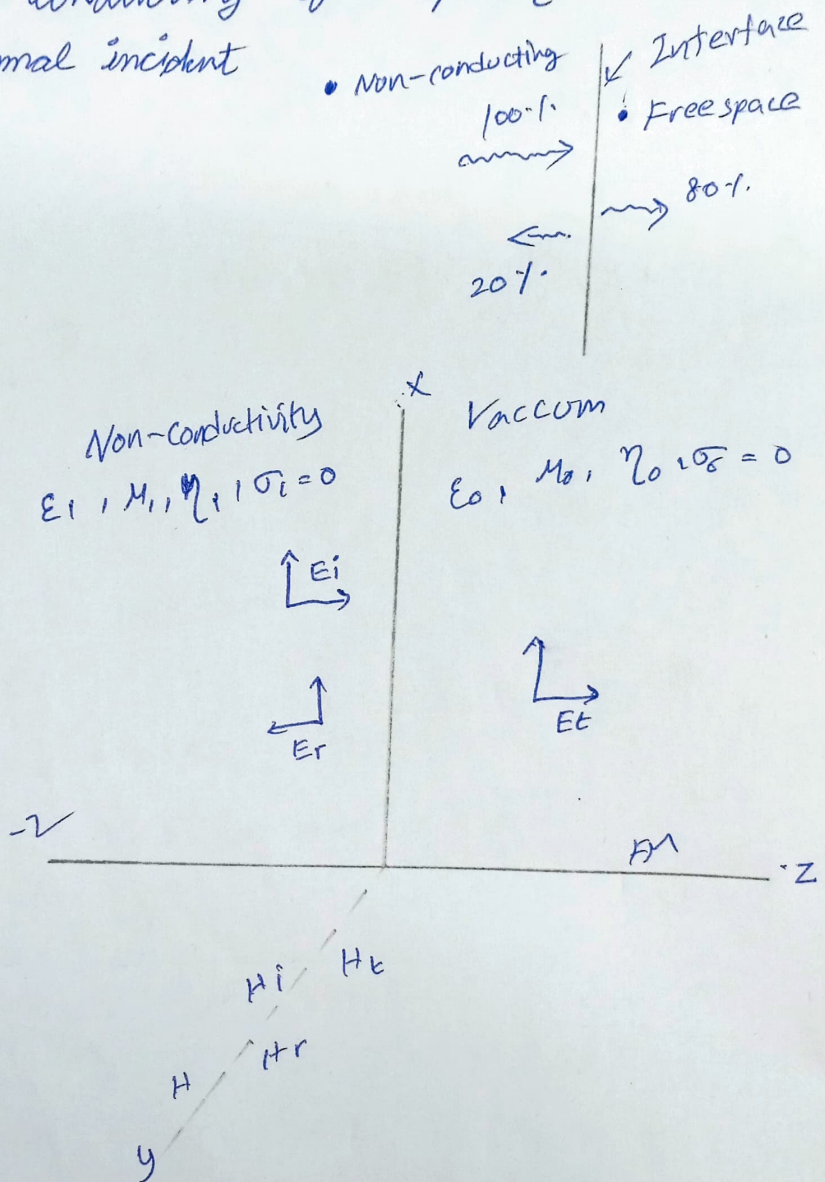


25/10/23

Reflection & Transmission of EM waves in non conducting - free space (vacuum) normal incident



$$E_i + E_r = E_t \rightarrow ①$$

$$H_i + H_r = H_t \rightarrow ②$$

$$\eta_1 = \frac{E_i}{H_i} \text{ (or) } H_i = \frac{E_i}{\eta_1} \rightarrow ③$$

From ②, we have

$$\frac{E_i}{\eta_1} + \frac{E_r}{\eta_1} = \frac{E_t}{\eta_0} \Rightarrow \frac{1}{\eta_1} [E_i + E_r] = \frac{E_t}{\eta_0}$$

$$\Rightarrow \frac{1}{\eta_1} [E_i - E_r] = E_t / \eta_0 \quad (E_r \text{ negative - reflection})$$

$$E_i - E_r = \frac{\eta_1}{\eta_0} E_t \rightarrow (4)$$

$$(3) + (4)$$

$$E_i + E_r + E_i - E_r = E_t + \frac{\eta_1}{\eta_0} E_t$$

$$2E_i = E_t + \frac{\eta_1}{\eta_0} E_t \Rightarrow = E_t \left[\frac{\eta_0 + \eta_1}{\eta_0} \right]$$

$$\textcircled{3} \rightarrow E_t = \left[\frac{2\eta_0}{\eta_0 + \eta_1} \right] E_i \Rightarrow \frac{E_t}{E_i} = \frac{2\eta_0}{\eta_0 + \eta_1} \rightarrow (6)$$

*
Transmitt

$$T = I_t / I_{i2} \Rightarrow I = \epsilon_0 / 2\eta_0 \text{ (free space)}$$

$$I_t = E_t^2 / 2\eta_0 \rightarrow (7)$$

$$I_t = E_i^2 / 2\eta_1 \rightarrow (8)$$

$$T = \frac{E_t^2 / 2\eta_0}{E_i^2 / 2\eta_1} = \left(\frac{E_t}{E_i} \right)^2 \frac{\eta_1}{\eta_0}$$

$$T = \left(\frac{2\eta_0}{\eta_0 + \eta_1} \right)^2 \frac{\eta_1}{\eta_0} \Rightarrow T = \frac{4\eta_0}{(\eta_0 + \eta_1)^2} \cdot \frac{\eta_1}{\eta_0}$$

$$T = 4\eta_0\eta_1 / (\eta_0 + \eta_1)^2 \rightarrow (9)$$

From (1)

$$E_i + E_r = \frac{2\eta_0}{\eta_0 + \eta_1} \cdot E_i$$

$$E_r = \frac{2\eta_0}{\eta_0 + \eta_1} E_i - E_i$$

$$= \left[\frac{2\eta_0}{\eta_0 + \eta_1} - 1 \right] E_i$$

$$E_r = \frac{2\eta_0 - \eta_0 - \eta_1}{\eta_0 + \eta_1} \cdot E_i \Rightarrow \frac{\eta_0 - \eta_1}{\eta_0 + \eta_1} = \frac{E_r}{E_i} \rightarrow (10)$$

Reflected

$$R = \frac{I_r}{I_i} = \frac{E_r^2 / 2\eta_1}{E_i^2 / 2\eta_1} = \left(\frac{E_r}{E_i} \right)^2$$

$$R = (\eta_0 - \eta_1)^2 / (\eta_0 + \eta_1)^2 \rightarrow (11)$$

$$= \frac{(\eta_0^2 + \eta_1^2 - 2\eta_0\eta_1)}{(\eta_0 + \eta_1)^2}$$

$$T + R = 1$$

$$= \frac{4\eta_0\eta_1}{(\eta_0 + \eta_1)^2} + \frac{\eta_0^2 - \eta_1^2 - 2\eta_0\eta_1}{(\eta_0 + \eta_1)^2}$$

$$= \frac{4\eta_0\eta_1 + \eta_0^2 - \eta_1^2 - 2\eta_0\eta_1}{(\eta_0 + \eta_1)^2}$$

$$\Rightarrow \frac{(\eta_0 + \eta_1)^2}{(\eta_0 + \eta_1)^2} = 1 \quad \therefore T + R = 1$$

total *