27-11-23

Part - 1

Day 54

Q1. Expand $e^x \cdot \cos y$ about $(0,0)$ upto 3rd term using taylor series.

Soln:

Given:

$f(x,y) = e^x \cos y$

$a = 0$	$b = 0$	$h = x - 0$
$h = x$	$K = y$	$K = y - 0$

$f(x,y) = f(a,b) + \frac{1}{1!}[hf_x + kf_y] + \frac{1}{2!}[h^2 f_{xx} + 2hk f_{xy} + k^2 f_{yy}] + \frac{1}{3!}[h^3 f_{xxx} + 3h^2 k f_{xxy} + 3hk^2 f_{xyy}]$

$+ \dots$

function	$(0,0)$
$f(x,y) = e^x \cos y$ $f(x) = e^x \cos y$ $f_y = e^x(-\sin y)$	$f(0,0) = e^0 \cos 0 = 1 \times 1 = 1$ $e^0 \cos 0 = 1$ $e^0(-\sin 0) = 0$
$f_{xx} = e^x \cos y$ $f_{xy} = e^x(-\sin y)$ $f_{yy} = -e^x \cos y$	$e^0 \cos 0 = 1$ $e^0(-\sin 0) = 0$ $-e^0(\cos 0) = -1$
$f_{xxx} = e^x \cos y$ $f_{xxy} = e^x(-\sin y)$ $f_{xyy} = e^x(-\cos y)$ $f_{yyy} = -e^x(-\sin y)$	$e^0 \cos 0 = 1$ $e^0(-\sin 0) = 0$ $e^0(-\cos 0) = -1$ $-e^0(\sin 0) = 0$

$f(x,y) = 1 + \frac{1}{1!}[x(1) + y \times 0] + \frac{1}{2!}[x^2(1) + 2xy(0) + y^2(-1)] + \frac{1}{3!}[x^2(1) + 3x^2y(0) + 3xy^2(-1) + y^3(0)] + \dots$

$f(x,y) = 1 + x \frac{1}{2}[x^2 - y^2] + \frac{1}{6}[x^3 - 3xy^2] + \dots$

25-11-23

Part - II

Day 54

Q3. Expand T.S $e^x \log(1+y)$ upto third term $(0,0)$ in power of x & y.

Soln: Given,

$f(x,y) = e^x \log(1+y)$

$a = 0$; $b = 0$

$h = x$; $k = y$.

function	at $(0,0)$
$f(x,y) = e^x \log(1+y)$	$f(0,0) = e^0 \log(1+0) = 0$
$f(x) = e^x \log(1+y)$	$e^0 \log(1+0) = 0$
$f(y) = e^x \frac{1}{1+y}(1)$	$e^0 \frac{1}{1+0} = 1$
$f_{xx} = e^x \log(1+y)$	$e^0 \log(1+0) = 0$
$f_{xy} = e^x \frac{1}{1+y}$	$e^0 \frac{1}{1+0} = 1$
$f_{yy} = e^x \left(\frac{-1}{(1+y)^2}\right)$	$e^0 \left(\frac{-1}{(1+0)^2}\right) = -1$
$f_{xxx} = e^x \log(1+y)$	0
$f_{xxy} = e^x \left(\frac{1}{1+y}\right)$	$e^0 \frac{1}{1+0} = 1$
$f_{xyy} = e^x \left(\frac{-1}{(1+y)^2}\right)$	$e^0 \left(\frac{-1}{(1+0)^2}\right) = -1$
$f_{yyy} = -e^x \left(\frac{-2}{(1+y)^3}\right)$	$-e^0 \left(\frac{-2}{(1+0)^3}\right) = +2$

$f(x,y) = f(a,b) = \frac{1}{1!}[hf_x + kf_y] + \frac{1}{2!}$
$[hf_{xx} + 2hkf_{xy} + k^2 f_{yy}] + \frac{1}{3!}[h^3_{xxx} + 3h^2 k f_{xxy}$
$+ 3hk^2 f_{xyy} + k^3_{yyy}$

$$f(x,y) = 0 + \frac{1}{1!}[x \times 0 + y \times (1)] + \frac{1}{2!}$$

$$[x^2(0) + 2xy(1) + y^2(-1)] + \frac{1}{3!}[x^3(0) +$$

$$3x^2y(1) + 3xy^2(-1) + y^3(2) + \ldots$$

$$f(x,y) = \frac{1}{1!}y + \frac{1}{2!}[2xy - y^2] + \frac{1}{3!}[3x^2y - 3y^2 +$$

$$2y^3] + \ldots$$

④ Find the Taylor series expansion of the function $f(x,y) = \sin x \sin y$ near the origin. (0,0)

Soln:

Given:

$f(x,y) = \sin x \sin y$

Function	(0,0)
$f(x,y) = \sin x \sin y$	0
$f_x = \cos x \sin y$	0
$f_y = \sin x \cos y$	0
$f_{xx} = -\sin x \sin y$	0
$f_{xy} = \cos x \cos y$	1
$f_{yy} = -\sin x \sin y$	0
$f_{xxx} = -\cos x \sin y$	0
$f_{xxy} = -\sin x \cos y$	0
$f_{xyy} = -\cos x \sin y$	0
$f_{yyy} = -\sin x \cos y$	0

$f(x,y) = f(a,b) + [h f_x(a,b) + k f_y(a,b)] + \frac{1}{2!}$
$[h^2 f_{xx}(a,b) + 2hk f_{xy}(a,b) + k^2 f_{yy}(a,b)] +$
$\frac{1}{3!}[h^3 f_{xxx}(a,b) + 3h^2 k f_{xxy}(a,b) + 3hk^2 f_{xyy}$
$(a,b) + k^3 f_{yyy}(a,b)] + \dots$

$h = x - a$ $\qquad$ $k = y - b$

$= x - 0$ $\qquad$ $= y - 0$

$\sin x \sin y = 0 + 0 + 0 \frac{1}{2!}[0 + 2xy + 0] + \frac{1}{3!}$
$(0 + 0 + 0 + 0) + \dots = xy$