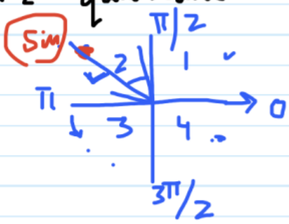
Trigonometry - Problems

Q1. If $\sin x = \frac{3}{5}$, $\cos y = -\frac{12}{13}$, x and y lie in 2^{nd} quadrant.

Find $\sin(x+y)$ ✓



$\rightarrow$

$$\sin(x+y) = \sin x \cos y + \cos x \sin y$$

$$\cos x = \sqrt{1-\sin^2 x} = \sqrt{1-\frac{9}{25}} = \sqrt{\frac{16}{25}} = \pm\frac{4}{5} \Rightarrow -\frac{4}{5}$$

$$\sin y = \sqrt{1-\cos^2 y} = \sqrt{1-\frac{144}{169}} = \sqrt{\frac{25}{169}} = \pm\frac{5}{13} \Rightarrow \frac{5}{13}$$ ✓

$$\sin(x+y) = \frac{3}{5}\left(\frac{-12}{13}\right) + \left(-\frac{4}{5}\right)\frac{5}{13}$$

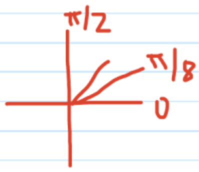
$$= -\frac{36}{65} - \frac{20}{65}$$

$$= \frac{-36-20}{65}$$

$$= -\frac{56}{65}$$ ✓

(2) Find $\tan\frac{\pi}{8}$

$$\tan\frac{\pi}{4} \quad (2x) \rightarrow \quad \frac{\pi}{8} \quad (x)$$



$$\Rightarrow \tan 2x = \frac{2\tan x}{1-\tan^2 x}$$

If $x = \frac{\pi}{8}$

$$ax^2+bx+c=0$$
$$x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$\tan\left(\frac{\pi}{4}\right) = \frac{2\tan\frac{\pi}{8}}{1-\tan^2\frac{\pi}{8}} \Rightarrow 1 = \frac{2\tan\frac{\pi}{8}}{1-\tan^2\pi/8}$$

$$\Rightarrow 1-\tan^2\pi/8 = 2\tan\pi/8 \Rightarrow \left(\tan\frac{\pi}{8}\right)^2 + 2\tan\frac{\pi}{8} - 1 = 0$$

$$\Rightarrow \tan\frac{\pi}{8} = \frac{-2\pm\sqrt{4+4}}{2} = -\frac{2\pm2\sqrt{2}}{2} = -1\pm\sqrt{2} \Rightarrow \sqrt{2}-1$$

③ Prove that : $2\cos\frac{\pi}{13}\cos\frac{9\pi}{13} + \cos\frac{3\pi}{13} + \cos\frac{5\pi}{13} = 0$

$\Rightarrow \cos x + \cos y = 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$ ✓

$$LHS = 2\cos\frac{\pi}{13}\cos\frac{9\pi}{13} + 2\cos\frac{4\pi}{13}\cos\frac{\pi}{13}$$

$$= 2\cos\frac{\pi}{13}\left(\cos\frac{9\pi}{13} + \cos\frac{4\pi}{13}\right)$$

$$= 2\cos\frac{\pi}{13}\left(2\cancel{\cos\left(\frac{\pi}{2}\right)}\cos\left(\frac{5\pi}{26}\right)\right)$$

$= 0$

$= RHS$

④ Prove that : $(\cos x + \cos y)^2 + (\sin x - \sin y)^2 = 4\cos^2\left(\frac{x+y}{2}\right)$

$$\Rightarrow LHS = \cos^2 x + \cos^2 y + 2\cos x\cos y + \sin^2 x + \sin^2 y - 2\sin x\sin y$$

$$= 1 + 1 + 2(\cos x\cos y - \sin x\sin y)$$

$$= 2 + 2\cos(x+y)$$

$\cos 2\theta = 2\cos^2\theta - 1$

$$= 2 + 2\left(2\cos^2\left(\frac{x+y}{2}\right) - 1\right)$$

$$= \cancel{2} + 4\cos^2\left(\frac{x+y}{2}\right) - \cancel{2}$$

$$= 4\cos^2\left(\frac{x+y}{2}\right)$$

$= RHS$

⑤ Prove that : $\dfrac{(\sin 7x + \sin 5x) + (\sin 9x + \sin 3x)}{(\cos 7x + \cos 5x) + (\cos 9x + \cos 3x)} = \tan 6x$

$$\Rightarrow LHS = \frac{2\sin 6x \cdot \cos x + 2\sin 6x \cdot \cos 3x}{2\cos 6x \cdot \cos x + 2\cos 6x \cdot \cos 3x}$$

$$= \frac{\cancel{2}\sin 6x\,(\cancel{\cos x + \cos 3x})}{\cancel{2}\cos 6x\,(\cancel{\cos x + \cos 3x})}$$

$= \tan 6x$

$= RHS$