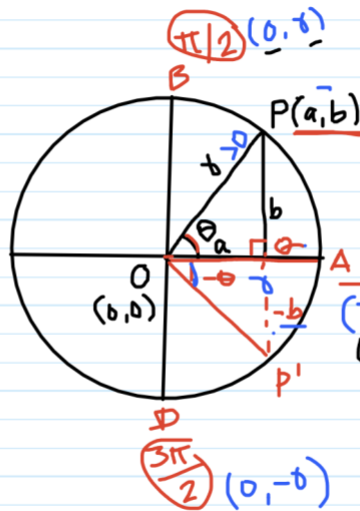


TRIGONOMETRIC FUNCTIONS - part 2

$$\sin \theta = \frac{b}{r}$$

$$\cos \theta = \frac{a}{r}$$

$(-r, 0)$



$$\theta \equiv 2\pi k + \theta$$

$k \in \mathbb{Z}$

$k \frac{\pi}{2} \rightarrow$ Quadrantal Angles

2	1
3	4

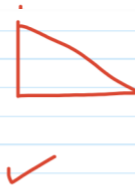
$$\sin(-\theta) = -\sin \theta$$

$$\cos(-\theta) = \cos \theta$$

$$* \sin^2 \theta + \cos^2 \theta = 1$$

$$= \frac{a^2 + b^2}{r^2}$$

$$= \frac{r^2}{r^2} = 1$$



$$\left. \begin{aligned} \tan \theta &= \sin \theta / \cos \theta = b/a \\ \sec \theta &= 1/\cos \theta = r/a \\ \operatorname{cosec} \theta &= 1/\sin \theta = r/b \\ \cot \theta &= 1/\tan \theta = a/b \end{aligned} \right\}$$

$$A[0] \Rightarrow \sin(0) = 0$$

$$(r, 0) \quad \cos(0) = r/r = 1$$

$$C[\pi] \Rightarrow \sin \pi = 0$$

$$(-r, 0) \quad \cos \pi = -r/r = -1$$

$$B[\pi/2] \Rightarrow \sin(\pi/2) = r/r = 1$$

$$(0, r) \quad \cos \pi/2 = 0/r = 0$$

$$D[3\pi/2] \Rightarrow \sin 3\pi/2 = -1$$

$$(0, -r) \quad \cos 3\pi/2 = 0$$

$$\boxed{\sin \theta = 0} \xrightarrow{+2k\pi} 0, \pi, 2\pi, 3\pi, \dots, n\pi \quad n \in \mathbb{Z}$$

$$\boxed{\cos \theta = 0} \xrightarrow{+2k\pi} \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \dots, \frac{(2n+1)\pi}{2} \quad n \in \mathbb{Z}$$

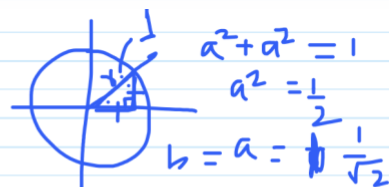
$$\boxed{\csc \theta = \frac{1}{\sin \theta}} \Rightarrow \theta \neq n\pi$$

$$\sec \theta = \frac{1}{\cos \theta} \Rightarrow \theta \neq \frac{(2n+1)\pi}{2}$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta} \Rightarrow \theta \neq \frac{(2n+1)\pi}{2} \quad \checkmark$$

$$\cot \theta = \frac{\cos \theta}{\sin \theta} \Rightarrow \theta \neq n\pi$$

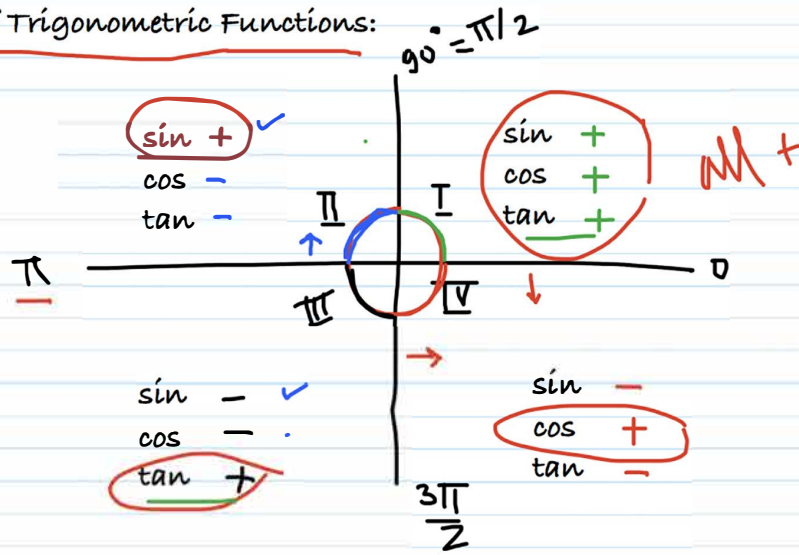
$$\begin{aligned} * \sin^2 \theta + \cos^2 \theta &= 1 \\ * \sec^2 \theta - \tan^2 \theta &= 1 \\ * \csc^2 \theta - \cot^2 \theta &= 1 \end{aligned}$$



$\theta \rightarrow$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$\sin \theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	0	-1	0
$\cos \theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	-1	0	1
$\tan \theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞	0	∞	0

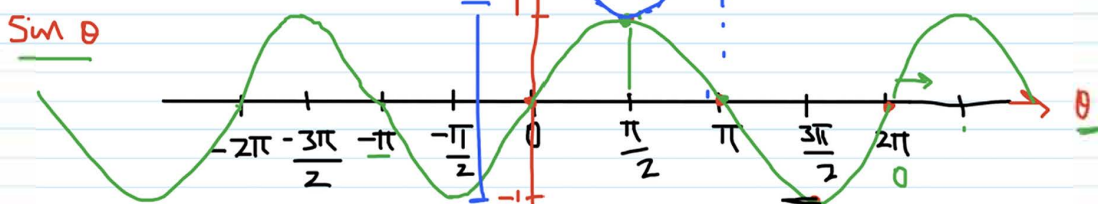


Sign of Trigonometric Functions:

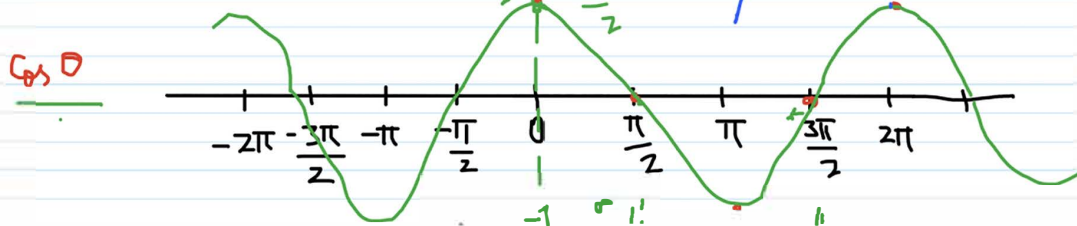


Graphs:

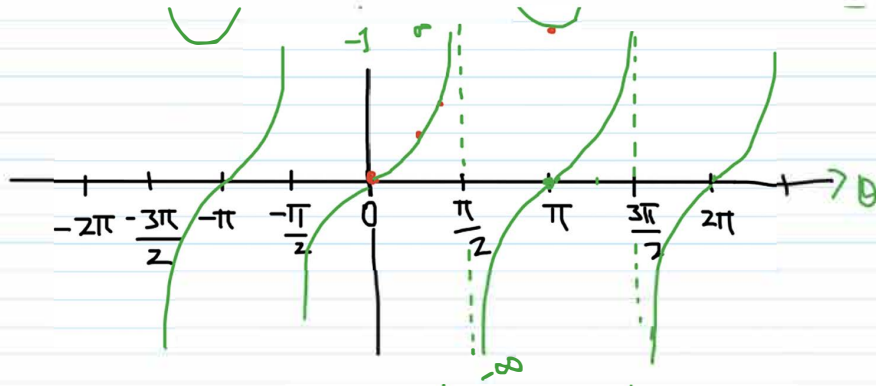
$\sin \theta$



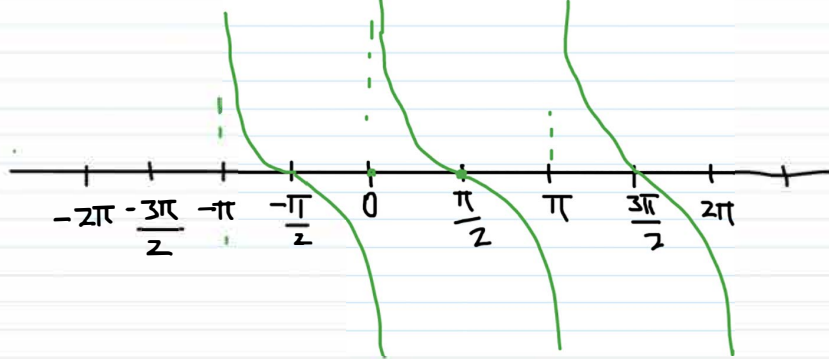
$\cos \theta$



$\tan \theta$



$\cot \theta$



Domain & Range

1. $\sin x \rightarrow$
2. $\cos x$
3. $\tan x$
4. $\operatorname{cosec} x$
5. $\sec x$
6. $\cot x$

Domain

\mathbb{R}

\mathbb{R}

$$\mathbb{R} - \frac{(2n+1)\pi}{2}$$

$$\mathbb{R} - n\pi$$

$$\mathbb{R} - \frac{(2n+1)\pi}{2}$$

$$\mathbb{R} - n\pi$$

Range

$$[-1, 1]$$

$$[-1, 1]$$

\mathbb{R}

$$(-\infty, -1] \cup [1, \infty)$$

)

\mathbb{R}

$$\mathbb{R} - (-1, 1)$$

$$n \in \mathbb{Z}$$

Ex1 - If $\cos x = -3/5$, x lies in the third quadrant. Find the values of other trigonometric functions

$$\sin x = \sqrt{1 - \cos^2 x} = \sqrt{1 - \frac{9}{25}}$$

$$= \pm \frac{4}{5}$$

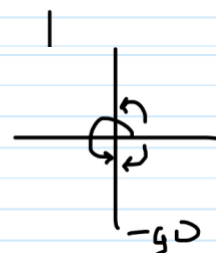
$$\sin x = -\frac{4}{5} \quad 1$$

$$\cos x = -\frac{3}{5} \quad 2$$

$$\sec x = -\frac{5}{3} \quad 3$$

$$\tan x = \frac{4}{3}$$

$$\cot x = \frac{3}{4} \quad 4$$



Ex2 - $\sin(-450^\circ)$

$$\sin(-450^\circ) = \sin(-450^\circ + 360^\circ)$$

$$= \sin(-90^\circ) = -\sin 90^\circ$$

$$= -1$$

$$\left. \begin{aligned} \sin(-\theta) &= -\sin(\theta) \\ \cos(-\theta) &= \cos(\theta) \end{aligned} \right\} \checkmark$$

$$\underline{\sin(\theta) = \sin(\theta + 2n\pi)} \quad \checkmark$$