25/10/23

Q) Find $\frac{dy}{dx}$ if $y = e^x - x$

Soln:

Given, $y = e^x - x$

$$\frac{dy}{dx} = e^x - 1$$

Q2) Find y' if $y = a^x$

Soln:

Given:

$y = a^x$

$y' \Rightarrow$

Taking log on both sides

$$\log y = \log a^x$$

$$\frac{1}{y} = x \log a$$

Diff w.r.to x, we get

$$\frac{1}{y} \cdot \frac{dy}{dx} = \log a \quad (1)$$

$$\frac{dy}{dx} = y \log a$$

$$\frac{dy}{dx} = a^x \log a$$

$\therefore \log(a,b) = \log a + \log b$

$\log(a/b) = \log a - \log b$

$\log a^x = x \log a$

Q3) Find y' if $y = 2^x$

Soln:

Given,

$$y = 2^x$$

$$\log y = \log 2^x$$

$$\frac{1}{y} \cdot \frac{dy}{dx} = x \log 2 \Rightarrow \frac{1}{y} \cdot \frac{dy}{dx} = 1 \log 2 \Rightarrow \frac{dy}{dx} = y \log 2$$

$$\Rightarrow \frac{dy}{dx} = 2^x \log 2$$

Q4) Product Rule (or) uv – method

$$d/dx\,(uv) = uv' + vu'$$

* Find $f'(x)$ if $f(x) = (x^3 + 2x)e^x$

Soln:

Given, $f(x) = (x^3 + 2x)e^x$

$$f'(x) = (x^3 + 2x)e^x + e^x(3x^2 + 2)$$

$$= e^x(x^3 + 2x + 3x^2 + 2)$$

Q5) Solve $\frac{d}{dx}(3x^5 \log x)$

Soln: $y = 3x^5 \log x$

$$\frac{dy}{dx} = (3x^5)\frac{1}{x} + \log x\,(15x^4)$$

$$= 3x^4 + 15x^4 \log x$$

Q6) Find dy/dx if $y = x^2e^{2x}(x^2+1)^4$

Soln:

Given, $y = x^2e^{2x}(x^2+1)^4$

* $u = x^2e^{2x}$

$$u' = x^2(2e^{2x}) + e^{2x}(2x)$$

$$u' = 2x^2e^{2x} + 2xe^{2x}$$

* $v = (x^2+1)^4$

$v' = 4(x^2+1)^3(2x+0)$

$= 8x(x^2+1)^3$

$\frac{dy}{dx} = uv' + vu'$

$= x^2e^{2x}(8x(x^2+1)^3) + (x^2+1)^4(2x^2e^{2x} + 2xe^{2x})$.

$= 8x^3e^{2x}(x^2+1)^3 + (x^2+1)^4(2x^2e^{2x} + 2xe^{2x})$

$= (x^2+1)^3(8x^3e^{2x} + (x^2+1)(2x^2e^{2x} + 2xe^{2x}))$.

$= e^{2x}(x^2+1)^3(8x^3 + (x^2+1)\cdot(2x^2+2x))$

$= e^{2x}(x^2+1)^3(8x^3 + 2x^4 + 2x^3 + 2x^2 + 2x)$

$= e^{2x}e^{2x}(x^2+1)^3(2x^4 + 10x^3 + 2x^2 + 2x)$

Q7) Find the derivatives of $y = x^4 - \sin x$

Soln:

Given, $y = x^4 - \sin x$

$\frac{dy}{dx} = 4x^3 - \cos x$

$= \sin x \sec^2 x + \sin x$

$= \sin(\sec^2 x + 1)$

Q8) If $y = \sin x \tan x$ then find dy/dx

Soln: $\frac{dy}{dx} = uv' + vu'$

$= (\sin x)(\sec^2 x) + \tan x(\cos x)$

$= \sin x \sec^2 x + \sin x/\cos x * \cos x$.

* Quotient Rule (or) u/v method

$\frac{d}{dx}\left(\frac{v}{u}\right) = \frac{vu' - uv'}{v^2}$

Q9) $\frac{d}{dx}\left(\frac{x^3}{3x-1}\right)$ Solve.

Soln:

$$\frac{dy}{dx} = \frac{vu' + uv'}{v^2}$$

$$= \frac{(3x-1)(3x^2) + (x^3)(3)}{(3x-1)^2}$$

$$= \frac{9x^3 - 3x^2 - 3x^3}{(3x-1)^2} = \frac{6x^3 - 3x^2}{(3x-1)^2}$$

$$= (3x^2(2x-1))/(3x-1)^2$$

Q10) If $f(x) = x^2/1+2x$ find $f'(x)$

Soln:

Given: $f(x) = x^2/1+2x$

$$f'(x) = vu' + uv'/v^2$$

$$= ((1+2x)(2x) + (x^2)(2))/(1+2x)^2$$

$$= 2x(1+x)/(1+2x)^2$$

Q11) $y = \frac{\sec x}{1+\tan x}$

Soln:

Given: $\frac{dy}{dx} = \frac{vu' + uv'}{v^2}$

$$= \frac{(1+\tan x)(\sec x \tan x) + (\sec x)(\sec^2 x)}{(1+\tan x)^2}$$

$$= \frac{\sec x(\tan x - 1)}{(1+\tan x)^2}$$