FUNCTIONS - Problems

Ex1 - Find the domain of $\neq 0$

$f(x) = (x^2 + 3x + 5)/(x^2 - 5x + 4)$

Sol:

$x^2 - 5x + 4 != 0$ $4x$

$\Rightarrow \underbrace{(x-4)}\underbrace{(x-1)} \neq 0$

$\Rightarrow x-4 \neq 0$ & $(x-1) \neq 0$

$\Rightarrow x \neq 4, \quad x \neq 1$

$\Rightarrow x \notin \{1, 4\}$ $\quad 1-5+4$

$$\boxed{\text{Domain} = \mathbb{R} - \{1, 4\}}$$

$16-20+4$

Ex2- Let $f = \{(1,1), (2,3), (0,-1), (-1,-3)\}$

It is linear function from Z to Z. Find f(x).

Sol:

Linear functions are of the form

$\boxed{f(x) = mx + c}$

Substituting the values of f for various values of x, we get:

(1,1) : 1 = m + c. ------ (1)

(0,-1): -1 = 0 + c. ------- (2)

From eq(2), c + 0 = -1

=> $\boxed{c = -1}$

So, substituting the value of c, in eq(1), we get:

1 = m -1

=> m - 1=1

=> $\boxed{m = 1+1 = 2}$

So, f(x) =mx + c

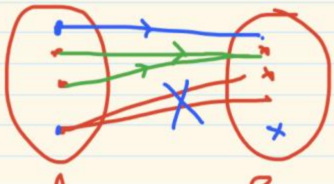
or, $\boxed{f(x) = 2x - 1}$ ✓

Ex3- The relation f is defined as

$$f(x) = \begin{cases} x^2, & 0 <= x <= 3 \\ 3x, & 3 <= x <= 10 \end{cases}$$

The relation g is defined by

$$g(x) = \begin{cases} x^2, & 0 <= x \boxed{<= 2} \\ 3x, & 2 <= x <= 10 \end{cases}$$



$f \neq (a,b), (a,c)$

Show that f is a function and g is not a function.

f: $x \to f$

$0 \to 0$

$1 \to 1$

$\boxed{2 \to 4}$

$3 \to 9$

$\cancel{3 \to 9}$

$4 \to 12$

$3 \to 9$

g: $x \to g(x)$

$0 \to 0$

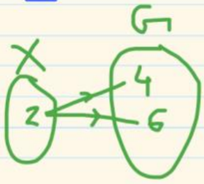
$1 \to 1$

$2 \to 4$

$2 \to 6$

$3 \to 9$

$4 \to 12$



For a relation from set A to B, to be a function, every element in A should have one and only one image in B.

$f = \{(0,0), (1,1), (2,4), (3,9), (4,12), (5,15), \ldots (10,30)\}$

$g = \{(0,0), (1,1), (2,4), (2,6), (3,9), (4,12), \ldots (10,30)\}$

In f, the first elements corresponding to all the the ordered pairs are distinct. So, f is a function.

But, in g element 2 appaears as first element in 2 ordered pairs namely: (2,4) and (2,6). So, 2 has 2 images defined under relation g. So, g is not a function.

Ex4 - Find the domain and range of the $\boxed{\text{real}}$ function defined by $f(x) = \sqrt{(x-1)}$

$\sqrt{\boxed{}} \geq 0$

if $\leq 0 \to$ a imaginary

$D \Rightarrow \quad x - 1 \geq 0$

$\Rightarrow x \geq 1$

Domain $= [1, \infty)$

Range $= [0, \infty)$

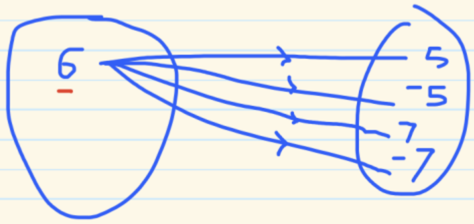
$\sqrt{2-1} = \sqrt{1}$

$\sqrt{\infty}$

Ex5 - f be the subset of Z x Z defined by:
f = {(ab, a+b) : a,b ∈ Z}
Is f a function from Z to Z?

$(-x)(-y) \Rightarrow x^2$

a b	ab	a+b
3, 2	6	5
-3, -2	6	-5
6 1	6	7
-6 -1	6	-7
⋮		



If we take any 2 integers a and b ∈ Z, then (ab, a+b) forms an ordered pair of relation defined by f.
If we take integers -a and -b ∈ Z, then (ab, -a-b) forms an ordered pair of relation defined by f.

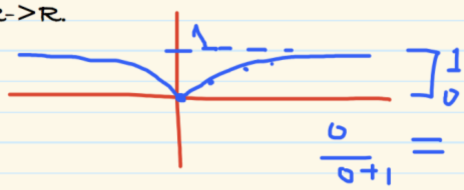
$ab \to a+b$, $ab \to -a-b$

At least for 2 ordered pairs in f (ab, a+b) and (ab, -a-b), ab is the first element. So, the relation defined by f is not a function.

For example, (6, 5) and (6, -5) are ordered pairs corresponding to integers 3,2 and -3, -2.

$\frac{\infty}{\infty+1} \approx 1$

Ex6 - Let f = { (x, $x^2/(1 + x^2)$) : x ∈ R. from R->R.
Find the range of f
Sol: Range = [0, 1)



$\frac{0}{0+1} = 0$