Reduction of Quadratic form to canonical form

- **Working Rule**

Step 1 : Quadratic form to matrix

Step 2 : Characteristic eqn

Step 3 : To find Eigen values

Step 4 : To find Eigen vectors

Step 5 : To find Normalized matrix N

Step 6 : To find N^T

Step 7 : $D = N^T A N$

Step 8 : To find Canonical form.

$$(y_1 \ y_2 \ y_3) \ D \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}$$

1) Reduce the Q.F

$$Q = 6x^2 + 3y^2 + 3z^2 - 4xy - 2yz + 4zx$$

into canonical form by an orthogonal transformation.

sln: Given,

$$Q = 6x^2 + 3y^2 + 3z^2 + 4xy - 2yz + 4zx$$

Step 1 : Q.F ⟹ Matrix

$$A = \begin{bmatrix} \text{coeff } x^2 & \text{coeff } \frac{1}{2}xy & \frac{1}{2}\text{Coeff } xz \\ \text{Coeff } \frac{1}{2}yx & \text{coeff } y^2 & \text{coeff } \frac{1}{2}yz \\ \text{Coeff } \frac{1}{2}zx & \text{coeff } \frac{1}{2}zy & \text{coeff } \frac{1}{2}z^2 \end{bmatrix}$$

$$A = \begin{bmatrix} 6 & \frac{1}{2}(-4) & \frac{1}{2}(4) \\ \frac{1}{2}(-4) & 3 & \frac{1}{2}(-2) \\ \frac{1}{2}(4) & \frac{1}{2}(-2) & 3 \end{bmatrix}^{-2 \quad 2}$$

$$= \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

Step : 2

$S_1 = 12$

$S_2 = \begin{vmatrix} 3 & -1 \\ -1 & 3 \end{vmatrix} + \begin{vmatrix} 6 & 2 \\ 2 & 3 \end{vmatrix} + \begin{vmatrix} 6 & -2 \\ -2 & 3 \end{vmatrix}$

$\quad = (9-1) + (18-4) + (18-4)$

$\quad = 8 + 14 + 14 = 36$

$S_3 = 6(9-1) + 2(-6-2) + 2(+2-6)$

$\quad = 6(8) + 2(-4) + 2(-4) = 48 - 8 - 8 = 32$

$\lambda^3 - S_1 \lambda^2 + S_2 \lambda - S_3 = 0$

$\lambda^3 - 12\lambda^2 + 36\lambda - 32 = 0$

Step : 3   (use calculator)

To find eigen values

$\lambda_1 = 8 \quad \lambda_2 = 2 \quad \lambda_3 = 2$

step : 4

To find eigen vectors

$(A - \lambda I) X = 0$