

```
In [20]: # We are aware that if one game is played, it is impossible to get two losses
         # in a row
         # Hence set  $F(1) = 0$  where  $F$  is the probability
```

```
In [21]: #  $F(2)$  can only happen when both games are losses. Since each loss is an independent event
         #  $F(2) = P_L * P_L$  where  $P_L = 0.2$  and  $P_W = 0.8$ 
```

```
In [22]: # A function can be created to recursively calculate  $F(82)$  by checking the previous outcomes
         # since we need to have two losses in a row.
         #  $F(n) = P_W * F(n-1) + P_L * (P_L + P_W * F(n-2))$ 
         # The first part of the equation states that if game  $n$  is a win, you multiply the probability it's a win times the
         # outcome of the previous game. The second part states that if game  $n$  is a loss, you the probability it's a loss
         # times the probability that the previous game was a loss (hence two losses in a row) plus the probability that
         # two games before it was a win and thus you continue recursively.
```

```
In [23]: # Using this formula and the constants for  $F(1)$  and  $F(2)$  we can iteratively find the solution using a loop
         # In addition, the formula can be solved analytically to get  $F(82)$  directly.
```

```
In [24]: import numpy as np
         # create an iterative list from games 3 to 82 to run the formula through
         games = np.arange(3,83)
         games
```

```
Out[24]: array([ 3,  4,  5,  6,  7,  8,  9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19,
                20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36,
                37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53,
                54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70,
                71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82])
```

```
In [25]: # create a list that will store the percentage of consecutive losses and hard
         # code the first two values
         consecutive_losses = []
         consecutive_losses.append(0)
         consecutive_losses.append(0.2*0.2)
```

```
In [26]: consecutive_losses
```

```
Out[26]: [0, 0.040000000000000001]
```

```
In [33]: # initialize variable
percent = 0
for game in games:
    #  $F(n) = P_W * F(n-1) + P_L * (P_L + P_W * F(n-2))$ 
    percent = 0.8 * consecutive_losses[-1] + 0.2 * (0.2 + 0.8 * consecutive_lo
sses[-2])
    consecutive_losses.append(percent)
```

In [32]: `consecutive_losses`

```
Out[32]: [0,
0.040000000000000001,
0.072000000000000001,
0.104000000000000001,
0.13472,
0.164416,
0.193088,
0.22077696000000002,
0.24751564800000003,
0.27333683200000003,
0.29827196928000005,
0.3223514685440001,
0.3456046899200001,
0.36805998690304015,
0.38974473990963215,
0.41068538983219216,
0.4309074702512949,
0.45043563857418667,
0.46929370609955656,
0.4875046670515152,
0.5050907266171412,
0.5220733280219554,
0.538473178676307,
0.5543102754245585,
0.5696039289278559,
0.5843727872102141,
0.5986348583966283,
0.612407532670937,
0.62570760348021,
0.638551288011518,
0.650954246966048,
0.6629316036546813,
0.6744979624383127,
0.6856674265353992,
0.6964536152184495,
0.7068696804204235,
0.7169283227712908,
0.7266418070843004,
0.736021977310847,
0.7450802709821657,
0.753827733155468,
0.7622750298815211,
0.7704324612100919,
0.778309973749117,
0.7859171727929084,
0.7932633340341855,
0.8003574148742137,
0.8072080653448407,
0.8138236386557468,
0.8202122013797719,
0.826381543288737,
0.8323391868517532,
0.8380923964076006,
0.843648187022361,
0.849013333043105,
0.8541943763580618,
0.8591976343733463,
```

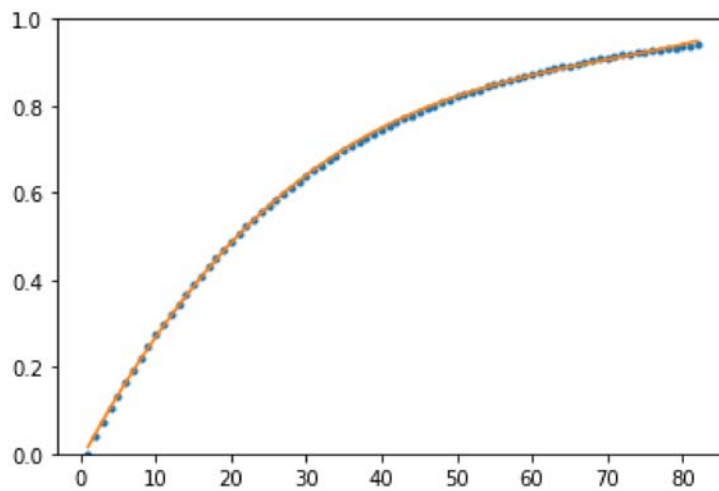
```

0.864029207715967,
0.8686949876725092,
0.8732006633725621,
0.8775517287256511,
0.881753489120131,
0.8858110678922091,
0.8897294125729883,
0.8935133009211442,
0.8971673467485937,
0.9006960055462581,
0.9041035799167816,
0.9073942248208267,
0.9105719526433464,
0.9136406380860094,
0.916604022891743,
0.919465720407156,
0.9222292199884037,
0.924897891255868,
0.927474988202839,
0.9299636531632102,
0.9323669206430224,
0.9346877210205315,
0.9369288841193089,
0.9390931426587322,
0.9411831355860754]

```

```
In [29]: full_games = np.arange(1,83)
```

```
In [31]: import matplotlib.pyplot as plt
%matplotlib inline
z = np.polyfit(full_games, consecutive_losses, 3)
p = np.poly1d(z)
xp = np.linspace(1, 82, 82)
_ = plt.plot(full_games, consecutive_losses, '.', xp, p(xp), '- ',)
plt.ylim(0,1)
plt.show()
```



```
In [ ]:
```