FUNCTIONS - Algebra of Real Functions

1. Addition of Real Functions:

$f : X \dashrightarrow R$, and
$g : X \dashrightarrow R$
f, and g are real functions, where $X \subset R$

Then, we'll define $(f + g) : X \to R$, by
$(f + g)(x) = f(x) + g(x)$, for all $x \in X$

$f(x) = x^2 \to 9$ (3)
$g \quad x^3 \to 27$
$(f+g)(3) = \underbrace{x^2 + x^3}$
$9 + 27 = 36$

2. Subtraction of real functions:

$f : X \to R$,
$g : X \to R$, Then
$(f - g)(x) = f(x) - g(x)$, for all $x \in X$

3. Multiplication of Real Functions: (pointwise multiplication)

$f : X \to R$,
$g : X \to R$, Then
$(fg) : X \to R$, is defined as:
$(fg)(x) = f(x)g(x)$, for all $x \in X$

4. Multiplication of a scalar (a real number):

$f : X \to R$,
c is a scalar like 3.5, -2.7, 5....
Then, the product $cf : X \to R$, defined by:
$(cf)(x) = c\,f(x)$, $x \in X$

$f(x) = x^2$
$f(5) = 25$
$c: 6$
$(6f)(5) = 150$

$cf = cx^2$
$\downarrow 6$
$6x^2$

5. Division/Quotient of two real functions:

Let $f : X \to R$,
$g : X \to R$, be two real functions.
Quotient (f/g) of f by g is defined as:
$(f/g)(x) = f(x)/g(x)$, where $g(x) \mathrel{!}= 0$

Ex1- Let $f(x) = x^2$, $g(x) = 2x + 1$ be two real functions. Find:

(a) $(f + g)(x)$,

(b) $(f - g)(x)$,

(c) $(fg)(x)$,

(d) $(f/g)(x)$

Sol:

(a) We know that for two real functions $f: X \to R$ and $g: X \to R$, for $X \subset R$

$(f + g)(x) = f(x) + g(x)$, for all $x \in X$.

Here, $f(x) = x^2$,

$g(x) = 2x + 1$,

So, $(f + g)(x) = f(x) + g(x)$,

$\Rightarrow (f + g)(x) = x^2 + 2x + 1$

(b) $(f - g)(x) = x^2 - (2x + 1)$

$= x^2 - 2x - 1$

(c) $(fg)(x) = (x^2)(2x + 1)$

$= 2x^3 + x^2$

(d) $(f/g)(x) = f(x)/g(x)$, where $g(x) \neq 0$,

$(f/g)(x) = x^2 / (2x + 1)$, where $(2x + 1) \neq 0$

$\Rightarrow (f/g)(x) = x^2 / (2x + 1)$, where $x \neq -1/2$

$2x + 1 \neq 0$

$2x \neq -1$

$\Rightarrow x \neq \frac{-1}{2}$

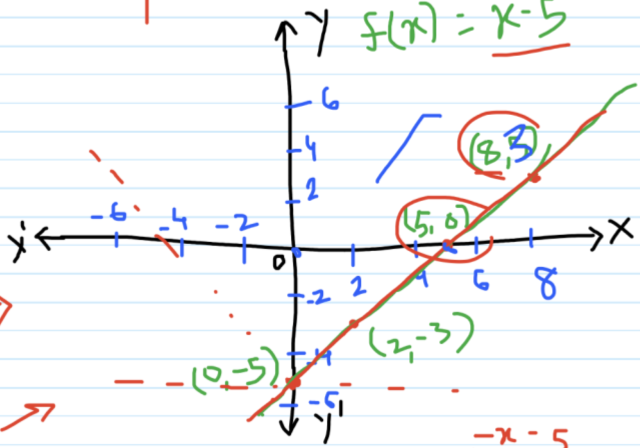
Ex2 - $f: R \to R$, defined as:

$f(x) = x - 5$.

Sketch the graph of $f(x)$

x	$f(x) = x - 5$
0	-5
2	-3
5	0
8	3

$\Rightarrow$



$-x - 5$

Linear Functions:

$\boxed{f(x) = mx + c}$, $x \in R$

$x - 5$

$m > 0$

$m < 0$

$m, c \to$ constants

$m = 1$, $c = -5$

$m = 0$

$(f/g)(x) = x^2 / (2x + 1)$,

$(8, 3)$, $(5, 0)$ find $f(x)$

$f(x) = mx + c$

Now substituting the value of x and f(x), from the given points

$3 = m8 + c.$ -----(1)

$0 = m5 + c.$ -----(2)

Subtract eq 2 from eq 1,

$3-0 = 8m + c - 5m - c$

$\Rightarrow 3 = 3m$

$\Rightarrow m = 1$

Now, substituting the value of m in eq. 1,

$3 = 8 + c$

$\Rightarrow c = 3 - 8 = -5$

$f(x) = x + (-5)$

$f(x) = x - 5$