1.

$$
\begin{aligned}
Fib(0) &= (\phi^0 - \gamma^0)/\sqrt{5} = 0 \\
Fib(1) &= (\phi^1 - \gamma^1)/\sqrt{5} = ((1+\sqrt{5})/2 - (1-\sqrt{5})/2)/\sqrt{5} = \sqrt{5}/\sqrt{5} = 1
\end{aligned}
$$

2.

$$
\begin{aligned}
Fib(n-1) + Fib(n-2) &= \frac{\phi^{n-1} - \gamma^{n-1}}{\sqrt{5}} + \frac{\phi^{n-2} - \gamma^{n-2}}{\sqrt{5}} \\
&= \frac{\phi^{n-1} - \gamma^{n-1} + \phi^{n-2} - \gamma^{n-2}}{\sqrt{5}} \\
&= \frac{(\phi^{n-1} + \phi^{n-2}) - (\gamma^{n-1} + \gamma^{n-2})}{\sqrt{5}}
\end{aligned}
$$

$$
\begin{aligned}
\phi^{n-1} + \phi^{n-2} &= (\frac{1+\sqrt{5}}{2})^{n-1} + (\frac{1+\sqrt{5}}{2})^{n-2} \\
&= (\frac{1+\sqrt{5}}{2} + 1)(\frac{1+\sqrt{5}}{2})^{n-2} \\
&= (\frac{3+\sqrt{5}}{2})(\frac{1+\sqrt{5}}{2})^{n-2} \\
&= (\frac{6+2\sqrt{5}}{4})(\frac{1+\sqrt{5}}{2})^{n-2} \\
&= (\frac{1+\sqrt{5}}{2})^2(\frac{1+\sqrt{5}}{2})^{n-2} \\
&= (\frac{1+\sqrt{5}}{2})^n \\
&= \phi^n
\end{aligned}
$$

$$
\begin{aligned}
\gamma^{n-1} + \gamma^{n-2} &= (\frac{1-\sqrt{5}}{2})^{n-1} + (\frac{1-\sqrt{5}}{2})^{n-2} \\
&= (\frac{1-\sqrt{5}}{2} + 1)(\frac{1-\sqrt{5}}{2})^{n-2} \\
&= (\frac{3-\sqrt{5}}{2})(\frac{1-\sqrt{5}}{2})^{n-2} \\
&= (\frac{6-2\sqrt{5}}{4})(\frac{1-\sqrt{5}}{2})^{n-2} \\
&= (\frac{1-\sqrt{5}}{2})^2(\frac{1-\sqrt{5}}{2})^{n-2} \\
&= (\frac{1-\sqrt{5}}{2})^n \\
&= \gamma^n
\end{aligned}
$$

$$
\begin{aligned}
Fib(n-1) + Fib(n-2) &= \frac{(\phi^{n-1} + \phi^{n-2}) - (\gamma^{n-1} + \gamma^{n-2})}{\sqrt{5}} \\
&= \frac{\phi^n - \gamma^n}{\sqrt{5}} \\
&= Fib(n)
\end{aligned}
$$

3.

$$Fib(n) = \frac{\phi^n - \gamma^n}{\sqrt{5}}$$

$$Fib(n) - \phi^n/\sqrt{5} = -\gamma^n/\sqrt{5}$$

just prove

$$\gamma^n/\sqrt{5} \leq 1/2$$

that's prove

$$\gamma^n \leq \sqrt{5}/2$$

we have

$$
\begin{gathered}
\gamma = (1 - \sqrt{5})/2 \approx -0.618 \\
\gamma^n < 1 \\
\sqrt{5}/2 \approx 1.118
\end{gathered}
$$

reference