UNIT-III

Shannon's theorems on channel capacity

The information is transmitted through the channel with rate R called information rate.

Shannon's theorem says that it is possible to transmit information with an arbitrarily small probability of error provided that information rate R is less than or equal to a rate C called channel capacity.

Statement:

$$R \leq C$$

There exists a coding technique such that the O/P of the source may be transmitted over the channel with a probability of error in the received message which may be made arbitrarily small.

This theorem says that if $R \leq C$ it is possible to transmit information without any error even if noise is present.

→ coding techniques are used to detect & correct the errors.

Negative Statement of Channel coding Theorem:

$R > C$

This statement says that If $R > C$, then every message will be in error.

Shannon Hartley theorem (or) Channel capacity theorem (or) Information capacity theorem:

When Shannon's theorem of channel capacity is applied specifically to a channel in which the noise is gaussian is known as Shannon Hartley theorem.

Statement:

The channel capacity of a white band limited gaussian channel is,

$C = B \log_2 (1 + S/N)$ bits/sec.

B - Channel Bandwidth.

S - signal power.

N - Noise power.

→ signal power is,

$$P = \int_{-B}^{B} \text{power spectral density}$$

→ Power spectral density of white noise is $\frac{N_0}{2}$

→ Noise power $N = \int_{-B}^{B} \frac{N_0}{2} df$

$$N = N_0 B.$$

Trade for B/w Bandwidth & Signal to Noise Ratio

Channel capacity of the gaussian channel is, $C = B \log_2 (1 + S/N)$

* Noiseless channel has infinity capacity

If there is no noise in the channel, then $N = 0$, $S/N = \infty$. Such channel is called noiseless channel.

$\therefore \quad C = B \log_2 (1+\infty) = \infty$

Thus noise channel is infinite capacity.

* Infinite bandwidth channel has limited capacity.

If bandwidth is infinite, channel capacity is limited.

noise power $N = N_0 B$.

If noise power increases, S/N - decreases

$\therefore$ If B approaches infinity, capacity does not approach infinity.

$$C_\infty = \lim_{B \to \infty} C = 1.44 \, S/N_0.$$

Linear Block codes:

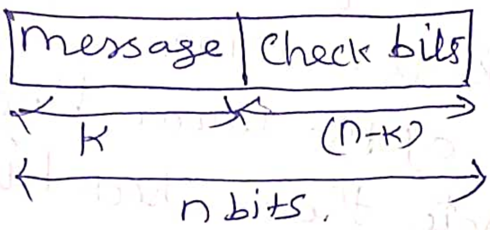
→ For the Block of k message bits, $(n-k)$ ⇒ Parity bits are added.

→ Total bits at the o/p channel encoder are n, such codes are called (n,k) block codes.

Systematic codes:

The message bits appear ~~to~~ at the beginning of the codeword.

message bits appear first & then check bits are transmitted. in a block.



Linear code:

A code is linear if the sum of any two codes vectors produces another code vector.

Code vector ⇒ $m_1\ m_2\ m_3 \ldots m_k$ (message bits)

check bits — $c_1, c_2, c_3, \ldots, c_q$

t.

→ ∴ Code vector,

$$X = (m_1\ m_2\ m_3 \cdots m_k\ c_1\ c_2 \cdots c_q)$$

here $q = n - k$.

↳ no. of redundant bits added by the encoder.

→ The above code vector can also be written as,

$$X = (M|C)$$

M = k bit message vector.

C = q bit check vector.

Check bits play the role of error detection & correction.

→ The Job of the linear block code is to generate those check bits.

Matrix Description of Linear Block Codes:

The code vector can be represented as, $X = MG$. → message vector

$$[X]_{1\times n} = [M]_{1\times k}\,[G]_{k\times n}$$

↓ Matrix form.

↳ Generator matrix.

→ The Generator matrix depends upon the linear block code.

It is represented as,

$$G = [I_k \mid P_{k \times q}]_{k \times n}.$$

I_k = $k \times k$ Identity matrix,

P = $k \times q$ - Submatrix.

→ The check vector can be operated as,

$$C = MP.$$

This can be written as,

$$\underbrace{[c_1 \; c_2 \; \cdots \; c_q]_{1 \times q}}_{\text{Check vector.}} = [m_1 \; m_2 \cdots m_k]_{1 \times k} \begin{bmatrix} P_{11} & P_{12} & \cdots & P_{1q} \\ P_{21} & P_{22} & \cdots & P_{2q} \\ \vdots & & & \\ P_{k1} & P_{k2} & & P_{kq} \end{bmatrix}_{k \times q}$$

→ Check vector can be obtained,

$$c_1 = m_1 P_{11} \oplus m_2 P_{21} \oplus \cdots \oplus m_k P_{k1}$$

$$c_2 = m_1 P_{12} \oplus m_2 P_{22} \oplus \cdots \oplus m_k P_{k2}$$

$$\vdots$$

all the additions are mod-2 additions.

→ Parity check matrix, (H)

$$H = [P^T : I_q]_{q \times n}.$$

P^T — transpose of P submatrix.

$$P = \begin{bmatrix} P_{11} & P_{12} & \cdots & P_{1q} \\ P_{21} & P_{22} & \cdots & P_{2q} \\ \vdots & & & \\ P_{K1} & P_{K2} & & P_{Kq} \end{bmatrix}_{(K \times q)}$$

$$P^T = \begin{bmatrix} P_{11} & P_{21} & \cdots & P_{K1} \\ P_{12} & P_{22} & \cdots & P_{K2} \\ P_{1q} & P_{2q} & \cdots & P_{Kq} \end{bmatrix}_{q \times K}.$$

$$\therefore [H]_{q \times n} = \begin{bmatrix} P^T : I_q \end{bmatrix}_{q \times n}.$$

The generator matrix for a (6,3) block code is given below. Find all code vectors of this code,

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

$n = 6,\ K = 3$

(1). To obtain P submatrix,

$$G = [I_k : P_{k \times q}]$$

$$I_k = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{3\times3}, \quad P_{k\times q} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}_{k \times q}$$

$k=3, \; q=3$

(2)

S.NO	Bits of message vector m_1	m_2	m_3
1	0	0	0
2	0	0	1
3	0	1	0
4	0	1	1
5	1	0	0
6	1	0	1
7	1	1	0
8	1	1	1

(3). Code vector $[c_1\ c_2\ c_3] = [m_1\ m_2\ m_3] \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

$$c_1 = (0 \times m_1) \oplus (1 \times m_2) \oplus (1 \times m_3) = m_2 \oplus m_3$$

$$c_2 = (1 \times m_1) \oplus (0 \times m_2) \oplus (1 \times m_3) = m_1 \oplus m_3$$

$$c_3 = (1 \times m_1) \oplus (1 \times m_2) \oplus (0 \times m_3) = m_1 \oplus m_2$$

S.NO	$m_1 m_2 m_3$ Bits of Message	Check bits c_1 c_2 c_3	Code vector
1	0 0 0	0 0 0	000 000
2	0 0 1	1 1 0	001110
3	0 1 0	1 0 1	010 101
4	0 1 1	0 1 1	011 011
5	1 0 0	0 1 1	100 011
6	1 0 1	1 0 1	101101
7	1 1 0	1 1 0	110110
8	1 1 1	0 0 0	111000

Hamming codes:

→ Hamming codes are (n, k) linear block codes.

→ These codes satisfy the following conditions.

- no. of check bits $q \geq 3$
- Block length $n = 2^q - 1$
- no. of message bits $k = n - q$
- min. distance $d_{min} = 3$

Error detection & correction capabilities of Hamming codes

Since the min distance of hamming code 3. It can be used to detect double errors or correct single errors.