16/11/23
Day 45

$$= \cos\theta \begin{vmatrix} r\cos\theta\ \cos\phi & -\sin\sin\phi \\ r\cos\theta\ \sin\phi & r\sin\theta\ \cos\phi \end{vmatrix} + r\sin\theta$$

$$\begin{vmatrix} \sin\theta\ \cos\phi & -\sin\sin\phi \\ \sin\theta\ \sin\phi & r\sin\theta\ \cos\phi \end{vmatrix}$$

$= \cos\theta\,[r^2\sin\theta\cos\theta\cos^2\phi + r^2\cos\theta\sin\theta\sin^2\phi]$

$+ r\sin\theta\,[r\sin^2\theta\cos^2\phi + r\sin^2\theta\sin^2\phi]$

$= \cos\theta\,[r^2\sin\theta\cos\theta\,(\cos^2\phi + \sin^2\phi)] + r\sin\theta$

$[r\sin^2\theta\,(\cos^2\phi + \sin^2\phi)]$

$= \cos\theta\,[r^2\sin\cos\theta\,(1)] + r\sin\theta\,[r\sin^2\theta\,(1)]$

$= r^2\sin\theta\cos^2\theta + r^2\sin^3\theta$

$= r^2\sin\theta\,(\cos^2\theta + \sin^2\theta)$

$$\frac{\partial(x, y, z)}{\partial(r, \theta, \phi)} = r^2\sin\theta$$

Qs. If $x + y + z = u$, $y + z = uv$, $z = uvw$ Prove that $\partial(x, y, z)/\partial(u, v, w) = u^2 v$

Soln: Given :

$y + z = uv$

$y = uv - z$

$y = uv - uvw$

- $\partial y / \partial u = v - vw$
- $\partial y / \partial v = u - uw$
- $\partial y / \partial w = -uv$

$x + y + z = u$

$\doteq u - (uv - uvw) - uvw$

$= u - uv + uvw - uvw$

$x = u - uv$

- $\partial x / \partial u = 1 - v$
- $\partial x / \partial v = -u$
- $\partial x / \partial w = 0$

$z = uvw$

- $\partial z / \partial u = vw$
- $\partial z / \partial v = uw$
- $\partial z / \partial w = uv$

$$\frac{\partial(x,y,z)}{\partial(u,v,w)} = \begin{vmatrix} 1-v & -u & 0 \\ v-vw & u-uw & -uv \\ vw & uw & uv \end{vmatrix}$$

$$= (1-v)\begin{vmatrix} u-uw & -uv \\ uw & uv \end{vmatrix} + u \begin{vmatrix} v-vw & -uv \\ vw & uv \end{vmatrix} + 0$$

$$= (1-v)\left[(u-uw)uv + u^2vw\right] + u\left[(v-vw)uv + uv^2w\right]$$

$$= (1-v)\left[u^2v - u^2vw + u^2vw\right] + u\left[uv^2 - uv^2w + uv^2w\right]$$

$$= (1-v)\left[u^2v\right] + u\left[uv^2\right]$$

$$= (1-v)u^2v + u^2v^2 = u^2v - u^2v^2 + u^2v^2$$

$$\partial(x,y,z)/\partial(u,v,w) = u^2v$$

#2 Maxima & Minima

• Working rule :

Step 1 : To find f_x & f_y

Step 2 : To find $A = f_{xx}$, $B = f_{xy}$, $C = f_{yy}$

Step 3 : To find stationary point $f_x = 0$; $f_y = 0$

Step 4 : To find maximum (or) minimum

$AC - B^2 > 0$	$A < 0$ (or) $B < 0$	maximum point
$AC - B^2 > 0$	$A > 0$ (or) $B > 0$	Minimum point
$AC - B^2 < 0$	$A > 0$ (or) $B > 0$	Saddle point
$AC - B^2 = 0$		Inconclusion

Step 5 : To find maximum value or minimum value

$f(a, b) = ?$