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Motion of the center of Mass

The motion of the CM is nothing but the force required to accelerate the system of particles with respect to the centre of mass.

$$CM = \sum_{i=1}^{n} m_i x_i \Big/ \sum_{i=1}^{n} m_i$$

(or) $$x_{CM} \Sigma_i m_i = \Sigma_i m_i x_i$$

Since $\Sigma_i m_i = M \Rightarrow$

$$M x_{CM} = m_1 x_1 + m_2 x_2 + m_3 x_3 + \ldots \rightarrow (1)$$

Differentiating (time)

$$M \frac{dx_{CM}}{dt} = m_1 \frac{dx_1}{dt} + m_2 \frac{dx_2}{dt} + m_3 \frac{dx_3}{dt} + \ldots$$

Once again Differ

$$M\frac{d^2x_{cm}}{dt^2} = m_1\frac{d^2x_1}{dt^2} + m_2\frac{d^2x_2}{dt^2} + \ldots\ldots \rightarrow (2)$$

Since acceleration $a = d^2x/dt^2$.

$\Rightarrow$ (2)

$$Ma_{cm} = ma_1 + m_2a_2 + ma_3 + \cdots \rightarrow (3)$$

According Newton's 2nd law $(F = ma)$

$$F_{cm} = F_1 + F_2 + F_3 \cdots$$

$$F_{cm} = \Sigma_i F_i \rightarrow (4)$$

It represents that the force on the CM is equal to the sum of the force acting on the system of particles. This force is required to move the particles with respect to the CM.

Force $\rightarrow$ Potential Energy $\Rightarrow$ Kinetic Energy

Rigid body:

A rigid body is an object which has definite shape and size and does not change due to external fore.

Rigid body can be defined as an extended objects in which the distance between particles is not altered during its motion.