25-11-23

Schrondinger's Time dependent wave Equation

Day 53

$$\psi = \psi_0 e^{-i\omega t} \rightarrow ①$$

$$\frac{\partial \psi}{\partial t} = \psi_0 (-i\omega) e^{-i\omega t} \rightarrow ②$$

$$\omega = 2\pi \nu$$

$$E = h\nu \Rightarrow \nu = E/h$$

from 2 we have

$$\frac{\partial \psi}{\partial t} = -\psi_0 i \cdot 2\pi \frac{E}{h} e^{-i\omega t}$$

$$\frac{\partial \psi}{\partial t} = -i \psi_0 \frac{E}{h/2\pi} e^{-i\omega t} = -i \frac{E}{\hbar} \psi$$

$$i \frac{\partial \psi}{\partial t} = -i^2 \left(\frac{E}{\hbar}\right) \psi$$

$$i\hbar \frac{\partial \psi}{\partial t} = E\psi \rightarrow ③$$

From Time independent Eqn.

$$\nabla^2 \psi + \frac{2m}{\hbar^2} (E - V) \psi = 0$$

$$\nabla^2 \psi + \frac{2m}{\hbar^2} (E\psi - V) \psi = 0 \rightarrow ④$$

From 4 we have,

$$\nabla^2 \psi = \frac{-2m}{\hbar^2}\left[i\hbar \frac{\partial \psi}{\partial t} - V\psi\right]$$

$$\hbar^2/2m \; \nabla^2 \psi = -i\hbar \frac{\partial \psi}{\partial t} - V\psi$$

$$h^2/2m \; \nabla^2 \psi + V\psi = -i\hbar \frac{\partial \psi}{\partial t}$$

$$\div \left(\frac{\hbar^2}{2m} \nabla^2 + V\right)\psi = i\hbar \frac{\partial \psi}{\partial t}$$

$$H\psi = E\psi$$

* Physical significance of wave function (ψ)

a) Variable
b) Relation (particle - wave)
c) complex quantity
d) Probability $P = \iiint_V \psi^2 \; || \; dx\,dy\,dz$