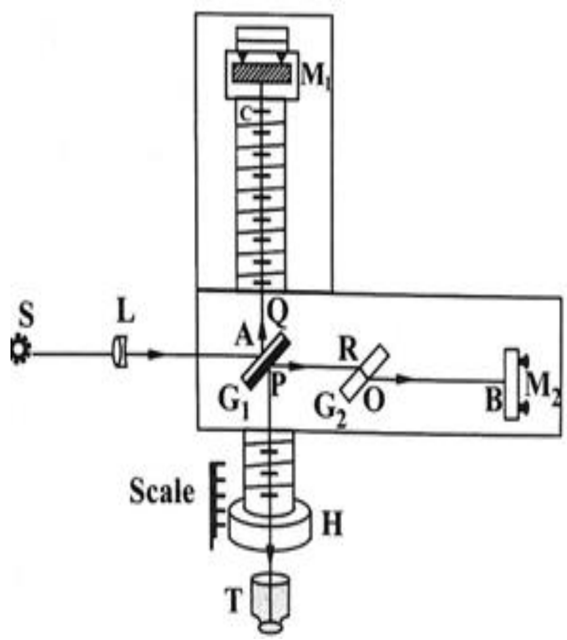
1. **Explain the principle, construction and working of Michelson's interferometer**

Principle

The phenomenon of interference

Construction



- It consists of two highly polished plane mirrors M_1 and M_2 which are equal to right angles to each other.
- Two optically flat glass plates G_1, and G_2, of same thickness and made up of same material placed parallel to each other.
- The plates are inclined at an angle of 45˚ with the mirrors M_1 and M_2 as shown in Fig.
- The plate G, is half silvered at the back so that the incident beam is divided into two beams viz, reflected beam and transmitted beam of equal intensity.
- The mirrors M_1 and M_2 are provided with screws on their backs, so that they can be adjusted exactly perpendicular to each other.
- The mirror M_1 is mounted on a carriage which can be moved forward and backward using the handle "H '. The distance at which the M_1 is moved can be read with the help of the scale as shown in Fig. The interference fringes can be observed in the field of view of the telescope T.

Working

- Light from a monochromatic source S is made parallel with the help of collimating lens L is allowed to fall on the semi silvered glass plate G_1.
- Partly reflected and partly transmitted rays travel along two mutually perpendicular paths and are reflected back by the mirror M_1 and M_2. These two rays again meet at glass plate G_1 and enter a short focus telescope T.
- The two rays cause the interference fringes in the field to view of the telescope. Hence a path difference can be introduced between the two reflected rays by moving the mirror M_1 .
- It is clear from the Fig. that a ray PC passes twice through the glass plate G_1 i.e, 1st through PQ and 2nd through QP, after reflection from the mirror M_1 , whereas the ray PB does not even passes once through G_1, even after reflection from the mirror M_2. Thus in the absence of the glass plate G_2 path traced by the beam between G_1 M_1, and G_1 M_2 are not equal.

- To equalize the path difference, a glass plate G_2 of same thickness and material as that of G_1 is introduced between G_1 and M_2. So that the ray PB will also pass twice, i.e, 1st through RO in glass plate G_2 and 2nd through OR in glass plate G_2 , after reflection from the mirror M_2. Since the glass plate G_2 is used to compensate the path difference between the two rays, it is called as a compensation plate. Thus the path of two rays viz PB and PC are made equal.

APPLICATIONS OF MICHELSON INTERFEROMETER

Michelson interferometer can be used to determine

1. The wavelength of the monochromatic light.
2. The refractive index and thickness of the various thin transparent materials.
3. The difference between the two neighbouring wavelengths or resolution of the spectral lines
4. For the measurement of the standard meter in terms of wavelength of light.

2. **Explain any two applications of Michelson's interferometer.**
   a. **DETERMINATION OF WAVELENGTH OF A MONOCHROMATIOC SOURCE OF LIGHT**

The Mirrors M_1 and M_2 are adjusted to get the circular fringes in the field of view of the telescope. Then the mirror M_1 moved and the number of fringes that cross the field of view is counted

If n is the number of fringes and d is the distance through which the mirror is moved, then the theory of circular fringes

$2d \cos\theta = n\lambda$

Since θ is very very small and is approximately 0

$\therefore 2d = n\lambda \quad (\because \cos 0 = 1)$

$d = \frac{n\lambda}{2}$

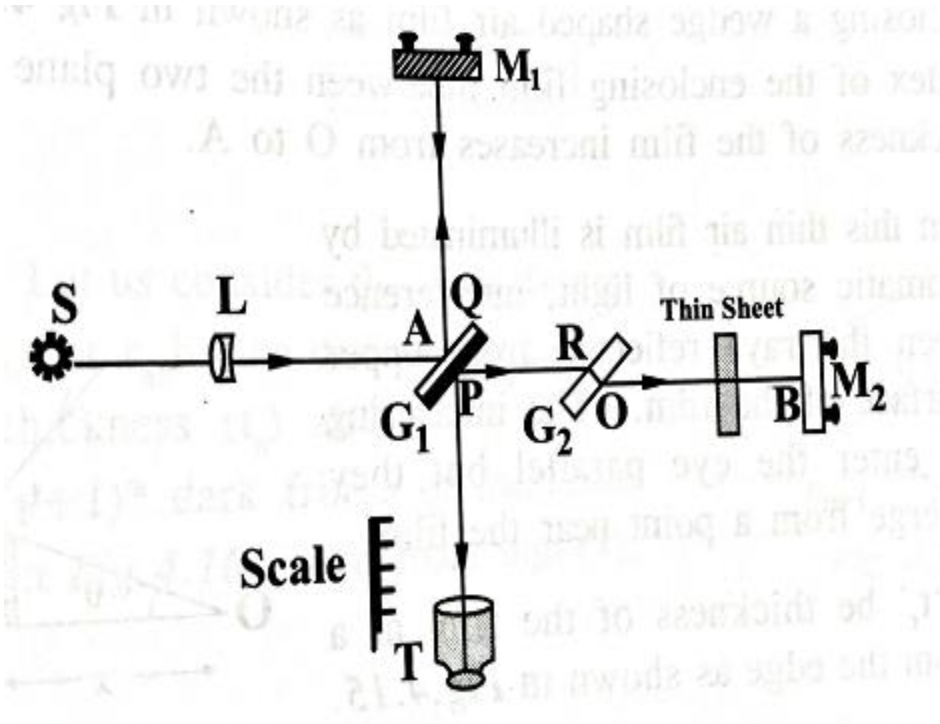
$\lambda = \frac{2d}{n}$

If n =1 i.e., only one fringe crosses the centre of field of view, then $d = \frac{\lambda}{n}$ i.e., the mirror moves through a distance which is equal to half of the wavelength

$\lambda = 2d$

b. DETERMINATION OF THICKNESS OF A THIN TRANSPARENT MEDIUM

Initially, the Michelson interferometer is set for localized fringes of white light. The central fringe in the field of view is made to coincide with the vertical wire and the position of the mirror M1, is noted. Now the thin transparent medium (sheet) of known refractive index (u) is introduced between the glass cross plate G, and the mirror M2, as shown in Fig.



Due to the introduction of thin sheet, a path difference is introduced and hence the central fringe is shifted from its original position to some other position.

In this position the path difference will be 2(μ-1) t.

The position of the mirror M_1 is adjusted until the central fringe again coincides with the cross wire. The distance'd' moved by the mirror M_1 is noted with the help of the scale.

Therefore, we can write the path difference 2(μ-1) t = nλ

Since $\lambda = \frac{2d}{n}$

We can write the path difference 2(μ-1) t = $\frac{n2d}{n}$

(or) The path difference 2(μ-1) t = 2d (or) (μ-1) t =d

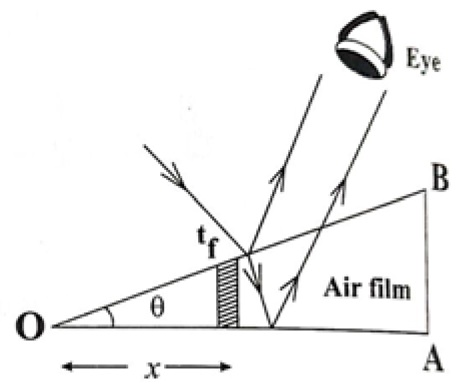
The thickness of the thin transparent sheet is $t = \frac{d}{(\mu-1)}$

From the above formula, the thickness of the thin transparent sheet can be found

- Explain with necessary theory of air wedge and hence explain how to find the thickness of the thin objects using air wedge method experimentally

Let us consider two plane surfaces OA and OB inclined at an angle θ, μ be the refractive index of the enclosing film in between the two plane surfaces OA and OB. The thickness of the film increases from O to A.

When this thin air film is illuminated by the monochromatic source of light, interference occurs between the rays reflected from upper and lower surface of the film. The interfering rays do not enter the eye parallel but they appear to diverge from a point near the film.



Let t be thickness of the film at a distance x from the edge as

The path difference for the reflected light $\Delta = 2\mu t_f \cos r$.

For normal incidence r = 0; cos 0 = 1

The path difference $(\Delta) = 2\mu t_f$

$$\left\{\begin{array}{l} \sin\theta = \frac{t_f}{x} \\ t_f = x\theta \end{array}\right.$$

If θ is very small then we can write $t_f = x\,\theta$

The path difference $(\Delta) = 2\,\mu\, x\, \theta$ …………….(1)

We know the condition for the formation of bright fringes due to the reflected light is

The path difference $\Delta = (2n+1)\,\lambda/2$ …………..(2)

From equations (1) and (2), we can write

The path difference $2\,\mu\, x\, \theta = (2n+1)\,\lambda/2$

Since the film enclosed is an air medium, the refractive index for air (μ) = 1

∴The path difference $2x\theta = (2n+1)\frac{\lambda}{2}$ ………(3)

Similarly, we know the condition for dark fringes due to reflected light is

The path difference $\Delta = n\lambda$ ………………….(4)

From equation (1) and (4), we can write

The path difference $2\,\mu\, x\, \theta = n\lambda$

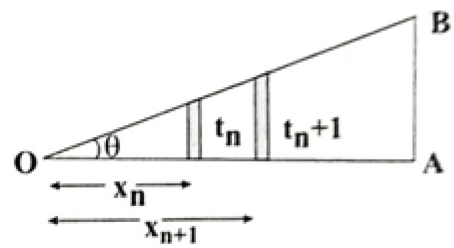
Since μ = 1 (for air), we have

$2x\theta = n\lambda$ …………………………..………..(5)

Fringe width (β)

Case (i):

Let us consider the dark fringe from the edge O. x_n be the distance of the n^{th} dark fringe of thickness (t_n) and $x_{(n+1)}$ be the distance of the $(n+1)^{th}$ dark fringe of thickness (t_{n+1}).



Then from equation (5) we can write,

$$x_n = \frac{n\lambda}{2\theta} \qquad \text{...(6)}$$

and

$$x_{(n+1)} = \frac{(n+1)\lambda}{2\theta} \qquad \text{.....................................(7)}$$

Fringe width (β), which is the distance any two consecutive bright (or) dark fringes can be got by subtracting eqn.(6)from eqn.(7)

$$\beta = x_{(n+1)} - x_n = \frac{(n+1)\lambda}{2\theta} - \frac{n\lambda}{2\theta}$$

$$\beta = \frac{\lambda}{2\theta} \qquad \text{..(8)}$$

Case (ii)

Similarly if we consider any two consecutive bright fringes, then fringe width β will be the same i.e. from equation (3) we can write

$$x_n = \frac{\lambda}{4\theta}(2n+1) \text{ and}$$

$$x_{(n+1)} = \frac{\lambda}{4\theta}(2n+3)$$

$$\beta = x_{n+1}.x_n = \frac{\lambda}{4\theta}(2n+3-2n-1)$$

$$\beta = \frac{\lambda}{2\theta} \qquad \text{..(9)}$$

AIRWEDGE – EXPERIMENT

Using the principle of airwedge, one who can **find the thickness of any thin materials**

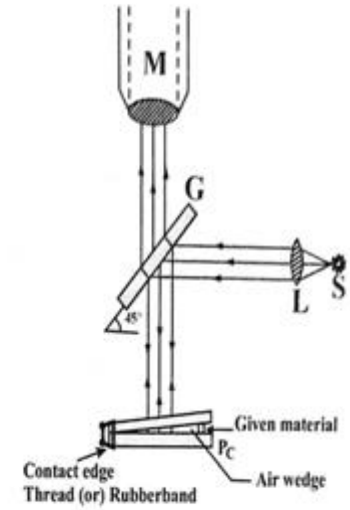
Principle

Two plane glass plates are inclined at an angle (θ) by introducing a thin material (e.g. hair), forming a wedge shaped air film . This film is illuminated by sodium light. Interference occurs between the two rays, one reflected from the front surface and the other by internal reflection at the back surface. Therefore straight line fringes parallel to the edge of the wedge are obtained. Using the theory of airwedge the thickness of the material can be determined.

Description

Two optically plane glass plates are placed one over the other and tied by means of a rubber band at one end. The given material of wire (or) paper is introduced on the other end, so that an airwedge is formed between the plates as shown in Fig.

This set up is placed on the horizontal bed plate of the travelling microscope. Light from the source ' S ' can be made to fall on the Air wedge setup with the help of a condensing lens (L) and 45° angled glass plate (G) .



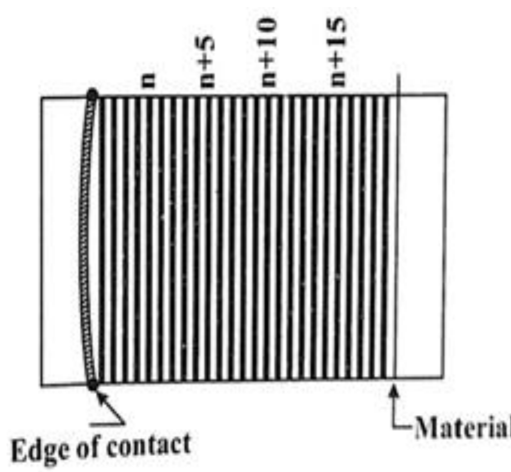
Working

Light from the sodium vapour lamp (S) is rendered parallel by means of a condensing lens (L). The parallel beam of light is incident on a plane glass plate (G) inclined at an angle of 45 ° and gets reflected. The reflected light is incident normally on the glass plates in contact P_C

Interference takes place between the light reflected from the top and bottom surfaces of the glass plates and is viewed through the travelling microscope (M)

Since the thickness of the material remains constant in the direction parallel to the thin edge of the wedge , a large number of equally spaced dark and bright straight line fringes are formed parallel to the edge of as a shown in Fig.

The microscope is adjusted so that the bright (or) dark fringe near the edge of contact is made to coincide with the vertical cross wire and this is taken as the nth fringe. The reading from the horizontal scale of the travelling microscope is noted.



The microscope is moved across the fringes using the horizontal transverse screw and the readings are taken when the vertical cross wire coincides with every successive 5 fringes (5, 10, 15, 20 ...). The width of 20 fringes is calculated and the width of one fringe is calculated. The mean of this gives the fringe width (β)

We know from the theory of Airwedge, the fringe width $\beta = \frac{\lambda}{2\theta}$ -- (1)

If ' l ' is the distance between the edge of contact and the material (say the distance of the last fringe) and t is the thickness of the material, then

For θ being very small, from the Fig

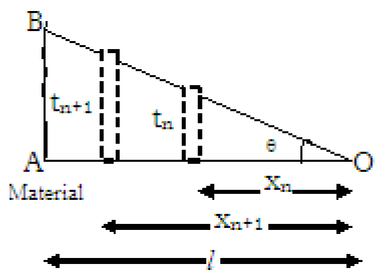
we can write $\theta = \frac{t}{l}$ ------(2)

Substituting equation (2) in (1) , we get

Fringe width $\beta = \frac{l\lambda}{2t}$

Thickness of the material (t) = $\frac{l\lambda}{2\beta}$

Thus, by finding l and β the thickness of any thin material can be determined using air wedge.



3. Derive the expression for Einstien's co-efficients and hence explain the existence of stimulated emission of radiation. Deduce the ratio of rates of stimulated and spontaneous emission of radiation

1. Stimulated Absorption:

The atoms in the ground state are raised to the excited state by taking energy from the incident photon. This process is called ***Stimulated absorption****.*

The rate of absorption

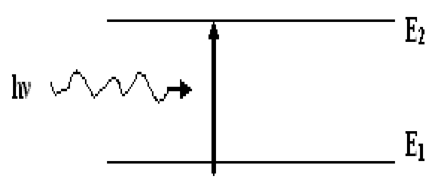
$Q\ N_{ab}\ \alpha\ N_1$

$N_{ab} = B_{12}N_1Q$ -------------------------1

N_1= Number of atoms in the ground state.

Q = Amount of supplied energy.

B = Einstein's co-efficient



2. Spontaneous emission:

After population inversion, the atoms in the excited state return to the ground state spontaneously by emitting their excess energy as photons hv. This process is called ***Spontaneous emission.***

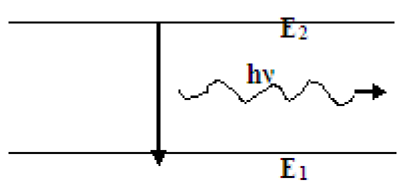
The rate of Spontaneous emission.

$N_{sp}\ \alpha\ N_2$

$N_{sp} = A_{21}\ N_2$ --------------------------------- 2

Where N_2 = Number of atoms in the excited state.

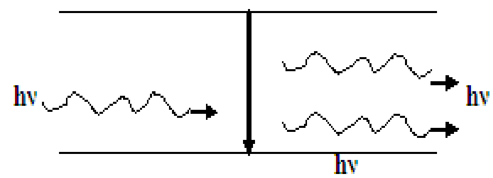
A = Einstein's co-efficient



3. Stimulated emission:

The atoms in the excited state interacts with the incident photons and move towards the ground state by emitting photons of energy hν. This is known as ***Stimulated emission****.*

The rate of Stimulated emission.



$N_{st} \propto N_2 Q$

$N_{st} = B_{21} N_2 Q$ -------------------- 3

Einstein's co-efficients :

A & B are called Einstein's co-efficient.

We know, at thermal equilibrium,

Rate of absorption = Rate of emission

$B_{12} N_1 Q = A_{21} N2 + B_{21} N_2 Q$ -------------------------- 4

$B_{12} N_1 Q - B_{21} N_2 Q = A_{21} N_2$ --------------------------5

Divide eqn. 5 by $B_{21} N_2$,

$$\frac{B_{12}N_1Q}{B_{21}N_2} - \frac{B_{21}N_2Q}{B_{21}N_2} = \frac{A_{21}N_2}{B_{21}N_2}$$

$$Q\left(\frac{B_{12}N_1}{B_{21}N_2} - 1 = \frac{A_{21}}{B_{21}}\right) \quad \text{----------------------------} \quad 6$$

$$Q = \frac{\frac{A_{21}}{B_{21}}}{\frac{B_{12}N_1}{B_{21}N_2} - 1} \quad \text{----------------------------------} \quad 7$$

According to Boltzmann's distribution law,

$$\frac{N_1}{N_2} = e^{h\nu/KT} \quad \text{------------------------} \quad 8$$

Put eqn, 8 in eqn. 7 and if $B_{12} = B_{21}$

$$Q = \frac{A_{21}/B_{21}}{e^{h\nu/KT} - 1}$$

$$Q = \frac{A_{21}}{B_{21}(e^{h\nu/KT} - 1)} \longrightarrow 9$$

According to Planck's quantum theory,

$$Q = \frac{8\pi\ h\,\nu^3}{C^3\ (e^{h\nu/KT} - 1)} \longrightarrow 10$$

Comparing equation no. 9 & 10, we get

$$\left[\frac{A_{21}}{B_{21}(e^{h\nu/KT} - 1)} = \frac{8\pi\ h\,\nu^3}{C^3\ (e^{h\nu/KT} - 1)}\right]$$

$$\frac{A_{21}}{B_{21}} = \frac{8\pi\ h\nu^3}{C^3} \longrightarrow 11$$

$$\frac{A_{21}}{B_{21}} \propto \nu^3 \longrightarrow 12$$

From the above equation we **infer** that

- The spontaneous emission is more predominate than the stimulated emission, since spontaneous emission process is directly proportional to ν^3, but only ordinary light is produced in this process.
- Therefore, it has to be established the population inversion to make stimulated emission as predominate to produce laser.

4. Explain in detail about the construction and working of Nd-YAG laser. Write its advantages and Uses

Nd-YAG Laser:

Introduction :

It is solid state laser

Nd – stands for Neodymium

YAG – stands for Yittrium Aluminium Garnet

Principle :

- The active medium is optically pumped by Krypton flash tube and Neodymium- Nd^{3+} ions are raised to excited level.
- During the transition from meta stable state to ground state, laser beam of wavelength 1.064 µm is emitted.

Characteristics :

Type : Four level solid state laser

Active medium : Nd-YAG rod

Pumping method : Optical pumping

Pumping source : Krypton flash tube

Optical resonator : Two reflecting mirrors at the ends of the rod- One end – fully silvered and Other end – partially silvered

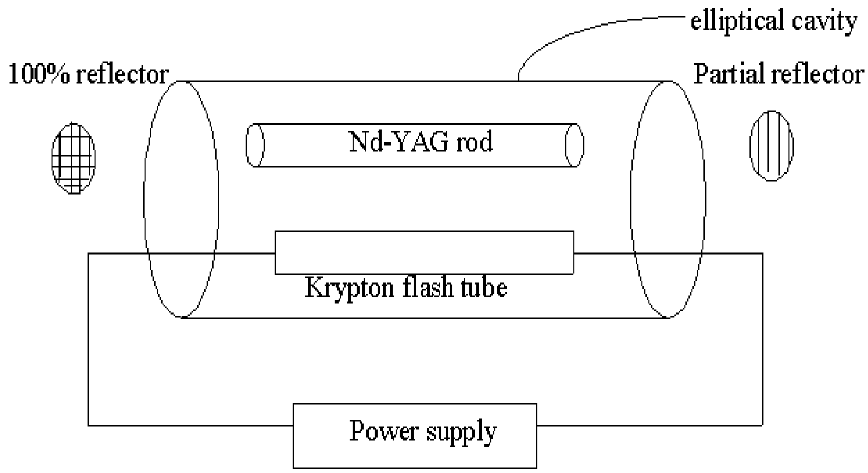
Power output : 70 W

Nature : Pulsed or Continuous

Wavelength : 1.06 µm

Construction:

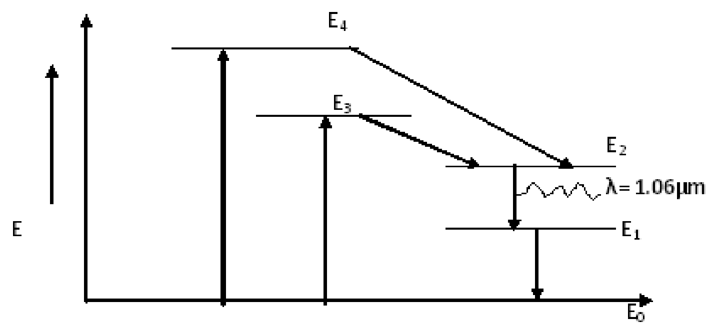
- The construction is shown in the figure.



- In active medium, a small amount of Y^{3+} is replaced by Nd^{3+}
- Active medium and pumping source are placed inside an elliptical cavity.
- The ends of the rod are polished and optical resonator set-up is formed.

Working:

- The energy level diagram is shown in figure
- Absorption $E_0 \longrightarrow E_4$ & $E_0 \longrightarrow E_3$
- These two transitions are absorption.
- Light photons are absorbed and Nd^{3+} ions are raised to excited states.



- Population Inversion : E_4 to E_2 & E_3 to E_2
- These are non-radioactive transitions.
- E_2 is a meta-stable state. Population inversion is achieved between $E_2 \longrightarrow E_1$

Laser action :

- $E_2 \longrightarrow E_1$ First, an ion makes a spontaneous transition from E_2 to E_1, emitting a photon of energy **hν.**
- This emitted photon leads to stimulated emission and laser action.
- $E_1 \longrightarrow E_0$ It is a non-radioactive transition.

Advantages :

- High energy output
- Repetition rate operation is high
- Easy to achieve population inversion

Disadvantages :

- Replacement of Nd^{3+} in YAG rod is a complicated process

Uses :

- It is widely used in engineering applications like drilling, trimming and micro-machining operations.
- It is used in medical applications like endoscopy, urology, etc.

5. Explain in detail about the fundamental modes of vibrations in CO_2 laser and its construction and working. Write its advantages and Uses

CO_2 Laser:

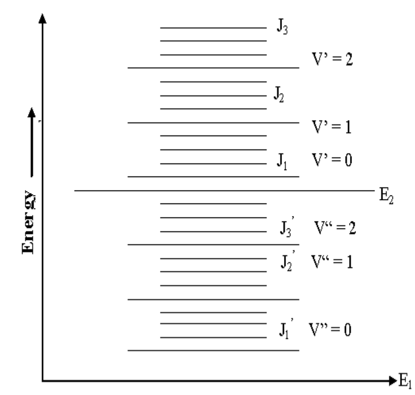
Introduction:

6. CO_2 laser is a molecular gas laser.
7. Photons are emitted during the transition between the vibrational states.
8. Energy level diagram is shown in the figure.

E_1 , E_2 are electronic energy levels

V', V" are vibrational energy levels

J , J' are rotational energy levels



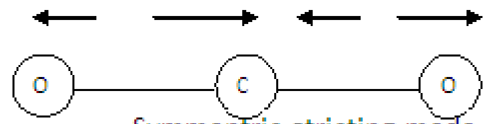
Fundamental modes of vibration of the CO_2 molecules:

1. Symmetric mode (1 0 0)
2. Bending mode (0 1 0) , (0 2 0)
3. Asymmetric mode (0 0 1) , (0 0 2)

Symmetric mode:

Carbon atom is stationary.

. O_2 atoms vibrate along the axis.



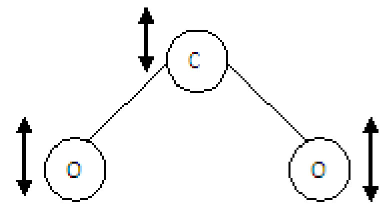
Symmentric stricting mode

Bending mode:

Carbon atom is stationary.

. The atoms are in perpendicular vibration.

. This gives rise to two quanta of frequency.

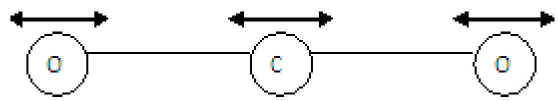


Bending mode

Asymmetric mode:

. All the three atoms will vibrate.

. The carbon atoms will vibrate in the opposite direction of O_2 vibration



Assymmentric stricting mode

Principle:

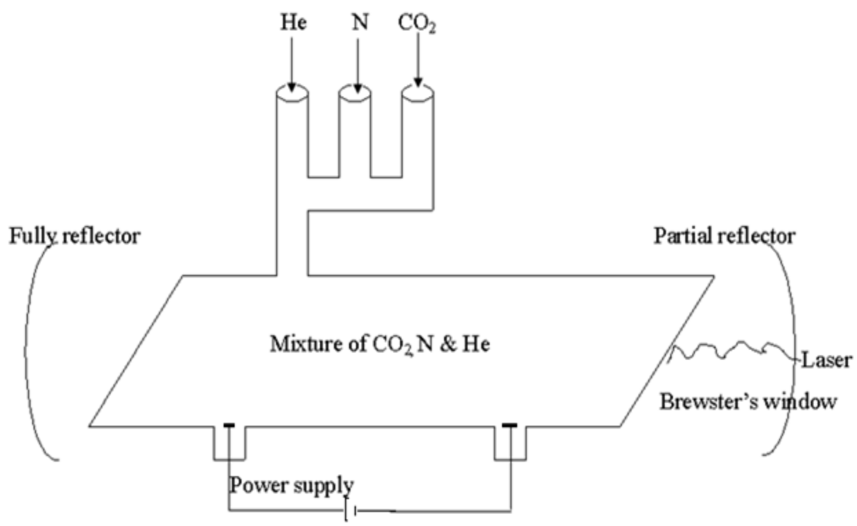
- The nitrogen atoms are initially raised to excited state.
- The nitrogen atoms deliver the energy to CO_2 atoms which has the very close energy level to it.
- Then, the transition takes place between the vibrational energy levels of the same electronic state of the CO_2 atoms and hence laser beam is emitted.

Characteristics :

- Type : Molecular gas laser
- Active medium : Mixture of CO_2 , N_2 and He
- Pumping method : Electric discharge method
- Optical resonator : Silicon mirrors coated with Al.
- Power output :10 KW
- Nature of output : Continuous
- Wavelength : 9.6 µm and 10.6 µm

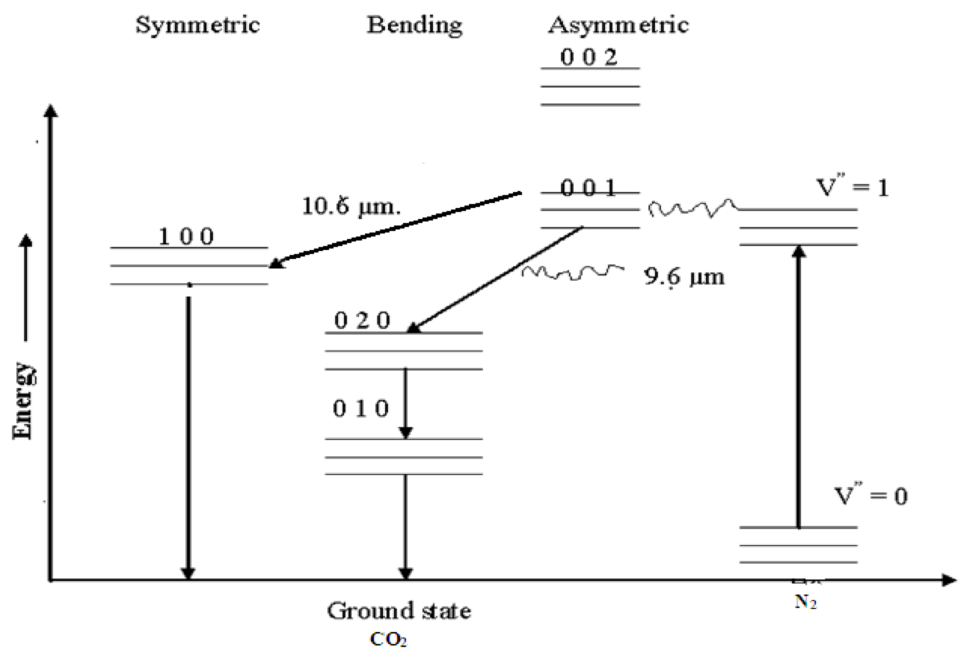
Construction:

- The construction is shown in figure
- Active Medium is placed in the discharge tube
- Nitrogen helps to increase the population of atoms in the upper level of CO_2.
- Helium is used to depopulate the atoms in the ground state.
- The discharge is produced by D.C. excitation
- Two silicon mirrors coated with Al, are used as optical resonators.



Working :

- Energy level diagram is shown in figure.
- N_2 atoms are raised to higher energy vibrational level.
- CO_2 atoms are raised through resonance energy transfer to (0 0 1)
- 0 0 1 $\longrightarrow$ 1 0 0: This transition generates a laser beam of wavelength 10.6 µm.
- 0 0 1 $\longrightarrow$ 0 2 0: This transition generates a laser beam of wavelength 9.6 µm.



Advantages :

- Construction is simple.
- Efficiency is high

Disadvantages :

- Operation temperature plays an important role.
- Exposure may damage our eyes.

Applications :

15. It is used in remote sensing.
16. It is used in various medical fields like neuro-surgery, microsurgery, treatment of liver etc.
17. It is used to perform bloodless operations.

- **Discuss in details the principle, construction and working of homo-junction and hetero-junction lasers with neat diagram. Write its merit, demerit and applications**

Homo-junction:

Introduction:

Homo-junction means, that p-n junction is formed by a single crystalline material.

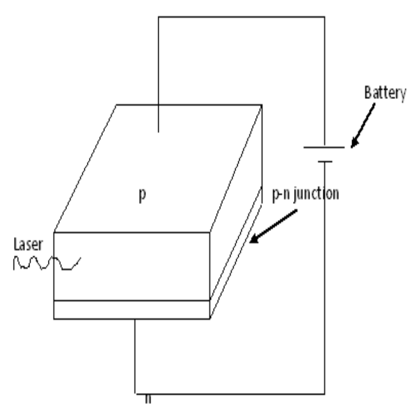
Principle:

When p-n junction is forward biased, the holes move towards ‘n’ region and electrons move towards ‘p’ region.

The recombination of charge carriers takes place in the junction region which results in laser radiation.

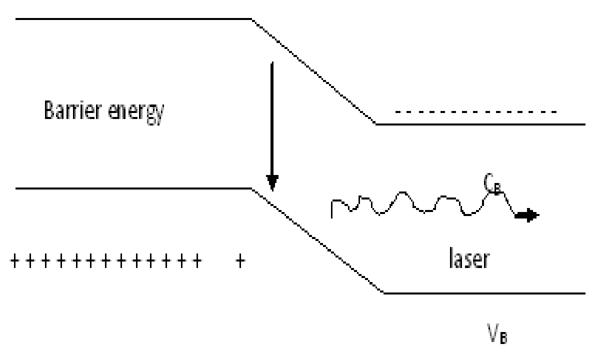
Construction:

The construction is shown in the figure.



- The active medium is a p-n junction diode made from a single crystal of GaAs.
- Voltage is applied to the crystal through the electrode fixed on the upper surface. The end faces of junction diode are well polished and parallel to each other. They act as an optical resonator through which the emitted light comes out.

Working



- The energy level diagram is shown in the figure.
- The p-n junction is forward biased.
- The electrons and holes are injected into the junction region in considerable concentration.

- The region around the junction contains a large number of electrons in the conduction band and large number of holes in the valance band.
- Population inversion is achieved. The electrons and holes recombine each other & produce light photons.
- These light photons stimulate further recombination and lead the emission of laser.
- Output wavelength is 8400 Ǻ.

Advantages:

- The experimental arrangement is simple.
- It exhibits high frequency.

Disadvantages:

- The purity and mono-chromaticity are poor.

Applications:

- It is used in fiber optic communication.
- It may be used as a pain killer.

Semi-conductor Laser: Hetero-junction:

Introduction:

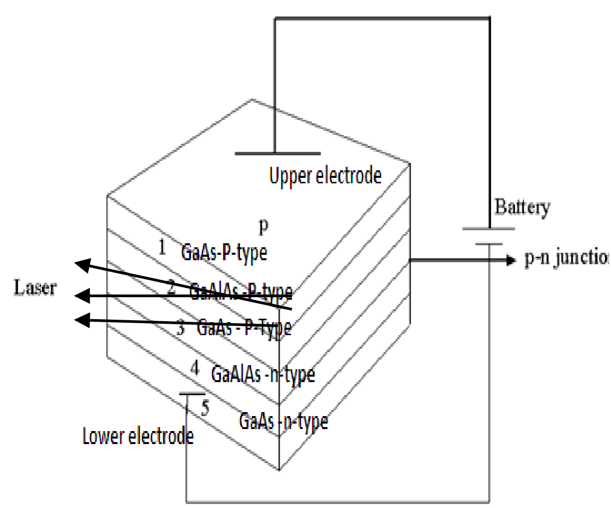
- Hetero-junction is a p-n junction which is formed by two different crystalline material.

Principle:

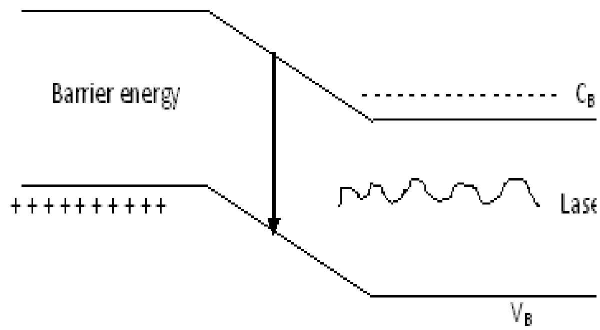
- *Population inversion is achieved in a forward biased hetero-junction diode.*
- *The recombination of charge carriers takes place in the junction region which results in laser action.*

Construction:

- The construction is shown in the figure.
- The active medium is a p-n junction diode from different materials GaAs & GaAlAs.
- Voltage is applied through electrodes
- The well polished end faces of the junction are used as optical resonator.



Working:



- The energy level diagram is shown in the figure.
- The function is forward biased.
- The region around the junction contains a large number of electrons in the conduction band and large number of holes in the valence band.
- The population inversion is achieved.
- Light photons are emitted due to recombination.
- These photons stimulate further recombinations and lead to the emission of laser.
- Output wavelength is $8000A^o$.

Advantages:

- It is a continuous laser.
- Purity and mono-chromacity are high.

Disadvantages :

- Construction is complicated.

Applications :

- It is used in fiber optic communication.
- It is used in holography.

Applications of Laser:

Industrial applications :

- Lasers are used in material processing.
- Lasers are used in printing.
- Lasers are used in testing of materials.
- Lasers are used in micro drilling on hard materials.
- Lasers are used in welding of two plates.

Medical applications :

- Lasers are used in eye treatment.
- Lasers are used in bloodless micro-surgery.
- Lasers are used in treatment of tumours.
- Lasers are used in cancer treatment.

1. **Give the Theory of Compton effect. Explain briefly about the experimental verification of Compton wavelength observed in various scattering angles.**

Compton Effect:

When a beam of monochromatic radiation such as X-rays, $\gamma -$ *rays etc., of high frequency is allowed to fall on a fine scatterer, the beam is scattered into two components viz.*

(i) One component having the same frequency (or) wavelength as that of the incident radiation, so called unmodified radiation

(ii) The other component having lower frequency or higher wavelength compared to incident radiation is called modified radiation.

This effect of scattering is called ***Compton Effect*** *and the shift is called* ***Compton shift***

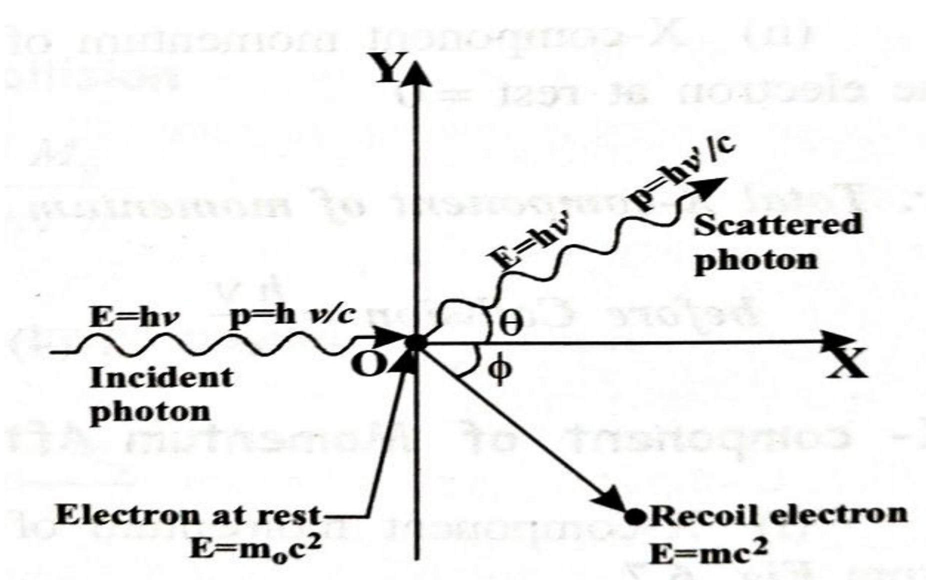
THEORY OF COMPTON EFFECT

Principle:

In Compton scattering the collision between a photon and an electron is considered. Then by applying the laws of conservation of energy and momentum, the expression for Compton wavelength is derived.

Assumptions

1. The collision occurs between the photon and electron in the scattering material,
2. The electron is free and is at rest before collision with the incident photon.



Energy before collision

If m_0 is rest mass of the electron, then

The total energy before collision $= h\nu + m_0c^2$ ------------------ 1

Energy after collision

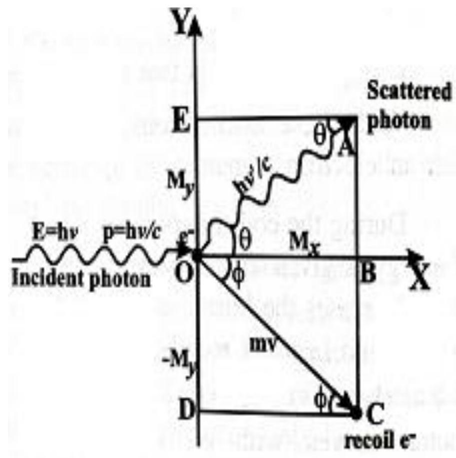
Total energy after collision = $h\nu' + mc^2$ ---------------- 2

According to conservation law of energy

Energy before collision = energy after collision

$h\nu + m_0c^2 = h\nu' + mc^2$ ---------------------------- 3

Momentum along X - Axis



Before collision

Total x – component momentum before collision $= \frac{h\nu}{c}$ --- 4

After collision

In ΔOAB,

$\cos\theta = \frac{M_x}{h\nu'/c}$

x – Component momentum of the scattered photon

$M_x = {h\nu'}/{c}\ \cos\theta$

In ΔOBC

$\cos\phi\ = \frac{M_x}{mv}$

x – Component momentum of the recoil electron

$M_x = mv\ \cos\phi$

Total x- component momentum after collision

$= {h\nu'}/{c}\ \cos\theta + mv\ \cos\phi$ ---------------------------- 5

According to the law of conservation of momentum

$\frac{h\nu}{c} = {h\nu'}/{c}\ \cos\theta + mv\ \cos\phi$ ---------------------------- 6

Along y Axis

Before Collision

Total y component momentum before collision = 0 ----- 7

After collision

From ΔOAE and ΔOCD

Total y - component momentum after collision

$= {h\nu'}/{c}\ \sin\theta - mv\ \sin\phi$ ---------------------------- 8

According to the conservation of law of momentum

$0 = {h\nu'}/{c}\ \sin\theta - mv\ \sin\phi$ ---------------------------- 9

From the equation (6)

$\frac{h\nu}{c} - h\nu'/_c \cos\theta = mv \cos\phi$

Or

$mcv \cos\phi = h(\nu - \nu' \cos\theta)$ ------------------------------ 10

From equation (9)

$mcv \sin\phi = h\nu' \sin\theta$ ------------------------------ 11

Squaring and adding (10) and (11)

$m^2c^2v^2(cos^2\phi + sin^2\phi) = h^2\left(\nu^2 - 2\nu\nu'' \cos\theta + \left(\nu'^2\right) cos^2\theta\right) + h^2\nu'^2 sin^2\theta$

Since $\cos^2 \phi + \sin^2 \phi = 1$

$m^2c^2v^2 = h^2\left(\nu^2 - 2\nu\nu'^2 \cos\theta + \left(\nu'^2\right)\right)$ ---------------------------------- 12

From the equation (3)

$mc^2 = m_0c^2 + h(\nu - \nu')$

Squaring on both sides we get

$$m^2c^4 = [m_0c^2 + h(\nu - \nu')]^2$$

$m^2c^4 = m_0^2c^4 + 2hm_0c^2(\nu - \nu') + h^2\left(\nu^2 - 2\nu\nu' + \nu'^2\right)$ ------------------13

Subtracting (12) from (13)

$m^2c^2(c^2 - v^2) = m_0^2c^4 + 2hm_0c^2(\nu - \nu') - 2h^2\nu\nu'(1 - \cos\theta)$ ----- 14

From theory of relativity

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Squaring and multiply c^2 on both sides

$m^2c^2(c^2 - v^2) = m_0^2c^4$ ------------------------------ 15

Equating (14) and (15)

$$m_0^2c^4 = m_0^2c^4 + 2hm_0c^2(\nu - \nu') - 2h\nu\nu'((1 - \cos\theta))$$

$$2hm_0c^2(\nu - \nu') = 2h^2\nu\nu'(1 - \cos\theta)$$

$$\frac{\nu - \nu'}{\nu\nu'} = \frac{h}{m_0c^2}(1 - \cos\theta)$$

$$\frac{1}{\nu'} - \frac{1}{\nu} = \frac{h}{m_0c^2}(1 - \cos\theta)$$

Multiply by ‘c’ on both sides

$$\frac{c}{\nu'} - \frac{c}{\nu} = \frac{h\,c}{m_0 c^2}(1 - \cos\theta)$$

$$\lambda' - \lambda = \frac{h}{m_0 c}(1 - \cos\theta)$$

Change in the wavelength

$$\Delta\lambda = \frac{h}{m_0 c}(1 - \cos\theta)$$

Special Cases

Case 1

When θ = 0, cos θ = 1

$$\Delta\lambda = \frac{h}{m_0 c}(1 - \cos\theta)$$

$$\Delta\lambda = \frac{6.625 \times 10^{-34}}{9.11 \times 10^{-31} \times 3 \times 10^{8}} \times (1 - 1)$$

$$\Delta\lambda = 0$$

This means that no scattering at the angle equal to zero

Case 2

When θ = 90^0, cos θ = 0

$$\Delta\lambda = \frac{6.625 \times 10^{-34}}{9.11 \times 10^{-31} \times 3 \times 10^{8}} \times (1 - 0)$$

$$\Delta\lambda = 0.024240\,\mathring{A}$$

This wavelength is called Compton wavelength, which has a good agreement with the experimental result.

case 3

When θ = 180^0, cos θ = -1

$$\Delta\lambda = \frac{6.625 \times 10^{-34}}{9.11 \times 10^{-31} \times 3 \times 10^{8}} \times (1 - (-1))$$

$$\Delta\lambda = 0.04848\,\mathring{A}$$

Thus for θ = 180^0 the shift in wavelength is found to be maximum

∴ When the angle of scattering θ various from 0 to 180°, the wavelength shifts from λ to λ + $\frac{2h}{m_0 c}$

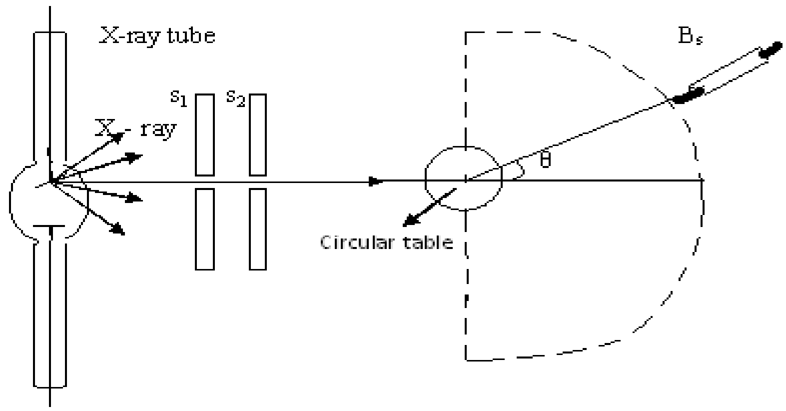
<u>EXPERIMENTAL DETRMINATION OF COMPTON EFFECT:</u>

Principle:

When a photon of energy 'hν' colloids with a scattering element, the scattered beam has two components viz., one of the same frequency (or) wavelength as that of the incident radiation and the other has lower frequency (or) higher wavelength compared to incident frequency (or) wavelength. This effect is called “Compton effect “and the shift in wavelength is called “Compton shift”.

Construction:

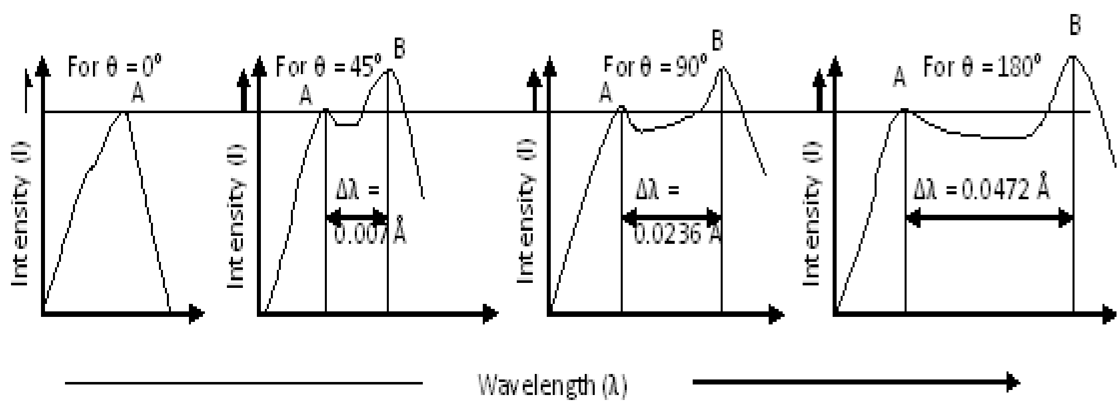
It consists of an X-ray tube for producing X-rays, scattering element, Bragg’s spectrometer and two slits s_1 & s_2 which helps to focus the X-rays on to the scattering element.



Working:

X-rays of monochromatic wavelength ‘λ’is produced from an X-ray tube and made to pass through the slits s_1 & s_2.These X-rays are made to fall on the scattering element. The scattered X-rays are received with the help of Bragg’s spectrometer and the scattered wavelength is measured.

The experiments are repeated for various scattering angles and the scattered wavelengths are measured. The experimental results are plotted as shown in the figure.



In this figure, when $\theta = 0^0$, the scattered radiation peak will be the same as that of the incident radiation, for $\theta = 90^0$, we get two peaks with $\Delta\lambda = 0.0236$, which has good agreement with the theoretical results. Hence this wavelength $\theta = 90\ ^0$is called Compton wavelength and $\Delta\lambda = 0.0236$ is called Compton shift.

2. What is Schrodinger wave equation? Derive Schrodinger's Time independent and dependant wave equations?

SCHRODINGER WAVE EQUATION

Schrodinger describes the wave nature of a particle in mathematical form and is known as ***Schrodinger wave equation.***

There are two types of wave equations.

1. Time Independent Schrodinger equation
2. Time dependent Schrodinger equation

Time Independent Schrodinger equation

According to de-Broglie a particle of mass 'm' moving with a velocity 'v' has the wavelength

$$\lambda = \frac{h}{mv} \longrightarrow 1$$

Classical wave equation is given by

$$\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2} = \frac{1}{V^2}\frac{\partial^2\psi}{\partial t^2} \longrightarrow 2$$

Where

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

It is called Laplacian operator.

The solution for the equation (2) is given by

$$\psi(x,y,z) = \psi_0(x,y,z)e^{-i\omega t}$$

$\psi = \psi_0 e^{-i\omega t}$ Where ψ_0 is the amplitude at the point

Differentiating ψ with respect to time we get

$$\frac{\partial\psi}{\partial t} = -i\omega\psi_0 e^{-i\omega t}$$

$$\frac{\partial^2\psi}{\partial t^2} = i^2\omega^2\psi_0 e^{-i\omega t}$$

$$\frac{\partial^2\psi}{\partial t^2} = -\omega^2\psi$$

Substituting the above in equation (2) we get

$$\nabla^2\psi = -\frac{\omega^2}{V^2}\psi$$

$$\nabla^2\psi + \frac{\omega^2}{V^2}\psi = 0$$

$$\nabla^2\psi + \frac{4\pi^2}{\lambda^2}\psi = 0$$

$$\left\{\begin{aligned} \omega &= 2\pi\nu \\ &= 2\pi\frac{c}{\lambda} \\ \lambda &= \frac{h}{mv} \end{aligned}\right.$$

$$\nabla^2\psi + \frac{4\pi^2 m^2 v^2}{h^2}\psi = 0 \longrightarrow 3$$

E is the total energy of the particle

$$E = K.E + P.E$$

$$E = \frac{1}{2}mv^2 + V$$

$$E - V = \frac{1}{2}mv^2$$

$$mv^2 = 2(E - V)$$

$$m^2v^2 = 2m(E - V)$$

Substituting the above in equation (3)

$$\nabla^2\psi + \frac{4\pi^2 2m(E - V)}{h^2}\psi = 0$$

$$\nabla^2\psi + \frac{8\pi^2 m(E - V)}{h^2}\psi = 0$$

$$\nabla^2\psi + \frac{2m}{\hbar^2}(E - V)\psi = 0 \qquad \therefore\ \hbar^2 = \left(\frac{h}{2\pi}\right)^2$$

This equation is known as Schrodinger Time independent equation for three dimensions.

For one dimensional wave equation:-

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}(E - V)\psi = 0$$

For a free particles (Electron):-

Potential energy V = 0

∴ The one dimensional wave equation becomes

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}E\psi = 0$$

Schrodinger Time Dependent equation

Schrodinger time dependent equation can be obtained from time independent equation by eliminating E.

Solution of classical wave equation

$$\psi(x, y, z) = \psi_0(x, y, z)e^{-i\omega t}$$

Differentiating ψ with respect to time ‘t’

$$\frac{\partial\psi}{\partial t} = -i\omega\psi_0 e^{-i\omega t}$$

$$\frac{\partial\psi}{\partial t} = -i2\pi\nu\ \psi_0 e^{-i\omega t}$$

$$\frac{\partial\psi}{\partial t} = -i2\pi\left[\frac{E}{h}\right]\ \psi_0 e^{-i\omega t} \qquad \left\{ \ E = h\nu \right.$$

$= -i\frac{E}{\hbar}\psi$ $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \nu = \frac{E}{h}$

Multiplying i on both sides

$i\frac{\partial\psi}{\partial t} = \frac{E}{\hbar}\psi$

$E\psi = i\hbar\frac{\partial\psi}{\partial t}$

Substituting Value of Eψ in Schrödinger time independent equation

$\nabla^2\psi + \frac{2m}{\hbar^2}\left(i\hbar\frac{\partial\psi}{\partial t} - V\psi\right) = 0$

$\nabla^2\psi = -\frac{2m}{\hbar^2}\left(i\hbar\frac{\partial\psi}{\partial t} - V\psi\right)$

Multiply by $\frac{-\hbar^2}{2m}$ on both sides, we have

$$\frac{-\hbar^2}{2m}\nabla^2\psi = \left(i\hbar\frac{\partial\psi}{\partial t} - V\psi\right)$$

$$\frac{-\hbar^2}{2m}\nabla^2\psi + V\psi = \left(i\hbar\frac{\partial\psi}{\partial t}\right)$$

This equation is known as Schrodinger time dependent wave equation.

$$\left[\frac{-\hbar^2}{2m}\nabla^2 + V\right]\psi = \left(i\hbar\frac{\partial\psi}{\partial t}\right)$$

$$H\psi = E\psi$$

Where E is the energy operator and H is called Hamiltonian operator

<u>Physical Significance of wave Function</u>

i) It is the variable quantity that is associated with a moving particle at any position (x, y, z) and at any time "t" and it relates the probability of finding the particle at that point and at that time.

ii) The wave function relates the particle and wave nature of matter statistically.

iii) It is a complex quantity and hence we cannot measure it accurately.

iv) The probability will have any value between zero to one.

v) If the particle is certainly to be found somewhere in space, then the probability value is equal to 1.

$p = \iiint \|\psi\|^2$ dx dy dz = 1

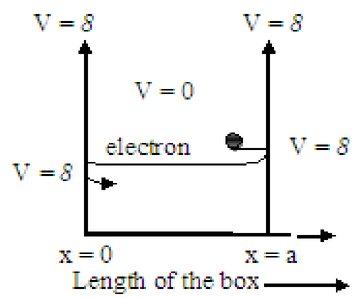
vi) If P = 0, then there is no chance for finding the particles. i.e. the particle is certainly not found within the given limit.

3. **Derive and expression of Schrodinger's one dimensional wave equation. From Schrodinger's Time independent wave equation, normalize the wave equation of electron trapped in a one dimensional potential well**

Particle in a One Dimension box

A particle in a one dimensional box is a fundamental quantum mechanical approximation describing the translator motion of a single particle confines inside an indefinitely deep well from which the particle cannot escape.

The particle in a box problem is a common application of quantum mechanical model to a simple system consisting of a particle (may be an electron) moving in a deep well where the particle cannot escape from it. Here the energy values E and wave function ψ is discussed. The following are needed to solve the quantum mechanical problem for a particle in a one dimensional box



a) Potential energy (V)
b) Schrodinger equation
c) Wave function
d) Allowed energies

Potential energy

Consider a particle of a mss 'm' is moving in an one dimensional box along the x axis. The potential energy is zero inside the box. The potential energy infinity outside the box

$$V = 0 \text{ for } 0 < x < a$$

$$V = \infty \quad \text{for } 0 \geq x \geq a$$

Since the particle cannot exist outside the box the wave function $\psi = 0$ when $0 \geq x \geq a$

Schrodinger Equation:-

To find the wave function of the particle within the box of length "a", let us consider time independent Schrodinger one dimensional equation for a particle moving with an energy 'E' and mass 'm' is given

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}(E - V)\psi = 0$$

Since the potential energy inside the well is zero i.e., V = 0 the particle has KE alone and thus it is named as free particles or free electron.

For free particles, Schrodinger equation can be written as

$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}E\psi = 0$

$\frac{d^2\psi}{dx^2} + k^2\psi = 0$ -- 1

Where $k^2 = \frac{2m}{\hbar^2}$ -- 2

Equation 1 is the second order differential equation and it should have solution with two arbitrary constants

$\psi(x) = A\, sin\, kx + B \cos kx$ --- 3

Where A and B are constants which are found by applying boundary conditions

i.e., V(x) = ∞ when x = 0 and x = a

Boundary condition 1: at x = 0 and V = ∞, there is no chance to find the particles inside the well ∴

$\psi(x) = 0$

Equation 3 becomes

$0 = A\, sin\, k0 + B \cos k0$

0 = 0 + B(1) $\therefore B = 0$

Boundary condition 2: at x = a and V = ∞, there is no chance to find the particles inside the well ∴

$\psi(x) = 0$

Equation 3 becomes

$0 = A\, sin\, ka + B \cos ka$

$0 = A\, sin\, ka\ 0$

$A \neq 0$: $\sin ka = 0$ We know $\sin n\pi = 0$

On comparison of the above two equations, we have

ka = nπ where n is an integer

or $k = \frac{n\pi}{a}$ -- 4

Substituting the above value in equation 3, we can write the wave function associated with the free electron in one dimensional well as

$\psi(x) = A \sin\frac{n\pi x}{a}$ --- 5

Energy of the particle (Electron)

We know from equation 2

$$k^2 = \frac{2m}{\hbar^2}$$

$$= \frac{2mE}{\frac{h^2}{4\pi^2}}$$

$k^2 = \frac{8\pi^2 mE}{h^2}$ --- 6

Squaring equation 4, we get

$k^2 = \frac{n^2\pi^2}{a^2}$ --- 7

Equating equation 6 and 7, we have

$\frac{8\pi^2 mE}{h^2} = \frac{n^2\pi^2}{a^2}$ --- 8

From equation 8 and 5 we can say that, for each value of n there is an energy level and the corresponding wave function, thus we can say that each value of E_n is known as Eign value and the corresponding value of Ψ_n is called as Eigen function

Allowed Energies

The energy which is allowed for a particle in a box is given by

$$E_n = \frac{n^2h^2}{8ma^2}$$

The lowest possible energy of a particle is a one dimensional box is not equal to zero. This is called zero point energy and means the particle never be at rest because it has minimum kinetic energy. The allowed energy levels obey Heisenberg uncertainty principle.

When n=1 $E_1 = \frac{h^2}{8ma^2}$

n= 2 $E_2 = \frac{4h^2}{8ma^2}$

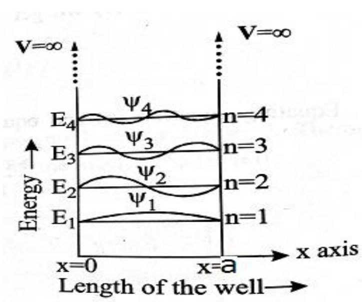
n= 3 $E_3 = \frac{9h^2}{a}$

n= 4 $E_4 = \frac{16h^2}{8ma^2}$

In general

$E_n = n^2E_1$ --------------- 9

It is found from the energy levels E_1, E_2, E_3, etc., the energy levels of an electron are Discrete. The various energy Eigen values and their corresponding Eigen functions of an electron enclosed in a one dimensional box is as shown in fig. Thus we have discrete energy values.



Normalisation of wave function

It is the process by which the probability (P) of finding the particle (like electron) inside the well can be done

If P = 1, then the particle is being inside the well

$\therefore P = \int_0^a A^2 sin^2 \frac{n\pi x}{a} dx = 1$-- 10

$$A^2 \int_0^a \frac{1-\cos 2n\pi x/a}{2} dx = 1$$

$$A^2 \left[\frac{x}{2} - \frac{1}{2}\frac{\sin 2n\pi x/a}{2n\pi/a}\right]_0^a = 1$$

$$A^2 \left[\frac{1}{2} - \frac{1}{2}\frac{\sin 2n\pi a/a}{2n\pi/a}\right] = 1$$

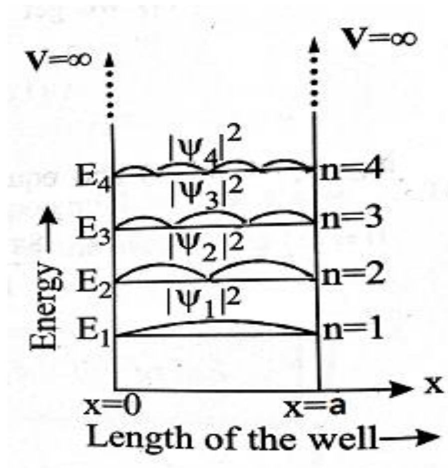
$A^2 \left[\frac{1}{2} - \frac{1}{2}\frac{\sin 2n\pi}{2n\pi/a}\right] = 1$ -- 11

We know $\sin n\pi = 0 \therefore \sin 2n\pi\ is\ also\ 0$

Equation 11 can be written as

$$\frac{A^2 a}{2} = 1$$

$$A = \sqrt{\frac{2}{a}}$$



Normalized wave function

$$\psi = \sqrt{\frac{2}{a}} \sin\left[\frac{n\pi}{a}\right] x$$

The normalized wave function and their energy values are as shown in the above fig.

1. **Explain the concept of Barrier penetration and quantum tunneling in detail with necessary sketch**

Barrier Penetration

If a particle with energy "E" is incident on a thin energy barrier of height V. greater than "E", then there is a finite probability of the particle to penetrate the barrier. This phenomenon is called barrier penetration and this effect is called Tunneling Effect (or) Quantum Tunneling.

According to classical mechanics, the Probability of a particle to penetrate / tunnel the barrier is zero, but according to Quantum mechanics it is finite.

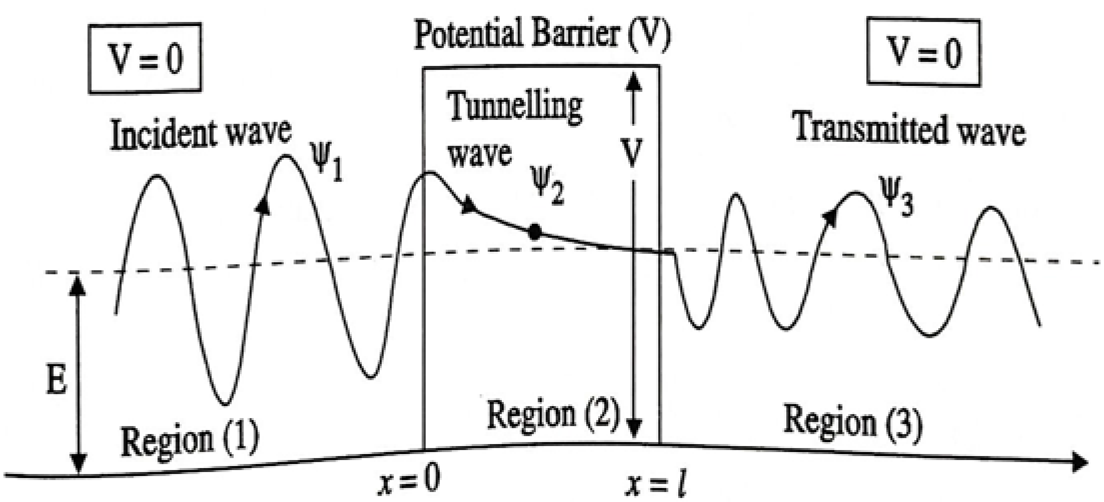
Concept of Barrier Penetration

Let us consider a particle of kinetic energy ' E which moves from the left (Region - 1) and strikes the potential barrier of height ' V as shown in Fig. If the kinetic energy (E) is lesser than the potential energy (V) (i.e) if E < V then , according to classical mechanics there is no chance for the particle to cross the potential barrier (V)

But, according to quantum mechanics , the particle has certain probability (few chances) to penetrate (or) cross the potential barrier (V) and comes out to Region - 3 , by tunneling the Region - 2 .

Derivation/ Proof

Let us consider such a particle with energy E < V incident from left side (Region - 1) and tunnel the Region - 2 of width / [x = 0 to l] and comes out as a transmitted wave in Region - 3, as shown in the below Fig.



Here the potential energy V= 0 on both sides of the barrier, which means, no forces will act upon the particle in Region-1and in Region-3.

The boundary conditions shall be written for various regions as

For Reion-1: when $x < 0$; $V = 0$

For Reion-2: when 0 < x < l; V=V

For Reion-3: when x > l; V= 0

Let Ψ_1, Ψ_2 and Ψ_3 be the wave functions in region 1,2and 3 respectively. Then the Schroedinger's wave equations for all the three regions shall be written as

For Region -1

$$\frac{d^2\psi_1}{dx^2} + \frac{2m}{\hbar^2}[E - V]\psi_1 = 0$$

Since V = 0 in Region -1 we can write the above equation as

$$\frac{d^2\psi_1}{dx^2} + \frac{2m}{\hbar^2}E\psi_1 = 0 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots..(1)$$

For Region-2

$$\frac{d^2\psi_2}{dx^2} - \frac{2m}{\hbar^2}[E - V]\psi_2 = 0$$

Since V = E in Region-2 we can write the above equation as

$$\frac{d^2\psi_2}{dx^2} - \frac{2m}{\hbar^2}[V - E]\psi_2 = 0 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

For Region -3

$$\frac{d^2\psi_3}{dx^2} + \frac{2m}{\hbar^2}[E - V]\psi_3 = 0$$

Since V= 0 in Region -3 we can write the above equation as

$$\frac{d^2\psi_3}{dx^2} + \frac{2m}{\hbar^2}E\psi_3 = 0 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(3)$$

Let $\frac{2mE}{\hbar^2} = \alpha^2$ and $\frac{2m}{\hbar^2}(V - E) = \beta^2$

Equations (1),(2) and (3) shall be written as

For Region-1 $\frac{d^2\psi_1}{dx^2} + \alpha^2\psi_1 = 0$ ……………..……(4)

For Region-2 $\frac{d^2\psi_2}{dx^2} + \beta^2\psi_2 = 0$ ……………..…..(5)

For Region-3 $\frac{d^2\psi_3}{dx^2} + \alpha^2\psi_3 = 0$ …..………..(6)

The solutions for equations (4),(5),and (6) and shall be written as

For Region-1 $\psi_1 = Ae^{i\alpha x} + Be^{-i\alpha x}$ …….……(7)

For Region-2 $\psi_2 = Fe^{\beta x} + Ge^{-\beta x}$ . . ……..…..(8)

For Region-3 $\psi_3 = Ce^{i\alpha x} + De^{-i\alpha x}$. ……..……(9)

Here A,B,C,D,F,G are the amplitudes of corresponding waves in various regions as shown in fig.

For Region-1

Let us discuss the behavior of wave function and the amplitudes in each Region, The wave function of the incident wave in Region-1 shall be written from equation (7) as

$$\psi_1(Incident) = Ae^{i\alpha x}$$

Where 'A' is the amplitude of the incident wave in Region-1

Since there are ample chances for the wave to get reflected within the Region -1 due to higher potential barrier (or) larger width of the barrier, the wave function of the reflected wave in Region -1 shall be written from equation (7) as

$$\psi_1(reflected) = Be^{-i\alpha x} \text{ -------------------------------- } 11$$

Where B is the amplitude of the reflected wave in Region-1

For Region-2

The wave function of the transmitted (or) tunneling wave at region -2 shall be written from equation (8) as

$$\psi_2 = Fe^{\beta x} + Ge^{-\beta x} \text{ ------------------------------------- } 12$$

Where β is the wave number given by $\beta = \frac{\sqrt{2m(V-E)}}{\hbar}$

F is the amplitude of the barrier penetrating wave (or) tunnelling wave in Region-2.

G is the amplitude of the reflected wave at the boundary between Region-1 and Region-2.

From eqn (12) we can see that the exponents are real quantities, so the wave function Ψ_2 will not oscillate and therefore does not represent a moving particle at Region-2.

Thus, the particle can either penetrate (or) tunnel through Region-2 and transmitted to Region-3 (or) it shall be reflected back to Region-1 itself. [Given by equation (11)]

Region-3

The wave function of the transmitted wave in Region-3 shall be written from equation (9) as

$$\Psi_3 \text{ (Transmitted)} = Ce^{i\alpha x} \text{ -------------------------------------- } (13)$$

Where C is the amplitude of the transmitted wave in Region-3

In Region-3, i.e., x > 1, there can be only transmitted wave and there will not be any reflected wave and therefore the amplitude of the reflected wave in Region-3 i.e., D = 0

Thus, the wave function of the reflected wave in Region-3 is also zero.

$$\Psi_3 \text{ (reflected)} = 0 \text{ -- } (14)$$

The transmission and reflection co-efficient shall be obtained as follows:

Transmission Co-efficient

We know that the probability density is the square of the amplitude of that function. Therefore the barrier transmission co-efficient (T) is the ratio between the square of the amplitude of the transmitted wave $|c|^2$ and the square of the amplitude of the incident wave $|A|^2$

The transmission co-efficient $T = \frac{|C|^2}{|A|^2} = \frac{4\sqrt{E}\sqrt{E-V}}{\left[\sqrt{E}+\sqrt{E-V}\right]^2}$ ------- (15)

Equation (15) is also called as the "Penetrability" of the barrier.

Reflection Co-efficient

The reflection co-efficient (R) for the barrier surface at x = 0 is the ratio between the square of the amplitude of the reflected wave B and the he amplitude of the incident wave | A |²

The reflection co-efficient R= $\frac{|B|^2}{|A|^2} = \left(\frac{\sqrt{E}-\sqrt{E-V}}{\left[\sqrt{E}+\sqrt{E-V}\right]^2}\right)^2$ -------- (16)

2. Explain the microscopic technique which uses quantum tunneling principle to scan the samples with a focused electron beam.

SCANNING TUNNELLING MICROSCOPE (STM)

Introduction

Principle

The basic principle used in scanning tunneling microscope (STM) is the **tunneling of electron between the sharp metallic tip of the probe and the surface of a sample**.

Construction

- The experimental setup consists of a probe as shown in Fig. in which a small thin metal wire is etched in such a way that the tip of the probe will have only one atom as shown in Fig

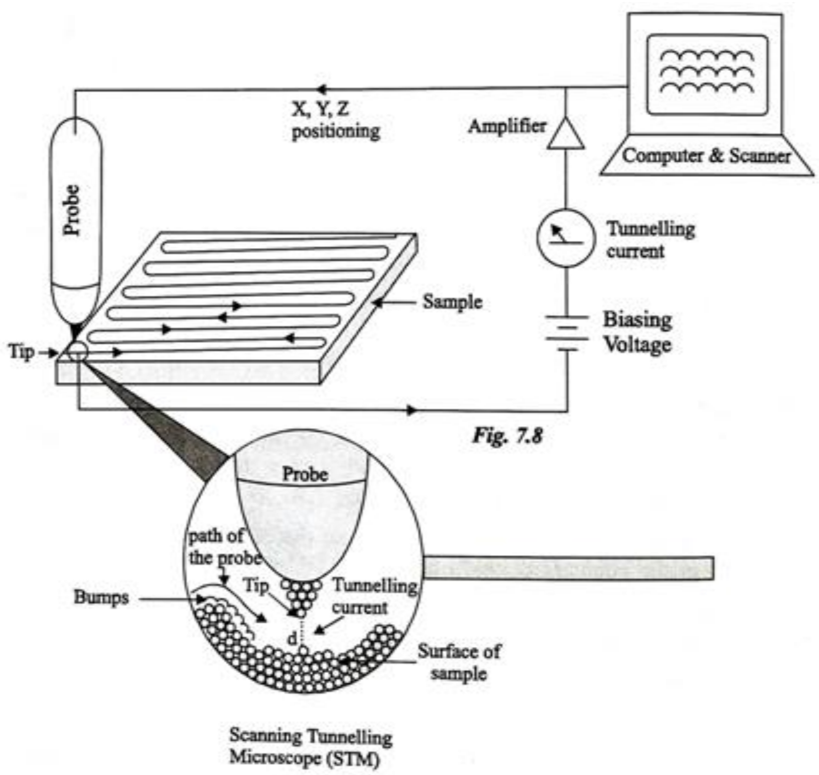


Fig. 7.8

Scanning Tunnelling Microscope (STM)

- The tip is tapered down to a single atom, so that it can follow even a small change in the contours of the sample.
- The tip is connected to the scanner and it can be positioned to X, Y, Z co - ordinates using a personal computer, as shown in Fig.
- The sample for which the image has to be recorded is kept below the tip of the probe at a particular distance (At least to a width of 2 atoms spacing) in such a way that the tip should not touch the sample. i.e., a small air gap should always be maintained between the tip of the probe and the sample, as shown in Fig.
- The computer is also used to record the path of the probe and the topography of the sample in grey -scale (or) colour
- Necessary circuit connections along with an amplifier are provided to measure the tunneling current in the circuit.
-

Working

- Circuit is switched ON and necessary biasing voltage is given to the probe.
- Due to biasing the electrons will tunnel (or) jump between the tip of the probe and the sample and therefore produces a small electric current called tunneling current , as shown in the above Fig
- The tunneling current flows through the circuit only if the tip is in contact with the sample through the small air gap at a distance ' d between them .
- The current produced is amplified and measured in the computer.
- It is found that the current increases (or) decreases based on the distance between the tip of the probe and the sample.
- The current in the circuit should be monitored in such a way that it should be maintained constant.
- Therefore, for maintaining the constant current , the distance (d) between the over the surface of the sample.
- The height fluctuations (d) between the tip and the sample is accurately as shown in Fig
- In a similar way the tip is scanned atom by atom and line by line of the sample between them and the topography of the sample is recorded in the computer.
- The STM does not show the picture of the atom, rather it records only the exact position of the atoms, more precisely the position of electrons.

Advantages

1. It can scan the positions & topography atom by atom (or) even electrons

2. It is the Latest technique used in Research laboratories for scanning the materials
3. Very accurate measurement shall be obtained
4. Magnification is up to nano - scale.

Disadvantages

1. Even a very small sound (or) vibrations will disturb the measurement setup.
2. It should be kept in vacuum, as even a single dust particle may damage the tip of the probe
3. Cost is high
4. More complexity

Applications

1. It is used to produce Integrated circuit.
2. It is used in biomedical devices.
3. Chemical and material sciences research labs are the major areas in which it is used
4. They are used in material science studies for both bump and flat surfaces.

3. Describe the principle and theory of Resonant Tunneling Diode with V-I characteristics. Write its advantages and disadvantages with their uses.

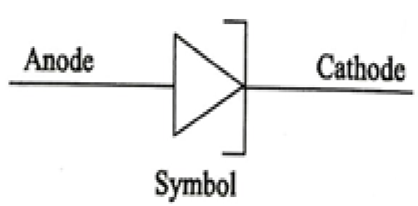
RESONANT TUNNELLING DIODE

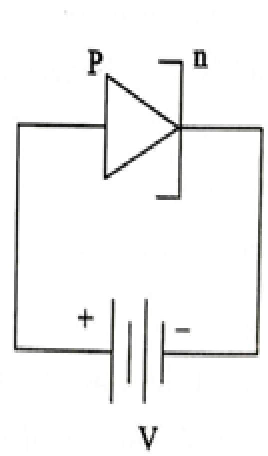
Principle

Resonant tunneling diode works on the principle of tunneling effect, in which the charge carriers cross the energy barrier (s) even with lesser energy than the barrier potential, quantum mechanically. The probability of tunneling increases with the decreasing barrier energy.

Symbol and Circuit diagram

The symbol and circuit diagram of a resonant tunneling diode is as shown in Fig. 1 and Fig. 2, respectively.



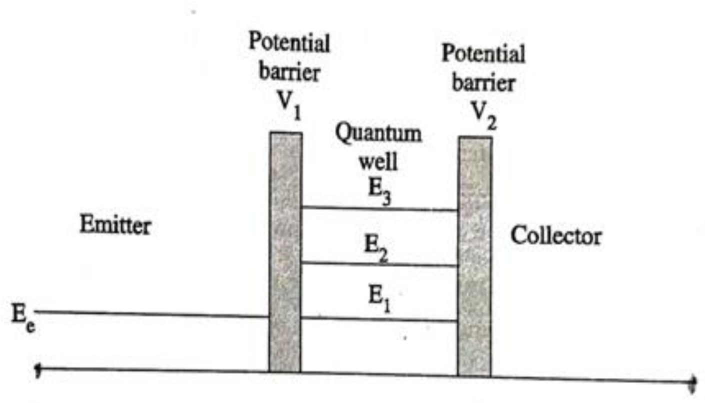


Theory

A resonant tunneling diode also called as Essaki diode is formed using and n - materials, with heavy doping say 1000 times larger than the conventional p-n junction diode. Due to heavy doping , the barrier potential decreases drastically, in turn will help the charge carriers to easily tunnel the junctions quantum mechanically.

Quantum Well Structure

A resonant tunneling diode (RTD) consists of a quantum well structure with discrete energy values E_1. E_2 etc, surrounded by two thin layers of potential barriers. (V1, and V_2) with emitter (in n - region) and collector (in p - region) on either side as shown in Fig.



V - I Characteristics

The diode is forward biased as During forward bias when voltage is increased, then the current in the diode varies at different resistance regions as follows.

Positive Resistance Region

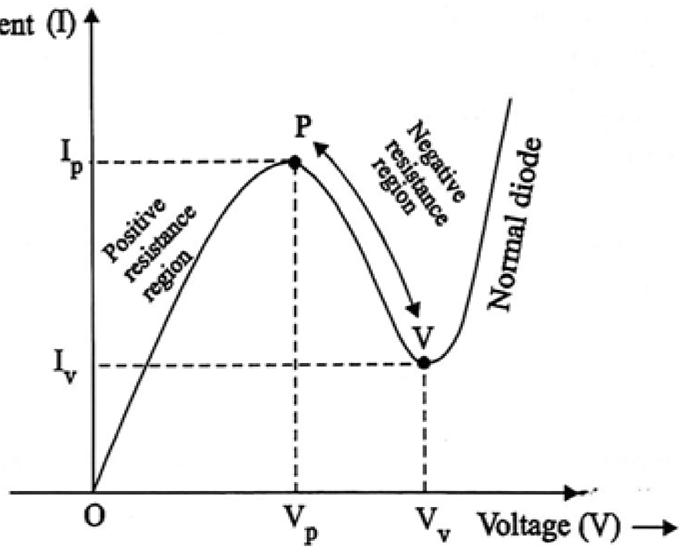
When a voltage is applied across the resonant tunneling diode , a terahertz wave is emitted and therefore at resonance , the energy value (E_1) in the quantum well becomes equal to the energy value (Ee) in the emitter side .

i.e., at low voltage and at resonance $E_1 \approx Ee$

Thus at $E_1 = Ee$ i.e. at resonance, the charge carriers tunnel the potential barriers (V_1, and V_2) and reaches collector region by the process called resonant tunneling.

Therefore the current increases rapidly due to tunneling effect and reaches the peak - point P as shown in Fig. and this current, is called **peak current (Ip)**. The voltage at which the diode reaches peak current is called **peak voltage** (V_p)

This region, where the current increases due to the increase in applied voltage is called **positive resistance region**.



Negative Resistance Region

When the voltage is further increased, then the terahertz wave dies out and now the energy value (E_1) in the quantum well becomes lesser than the energy value (E) in the emitter side

ie.. At higher voltage = $E1 < E_e$

However, since the quantum well has discrete energy values, the energy value (E_2) in the quantum well is still larger than the energy value (E_e) in the emitter side as shown in Fig.

i.e. $E_2 > E_e$

Therefore, the charge carriers cannot tunnel the potential barriers and thus the current in the diode decreases, and reaches the valley point V as shown in Fig. 7.13

This region where the current decreases due to increases in applied voltage is called negative resistance region.

This minimum current is called valley (I_v) and the corresponding voltage is called valley voltage (V_v).

Normal Diode

Now, ***when the applied voltage is further increased beyond the valley point in such a way that energy value (E2) in the quantum well becomes equal, Now, when the applied voltage is to the energy value (Ee) in the emitter side, then the current again increases and therefore the resonant tunneling diode behaves as a normal diode*** as shown in Fig.

Thus, the current in resonant tunneling diode is due to 3 components via.

(i) Tunneling current (I_T)

(ii) Diode current (I_D) and

(iii) Excess current (I_E)

Therefore, total current I_{Total} $I_T + I_D + I_E$

Advantages

1. Cost and noise is low.
2. Fabrication is very simple.
3. Operation speed is very high.
4. Power dissipation is low and hence it is environmental friendly device.

Disadvantages

1. Since it is a two terminal device, it is difficult to isolate the input and output.
2. It is a low output swing device.

Applications

1. As resonant tunneling diode has both positive resistance (From point O to Pl and negative resistance [From point P to V , it has many applications in the switching devices .
2. It can be used as normal diodes also.
3. They are used as high frequency microwave oscillators.
4. When resonant tunneling diode is operated under negative resistance region then it can be used as an oscillator (or) a switch .
5. Resonant tunneling diode (RTD) are used in memory calls, multivalued logic circuit devices etc.

4. Write a brief note on Bloch's theorem for particles in a periodic potential and Kronig Penny model

Bloch theorem

Bloch theorem is a mathematical statement of an electron (particle) wave function moving in a perfectly periodic potential. These functions are called Bloch functions.

Explanation

Let us consider a particle like electron moving in a periodic potential

The one dimensional Schrodinger wave equation can be written for the above

$$\frac{d^2\Psi(x)}{dx^2} + \frac{2m}{\hbar^2}[E - V(X)]\Psi(x) = 0 \text{ ------------------------------------ } 1$$

The PE of an electron if it moves along x-direction,

V(x) – V (x+a) -- 2

Where a is the periodicity of the potential

The solution of eqn 1 can be got

Ψ(x) = e^{ikx} u_k (x) - Bloch theorem ------------------------------------ 3

u_k (x) = u_k (x+a) – Bloch function ------------------------------------- 4

Here e^{ikx} & u_k (x) represent plane wave and periodic function respectively

Proof

If eqn 1 has the solutions with property of eqn 2, we can write the property of the Bloch function, i.e., eqn 3 as

$$\boldsymbol{\Psi(x+a) = e^{ik(x+a)}\, u_k(x+a)}$$

$$\boldsymbol{\Psi(x+a) = e^{ikx} e^{ika}\, u_k(x+a)}$$

Since $u_k(x+a)$, the above eqn can be written as

$$\boldsymbol{\Psi(x+a) = e^{ikx} e^{ika}\, u_k(x)}$$

Since Ψ(x) = e^{ikx} u_k (x), the above eqn can be written as

$\boldsymbol{\Psi(x+a) = e^{ika}\, \Psi(x)}$ **-- 5**

Ψ(x) is a single valued function, then we can write

$\boldsymbol{\Psi(x) = \Psi(x+a)}$

Thus Bloch Theorem is proved

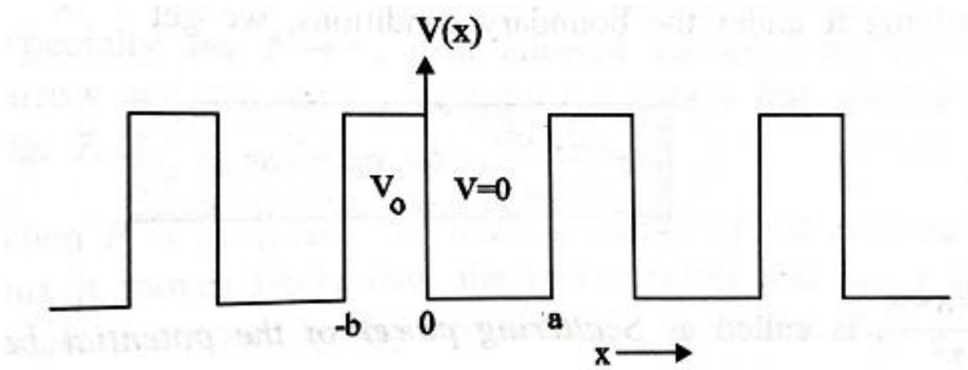
The above eqn is similar to eqn 2 and 4, i.e., If the potential is a function of "x" and "a", then the wave function is also a function of "x" and "a".

BEHAVIOUR OF AN ELECTRON IN A PERIODIC POTENTIAL -THE KRONIG PENNEY MODEL

Kronig and Penney treated a simplest example for one dimensional periodic potential. In this model it is assumed that the potential energy of an electron has the form of a periodic array of square wells as shown in Fig.

Here we have two regions. viz.

Region (i) In this region, between the limits 0 < x < a, the potential energy is zero and hence the electron is assumed to be a free particle.



Therefore, The one dimensional Schroedinger wave equation for a free particle is

$$\frac{d^2\Psi}{dx^2}+\frac{2m}{\hbar^2}[E-0)]\Psi = 0$$

$$\frac{d^2\Psi}{dx^2}+\alpha^2\Psi = 0 \quad \text{----------------------------------- (1)}$$

Where $\alpha^2 = \frac{2mE}{\hbar^2}$

Region (ii) In this region between the limits -b < x < 0, the potential energy of the electron is V_o

∴ The one dimensional Schroedinger wave equation is

$$\frac{d^2\Psi}{dx^2}+\frac{2m}{\hbar^2}[E-V_o)]\Psi = 0$$

$$\frac{d^2\Psi}{dx^2}-\beta^2\Psi = 0 \text{ --------------------------------------- (2)}$$

Where $\beta^2 = \frac{2m}{\hbar^2}[V_o{}_-E)]$ (since E < V_o

For both the region, the appropriate solution suggested by Bloch is of the form,

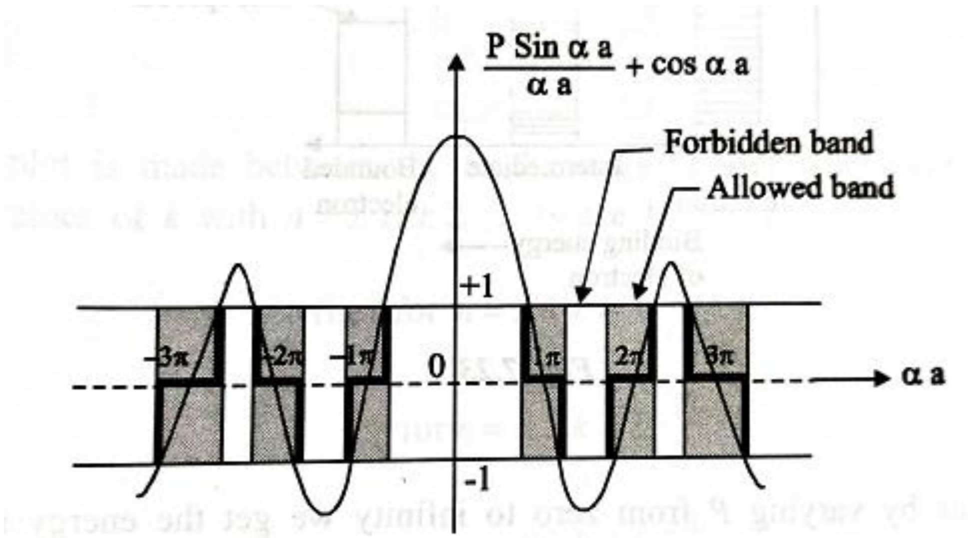
$\Psi(x) = e^{ikx} u_k(x)$ --- (3)

Differentiating equation (3) and substituting it in equation (1) and (2) and then further solving it under the boundary conditions, we get

$$P\,\frac{\sin\alpha a}{\alpha a}+\cos\alpha a = \cos ka \text{ -------------------------- (4)}$$

where $P = \frac{mV_Oba}{\hbar^2}$ *is called as **Scattering power** of the potential barrier, which is the measure of the strength with which the electrons are attracted by the positive ions.*

In equation (4), there are only two variables (i.e) α and k. We know cos ka can take values only from -1 to 1. Therefore the left hand side of equation (4) must also fall in this range. A plot is made between the LHS of equation (4) and ca for a value of $P = \frac{3\pi}{2}$ (arbitrary) as shown in Fig.



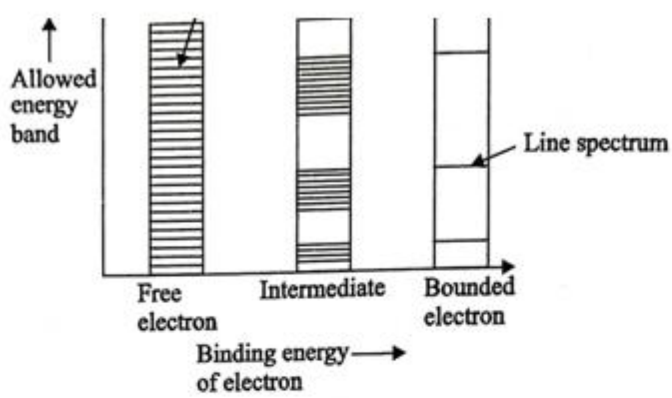
Conclusions

From the above Fig. the following conclusions can be made.

(1) The energy spectrum has a number of allowed energy bands denoted by solid horizontal line separated by forbidden band gaps denoted by dotted lines.

(ii) The width of allowed energy band (shaded portion) increases with the increase in αa.

(iii) When P is increased, the binding energy of the electrons with the lattice points is also increased. Therefore the electron will not be able to move freely and hence the width of the allowed energy band is decreased. Especially for P→ ∞, then allowed energy band becomes infinitely narrow and the energy spectrum becomes a line spectrum as shown in Fig.



(iv) When P is decreased, the binding energy of the electron decreases and thus it moves freely over the lattice points and hence we get a wide range of allowed energy levels as shown in Fig.

(v) Thus by varying P from zero to infinity, we get the energy spectra of all ranges.