# TRIGONOMETRIC EQUATIONS

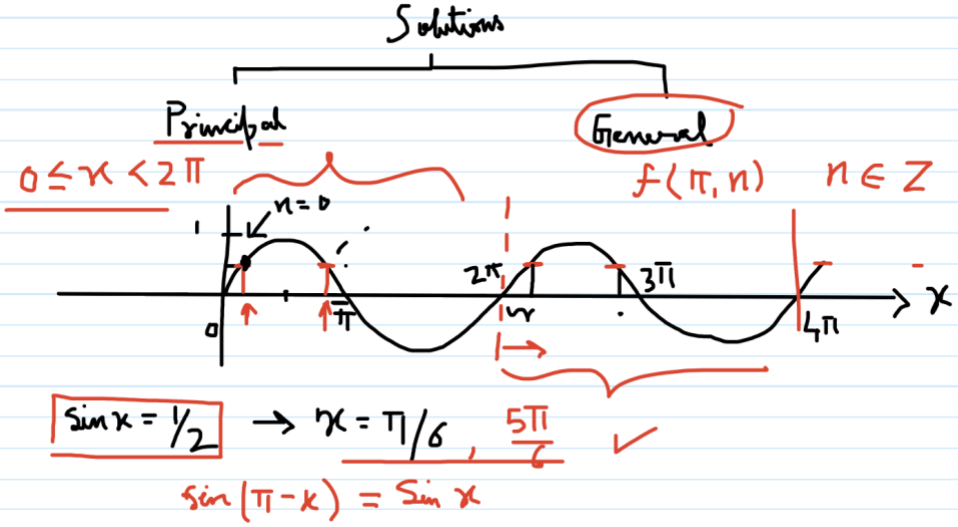
✓ $2x + 3 = 5$ — variable x is a linear function

✓ $x^2 + 2x + 1 = 9$ — x is a quadratic function

✓ $x^5 + - -$

✓ $\sin 2x = 1/2$ — variable x is a trigonometric function ✓

Equations involving trigonometric functions of a variable. - Trigonometric Equations




$\sin x = 1/2 \rightarrow x = \pi/6, \frac{5\pi}{6}$ ✓

$\sin(\pi - x) = \sin x$

① $\sin x = \sin y$, $x, y \in \mathbb{R}$

$\Rightarrow \boxed{x = n\pi + (-1)^n y}$, $n \in Z$

**Proof:** $\sin x = \sin y$

$\Rightarrow \sin x - \sin y = 0$

$\Rightarrow 2\cos\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right) = 0$

$\Rightarrow \cos\left(\frac{x+y}{2}\right) = 0$ or $\sin\left(\frac{x-y}{2}\right) = 0$

$\cos x = 0 \Rightarrow x = (2n+1)\frac{\pi}{2}$

$\sin x = 0 \Rightarrow x = n\pi$

$n \in Z$

$\frac{x+y}{2} = (2n+1)\pi$ or $\frac{x-y}{2} = n\pi$

$\Rightarrow \quad x = (2n+1)\pi - y \quad$ or $\quad x = 2n\pi + y$

$\Rightarrow \quad x = k\pi + (-1)^k y$

$x = (\text{odd})\pi + (-1)^{\text{odd}} y$ or $(\text{even})\pi + (-1)^{\text{even}} y$

$x = n\pi + (-1)^n y$ ; $n$ : odd / even

$x = (2n+1)\pi + (-1)^{2n+1} y$

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(2) If $\cos x = \cos y$. Then

$x = 2n\pi \pm y$

$x, y \in \mathbb{R}$, $n \in \mathbb{Z}$

$\rightarrow \quad \cos x - \cos y = 0$

$\Rightarrow -2 \sin\left(\frac{x+y}{2}\right) \sin\left(\frac{x-y}{2}\right) = 0$

$\Rightarrow \quad \frac{x+y}{2} = n\pi$ or $\frac{x-y}{2} = n\pi$

$\Rightarrow \quad x = 2n\pi - y$ or $x = 2n\pi + y$

$\Rightarrow \quad x = 2n\pi \pm y$

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(3) If $\tan x = \tan y$, then

$x = n\pi + y$

$\rightarrow \quad \tan x - \tan y = 0$

$\Rightarrow \frac{\sin x}{\cos x} - \frac{\sin y}{\cos y} = 0$

$\Rightarrow \frac{(\sin x \cos y - \cos x \sin y)}{\cos x \cos y} = 0$

$\Rightarrow \quad \sin(x-y) = 0$

$\Rightarrow \quad x - y = n\pi$

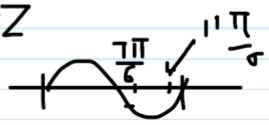
$\Rightarrow \quad x = n\pi + y$

Ex1: $\sin x = -\frac{1}{2}$. Find $x$. General soln

$\Rightarrow \sin x = -\sin\left(\frac{\pi}{6}\right) = \sin\left(\pi + \frac{\pi}{6}\right)$

$\Rightarrow \sin x = \sin\left(\frac{7\pi}{6}\right)$

$\Rightarrow \left[x = n\pi + (-1)^n \frac{7\pi}{6}\right]$ ✓ $n \in \mathbb{Z}$




② $\cos x = \frac{1}{2}$: Find g.s.

$\rightarrow \cos x = \cos\frac{\pi}{3}$

$\Rightarrow \left[x = 2n\pi \pm \frac{\pi}{3}\right]$ $n \in \mathbb{Z}$

③ Solve $\tan 2x = -\cot\left(x + \frac{\pi}{3}\right)$

$\rightarrow \tan 2x = \tan\left(\frac{\pi}{2} + x + \frac{\pi}{3}\right) = \tan\left(x + \frac{5\pi}{6}\right)$

$\Rightarrow 2x = n\pi + x + \frac{5\pi}{6}$

$\Rightarrow \left[x = n\pi + \frac{5\pi}{6}\right]$ $n \in \mathbb{Z}$