

08-12-23

Bloch's Theorem

Day 63

- Mathematical statement of electron function moving in a periodical potential.

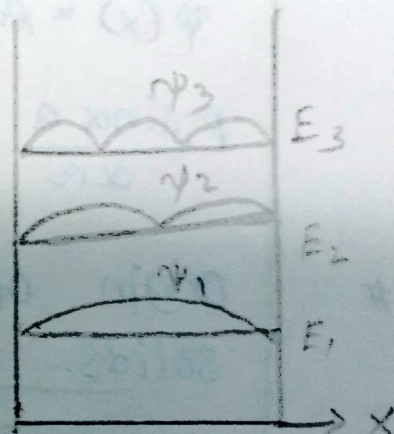
$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2} (E - V) \psi = 0 \rightarrow \textcircled{1}$$

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2} E \psi = 0$$

$$V(x) = V(x+a) \rightarrow \textcircled{2} \text{ # theorem}$$

Where, a - Periodicity of Potential

$$\psi(x) = e^{ikx} u_k(x)$$



$$\therefore k = \frac{2m}{\hbar^2} (E - V)$$

$$\psi(x+a) = e^{ik(x+a)} u_k(x+a) \rightarrow (3) \quad \# \text{ Function}$$

$$\psi(x+a) = e^{ikx} \cdot e^{ika} \cdot u_k(x+a)$$

$$\psi(x+a) = \psi(x) \cdot e^{ika} \rightarrow (4) \quad \# \text{ Proof}$$

$$\psi(x+a) = \psi(x) \rightarrow (5)$$

Depends on \rightarrow direction, Periodical

Behaviour of an electron in a periodic potential - The Kronig Penney model (Qualitative Treatment)

$$\frac{d^2\psi(x)}{dx^2} + \frac{2m}{\hbar^2} (E - V) \psi(x) = 0 \rightarrow (1)$$

$$\frac{d^2\psi(x)}{dx^2} + \frac{2m}{\hbar^2} E \psi(x) = 0 \quad \therefore V = 0$$

$$\frac{d^2\psi}{dx^2} + \alpha^2 \psi(x) = 0 \rightarrow (2) \quad \therefore \frac{2m}{\hbar^2} E = \alpha^2$$

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2} (V - E) \psi(x) = 0 \quad \therefore V > E$$

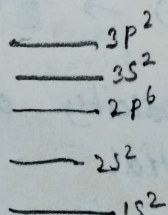
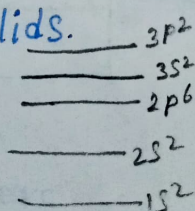
$$\frac{d^2\psi}{dx^2} - \beta^2 \psi(x) = 0 \rightarrow (3) \quad \frac{2m}{\hbar^2} (V - E) = \beta^2$$

$$\psi(x) = \psi(x+a)$$

$$\psi(x) = A \sin Kx + B \cos Kx$$

$$p \frac{\sin \alpha a}{\alpha a} + \cos \alpha a = \cos Ka \quad \therefore p = \frac{mV_0 ba}{\hbar^2}$$

origin of energy band formation in solids.



(2) ... (n)

