Inclusive ratio:

$$R_\tau = 12\pi S_{EW} \int_0^{m_\tau} \frac{ds}{m_\tau^2} \left(1 - \frac{s}{m_\tau^2}\right) \left[\left(1 + 2\frac{s}{m_\tau^2}\right) \operatorname{Im} \Pi^{(1)}(s) + \operatorname{Im} \Pi^{(0)}(s)\right]$$

(1)

OPE:

$$\Pi_{OPE}^{(1+0)}(s) = \sum_{k=0}^{\infty} \frac{C_{2k}(s)}{(-s)^k}$$

(2)

Countour integral:

$$\int_{s_{th}}^{s_0} \frac{ds}{s_0} \omega(s) \operatorname{Im} \Pi_{V/A}(s) = \frac{i}{2} \oint_{|s|=s_0} \frac{ds}{s_0} \omega(s) \Pi_{V/A}(s)$$

(3)

chisquared:

$$\chi^2(\alpha) = (I_i^{exp} - I_i^{th}(\alpha)) C_{ij}^{-1} (I_j^{exp} - I_j^{th}(\alpha))$$

(4)

used weights

$$\omega_{wk}(s) = \left(1 - \frac{s}{m_\tau^2}\right)^{2+k} \left(\frac{s}{m_\tau^2}\right)^l \left(1 + \frac{2s}{m_\tau^2}\right)$$

(5)

$(k, l) = (0, 0), (1, 0), (1, 1), (1, 2), (1, 3)$
weight to OPE:

$$\frac{1}{2\pi i s_0} \oint_{|s|=s_0} ds \left(\frac{s}{s_0}\right)^n \frac{C_{2k}}{(-s)^k} = (-1)^{n+1} \frac{C_{2(n+1)}}{s_0^{n+1}} \delta_{k,n+1}$$

(6)

implying that an n-th degree monomial in the weight $\omega(s/s_0)$ selects the $D = 2k = 2(n+1)$ term in the OPE.

$$A_{00,V/A}^{ALEPH} = A_{00,V/A}^{ALEPH}(a_s, \mathcal{O}_{6,V/A})$$
$$A_{10,V/A}^{ALEPH} = A_{10,V/A}^{ALEPH}(a_s, \langle a_s GG \rangle, \mathcal{O}_{6,V/A}, \mathcal{O}_{10,V/A})$$
$$A_{11,V/A}^{ALEPH} = A_{11,V/A}^{ALEPH}(a_s, \langle a_s GG \rangle, \mathcal{O}_{6,V/A}, \mathcal{O}_{10,V/A} \mathcal{O}_{12,V/A})$$
$$A_{12,V/A}^{ALEPH} = A_{12,V/A}^{ALEPH}(a_s, \mathcal{O}_{6,V/A}, \mathcal{O}_{10,V/A}, \mathcal{O}_{12,V/A}, \mathcal{O}_{14,V/A})$$
$$A_{13,V/A}^{ALEPH} = A_{13,V/A}^{ALEPH}(a_s, \mathcal{O}_{10,V/A}, \mathcal{O}_{12,V/A}, \mathcal{O}_{14,V/A}, \mathcal{O}_{16,V/A})$$

(7)

α_s

Channel	$\alpha_s(m_\tau^2)$	$\langle a_s GG \rangle$
V+A (FOPT)	$0.319^{+0.010}_{-0.009}$	-3
V+A (CIPT)	0.339	-16