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Properties of Eigen value.

- i) Sum of the Eigen value = Sum of the main diagonal
- ii) Product of the Eigen value = $|A|$
- iii) λ is the eigen value of 'A', Then λ^{-1} is the Eigen value of A^{-1}
- iv) λ is the eigen value of 'A', then λ^2 is the eigen value of A^2
- v) If λ is the eigen value of A, then λ^3 is the eigen value of A^3 .
- vi) If λ is the eigen value of A, then 5λ is the eigen value of $5A$.
- vii) If λ is the eigen value of A, then $2A - I$
- viii) Eigen value of $A =$ Eigen value of A^T .

Q1. Find the sum and product of the Eigen values of the matrix. $\begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix}$

Soln : Given,

$$\begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix}$$

$$\text{Sum} = (-1) + (-1) + (-1) = -3$$

$$\text{Product} = (-1) \times (-1) \times (-1) = -1$$

$$= -1 \begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} - 1 \begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} + 1 \begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix}$$

$$= -1(1-1) - 1(-1-1) + 1(1+1)$$

$$= -1(-2) + 2$$

Q₂: Find the Sum and product of the Eigen values

$$\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$$

Soln: Sum = $(-2) + 1 + (0) = \boxed{-1}$

$$\text{Product} = -2 \begin{vmatrix} 1 & -6 \\ -2 & 0 \end{vmatrix} - 2 \begin{vmatrix} 2 & -6 \\ -1 & 0 \end{vmatrix} + (-3) \begin{vmatrix} 2 & 1 \\ -1 & -2 \end{vmatrix}$$

$$= -2(0 + 12) - 2(0 + 6) + 3(-4 + 1)$$

$$= -24 + 12 + 9$$

$$\text{Product} = 36 - 9 = \boxed{45}$$

Q₃: If 2, 2, 3 are the eigen values of the matrix

$A = \begin{bmatrix} 3 & 10 & 5 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{bmatrix}$ then find the eigen values of A^{-1} , A^T .

Soln: Given,

$$A = \begin{bmatrix} 3 & 10 & 5 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{bmatrix}$$

$$A^{-1} = \frac{1}{2}, \frac{1}{2}, \frac{1}{3}$$

$$A^T = 2, 2, 3$$

Q₄: Find the eigen value of the inverse of the matrix. $A = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 3 & 4 \\ 0 & 0 & 4 \end{bmatrix}$

Soln: The given matrix is \therefore upper / lower triangular \Rightarrow diag.
an upper triangular matrix

2, 3, 4 is an Eigen values of A

$\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ is an Eigen value of A^{-1}