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Inertia :

The property due to which a body does not change its state of rest or motion is called Inertia.

Moment of Inertia :

The summation of "Product of the mass and square of the perpendicular distance" of different particles of the body from the axis of rotation. UNIT : $Kg\ m^2$ $I = MR^2$

Theorems of moment of Inertia [M.I]

1) Parallel axis theorem
2) Perpendicular axis theorem

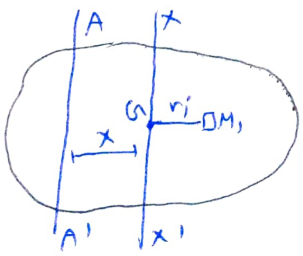
1) Parallel axis theorem :

Theorem :

It states that moment of inertia with respect to any axis to equal to the sum of moment of inertia with respect to a parallel axis passing through the centre of mass and the product of mass and square of the perpendicular distance between the parallel axes.

Proof :

Let us consider a body of mass 'M' for which the centre of mass axis at G. Let AA' be the any axis parallel to xx' passing through G. Let 'x' be perpendicular distance between the parallel axis AA' and XX' as show in fig



The body consists 'n' number of particles with different masses and at different distance from the XX' axis. Let M_i be the mass of one such particle in the body, located at a distance r_i from the XX' axis.

The moment of inertia of this particle with respect to XX' axis is $\therefore I = M_i r_i^2$

$$dI_{XX'} = M_i r_i^2 \rightarrow (1)$$

This moment of inertia of the entire body with respect to XX' axis is.

$$I_{XX'} = \Sigma dI_{XX'} = \Sigma M_i r_i^2 \rightarrow (2)$$

By, the moment of inertia of this particle with respect to AA' axis is

$$dI_{AA} = M_i (r_i + x)^2 \ldots\ldots\rightarrow (3)$$

The moment of Inertia of the entire body with respect to AA' axis is

$$I_{AA'} = \Sigma dI_{AA'} = \Sigma M_i (r_i + x)^2$$

$$I_{AA'} = \Sigma M_i r_i^2 + \Sigma 2 m_i r_i x + \Sigma M_i x^2 \rightarrow (4)$$

Sub. eqn. (2) in eqn. (4) we, get

$$I_{AA'} = I_{XX'} = 2x \Sigma M_i r_i + Mx^2 \rightarrow (5)$$

$$M_i r_i = 0$$

$$M = m \text{ ; (mass of the body)}$$

$$I_{AA'} = I_{XX'} + Mx^2$$

The above eqn. represent parallel axis theorm.