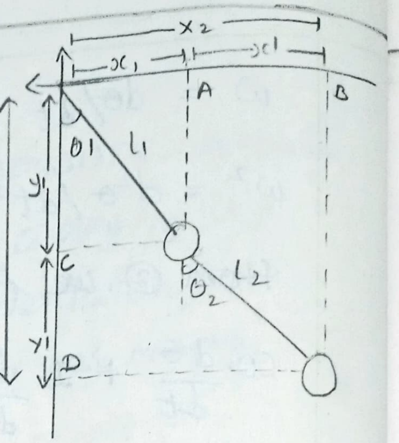
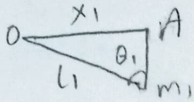
12/10/23 Day 21

Double Pendulum

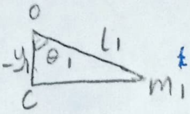


a) Displacement :



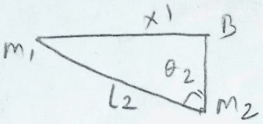
$$\sin\theta_1 = \frac{x_1}{l_1}$$

$$x_1 = l_1 \sin\theta_1 \quad (1)$$



$$\cos\theta_1 = -y_1/l_1$$

$$+y_1 = -l_1 \cos\theta_1 \quad (2)$$



$$x_2 = x_1 + x' \quad (3)$$

$$\sin\theta_2 = x'/l_2$$

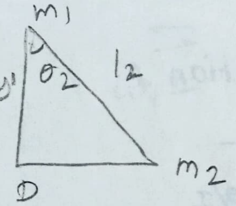
$$x' = l_2 \sin\theta_2 \rightarrow (4)$$

Sub eqn (4) in (3)

$$x_2 = x_1 + l_2 \sin\theta_2 \rightarrow (5)$$

Sub eqn (1) in (5)

$$x_2 = l_1 \sin\theta_1 + l_2 \sin\theta_2 \rightarrow (6)$$



$$y_2 = y_1 + y' \rightarrow (7)$$

$$\cos\theta_2 = -y'/l_2$$

$$-y' = -l_2 \cos\theta_2 \rightarrow (8)$$

Sub eqn (8) in (7)

$$y_2 = y_1 + (-l_2 \cos\theta_2) = y_1 - l_2 \cos\theta_2 \rightarrow (9)$$

Sub eqn (2) in (9)

$$y_2 = -l_1 \cos\theta_1 - l_2 \cos\theta_2 \rightarrow (10)$$

* Eqn (1), (2), (6) and (10) ⇒ displacement

b) Velocity :

- Diff eqn (1)

$$V_{x1} = \frac{dx_1}{dt} = \frac{d(l_1 \sin\theta_1)}{dt} \qquad V_{x1} = l_1 \cos\theta_1 \frac{d\theta_1}{dt}$$

since $\frac{d\theta_1}{dt} = \dot{\theta}$ ⇒ $V_{x1} = l_1 \cos\theta_1 \dot{\theta}_1 \rightarrow (11)$

- Diff eqn (2)

$$V_{y1} = \frac{dy_1}{dt} = \frac{d(-l_1 \cos\theta_1)}{dt} \qquad V_{y1} = l_1 \sin\theta_1 \frac{d\theta_1}{dt}$$

Since $\frac{d\theta_1}{dt} = \dot{\theta}$ ⇒ $V_{y1} = l_1 \sin\theta_1 \dot{\theta}_1 \rightarrow (12)$

- Diff eqn (6)

$$V_{x2} = \frac{dx_2}{dt} = \frac{d(l_1 \sin\theta_1 + l_2 \sin\theta_2)}{dt}$$

$$V_{x2} = l_1 \cos\theta_1 \dot{\theta}_1 + l_2 \cos\theta_2 \dot{\theta}_2 \rightarrow (13)$$

- Diff eqn (10)

$$V_{y2} = \frac{dy_2}{dt} = \frac{d(-l_1 \cos\theta_1 - l_2 \cos\theta_2)}{dt}$$

$$V_{y2} = l_1 \sin\theta_1 \dot{\theta}_1 + l_2 \cos\theta_2 \dot{\theta}_2 \rightarrow (14)$$

* Eqn (11), (12), (13) and (14) ⇒ Velocity

c) Kinetic energy

$$T = \sum_{i=1}^{2} \frac{1}{2} m_i (V_{xi}^2 + V_{yi}^2)$$

$$= \frac{1}{2} m_1 (V_{x1}^2 + V_{y1}^2) + \frac{1}{2} m_2 (V_{x2}^2 + V_{y2}^2) \rightarrow (15)$$

$$= \frac{1}{2} m_1 (l_1^2 \sin\theta_1 \dot{\theta}_1 + l_2^2 \sin\theta_2 \dot{\theta}_2) + \frac{1}{2} m_2 [(l_1 \cos\theta_1 \dot{\theta}_1 + l_2 \cos\theta_2 \dot{\theta}_2)^2 + (l_1 \sin\theta_1 \dot{\theta}_1 + l_2 \sin\theta_2 \dot{\theta}_2)^2] \rightarrow (16)$$

d) Potential energy

$$V = m_1 g y_1 + m_2 g y_2 \rightarrow (17)$$

$$V = m_1 g l_1 \cos\theta_1 - m_2 g (l_1 \cos\theta_1 + l_2 \cos\theta_2) \rightarrow (18)$$

e) Lagrangian ⇒ $L = T - V$

$$L = \frac{1}{2} m_1 (l_1^2 \sin\theta_1 \dot{\theta}_1 + l_2^2 \sin\theta_2 \dot{\theta}_2) + \frac{1}{2} m_2 [(l_1 \cos\theta_1 \dot{\theta}_1 + l_2 \cos\theta_2 \dot{\theta}_2) + (l_1 \sin\theta_1 \dot{\theta}_1 + l_2 \sin\theta_2 \dot{\theta}_2)^2] - m_1 g l_1 \cos\theta_1 - m_2 g (l_1 \cos\theta_1 + l_2 \cos\theta_2)$$