25-11-23

Day 53

Taylor series

$f(x,y) = f(a,b) + \frac{1}{1!}\left[hf_x^{(a,b)} + kf_y^{(a,b)}\right] + \frac{1}{2!}$
$\left[hf_{xx} + 2hkf_{xy} + k^2 f_{yy}\right] + \frac{1}{3!}\left[h^3 f_{xxx} + 3h^2 k f_{xxy} +\right.$
$\left. 3hk^2 f_{xyy} + k^3 f_{yyy}\right]$

Q: Expand $e^x \cdot \cos y$ about $(0, \pi/2)$ upto 3rd term using taylor series.

Soln: $h = x - a$; $k = y - b$

Given,

$f(x,y) = e^x \cos y$

$a = 0$; $b = \pi/2$

$h = x - a$ $\quad$ $k = y - b$
$= x - 0$ $\quad$ $= y - \pi/2$

function	Value at $(0, \pi/2)$
$f(x,y) = e^x \cos y$ $f(x) = e^x \cos y$ $f(y) = e^x(-\sin y)$	$e^0 \cos \pi/2 = 0$ 0 $e^0(-\sin \pi/2)$
$f_{xx} = e^x \cos y$ $f_{xy} = e^x(-\sin y)$ $f_{yy} = -e^x \cos y$	0 -1 $-e^0 \cos \pi/2 = 0$
$f_{xxx} = e^x \cos y$ $f_{xxy} = e^x(-\sin y)$ $f_{xyy} = -e^x(\cos y)$ $f_{yyy} = e^x(\sin y)$	0 -1 $-e^0 \cos \pi/2 = 0$ 1

$$f(x,y) = 0 + \frac{1}{1!}[x \times 0 + (y - \pi/2)(-1) + \frac{1}{2!}$$
$$[x^2 \times 0 + 2x(y-\pi/2)(-1) + (y-\pi/2)^2(0)] +$$
$$\frac{1}{3!}[x^3 \times 0 + 3x^2(y-\pi/2)(-1) + 3 \times x \times (y-\pi/2)^2$$
$$(0) + (y-\pi/2)^3(1)]$$

$$f(x,y) = -(y-\pi/2) + \frac{1}{2}(-2x(y-\pi/2)) + \frac{1}{6}$$
$$[-3x^2(y-\pi/2) + (y-\pi/2)^3]$$

$$= -(y-\pi/2) - x(y-\pi/2) - \frac{3}{6}x^2(y-\pi/2) +$$
$$\frac{1}{6}(y-\pi/2)^3$$

$$f(x,y) = -(y-\pi/2)(1+x) - \frac{x^2}{2}(y-\pi/2) + \frac{1}{6}$$
$$(y-\pi/3)^3$$