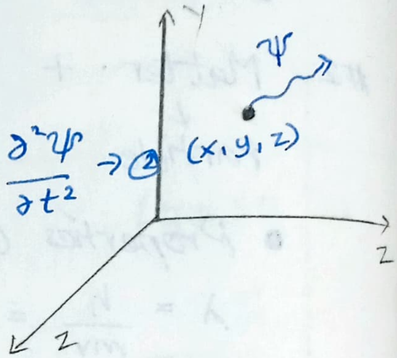
24-11-23

Day 52

Schrondinger's wave Equation

- Time independent

$$\lambda = \frac{h}{mv} \rightarrow ①$$



$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} = \frac{1}{v^2}\frac{\partial^2 \psi}{\partial t^2} \rightarrow ②$$

$$\nabla^2 \psi = \frac{1}{v^2}\frac{\partial^2 \psi}{\partial t^2} \rightarrow ③$$

$$\therefore \nabla^2 = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)\psi$$

$$\psi(x,y,z) = \psi_0(x,y,z) \cdot e^{-i\omega t} \rightarrow ④$$

$$\frac{\partial \psi}{\partial t} = \psi_{0(x,y,z)}(-i\omega)\, e^{-i\omega t} \rightarrow ⑤$$

$$\frac{\partial^2 \psi}{\partial t^2} = \psi_0(-i\omega)(-i\omega)\, e^{-i\omega t} \Rightarrow = -\psi_0\,\omega^2 e^{-i\omega t}$$

$$\frac{\partial^2 \psi}{\partial t^2} = -\psi\omega^2 \rightarrow ⑥$$

from eqn ③

$$\nabla^2 \psi = \frac{1}{v^2} \cdot (-\psi\omega^2) = -\frac{1}{v^2}\psi\omega^2$$

$$= -\frac{1}{v^2}\frac{4\pi^2 v^2}{\lambda^2}\psi = \frac{4\pi^2}{\lambda^2}\psi$$

$$= -\frac{4\pi^2 m^2 v^2}{h^2}\psi$$ ∴ Debroglie eqn.

$$= -\frac{m^2v^2}{h^2/4\pi^2}\psi = -\frac{m^2v^2}{\hbar^2}\psi \rightarrow ⑦$$

Total Energy

$$E_{tot} = K.E + P.E = \frac{1}{2}mv^2 + V$$

$$\Rightarrow 2(E-V) = mv^2 \Rightarrow 2m(E-V) = m^2v^2 \rightarrow ⑧$$

From ① we have,

$$\nabla^2 \psi = -\frac{2m}{\hbar^2}(E - V)\psi$$

*. $$\nabla^2 \psi + \frac{2m}{\hbar^2}(E - V)\psi = 0$$ (3 dimension)

- One dimension

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}(E - V)\psi = 0$$

- for free particles

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}(E\psi) = 0$$