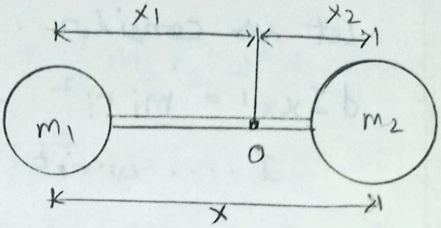
Moment of inertia of a Diatomic molecule.



$x = x_1 + x_2 \rightarrow (1)$

Centre of mass of syst

$m_1 x_1 = m_2 x_2 \rightarrow (2)$

from eqn (1)

$x_2 = x - x_1 \rightarrow (3)$

Sub eqn (3) in (2)

$m_1 x_1 = m_2 (x - x_1)$

$m_1 x_1 = m_2 x - m_2 x_1$ (or) $m_1 x_1 + m_2 x_1 = m_2 x$

$(m_1 + m_2) x_1 = m_2 x$

$x_1 = m_2 x / (m_1 + m_2) \rightarrow (4)$

from eqn (1)

$x_1 = x - x_2 \rightarrow (5)$

Subst eqn (5) in (2)

$m_1 x_1 = m_2 x_2 \Rightarrow m_1 (x - x_2) = m_2 x_2$

$x_2 = m_1 x / (m_1 + m_2) \rightarrow (6)$

a) Moment of Inertia.

$I = m_1 x_1^2 + m_2 x_2^2 \rightarrow (7)$

Sub eqn (4) and (6) in (7)

$$I = m_1 \left[\frac{m_2 x}{m_1 + m_2}\right]^2 + m_2 \left[\frac{m_1 x}{(m_1 + m_2)}\right]^2$$

$$I = \frac{m_1 m_2}{m_1 + m_2} x^2 \rightarrow (8)$$

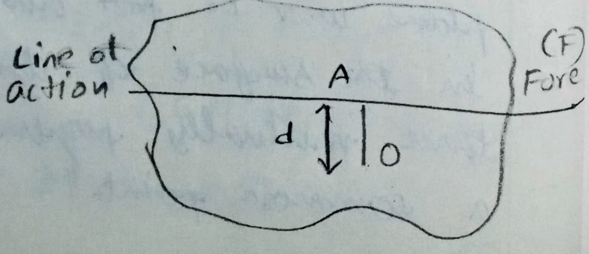
Since $\mu = m_1 m_2 / m_1 + m_2$

we can write

$I = \mu x^2 \rightarrow (9)$

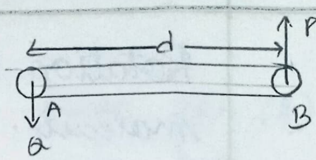
b) Moment

$M_O = F \times d$



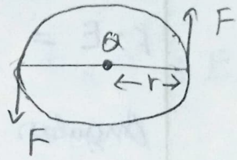
c) Couple:

Couple $= M_A = M_B = P \times d$

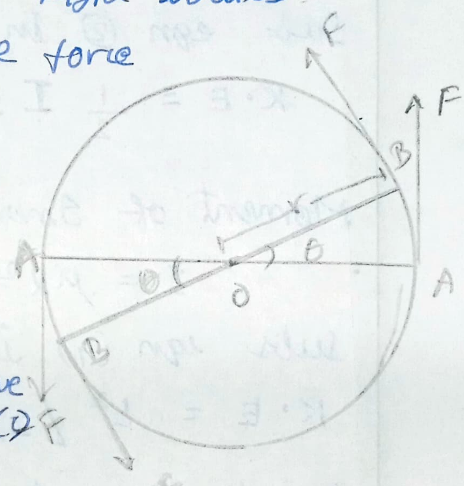


d) Torque

τ = Force x Radius



Rotation Dynamics of Rigid bodies



- F - The distance moved by the force
- θ - length of the arc AB
- work done (couple) $= 2Fr\theta$
- Length of arc AB $= r\theta$
- Work done (single force) $= Fr\theta$
- Fr = moment of couple / torque
- work done by the torque $= \tau\theta$
- $\tau = dL/dt$

Conservation of Angular momentum

The relation between torque (τ) & Angular momentum (L)

$$\tau = dL/dt$$

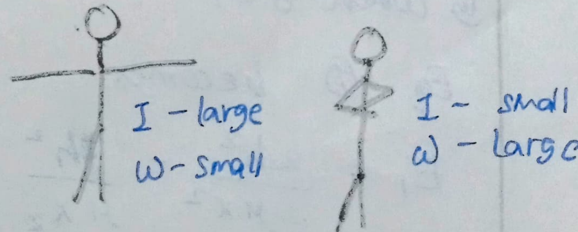
If no net external force acting on the body

$$\tau_{net} = 0$$

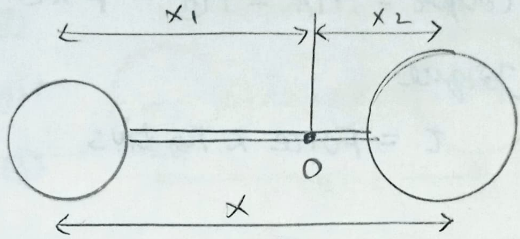
$$\tau_{net} = 0 \Rightarrow dL/dt = 0$$

$$I \propto \frac{1}{\omega}$$

Eg: Ice dancer.



Rotation Energy State of a Rigid Diatomic molecule.



$K \cdot E = \frac{1}{2} I \omega^2 \rightarrow ①$

Angular momentum

$L = I\omega$ (or) $\omega = L/I \rightarrow ②$

Sub eqn ② in eqn ①

$K \cdot E = \frac{1}{2} I \frac{L^2}{I^2} \Rightarrow = \frac{1}{2} \frac{L^2}{I} = \frac{L^2}{2I} \rightarrow ③$

Moment of Inertia

$I = \mu x^2 \rightarrow ④$ ∴ μ is the reduced mass.

Subs eqn (4) in (3)

$K \cdot E = L^2 / 2\mu x^2 \rightarrow ⑤$

Based on quantum theory.

The angular momentum 'z'

$\Rightarrow L = \sqrt{J(J+1)}\, h \rightarrow ⑥$

Subs eqn (6) in (5)

$K \cdot E\ (E_J) = (\sqrt{J(J+1)})^2 h^2 / 2\mu x^2$

$E_J = J(J+1) h^2 / 2\mu x^2 \rightarrow ⑦$

* Special cases :

i) When $J = 0$

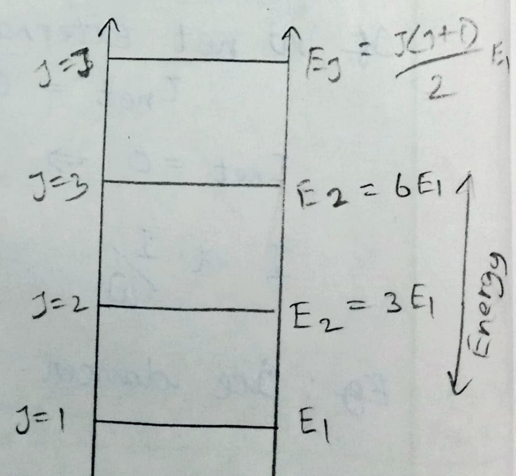
Eq ⑦ $\Rightarrow E_0 = 0$

ii) when $J = 1$

Eq ⑦ becomes

$E_1 = \frac{2h^2}{2\mu x^2} = \frac{h^2}{\mu x_2}$

$\rightarrow ⑧$



iii) When $J = 2$

Eqn ⑦ becomes

$E_2 = 2(3) h^2 / 2\mu x^2$

$E_2 = 6h^2 / 2\mu x^2 \cdot \rightarrow$ ⑨

From eqn (8) and (9)

$\Rightarrow E_2 = 3 E_1$

iv) When $J = 3$

Eqn (7) becomes

$E_3 = 3(4) h^2 / 2\mu x^2 = 6h^2 / \mu x^2$

$= 6 E_1$

$$E_J = \frac{J(J+1)}{2} E_1$$

∴ Quantized and discrete