

current-current two-point functions

$$\begin{aligned}
\Pi_{\mu\nu}(q) &= i \int d^4x e^{iqx} \langle 0 | T \left\{ \mathcal{J}_{ij}^\mu(x) \mathcal{J}_{ij}^{\nu\dagger}(0) \right\} \rangle \\
&= \left(q_\mu q_\nu - q^2 g_{\mu\nu} \right) \Pi_{ij,\mathcal{J}}^{(1)}(q^2) + q^\mu q^\nu \Pi_{ij,\mathcal{J}}^{(0)}(q^2) \\
&= \left(q_\mu q_\nu - q^2 g_{\mu\nu} \right) \Pi^{(1+0)}(q^2) + q^2 g_{\mu\nu} \Pi^{(0)}(q^2)
\end{aligned} \tag{1}$$

Inclusive ratio:

$$R_\tau = \frac{\Gamma[\tau^- \rightarrow \nu_\tau \text{hadrons}]}{\Gamma[\tau^- \rightarrow \nu_\tau e^- \bar{\nu}_e]} \tag{2}$$

$$R_\tau = 12\pi S_{EW} \int_0^{m_\tau} \frac{ds}{m_\tau^2} \left(1 - \frac{s}{m_\tau^2} \right) \left[\left(1 + 2 \frac{s}{m_\tau^2} \right) \text{Im} \Pi^{(1)}(s) + \text{Im} \Pi^{(0)}(s) \right] \tag{3}$$

$$\Pi^{(J)}(s) \equiv |V_{uq}|^2 \left(\Pi_{ud,V}^{(J)} + \Pi_{ud,A}^{(J)}(s) \right) \tag{4}$$

OPE:

$$\Pi_{OPE}^{(1+0)}(s) = \sum_{k=0}^{\infty} \frac{C_{2k}(s)}{(-s)^k} \tag{5}$$

$$\mathcal{O}_{4,V/A} = \frac{1}{12} \left[1 - \frac{11}{18} a_s \right] \langle a_s G G \rangle + \left[1 + \frac{\pm 36 - 23}{27} a_s \right] \langle (m_u + m_d) \bar{q} q \rangle \tag{6}$$

Contour integral:

$$\int_{s_{th}}^{s_0} \frac{ds}{s_0} \omega(s) \text{Im} \Pi_{V/A}(s) = \frac{i}{2} \oint_{|s|=s_0} \frac{ds}{s_0} \omega(s) \Pi_{V/A}(s) \tag{7}$$

adler function

$$D(s) \equiv -s \frac{d\Pi^{PT}}{ds} = \frac{1}{12\pi^2} \sum_{n=0}^{\infty} a_\mu^n \sum_{k=1}^{n+1} k c_{nk} \ln \left(\frac{-s}{\mu^2} \right) \tag{8}$$

contour integral in terms of adler

$$A^{\omega,PT} = \frac{i}{2s_0} \oint_{|s|=s_0} \frac{ds}{s} [W(s) - W(s_0)] D(s) \tag{9}$$

spectral function:

$$\rho(s) \equiv \frac{1}{\pi} \text{Im} \Pi(s) \tag{10}$$

aleph:

$$\begin{aligned}
v_1(s) &\equiv \frac{m_\tau^2}{6|V_{ud}|^2 S_{EW}} \frac{B(\tau^- \rightarrow V^- \nu_\tau)}{B(\tau^- \rightarrow e^- \bar{\nu}_e \nu_\tau)} \frac{dN_V}{N_V ds} \left[\left(1 - \frac{s}{m_\tau^2}\right)^2 \left(1 + \frac{2s}{m_\tau^2}\right) \right]^{-1} \\
a_1(s) &\equiv \frac{m_\tau^2}{6|V_{ud}|^2 S_{EW}} \frac{B(\tau^- \rightarrow A^- \nu_\tau)}{B(\tau^- \rightarrow e^- \bar{\nu}_e \nu_\tau)} \frac{dN_A}{N_A ds} \left[\left(1 - \frac{s}{m_\tau^2}\right)^2 \left(1 + \frac{2s}{m_\tau^2}\right) \right]^{-1} \\
a_0(s) &\equiv \frac{m_\tau^2}{6|V_{ud}|^2 S_{EW}} \frac{B(\tau^- \rightarrow \pi^- \nu_\tau)}{B(\tau^- \rightarrow e^- \bar{\nu}_e \nu_\tau)} \frac{dN_A}{N_A ds} \left(1 - \frac{s}{m_\tau^2}\right)^2
\end{aligned} \tag{11}$$

$$\begin{aligned}
\text{Im } \Pi_{\bar{u}d,V}^{(1)}(s) &= \frac{1}{2\pi} v_1(s) \\
\text{Im } \Pi_{\bar{u}d,A}^{(1)}(s) &= \frac{1}{2\pi} a_1(s) \\
\text{Im } \Pi_{\bar{u}d,A}^{(0)}(s) &= \frac{1}{2\pi} a_0(s)
\end{aligned} \tag{12}$$

chisquared:

$$\chi^2(\alpha) = (I_i^{exp} - I_i^{th}(\alpha)) C_{ij}^{-1} (I_j^{exp} - I_j^{th}(\alpha)) \tag{13}$$

$$\begin{aligned}
I_{i=kl}^{exp}(s_k, \omega_l) &= \int_{s_{th}}^{s_k} \frac{ds}{s_k} \omega_l(s) \text{Im } \Pi_{V/A}(s) \\
I_{i=kl}^{th}(s_k, \omega_l) &= \frac{i}{2s_k} \oint_{|s|=s_k} \frac{ds}{s} [W_l(s) - W_l(s_k)] D(s)
\end{aligned} \tag{14}$$

used weights

$$\omega_{wk}(s) = \left(1 - \frac{s}{m_\tau^2}\right)^{2+k} \left(\frac{s}{m_\tau^2}\right)^l \left(1 + \frac{2s}{m_\tau^2}\right) \tag{15}$$

$$(k, l) = (0, 0), (1, 0), (1, 1), (1, 2), (1, 3)$$

weight to OPE:

$$\frac{1}{2\pi i s_0} \oint_{|s|=s_0} ds \left(\frac{s}{s_0}\right)^n \frac{C_{2k}}{(-s)^k} = (-1)^{n+1} \frac{C_{2(n+1)}}{s_0^{n+1}} \delta_{k,n+1} \tag{16}$$

implying that an n -th degree monomial in the weight $\omega(s/s_0)$ selects the $D = 2k = 2(n+1)$ term in the OPE.

$$\frac{s}{s_0} + \left(\frac{s}{s_0}\right)^2 + \left(\frac{s}{s_0}\right)^3 + \dots \tag{17}$$

$$\begin{aligned}
A_{00,V/A}^{ALEPH} &= A_{00,V/A}^{ALEPH}(a_s, \mathcal{O}_{6,V/A}) \\
A_{10,V/A}^{ALEPH} &= A_{10,V/A}^{ALEPH}(a_s, \langle a_s GG \rangle, \mathcal{O}_{6,V/A}, \mathcal{O}_{10,V/A}) \\
A_{11,V/A}^{ALEPH} &= A_{11,V/A}^{ALEPH}(a_s, \langle a_s GG \rangle, \mathcal{O}_{6,V/A}, \mathcal{O}_{10,V/A}, \mathcal{O}_{12,V/A}) \\
A_{12,V/A}^{ALEPH} &= A_{12,V/A}^{ALEPH}(a_s, \mathcal{O}_{6,V/A}, \mathcal{O}_{10,V/A}, \mathcal{O}_{12,V/A}, \mathcal{O}_{14,V/A}) \\
A_{13,V/A}^{ALEPH} &= A_{13,V/A}^{ALEPH}(a_s, \mathcal{O}_{10,V/A}, \mathcal{O}_{12,V/A}, \mathcal{O}_{14,V/A}, \mathcal{O}_{16,V/A})
\end{aligned} \tag{18}$$

α_s

Channel	$\alpha_s(m_\tau^2)$	$\langle a_s GG \rangle$
V+A (FOPT)	$0.319^{+0.010}_{-0.009}$	-3
V+A (CIPT)	0.339	-16