current-current two-point functions

$$\Pi_{\mu\nu}(q) = i \int d^4 x e^{iqx} \langle 0|T\left\{\mathcal{J}_{ij}^{\mu}(x)\mathcal{J}_{ij}^{\nu\dagger}(0)\right\}\rangle$$
$$= \left(q_\mu q_\nu = q^2 g_{\mu\nu}\right)\Pi_{ij,\mathcal{J}}^{(1)}(q^2) + q^\mu q^\nu \Pi_{ij,\mathcal{J}}^{(0)}(q^2) \tag{1}$$
$$= \left(q_\mu q_\nu - q^2 g_{\mu\nu}\right)\Pi^{(1+0)}(q^2) + q^2 g_{\mu\nu}\Pi^{(0)}(q^2)$$

Inclusive ratio:
$$R_\tau = \frac{\Gamma[\tau^- \to \nu_\tau \text{hadrons}]}{\Gamma[\tau^- \to \nu_\tau e^- \bar{\nu}_e]} \tag{2}$$

$$R_\tau = 12\pi S_{EW} \int_0^{m_\tau} \frac{ds}{m_\tau^2}\left(1 - \frac{s}{m_\tau^2}\right)\left[\left(1 + 2\frac{s}{m_\tau^2}\right)\text{Im}\,\Pi^{(1)}(s) + \text{Im}\,\Pi^{(0)}(s)\right] \tag{3}$$

$$\Pi^{(J)}(s) \equiv \left|V_{uq}\right|^2\left(\Pi_{ud,V}^{(J)} + \Pi_{ud,A}^{(J)}(s)\right) \tag{4}$$

OPE:
$$\Pi_{OPE}^{(1+0)}(s) = \sum_{k=0}^{\infty}\frac{C_{2k}(s)}{(-s)^k} \tag{5}$$

Countour integral:
$$\int_{s_{th}}^{s_0}\frac{ds}{s_0}\omega(s)\,\text{Im}\,\Pi_{V/A}(s) = \frac{i}{2}\oint_{|s|=s_0}\frac{ds}{s_0}\omega(s)\Pi_{V/A}(s) \tag{6}$$

chisquared:
$$\chi^2(\alpha) = (I_i^{exp} - I_i^{th}(\alpha))C_{ij}^{-1}(I_j^{exp} - I_j^{th}(\alpha)) \tag{7}$$

used weights
$$\omega_{wk}(s) = \left(1 - \frac{s}{m_\tau^2}\right)^{2+k}\left(\frac{s}{m_\tau^2}\right)^l\left(1 + \frac{2s}{m_\tau^2}\right) \tag{8}$$

$(k,l) = (0,0),(1,0),(1,1),(1,2),(1,3)$
weight to OPE:
$$\frac{1}{2\pi i s_0}\oint_{|s|=s_0}ds\left(\frac{s}{s_0}\right)^n\frac{C_{2k}}{(-s)^k} = (-1)^{n+1}\frac{C_{2(n+1)}}{s_0^{n+1}}\delta_{k,n+1} \tag{9}$$

implying that an n-th degree monomial in the weight $\omega(s/s_0)$ selects the $D = 2k = 2(n+1)$ term in the OPE.

$$A_{00,V/A}^{ALEPH} = A_{00,V/A}^{ALEPH}(a_s, \mathcal{O}_{6,V/A})$$
$$A_{10,V/A}^{ALEPH} = A_{10,V/A}^{ALEPH}(a_s, \langle a_s GG \rangle, \mathcal{O}_{6,V/A}, \mathcal{O}_{10,V/A})$$
$$A_{11,V/A}^{ALEPH} = A_{11,V/A}^{ALEPH}(a_s, \langle a_s GG \rangle, \mathcal{O}_{6,V/A}, \mathcal{O}_{10,V/A} \mathcal{O}_{12,V/A}) \qquad (10)$$
$$A_{12,V/A}^{ALEPH} = A_{12,V/A}^{ALEPH}(a_s, \mathcal{O}_{6,V/A}, \mathcal{O}_{10,V/A}, \mathcal{O}_{12,V/A}, \mathcal{O}_{14,V/A})$$
$$A_{13,V/A}^{ALEPH} = A_{13,V/A}^{ALEPH}(a_s, \mathcal{O}_{10,V/A}, \mathcal{O}_{12,V/A}, \mathcal{O}_{14,V/A}, \mathcal{O}_{16,V/A})$$

α_s

Channel	$\alpha_s(m_\tau^2)$	$\langle a_s GG \rangle$
V+A (FOPT)	$0.319^{+0.010}_{-0.009}$	-3
V+A (CIPT)	0.339	-16