

①  
2m  
13/10/23

The eigen values and eigen vectors of  $2 \times 2$  matrix is given by  $\lambda_1 = 8$ ;  $\lambda_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ ,  $\lambda_2 = 4$ ;  $x_2 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$   
Find the corresponding matrix.

Day 23

Soln:  $D = N^T A N$

$$N^T A N = D$$

$$A = N^T D N$$

$$N = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$$

$$N^T = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$$

$$D = \begin{bmatrix} 8 & 0 \\ 0 & 4 \end{bmatrix} \therefore D = \begin{bmatrix} 8 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} \rightarrow \text{normal}$$

$$A = N^T D N$$

$$= \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{bmatrix} \begin{bmatrix} 8 & 0 \\ 0 & 4 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{bmatrix}$$

$$= \begin{bmatrix} 8/\sqrt{2} & 4/\sqrt{2} \\ 8/\sqrt{2} & -4/\sqrt{2} \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{bmatrix}$$

$$A = \begin{bmatrix} 8/\sqrt{2} & 4/\sqrt{2} \\ 8/\sqrt{2} & -4/\sqrt{2} \end{bmatrix} \times \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{bmatrix}$$

$$= \begin{bmatrix} 8/2 + 4/2 & 8/2 - 4/2 \\ 8/2 - 4/2 & 8/2 + 4/2 \end{bmatrix} = \begin{bmatrix} 12/2 & 4/2 \\ -4/2 & 12/2 \end{bmatrix}$$

$$A = \begin{bmatrix} 6 & 2 \\ 2 & 6 \end{bmatrix}$$

②

Find the index and signature of the Quadratic form.  $x_1^2 + 2x_2^2 - 3x_3^2$

Index = number of positive term  
= 2

Signature = number of +ve term - number of -ve term  
= 2 - 1 = 1