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Q2. Use Cayley Hamilton Theorem find the value of the matrix given by $f(A) = A^8 - 5A^7 + 7A^6 -$

i) $3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I$

ii) $g(A) = A^8 - 5A^7 + 7A^6 - 3A^5 + 8A^4 - 5A^3 + 8A^2 -$

$2A + I$

If the matrix $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$

Soln: $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$

i)

$f(A) = A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 -$
$2A + I$

- Characteristic eqn: $\lambda^3 - S_1\lambda^2 + S_2\lambda - S_3 = 0$

$S_1 = 5$

$S_2 = \begin{vmatrix} 1 & 0 \\ 1 & 2 \end{vmatrix} + \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} + \begin{vmatrix} 2 & 1 \\ 0 & 1 \end{vmatrix}$

$= (2-0) + (4-1) + (2-0) = 2 + 3 + 2 = 7$

$S_3 = 2(2-0) - 1(0-0) + 1(2-0)$

$= 2(2) - 1 = 4 - 1 = 3$

$$\lambda^3 - 5\lambda^2 + 7\lambda - 3 = 0$$

let $\lambda = A$

$$A^3 - 5A^2 + 7A - 3I = 0$$

$$
\begin{array}{r|l}
 & A^5 + A \\
\hline
A^3 - 5A^2 + 7A - 3I & A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I \\
 & A^8 - 5A^7 + 7A^6 - 3A^5 \\
 & (-)\ (+)\ (-)\ (+) \\
\hline
 & A^4 - 5A^3 + 8A^2 - 2A + I \\
 & A^4 - 5A^3 + 7A^2 - 3A \\
 & (-)\ (+) \\
\hline
 & A^2 + A + I
\end{array}
$$

$$f(A) = (A^5 + A)(A^3 - 5A^2 + 7A - 3I) + (A^2 + A + I)$$

$$= (A^5 + A) \times 0 + (A^2 + A + I)$$

$$f(A) = A^2 + A + I$$

$$= \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix} \times \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix} + \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 4 & 4 \\ 0 & 1 & 1 \\ 4 & 4 & 5 \end{bmatrix} + \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$f(A) = \begin{bmatrix} 8 & 5 & 5 \\ 0 & 3 & 0 \\ 5 & 5 & 8 \end{bmatrix}$$