23-11-'23

Day 51

① Find the maxima & minima for $f(x,y) = x^3 + y^3 - 3x - 12y + 20$

Soln:. Given,

$$f(x,y) = x^3 + y^3 - 3x - 12y + 20$$

Step 1: $f(x) = 3x^2 + 0 - 3(1) - 0 + 0$

$f(x) = 3x^2 - 3$

$f(y) = 0 + 3y^2 - 0 - 12 + 0$

$f(y) = 3x^2 - 12$

Step 2:

$A = f_{xx}$	$B = f_{xy}$	$C = f_{yy}$
$A = 6x - 0$	$B = 0$	$C = 6y$
$A = 6x$		

Step 3 :

$f_x = 0$

$3x^2 - 3 = 0$

$3x^2 = 3$

$x^2 = 3/3 = 1$

$x = \pm 1$

$x = 1, x = -1$

$f_y = 0$

$3y^2 - 12 = 0$

$3y^2 = 12$

$y^2 = 12/3$

$y^2 = 4$

$y = \pm 2$

$y = 2, y = -2$

Stationary points are

$(x, y) = (1, 2), (1, -2), (-1, 2), (-1, -2)$

Step 4 :

points	$(1,2)$	$(1,-2)$	$(-1,2)$	$(-1,-2)$
$A = 6x$	$6 > 0$	$6 > 0$	$-6 < 0$	$-6 < 0$
$B = 0$	0	0	0	0
$C = 6y$	$12 > 0$	$-12 < 0$	$12 > 0$	$-12 < 0$
$AC - B^2 = +36xy$	$72 > 0$	$-72 < 0$	$-72 < 0$	$72 > 0$

Conclusion :

min point, saddle point, max point, saddle point

Step 5 :

Minimum point is $(1, 2)$

Maximum point is $(-1, 2)$

- $f(x, y) = x^3 + y^3 - 3x - 12y + 20$

$f(1, 2) = (1)^2 + (2)^3 - 3(1) - 12(2) + 20 = -27 + 29$

$f(1,2) = 2$ is minimum value

- $f(-1,2) = (-1)^3 + (2)^3 - 3(-1) - 12(2) + (20)$

$= -25 + 31$

$f(-1,2) = 6$ is maximum value