14/11/23 Limit of function Day 43

odd and even function

* Odd function

$f(x)$ is odd function if $f(-x) = -f(x)$

* Even function

$f(x)$ is even function if $f(-x) = f(x)$

Q1. $f(x) = x/x^2+1$

$f(-x) = -x/(-x)^2+1$

$f(-x) = -x/x^2+1$

$\therefore f(-x) = -f(x)$

The given fuenetion is odd function.

Q2. $f(x) = x^2+1$

$f(-x) = (-x)^2+1$

$= x^2+1 \quad \therefore f(-x) = f(x)$

The given function is even function.

Q3. $f(x) = \cos x$

$f(-x) = \cos(-x)$

$f(-x) = \cos x$ $\quad \therefore f(-x) = f(x)$

The given function is even.

Q4. $f(x) = \sin x$

$f(-x) = \sin(-x)$

$f(-x) = -\sin x$

$f(-x) = -f(x)$

The given function is odd.

Q5. Evaluate $\lim_{t \to 1} \frac{t^4-1}{t^3-1}$

Soln :

Given :

$$\lim_{t \to 1} \frac{t^4-1}{t^3-1} = \lim_{t \to 1} \frac{(t^2)^2 - 1^2}{t^3 - 1^3}$$

$$= \lim_{t \to 1} \frac{(t^2+1)(t^2-1)}{(t-1)(t^2+t+1)}$$

$\because (a^2-b^2) = (a+b)(a-b)$

$\because (a^3-b^3) = (a-b)(a^2+ab+b)$

$\because (a^3+b^3) = (a+b)(a^2-ab+b)$

$$= \lim_{t \to 1} \frac{(t^2+1)(t+1)(t-1)}{(t-1)(t^2+t+1)}$$

$$= \lim_{t \to 1} \frac{(t^2+1)(t+1)}{t^2+t+1}$$

$$= \frac{(1^2+1)(1+1)}{(1)^2+1+1} = \frac{4}{3}$$

$$\lim_{t \to 1} \frac{t^4-1}{t^3-1} = \frac{4}{3}$$

Q6 : Evaluate $\lim_{x \to 5} (2x^2 - 3x + 4)$

Soln :

Given

$$\lim_{x \to 5} (2x^2 - 3x + 4)$$

$$= 2(5)^2 - 3(5) + 4 = 50 - 15 + 4$$

$$\lim_{x \to 5} (2x^2 - 3x + 4) = 39$$

Q7. Evaluate : $\lim_{x \to 1} \frac{(x^2 - 4x)}{x^2 - 3x - 4}$

Soln :

$$\lim_{x \to 1} \frac{(x^2 - 4x)}{x^2 - 3x - 4}$$

$$= \lim_{x \to 1} \frac{x(x-4)}{(x-4)(x+1)}$$

$$= \lim_{x \to 1} \frac{x}{x+1}$$

$$= \frac{1}{1+1} = \frac{1}{2}$$

$$\lim_{x \to 1} \frac{(x^2 - 4x)}{x^2 - 3x - 4} = \frac{1}{2}$$

Q8. Evaluate $\lim_{x \to 1} \frac{2-x}{(x-1)^2}$

Soln :

Given ; $\lim_{x \to 1} \frac{2-x}{(x-1)^2}$ $\quad \because \left(\frac{1}{0}\right)$ form (∞)

Q9. If $f(x) = 3x^2 - x + 2$. Find $f(a), f(-a)$, $f(a+1), f(a^2)$.

Soln:

Given:

$$f(x) = 3x^2 - x + 2$$

i) $f(a)$

$$3a^2 - a + 2$$

ii) $f(-a)$

$$3a^2 + a + 2$$

iii) $f(a+1)$

$$= 3(a+1)^2 - (a+1) + 2$$

$$= 3(a^2 + 1 + 2a) - a - 1 + 2$$

$$= 3a^2 + 3 + 6a - a - 1 + 2$$

$$= 3a^2 + 5a + 4$$

iv) $f(a^2)$

$$= 3a^2 - a + 2$$

Domain & Range

Q10. If $f(x) = x^2$, find domain & range

Soln:

Given:

$$f(x) = x^2$$

$$y = x^2$$

x	$-\infty$	....	-2	-1	0	1	2	....	∞
$y=f(x)$	∞	...	4	1	0	1	4	....	∞

Domain $(-\infty, \infty)$

Range $[0, \infty)$

Q11. If $f(x) = 1+x^2$, find domain & range

Soln:

Given: $f(x) = 1+x^2$

$$y = 1+x^2$$

x	$-\infty$	..	-2	-1	0	1	2	...	∞
$y=1+x^2$	$+\infty$	..	5	2	1	2	5	...	∞

Domain $(-\infty, \infty)$

Range $[1, \infty)$