

M.Phil/Ph. D. Entrance Examination-2018

PHYSICS

(Part-II)

ROLL NO.

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Signature of Candidate

Signature of Invigilator

Timings: Two Hours

Total Marks: 100

(Part II(a) – 50 marks)

(Part II (b) – 50 marks)

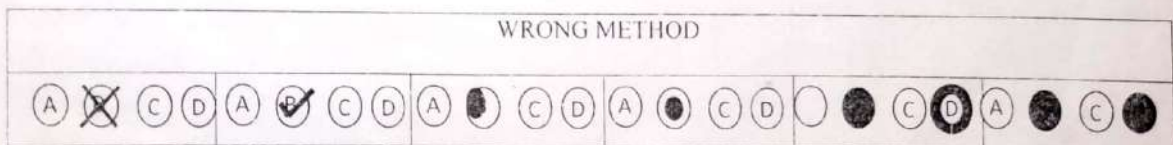
Instructions to Candidates

1. Do not write your name or put any other mark of identification anywhere in the OMR Answer Sheet. **IF ANY MARK OF IDENTIFICATIONS IS DISCOVERED ANYWHERE IN OMR ANSWER SHEET, the OMR sheet will be cancelled, and will not be evaluated.**
2. This Question Booklet contains this cover page and a total of **50 Multiple Choice Questions of 1 mark each in Part II (a).** Space for rough work has been provided at the end. Available space on each page may also be used for rough work.
3. The candidate may carry this booklet.
4. There is no negative marking in Multiple Choice Questions.
5. **USE OF CALCULATOR IS NOT PERMITTED.**
6. **USE/POSSESSION OF ELECTRONIC GADGETS LIKE MOBILE PHONE, iphone, iPad, pager ETC. is not permitted.**
7. Candidate should check the serial order of questions at the beginning of the test. If any question is found missing in the serial order or any other abnormality, it should be immediately brought to the notice of the Invigilator. No page should be torn out from this question booklet.
8. Answers must be marked in the OMR answer sheet which is provided separately. OMR answer sheet must be handed over to the invigilator before you leave the seat.
9. The OMR answer sheet should not be folded or wrinkled. The folded or wrinkled OMR/Answer Sheet will not be evaluated.
10. Write your Roll Number in the appropriate space (above) and on the OMR Answer Sheet. Any other details, if asked for, should be written only in the space provided.
11. Use Black or Blue Ball Pen only for filling the ovals/circles in OMR Answer Sheet while answering the Questions.
12. There are four alternative answers to each question marked A, B, C and D. Select one of the answers you consider most appropriate and fill up the corresponding oval/circle in the OMR Answer Sheet provided to you. For your Choice of answers darken the correct oval/circle completely. If the correct answer is 'B', the corresponding oval/circle should be completely filled and darkened as shown below.

CORRECT METHOD

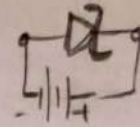


WRONG METHOD



1. Avalanche breakdown occurs when
- the forward current becomes excessive.
 - the forward bias exceeds a certain value.
 - ☒ the reverse bias exceeds a certain value.
 - the potential barrier is reduced to zero.

V_Z > 8V Breakdown
occurs as a
result of
collision b/w
high energy carriers



2. A tunnel diode

P-N junction
low power device



- works by quantum mechanical tunneling exhibited by gallium arsenide only.
- ☒ uses a high doping level so as to provide a narrow junction.
- is a point-contact device with large reverse resistance.
- has a tiny hole through its centre so as to facilitate tunneling.

operates in
both FB & RB
Generally used
in FB

3. The potential-divider bias is more commonly used because it

- uses minimum circuit components.
- uses only one battery.
- does not reduce the input and output impedance drastically.
- ☒ stabilizes the collector current.

4. A Schmitt trigger circuit is used for

- ☒ shaping an input pulse into a square pulse.
- producing a delayed pulse.
- generating a ramp
- differentiating a pulse

(+ve feedback used)
(comparator)
(Relaxation Osc.)

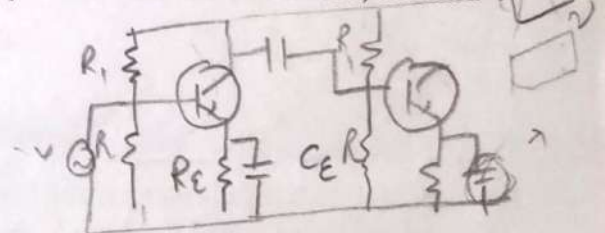


5. A field effect transistor

- ☒ involves three P-N junctions.
- incorporates a forward biased P-N junction.
- operates on the principle of variation of a magnetic field.
- operates on the principle of the depletion layer width variation with the junction reverse voltage.

6. In an R-C coupled amplifier, if the emitter by-pass capacitor C_e is disconnected, the mid-band gain of the stage will

- increase
- decrease
- ☒ remain unchanged
- become unstable.



7. The commutator $[x, p^2]$, where x and p are the position and momentum operators respectively, is

- ☒ $2i\hbar p$
- $i\hbar$
- $2i\hbar xp$
- $-i\hbar p$

$$[x, p^2] = [x, p]p + p[x, p]$$

$$= i\hbar p + p(i\hbar) = 2i\hbar p$$

8. If the potential is symmetric, then the wave function representing a particle is

- A. symmetric
- B. antisymmetric
- C. either symmetric or antisymmetric
- D. neither symmetric or antisymmetric

9. The correct representation of J^2 operator for $j = 1$ is given by

A. $2\hbar^2 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$

☒ B. $2\hbar^2 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

C. $\hbar\sqrt{2} \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$

D. $\hbar\sqrt{2} \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$

$J = L + S$
 $J \cdot J = (L+S) \cdot (L+S)$

$J(J+1)\hbar^2$
 $J=1, L=1, S=0$

10. The total energy operator is represented as

A. $i\hbar \frac{\partial}{\partial x}$

☒ B. $-i\hbar \frac{\partial}{\partial t}$

C. $-i\hbar \frac{\partial}{\partial x}$

D. $i\hbar \frac{\partial}{\partial t}$

$E = -\frac{13.6}{n^2} \text{ eV}$

11. Which of the following is not a Pauli matrix

☒ A. $\begin{pmatrix} 0 & i \\ 1 & 0 \end{pmatrix}$

B. $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

C. $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

D. $\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$

$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
 $\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$
 $\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

12. Which of the following wave functions can be a solution of the Schrodinger equation for all values of x and positive k

A. Ae^{kx}

B. Ae^{-kx}

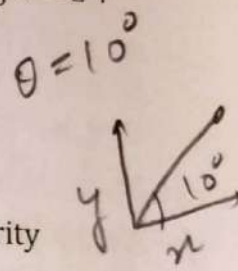
C. Ae^{kx^2}

☒ D. Ae^{-x^2}

$\psi = \phi$
 $E \phi = 0$

13. Two curves intersect each other at an angle of 10° in a complex plane. What will be the corresponding angle between the transformed curves under a mapping $w = z^2$?

A. 40°
 B. 20°
 C. 10°
 D. 5°



14. Which of the functions of a complex variable z has an essential singularity

A. $1/z$
 B. $1/[\sin(z)]$
 C. $1/[\tan(z)]$
 D. $\tan(1/z)$

$$\frac{1}{\sin z} = \frac{1}{2 - 2i + \frac{25}{3!} - \frac{25}{5!} \dots}$$

15. In a permutation group of n objects which of the following is a subgroup

☒ A. set of all transpositions
☐ B. set of all non-transpositions
☐ C. set of all even permutations
☐ D. set of all odd permutations

16. Consider the differential equation $\frac{d^2y}{dx^2} + y = 0$. A possible solution is

☒ A. $y = \cos x$
☐ B. $y = \tan x$
☐ C. $y = x$
☐ D. $y = e^x$

$$m^2 + 1 = 0$$

$$m = \pm i$$

$$y = \cos x$$

$$\frac{dy}{dx} = -\sin x$$

$$\frac{d^2y}{dx^2} = -\cos x$$

17. Laplace equation is an equation of the type

☒ A. homogeneous
☐ B. non-homogeneous
☐ C. non-linear
☐ D. first order

$$\Delta \phi = 0$$

$$I = \frac{2-A}{2}$$

$$2 \sec x \tan x + \tan x$$

$$e^x \cdot e^x$$

18. The intrinsic electric dipole moment of a nucleus ${}_Z X^A$

A. increases with Z but independent of A
 B. decreases with Z but independent of A
 C. is always zero
 D. increases with Z and A

$$\vec{I} = \frac{2-A}{2}$$

19. The radius of a ${}_{29}\text{Cu}^{64}$ nucleus is measured to be 4.8×10^{-13} cm. The radius of ${}_{12}\text{Mg}^{27}$ nucleus can be estimated to be

A. 2.86×10^{-13} cm
 B. 5.2×10^{-13} cm
 C. 8.6×10^{-13} cm
☒ D. 3.6×10^{-13} cm

$$R = R_0 A^{1/3}$$

$$4.8 \times 10^{-13} = R_{\text{Cu}} = R_0 (64)^{1/3} = 4R_0, \quad R_0 = \frac{4.8 \times 10^{-13}}{4}$$

$$R_{\text{Mg}} = 1.2 \times 10^{-13} (27)^{1/3} = 3.6 \times 10^{-13} \text{ cm}$$

$$R_0 = 1.2 \times 10^{-13}$$

20. A particle is a composite state of three quarks u, d and s. The possible values of electric charge, spin and strangeness, respectively, may be

- A. $1, \frac{1}{2}, -1$
 B. $0, 0, -1$
 C. $0, \frac{1}{2}, -1$
 D. $-1, -\frac{1}{2}, 1$

Handwritten calculation for quark composition uds :

$$\begin{array}{r} \frac{2}{3} + \frac{1}{3} - \frac{1}{3} = \frac{2}{3} \\ \frac{1}{2} + \frac{1}{2} - \frac{1}{2} = \frac{1}{2} \\ 0 + 0 - 1 = -1 \end{array}$$

21. A baryon X decays by strong interaction as $X \rightarrow \Sigma^+ + \pi^- + \pi^0$, where Σ^+ is the member of the isotriplet ($\Sigma^+, \Sigma^0, \Sigma^-$). The third component I_3 of the isospin of X is

- A. 0
 B. $\frac{1}{2}$
 C. 1
 D. $\frac{3}{2}$

Handwritten calculation for isospin:

$$I_3 = \frac{1}{2} - \frac{1}{2} - \frac{1}{2} = -\frac{1}{2}$$

22. What should be the minimum energy of a photon for it to split an α particle at rest into a tritium and a proton? (The masses of ${}^4_2\text{He}$, ${}^3_1\text{H}$ and ${}^1_1\text{H}$ are 4.0026 amu, 3.0161 amu and 1.0073 amu respectively, and 1 amu = 938 MeV)

- A. 32.2 MeV
 B. 19.3 MeV
 C. 3 MeV
 D. 931.5 MeV

Handwritten calculation for minimum energy:

$$E = (4.0026 - 3.0161 - 1.0073) \times 938 = 0.0208 \times 938 = 19.3 \text{ MeV}$$

23. According to the shell model the total angular momentum and the parity of the ground state of the ${}^7_3\text{Li}$ nucleus is

- A. $3/2$ with negative parity
 B. $3/2$ with positive parity
 C. $1/2$ with positive parity
 D. $7/2$ with negative parity

Handwritten calculation for angular momentum and parity:

$$J = 0 + 0 + 0 + 1 = 1$$

$$P = (-1)^1 = -$$

24. Ruby is a three level laser in which concentration of Cr^{+3} ions in the crystal is 1.6×10^{19} ions cm^{-3} . Considering the average pumping frequency of 6.25×10^{14} Hz and spontaneous emission time of the upper level 3 msec, the minimum power per unit volume required to maintain population inversion in this three level laser system is

- A. 1500 Wcm^{-3}
 B. 1100 Wcm^{-3}
 C. 150 Wcm^{-3}
 D. 110 Wcm^{-3}

Handwritten calculation for minimum power per unit volume:

$$P = \frac{h\nu}{\tau} N = \frac{6.626 \times 10^{-34} \times 6.25 \times 10^{14}}{3 \times 10^{-3}} \times 1.6 \times 10^{19} = 1100 \text{ Wcm}^{-3}$$

25. A three level system of atoms has N_1 atoms in level E_1 , N_2 atoms in level E_2 , and N_3 atoms in level E_3 , (with $N_2 > N_1 > N_3$ and $E_3 > E_2 > E_1$). Laser emission is possible between the levels

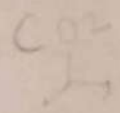
- A. $E_3 \rightarrow E_1$
 B. $E_3 \rightarrow E_2$
 C. $E_2 \rightarrow E_1$
 D. $E_2 \rightarrow E_3$

Handwritten diagram of energy levels and transitions:

$$\begin{array}{l} E_3 \xrightarrow{A} E_1 \\ E_3 \xrightarrow{B} E_2 \\ E_2 \xrightarrow{C} E_1 \end{array}$$

26. The number of fundamental vibrational modes of CO_2 molecule is

- A. four: 2 are Raman active and 2 are infrared active
- B. four: 1 is Raman active and 3 are infrared active
- C. three: 1 is Raman active and 2 are infrared active
- D. three: 2 is Raman active and 1 is infrared active

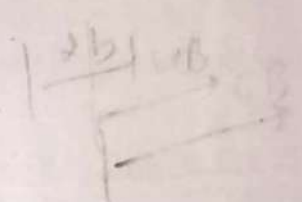


27. An optical resonator consists of two spherical mirrors of radii R_1 and R_2 separated by a distance d . The condition for confinement of paraxial rays in the resonator is:

- A. $0 \leq \left(1 - \frac{d}{R_1}\right) \left(1 - \frac{d}{R_2}\right) \leq 1$
- B. $0 \leq \left(1 + \frac{d}{R_1}\right) \left(1 + \frac{d}{R_2}\right) \leq 1$
- C. $0 \leq \left(1 - \frac{d}{R_1}\right) \left(1 + \frac{d}{R_2}\right) \leq 1$ ✓
- D. $0 \leq \left(1 + \frac{d}{R_1}\right) \left(1 - \frac{d}{R_2}\right) \leq 1$

28. The separation between the first Stokes and the first anti-Stokes lines of the rotational Raman spectrum in terms of the rotational constant B is

- A. $2B$
- B. $4B$ ✓
- C. $6B$
- D. $12B$



29. The light emitted from a Nd:YAG laser of wavelength λ is a Gaussian beam with beam waist radius W_0 having a Rayleigh range of Z_0 . The relation between Rayleigh range and beam waist radius of the beam is

- A. $Z_0 = \frac{\pi W_0^2}{\lambda}$
- B. $Z_0 = \frac{W_0^2}{\pi^2 \lambda}$
- C. $Z_0 = \frac{\pi W_0^3}{\lambda^2}$
- D. $Z_0 = \sqrt{\frac{\pi W_0^3}{\lambda}}$

30. If the wavelength of the first line of Balmer series in hydrogen is λ , then the wavelength of the first line of the Lyman series is

- A. $\left(\frac{27}{5}\right)\lambda$
- B. $\left(\frac{5}{27}\right)\lambda$
- C. $\left(\frac{32}{27}\right)\lambda$
- D. $\left(\frac{27}{32}\right)\lambda$

$$\frac{1}{\lambda} = R \left[\frac{1}{n_1^2} - \frac{1}{n_2^2} \right]$$

$$\frac{1}{\lambda} = R \left[\frac{1}{1^2} - \frac{1}{4} \right]$$

$$\frac{1}{\lambda} = \frac{1}{\lambda_B} = R \left[\frac{1}{(2)^2} - \frac{1}{(3)^2} \right] = R \left[\frac{1}{4} - \frac{1}{9} \right]$$

$$R \left[\frac{24}{36} \right]$$

$$\frac{1}{\lambda} = \frac{36}{5} R \left[\frac{1}{1} - \frac{1}{4} \right]$$

$$\frac{1}{\lambda} = \frac{36}{5} R \left[\frac{3}{4} \right] = \frac{27}{5} R$$

$$R = \frac{5}{27} \cdot \frac{1}{\lambda}$$

$$\frac{1}{\lambda} = R \left[\frac{1}{1^2} - \frac{1}{4^2} \right]$$

$$R = \frac{5}{36} \cdot \frac{1}{\lambda}$$

31. If R_1 is the value of the Rydberg constant assuming mass of the nucleus to be infinitely large compared to that of an electron and if R_2 is taking nuclear mass to be 7500 times the mass of the electron, then the ratio of $\frac{R_2}{R_1}$ is

- A. a little less than unity
- B. a little more than unity
- ☒ C. infinitely small
- D. infinitely large

32. For a multi-electron atom l , L and S specify the one electron orbital angular momentum, total angular momentum and total spin angular momentum respectively. The selection rules for electric dipole transition between two electronic energy levels, specified by l , L and S are

- A. $\Delta L = 0, \pm 1; \Delta S = 0; \Delta l = 0, \pm 1$
- ☒ B. $\Delta L = 0, \pm 1; \Delta S = 0; \Delta l = \pm 1$
- C. $\Delta L = 0, \pm 1; \Delta S = \pm 1; \Delta l = 0, \pm 1$
- D. $\Delta L = 0, \pm 1; \Delta S = \pm 1; \Delta l = \pm 1$

33. The degeneracy of the J-states arising from $3p$ term with spin-orbit interaction are

- ☒ A. 1, 3, 5
- B. 1, 2, 3
- C. 3, 5, 7
- D. 2, 6, 10

34. The Lande g -factor for the $3p_1$ level of an atom is

- A. $\frac{1}{2}$
- ☒ B. $\frac{3}{2}$
- C. $\frac{5}{2}$
- D. $\frac{7}{2}$

35. The spectral term for the atom with 70% filled sub-shell and only $s = \frac{3}{2}$ is

- A. $3p_0$
- B. $3f_{1/2}$
- ☒ C. $4f_{9/2}$
- D. $4p_{1/2}$

36. A charge density is described by the distribution $\rho(r) = \epsilon_0 A [(2/r) - 1] \exp[-r]$, where A is a positive constant. What is the corresponding electrostatic potential?

- A. $V(r) = A \exp(-r)$
- B. $V(r) = \epsilon_0 A \exp(-r)$
- C. $V(r) = (A/\epsilon_0) \exp(-r)$
- D. $V(r) = [A \exp(-r)]/r$

37. A solid non-conducting sphere of radius R has a charge density $\rho(r) = \rho_0(1 - r/R)$, where ρ_0 is a constant. What is the magnitude of the electric field at $r = R/2$ as compared to the field $E(R)$ on the surface?

- A. $E(R)/2$
 B. $3E(R)/2$
 C. $4E(R)/3$
 D. $5E(R)/4$

Poynting vector $\vec{S} = \frac{\vec{E} \times \vec{B}}{\mu_0}$
 $(1 = \frac{1}{n}) \quad |\vec{R}| = n|\vec{K}_0|$

38. A pair of infinitely long parallel wires separated by a distance r carry currents I_1 and I_2 . What is the force exerted on one wire by the other (K is a constant)?

- A. KI_1I_2/r^2
 B. KI_1I_2/r
 C. $K(I_1 + I_2)/r^2$
 D. $K(I_1 + I_2)/r$

$\hat{i} \times \hat{j} = -\hat{j}$
 $\hat{j} \times \hat{k} = \hat{i}$

$\vec{K} \times \vec{E} = (0.50\hat{i} + 0.60\hat{j})$
 $\epsilon_0 \omega^2 [n(0.5)] \hat{k}$

39. An electromagnetic wave has an electric field $\vec{E} = E_0 \cos[\pi(0.5x + 0.6y - 500t)]\hat{k}$. What is the corresponding magnetic field \vec{B} ?

- A. $\vec{B} = (E_0/500) \cos[\pi(0.5x + 0.6y - 500t)](\hat{i} + \hat{j})$
 B. $\vec{B} = (E_0/500\pi) \cos[\pi(0.5x + 0.6y - 500t)](\hat{i} + \hat{j})$
 C. $\vec{B} = (E_0/500) \cos[\pi(0.5x + 0.6y - 500t)](0.6\hat{i} - 0.5\hat{j})$
 D. $\vec{B} = (E_0/500\pi) \cos[\pi(0.5x + 0.6y - 500t)](6\hat{i} - 5\hat{j})$

$\vec{k} = 0.5\pi\hat{i} + 0.6\pi\hat{j}$

$\vec{B} = \frac{\vec{k} \times \vec{E}}{\omega}$

$\vec{E} = \frac{c\vec{B}}{\omega}$

$\frac{E_0}{3 \times 10^8}$

40. Which of the following expressions are valid Maxwell's equations for electromagnetic plane waves propagating in a linear, homogeneous and isotropic material medium, where, ω = angular frequency, \vec{k} = propagation vector, ϵ = electric permittivity, μ = magnetic permeability

$\nabla = i\vec{k}$ $\partial/\partial t = -i\omega$

- A. $\vec{k} \cdot \vec{E} = 0$; $\vec{k} \cdot \vec{H} = 0$; $\vec{k} \times \vec{E} = \omega\mu\vec{H}$; $\vec{k} \times \vec{H} = -\omega\epsilon\vec{E}$
 B. $\vec{k} \cdot \vec{E} = 0$; $\vec{k} \cdot \vec{H} = 0$; $\vec{k} \times \vec{E} = -\omega\epsilon\vec{H}$; $\vec{k} \times \vec{H} = \omega\mu\vec{E}$
 C. $\vec{k} \cdot \vec{E} = 0$; $\vec{k} \cdot \vec{H} = 0$; $\vec{k} \times \vec{E} = -\omega\mu\vec{H}$; $\vec{k} \times \vec{H} = \omega\epsilon\vec{E}$
 D. $\vec{k} \cdot \vec{E} = 0$; $\vec{k} \cdot \vec{H} = 0$; $\vec{k} \times \vec{E} = \omega\epsilon\vec{H}$; $\vec{k} \times \vec{H} = -\omega\mu\vec{E}$

$\epsilon = \mu\eta$

$\nabla \times \vec{B} = \mu\epsilon \frac{\partial \vec{E}}{\partial t}$

$i\vec{k} \times \vec{B} = -\mu\epsilon i\omega \vec{E}$

$\vec{k} \times \vec{B} = -\mu\epsilon \omega \vec{E}$

$\nabla \cdot \vec{E} = 0$

$\nabla \cdot \vec{B} = 0$

$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

$i\vec{k} \times \vec{E} = -i\omega \vec{B}$

$\vec{k} \times \vec{E} = -\omega \vec{B}$

$\vec{k} \times \vec{E} = -\omega \vec{B}$

41. Which of the following expressions of the scalar potential ϕ and vector potential \vec{A} are valid gauge transformations?

- A. $\phi' = \phi + ax$, $\vec{A}' = \vec{A} + at\hat{k}$
 B. $\phi' = \phi + ax$, $\vec{A}' = \vec{A} + at\hat{i}$
 C. $\phi' = \phi + ax$, $\vec{A}' = \vec{A} - at\hat{k}$
 D. $\phi' = \phi + ax$, $\vec{A}' = \vec{A} - at\hat{i}$

42. The pair of values of PV for an ideal gas and a gas of photons with internal energy U is given by

- A. $\frac{1}{3}U, \frac{2}{3}U$
 B. $\frac{2}{3}U, \frac{1}{3}U$
 C. $\frac{3}{2}U, \frac{1}{3}U$
 D. $\frac{1}{3}U, \frac{3}{2}U$

43. In grand canonical ensemble, a system S of fixed volume is in contact with a large reservoir S' . Then

- A. S can exchange only particles with S' .
- ☒ B. S can exchange only energy with S' .
- C. S can exchange both particles and energy with S' .
- D. S can exchange neither particles nor energy with S' .

44. For a classical idea gas with density $\rho = \frac{N}{V}$ and de-Broglie thermal wavelength

$\lambda_T = \sqrt{\frac{h^2}{2\pi mkT}}$, the quantum statistics goes over to the classical Maxwell-Boltzmann statistics under the condition

- A. $\rho\lambda_T^3 \gg 1$
- B. $\rho\lambda_T^3 = 1$
- ☒ C. $\rho\lambda_T^3 \ll 1$
- D. $\rho\lambda_T^3 = 0$

45. A System containing N non-interacting localized particles of spin- $\frac{1}{2}$ and magnetic moment μ each is kept in constant external magnetic field B and in thermal equilibrium at temperature T . The magnetization of the system is

- A. $N\mu \sinh\left(\frac{\mu B}{k_B T}\right)$
- B. $N\mu \cosh\left(\frac{\mu B}{k_B T}\right)$
- ☒ C. $N\mu \coth\left(\frac{\mu B}{k_B T}\right)$
- ☒ D. $N\mu \tanh\left(\frac{\mu B}{k_B T}\right)$

46. A non-holonomic constraint may be expressed in the form of

- A. an equality
- ☒ B. an inequality
- C. Both of the above
- D. None of the above

47. The Hamiltonian H of a system does not involve a particular generalized coordinate q_k implying $\frac{\partial H}{\partial q_k} = 0$. Such a coordinate is called

- A. a cyclic coordinate only
- B. a ignorable coordinate only
- ☒ C. cyclic as well as ignorable
- D. none of the above

48. The differential equation for the orbit with integrable power-law potential $V(r)$ is given by

A. $\frac{d^2 u}{d\theta^2} + u = -\frac{m}{l^2} \frac{d}{du} V\left(\frac{1}{u}\right)$

B. $\frac{d^2 u}{d\theta^2} - u = +\frac{m}{l^2} \frac{d}{du} V\left(\frac{1}{u}\right)$

C. $\frac{d^2 u}{d\theta^2} + u = +\frac{m}{l^2} \frac{d}{du} V\left(\frac{1}{u}\right)$

D. $\frac{d^2 u}{d\theta^2} - u = -\frac{m}{l^2} \frac{d}{du} V\left(\frac{1}{u}\right)$

with $u = \frac{1}{r}$, l and m being the angular momentum and the mass respectively.

49. A rigid body is rotating about its centre of mass, fixed at the origin, with an angular velocity $\vec{\omega}$ and angular acceleration $\vec{\alpha}$. If the torque acting on it is $\vec{\tau}$ and its angular momentum \vec{L} , the rate of change of its kinetic energy is

A. $\frac{1}{2} \vec{\tau} \cdot \vec{\omega}$

B. $\frac{1}{2} \vec{L} \cdot \vec{\omega}$

C. $\frac{1}{2} (\vec{\tau} \cdot \vec{\omega} + \vec{L} \cdot \vec{\alpha})$

D. $\frac{1}{2} \vec{L} \cdot \vec{\alpha}$

50. A cyclic group is always

A. Abelian

B. Non-Abelian

C. of prime order

D. of non-prime order

All element generated from single element of the group

which satisfy the condition $x^2 = E$ (identity)

PART-II(B)

Max Marks : 50

Instruction: Answer any five questions. Each question carries 10 marks.

1. Find all poles of the complex function $f(z) = 1/(z^4+17)^2$ and classify them. Calculate the integral of $f(z)$ around a closed contour $|z| = 2$ evaluated in counter clockwise direction.
2. List all the elements of the group of symmetries of an equilateral triangle C_3 . Write complete group multiplication table of C_3 .
3. The wave function of an electron in the ground state of the Hydrogen atom is given by $\psi = \frac{e^{-r/a}}{\sqrt{\pi a^3}}$. Find the expectation values $\langle r \rangle$ and $\langle r^2 \rangle$.
4. Find the expectation values of the kinetic and potential energies in the n^{th} state of the harmonic oscillator.
5. Draw the volt-ampere characteristics of a tunnel diode. Explain the occurrence of negative differential resistance in the characteristics of a tunnel diode. Write some of the uses of the tunnel diode.
6. Prove that both the stability and the bandwidth of an amplifier increase by employing negative feedback.

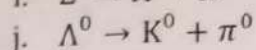
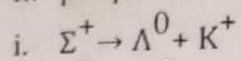
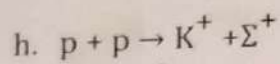
- 7 (i) The binding energy (in MeV) of a light nucleus (Z, A) is given by the approximate formula

$$B(A, Z) \approx 16A - 20A^{\frac{2}{3}} - \frac{3}{4}Z^2A^{-\frac{1}{3}} + 30 \frac{(N-Z)^2}{A} \quad (5)$$

where $N=A-Z$ is the neutron number. Find the value of Z for the most stable isobar for a given A .

- (ii) In a fission of a nucleus of atomic number A_0 (mass M_0) into two nuclei A_1 and A_2 (masses M_1 and M_2) the energy released is $Q = M_0c^2 - M_1c^2 - M_2c^2$. Express Q in terms of B . (5)
8. Consider the following high energy reactions or particle decays. For each case indicate whether they are allowed or forbidden. If forbidden, state the law that is violated and if allowed state the type of interaction.

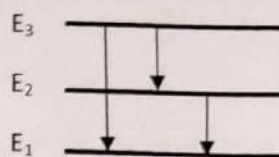
- a. $\pi^- + p \rightarrow \pi^0 + n$
- b. $\rho^0 \rightarrow \pi^+ + \pi^-$
- c. $p + n \rightarrow \Sigma^+ + \Lambda^0$
- d. $\Lambda^0 \rightarrow K^0 + \pi^0$
- e. $\pi^0 \rightarrow \gamma + \gamma$
- f. $\mu^- \rightarrow e^- + \nu_\mu + \bar{\nu}_e$
- g. $\mu^+ \rightarrow e^+ + \gamma$



9. The figure shows the energy levels of 3 level system with Einstein A coefficients

$$A_{32} = 7 \times 10^7 \text{ s}^{-1}, A_{31} = 1 \times 10^7 \text{ s}^{-1}, \text{ and } A_{21} = 1 \times 10^8 \text{ s}^{-1}.$$

(4+4+2)



- (i) Calculate the lifetime of level 3.
 - (ii) At time $t=0$, N atoms/cm³ are excited to level 3 by the external pumping, obtain the expression for the rate of change of population of the level 3.
 - (iii) Can this system be used for laser action between levels 2 and 1? Give the reason.
- 10 (i) A commercially available ruby laser amplifier using a 2.3 cm long rod has a small signal gain of 10. What is the small signal gain of a 20 cm long rod? Neglect gain saturation effects. (5)
- (ii) Briefly explain various mechanisms for line-shape broadening in atoms and molecules. (5)
- 11 (i) If the doublet splitting of the first excited state, $2^2p_{3/2} - 2^2p_{1/2}$, of He^+ is 5.84 cm^{-1} , calculate the corresponding separation for H.
- (ii) Calculate the possible orientations of spin vector \vec{S} with respect to the magnetic field \vec{B} .
12. A particle with charge q is initially at rest at a height h above an infinite, grounded, conducting plane. What is the force acting on the particle? If the particle is now released, what will be its speed u when it reaches a height $h/2$ above the plane? At what distance from the plane will the particle reach the speed $2u$?
13. A point charge q is moving in a straight line motion i.e., its velocity \vec{v} and acceleration \vec{a} are instantaneously collinear. Find the angular distribution of radiation and the angle at which the maximum radiation is emitted.
14. In the two-state model system, each particle can either be in its ground state with energy zero or in the excited state with energy ϵ . For an assembly of N such particles with internal energy U , enumerate the number of microstates and hence write down the entropy $S(U, V, N)$. Obtain $U(T, V, N)$ and the heat capacity.
15. Consider the electromagnetic radiation contained within a closed vessel of volume V , which we may think of as a cubical cavity with perfectly conducting walls. Calculate the Helmholtz free energy and hence show that the internal energy is given by

$$\frac{\pi^2 k_B^4}{15 h^3 c^3} V T^4$$

$$T \propto \frac{1}{n}$$

16. Show that for a system of one degree of freedom the transformation

$$Q = \tan^{-1} \left(\frac{\alpha q}{p} \right)$$

$$P = \frac{1}{2} (\alpha q^2 + p^2)$$

is canonical where α is an arbitrary constant of suitable dimensions.

17. A dynamical system has the Lagrangian

$$L = \dot{q}_1^2 + \frac{\dot{q}_2^2}{a + bq_1^2} + k_1 q_1^2 + k_2 \dot{q}_1 \dot{q}_2$$

where a , b , k_1 and k_2 are constants. Find the equations of motion in the Hamiltonian formulation.