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Paper Code No- Am27/I

Question Booklet No 627112

# ENTRANCE EXAMINATION-2016

## M.Sc Mathematics with Computer Science

## SET B

ROLL NO. M 2 7 1 1 5 0 7

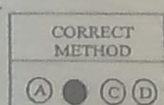
Signature of Invigilator

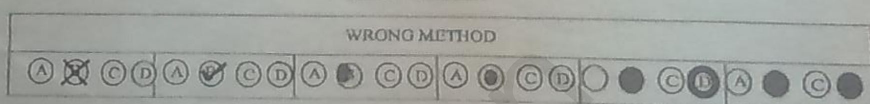
Time: 1 Hour 45 Minutes

Total Marks: 100

### Instructions to Candidates

1. Do not write your name or put any other mark of identification anywhere in the OMR Answer Sheet. **IF ANY MARK OF IDENTIFICATIONS IS DISCOVERED ANYWHERE IN OMR ANSWER SHEET, the OMR sheet will be cancelled, and will not be evaluated.**
2. This Question Booklet contains this cover page and a total of **100 Multiple Choice Questions of 1mark.** Space for rough work has been provided at the beginning and end. Available space on each page may also be used for rough work.
3. Each correct answer carries one mark.
4. There is negative marking for Multiple Choice Questions. For each wrong answer, 0.25 marks will be deducted.
5. USE OF CALCULATOR IS PERMITTED.
6. USE/POSSESSION OF ELECTRONIC GADGETS LIKE MOBILE PHONE, iPhone, iPad, pager ETC. is not permitted.
7. Candidate should check the serial order of questions at the beginning of the test. If any question is found missing in the serial order, it should be immediately brought to the notice of the Invigilator. No pages should be torn out from this question booklet.
8. Answers must be marked in the OMR answer sheet which is provided separately. OMR answer sheet must be handed over to the invigilator before you leave the seat.
9. The OMR answer sheet should not be folded or wrinkled. The folded or wrinkled OMR/Answer Sheet will not be evaluated.
10. Write your Roll Number in the appropriate space (above) and on the OMR Answer Sheet. Any other details, if asked for, should be written only in the space provided.
11. There are four alternative answers to each question marked A, B, C and D. Select one of the answers you consider most appropriate and fill up the corresponding oval/circle in the OMR Answer Sheet provided to you. The correct procedure for filling up the OMR Answer Sheet is mentioned below.
12. Use Black or Blue Ball Pen only for filling the ovals/circles in OMR Answer Sheet while answering the Questions. For your Choice of answers darken the correct oval/circle completely. If the correct answer is 'B', the corresponding oval/circle should be completely fill and darkened as shown below.



**SET-B**

**M.Sc.( Mathematics with Computer Science) Entrance Test 2016**

Note: Choose and tick the correct answer out of the choices provided:

**Q.1.**
If the equation $M(x,y)dx + N(x,y)dy = 0$ is homogeneous and $(Mx + Ny) \neq 0$, then an integrating factor of $M(x,y)dx + N(x,y)dy = 0$ is

(A) $(Mx + Ny)$
(B) $(My - Nx)$
(C) $\frac{1}{(My-Nx)}$
(D) $\frac{1}{(Mx+Ny)}$

**Q.2.**
An integrating factor of the differential equation $(x^2 + y^2 + x)dx + xydy = 0$ is

(A) $x$
(B) $\frac{1}{x}$
(C) $\log x$
(D) None of these

$\frac{\partial M}{\partial y} = 2y$ $\quad \frac{1}{N}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) = \frac{y}{xy}$ $\quad = e^{\log}$

$\frac{\partial N}{\partial x} = y$ $\quad IF = e^{\int \frac{1}{x}dx}$

$(x + \frac{y^2}{x} + 1) + y$

$\frac{\partial M}{\partial y} = \frac{2y}{x}$

$\frac{\partial M}{\partial y} = 2y$ $\quad y$

$\frac{\partial N}{\partial x} = y$ $\quad \frac{1}{N}($

**Q.3.**
If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, then div $\vec{r}$ is

(A) 0
(B) 1
(C) 2
(D) 3

**Q.4.**
Which of the following is correct?

(A) The sequence $\{S_n\}$, where $\{S_n\} = 1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \cdots$ is divergent
(B) The sequence $\{S_n\}$, where $\{S_n\} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots$ is convergent
(C) The sequence $\{S_n\}$, where $\{S_n\} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots$ is divergent
(D) None of these

**Q.5.**
Stoke's theorem is

(A) $\oint_C \vec{F}.\hat{n}\, dS = \iint_S curl \vec{F}.\hat{n} dS$
(B) $\oint_C \vec{F}.\hat{n}\, dS = \iint_S \hat{n}.div \vec{F} dS$

(C) $\iint_S \vec{F}.\hat{n}dS = \iiint_V div\,\vec{F}\,dV$
(D) $\oint_C \vec{F}.d\vec{r} = \iint_S curl\,\vec{F}.\hat{n}dS$

**Q.6.**
The perimeter of the curve $r = 2\cos\theta$ is
(A) $\frac{\pi}{2}$
(B) $\pi$
(C) $2\pi$
(D) $\frac{3\pi}{2}$

**Q.7.**
If an alternating series $\sum_{n=1}^{\infty}(-1)^{n-1}u_n$ satiesfies $u_{n+1} \leq u_n \forall n$ and $\lim_{n\to\infty} u_n = 0$, then the series $\sum_{n=1}^{\infty}(-1)^{n-1}u_n$ converges.
This theorem is known as
(A) Cauchy's test
(B) Leibnitz test
(C) d'Alembert's test
(D) Rabbe's test

**Q.8.**
The solution of $y = p(x-b) + \frac{a}{p}$ is
(A) $y = p(x-b) + \frac{a}{c}$
(B) $y = c(x-b) + \frac{a}{c}$
(C) $x = c(y-b) + \frac{a}{c}$
(D) None of these

$y = c(x-b) + \frac{a}{c}$

**Q.9.**
Identify the Newton -Raphson iteration scheme for finding the square root of 3
(A) $x_{n+1} = \frac{3}{x_n}$
(B) $x_{n+1} = \frac{1}{2}(x_n - \frac{3}{x_n})$
(C) $x_{n+1} = \frac{1}{2}(x_n + \frac{3}{x_n})$
(D) $x_{n+1} = \frac{1}{3}(x_n + \frac{3}{x_n})$

$x_n^2 = \sqrt{3}$

**Q.10.**
The line $y = x+1$ is resolved about x-axis. The volume of solid of revolution formed by revolving the area covered by the given curve, x-axis and the lines $x = 0$, $x = 2$ is
(A) $\frac{26\pi}{3}$

(B) $\frac{19\pi}{3}$
(C) $\frac{17\pi}{3}$
(D) None of these

**Q.11.**
If the complex number $z = x + iy$ satisfies $|z^2 - 1| = |z^2| + 1$, then $z$ lies on
(A) the real axis
(B) the imaginary axis
(C) a circle
(D) an ellipse.

**Q.12.**
Which of the following statement is not true?
(A) Every field is an integral domain.
(B) Every field is a ring
(C) Every finite integral domain is a field.
(D) Every integral domain is a field.

**Q.13.**
For the linear programming problem
Minimize $z = x - y$ subject to $2x + 3y \leq 6, 0 \leq x \leq 3, 0 \leq y \leq 3$,
the number of extreme points of its feasible region and the number of basic feasible solutions respectively, are
(A) 3 and 5
(B) 3 and 3
(C) 4 and 4
(D) 4 and 5

**Q.14.**
The function $f(z) = \bar{z}$, is
(A) differentiable at $(0, 0)$
(B) differentiable at $(0, 1)$
(C) differentiable at $(1, 0)$
(D) nowhere differentiable

**Q.15.**
The function $f(z) = x^3 + 3xy^2 + i(y^3 + 3x^2y)$ is
(A) differentiable and analytic at $(0, 0)$
(B) differentiable but not analytic at $(0, 0)$
(C) analytic but not differentiable at $(0, 0)$
(D) None of these

**Q.16.**
If $z = \frac{1-i\sqrt{3}}{1+i\sqrt{3}}$, then argument of $z$ is
(A) $\frac{\pi}{3}$
(B) $\frac{2\pi}{3}$
(C) $\frac{4\pi}{3}$
(D) $\frac{5\pi}{3}$.

**Q.17.**
If $f = (2,3)$ and $g = (4,5)$ be two permutations on five symbols 1, 2, 3, 4, 5, then $g \circ f$ is

$g \circ f = (4,5)\cdot(2,3)$

$= \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & 5 & 4 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 3 & 2 & 4 & 5 \end{pmatrix}$

$= \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 3 & 2 & 5 & 4 \end{pmatrix}$

(A) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 3 & 5 & 4 & 2 \end{pmatrix}$
(B) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & 5 & 4 \end{pmatrix}$
(C) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 3 & 2 & 4 & 5 \end{pmatrix}$
(D) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 3 & 2 & 5 & 4 \end{pmatrix}$

**Q.18.**
The value of $\alpha$ for which $G = \{\alpha, 1, 3, 9, 19, 27\}$ is a cyclic group under multiplication modulo 56 is

$z_{56} = 2^2 \cdot 14 = 2^3 \cdot 7$

(A) 25
(B) 35
(C) 5
(D) 15

**Q.19.**
The value of the integral $\int_0^{1+i} z^2 dz$ is

$= \left[\frac{z^3}{3}\right]_0^{1+i}$

$= \frac{1}{3}(1+i)^3$

(A) $\frac{1}{3}(1+i)^3$
(B) $\frac{1}{3}(1+i)^2$
(C) $\frac{2}{3}(1+i)^3$
(D) None of these

**Q.20.**
Stationary point of the function $f(x,y) = 6x - 4y - x^2 - 2y^2$ is

$f_x = 6 - 2x = 0 \quad 2(3-x) = 0$

$f_y = -4 - 4y \quad -4(1+y) = 0$

(A) $(-1,3)$
(B) $(3,-1)$
(C) $(1,3)$
(D) $(3,1)$

**Q.21.**
Let $A = \{\begin{pmatrix} a & b \\ 0 & c \end{pmatrix} : a, b, c \in R\}$ be the ring under matrix addition and multiplication. Then the subset $\{\begin{pmatrix} 0 & p \\ 0 & 0 \end{pmatrix} : p \in R\}$ is

$\begin{pmatrix} 0 & p \\ 0 & 0 \end{pmatrix}\begin{pmatrix} a & b \\ 0 & c \end{pmatrix} = \begin{pmatrix} 0 & pc \\ 0 & 0 \end{pmatrix}$

(A) is a maximal ideal of A
(B) is a prime ideal but not a maximal ideal of A
(C) an ideal but not a prime ideal of A
(D) not an ideal of A

**Q.22.**
Which of the following statement is not true?
(A) The order of the subgroup of a finite group divides the order of the group.
(B) Every group of finite order may or may not be cyclic.
(C) Every cyclic group is abelian.
(D) If m is a divisor of the order of a group $G$, then $G$ must have a subgroup of order m.

**Q.23.**
The dimension of the null space of the linear transformation $T : R^3 \to R^2$ defined by $T(a, b, c) = (a + b, a - c)$ is
(A) 1
(B) 2
(C) 3
(D) None of these

**Q.24.**
The set of all matrices of the form $\{\begin{pmatrix} 0 & x \\ 0 & y \end{pmatrix} : x, y \in Q\}$ under the two operations of matrix addition and matrix multiplication is
(A) a ring with unity
(B) commutative ring
(C) non-commutative ring with zero divisors
(D)None of these

$bx = 0$
$\Rightarrow b = 0$ or $x = 0$

**Q.25.**
The value of $\frac{1}{2\pi i} \int_{|z|=3} \frac{e^z}{z-2}$ is

(A) 0
(B) 1
(C) $e^2$
(D) $e^3$.

**Q.26.**
If the axes are rectangular and P is the point $(2,3,-1)$, then the equation of the plane through P at right angles to OP is

$2x + 3y - z$

$OP = \frac{4+9+1}{\sqrt{16+}}$

(A) $3x + 2y - z - 14 = 0$
(B) $2x + 3y - z - 14 = 0$
(C) $2x + 3y + z - 14 = 0$
(D) $3x + 2y + z - 14 = 0$

**Q.27.**
Equation of the line through the point $(1,2,3)$ and parallel to the line $x - y + 2z = 5$, $3x + y + z = 6$ is

$x-1 = y-2 = z-3$

$3x + y + z = 6$

$x - y + 2z = 5$

$2x + y + z$

(A) $\frac{x-1}{-3} = \frac{y-2}{5} = \frac{z-3}{4}$
(B) $\frac{x-1}{-3} = \frac{y-2}{4} = \frac{z-3}{5}$
(C) $\frac{x-1}{5} = \frac{y-2}{-3} = \frac{z-3}{4}$
(D) None of these

**Q.28.**
Real asymptotes of the curve $(x^2 - 2ax)(x^2 + y^2) = b^2x^2$ are
(A) $y = 0$ and $x + 2a = 0$
(B) $x = 0$ and $x + 2a = 0$
(C) $x = 0$ and $x - 2a = 0$
(D) $y = 0$ and $x - 2a = 0$

**Q.29.**
The interval for which Lagrange's Mean Value Theorem is applicable to the function $f(x) = \sin\frac{1}{x}$ is

$\sin\frac{1}{c} = \frac{\sin\frac{1}{b} - \sin\frac{1}{a}}{\frac{1}{b} - \frac{1}{a}}$

$=$

(A) $[-1,1]$
(B) $[-2,2]$
(C) $[-3,3]$
(D) $[2,3]$

**Q.30.**
Radius of curvature at the point $(0,0)$ of the curve $3x^2 + xy + y^2 - 4x = 0$ is
(A) 1
(B) 2
(C)3
(D) None of these

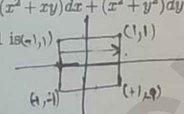
**Q.31.**
The plane containing principal normal and binormal is called

(A) tangent plane
(B) normal plane
(C) osculating plane
(D) rectifying plane

**Q.32.**
The limiting point of the sphere $x^2 + y^2 + z^2 + 2ux + d = 0$ exists only if
(A) d is zero
(B) d is negative
(C) d is positive
(D) None of these

**Q.33.**
Which of the following set is neither open nor closed ?
(A) $\mathbb{N}$
(B) $\mathbb{Z}$
(C) $\mathbb{R}$
(D) $S = \{\frac{1}{n} : n \in \mathbb{N}\}$

**Q.34.**
The value of the line integral $\int_C [(x^2 + xy)dx + (x^2 + y^2)dy]$, where $C$ is the square formed by the lines $x = \pm 1$ and $y = \pm 1$ is
(A) 0
(B) 1
(C) 2
(D) 3



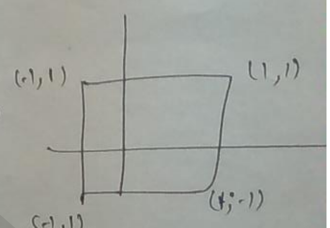

$\int_{y=-1}^{1}\int_{x=-1}^{1} (x - 2x)\,dx\,dy$

$\int_{-1}^{1} \Big\{ -\left[\frac{x^2}{2}\right]_{-1}^{1} = -\int_{-1}^{1} \frac{1}{2}$

$= 0$.

**Q.35.**
The value of the integral $\int\int (x^2 + y^2)dxdy$ over the region bounded by $xy = 1$, $y = 0$, $y = x$ and $x = 2$ is
(A) $\frac{23}{27}$
(B) $\frac{4}{3}$
(C) $\frac{37}{24}$
(D) $\frac{47}{24}$

**Q.36.**
Which of the following are not prime ideal of $\mathbb{Z}$?
1. $3\mathbb{Z}$ 2. $4\mathbb{Z}$ 3. $5\mathbb{Z}$ 4. $6\mathbb{Z}$
(A) 1 and 2
(B) 1 and 3






$\int^{1}\int^{1} (2x - x)\,dx\,dy$

(C) 3 and 4
(D) 2 and 4

**Q.37.**
If $a * b = a + b - ab$ for all $a, b \in R$, then which of the following statement is correct?
(A) $(R - \{-1\}, *)$ is a group.
(B) $(R - \{1\}, *)$ is a group.
(C) $(R - \{0\}, *)$ is a group.
(D) $(R, *)$ is a group.

**Q.38.**
The number of homomorphism that can be defined from $Z_6$ to $Z_{12}$ is
(A) 6
(B) 12
(C) 72
(D)None of these.

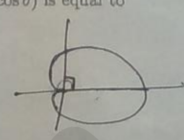
$f : Z_6 \longrightarrow Z_{12}$

**Q.39.**
Which of the following set is a basis for the subspace $A = \{\begin{pmatrix} x & y \\ 0 & t \end{pmatrix} : x+2y+t = 0, y+t = 0\}$ of the vector space of all real $2 \times 2$ matrices ?
(A) $\{\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}\}$
(B) $\{\begin{pmatrix} 2 & 1 \\ 0 & -1 \end{pmatrix}, \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}\}$
(C) $\{\begin{pmatrix} -1 & 1 \\ 2 & -1 \end{pmatrix}\}$
(D) $\{\begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}\}$

$y = -t$
$x - 2t + t = 0$
$x = t$
$\begin{bmatrix} t & -t \\ 0 & t \end{bmatrix} = \{\begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}\}$

**Q.40.**
For a positive term series $\sum a_n$, the ratio test states that
(A) the series converges, if $\lim_{n\to\infty} (\frac{a_{n+1}}{a_n}) < 1$
(B) the series converges, if $\lim_{n\to\infty} (\frac{a_{n+1}}{a_n}) > 1$
(C)the series converges, if $\lim_{n\to\infty} (\frac{a_{n+1}}{a_n}) = 1$
(D) None of these

**Q.41.**
The area of the cardioid $r = a(1 + \cos\theta)$ is equal to
(A) $2\int_0^{\pi} \int_{r=0}^{(1+\cos\theta)}$
(B) $2\int_0^{\pi} \int_{r=0}^{a(1+\cos\theta)}$
(C) $2\int_0^{\frac{\pi}{2}} \int_{r=0}^{a(1+\cos\theta)}$
(D) $2\int_0^{\frac{\pi}{4}} \int_{r=0}^{a(1+\cos\theta)}$

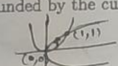


**Q.42.**
Maximum value of directional derivative of $2x^2 - y - z^4$ at the point $(2, -1, 1)$ is
(A) 1
(B) 9
(C) 33
(D) None of these

**Q.43.**
If curl $\vec{V}$ of any vector $\vec{V}$ is zero, then it is
(A) invariant
(B) rotational
(C) irrotational
(D) solenoidal

$x^2 = y$

**Q.44.**
The area bounded by the curves $y^2 = x$ and $x^2 = y$ is
(A) $\frac{1}{3}$
(B) $\frac{2}{3}$
(C) 0
(D) 1

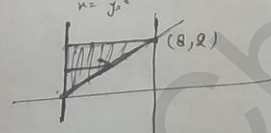



$x^2 = \sqrt{x}$
$x^4 = x$
$x(x^3-1)$

$= \int_{x=0}^{1}\int_{y=x^2}^{\sqrt{x}} dy\,dx$
$= \int_0^1 (\sqrt{x} - x^2)\,dx$
$= \left\{\frac{2}{3}x^{3/2} - \frac{x^3}{3}\right\}_0^1 =$

**Q.45.**
Changing the order of integration of $\int_0^8 \int_{\frac{x}{4}}^{2} f(x,y)dydx$ leads to $\int_a^b \int_c^d f(x,y)dxdy$, then the value of d is
(A) $4y$
(B) $16y^2$
(C) $x$
(D) 8

$x = y^4$




$\int_{y=0}^{2}\int_{x=0}^{4y}$

**Q.46.**
Gauss divergence theorem relates
(A) surface and volume integral
(B) line and volume integral
(C) line and surface integral
(D) All of these

**Q.47.**
If $f'(c)$ exists, then

$$\lim_{h\to 0}\frac{f(c+h) - f(c-h)}{h}$$

$= \lim_{h\to 0}\big(f(c+h) -$

$f'(c) = \lim_{h\to 0}\frac{f(c+h) - f(c)}{h}$

$\frac{(f(c-h) - f(c}{h}$

$= f'(c) - f'(c$

$h\to 0$

is equal to
(A) $f'(c)$
(B) $2f'(c)$
(C) $\frac{f'(c)}{2}$
(D) None of these

Q.48.
The equation $x^3 - x^2 + 4x - 4 = 0$ is to be solved using Newton-Raphson formula method. If $x = 2$ is taken as the initial approximation of the solution, then the next approximation using this method will be
(A) 1
(B) $\frac{2}{3}$
(C) $\frac{3}{2}$
(D) $\frac{4}{3}$

Q.49.
The partial differential equation $f_{xx} + 2f_{xy} + 4f_{yy} = 0$ is
(A) hyperbolic at all points
(B) parabolic at all points
(C) elliptic at all points
(D) None of these

$a \quad h \quad b$

$h^2 = 4$

$ab = 8$

$h^2 - ab = -4 < 0$

Q.50.
For the partial differential equation $xp + yq = pq$, the Charpit's auxiliary equations are
(A) $\frac{dx}{-(x-q)} = \frac{dy}{(y-p)} = \frac{dz}{-p(x-q)-q(y-p)} = \frac{dp}{p} = \frac{dq}{q}$
(B) $\frac{dx}{-(x-q)} = \frac{dy}{-(y-p)} = \frac{dz}{p(x-q)-q(y-p)} = \frac{dp}{p} = \frac{dq}{q}$
(C) $\frac{dx}{-(x-q)} = \frac{dy}{-(y-p)} = \frac{dz}{-p(x-q)-q(y-p)} = \frac{dp}{p} = \frac{dq}{q}$
(D) None of these

$\frac{dp}{\frac{\partial F}{\partial x}} = \frac{dq}{\frac{\partial F}{\partial y}} = \frac{dx}{-\frac{\partial F}{\partial x}} = \frac{dy}{-\frac{\partial F}{\partial y}}$

$= \frac{dz}{p\frac{\partial F}{\partial x} + q\frac{\partial F}{\partial y}}$

$\frac{dP}{p} = \frac{dq}{q}$

Q.51.
The complementary function of the partial differential equation $(D^2 + DD' - 6D'^2)z = x + y$ is
(A) $\phi_1(y + 3x) + \phi_2(y - 2x)$
(B) $\phi_1(y - 3x) + \phi_2(y + 2x)$
(C) $\phi_1(3y - x) + \phi_2(2y + x)$
(D) None of these

$m^2 + m - 6 = 0$

$m = \frac{-1 \pm \sqrt{1+24}}{2}$

$\phi_1(y - 3x) + \phi_2(y + 2x)$

Q.52.
The expression $\frac{1}{D^2+9} x \sin x$ is equal to
(A) $\frac{\sin x}{8} - \frac{\cos x}{32}$

$= \left(x - \frac{F'(D)}{F(D)}\right)\frac{1}{F(D)} V$

$= \left(x - \frac{2D}{D^2+9}\right)\frac{1}{D^2+9} \sin x$

$= \left(x - \frac{2D}{8}\right)\left(\frac{1}{8}\sin x\right)$

(B) $\frac{x\sin x}{8} + \frac{\cos x}{32}$
(C) $\frac{x\sin x}{8} - \frac{x\cos x}{32}$
(D) $\frac{x\sin x}{8} - \frac{\cos x}{32}$

**Q.53.**
The function $f(x) = x|x|$ is
(A) not monotonic
(B) strictly decreasing
(C) differentiable $\forall x \in R$ except $x = 0$
(D) differentiable $\forall x \in R$ .

$-x^2 \quad x^2$
$-2x \quad 0 \quad 2x$
$0$

**Q.54.**
Let $f(x+y) = f(x).f(y)$ for all x and y. If $f(3) = 3$ and and $f'(0) = 11$, then $f'(3)$ is equal to
(A) 33
(B) 28
(C) 22
(D) None of these

$f(3) = 3$
$f'(0) = 11$
$f'(3) = \lim_{x\to 3} \frac{f(x+h) - f(x)}{h}$
$=$

**Q.55.**
If $\alpha$ is a repeated root of the polynomial $f(x) = 0$, then
(A) $f(\alpha) = 0$ but $f'(\alpha) \neq 0$
(B) $f(\alpha) = 0$ and $f'(\alpha) = 0$
(C) $f(\alpha) \neq 0$ but $f'(\alpha) = 0$
(D) $f(\alpha) \neq 0$ and $f'(\alpha) \neq 0$

$f(x) = 0$
$(x-\alpha)^2 = 0$

**Q.56.**
The wronskian of the solutions of $\frac{d^2y}{dx^2} + 4y = 0$ is
(A) 4
(B) 2
(C) 1
(D) doesn't exist

$m^2 + 4 = 0$
$m = \pm 2i$
$y = c_1 \cos 2x + c_2 \sin 2x$
$W = \begin{vmatrix} \cos 2x & \sin 2x \\ -2\sin 2x & 2\cos 2x \end{vmatrix}$

**Q.57.**
If
$$f(x,y) = \begin{cases} \frac{xy}{x^2+y^2}, & when\ (x,y) \neq 0 \\ 0, & when\ (x,y) = 0, \end{cases}$$
then
(A) both $f_x(0,0)$ and $f_y(0,0)$ exist
(B) $f_x(0,0)$ exists but not $f_y(0,0)$

$f_x(0,0) =$

$\lim_{x\to 0}$

(C) $f_y(0,0)$ exists but not $f_x(0,0)$
(D) None of these

**Q.58.**
If $u = f(y+ax) + \phi(y-ax)$, then $\frac{\partial^2 u}{\partial x^2}$ equals
(A) $\frac{\partial^2 u}{\partial y^2}$
(B) $-a^2\frac{\partial^2 u}{\partial y^2}$
(C) $a\frac{\partial^2 u}{\partial y^2}$
(D) $a^2\frac{\partial^2 u}{\partial y^2}$

**Q.59.**
The figure bounded by the line $y = x+2$ and the parabola $y = x^2$ is revolved about the x-axis. Then the volume of solid generated is
(A) $\frac{138\pi}{5}$
(B) $\frac{79\pi}{8}$
(C) $\frac{72\pi}{5}$
(D) None of these

**Q.60.**
The trapezoidal rule of integration, when applied to $\int_a^b f(x)dx$, will give the exact value of the integral
(A) if $f(x)$ is linear function of x
(B) if $f(x)$ is a quadratic function of x
(C) for any $f(x)$
(D) None of these

**Q.61.**
The value of $\frac{\Delta^2}{E}(x^3)$ is
(A) $2x$
(B) $3x$
(C) $6x$
(D) None of these

**Q.62.**
In the system of particles, suppose we do not assume that the internal forces come in pairs. Then, the fact that the sum of internal forces is zero follows from
(A) conservation of angular momentum
(B) conservation of energy
(C) Newton's second law
(D) principle of virtual displacement

**Q.63.**
A particle is constrained to move along the inner surface of a fixed hemispherical bowl. The number of degree of freedom of the particle is
(A) 1
(B) 2
(C) 3
(D) 4

**Q.64.**
If A and B are two subsets of a metric space $(X, d)$, then which of the following is not necessarily true?
(A) $(A \cap B)^{\circ} = A^{\circ} \cap B^{\circ}$
(B) $A^{\circ} \cup B^{\circ} \subset (A \cup B)^{\circ}$
(C) $\overline{A} \cup \overline{B} = \overline{A \cup B}$
(D) $\overline{A} \cap \overline{B} \subset \overline{A \cap B}$

$A^{\circ} \cap B^{\circ} = (A \cap B)^{\circ}$
$A^{\circ} \cup B^{\circ} \subseteq (A \cup B)^{\circ}$
$A \cap B \subset A$
$\overline{A \cap B} \subset \overline{A}$
$\overline{A \cap B} \subset \overline{A} \cap \overline{B}$

**Q.65.**
General value of $(-i)^{(-i)}$ equals
(A) $e^{(4n-1)\frac{\pi}{2}}$
(B) $e^{(4n+1)\frac{\pi}{2}}$
(C) $e^{(2n-1)\frac{\pi}{2}}$
(D) $e^{(2n+1)\frac{\pi}{2}}$

**Q.66.**
The differential equation $2ydx - (3y - 2x)dy = 0$ is
(A) homogeneous and linear but not exact
(B) homogeneous and exact but not linear
(C) exact and linear but not homogeneous
(D) exact, homogeneous and linear

$2ydx + (2x - 3y)dy = 0$
$\frac{\partial M}{\partial y} = 2 \quad \frac{\partial N}{\partial x} = 2$
$\frac{dy}{dx} = \frac{2y}{-2x+3y}$
$\frac{dx}{dy} = \frac{3y-2x}{2y}$

**Q.67.**
Which of the following statement is correct?
(A) If an Linear Programming Problem (LPP) has unbounded solution, then its dual is infeasible
(B) If an LPP has unbounded solution, then its dual also has unbounded solution
(C) If an LPP is infeasible, then its dual always has unbounded solution
(D) If an LPP is infeasible, then its dual is also infeasible

$x = (3y - 2x)dy$
$3y - 2x) = 2y$

$\frac{xdy - ydx}{x^2}$

**Q.68.**
The general solution of the differential equation $xdy - ydx = (x^2 + y^2)dx$ is

$(x^2 + y^2 + y)dx - xdy = 0$

$\frac{\partial M}{\partial y} = 2y + 1$

$\frac{\partial N}{\partial x} = -1$

(A) $y = x\tan(x + c)$
(B) $y = x\tan^{-1}(x + c)$
(C) $x = y\tan(x + c)$
(D) None of these

**Q.69.**
If $f = \frac{x^2(x^2-y^2)^3}{(x^2+y^2)^3}$, then $x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y}$ is equal to

$= 4$

(A) $7f$
(B) $8f$
(C) $2f$
(D) $4f$

**Q.70.**
The binomial distribution whose mean is 10 and standard deviation is $2\sqrt{2}$ is
(A) ${}^{50}C_r(\frac{1}{5})^r(\frac{4}{5})^{50-r}, r = 0, 1, 2 \cdots, 50$
(B) ${}^{25}C_r(\frac{1}{5})^r(\frac{4}{5})^{25-r}, r = 0, 1, 2 \cdots, 50$
(C) ${}^{75}C_r(\frac{1}{5})^r(\frac{4}{5})^{75-r}, r = 0, 1, 2 \cdots, 50$
(D) None of these

**Q.71.**
Which of the following limit exists?

(A) $\lim_{(x,y)\to(0,0)} (\frac{2xy}{x^2 + y^2})$ $\quad y = mx$

(B) $\lim_{(x,y)\to(0,0)} (\frac{xy^3}{x^2 + y^6})$ $\quad x = my^3$

(C) $\lim_{(x,y)\to(0,0)} (\frac{xy}{\sqrt{x^2 + y^2}})$ $= \frac{x \cdot mx}{\sqrt{x^2 + m^2x^2}} = \frac{mx}{\sqrt{1+m^2}} = 0$

(D) $\lim_{(x,y)\to(0,0)} \{\frac{x^2y^2}{x^2y^2 + (x^2 - y^2)^2}\}$ $\quad \frac{x^2(mx)^2}{x^2(mx)^2 + (x^2 - m^2x^2)^2} =$

**Q.72.**
If the random variables X and Y have the joint density function

$$f(x, y) = \begin{cases} \frac{3}{4} + xy, & 0 < x < 1, 0 < y < 1 \\ 0, & \text{otherwise,} \end{cases}$$

then the conditional probability function of Y given X is

(A) $f(x, y) = \begin{cases} \frac{3+4xy}{3+2x}, & 0 < y < 1 \\ 0, & \text{otherwise} \end{cases}$

(B) $f(x,y) = \begin{cases} \frac{3+4xy}{3+2y}, & 0 < y < 1 \\ 0, & \text{otherwise} \end{cases}$

(C) $f(x,y) = \begin{cases} \frac{4+3xy}{3+2x}, & 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$

(D) $f(x,y) = \begin{cases} \frac{4+3xy}{3+2y}, & 0 < y < 1 \\ 0, & \text{otherwise} \end{cases}$

**Q.73.**
One series containing numbers has mean 8, variance 24 and the second series containing 5 numbers has mean 8, variance 18. Then the variance of the combined data is
(A) $\frac{81}{4}$
(B) $\frac{43}{2}$
(C) 26
(D) None of these.

**Q.74.**
If the lines of regression are $4x + 3y + 7 = 0$ and $3x + 4y + 8 = 0$, then correlation coefficient between x and y is
(A) $\frac{5}{4}$
(B) $\frac{41}{50}$
(C) $-\frac{1}{4}$
(D) $-\frac{3}{4}$

**Q.75.**
The set $X = R$ with metric $d(x,y) = \frac{|x-y|}{1+|x-y|}$ is
(A) bounded
(B) unbounded
(C) compact
(D) None of these

**Q.76.**
Which of the following statements is not correct?
(A) The real line $\mathbb{R}$ with the usual metric is not compact
(B) The set $\mathbb{Z}$ of integers with the usual metric is not compact
(C) The open interval $(0,1)$ with the usual metric is not compact
(D) The subset $\{1, \frac{1}{2}, \frac{1}{3}, \cdots\}$ of $\mathbb{R}$ is compact.

**Q.77.**
The equation $\frac{x^2}{12-k} - \frac{y^2}{8-k} = 1$ represents
(A) a hyperbola if $k < 8$
(B) an ellipse if $k > 8$

(C) a hyperbola if $8 < k < 12$
(D) None of these

**Q.78.**
Which of the following subset of $\mathbb{R}$ with the usual metric $d(x,y) = |x-y|$ is not a neighbourhood of 1?
(A) $[0,2] - \frac{3}{2}$
(B) $[1,2]$
(C) $(0,2)$
(D) $\mathbb{R}$

**Q.79.**
The closed interval $[0,1]$ is
(A) compact and connected
(B) compact but not connected
(C) connected but not compact
(D) neither compact nor connected

**Q.80.**
The equation of tangent to the curve $x = t^3 - 4$, $y = 2t^2 + 1$ and the point where $t = 2$ is
(A) $2x - 3y + 19 = 0$
(B) $2x - 3y - 19 = 0$
(C) $2x + 3y - 19 = 0$
(D) None of these

$\frac{dy}{dt} = 4t$ $\quad \frac{dy}{dx} = \frac{4t}{3t^2} \Rightarrow \frac{4}{3t}$

$\frac{dx}{dt} = 3t^2$

$(y - 9) = \frac{2}{3}(x - 4)$

$3y - 27 = 2x - 8$

$2x - 3$
$2x - 3$

**Q.81.**
The asymptotes of a circle
(A) are parallel to x-axis
(B) are parallel to y-axis
(C) do not exist
(D) None of these

**Q.82.**
Let $x \oplus y = 3xy$ for all $x, y \in R - \{0\}$. The inverse of the element 3 in the group $(R - \{0\}, \oplus)$ is
(A) $\frac{1}{3}$
(B) $\frac{1}{9}$
(C) $\frac{1}{27}$
(D) None of these

**Q.83.**
Let V and W be vector spaces over a field F with $dim_F V = m$ and $dim_F W = n$,where m and n both are finite. Then $dim_F Hom(V, W)$ is
(A) $|m-n|$
(B) $\frac{m}{n}$
(C) $m+n$
(D) $mn$

$\dim V = m$
$\dim W = n$

**Q.84.**
For a Poisson variate X, if $P(X=2) = 3P(X=3)$, then the mean of X is
(A) 1
(B) $\frac{1}{2}$
(C) $\frac{1}{3}$
(D) $\frac{1}{4}$

**Q.85.**
Let $A$ be a $n \times n$ matrix such that $xA^Tx > 0$ for every non zero vector x in $R^n$. Which of the following statement is true?
(A) Exactly one eigenvalue of A is zero.
(B) More than one eigenvalues of A are zero.
(C) All eigenvalues of A are negative.
(D) All eigenvalues of A are positive.

**Q.86.**
The angle at which the radius vector cuts the curve $r = a(1+\cos\theta)$ is
(A) $\pi + \theta$
(B) $\pi + \frac{\theta}{2}$
(C) $\frac{\pi}{2} + \theta$
(D) $\frac{\pi}{2} + \frac{\theta}{2}$

**Q.87.**
The dimension of the subspace

$$W = \{(a,b,c,d) : a+c+d = 0, b+c+d = 0\}$$

of $R^4$ is
(A) 1
(B) 2
(C) 3
(D) 4

$\begin{pmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{pmatrix}$

$d = -(b+c)$
$a + c - (b+c) = 0$
$a - b = 0$
$a = b$
$(b, b, c, -(b+c))$

$T(5,6) = 5(2,3) - 4(1,4)$
$= (10,15) + (-4, -16)$
$= (6,-1)$

$T(5,6) = 7($

$2a + 4b = 6$

$\left(\begin{array}{cc|c} 1 & 1 & 5 \\ 2 & 4 & 6 \end{array}\right) \sim \left(\begin{array}{cc|c} 1 & 1 & 5 \\ 0 & 2 & -4 \end{array}\right)$

$2b = -4$
$b = -2$
$a = 7$

**Q.88.**
Let $T : R^2 \to R^2$ be a linear transformation such that $T((1,2)) = (2,3)$ and $T((0,1)) = (1,4)$. Then $T((5,6))$ is

~~$(5,6) = a(2,3) + b($~~
$(5,6) = a(1,2) + b(0,1) = (a, 2a+b)$
$a = 5$
$2 \times 5 + b = 6$
$b = -4$

$T(5,6) = 5(2,3) - 4(1,$
$= (10,15) + (-4$
$= (6,-1)$

(A) $(-1,6)$
(B) $(1,-6)$
(C) $(31,6)$
(D) $(6,31)$

**Q.89.**
A box contain 10 mangoes out of which 4 are rotten. 2 mangoes are taken together. If one of them is found to be good, then probability that the other is also good is
(A) $\frac{1}{3}$
(B) [illegible]
(C) [illegible]
(D) [illegible]

**Q.90.**
The angle between the tangents drawn from the point $(-1,7)$ to the circle $x^2 + y^2 = 25$ is
(A) $\frac{\pi}{2}$
(B) $\frac{\pi}{3}$
(C) $\frac{\pi}{4}$
(D) $\tan^{-1}\sqrt{2}$

**Q.91.**
If X is a Poisson random variate with mean 3, then $P(|X-3| < 1)$ is equal to
(A) [illegible] $e^{-3}$
(B) [illegible] $e^{-3}$
(C) [illegible] $e^{-3}$
(D) $3e^{-3}$.

**Q.92.**
A coin is tossed 5 times and the probability of getting head is $\frac{1}{2}$. The probability of exactly two heads is
(A) [illegible]
(B) [illegible]
(C) $\frac{1}{16}$
(D) $\frac{5}{16}$

**Q.93.**
Let $f : \mathbb{R} \to \mathbb{R}$ be a differentiable function with $f(0) = 0$. If $1 < f'(x) < 2$, for all x in $R$, then which of the following statement is true on $(0, \infty)$?

$f'(x) > 0$

(A) $f$ is periodic
(B) $f$ is unbounded
(C) $f$ is increasing and bounded
(D) None of these

**Q.94.**
The limit superior and limit inferior of the sequence $\{\frac{(-1)^n}{n^2}\}$ are respectively equal to
(A) 1, 0
(B) 0, 0
(C) −1, 0
(D) −1, 1

$a_{2m} = \frac{1}{(2m)^2} = 0$
$a_{2m+1} = -\frac{1}{(2m+1)^2} = 0$

**Q.95.**
The series $\sum_{n=1}^{\infty}(-1)^n \frac{x^n}{n}$, for $|x| > 1$ is
(A) convergent
(B) divergent
(C) oscillatory
(D) None of these

$\lim_{n\to\infty} \frac{x^n}{n}$
$a_{2m} = \frac{x^{2m}}{2m} = \lim_{m\to\infty} \frac{x^{2m}}{2m} = \infty$
$a_{2m+1} = -$ ... $= -\infty$

**Q.96.**
Which of the following is incorrect?
(A) every sequence has a monotonic subsequence
(B) every bounded sequence has a convergent subsequence
(C) every sequence has a countable number of terms
(D) every sequence has a limit point

**Q.97.**
Which of the following sequence is convergent?
(A) $S_n = \{1 + \frac{1}{n}\}$
(B) $S_n = \{\frac{(-1)^n}{n^2}\}$ $\quad (-\frac{1}{1}, \frac{1}{2^2}, -\frac{1}{9}, \frac{1}{16}, \dots)$
(C) $S_n = \{(-1)^n\}$ ×
(D) $S_n = \{(-1)^n(1 + \frac{1}{n})\}$

**Q.98.**
If

$$f(x) = \begin{cases} 1 + x, & when\, 0 \le x < \frac{1}{2}, \frac{1}{2} < x \le 1 \\ 0, & when\ x = \frac{1}{2}, \end{cases}$$

$\lim (1+x) = \frac{3}{2} \ne f(\frac{1}{2})$

is defined on the interval $[0,1]$, then
(A) $f$ is Riemann integrable on $(0,1)$
(B) $f$ is not Riemann integrable on $(0,1)$
(C) $f$ is unbounded on $[0,1]$
(D) None of these

**Q.99.**

$$\lim_{n\to\infty}\left\{\frac{1^2}{n^3}+\frac{2^2}{n^3}+\frac{3^2}{n^3}+\cdots+\frac{(n-1)^2}{n^3}\right\}$$

$\lim$

$\frac{1}{n^3} \sum$

is equal to
(A) 0
(B) 1
(C) $\frac{1}{3}$
(D) $\frac{2}{3}$

**Q.100.**
Let X be a continuous random variable with probability density function

$$f(x)=\frac{1}{\sqrt{2\pi}}e^{\frac{-x^2}{2}}, -\infty < x < \infty$$

and let $Y = X + 1$. Then the expected value of Y is
(A) 0
(B) 1
(C) $\pi$
(D) $\sqrt{\pi+1}$

$\sum_{r=1}^{(n-1)} \frac{r^2}{n^3}$

$\int_0^1 x^2\,dx$

$\left[\frac{x^3}{3}\right]_0^1$

$= \frac{1}{3}$