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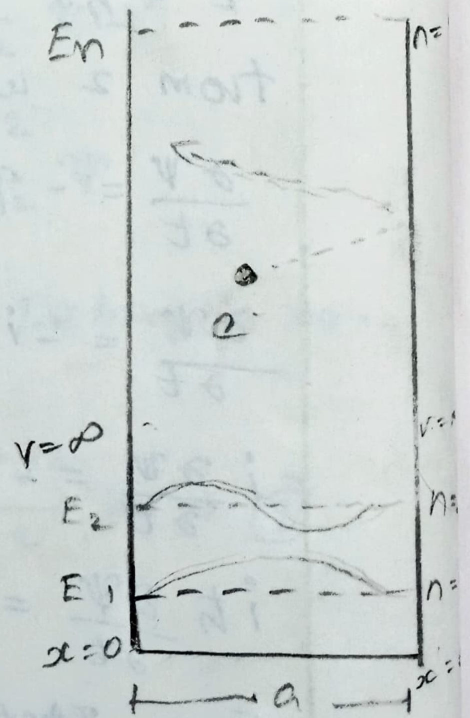
Application of schrondilger's wave Equation

Pay 54



- One dimension

i) Potential Energy

a) $0 < x < a \Rightarrow v = 0$

b) $0 \geq x \geq a \Rightarrow v = \infty$

ii) Schrondinger one dimensional

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}(E-v)\psi = 0 \rightarrow ①$$

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}(E)\psi = 0 \rightarrow ②$$

From ②

$$\frac{d^2\psi}{dx^2} + K^2\psi = 0 \rightarrow ③$$

Since $K^2 = \frac{2m}{\hbar^2}E \rightarrow ④$

Soln for Eqn ③

$\psi(x) = A \sin Kx + B \sin Kx$

$\therefore x = 0 ; \psi = 0$

$0 = A \sin K0 + B \sin K0$

$0 = 0 + B \Rightarrow \therefore B = 0 \rightarrow ⑥$

$\therefore x = a ; \psi = 0$

$0 = A \sin K0 + B \cos Ka$

$0 = A \sin K \cdot a$

$A \neq 0$

$\sin Ka = 0$

$\sin \pi = 0 \Rightarrow \sin n\pi = 0$

Where, $n = 1, 2, 3, \dots$

$Ka = n\pi$

$K = n\pi / a$

$K^2 = n^2\pi^2 / a^2 \rightarrow ⑦$

we Known

$K^2 = 2m/\hbar^2 \cdot E$

$$K^2 = \frac{2m}{h^2/4\pi^2} E = \frac{8\pi^2 m}{h^2} \cdot E \rightarrow ⑧$$

$$\frac{8\pi^2 m}{h^2} E = \frac{n^2\pi^2}{a^2}$$

$$E = \frac{n^2 h^2 \pi^2}{8\pi^2 m a^2}$$

*. $E_n = n^2 h^2 / 8 m a^2 \rightarrow ⑨$

$$\psi(x) = A \sin \frac{n\pi}{a} \cdot x \rightarrow ⑩$$

* Energy of the electron

from eqn 8 $\Rightarrow K^2 = 2m/\hbar^2 = \frac{2mE}{h^2/4\pi^2}$ $\because$ Eqn 8.

$K^2 = 8\pi^2 mE / h^2 \rightarrow$ (11)

Squaring $\Rightarrow K = n\pi/a \Rightarrow K^2 = n^2\pi^2/a^2 \rightarrow$ (12)

Eqn (11) and (12)

$$\frac{8\pi^2 mE}{h^2} = \frac{n^2\pi^2}{a^2}$$

* Allowed energies In General

$E_n = n^2h^2/8ma^2$ $E_n = n^2E_1 \rightarrow$ (13)

* Normalisation of wave function

if $P = 1$

$$P = \int_0^a A^2 \sin^2 \frac{n\pi x}{a}\, dx = 1 \rightarrow (14)$$

$$A^2 \int_0^a \frac{1 - \cos 2n\pi x/a}{2}\, dx = 1$$

$$A^2 \left[\frac{x}{2} - \frac{1}{2}\frac{\sin 2n\pi x/a}{2n\pi/a}\right]_0^a = 1$$

$$A^2 \left[\frac{1}{2} - \frac{1}{2}\frac{\sin 2n\pi a/a}{2n\pi/a}\right] = 1$$

$$A^2 \left[\frac{1}{2} - \frac{1}{2}\frac{\sin 2n\pi}{2n\pi/a}\right] = 1 \rightarrow (15)$$

$\because \sin n\pi = 0 \therefore \sin 2n\pi = 0$

$$\frac{A^2 a}{2} = 1 \Rightarrow A = \sqrt{2/a}$$

$$\psi = \sqrt{2/a}\,\sin\left[\frac{n\pi}{a}\right]x.$$