Trigonometry - Problems

① P.T. $3\sin\frac{\pi}{6}\sec\frac{\pi}{3} - 4\sin\frac{5\pi}{6}\cot\frac{\pi}{4} = 1$

$$\text{LHS} = 3\cdot\frac{1}{2}\cdot 2 - 4\sin\left(\pi-\frac{\pi}{6}\right)\cdot 1$$
$$= 3 - 4\sin\left(\frac{\pi}{6}\right)$$
$$= 3 - \cancel{4}^{2}\cdot\frac{1}{\cancel{2}}$$
$$= 3 - 2$$
$$= 1$$
$$= \text{RHS}$$

② Find $\tan 15^\circ$

0, 30, 45, 60, 90 . . . .

$$\rightarrow \tan(60^\circ - 45^\circ)$$
$$= \frac{\tan 60^\circ - \tan 45^\circ}{1 + \tan 60^\circ \tan 45^\circ}$$
$$= \frac{\sqrt{3}-1}{1+\sqrt{3}} = \frac{\sqrt{3}-1}{\sqrt{3}+1}$$

$$\frac{1-\frac{1}{\sqrt{3}}}{1+\frac{1}{\sqrt{3}}} = \frac{\frac{\sqrt{3}-1}{\cancel{\sqrt{3}}}}{\frac{\sqrt{3}+1}{\cancel{\sqrt{3}}}} = \frac{\sqrt{3}-1}{\sqrt{3}+1}$$

③ P.T. $\frac{\sin(x+y)}{\sin(x-y)} = \frac{\tan x + \tan y}{\tan x - \tan y}$

$$\text{LHS} \quad \frac{\sin x\cos y + \cos x\sin y}{\sin x\cos y - \cos x\sin y} = \frac{\frac{\sin x\cos y + \cos x\sin y}{\cos x\cos y}}{\frac{\sin x\cdot\cos y - \cos x\sin y}{\cos x\cdot\cos y}}$$

$$= \frac{\frac{\sin x}{\cos x} + \frac{\sin y}{\cos y}}{\frac{\sin x}{\cos x} - \frac{\sin y}{\cos y}} = \frac{\tan x + \tan y}{\tan x - \tan y} = \text{RHS}$$

④ P.T. $\underbrace{\cos\left(\frac{\pi}{4}+x\right)}_{}^{A} + \cos\left(\frac{\pi}{4}-x\right)^{B} = \sqrt{2}\cos x$

$$LHS = \cos\left(\frac{\pi}{4}\right)\cos x - \cancel{\sin\frac{\pi}{4}\sin x} + \cos\frac{\pi}{4}\cos x + \cancel{\sin\frac{\pi}{4}\sin x}$$

$$= 2\cos\frac{\pi}{4}\cos x$$

$$= \sqrt{2}\cancel{2}\cdot\frac{1}{\cancel{\sqrt{2}}}\cos x$$

$$2\cos\left(\frac{\pi}{4}\right)\cos x$$

$$= \sqrt{2}\cos x$$

$$= RHS$$

Method II $\cos x + \cos y = 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$ ✓

⑤ Prove $\cos\left(\frac{\pi}{4}-x\right)\cos\left(\frac{\pi}{4}-y\right) - \sin\left(\frac{\pi}{4}-x\right)\sin\left(\frac{\pi}{4}-y\right)$

$$= \sin(x+y)$$

$$LHS = \cos\left(\frac{\pi}{4}-x + \frac{\pi}{4}-y\right)$$

$$= \cos\left(\frac{\pi}{2}-(x+y)\right) \qquad - \left[\cos\left(\frac{\pi}{2}-x\right) = \sin x\right]$$

$$= \sin(x+y)$$

$$= RHS$$

⑥ Prove $\dfrac{\tan\left(\frac{\pi}{4}+x\right)}{\tan\left(\frac{\pi}{4}-x\right)} = \left(\dfrac{1+\tan x}{1-\tan x}\right)^2$

$$LHS = \frac{\dfrac{\tan\frac{\pi}{4}+\tan x}{1-\tan\frac{\pi}{4}\tan x}}{\dfrac{\tan\frac{\pi}{4}-\tan x}{1+\tan\frac{\pi}{4}\tan x}} = \frac{\dfrac{1+\tan x}{1-\tan x}}{\dfrac{1-\tan x}{1+\tan x}} \qquad \left[\frac{\frac{a}{b}\times bd}{\frac{c}{d}\times bd} = \frac{ad}{bc}\right]$$

$$= \frac{1+\tan x}{1-\tan x}\times\frac{1+\tan x}{1-\tan x}$$

$$= \left(\frac{1+\tan x}{1-\tan x}\right)^2 = RHS$$

(7) Prove: $\underline{\sin(n+1)x \sin(n+2)x} + \underline{\cos(n+1)x\cos(n+2)x}$ ✓
$= \cos x$

$$\begin{aligned}
\text{LHS} &= \cos[(n+1)x - (n+2)x] \checkmark \\
&= \cos(\cancel{nx} + x - \cancel{nx} - 2x) \\
&= \cos(-x) \\
&= \cos x \\
&= \text{RHS}
\end{aligned}$$

(8) Prove: $\sin^2 6x - \sin^2 4x = \sin 2x \sin 10x$

$$\begin{aligned}
\text{LHS:} & \ (\sin 6x + \sin 4x)(\sin 6x - \sin 4x) \checkmark \\
&= \left[2\sin\left(\frac{6x+4x}{2}\right)\cos\left(\frac{6x-4x}{2}\right)\right]\left[2\cos\left(\frac{6x+4x}{2}\right)\sin\left(\frac{6x-4x}{2}\right)\right] \checkmark \\
&= (2\sin 5x \cdot \cos x)(2\cos 5x \sin x) \checkmark \\
&= (2\sin 5x\cos 5x)(2\sin x\cos x) \\
&= \sin 10x \sin 2x \\
&= \sin 2x \sin 10x \\
&= \text{RHS}
\end{aligned}$$

$\boxed{\sin 2x = 2\sin x\cos x}$

Hence proved.