I ended up working on organizing my files (still need to finish) and then working on computing/proving some things about when order of the cars matters

Typed board work

Making distance displaced precise
Previously, we talked about distance that a car has been displaced as a potential "permutation"-statistic (parking-statistic). One thing that we didn't make precise was how cars which are not parked count towards this. There are two logical choices: inclusive distance and exclusive distance. For inclusive distance, the cars which cannot park count as having a distance displaced all the way to the end of the parking lot. For exclusive distance, we ignore these cars.

Exclusive distance is not invarient under the action of S_n. For the notation, subscript indexes when the car gets to attempt to park, superscripts indicate the desired spot. Counter example:

$$2\,2\,3\,4 \rightarrow \_\_\ \underline{c_3^2}\ \underline{c_2^3}\ \underline{c_1^4} \qquad \text{exclusive distance: } 0$$
$$4\,3\,2\,2 \rightarrow \_\_\ \underline{c_1^2}\ \underline{c_2^2}\ \underline{c_3^3} \qquad \text{exclusive distance: } 2$$

Order invariance of parkability for 1-element subset preferences
Note that S_n is generated by adjacent transpositions, so to show that something is invarient under the action of S_n, we just need to show that it is invarient under the action of adjacent transpositions. Consider the action of the transposition $(i\ (i+1))$. The first $i-1$ cars will have parked in the same way, so we think of this as two different cars trying to park given a set of occupied spots.

Given the spots occupied, the ith car and $i+1$th car each have a preferred spot p_i and p_{i+1} respectively. If $p_i \neq p_{i+1}$, it doesn't matter what order these two cars park since they want to park in different spots. However if there is a collision and $p_i = p_{i+1}$, then the collision will change the outcome of which car gets to park where. However, a collision will not change which spots are ultimately occupied. Additionally, it will not change the cumulative distance that the cars are displaced

2-element subsets depend on the order of the cars Example:

$$(2,4)(1,2)(1)(4) \rightarrow \ \underline{(1)}\ \underline{(1,2)}\ \_\_\ \underline{(4)} \qquad \text{parkability: } 3$$
$$(1,2)(2,4)(1)(4) \rightarrow \underline{(1)}\ \underline{(2,4)}\ \underline{(1,2)}\ \underline{(4)} \qquad \text{parkability: } 4$$

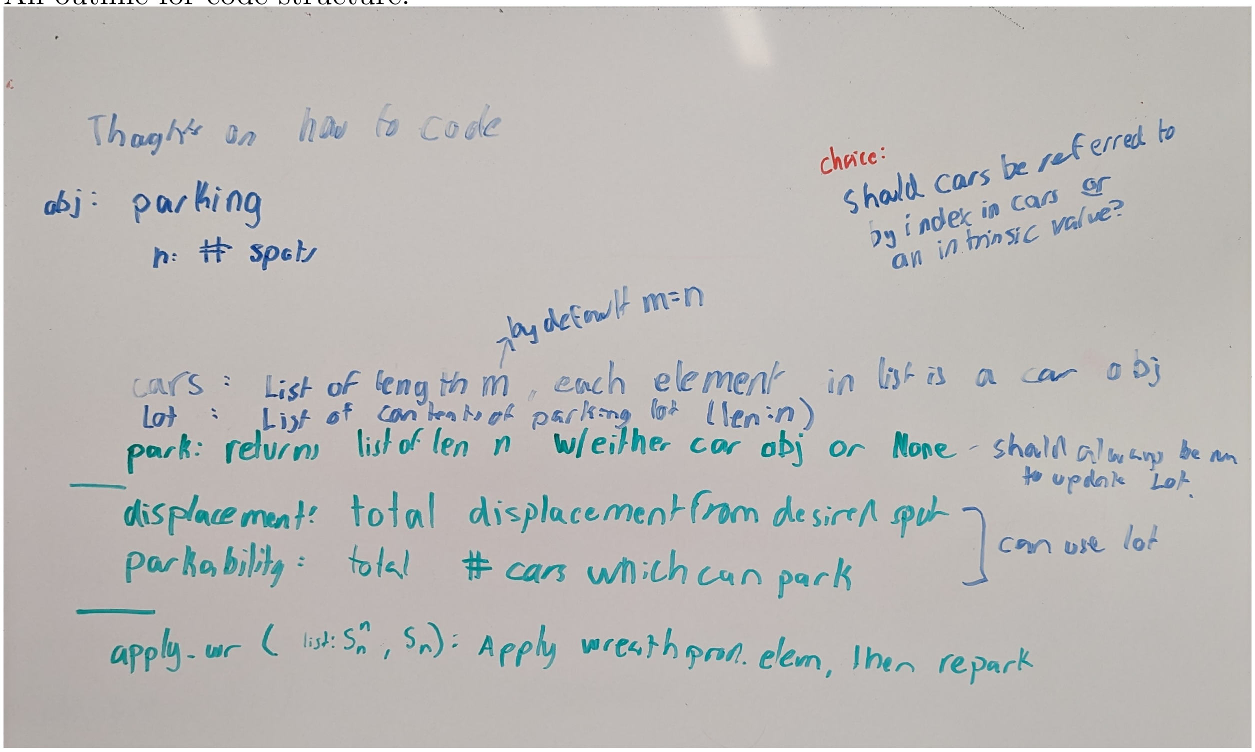
I think this might be the smallest example of order changing parkability

Future directions/things I didn't get done/questions

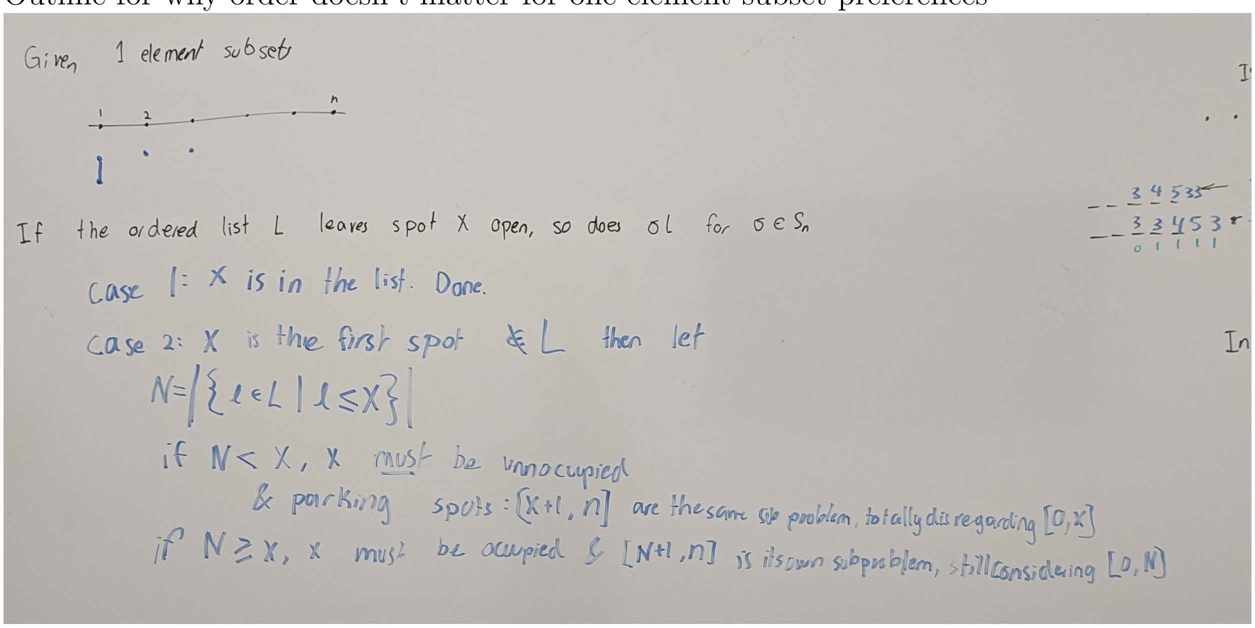
- Think about the submodule of $S_n \wr S_n$ where parkability for 1 element subset preferences is constant
- Read more on wreath product rep theory (along the way learn more about Gelfand pairs (related to two sided cosets, algebra connection with commutative algebra) and spherical functions (related to multiplicity free-ness and modules whose representations only need one column of an irreducible representation)
- Read Theses
- When can graph notion of local and rep theory notion of local match up? When can graph notion of local provide a finer categorization of local? Does that relate to when $G \subset Aut(\Gamma)$ (ex:$\mathbb{Z}_2^n$) versus $G = Aut(\Gamma)$
- Develop better more consistent notation and names for parking function adjacent things

Photos from board work

An outline for code structure:



Outline for why order doesn't matter for one element subset preferences



Total distance from desired spot is order invariant (given the right way of counting distance). And parkability is not invariant under action of subgroup $e \wr S_n$ (aka depends on order of cars).