12/10/23

Day 22

Q5. ~~Two~~ Two of the Eigen values of $A = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$ are 3 and 6. Find the Eigen value of A^{-1}.

Soln:

$\lambda_1 = 3$; $\lambda_2 = 6$; $\lambda_3 = ?$

Eigen Valuse :

Sum = $3 + 5 + 3$.

$\lambda_1 + \lambda_2 + \lambda = 11$

$3 + 6 + \lambda_3 = 11$

$\lambda_3 = 11 - 9 = 2$

3, 6, 2 are Eigen value of A

$\frac{1}{3}, \frac{1}{6}, \frac{1}{2}$ are Eigen value of A^{-1}.

Q6. The product of two eigen value of matrix $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ is 16. Find the 3rd Eigen Value.

Soln: $\lambda_1, \lambda_2, \lambda_3$ are eigen values of A.

Given,

Product = 16

$\lambda_1 \lambda_2 = 16$

Product of the eigen value = $|A|$

$\lambda_1 \times \lambda_2 \times \lambda_3 = |A|$

$$16 \times \lambda_3 = \begin{vmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{vmatrix}$$

$$= 6\begin{vmatrix} 3 & -1 \\ -1 & 3 \end{vmatrix} + 2\begin{vmatrix} -2 & -1 \\ 2 & 3 \end{vmatrix} + 2\begin{vmatrix} -2 & 3 \\ 2 & -1 \end{vmatrix}$$

$= 6(9-1) + 2(-6+2) + 2(2-6)$

$= (6 \times 8) + 2(-4) + 2(-4)$

$= 48 - 8 - 8 = 40 - 8$

$16 \times \lambda_3 = 32 \Rightarrow \lambda_3 = \cancel{32}^{2}/\cancel{16}_{1}$ $\therefore \lambda_3 = 2$

Q7. If two eigen value of matrix $A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$ are equal to 1 each find the eigen value of A^{-1}.

Soln: Given,

$$A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$$

$\lambda_1 = 1$; $\lambda_2 = 1$; $\lambda_3 = ?$

Sum $= 2 + 3 + 2$

$\lambda_1 + \lambda_2 + \lambda_3 = 7$

$1 + 1 + \lambda_3 = 7$

$2 + \lambda_3 = 7$

$\lambda_3 = 7 - 2 = 5$

1, 1, 5 are Eigen value of A :

1, 1, 1/5 are Eigen value of A^{-1}

Q8. Write the uses of Cayley Hamilton theorem.

* To find the A^{-1}.
* To find higher powe : A^4, A^5 ....

Q9. State CHT.

" Every square matrix satisfies its own characteristic equation "

Q10. Find the eigen value and eigen vectors of the matrix $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$

Soln: Given,

$$A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$$

i) Char eqn.

$S_1 = -1$

$S_2 = (0 - 12) + (0 - 3) + (-2 - 4) = 12 + 3 + 6 = 21$

$S_3 = -2(0-12)+2(0-6)-3(-4+1)$
$= -2(-12)-2(6)-3(-3)$
$= 24+12+9 = 45$

$\therefore \lambda^3 - (-1)\lambda^2 + (21)\lambda - 45 = 0$

$\lambda^3 + \lambda^2 - 21\lambda - 45 = 0$

Eigen Values are :

$\lambda = 5, -3, -3$

To find eigen vector

$(A - \lambda I)X = 0$

$$\left[\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\right] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -2-\lambda & 2 & -3 \\ 2 & 1-\lambda & -6 \\ -1 & -2 & 0-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Case 1 : $\lambda = 5$

$$\begin{bmatrix} -2-5 & 2 & -3 \\ 2 & 1-5 & -6 \\ -1 & -2 & 0-5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -7 & 2 & -3 \\ 2 & -4 & -6 \\ -1 & -2 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$-7x_1 + 2x_2 - 3x_3 = 0$
$2x_1 - 4x_2 - 6x_3 = 0$
$-x_1 - 2x_2 - 5x_3 = 0$

$$\frac{x_1}{\begin{vmatrix} -4 & -6 \\ -2 & -5 \end{vmatrix}} + \frac{x_2}{\begin{vmatrix} -6 & 2 \\ -5 & -1 \end{vmatrix}} + \frac{x_3}{\begin{vmatrix} 2 & -4 \\ -1 & -2 \end{vmatrix}}$$

$$\frac{x_1}{(20-12)} + \frac{x_2}{(6+10)} + \frac{x_3}{(-4+4)}$$

$$\frac{x_1}{8} = \frac{x_2}{16} = \frac{x_3}{-8} \Rightarrow \frac{x_1}{1} = \frac{x_2}{2} = \frac{x_3}{-1}$$

$x_1 = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$

case 2: $\lambda = -3$

$$\begin{bmatrix} -2+3 & 2 & -3 \\ 2 & 1+3 & -6 \\ -1 & -2 & 0+3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & -3 \\ 2 & 4 & -6 \\ -1 & -2 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$x_1 + 2x_2 - 3x_3 = 0$

$2x_1 + 4x_2 - 6x_3 = 0$

$-x_1 - 2x_2 + 3x_3 = 0$

Put $x_1 = 0$

$x_1 + 2x_2 - 3x_3 = 0$

$0 + 2x_2 - 3x_3 = 0$

$2x_3 = 3x_3$

$\frac{x_2}{3} = \frac{x_3}{2}$

$$x_2 = \begin{bmatrix} 0 \\ 3 \\ 2 \end{bmatrix}$$

case 3 : $\lambda = -3$

let $x_2 = 0$

$x_1 + 2x_2 - 3x_3 = 0$

$x_1 + 0 - 3x_3 = 0$

$-3x = -x_1$

$3x_3 = x_1$

$\frac{x_3}{1} = \frac{x_1}{3}$

$$x_3 = \begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix}$$