

## Table of Contents

- First of all
- Operation
- Programming Implementation
  - Implemente the aboves steps in java via the BigInteger class.
  - For those who want to implement all by themselves.

## First of all

This is my operating system curriculum design, I wrote it just because I have to chose one topic to wrote. So I chose the topic is an analog implementation of a public encryption algorithm. I do know that there is no difficulty, but there is no difficulty in other topics. It's here for the convenience of those who want to know how RSA(encryption algorithm) works.

## Operation

See the wiki for details.

In simple terms, there are five steps that require generating three numbers.

1. Generate two unequal large prime, denoted as  $p$  and  $q$ .
2. Calculate  $n = pq$ .
3. According to the Euler function, calculate  $\varphi(n)$ , denoted as  $r$ .

$$r = \varphi(n) = \varphi(pq) = \varphi(p)\varphi(q) = (p-1)(q-1)$$

4. Generate a number  $e$  and  $e$  is coprime to  $r$ , and  $1 < e < r$ .
5. Generate a number  $d$  and  $ed \equiv 1 \pmod{r}$ , which means  $(ed-1) \pmod{r} = 0$ ,  $d$  is the modular multiplicative inverse of  $e$  about  $r$ .

Then  $(n, e)$  is public key, and  $(n, d)$  is private key.

Let  $m$  be the information that needs to be encrypted, and record  $c$  an the encrypted cipertext( $m$  and  $c$  are both large numbers smaller than  $n$ ).

then

$$\begin{aligned}c &= m^e \pmod{n} \\ m &= c^d \pmod{n}\end{aligned}$$

## Programming Implementation

Implemente the aboves steps in java via the `BigInteger` class.

1. For generating large prime, Use `new BigInteger(BitLength, Certainty, Random)`
2. For generating  $d$ , Use `e.modInverse(n)`.
3. For calculae  $c$  and  $m$ , Use `m.modPow(e, n)` and `c.modPow(c, n)`.

**For those who want to implement all by themselves.**

See those links.

1. For testing if a integer is a prime, see Fermat's little theorem
2. For calculating modular multiplicative inverse of  $e$  about  $r$ , see extended Euclidean algorithm
3. For calculating modulo operation of high-order of large integer, see Montgomery modular multiplication

Other algorithms may also work.