\Rightarrow \frac{n!}{\prod_{p\in\lambda} p!} \quad \text{preference lists}$$

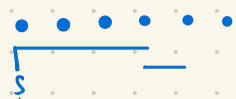
Note: Knowing $N_\lambda(d)$ would give a different way of counting the number of defect d preference lists

$$\#\text{ defect } d \text{ pref. lists} = \sum_{\lambda\vdash n} \frac{n!}{\left[\prod_{p\in\lambda} p!\right]} N_\lambda(d)$$

This is deeply unhelpful at the moment since $N_\lambda(d)$ is tricky.

For Partition $\lambda=2,n-2$.

$$N_\lambda(0) = (n-2) + \overset{\text{large part second}}{2} = n$$



$$N_\lambda(1) = \sum_{s=1}^{2}\begin{cases}1 & s=1\\ n-2 & s=2\end{cases} + \sum_{s=3}^{4}\begin{cases}1 & s=3\\ 2 & s=4\end{cases} = n+2 \text{ for } n>4$$

$$N_\lambda(2) = \overset{s=3}{(n-2)-1} + \sum_{s=4}^{5}\begin{cases}1 & s=4\\ \underset{\hookrightarrow=2+1}{3} & s=5\end{cases} = n+1$$

$$N_\lambda(3) = \overset{s=4}{(n-2)-2} + \overset{s=5}{1} + \overset{s=6}{4} = n+1$$

Thinking on bases

for S_n acting on a single element,

$$\begin{bmatrix} 1 & 1 & -1 \\ 1 & 0 & 2 \\ 1 & -1 & -1 \end{bmatrix}$$

is a natural basis because it separates the action of S_n (for this module) into irreducibles (and also probably some voting theory b/c col 2)

Is there a good way to take this basis and use it to construct a basis for S_n acting on itself?
If so, that could let a nice basis for pref. lists give information about $C_n \wr S_m$

For $n=m=3$. Current basis: χ_g, $g \in C_3^3$
new basis

these are the actually good ones. indexing was backwards grr..

	χ_{000}
	$\chi_{001}+\chi_{010}+\chi_{100}$
$\chi_{010}+\chi_{001}$	$\chi_{010}+\chi_{100}$
χ_{001}	χ_{100}
	$\chi_{002}+\chi_{020}+\chi_{200}$
$\chi_{002}+\chi_{020}$	$\chi_{020}+\chi_{200}$
χ_{002}	χ_{200}
	$\chi_{011}+\chi_{101}+\chi_{110}$
$\chi_{101}+\chi_{110}$	$\chi_{011}+\chi_{101}$ $\quad \chi_{110}+\chi_{101}$
χ_{110}	χ_{011}
	$\chi_{012}+\chi_{102}+\chi_{021}+\chi_{201}+\chi_{120}+\chi_{210}$
$\chi_{201}+\chi_{210}$	$\chi_{120}+\chi_{210}$ or $\chi_{120}+\chi_{210}+w(\chi_{201}+\chi_{021})+w^2(\chi_{012}+\chi_{102})$
$\chi_{120}+\chi_{102}$	$\chi_{021}+\chi_{201}$
χ_{012}	χ_{120}
χ_{120}	χ_{012} cycles - coset reps of S_2
χ_{201}	χ_{201}
	$\chi_{022}+\chi_{202}+\chi_{220}$
$\chi_{220}+\chi_{202}$	$\chi_{022}+\chi_{202}$
χ_{220}	χ_{022}
	χ_{111}
	$\chi_{112}+\chi_{121}+\chi_{211}$
$\chi_{112}+\chi_{121}$	$\chi_{121}+\chi_{211}$
χ_{112}	χ_{211}
	$\chi_{221}+\chi_{212}+\chi_{122}$
$\chi_{221}+\chi_{212}$	$\chi_{212}+\chi_{122}$
χ_{221}	χ_{122}
	χ_{222}

Change of Basis: Character basis → new basis