05/10/23

$$(A - \lambda I)X = 0$$

$$\left(\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right) \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = 0$$

case 1 ;

$\lambda = 8$

$$\left(\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} - 8\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\right)\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 6-8 & -2 & 2 \\ -2 & 3-8 & -1 \\ 2 & -1 & 3-8 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \therefore \begin{pmatrix} 6-\lambda & -2 & 2 \\ -2 & 3-\lambda & -1 \\ 2 & -1 & 3-\lambda \end{pmatrix}$$

$$\begin{pmatrix} -2 & -2 & 2 \\ -2 & -5 & -1 \\ 2 & -1 & -5 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$-2x_1 - 2x_2 + 2x_3 = 0$$
$$-2x_1 - 5x_2 - x_3 = 0$$
$$2x_1 - x_2 - 5x_3 = 0$$

Cross multiplication role, weget,

$$= \frac{x_1}{\begin{vmatrix} -2 & 2 \\ -5 & -1 \end{vmatrix}} + \frac{x_2}{\begin{vmatrix} 2 & -2 \\ -1 & -2 \end{vmatrix}} + \frac{x_3}{\begin{vmatrix} -2 & -2 \\ -2 & -5 \end{vmatrix}}$$

$$= \frac{x_1}{2+10} + \frac{x_2}{-4+2} + \frac{x_3}{10-4}$$

$$= \frac{x_1}{12} \pm \frac{x_2}{-6} \mp \frac{x_3}{6}$$

$$= \frac{x_1}{2} = \frac{x_2}{-1} = \frac{x_3}{1}$$

$$x_1 = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$$

case 2 :

$\lambda = 2$

$$\left(\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} - 2\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\right)\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{pmatrix} 6-2 & -2 & 2 \\ -2 & 3-2 & -1 \\ 2 & -1 & 3-2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 4 & -2 & 2 \\ -2 & 1 & -1 \\ 2 & -1 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$-4x_1 - 2x_2 + 2x_3 = 0$

$-2x_1 + x_2 - x_3 = 0$

$2x_1 - x_2 + x_3 = 0$

By (x)

$$= \frac{x_1}{\begin{vmatrix} -2 & 2 \\ 1 & -1 \end{vmatrix}} + \frac{x_2}{\begin{vmatrix} 2 & -4 \\ -1 & -2 \end{vmatrix}} + \frac{x_3}{\begin{vmatrix} -4 & -2 \\ -2 & 1 \end{vmatrix}}$$

$$= \frac{x_1}{2+2} + \frac{x_2}{-2-4} + \frac{x_3}{-4-2}$$

$$= \frac{x_1}{4} + \frac{x_2}{-6} + \frac{x_3}{-6}$$

$$= \frac{x_1}{2} + \frac{x_2}{-3} + \frac{x_3}{-3}$$

$$x_2 = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

Put $x_1 = 0$

$2(0) - x_2 + x_3 = 0$

$-x_2 + x_3 = 0$

$\frac{x_2}{-1} \pm \frac{x_3}{-1} =$

$$x_2 = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

Case 3:

$$x_3 = \begin{pmatrix} l \\ m \\ n \end{pmatrix}$$

$x_1^T x_3 = 0 \quad (2 \ -1 \ 1)\begin{pmatrix} l \\ m \\ n \end{pmatrix} = 0 \Rightarrow 2l - m + n = 0$

$x_2^T x_3 = 0 \quad (0 \ 1 \ 1)\begin{pmatrix} l \\ m \\ n \end{pmatrix} = 0 \Rightarrow 0l + m + n = 0$

$$\Rightarrow \frac{l}{\begin{vmatrix} -1 & 1 \\ 1 & 1 \end{vmatrix}} = \frac{m}{\begin{vmatrix} 1 & 2 \\ 1 & 0 \end{vmatrix}} = \frac{n}{\begin{vmatrix} 2 & -1 \\ 0 & 1 \end{vmatrix}} \Rightarrow \frac{l}{-2} = \frac{m}{-2} = \frac{m}{2}$$

$x \, l \, y(-2) \quad \frac{l}{1} = \frac{m}{1} = \frac{n}{-1}$

$$x_3 = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$$

Step 5 : To find Normalized matrix N

$$N = \begin{pmatrix} \frac{2}{\sqrt{6}} & 0/\sqrt{2} & 1/\sqrt{3} \\ -1/\sqrt{6} & 1/\sqrt{2} & 1/\sqrt{3} \\ 1/\sqrt{6} & 1/\sqrt{2} & -1/\sqrt{3} \end{pmatrix}$$

$x_1 = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$

$= \sqrt{2^2 + (-1)^2 + (1)^2}$

$x_2 = \ldots$

$x_3 = \ldots$

Step 6 : N^T

$$N^T = \begin{bmatrix} 2/\sqrt{6} & -1\sqrt{6} & 1/\sqrt{6} \\ 0\sqrt{2} & 1\sqrt{2} & 1\sqrt{2} \\ 1\sqrt{3} & 1\sqrt{3} & -1\sqrt{3} \end{bmatrix}$$

Step 7 : $D = N^T A N$

$$D = \begin{bmatrix} 2\sqrt{6} & -1\sqrt{6} & 1\sqrt{6} \\ 0\sqrt{2} & 1\sqrt{2} & 1\sqrt{2} \\ 1\sqrt{3} & 1\sqrt{3} & -1\sqrt{3} \end{bmatrix} \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} \begin{bmatrix} 2/\sqrt{6} & 0/\sqrt{2} & 1/\sqrt{3} \\ -1/\sqrt{6} & 1/\sqrt{2} & 1/\sqrt{3} \\ 1/\sqrt{6} & 1/\sqrt{2} & -1/\sqrt{3} \end{bmatrix}$$

$$D = \begin{bmatrix} 8 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

Step 8 : Canonical form

$$(y_1 \; y_2 \; y_3) \, D \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}$$

$$(y_1 \; y_2 \; y_3) \begin{pmatrix} 8 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} \quad \boxed{8y_1^2 + 2y_2^2 + 2y_3^2}$$

Additional

* Rank = no of terms in canonical form

$\boxed{\text{Rank} = 3}$ (for $8y_1^2 + 2y_2^2 + 2y_3^2$)

* Index = no of +ve terms

$\boxed{\text{Index} = 3}$

* signature = no. of +ve terms − (−ve) terms

signature = 3 − 0 = 3

* Nature of Quadratic form:

= definite (because of +ve terms in all degree) $\Rightarrow 8y_1^2 + 2y_2^2 + 2y_3^2$

(If −ve $\Rightarrow$ Indefinite)