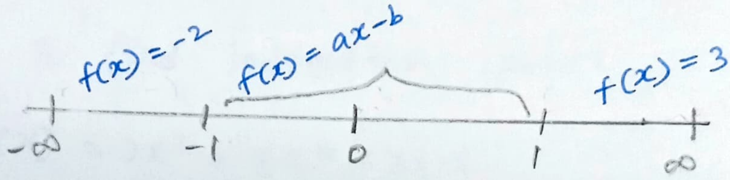
Q1) Find 'a' and 'b' if

$$f(x) = \begin{cases} -2, & x \le -1 \\ ax-b, & -1 < x < 1 \\ 3, & x \ge 1 \end{cases}$$, f is continuous at x ?

Soln.



* At $x = -1$

$f(-1^-) = \lim_{x \to -1} (-2) = -2$

$f(-1) = \lim_{x \to -1} (-2) = -2$

$f(-1^+) = \lim_{x \to -1} (ax-b) = -a-b$

$-a-b = -2 \Rightarrow a+b = 2 \rightarrow ①$

Given :

f(x) is continuous

f(-1) is continuous

$\therefore f(-1^-) = f(-1) = f(-1^+)$

$f(1^-) = f(1) = f(1^+)$

* At $x = 1$

$f(1^-) = \lim_{x \to 1^-} (ax-b) = a-b$

$f(1) = \lim_{x \to 1} 3 = 3$

$f(1^+) = \lim_{x \to 1^+} 3 = 3$

Given :

f is continuous at $x = 1$

$\therefore f(1^-) = f(1) = f(1^+)$

$a-b = 3 \rightarrow ①$

① & ②

$a+b = 2$
$a-b = 3$

$2a = 5$

$a = 5/2$

Sub $a = \frac{5}{2}$ in ①

$\frac{5}{2} + b = 2$

$b = 2 - \frac{5}{2}$

$= \frac{4-5}{2}$

$b = -1/2$

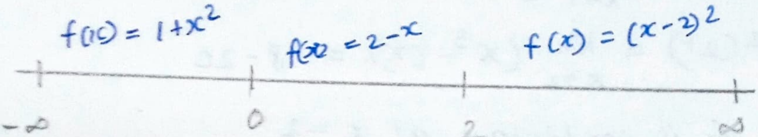
01/11/23

Day 35

Q2) Continuous

$$f(x) = \begin{cases} 1+x^2, & x \le 0 \\ 2-x, & 0 < x \le 2 \\ (x-2)^2, & x > 2 \end{cases}$$

Sol/n:



* At $x=0$

$f(0^-) = \lim_{x \to 0} (1+x^2) = 1$

$f(0) = \lim_{x \to 0} (1+x^2) = 1$

$f(0^+) = \lim_{x \to 0} (2-x) = 2$

$f(0^-) = f(0) \neq f(0^+)$

* f is continuous on Left hand limit at $x=0$
* f is discontinuous on right hand limit at $x=0$.

* At $x=2$

$f(2^-) = \lim_{x \to 2^-} (2-x) = 0$

$f(2) = \lim_{x \to 2} (2-x) = 0$

$f(2^+) = \lim_{x \to 2} (x-2)^2 = 0$

$f(2^-) = f(2) = f(2^+)$

$\therefore$ f is continuous at $x=2$

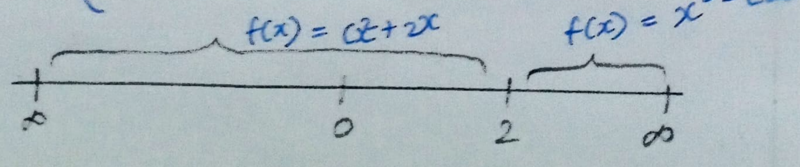
$\therefore$ Domain $(0,2) \cup (2,\infty)$

Q3) For what value c

$$f(x) = \begin{cases} cx^2+2x, & x < 2 \\ x^2-cx, & x \ge 2 \end{cases}$$ is continuous

Soln: Given,

$$f(x) = \begin{cases} cx^2+2x, & x < 2 \\ x^2-cx, & x \ge 2 \end{cases}$$



At $x = 2$

$f(2^-) = \lim_{x \to 2} cx^2 + 2x = 4c + 4$

$f(2) = \lim_{x \to 2} (x^3 - cx) = 8 - 2c$

$f(2^+) = \lim_{x \to 2} (x^3 - cx) = 8 - 2c$

f is continuous at $x = 2$

$f(2^-) = f(2) = f(2^+)$

$4c + 4 = 8 - 2c$

$4c + 2c = 8 - 4 \Rightarrow 6c = 4 \Rightarrow c = 4/6 = 2/3$