PH3151

&

Engineering Physics

Course material

First semester

(Common to all branches of BE/B.Tech)

R- 2021

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PH3151 ENGINEERING PHYSICS

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COURSE OBJECTIVES:

- To make the students effectively achieve an understanding of mechanics.
- To enable the students to gain knowledge of electromagnetic waves and its applications.
- To introduce the basics of oscillations, optics and lasers.
- Equipping the students to successfully understand the importance of quantum physics.
- To motivate the students towards the applications of quantum mechanics.

UNIT I MECHANICS **9**

Multi-particle dynamics: Center of mass (CM) – CM of continuous bodies – motion of the CM – kinetic energy of the system of particles. Rotation of rigid bodies: Rotational kinematics – rotational kinetic energy and moment of inertia - theorems of M .I –moment of inertia of continuous bodies – M.I of a diatomic molecule - torque – rotational dynamics of rigid bodies – conservation of angular momentum – rotational energy state of a rigid diatomic molecule - gyroscope - torsional pendulum – double pendulum –Introduction to nonlinear oscillations.

UNIT II ELECTROMAGNETIC WAVES **9**

The Maxwell's equations - wave equation; Plane electromagnetic waves in vacuum, Conditions on the wave field - properties of electromagnetic waves: speed, amplitude, phase, orientation and waves in matter - polarization - Producing electromagnetic waves - Energy and momentum in EM waves: Intensity, waves from localized sources, momentum and radiation pressure - Cell-phone reception. Reflection and transmission of electromagnetic waves from a non-conducting medium-vacuum interface for normal incidence.

UNIT III OSCILLATIONS, OPTICS AND LASERS **9**

Simple harmonic motion - resonance –analogy between electrical and mechanical oscillating systems - waves on a string - standing waves - traveling waves - Energy transfer of a wave - sound waves - Doppler effect. Reflection and refraction of light waves - total internal reflection - interference –Michelson interferometer –Theory of air wedge and experiment. Theory of laser - characteristics - Spontaneous and stimulated emission - Einstein's coefficients - population inversion - Nd-YAG laser, CO2 laser, semiconductor laser –Basic applications of lasers in industry.

UNIT IV BASIC QUANTUM MECHANICS **9**

Photons and light waves - Electrons and matter waves –Compton effect - The Schrodinger equation (Time dependent and time independent forms) - meaning of wave function - Normalization –Free particle - particle in a infinite potential well: 1D,2D and 3D Boxes- Normalization, probabilities and the correspondence principle.

UNIT V APPLIED QUANTUM MECHANICS **9**

The harmonic oscillator(qualitative)- Barrier penetration and quantum tunneling(qualitative)- Tunneling microscope - Resonant diode - Finite potential wells (qualitative)- Bloch's theorem for particles in a periodic potential –Basics of Kronig-Penney model and origin of energy bands.

TOTAL: 45 PERIODS

COURSE OUTCOMES:

After completion of this course, the students should be able to

- Understand the importance of mechanics.
- Express their knowledge in electromagnetic waves.
- Demonstrate a strong foundational knowledge in oscillations, optics and lasers.
- Understand the importance of quantum physics.
- Comprehend and apply quantum mechanical principles towards the formation of energy bands.

TEXT BOOKS:

1. D.Kleppner and R.Kolenkow. An Introduction to Mechanics. McGraw Hill Education (Indian Edition), 2017.
2. E.M.Purcell and D.J.Morin, Electricity and Magnetism, Cambridge Univ.Press, 2013.
3. Arthur Beiser, Shobhit Mahajan, S. Rai Choudhury, Concepts of Modern Physics, McGraw-Hill (Indian Edition), 2017.

REFERENCES:

1. R.Wolfson. Essential University Physics. Volume 1 & 2. Pearson Education (Indian Edition), 2009.
2. Paul A. Tipler, Physic – Volume 1 & 2, CBS, (Indian Edition), 2004.
3. K.Thyagarajan and A.Ghatak. Lasers: Fundamentals and Applications, Laxmi Publications, (Indian Edition), 2019.
4. D.Halliday, R.Resnick and J.Walker. Principles of Physics, Wiley (Indian Edition), 2015.
5. N.Garcia, A.Damask and S.Schwarz. Physics for Computer Science Students. Springer- Verlag, 2012.

UNIT - I

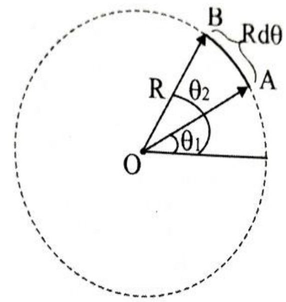
MECHANICS

Multi-particle dynamics: Center of mass (CM) – CM of continuous bodies – motion of the CM – kinetic energy of the system of particles. Rotation of rigid bodies: rotational kinematics – rotational kinetic energy and moment of inertia - theorems of M .I – moment of inertia of continuous bodies – M.I of a diatomic molecule - torque – rotational dynamics of rigid bodies – conservation of angular momentum – rotational energy state of a rigid diatomic molecule - gyroscope - torsional pendulum – double pendulum –Introduction to nonlinear oscillations.

BASIC DEFINITION:

Let us discuss about some of the basic definitions related to motion of the particles in a circular (or) rotation motion.

(i) ANGULAR DISPLACEMENT



The change in position of the particle moving in a circular path with respect to an angle (dθ) is called ***angular displacement****.*

Angular displacement $d\theta = (\theta_1 - \theta_2)$

Unit for angular displacement is Radian

The relation between angular displacement (dθ) and liner displacement (*l*) is given by, Arc length $l = R\, d\theta$

(ii) ANGULAR VELOCITY

The rate of change of angular displacement is called ***angular velocity****.*

Angular velocity $\omega = d\theta/dt$

Unit for angular velocity is Rad S^{-1}

The relation between angular velocity (ω) and liner velocity (v) is given by, $V = r\,\omega$

(iii) ANGULAR ACCELERATION

The rate of change of angular velocity is called ***angular acceleration****.*

Angular acceleration $\alpha = d\omega/dt$ (or) $d^2\theta/dt^2$

Unit for angular velocity: Rad S^{-2}

(iv) ANGULAR MOMENTUM

The moment of inertia times of angular velocity of the particle is called angular momentum. Angular momentum $L = I\,\omega$

Unit for angular momentum is $kgm^2\ S^{-1}$

(v) INERTIA

Inertia *is defined as the tendency of an object to maintain its state of rest or uniform motion along the same direction. Inertia is a resisting capacity of an object to alter its state of rest and motion* (direction and /or magnitude)

MULTI -PARTICLE DYNAMICS [DYNAMICS IN A SYSTEM OF PARTICLES]

Dynamics

Dynamics is the study of motion of bodies under the action of forces.

Example: The moon affecting the ocean waves is the best example for the external force that acts on a body.

Multi -particle Dynamics

Motion in respect of a group of particles in which the separation between the particles will be very small i.e., the distance between the particles will be negligible.

Explanation

In dynamics, we use to study all the physical parameters by considering an object as a point mass and its shape and size is ignored.

But, in real world problems, objects will execute rotation motion also along with translational motion, example if we throw a chalk -piece in air, it has combined motion of both translation and rotational.

As both the translational motion and the rotational motion depend on the shape and size of the object, both cannot be ignored even if it is negligible.

In the universe most of the object consists of many particles (Multi -particle) which have equal (or) unequal mass. If an object consists of multi-particles for which, if the size and shape is negligible, then, it is called system of particles (extended object).

CENTRE OF MASS (CM)

(i) A system consists of many particles with different masses and different position from the reference point.

(ii) The mass of the system is equal to the sum of the mass of each particle in the system.

Therefore, *if the mass of the entire particles of the system (system of particles) is concentrated at a particular point that point is called* ***centre of mass*** *of the system.*

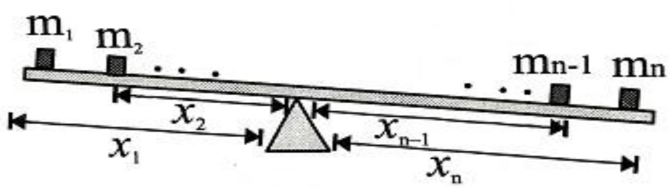
CENTER OF MASS IN A ONE DIMENSION SYSTEM

Definition

If the mass of the entire in the system is concentrated at a particular point, then that point is called ***centre of mass*** *of the system.*

Explanation

Let us consider a fulcrum placed along the x-axis which is not at equilibrium position,



Sea-saw board

The position of masses $m_1, m_2, m_3, m_{n-1}, \ldots\ldots\ldots\ldots m_n$ at a distance $x_1, x_2, x_3 \ldots\ldots x_{n-1}, x_n$ respectively from the supporting point (or) fulcrum.

The tendency of a mass to rotate with respect to origin or supporting point is called moment of mass.

If the moments on both sides are equal, then the system is said to be in equilibrium. The total moments with respect to the fulcrum is

$$m_1 x_1 + m_2 x_2 + m_3 x_3, \ldots\ldots m_{n-1} x_{n-1} + m_n x_n = \sum_{i=1}^{n} m_i x_i = 0 \quad \ldots\ldots\ldots(1)$$

If the total moment is equal to zero, then the centre of mass will lie at the supporting point (or) fulcrum and the system is said to be in equilibrium.

But from the above fig, we can see that the total moment of the system is not equal to zero (unbalanced position), therefore the fulcrum should be adjusted (to a distance X) with respect to the centre of mass area in order to get the balanced position of the system and to reach equilibrium condition (balanced position) as show in fig:

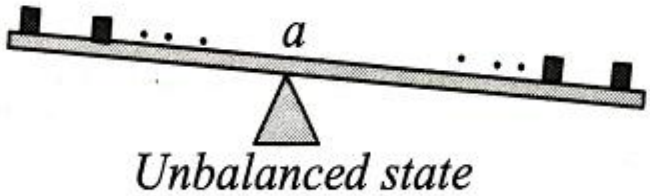
Under equilibrium conditions, we can write equation (1) as

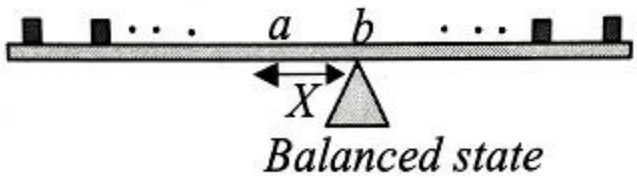
$$\sum_{i=1}^{n} m_i x_i - \sum_{i=1}^{n} m_i X = 0$$

$$X = \frac{\sum_{i=1}^{n} m_i x_i}{\sum_{i=1}^{n} m_i} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

$\sum_{i=1}^{n} m_i$ is the mass of the system

$\sum_{i=1}^{n} m_i x_i$ is the moment of the system





The distance moved to obtain equilibrium position (or) so called the centre of mass in a one dimensional system is given by,

$$X = \frac{m1\ x1\ + m2\ x2 +\ m3\ x3, \ldots\ldots}{m1\ + m2\ +\ m3, \ldots\ldots} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

CENTER OF MASS OF CONTINUOUS BODIES

When a system contains 'n' number of particles, where the mass and position of each particle is represented by m_i and r_i respectively, then

Then centre of mass of the system

$$\overrightarrow{r_{cm}} = \sum_i m_i \vec{r_i} \Big/ \sum_i m_i \quad \text{------------------------} \ (1)$$

Equation (1) represents the summation of the centre of mass of a system. However, this equation will not hold good for continuous bodies, because a continuous body will have infinitesimal small region.

Let us consider the mass of the one such small region 'dm' and its position 'r' if the elemental mass m_i is arbitrarily very small in the region i.e., if m_i tends to zero, then equation (1)will become an integral over the entire volume of the body.

$$\vec{\boldsymbol{r}}_{cm} = \lim_{m_{i->0}} \frac{\sum_i m_i \bar{r}_i}{\sum_i m_i}$$

$$\vec{\boldsymbol{r}}_{cm} = \frac{\int \vec{r}\, dm}{M} \quad \text{-----------------------------}(2)$$

Equation (2) represents the center of mass of continuous bodies

MOTION OF THE CENTER OF MASS [CM]

*The **motion of the centre of mass** is nothing but the force required to accelerate the system of particles with respect to the centre of mass.*

The motion of the centre of mass shall be obtained as follows.

Let us consider an external force 'F' acting on the system of particles along the x-axis.

The centre of mass of the system along x-axis shall be written as

$$x_{cm} = \sum_i \frac{m_i \ x_i}{m_i}$$

(or) $x_{cm} \sum_i m_i = \sum_i m_i \, x_i$

Since $\sum_i m_i = M$, we can write

$$Mx_{cm} = m_1x_{1+} m_2x_{2+} m_3x_3 + \quad \text{--------------------------------(1)}$$

Differentiating once again with respect to time ,we get

$$M \frac{dx_{cm}}{dt} = m_1 \frac{dx_1}{dt} + m_2 \frac{dx_2}{dt} + m_3 \frac{dx_3}{dt} + \ldots\ldots\ldots$$

Differentiating once again with respect to time

$$M \frac{d^2x_{cm}}{dt^2} = m_1 \frac{d^2x_1}{dt^2} + m_2 \frac{d^2x_2}{dt^2} + m_3 \frac{d^2x_3}{dt^2} + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

Since, acceleration $a = d^2x/dt^2$, therefore equation (2) shall be written as

$$M\, a_{cm} = m_1a_1 + m_2\, a_{2\,+}\, m_3a_3 + \ldots\ldots \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \ (3)$$

According to Newton's second law, we know F = ma

Equation (3) shall be written as

$$F_{cm} = F_1+F_2+F_3+\ldots\ldots$$

$$F_{cm} = \sum_i F_i \quad \text{------------------------ (4)}$$

Equation (4) represents that the force on the centre of mass is equal to the sum of the force that acting on the system of particles. This forces is required to move the particles with respect to the center of mass.

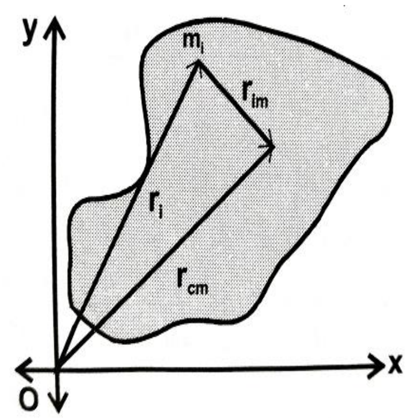
KINETIC ENERGY OF SYSTEM OF PARTICLES

Let us consider a multi particle system with 'n' number of particles in which each particle is moving with come velocity. Let r_i be the displacement and v be the velocity of the i^{th} particle at any instant of time as shown in fig.

Then, the kinetic energy of the i^th^ particle shall be written as

$E_k = \sum_i \frac{1}{2} m_i v_i^2$ -------------------------------------- (1)

If v the velocity of centre of mass with respect to the origin 'o' and v cm is the velocity of i^th^ particle with respect to centre of mass. Then, the velocity of the i^th^ particle can be written as



$V_i = V_{cm} + V_{im}$ --(2)

Substituting equation (2) in equation (1),we get

$E_k = \sum_i \frac{1}{2} m_i (v_{cm} + v_{im})^2$

$E_k = \sum_i \frac{1}{2} m_i (v^2_{cm} + v^2_{im} + 2v_{cm}.v_{im})$ (or)

$E_k = \frac{1}{2}\sum_i m_i V_{cm}^2 + \frac{1}{2}\sum_i m_i V_{im}^2 + \frac{1}{2}\sum_i m_i V_{cm} V_{im})$ -----(3)

Here, $\sum_i m_i$ =M

The total momentum with respect of centre of mass of the system,

i.e., $\sum_i m_i V_{im} = 0$

Therefore, eqution (3) becomes

$E_k = \frac{1}{2} M v^2 + \frac{1}{2} \sum_i m_i v^2_{im}$ ·········· (4)

Equation 4 represents kinetic energy of system of particles.

In equation 4, we have two kinetic energy terms

Term 1; $1/2Mv^2cm$ which represent the kinetic energy of centre of mass of the system and

Term 2: ½ $\sum_i m_i v^2_{im}$ which represents the sum of kinetic energy of all particles (moving with centre of mass) with respect to the origin

ROTATION OF RIGID BODIES

Rigid body

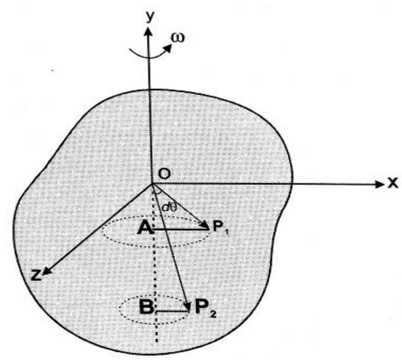
A rigid body is an object which has definite shape and size and does not change due to external force. In other words, ***Rigid body can be defined as an extended object in which the distance between particles is not altered during its motion.***

Rotational motion

A rotational motion in a rigid body may be considered as a stationary motion and here, the rotation is caused by a couple acting on the body. Its state can be changed only by applying a couple (or) set of couples.

Explanation

Let us consider two particles say $p_1$and p_2 which revolves in a circular path about the point A and B respectively.



Here, it is found that the centre of each circle lies on OY and the radii of these circles. (AP1 and BP2) will be equal to perpendicular distance from the axis OY.

We know that in rotational motion, though the particle will have different linear velocities, they will have same angular velocity. Therefore all the particles will rotate through an angle (dθ) in a small interval of time dt

Angular velocity $\omega = \frac{d\theta}{dt}$

The corresponding angular acceleration i.e. rate of change of angular velocity is

$$\alpha = \frac{d\omega}{dt}$$

ROTATIONAL KINEMATICS

Rotational kinematics describes the inter-relationship between the angular displacement, angular velocity and angular acceleration with respect to the time.

It describes the rotational motion of the particles without considering the mass (or) forces that affect the rotation.

Kinematics of rotational motion for constant angular acceleration with respect to an axis of rotation is analogue to kinematics of linear motion.

The equations governing the linear motion and rotational motion with various relationship between displacement, velocity ,acceleration and time are provided in the table as follows.

S.NO	Linear motion	Rotational motion
1	$V_f = V_i + \alpha\, t$	$\omega_f = \omega_i + \alpha t$
2	$S = V_i t + 1/2\, \alpha\, t^2$	$\theta = \omega_i t + \frac{1}{2}\alpha\alpha t^2$
3.	$V^2_f = V_i^2 + 2aS$ Here, V_i is the initial velocity at t=0 V_f is the final velocity at 't' a is the acceleration t is the time S is the displacement	$\omega_f^2 = \omega_i^2 + 2\alpha\theta$ Here, ω_i is the initial angular velocity at t=0 ω_f is the final angular velocity at 't' α is the angular acceleration t is the time θ is the angular displacement

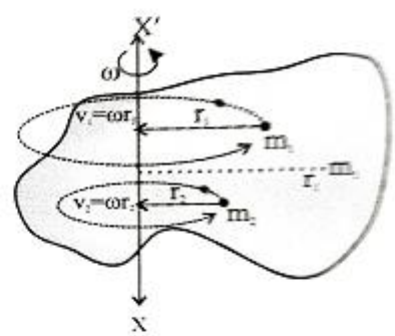
ROTATIONAL KINETIC ENERGY

Definition:- *Rotational Kinetic energy of a rigid body is defined as the sum of the kinetic energy of all the particles in the rigid body rotating about the axis of rotation*

Let us consider a rigid body rotation about an axis XX with constant angular velocity ω as shown in fig.

All particles in rigid body have the same angular velocity 'ω' but with different linear velocity 'v' here the velocity 'v' varies with radial distance from the axis XX'

Let v_1, v_2, …….v_i be the linear velocity of the particle of masses m_1 m_2…..m_i rotating about the axis of rotation at distance r_1, r_2……..r_i respectively.



The K.E of the particle with mass $m_1 = ½\, m_1 v_i^2$

The K.E of the particle with mass $m_2 = ½\, m_2 v_2^2$

The K.E of the particle with mass $m_i = ½\, m_i v_i^2$

Total K.E = $½\, m_1 v_i^2 + ½\, m_2 v_2^2 + \ldots + ½\, m_i v_i^2$ ……………..……(1)

Since all the particles move with same angular velocities (ω) but with different linear velocities (v_1, v_2,……. v_i) at different distance (r_1, r_2 …… r_i,) form the axis of rotation.

$$v_1 = r_1\omega : v_2 = r_2\omega : \ldots.\ v_i = r_i\omega$$

Equation (1) becomes

$$\text{Total K.E} = \frac{1}{2}m_1r_1^2\omega^2 + \frac{1}{2}m_2r_2^2\omega^2 + \;\dots\dots.. + \frac{1}{2}m_ir_i^2\omega^2$$

$$\text{Total K.E} = \frac{1}{2}(\textstyle\sum_i m_i\mathbf{r}_i^2)\,\omega^2 \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots(2)$$

The moment of inertia of body

$$I=\textstyle\sum_i m_I r_i^2 \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots(3)$$

Equation (2) becomes,

$$\boxed{\text{Total Kinetic energy} = \frac{1}{2}I\,\omega^2} \quad \text{---.-----(4)}$$

Equation 4 represents the rotational Kinetic Energy of the particles i.e., rigid boidy

MOMENT OF INERTIA (M.I)

Definition

Moment of inertia of a body about an axis is defined as the summation of "product of the mass and square of the perpendicular distance" of different particles of the body from the axis of rotation. UNIT : kgm2

Concept

The property due to which a body does not change its state of rest or motion is called 'Inertia'.

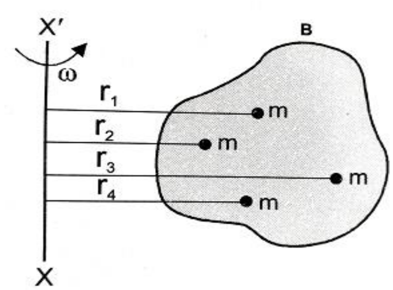
For the motion in a straight line, inertia depends on the mass of the body i.e., if the mass is more, then the inertia will be more.

However, when a body moves about an axis the kinetic energy of its rotation not only depends on its mass and angular velocity, but also depends on the axis about which the rotation is taking place.

Thus the angular inertia not only depends on the mass, but also depends on the square of the distances of particle from the axis of rotation.

PROOF

Let us consider a rigid body 'B' which consists of 'n' number of particles located at different distances from the axis of rotation XX' as shown in fig.



The moment of inertia of the particle $1 = m_1r_1^2$

The moment of inertia of the particle $2 = m_2r_2^2$

We can get the moment of inertia of the entire rigid body by summing the moment of inertia of all particles. I = $\sum m_i r_i^2$ …………………(1)

Equation (1) represents the moment of inertia of rigid body.

THEOREMS OF MOMENT OF INIERTIA [M.I]

The moment of inertia is not only depends on the rotation of axis but also depends on the orientation of the body with respect to the axis, which is different for different axis of the same body .

Based on the orientation of the body and with respect to the rotating axis moment of inertia shall be calculated for various bodies by using the following theorems,

1. Parallel axis theorem
2. Perpendicular axis theorem.

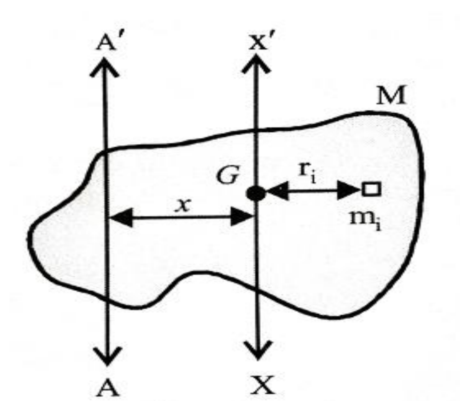
PARALLEL AXIS THEOREM

Theorem:

It states that moment of inertia with respect to any axis is equal to the sum of moment of inertia with respect to a parallel axis passing through the center of mass and the product of mass and square of the perpendicular distance between the parallel axes .

proof

let us consider a body of mass 'M' for which the centre of mass axts at G. let AA' be the an axis parallel to xx' passing through G. let 'x' be perpendicular distance between the parallel axis AA' and XX' as shown in Fig.



The body consists of 'n' number of particles with different masses and at different distances from the XX' axis. Let m_i be the mass of one such particle in the body, located at a distance r_i from the XX' axis

The moment of inertia of this particle with respect to XX' axis is

$dI_{XX'} = m_i r_i^2$ ………………………………(1)

The moment of inertia of the entire body with respect to XX' axis is

$$I_{XX'} = \sum dI_{XX'} = \sum m_i r_i \quad \ldots\ldots\ldots (2)$$

Similarly, the moment of inertia of this particle with respect to AA' axis is

$$dI_{AA'} = m_i (r_i + x)^2 \quad \ldots\ldots\ldots (3)$$

The moment of inertia of the entire body with respect to AA' axis is

$$I_{AA'} = \sum dI_{AA'} = \sum m_i (r_i + x)^2$$

$$I_{AA'} = \sum m_i r_i^2 + \sum 2 m_i r_i x + \sum m_i x^2 \quad \ldots\ldots\ldots (4)$$

Substituting equation (2) in equation (4) we get,

$$I_{AA'} = I_{XX'} + 2x \sum m_i r_i + M x^2 \quad \ldots\ldots\ldots (5)$$

Where, $M = m_i$ [Mass of the body]

According to centre of mass for a rigid body $m_i r_i = 0$

Equation (5) as

$$\boxed{I_{AA'} = I_{XX'} + M x^2} \quad \ldots\ldots\ldots (6)$$

The above equation represents parallel axis theorem.

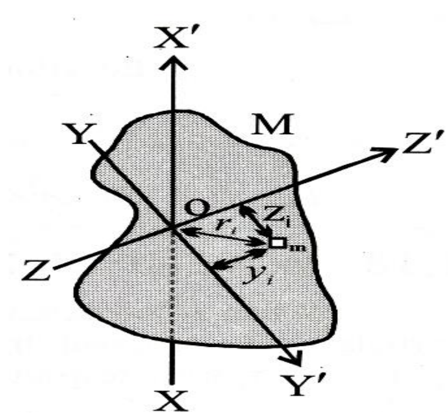
PERPENDICULAR AXIS THEOREM

Theorem

It states that the moment of inertia of a thin plane body with respect to an axis perpendicular to the thin plane surface is equal to the sum of the moment of inertia of a thin plane with respect to two perpendicular axes lying in the surface of the plane and these three mutually perpendicular axes meet at a common point.

Proof:

Let us consider the thin plane body of mass M and three mutually perpendicular axes XX' YY'&ZZ' passing through the point 'O'



Let YY'&ZZ' axis lie in the surface of the thin plane, and XX' axis lies perpendicular to plane surface as show in Fig. The moment of inertia of the thin plane with respect to XX' axis is

$$dI_{xx'} = m_i r_i^2 \quad \ldots\ldots\ldots (1)$$

The moment of inertia of the entire body with respect to the axis XX' is

$I_{xx'} = \sum m_i r_i^2$ ……………………………………(2)

From the Fig ,we can write, $r_i^2 = y_i^2 + z_i^2$ ……..(3)

Substituting equation (3) in equation (2), we get

$$I_{xx'} = \sum m_i (y_i^2 + z_i^2)$$

$$I_{xx'} = \sum m_i y_i^2 + \sum m_i z_i^2 \quad ………….(4)$$

We know that

The moment of inertia of a thin plane with respect to YY' axis is $I_{YY'} = \sum m_i y_i^2$ and

The moment of inertia of a thin plane with respect to YY' axis is $I_{ZZ'} = \sum m_i z_i^2$

Equation (4) becomes

$$\boxed{I_{XX'} = I_{YY'} + I_{ZZ'}}$$

The above equation is representing for perpendicular axis theorem.

MOMENT OF INERTIA OF CONTINUOUS BODIES

When a body contains 'n' number of particles where the mass of each particle is represented by m_1 ,m_2………mi and its position ils represented by r_1,r_2,with respect to the rotation axis ,then , The moment of inertia of the body $I = \sum m_i r_i^2$…………………..(1)

Equation (1) represents the summation of moment of inertia of a system. However this equation will not hold good for a continuous body, because a continuous body will have infinitesimal small regions.

Let us consider the mass of one such small region 'dm' and its position is 'r'

If the elemental mass mi is arbitrarily very small in the region (m_i tends to zero),the equation (1) will become an integral over the entire volume of the body .

$$I = \lim_{mi \to 0} \sum_i m_i r_i^2 = \int dm r^2$$

$$I = \int dm r^2 \quad ……………………………….(2)$$

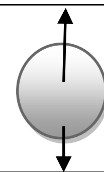
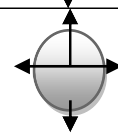
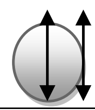
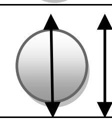
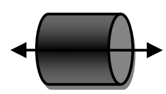
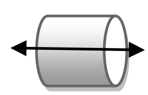
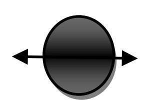
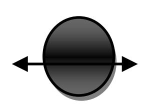
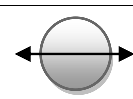
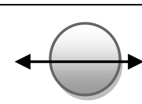
The above equation represents the moment of inertia of continuous body

The moment of inertia of continuous body method is used to find the moment of inertia for various bodies with different shapes

- Circular Ring
- Circular disc
- Solid cylinder
- Hollow cylinder
- Solid sphere
- Hollow sphere etc.,

MOMENT OF INERTIA OF FEW OBJECTS IN DIFFERENT AXIS OF ROTATION AND ITS VALUES

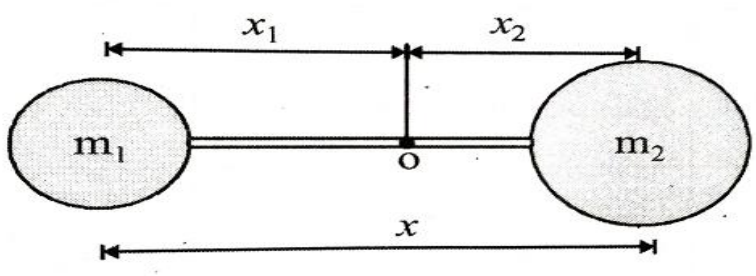
The moment of inertia for various objects with various rotational axis along with their structure is given in the table below,

S.No	Object	Rotating axis	Structure	M.I(I) kgm^2
1	Circular disc	Through COM(center of the disc) & $\perp$ to disc plane		$I = \frac{1}{2}MR^2$
2	Circular disc	Through diameter of the disc & $\|^{rl}$ to disc plane		$I = \frac{1}{4}MR^2$
3	Circular disc	Through edge of the disc)& $\perp$ to disc plane		$I = \frac{3}{4}MR^2$
4	Circular disc	Through edge of the disc & $\|^{rl}$ to disc plane		$I = 4MR^2$
5	Solid cylinder	Through center and along the central axis of the cylinder		$I = \frac{1}{2}MR^2$
6	Hollow cylinder	Through the center and along the central axis of the cylinder		$I = \frac{1}{2}M(R_1^2 + R_2^2)$
7	Solid sphere	Through diameter of the solid sphere		$I = \frac{2}{5}MR^2$
8	Solid sphere	Through tangent of the solid sphere		$I = \frac{7}{5}MR^2$
9	Hollow sphere	Through diameter of the Hollow sphere		$I = \frac{2}{3}MR^2$
10	Hollow sphere	Through tangent of the Hollow sphere		$I = \frac{5}{3}MR^2$

MOMENT OF INERTIA OF A DIATOMIC MOLECULE

Let us consider a rigid diatomic molecule containing two atoms of masses m_1 and m_2, separated by a distance 'x'.

The centre of mass of the system diatomic molecule lies between the two atoms and is denoted by the point 'O'. Let x_1 and x_2 be the distance of two atoms from the point 'O'



Rigid diatomic molecule

From fig, we can write $x = x_1+x_2$ ……………….………..….(1)

Since the system is balanced with respect to the center of mass ,

$m_1x_1 = m_2 x_2$ …………………………...…..……(2)

From equation (1) we can write

$x_2 = x-x_1$ …………………………..……..….(3)

substituting equation (3)in eqn (2)we get

$m_1 x_1 = m_2 (x-x_1)$

$m_1 x_1= m_2x- m_2x_1$ (or)

$m_1 x_1+ m_2x_1= m_2x$ (or)

$(m_1+m_2) x_1 = m_2x$

$$\boxed{x_1 = \frac{m_2\, x}{(m_1+m_2)}}$$ ………………………..(4)

From equation (1) we can write,

$x_1 = x – x_2$ ……………………….(5)

similarly, by substituting equation (5)in eqn (2)we get

$m_1 (x-x_2) = m_2 x_2$

$$\boxed{x_1 = \frac{m_1\, x}{(m_1+m_2)}}$$ ……………………..…(6)

Moment of inertia

The moment of inertia (I) of a diatomic molecule with respect to an axis passing through center of mass of the system shall be written as

$$I = m_1 x_1^2 + m_2 x_2^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(7)$$

Substituting equation (4) and equation (6) in equation (7),we get

$$I = m_1\left[\frac{m_2\ x}{m_1+m_2}\right]^2 + m_2\left[\frac{m_1\ x}{m_1+m_2}\right]^2$$

$$I = \frac{m_1 m_2}{m_1+m_2} x^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(8)$$

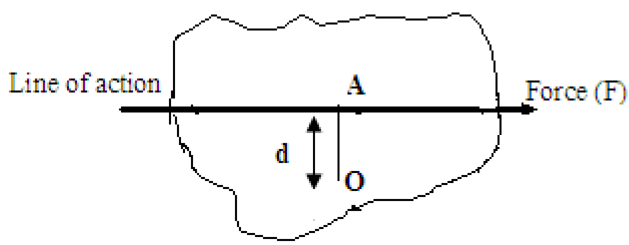
Since , $\mu = \frac{m_1 m_2}{m_1+m_2}$ is called the reduced mass of the system , we can write equation (8) as

$$I = \mu\, x^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(9)$$

Equation (9) represents the moment of inertia of diatomic molecule.

MOMENT

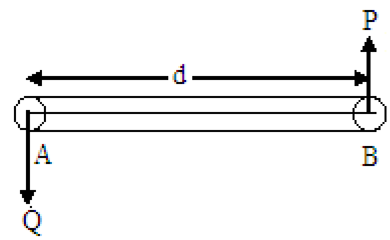
Moment of force:-The product of the magnitude of the force and the perpendicular distance from the point to the line of action of force.



The moment of force F about O is $M_o = F \times d$

COUPLE

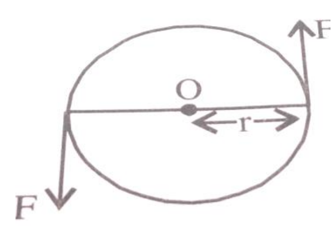
A couple constitutes a pair of two equal and opposite forces acting on a body in such a way that the lines of action of the two forces are not in same straight line.



Couple = $M_A = M_B$ = p x d

TORQUE

Torque is defined as the moment of force acting on the body in rotational motion with respect to the fixed point.



Torque τ = force x Radius

ROTATION DYNAMICS OF RIGID BODIES

Dynamic of rigid bodies

The dynamics of rigid bodies is the study of effect force and couple and its variation with respect to the rigid body

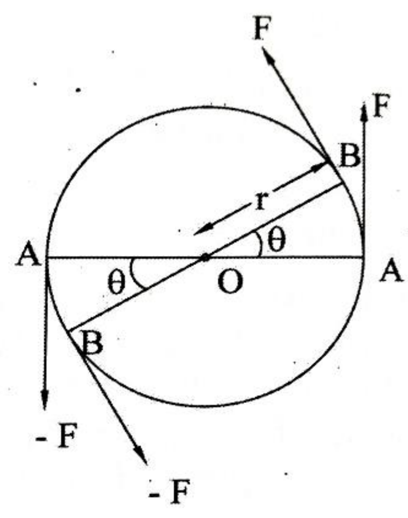
Concept of Rotational dynamics

- Dynamics is the movement of the rigid body under the force, which depends on where the force is acting and the state of restriction of the object.
- If the object has no restriction and force is acting through the center of gravity, then the movement of the object is purely translational as explained by Newton law of motion.

If the object is under the restriction (it is rigidly fixed at one point, called pivot) and if the force is acting in such way that the line of force is not passing through the pivot .This is the concept of rotational dynamics.

Explanation

Let us consider two equal and opposite forces F and –F acting tangentially with respect to the pivot 'O' on the rim of a circular disc from the extremities of diameter as shown in fig.



F - the distance moved by the force

θ - length of the arc AB

Work done by two forces constituting a couple $= 2Fr\,\theta$

The length of the arc AB $= r\theta$

Work done by a single force $= F\,r\,\theta$

Fr is the moment of the couple (or) torque (τ)

Work done by the torque $= \tau\,\theta$

If L is the angular momentum of the rotating body then the relation between torque and angular momentum

Torque $\tau = \frac{dL}{dt}$

Special case

If the object rotation is anti –clock wise, the direction of torque to outward. If the object rotation is clock wise , then the direction of torque is inward.

Rotational dynamics of rigid bodies

The rotational dynamics of rigid bodies are described by the laws of kinematics and the applications of Newton's laws for linear motion and rotational motion

Example

1. Torsional pendulum
2. Double pendulum
3. Gyroscope

CONVERVATION OF ANGULAR MOMENTUM

The relation between torque (τ) and angular momentum (L) is

$$\text{Torque } \tau = \frac{dL}{dt}$$

If no net external torque is acting on the body if $\tau_{net} = 0$

$$\tau_{net} = 0 \Rightarrow \frac{dL}{dt} = 0$$

The above equation is known as the law of conservation of angular momentum

$$I \,\alpha\, \frac{I}{\omega}$$

A rigid body when the moment of inertia increases, then, the angular velocity will decrease, if the external net torque is zero.

Illustration to explain the law of conservation of momentum

Let us one ice dancer dancing as shown in fig. From the fig A, we can observe that the dancer spins slowly when the hands are stretched out. This is due to moment of inertia increases, thus angular velocity (ω) decreses resulting in slower spin

From the fig B, we can observe that the dancer spins faster when the hands are brought closer to the body.

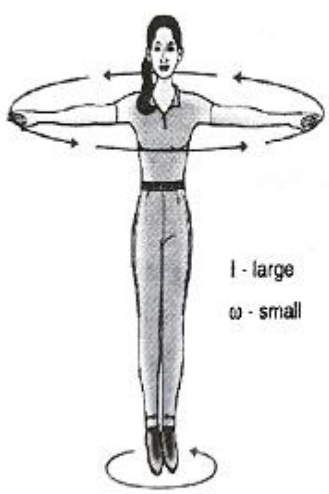


Fig. (A)

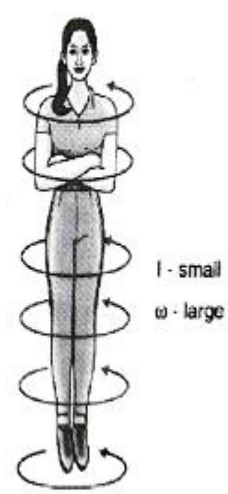


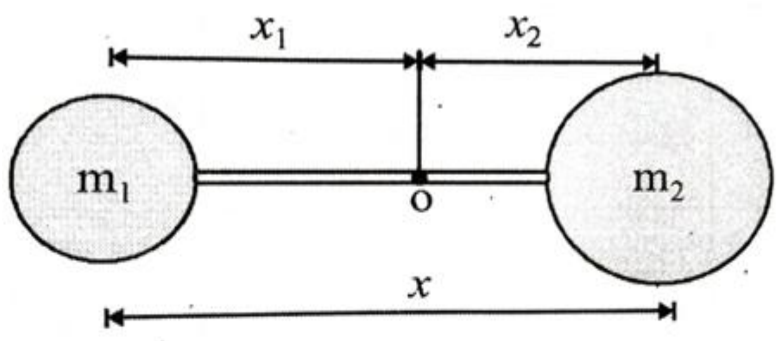
Fig. (B)

This is due to moment of inertia decreases, thus angular velocity (ω) increases resulting in faster spin

ROTATION ENERGY STATE OF A RIGID DIATOMIC MOLECULE

Let us consider a rigid diatomic molecule having two atoms of masses m_1 and m_2 connected by a weightless rod of length 'x'

This rigid diatomic molecule rotates with an angular velocity ω,with respect to an axis through the centre of mass 'O' and is perpendicular to the connecting rod as shown in fig.



We know that, the kinetic energy of rotationg diatomic molecule is

$$K.E = \frac{1}{2} I\omega^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots(1)$$

The angular momentum of a rotating body is

$$L = I\omega \quad \text{(or)}$$

$$\omega = \frac{L}{I} \quad \ldots\ldots\ldots\ldots\ldots (2)$$

Substituting equation (2) in equation (1), we get

$$K.E = \frac{1}{2}\, I\, \frac{L^2}{I^2}$$

$$K.E = \frac{1}{2}\, \frac{L^2}{I}$$

$$K.E = \frac{L^2}{2I} \quad \ldots\ldots\ldots\ldots (3)$$

The moment of inertia of a rotating diatomic molecule is

$$I = \mu x^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

μ= is the reduced mass

Substituting equation (4) in equation (3) we get

$$\text{Kinetic energy (K.E)} = \frac{L^2}{2\mu x^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots(5)$$

Equation (5) represents the classical equation for kinetic energy of a rigid diatomic molecule, in which the energy levels are continuous for all possible valves of 'L'.

But according to quantum mechanics, we know that the energy values are discrete.

∴ Based on the quantum theory, the angular momentum 'z' shall be written as

$$L = \sqrt{J(J+1)}\,\hbar \quad \ldots\ldots\ldots\ldots\ldots\ldots(6)$$

Where J is the total angular momentum quantum number and its values are 0,1,2,3,….. and so on

Substituting equation (6) in equation (5)we get

$$K.E\ (E_J) = \frac{\left(\sqrt{J(J+1)}\right)^2 \hbar^2}{2\mu X^2}$$

$$E_J = \frac{J(J+1)\,\hbar^2}{2\mu X^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (7)$$

Thus, equation (7) represents the rotational kinetic energy a rigid diatomic molecule quantum mechanically.

Special cases

(i) When J=0 ,equation (7) becomes $E_0 = 0$

(ii) When J=1,equation (7) becomes

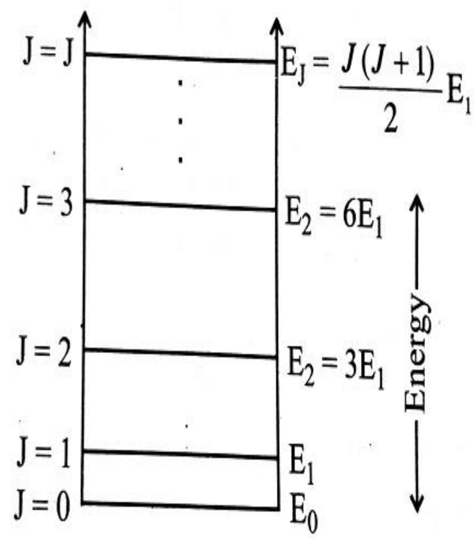
$$E_1 = \frac{2h^2}{2\mu x^2}$$

$$E_1 = \frac{h^2}{\mu x^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(8)$$

(iii)When J=2, equation (7) becomes

$$E_2 = \frac{2(3)h^2}{2\mu x^2}$$

$$E_2 = \frac{3h^2}{\mu x^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(9)$$



From eqn (8) and eqn (9), we can write

$$E_2 = 3E_1$$

When J=3 equation (7) becomes

$$E_3 = \frac{3(4)h^2}{2\mu x^2}$$

$$E_3 = \frac{6h^2}{\mu x^2}$$

$$E_3 = 6\,E_1$$

$$\boxed{E_J = \frac{J(J+1)}{2} E_1}$$

From the above results**, we can confirm that rotational kinetic energy of rigid diatomic molecule is quantized and discrete.**

GYROSCOPE

A gyroscope is a device which is used to measure (or) maintain the angular velocity and orientation, without changing its magnitude.

The device has a spinning wheel (or) disc mounted on a base in which an axis of rotation

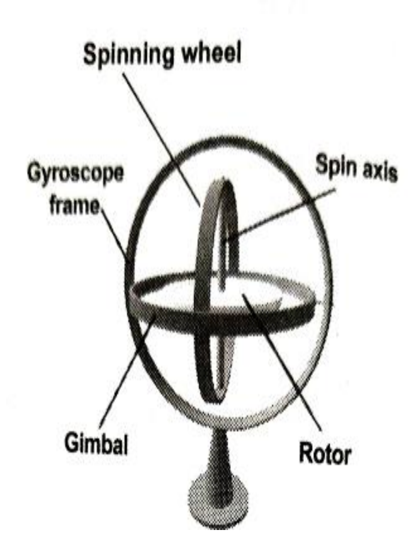
Principle:

The main principle used in gyroscope is the product of angular momentum which is experienced by the torque on the wheel.

Types:

There are different types of gyroscope

- Mechanical gyroscope
- Optical gyroscope
- Gas bearing gyroscope



Design

It consists of four main parts as shown in fig.

1. Rotor
2. Gimbals spinning wheel
3. Spinning wheel
4. Gyroscope frame with base.

In gyroscope the massive rotor is fixed on the supporting rings known as gimbals. The rotor will have three degrees of rotation, which will be helpful to the following parameters.

i. Angular velocity (ω)
ii. Angular momentum (L) and
iii. Torque (τ) of the rotational motion.

The above three parameters are inter related. Here the direction of angular momentum acts in the same direction as that of the rotational axis in symmetrical bodies.

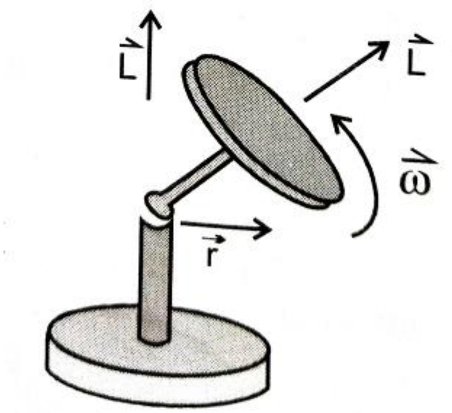
Working

Without spinning

If there is no spinning of wheel (L=0) the free end only move to horizontal plane (xy plane) due to gravitational force .

With spinning

If there is spinning of wheel, the free end moves towards downward direction combined with the spin of the wheel about the axis.



Downward force W = mg will act at a distance 'r' and will produce a torque which in turn will produce an angular momentum simultaneously and rotates the spinning wheel along the horizontal plane

Therefore, gyroscope movement steadily increases depends upon time interval in horizontal direction based on the equation,

$$\sum \tau = \frac{dL}{dt} \quad \text{or} \sum \tau \, dt = dL \qquad \text{…………………..(1)}$$

Due to the constant direction, torque and angular momentum will alter its direction without change of magnitude. As a result, the axis of rotation of wheel does not fall. Thus the gyroscope maintains its orientation even though the base is moved to any place.

Applications

- Gyroscopes are used in the following areas.
- They are used as compasses in boats, aeroplanes air craft's etc,
- Gyroscope is used in spacecraft in order to navigate the spacecraft to the desired target.
- It is also used to stabilize the shops, satellites, ballistic missiles, etc.,
- Gyroscopes are used in gyroscopes for maintaining the direction in tunnel mining.
- In recent days, gyroscopes along with accelerometers are used in smart phones for providing excellent motion sensing.

TORSIONAL PENDULUM

The torsion involves shearing strain and hence the modulus involved is the Rigidity modulus.

Principle

When a disc (torsion pendulum) is rotated in a horizontal plane, the disc executes simple harmonic oscillation due to the restoring couple produced in the wire.

Description

A torsion pendulum consists of a wire with one end fixed to a split chuck and the other end fixed to the centre of the circular disc of radius R.

L the distance between the chuck ends to the disc

r the radius of the suspended wire.

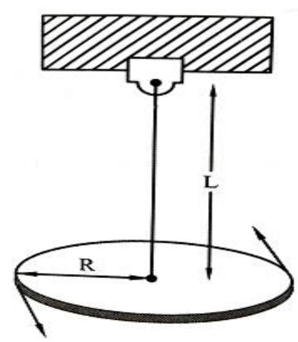
Working

The circular disc is rotated in horizontal plane so that the wire is twisted through an angle 'θ'

The various elements of the wire will undergo shearing strain and a restoring couple is produced. Now if the disc is released, the disc will produce torsion oscillations.

The couple action on the disc produces an angular acceleration in it which is proportional to the angular displacement and is always directed towards its mean position.

Form the law of conservation of energy the total energy of the system is conserved.



Total energy of the torsion pendulum = potential energy (P.E) + kinetic energy (K.E) ------------- (1)

Total energy confined to the wire is equal to the work done in twisting the disc

Restoring couple (P.E) through an angle $\theta = \int_0^{\theta} moment\ of\ couple\ X\ d\,\theta$ P.E $= \int_0^{\theta} C\theta\, d\theta$

P.E $= \frac{C\,\theta^2}{2}$ ……………………………………………..(2)

ω = is th angular velocity with the disc oscillates, due to the resorting couple

The kinetic energy confined to the rotation disc (Deflecting couple) = $\frac{1}{2} I\omega^2$

K.E = $\frac{1}{2} I\omega^2$ ……………………………………..…..…….(3)

I is the moment of inertia of inertia of the circular disc

Total energy T = $\frac{C\theta^2}{2} + \frac{I\omega^2}{2}$ = Constant ……………………..(4)

Differentiating equation (4) with respect to time 't' we get,

C θ $\frac{d\theta}{dt}$ + I ω $\frac{d\omega}{dt}$ = 0 …………….……………..(5)

Since the angular velocity $\omega = \frac{d\theta}{dt}$ and Angular acceleration $\frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$

We can write eqn(5) as

$$C\,\theta \frac{d\theta}{dt} + I\,\frac{d\theta}{dt}\cdot\frac{d^2\theta}{dt^2} = 0 \qquad \text{or} \; \frac{d\theta}{dt}\left[C\,\theta + I\,\frac{d^2\theta}{dt^2}\right] = 0$$

Here , $\frac{d\theta}{dt} \neq 0$

Angular acceleration $\frac{d^2\theta}{dt^2} = \frac{-C\,\theta}{I}$ ………….…...(6)

Negative sign indicates that couple tends to decrease the twist on the wire

Period of oscillation

The time period of oscillation $T = 2\pi\sqrt{\frac{Displacement}{Acceleration}} = \pi\sqrt{\frac{\theta}{C\theta/I}}$

The time period of torsion oscillation $T = 2\pi\sqrt{\frac{I}{C}}$ ….……….(7)

Frequency of oscillation $f = \frac{1}{2\pi}\sqrt{\frac{C}{I}}$.: $f = \frac{1}{T}$

Rigidity modulus of the wire

If ‘r’ is the radius of the wire and ‘L’ is the length of the wire suspended, then we know

The torque per unit twist $C = \frac{n\pi r^4}{2L}$ ……………………….(8)

Substituting eqn(7) in eqn (8) we get

$$T = 2\pi\sqrt{\frac{I\,2L}{n\pi r^4}} \quad : \; T^2 = \frac{4\pi^2 2LI}{\pi n r^4}$$

(or)

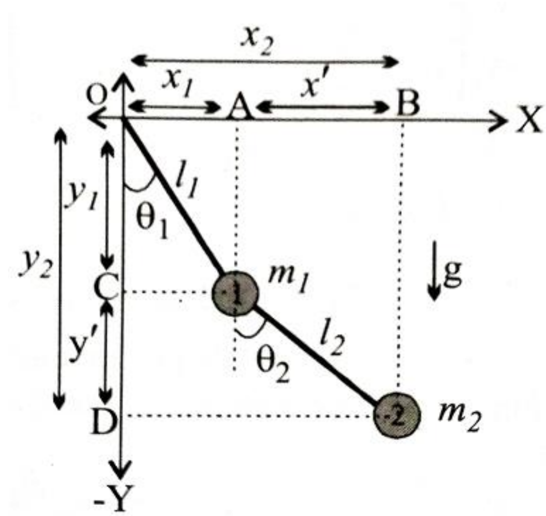
$$\boxed{\text{Rigidity modulus of the wire (n)} = \frac{8\pi\, I\, L}{T^2 r^4}\ \text{Nm}^{-2}}$$

Thus the torsion pendulum shall be used to find the rigidity modulus for various materials.

DOUBLE PENDULUM

Double pendulum consists of two pendulums in which one pendulum is attached to the end of the other pendulum. If the motion is small then the pendulum behaves as a simple pendulum. If the motion is large then it behaves as a chaotic (disorder) system.

Description

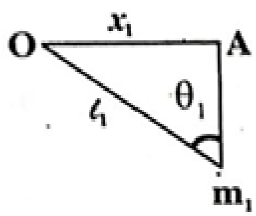


Let us consider a double pendulum suspended to a point 'o' which consists of pendulum-1 of mass $m_1$and pendulum-2 of mass m_2 as shown in fig. Let l_1 be the length of pendulum-1, l_2 be the length of pendulum-2 ,θ_1 and θ_2 - oscillate at an angle respectively

Derivations

Displacement

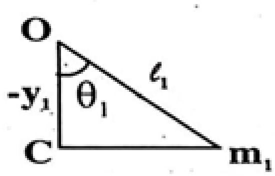
Let x_1 (OA) and x_2 (OB) be the displacement of pendulum -1 and pendululm-2 respectively, along the x_1 -axis and let $y_1$and y_2 be the displacement of pendulum -1 and pendulum -2 respectively, along the negative y-axis.



Then, from the Fig. we can write

$$\sin\theta_1 = \frac{x_1}{l_1}$$

$$x_1 = l_1 \sin\theta_1 \quad \text{……………..(1)}$$



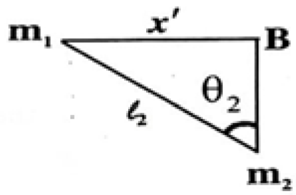
Similarly from fig .we can write

$$\cos\theta_1 = -\frac{y_1}{l_1}$$

$$y_1 = -l_1 \cos\theta_1 \quad \text{……………(2)}$$

Here the negative sign is imposed due to negative y-direction.

Since the displacement of pendulum -2 depends on pendulum-1, from fig we can write the displacement of pendulum -2 along x-axis as



$$x_2 = x_1 + x' \quad \text{………………...(3)}$$

From Fig, we can write

$$\sin\theta_2 = \frac{x'}{l_2}$$

$$x' = l_2 \sin\theta_2 \quad \text{…...…………..(4)}$$

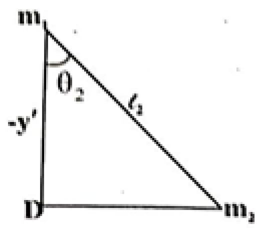
Substituting eqn (4) in eqn (3), we get

$$x_2 = x_1 + l_2 \sin\theta_2 \quad \text{...} (5)$$

Substituting eqn (1) in eqn (5), we get

$$x_2 = l_1 \sin\theta_1 + l_2 \sin\theta_2 \quad \text{...} (6)$$

Similarly we can write the displacement of pendulum-2 along 'y'-axis as



$$y_2 = y_1 + y' \quad \text{.........................} (7)$$

From the fig, we can write

$$\cos\theta_2 = -\frac{y'}{l_2}$$

$$y' = -l_2 \cos\theta_2 \quad \text{.........................} (8)$$

Substituting eqn (8) in eqn (7), we get

$$y_2 = y_1 - l_2 \cos\theta_2 \text{---.--------} (9)$$

Substituting eqn (2) in eqn (9), we get

$$y_2 = -l_1 \cos\theta_1 - l_2 \cos\theta_2 \quad \text{...} (10)$$

Equations (1), (2), (6), and (10) represent the displacement at various positions of the double pendulum.

Velocity:

Differentiating equation (1) we get

$$v_{x1} = \frac{dx_1}{dt} = \frac{d(l_1 \sin\theta_1)}{dt}$$

$$v_{x1} = l_1 \cos\theta_1 \frac{d\theta_1}{dt} \qquad \qquad since\ \frac{d\theta_1}{dt} = \dot{\theta}$$

$$v_{x1} = l_1 \cos\theta_1 \dot{\theta}_1 \quad \text{...} (11)$$

Differentiating equation (2) we get

$$v_{y1} = l_1 \sin\theta_1 \dot{\theta}_1 \quad \text{...}(12)$$

Differentiating eqn 6, we get

$$v_{x2} = \frac{dx_2}{dt} = \frac{d(l_1 \sin\theta_1 + l_2 \sin\theta_2)}{dt}$$

$$v_{x2} = l_1 \cos\theta_1 \dot{\theta}_1 + l_2 \cos\theta_2 \dot{\theta}_2 \quad \text{...................................}(13)$$

Differentiating eqn 10, we get