06/10/23

3. Eigen values & Eigen vectors.

1. Find the Eigen values and Eigen vectors for the matrix $\begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$

Soln: Given,

$$\begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$$

i) $\therefore \lambda^3 - S_1\lambda^2 + S_2\lambda - S_3 = 0$

$S_1 = 6$

$S_2 = \begin{vmatrix} 2 & 1 \\ 2 & 3 \end{vmatrix} + \begin{vmatrix} 1 & -1 \\ 2 & 3 \end{vmatrix} + \begin{vmatrix} 1 & 0 \\ 1 & 2 \end{vmatrix}$

$= (6-2) + (3-2) + (2-0)$

$= 4 + 1 + 2 = 11$

$S_3 = 1(6-2) + 1(2-4) = 4 + 2 = 6$

$\lambda^3 - 6\lambda^2 + 11\lambda - 6 = 0$

ii) $\lambda_1 = 1, \lambda_2 = 3, \lambda_3 = 2$ (Eigen values)

iii) Eigen vectors

$(A - \lambda I)X = 0$

$$\left(\begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\right) \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

case 1 : $\lambda = 3$

$$\begin{bmatrix} 1-3 & 0 & -1 \\ 1 & 2-3 & 1 \\ 2 & 2 & 3-3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$\begin{bmatrix} -2 & 0 & -1 \\ 1 & -1 & 1 \\ 2 & 2 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$-2x_1 + 0x_2 - x_3$

$x_1 - x_2 + x_3$

$2x_1 + 2x_2 + 0x_3$

$$\frac{x_1}{\begin{vmatrix} 0 & -1 \\ -1 & 1 \end{vmatrix}} = \frac{x_2}{\begin{vmatrix} -1 & -2 \\ 1 & 1 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} -2 & 0 \\ 1 & -1 \end{vmatrix}}$$

$$= \frac{x_1}{(0+1)} = \frac{x_2}{(-1+2)} = \frac{x_3}{(2-0)} = \begin{pmatrix} +1 \\ -1 \\ -2 \end{pmatrix}$$

case 2 ; $\lambda = 2$

$$\begin{bmatrix} 1-2 & 0 & -1 \\ 1 & 2-2 & 1 \\ 2 & 2 & 3-2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$\begin{bmatrix} -1 & 0 & -1 \\ 1 & 0 & 1 \\ 2 & 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$\frac{x_1}{\begin{vmatrix} 0 & 1 \\ 2 & 1 \end{vmatrix}} = \frac{x_2}{\begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} 1 & 0 \\ 2 & 2 \end{vmatrix}}$$

$$\frac{x_1}{(0-2)} = \frac{x_2}{(2-1)} = \frac{x_3}{(2-0)}$$

$$\frac{x_1}{-2} = \frac{x_2}{-1} = \frac{x_3}{2} = x(-) = \cancel{\begin{pmatrix} -2 \\ -1 \\ 2 \end{pmatrix}} = \begin{pmatrix} 2 \\ -1 \\ -2 \end{pmatrix}$$

case 3 : $\lambda = 1$

$$\begin{bmatrix} 1-1 & 0 & -1 \\ 1 & 2-1 & 1 \\ 2 & 2 & 3-1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$\begin{bmatrix} 0 & 0 & -1 \\ 1 & 1 & 3 \\ 2 & 2 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$0x_1 + 0x_2 - x_3$

$x_1 + x_2 + 3x_3$

$2x_1 + 2x_2 + 2x_3$

$$\frac{x_1}{\begin{vmatrix} 0 & -1 \\ 1 & 3 \end{vmatrix}} = \frac{x_2}{\begin{vmatrix} -1 & 0 \\ 3 & 1 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} 0 & 0 \\ 1 & 1 \end{vmatrix}}$$

$$\frac{x_1}{(0-1)} = \frac{x_2}{(-1-0)} = \frac{x_3}{(0-0)}$$

$$X_3 = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$$