

16/10/23

Q1. Find the eigen value of  $3A + 2I$  of the matrix  $A = \begin{bmatrix} 5 & 4 \\ 0 & 2 \end{bmatrix}$

Soln: 5, 2 are Eigen values of  $A$   $\therefore$  Upper triangular matrix

$$\Rightarrow 3(5) + 2 = 17$$

$$\Rightarrow 3(2) + 2 = 8$$

17, 8 are Eigen values of  $3A + 2I$

## UNIT - 2

### Differential Calculus

Syllabus:

- Representation of functions - limits of a function - continuity - derivation - differentiation rules (sum, product, quotient, chain rules) - implicit differentiation - logarithmic differentiation - applications: maxima and minima of functions of one variable

#### Differentiation

#### Integration

a)  $\frac{d}{dx}(x^n) = nx^{n-1} \quad (1)$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$

Eg:

- $d/dx(x^2) = 2x \quad (1)$
- $d/dx(x) = 1$
- $d/dx(\text{constant}) = 0$

Eg:

- $\int x^2 dx = x^3/3$
- $\int dx = x$

b)  $\frac{d}{dx}(e^{ax}) = a e^{ax}$

$$\int e^{ax} dx = \frac{e^{ax}}{a}$$

c)  $\frac{d}{dx}(\sin ax) = a \cos ax$

$$\int \sin ax dx = -\frac{\cos ax}{a}$$

d)  $\frac{d}{dx}(\cos ax) = -a \sin ax$

$$\int \cos ax dx = \frac{\sin ax}{a}$$



$$e) \frac{d}{dx} (\tan ax) = a \sec^2 ax \quad \int \tan ax dx = \frac{1}{a} \log (\sec ax)$$

$$f) \frac{d}{dx} (\cot ax) = -a \operatorname{cosec}^2 ax \quad \int \cot ax dx = \frac{1}{a} \log (\sin ax)$$

$$g) \frac{d}{dx} (\operatorname{cosec} ax) = -a \operatorname{cosec} ax \cot ax \quad \int \operatorname{cosec} ax dx = \frac{1}{a} \log (\operatorname{cosec} ax - \cot ax)$$

$$h) \frac{d}{dx} (\sec ax) = a \sec ax \tan ax \quad \int \sec ax dx = \frac{1}{a} \log (\sec ax + \tan ax)$$

$$\text{ref: } \cot x = \frac{1}{\tan x}, \tan x = \frac{1}{\cot x}$$

$$\operatorname{cosec} x = \frac{1}{\sin x}, \sec x = \frac{1}{\cos x}$$

$$i) \frac{d}{dx} (\log x) = \frac{1}{x} \quad \int \frac{1}{x} dx = \log x$$

Q1:

$$i) \text{ Differentiate } \frac{d}{dx} (x^2 + e^{2x} - \log x)$$

$$= 2x(1) + 2e^{2x} - \frac{1}{x}$$

$$ii) \frac{d}{dx} (2x^2 + 2e^{2x} - 2 \log x)$$

$$= 4x(1) + 4e^{2x} - 2\left(\frac{1}{x}\right) = 4x + 4e^{2x} - \frac{2}{x}$$

uv method:

$$\frac{d}{dx} (uv) = u \frac{dv}{dx} + v \frac{du}{dx}$$

$$\frac{d}{dx} (uv) = u v' + v u'$$

$$j) \text{ Eg: } \frac{d}{dx} (x^2 e^{2x}) = (x^2) (2e^{2x}) + (e^{2x}) (2x)$$



$$= e^{2x} (2x^2 + 2x)$$

$$= 2x e^{2x} (x+1)$$

Ex:  $\frac{d}{dx} (x e^{3x})$

$$\bullet = \frac{d}{dx} (x) + \frac{d}{dx} (e^{3x}) = (1) + 3e^{3x} = 3e^{3x}$$

$$\bullet \frac{d}{dx} (uv) = uv' + vu'$$

$$= (x) (3e^{3x}) + (e^{3x}) (1)$$

$$= e^{3x} (3x + 1)$$