26/9/23

Day 12

xi) Symmetric matrix

A matrix A is said to be symmetric if

$A = A^T$

xii) Skew symmetric matrix

The matrix A is said to be skew symmetric if

$$A = -A^T$$

xiii) Singular matrix

A matrix is said to be singular if Determinant of A is zero.

$$|A| = 0.$$

14. Non-Singular matrix

The matrix is said to be Non-Singular if $|A| \neq 0$.

15. Inverse of a matrix

$A^{-1} = 1/|A| \operatorname{adj} A$

adjA ⇒ A^T

⇒ Element minor cofactor

16. Orthogonal matrix

The matrix A is said to be orthagonal if $AA^T = A^TA = I$

17. Conjugate matrix (a+ib)

$$A = \begin{pmatrix} 1+i & 2-3i \\ 7+2i & -i \end{pmatrix} \Rightarrow \bar{A} = \begin{pmatrix} 1-i & 2+3i \\ 7-2i & i \end{pmatrix}$$

18. Unitary matrix

$(\bar{A})^T A = I$

The matrix A is said to be unitary matrix if Conjugate of A of Transpose × A results Identity matrix.

19. Hermition matrix

The matrix A is said to be Hermition if $(\bar{A})^T = A$

20. Skew Hermition matrix

$(\bar{A})^T = -A$

Characteristic Equation

* 2×2 Matrix

$\lambda^2 - S_1\lambda + S_2 = 0$

* 3×3 Matrix

$$\lambda^3 - S_1\lambda^2 + S_2\lambda - S_3 = 0$$

2×2 S_1 = Sum of the main diagonal elements

$S_2 = |A|$

3×3 S_1 = Sum of the main diagonal elements.

S_3 = Sum of the minors of the main diagonal elements

$S_4 = |A|$

Q1 Find the characteristic equation of the matrix

$A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$

Soln: The characteristic eqn is

$\therefore \lambda^2 - S_1\lambda + S_2 = 0$

$S_1 = 1+4 = 5$

$S_2 = -3$

$\lambda^2 - 5\lambda + (-3) = 0 \Rightarrow \lambda^2 - 5\lambda - 3 = 0$

Q2 Find the characteristic eqn of the matrix

$A = \begin{bmatrix} -1 & 3 \\ 2 & -4 \end{bmatrix}$

Soln: The characteristic eqn is

$\therefore \lambda^2 - S_1\lambda + S_2 = 0$

$S_1 = -5$

$S_2 = -2$

$\lambda^2 + 5\lambda - 2 = 0$

Q3 Find the characteristic eqn of the matrix

$A = \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}$

Soln: $\therefore \lambda^2 - S_1\lambda + S_2 = 0$

$S_1 = 3$; $S_2 = 2$ $\therefore \lambda^2 - 3\lambda + 2 = 0$

Q4 Find the characteristic eqn of the matrix

$$A = \begin{bmatrix} 2 & -3 & 1 \\ 3 & 1 & 3 \\ -5 & 2 & -4 \end{bmatrix}$$

Soln :

$$A = \begin{bmatrix} 2 & -3 & 1 \\ 3 & 1 & 3 \\ -5 & 2 & -4 \end{bmatrix}$$

$$\therefore \lambda^3 - S_1\lambda^2 + S_2\lambda - S_3 = 0$$

* $S_1 = -1$
* $S_2 = \begin{vmatrix} 1 & 3 \\ 2 & -4 \end{vmatrix} + \begin{vmatrix} 3 & 3 \\ -5 & -4 \end{vmatrix} + \begin{vmatrix} 3 & -3 \\ -5 & 2 \end{vmatrix}$

$= (-4-6) + (-8+5) + (2+9)$

$= -10 + (-3) + 11 = -13 + 11 = -2$

* $S_3 = 2(-4-6) + 3(-12+15) + 1(2+9)$

$= 2(-10) + 3(-3) + 1(11) = -20 + 9 + 11 = 0$

$$\lambda^3 - S_1\lambda^2 + S_2\lambda - S_3 = 0$$

$$\lambda^3 - (-1)\lambda^2 + (-2)\lambda - 0 = 0$$

$$\lambda^3 + \lambda^2 - 2\lambda = 0$$

$$\lambda(\lambda^2 + \lambda - 2) = 0$$

① $A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$

Soln: $A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$

$$\therefore \lambda^3 - S_1\lambda^2 + S_2\lambda - S_3 = 0$$

$S_1 = 7$

$S_2 = \begin{vmatrix} 3 & 1 \\ 2 & 2 \end{vmatrix} + \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} + \begin{vmatrix} 2 & 3 \\ 1 & 2 \end{vmatrix}$

$= (6-2) + (2-1) + (2-3) = 4 + 1 + 1 = 4$

$$S_3 = 2(6-2) - 2(2-1) + 1(2-3)$$

$$= 2(4) - 2(1) + 1(-1) = 8 - 2 - 1 = 5$$

$$\lambda^3 - S_1\lambda^2 + S_2\lambda - S_3 = 0$$

$$\lambda^3 - 7\lambda^2 + 4\lambda - 5 = 0$$