current-current two-point functions

$$\begin{aligned}\Pi_{\mu\nu}(q) &= i\int \mathrm{d}^4\, x e^{iqx}\langle 0|T\left\{\mathcal{J}^{\mu}_{ij}(x)\mathcal{J}^{\nu\dagger}_{ij}(0)\right\}\rangle \\ &= \left(q_\mu q_\nu = q^2 g_{\mu\nu}\right)\Pi^{(1)}_{ij,\mathcal{J}}(q^2) + q^\mu q^\nu \Pi^{(0)}_{ij,\mathcal{J}}(q^2) \\ &= \left(q_\mu q_\nu - q^2 g_{\mu\nu}\right)\Pi^{(1+0)}(q^2) + q^2 g_{\mu\nu}\Pi^{(0)}(q^2)\end{aligned} \tag{1}$$

Inclusive ratio:

$$R_\tau = \frac{\Gamma[\tau^- \to \nu_\tau \text{hadrons}]}{\Gamma[\tau^- \to \nu_\tau e^- \bar{\nu}_e]} \tag{2}$$

$$R_\tau = 12\pi S_{EW}\int_0^{m_\tau}\frac{\mathrm{d}s}{m_\tau^2}\left(1-\frac{s}{m_\tau^2}\right)\left[\left(1+2\frac{s}{m_\tau^2}\right)\operatorname{Im}\Pi^{(1)}(s) + \operatorname{Im}\Pi^{(0)}(s)\right] \tag{3}$$

$$\Pi^{(J)}(s) \equiv \left|V_{uq}\right|^2\left(\Pi^{(J)}_{ud,V} + \Pi^{(J)}_{ud,A}(s)\right) \tag{4}$$

OPE:

$$\Pi^{(1+0)}_{OPE}(s) = \sum_{k=0}^{\infty}\frac{C_{2k}(s)}{(-s)^k} \tag{5}$$

fourth order:

$$\mathcal{O}_{4,V/A} = \frac{1}{12}\left[1-\frac{11}{18}a_s\right]\langle a_s GG\rangle + \left[1+\frac{\pm 36-23}{27}a_s\right]\langle (m_u+m_d)\bar{q}q\rangle \tag{6}$$

Countour integral:

$$\int_{s_{th}}^{s_0}\frac{\mathrm{d}s}{s_0}\omega(s)\operatorname{Im}\Pi_{V/A}(s) = \frac{i}{2}\oint_{|s|=s_0}\frac{\mathrm{d}s}{s_0}\omega(s)\Pi_{V/A}(s) \tag{7}$$

adler function

$$D(s) \equiv -s\frac{\mathrm{d}\Pi^{PT}}{\mathrm{d}s} = \frac{1}{12\pi^2}\sum_{n=0}^{\infty}a_\mu^n\sum_{k=1}^{n+1}kc_{nk}\ln\left(\frac{-s}{\mu^2}\right) \tag{8}$$

contour integral in terms of adler

$$A^{\omega,PT} = \frac{i}{2s_0}\oint_{|s|=s_o}\frac{\mathrm{d}s}{s}\left[W(s)-W(s_0)\right]D(s) \tag{9}$$

chisquared:

$$\chi^2(\alpha) = (I_i^{exp} - I_i^{th}(\alpha))C_{ij}^{-1}(I_j^{exp} - I_j^{th}(\alpha)) \tag{10}$$

used weights

$$\omega_{wk}(s) = \left(1-\frac{s}{m_\tau^2}\right)^{2+k}\left(\frac{s}{m_\tau^2}\right)^l\left(1+\frac{2s}{m_\tau^2}\right) \tag{11}$$

$(k,l) = (0,0),(1,0),(1,1),(1,2),(1,3)$

weight to OPE:

$$\frac{1}{2\pi i s_0}\oint_{|s|=s_0} \mathrm{d}s \left(\frac{s}{s_0}\right)^n \frac{C_{2k}}{(-s)^k} = (-1)^{n+1}\frac{C_{2(n+1)}}{s_0^{n+1}}\delta_{k,n+1} \tag{12}$$

implying that an n-th degree monomial in the weight $\omega(s/s_0)$ selects the $D = 2k = 2(n+1)$ term in the OPE.

$$\begin{aligned}
A^{ALEPH}_{00,V/A} &= A^{ALEPH}_{00,V/A}(a_s, \mathcal{O}_{6,V/A}) \\
A^{ALEPH}_{10,V/A} &= A^{ALEPH}_{10,V/A}(a_s, \langle a_s GG\rangle, \mathcal{O}_{6,V/A}, \mathcal{O}_{10,V/A}) \\
A^{ALEPH}_{11,V/A} &= A^{ALEPH}_{11,V/A}(a_s, \langle a_s GG\rangle, \mathcal{O}_{6,V/A}, \mathcal{O}_{10,V/A}\mathcal{O}_{12,V/A}) \\
A^{ALEPH}_{12,V/A} &= A^{ALEPH}_{12,V/A}(a_s, \mathcal{O}_{6,V/A}, \mathcal{O}_{10,V/A}, \mathcal{O}_{12,V/A}, \mathcal{O}_{14,V/A}) \\
A^{ALEPH}_{13,V/A} &= A^{ALEPH}_{13,V/A}(a_s, \mathcal{O}_{10,V/A}, \mathcal{O}_{12,V/A}, \mathcal{O}_{14,V/A}, \mathcal{O}_{16,V/A})
\end{aligned} \tag{13}$$

α_s

Channel	$\alpha_s(m_\tau^2)$	$\langle a_s GG\rangle$
V+A (FOPT)	$0.319^{+0.010}_{-0.009}$	-3
V+A (CIPT)	0.339	-16