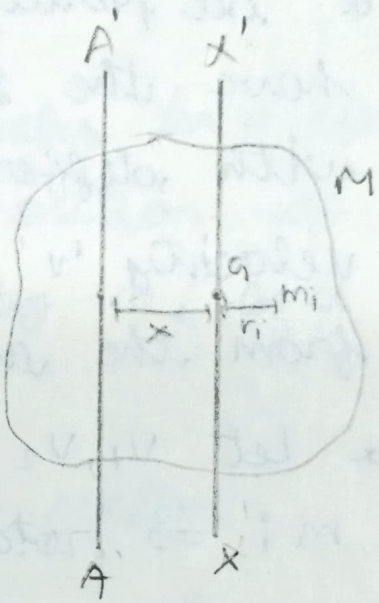
06/10/23

Theorems

1. Parallel axis theorem



- Let us consider a body of mass 'M' .....

The moment of I w.r.t xx'

$$dI_{xx'} = m_i r_i^2 \rightarrow (1)$$

..... Entire body

$$I_{xx'} = \Sigma dI_{xx'} = m_i r_i \rightarrow (2)$$

..... w.r.t Ax'

$$dA_{xx'} = m_i (r_i + x)^2 \rightarrow (3)$$

..... Entire body

$$I_{AA'} = \Sigma dI_{AA'} = \Sigma m_i (r_i + x)^2 \rightarrow (4)$$

$$I_{AA'} = \Sigma m_i r_i^2 + \Sigma 2 m_i r_i x + \Sigma m_i x^2 \rightarrow (5)$$

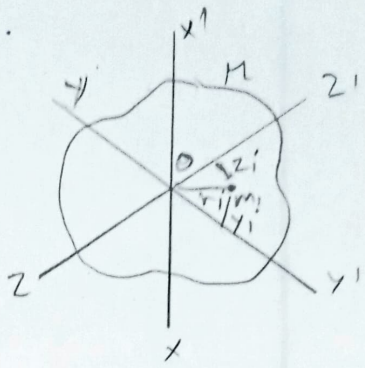
Sub eqn (2) in (4).

$$I_{AA'} = I_{xx'} + 2x \Sigma m_i r_i + M x^2 \rightarrow (5)$$

where $M = m_i$ (CM for R.B $\Rightarrow m_i r_i = 0$)

$$I_{AA'} = I_{xx'} + M x^2 \rightarrow (6)$$

2. Perpendicular axis theorem.



Let us consider.

$dI_{xx'} = m_i r_i^2 \rightarrow (1)$

I .... w.r.t xx'

$I_{xx'} = \Sigma m_i r_i^2 \rightarrow (2)$

From fig $r_i^2 = y_i^2 + z_i^2 \rightarrow (3)$

Sub (3) in (2)

$I_{xx'} = \Sigma m_i (y_i^2 + z_i^2)$

$I_{xx'} = \Sigma m_i y_i^2 + \Sigma m_i z_i^2 \rightarrow (4)$

The moment of I of a thin plane w.r.t yy' axis is $I_{yy'} = \Sigma m_i y_i^2$

The . . . . . -

zz' axis is $I_{zz'} = \Sigma m_i z_i^2$

$I_{xx'} = I_{yy'} + I_{zz'}$

1. PAT - The moment of inertia respect to any axis is equal to the sum of moment of inertia with respect to parallel axis passing through the centre of mass and the product of mass and square of the perpendicular distance between the parallel axis.

2. PE·A·T - The moment of inertia of a thin plane body w.r.t an axis perpendicular to the thin plane surface is equal to the sum of the moment of inertia of a thin plane w.r.t to ~~per~~ two perpendicular axes lying in the surface of the plane and these three mutually perpendicular axes meet at a common point.