

18/10/23

Differentiate

Day 28

Q1. $f(x) = x^{1000}$, find $f'(x)$

i) Soln:

$$\begin{aligned}\text{Given, } f(x) &= x^{1000} \\ f'(x) &= 1000x^{999}\end{aligned}$$

ii) $y = \sqrt{x}$.

Soln: Given, $y = \sqrt{x}$

$$\begin{aligned}y &= x^{1/2} \\ \frac{dy}{dx} &= \frac{1}{2} x^{1/2-1} = \frac{1}{2} x^{-1/2}\end{aligned}$$

$$y' = \frac{1}{2} \frac{1}{x^{1/2}} = \frac{1}{2} \frac{1}{\sqrt{x}} = \frac{1}{2\sqrt{x}}$$

iii) $y = x^8 + 12x^5 - 4x^4 + 10x^3 - 6x + 7$

$$\frac{dy}{dx} = 8x^7 + 60x^4 - 16x^3 + 30x^2 - 6$$

iv) $y = x^2(1-2x)$

Soln: $x^2 - 2x^3$

$$\frac{dy}{dx} = 2x - 6x^2 = 2x(1-3x)$$

$$v) f(x) = 10x^{10} + 3x^5 - x$$

$$f(x) = 10x^{10} + 5x^5 - x$$

$$= 100x^9 + 25x^4$$

$$f'(x) = 100x^9 + 15x^4 - x^0 + (-1)x^{-1} = 0$$

$$vi) f(x) = 2x - 5x^{3/4}$$

$$f'(x) = 2 - \frac{15}{4}x^{-1/4} = 2 - \frac{15}{4x^{1/4}} = \frac{.8x^{1/4} - 15}{4x^{-1/4}}$$

$$vii) y = 3x^2 - x^3$$

$$= 6x - 3x^2 = 3x(2-x)$$

Q2. Equation of motion of a particle is $s = t^3 - 3t$, where s is in meters and t is in seconds. Find the velocity and acceleration as Functions of t .

Soln: Given, $s = t^3 - 3t$

$$a) \bullet v = \frac{ds}{dt} = 3t^2 - 3$$

$$\bullet A = \frac{dv}{dt} = 3(2t) - 0 = 6t$$

b) If $t = 2s$ [The acceleration after 2s]

$$A = 6t = 6 \times 2 = 12 \text{ m/s}^2$$

c) F. The acceleration when the velocity is 0

$$A = \left(\frac{dv}{dt}\right)_{v=0}$$

$$A = (6t) = (12t) = 6$$

$$\cancel{3t^2 - 3} \quad v = 0$$

$$\cancel{3t^2 - 3} = 0 \Rightarrow 3(t^2 - 1) = 0 \Rightarrow t^2 - 1 = 0$$

$$t^2 = 1 \Rightarrow t = \pm 1$$

Q3. Find the points on the curve $y = 2x^3 + 3x^2 - 12x + 1$ where the tangent is horizontal

Soln :

Given,

$$y = 2x^3 + 3x^2 - 12x + 1$$

$$\frac{dy}{dx} = 6x^2 + 6x - 12$$

$$\text{Put, } 6x^2 + 6x - 12 = 0$$

$$x = 1, -2$$