18/10/23

Day 28

iii) plane EM waves in terms of Magnetic field in vacam.

- $\vec{\nabla} \times \vec{B} = \varepsilon_0 \mu_0 \, \partial \vec{E} / \partial t$

Taking curl on both sides

- $\vec{\nabla} \times (\vec{\nabla} \times \vec{B}) = \vec{\nabla} \times \left( \varepsilon_0 \mu_0 \frac{\partial \vec{E}}{\partial t} \right) = \varepsilon_0 \mu_0 \frac{\partial (\vec{\nabla} \times \vec{E})}{\partial t}$

vector identity

- $\nabla \cdot (\nabla \cdot B) - \nabla^2 B = \varepsilon_0 \mu_0 \, \varepsilon_0 \mu_0 \frac{\partial}{\partial t} \left[ -\frac{\partial \vec{B}}{\partial t} \right]$

$$= -\varepsilon_0 \mu_0 \frac{\partial^2 \vec{B}}{\partial t^2}$$

$\therefore (\nabla \cdot B) = 0$

- $\nabla^2 \vec{B} - \varepsilon_0 \mu_0 \frac{\partial^2 \vec{B}}{\partial t^2} = 0$

Max Eqn

$*\ \nabla \cdot B = 0$ $\quad *\ \nabla \cdot E = 0$

$*\ \nabla \times \vec{B} = \varepsilon_0 \mu_0 \, \partial \vec{E} / \partial t$ $\quad *\ \nabla \times \vec{E} = -\partial \vec{B} / \partial t$

iv) Speed

$\nabla^2 f - \varepsilon_0 \mu_0 \, \partial^2 f / \partial t^2 = 0$

$\nabla^2 \psi - \frac{1}{v^2} \frac{\partial^2 \psi}{\partial t^2} = 0$

$\Rightarrow \frac{1}{v^2} = \varepsilon_0 \mu_0 \Rightarrow v^2 = 1/\varepsilon_0 \mu_0 \Rightarrow v = 1/\sqrt{\varepsilon_0 \mu_0} = c$

$\Rightarrow v = 1/\sqrt{8.8541 \times 10^{-2} \times 4\pi \times 10^{-7}}$

$\Rightarrow v = 2.998 \times 10^8 \ ms^{-1} = c$ (Velocity of light)

$\therefore \varepsilon_0 \mu_0 = 1/c^2$

- $\nabla^2 \times \vec{B} - 1/c^2 \frac{\partial^2 \vec{B}}{\partial t^2} = 0$ $\quad$ • $\nabla^2 \times E - 1/c^2 \frac{\partial^2 \vec{E}}{\partial t^2} = 0$

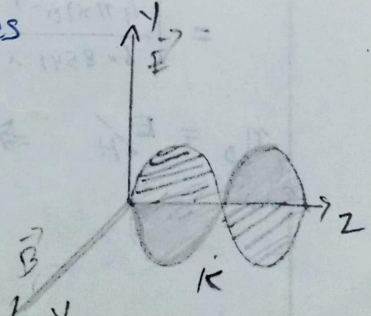
v) orientation of plane EM waves

$E_{(r,t)} = E_0 e^{i(kr - \omega t)}$

$\therefore \vec{k} = k\hat{n}$

$k = 2\pi/\lambda$

$ik = \nabla \ ; \ -i\omega = \frac{\partial}{\partial t}$



$$\nabla^2 E - \varepsilon_0 \mu_0 \frac{\partial^2 \vec{E}}{\partial t^2} = 0$$

i) $\nabla \cdot \vec{E} = 0$

$i\vec{K} \cdot \vec{E} = 0$

$\vec{K} \cdot \vec{E} = 0$

ii) $\nabla \cdot \vec{B} = 0$

$i\vec{K} \cdot \vec{B} = 0$

$\vec{K} \cdot \vec{B} = 0$

iii) $\nabla \times \vec{E} = \frac{-\partial \vec{B}}{\partial t}$

$i\vec{K} \times \vec{E} = -(-i\omega)\vec{B}$

$\vec{K} \times \vec{E} = \omega \vec{B}$

$\vec{K} \times \vec{E} = \vec{B} \cdot \omega$

iv) $\nabla \times \vec{B} = \varepsilon_0 \mu_0 \frac{\partial \vec{E}}{\partial t}$

$i\vec{K} \times \vec{B} = \varepsilon_0 \mu_0 (-i\omega)\vec{E}$

$(\vec{B} \times \vec{K}) = \varepsilon_0 \mu_0 \omega \vec{E}$

Q) Phase & Amplitude

$\vec{K} \cdot K\hat{n}$

$K \cdot \hat{n} \times \vec{E} = \omega \vec{B}$

$\hat{n} \times \vec{E} = \frac{\omega}{K} \cdot \vec{B} \Rightarrow \hat{n} \times \vec{E} = c \cdot \vec{B}$

$\vec{B} = \frac{1}{c} \cdot \hat{n} \times \vec{E}$

$\mu_0 H = \frac{1}{c} \cdot \hat{n} \times \vec{E}$ $\quad \therefore B = \mu_0 H$

$\mu_0 H = \frac{1}{c} |\hat{n} \times \vec{E}| \Rightarrow \frac{1}{c} E = \mu_0 H$

$\Rightarrow E/H = \mu_0 c$

$\eta_0 = E/H = \mu_0 c$

$$\frac{E}{H} = \mu_0 \frac{1}{\varepsilon_0 \mu_0} = \frac{\sqrt{\mu_0}\sqrt{\mu_0}}{\sqrt{\varepsilon_0}\sqrt{\mu_0}} = \sqrt{\frac{\mu_0}{\varepsilon_0}}$$

$$= \sqrt{\frac{4\pi \times 10^{-7}}{8.8541 \times 10^{-2}}} = 376.6 \text{ ohm } (\Omega)$$

$\eta_0 = E/H \Rightarrow E = \eta_0 H$

In medium

Speed

$$V^2 = \frac{1}{\varepsilon_0 \mu_0} \Rightarrow V = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} = c$$

$\therefore \varepsilon = \varepsilon_r \varepsilon_0$

$\therefore \mu = \mu_r \varepsilon_0$

$$c = \frac{1}{\sqrt{\varepsilon \mu}} = \frac{1}{\sqrt{\varepsilon_0 \varepsilon_r \cdot \mu_0 \mu_r}}$$