M.Phil/Ph.D. Entrance Examination-2017

THEORETICAL PHYSICS

[Part-II (b)]

Max. Marks: 50

Note: Attempt any 5 questions. All questions carry equal marks.

1. A particle of mass m collides elastically with a stationary particle of equal mass. The incident particle has a kinetic energy K_1. What is its kinetic energy after collision if the scattering angle is θ?

2. Consider the motion of earth around sun and assume that the orbit is circular. Suppose the Sun loses its mass very slowly from an original mass M_1 to a mass M_2. Suppose the initial radius of orbit for earth was R_1. What is the final radius of orbit for earth?

3. Find Heisenberg equations of motion for operators $p(t)$ and $q(t)$ for one dimensional harmonic oscillator with Hamiltonian $H = \frac{\hbar\omega}{2}(p^2 + q^2)$ where p and q are the unitless momentum and position operators satisfying $[p, q] = i$

4. The normalized wavefunction of a particle moving along x-axis is given by

$$\psi(x) = \left(\frac{1}{\alpha\sqrt{\pi}}\right)^{1/2} e^{(i/\hbar)p_0 x - x^2/2\alpha^2}$$

Calculate the average momentum of the particle.

5. The electron in an hydrogen atom occupies the orbital state $|2, -1\rangle$ and spin state $|\frac{1}{2}, \frac{1}{2}\rangle$. Find the total angular momentum states. If we measure J^2 (where J is total angular momentum) what values we might get ?

6. Find the Helmholtz free energy of a system in canonical ensemble, whose canonical partition function is given by $Z = \left(\frac{2}{\hbar\beta\omega}\right)^{3N}$ where $\beta = 1/kT$.

7. Consider N particles of spin 1/2, in a magnetic field B, with the Hamiltonian

$$\hat{H} = -\frac{2\mu_B B}{\hbar} S_z$$

Use quantum canonical partition function to calculate the average energy.

[P.T.O.]

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8. Find the value of integral $\int \frac{3z^2+2}{(z-1)(z^2+9)} dz$, taken counter clockwise around the circle $|z-2| = 2$.

9. Find eigenvlaues and eigenvectors of the matrix

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 1 & 0 \\ 1 & 2 & 1 \end{bmatrix}$$

10. A sphere of radius R carries a polarization $\vec{P} = k\vec{r}$, where k is a constant and $\vec{r}$ is the vector from the center. Find the field inside the sphere.

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M.Phil/Ph.D. Entrance Examination-2017

THEORETICAL PHYSICS

[Part-II (a)]

ROLL NO. P 5 3 5 4 0 0 0 6

Signature of Candidate

Signature of Invigilator

Timings: Two Hours

Total Marks: 100
(Part II(a) – 50 marks)
(Part II (b) – 50 marks)

Instructions to Candidates

1. Do not write your name or put any other mark of identification anywhere in the OMR Answer Sheet. **IF ANY MARK OF IDENTIFICATIONS IS DISCOVERED ANYWHERE IN OMR ANSWER SHEET, the OMR sheet will be cancelled, and will not be evaluated.**
2. This Question Booklet contains this cover page and a total of **50 Multiple Choice Questions of 1 mark each.** Space for rough work has been provided at the end. Available space on each page may also be used for rough work.
3. The candidate may carry this booklet.
4. There is no negative marking in Multiple Choice Questions.
5. USE OF CALCULATOR IS NOT PERMITTED.
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CORRECT METHOD

(A) ● (C) (D)

WRONG METHOD

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Note: *(i) The paper has two part viz Part-II (a) and Part-II (b). (ii) Part-II (a) has 50 Multiple Choice Questions (MCQ) of 1 Marks each and* ***All questions are compulsory from Part II (a).*** *(iii) Part-II (b) has 10 Question of 10 marks each.* ***Attempt any 5 questions from Part-II (b).***

1. The differential equation $e^{x^2}\frac{dy}{dx} + 5y = x^3y$ is

   (a) Separable and not linear

   (b) Linear and not separable

   (c) Both separable and linear

   (d) Neither separable nor linear

2. The general solution to $\frac{d^2x}{dt^2} + 6\frac{dx}{dt} + x = 0$ is $x = Ate^{-3t} + Be^{-3t}$. Which of the following is correct?

   (a) As $t \to \infty$, $x \to A$ for any value of B

   (b) As $t \to \infty$, $x \to 0$ for any values of A and B

   (c) The behaviour of x as $t \to \infty$ depends on the values of A and B

   (d) As $t \to \infty$, $x \to \infty$ for any values of A and B

3. If $F(s)$ be the Laplace transform of $f(t)$, then Laplace transform of $f(at)$.

   (a) $F(as)$

   (b) $F(\frac{s}{a})$

   (c) $\frac{F(as)}{a}$

   (d) $\frac{1}{a}F(\frac{s}{a})$

4. If U and W are subspace of a vector space V, then $U \cap W$ is

   (a) Superset of $U \cup W$ (b) Subspace of V

   (c) Superset of $U \cap W$ (d) None of these.

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5. The set $\{v_1, v_2\}$ in a vector space V is linear dependent iff

(a) $v_1 = av_2$, a is a scalar (b) $v_1 \neq av_2$, a is a scalar

(c) both v_1 and v_2 are non-zero (d) None of these

6. The basis B of a vector space V is a

(a) Linear independent set (b) Linear dependent set

(c) $B = \{\bar{0}\}$ (d) none of these.

7. The function $f(z) = \frac{\sin z}{z}$ has a singularity at $z = 0$, which is

(a) Removable (b) Essential (c) Simple Pole (d) Isolated.

8. The Laplace transformation of $e^{at}\sin bt$ is

(a) $\frac{b}{(s+a)^2+b^2}$ (b) $\frac{b}{(s-a)^2+b^2}$ (c) $\frac{b}{(s+a)^2-b^2}$ (d) $\frac{b}{(s-a)^2-b^2}$.

9. The radius of convergence of the Taylor series expansion of the function $f(x) = \frac{1}{\cosh x}$ around $x = 0$ is

(a) 0 (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{2}$ (d) 1.

10. Let $P_n(x)$ is the Legendre polynomial of degree n, then $\int_{-1}^{+1} P_n(x)P_m(x)dx$ is

(a) $\frac{2}{2n-1}\delta_{nm}$ (b) $\frac{2}{2n-1}\delta_{nm}$ (c) $\frac{1}{2n+1}\delta_{nm}$ (d) $\frac{1}{2n-1}\delta_{nm}$.

11. In a canonical ensemble, a system A of fixed volume is in contact with a large reservoir B. Then

(a) A can exchange only energy with B

(b) A can exchange only particles with B

(c) A can exchange neither energy nor particles with B

(d) A can exchange both energy and particles with B

12. Probability of a closed system to be found in a particular macrostate is proportional to

(a) the number of microstates in it

(b) the energy of the state

(c) inverse of the energy of the state

(d) the temperature

13. If the energy of a quantum harmonic oscillator is $E_n = n\hbar\omega$, $n = 0, 1, 2, \ldots \infty$, the partition function of the oscillator is

(a) $e^{-\hbar\omega/kT}$

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(b) $e^{\beta\hbar\omega}$

(c) $\frac{1}{e^{-\hbar\omega/kT}-1}$

(d) $\frac{1}{1-e^{-\hbar\omega/kT}}$

14. When two systems, 1 and 2, are brought in thermal contact with each other, equilibrium is reached when

(a) $E_1 = E_2$

(b) $\frac{\partial E_1}{\partial T_1} = \frac{\partial E_2}{\partial T_2}$

(c) $\frac{\partial S_1}{\partial T_1} = \frac{\partial S_2}{\partial T_2}$

(d) $\frac{\partial S_1}{\partial E_1} = \frac{\partial S_2}{\partial E_2}$

$dS = \frac{dQ}{T}$ $\frac{1}{T} = \frac{dE}{dS}$

dS_1 $\frac{dE}{dT}$

15. Gibb's paradox is resolved by taking into account the fact that

(a) gas atoms are identical particles

(b) gas atoms are moving particles

(c) gas atoms are distinguishable particles

(d) gas atoms are charge-neutral particles

16. In canonical ensemble, average energy is given by

(a) $\langle E\rangle = \frac{\partial Z}{\partial T}$

(b) $\langle E\rangle = \frac{\partial Z}{\partial \beta}$

(c) $\langle E\rangle = \frac{\partial \log(Z)}{\partial T}$

(d) $\langle E\rangle = -\frac{\partial \log(Z)}{\partial \beta}$

17. In canonical ensemble, when the number of particles is increased, the relative energy fluctuation, $\sqrt{\langle E^2\rangle - \langle E\rangle^2}/\langle E\rangle$

$\frac{(\Delta E)^2}{\langle E\rangle} \propto \frac{1}{\sqrt{N}}$ 0 ∞

(a) becomes larger

(b) becomes smaller

(c) remains same

(d) becomes imaginary

18. N noninteracting spin-1 particles are sitting on fixed lattice sites. The entropy of this system is given by

$-\frac{\hbar\omega}{2}$

$(2S+1)^N$

3^N

$K \log \Omega$

$NK \log 3$

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(a) $S = Nk\log 2$

(b) $S = 2k\log(N!)$

(c) $S = 3k\log(N)$

(d) $S = Nk\log(3)$

19. If two systems have their number of microstates given by Ω_1 and Ω_2, respectively, the number of microstates of the combined system is given by

(a) $\Omega_1 + \Omega_2$

(b) $\log(\Omega_1 + \Omega_2)$

(c) $\sqrt{\Omega_1^2 + \Omega_2^2}$

(d) $\Omega_1\Omega_2$

20. In canonical ensemble, the Helmholtz free energy is given by

(a) $F = k\log E$

(b) $F = -kT\log E$

(c) $F = -kT\log Z$

(d) $F = k\log Z$

21. Given a force law $\vec{F} = Ar^n\hat{r}$, what is the dimension of A?

$MLT^{-2} = A L^n$

$ML^{1-n}T^{-2}$

(a) $[M][L]^{1-n}[T]^{-2}$.

(b) $[M][L]^{n-1}[T]^{-2}$.

(c) $[M][L]^{n-2}[T]^{-2}$.

(d) $[M][L]^{2-n}[T]^{-2}$.

22. A block of mass m and contact area A slides down an inclined plane with friction, covering a distance d in time T. How much time does it take for a different block with same mass and composition but with contact area $A/2$, to slide down the same distance?

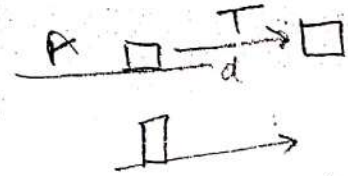
$T_1 = \frac{d+A}{S}$

(a) $\sqrt{T}$.

(b) T.

(c) T^2.

(d) $T^{3/2}$.



$T_2 = \frac{\frac{A}{2}+d}{S}$

$\frac{A}{2}+d$

$T_2 = \frac{\frac{A}{2}+d}{d+\frac{A}{T}}$

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23. A planet is moving around star in a circular orbit so that its kinetic energy is K. What is its potential energy? (assume that the zero potential energy is at infinity)

(a) $-2K$.

(b) $-K/2$.

(c) $-K$.

(d) $-K/4$.

24. An atom of mass M_1 in the ground state is hit by a particle with mass m_1. Provided that the energy of the first excited state of the atom is δE, find the minimal initial energy E_0 of the particle required to excite the atom.

(a) $E_0 \geq \delta E(1 + 2m_1/M_1)$.

(b) $E_0 \geq \delta E(1 + m_1/M_1)$.

(c) $E_0 \geq 2\delta E$.

(d) $E_0 \geq \delta E(1 + M_1/m_1)$.

25. Two identical planets A and B are in circular motion around a star. The orbital radius of the planet A is twice that of planet B. The angular momentum of planet A to that planet B is given by

(a) 1/2.

(b) $\sqrt{2}$.

(c) $1/\sqrt{2}$.

(d) 2.

26. A relativistic particle has total energy 6.0 Gev. and momentum 5.9 Gev/c. It's rest mass is approximately given by

(a) 1.2 Gev/c^2.

(b) 1.0 Gev/c^2.

(c) 0.5 Gev/c^2.

(d) 1.5 Gev/c^2.

27. A uniform solid disk starts from rest and rolls down on an inclined plane without slipping. After sometime, what fraction of the disk's total kinetic energy is its rotational kinetic energy?

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(a) 1/3.

(b) 1/2.

(c) 1/4.

(d) 2/3.

28. Why do raindrops fall with constant speed during the late stage of their descent?

(a) the gravitational force is same for all drops.

(b) Air resistance balances the gravitational force on a drop.

(c) The gravitational force is negligible for small objects like rain drops.

(d) the gravitational force can not increase the speed of falling objects to more than $9.8 m/s$.

29. A mass M is undergoing simple harmonic oscillations with a frequeucy ν_1. A force F proportional to the speed of the particle v_1, $F = -\alpha v_1$ (where α is a constant) is introduced. If the particle continues to oscillate, the frequency is

(a) smaller than ν_1.

(b) same as ν_1.

(c) larger than ν_1.

(d) dependent inversely on α.

30. A particle with unit mass is undergoing motion in x-direction with its velocity varies as $v(x) = \alpha x^{-p}$ where α and p are two arbitrary constants. What is the acceleration of the particle as a function of x?

(a) $-p\alpha^2 x^{-2p-1}$.

(b) $-p\alpha^2 x^{-2p}$.

(c) $-p\alpha^2 x^{-p}$.

(d) $-p\alpha^2 x^{-p-1}$.

31. Two physical states are denoted by ket $|\alpha\rangle$ and ket $|\beta\rangle$. Then in general

(a) $\langle\beta|\alpha\rangle = \langle\alpha|\beta\rangle$.

(b) $\langle\alpha|\alpha\rangle = \langle\beta|\beta\rangle$.

(c) $\langle\alpha|\beta\rangle = \langle\beta|\alpha\rangle^*$.

(d) $\langle\alpha|\beta\rangle = 0$.

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32. Which of the [illegible] particle of [illegible] in orbital state $l = 3$ and total angular momentum [illegible]

(a) $J^2 = 16\hbar^2$.

(b) $J^2 = 6\hbar^2$.

(c) $J^2 = 5\hbar^2$.

(d) $J^2 = 18\hbar^2$.

$S = 1$. $l = 3$

$J = (1+3)$ to $|1-$

$2+3+4$ 4, 2

4 3 2 6

33. According to the uncertainty principle in quantum mechanics

(a) The uncertainties in product of position and momentum are always at least Planck's constant h.

(b) The uncertainties in the product of the position and momentum are always at most Planck's constant h divided by 2π.

(c) The uncertainties in the product of the position and momentum are at least Planck's constant h when measured simultaneously.

(d) The uncertainties in the product of the position and momentum are at least Planck's constant h divided by 2π when measured simultaneously.

34. The antiparticle was predicted theoretically by analyzing a solution of

(a) Dirac's equation of motion

(b) Newton's equation of motion

(c) Schrodinger's equation of motion

(d) None of the above.

35. Which of the following is correct in Schrödinger equation?

(a) Time is an operator

(b) Probability density is not positive definite

(c) This satisfies the equation of continuity

(d) This is invariant under Lorentz transformation

36. Let an operator corresponding to a physically observable quantity be represented by a complex symmetric matrix A. Then the eigenvalues of the matrix $A^\dagger A$ is

(a) always complex

(b) always 1

$A^\dagger$ A

$\begin{pmatrix} a' & b' \\ c' & d' \end{pmatrix}\begin{pmatrix} a & c \\ b & d \end{pmatrix}$

$p = -i\hbar \frac{\partial}{\partial x}$.

$a'a =$

$-3i \cdot 3i$

$\left(i\hbar \frac{\partial}{\partial x}\right)\left(-i\hbar \frac{\partial}{\partial x}\right)$

$\hbar^2 \frac{\partial^2}{\partial x^2}$

$a'a + b'b$ $a'c + b'd$

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(c) Either real or complex

(d) always real

37. If the bound state wavefunction corresponding to a Schrödinger equation is found to be of even parity then which of the following is correct ?

(a) The potential $V(-x) = V(x)$ but there is no condition on energy

(b) It could be the first excited state but there is no condition on the potential

(c) It could be the ground state and the potential $V(-x) = V(x)$

(d) It could be the ground state but there is no condition on the potential

38. Which of the following is true for relativistic Klein-Gordon and Dirac equation of motion?

(a) Both of these equations are not suitable for spin 1/2 particle.

(b) Both of these equations are suitable for spin 1/2 particle.

(c) Klein-Gordon equation is valid for particle but Dirac equation is valid only for antiparticle.

(d) Klein-Gordon equation is suitable for massive spin 0 particle but Dirac equation is not suitable for the same particle.

39. Twelve equal charges, q, are situated at the corners of a regular 12-sided polygon (for instance, one on each numeral of a clock face). What is the net force on a test charge Q at the center? (a)12 (b) -12 (c) 0 (d) none of these.

40. Suppose the electric field in some region is found to be $\vec{E} = kr^3\hat{r}$, in spherical coordinates (k is some constant). The charge density ρ is (a)ϵ_0 (b) $\epsilon_0/3$ (a)$3\epsilon_0$ (d) none of these. Here the constant ϵ_0 is called the permitivity of free space.

41. Using the Gauss's law, the electric field inside a spherical shell of radius R, which carries a uniform surface charge density σ, is (a) 0 (b) 3σ (C) $\sigma/3$ (d) $\pi\sigma/3$

42. If the electric field in some region is given (in spherical coordinates) by the expression $E(r) = \frac{Ar^2 + B\sin\theta\cos\phi}{r}$. What is the charge density?

(a) $\frac{\epsilon(A - B\sin\phi)}{r^2}$ (b) $\frac{\epsilon(A - B\cos\phi)}{r^2}$ (c) $\frac{\epsilon(A - B\sin\phi)}{2r^2}$ (d) $\frac{\epsilon(A - B\cos\phi)}{2r^2}$

Here A and B are constants.

43. The curl of $\vec{V} = -4\hat{i} + x\hat{j}$ is

(a) $\hat{i}$ (b) $\hat{j}$ (c) $2\hat{k}$ (d) $\hat{k}$

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44. When distance between two equal charges is decreased to half and their magnitude of charge is also decreased to half, the force between them

(a) remains unchanged (b) reduces to half (c) becomes double (d) becomes four times.

45. If a coil does not consume any power, it should be

(a) resistive (b) pure capacitance (c) inductance (d) None of these.

46. Two charged particles (a and b) with same mass and charge move in circular orbits which are perpendicular to the magnetic field. If $v_a > v_b$ then

(a) a takes less time to complete revolution (b) b takes less time to complete revolution (c) Both take same time (d) None of these.

47. Relativistic motion of a charged particle in parallel electric and magnetic field is described by

(a) circle (b) parabola (c) trochoid (d) helical line.

48. A linear map $T : U \to V$ is 1-1

(a) N(T) is zero subspace of U (b) R(T) is zero subspace of V (c) N(T) = U (d) R(T) = V .

49. Let $P_n(x)$ be a Legendre's Polynomial of degree n, then $P_n(-1)$.

(a) 1 (b) $(-1)^n$ (c) -1 (d) None of these.

50. Gauss's law in differential form is give by

(a) $\nabla \times \vec{E} = \frac{\rho}{\epsilon_0}$ (b) $\nabla . \vec{E} = \frac{\rho}{\epsilon_0}$ (c) $|\vec{E}| = \frac{\rho}{\epsilon_0}$ (d) None of these.

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Question Booklet No: 053542

SET - B

M.Phil/Ph.D. Entrance Examination-2017

NATURAL SCIENCES

Part I

ROLL NO. P 5 3 5 4 0 0 6

Signature of Candidate

Signature of Invigilator

Timings: Two Hours

Total Marks: 100

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CORRECT METHOD

(A) ● (C) (D)

WRONG METHOD

(A) ⊗ (C) (D) (A) ✓ (C) (D) (A) ● (C) (D) (A) ● (C) (D) ( ) ● (C) (D) (A) ● (C) ●

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Directions (Questions 1-4): Read the following passage and answer the items that follow. Your answers should only be based on the passage and you are not supposed to use your general knowledge/information in order to answer the items that follow the passage.

Recent technological advancement in manned and unmanned undersea vehicles overcomes some of the limitations of the divers' equipment. Without a vehicle, divers often become sluggish and their mental concentration was limited. Because of undersea pressure that affected their mind, concentration among divers was difficult or impossible. But today, oceanographers make observations by means of instruments that are lowered into the oceans or form samples taken from the water. Direct observations of the ocean floor are made not only by divers but also by deep diving submarines. Some of these submarines can dive to depths more than several miles and cruise at depths of 15000 feet. Ratio equipped buoys can be operated by remote control in order to transmit information back to land based laboratories including data about water temperature, currents and weather. Some of mankind's most serious problems, especially those concerning energy and food may be solved with the help of observations made possible by these undersea vehicles.

Q1. What does the passage talk about?

A. Manned and unmanned submarines
B. Oceanographers and their limitations
C. Sluggish brain activity in divers
D. Observation of the seabed using latest technology

Q2. Why do divers find it hard to concentrate underwater?

A. The marine flora and fauna distract them
B. They do not think clearly
C. They become slow due to tiredness
D. The underwater pressure affects their minds

Q3. Which of the following statements about submarines is true according to the passage?

A. Not all submarines can observe the ocean floor
B. Submarines may be manned and unmanned
C. Both (1) and (2)
D. None of the above

Q4. Which of the following statement is the most similar to the ratio-equipped buoy described in the passage?

A. A robot
B. Hubble space telescope
C. An MRI machine
D. None of the above

Directions (Question 5): The question below consists of a main statement followed by four numbered statements; select the one that logically follows the statement

Q5. Unless I have money I cannot enjoy my Sunday

A. I have money so I can enjoy my Sunday
B. I can enjoy my Sunday means I have money
C. I do not have money implies I cannot enjoy my Sunday
D. Both (2) and (4)

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Directions (Question 6): Select the one which is different from the other three responses.

Q6. Find out the set of numbers amongst the four set of numbers given in the alternatives which is most likely the set given in the question
Given set 3, 8, 16

A. (2,10,16)
B. (12,72,96).
C. (4,24,48)
D. (6,42,48)

Directions (Questions 7-8): In each of the following letter series, some of the letter series are missing which are given in that order as one of the alternatives below it. Choose the correct alternative.

Q7. ab abab ba bab
A. abbba
B. abbab
C. baabb
D. bbaba

Q8. a n b ncb ncb
A. abbcc
B. abcbcb
C. bacbab
D. bcabab

Q9. Where do you find the clock and calendar in/on your computer?
A. Task Bar
B. Recycle Bin
C. Library
D. Start Menu

Q10. Which one of the following memories is not primary memory of a computer?
A. ROM
B. RAM
C. EEPROM
D. CD-ROM

Q11. A person use his/her expertise to gain access to other people's computer to get information illegally or do damage is called as
A. Spammer
B. Programmer
C. Hacker
D. Instant messenger

Q12. Jamia Millia Islamia came into existence in?
A. 1950
B. 1940
C. 1980
D. 1920

Q13. Who said "They built up the Jamia Millia stone by stone and sacrifice by sacrifice"?
A. Sarojini Naidu
B. Dr. A.P.J. Abdul Kalam
C. Mahatma Gandhi
D. Jawaharlal Nehru

Q14. Who was the first Vice-Chancellor of Jamia Millia Islamia?
A. Hakim Ajmal Khan
B. Allama Iqbal
C. Mohamed Ali Jauhar
D. None of these

Q15. In Urdu language, Jamia means '_____' and Millia means '_____'.
A. University, National
B. National, University
C. University, International
D. College, National

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Q16. Which of the following is **not** correctly abbreviated?

a. GATE: Gratitude Aptitude Test in Engineering
b. GSM: Global System for Mobile Communication
c. MODEM: Modulator demodulator
d. ISP: International Services Program

A. Only b
B. Only d
C. Only c and d
D. All of these

Directions (Questions 17-18): In the following questions choose the missing word in place of sign "?" On the basis of the relationship between the words given on the left/right hand of sign ":" :

Q17. Doctor : Nurse : ? : Follower

A. Worker
B. Employer
C. Union
D. Leader

Q18. Foundation : Edifice : Constitution : ?

A. Government
B. Stage
C. Nation
D. Cabinet

Q19. Which of the following groups have only output devices?

A. Scanner, Keyboard, Printer
B. Mouse, Printer, Monitor
C. Keyboard, Monitor, Printer
D. Plotter, Printer, Monitor

Q20. Which one of the following applications is commonly used to prepare a presentation or slide show?

A. Photoshop
B. Power point
C. Internet Explorer
D. Outlook express

Directions (Questions 21-25): Read the following passage and answer the items that follow. Your answers should only be based on the passage and you are not supposed to use your general knowledge/information in order to answer the items that follow the passage.

The capitalist system of society does not foster healthy relations among human beings. A few people owe all the means of production and others-though nominally few have to sell their labour under conditions imposed upon them. The emphasis of capitalism being on the supreme importance of material with the intensity of its appeal is to acquisitive intensity. It promotes worship of economic power with little regard to the means employed for its acquisition and the end that it serves. By its exploitation of human beings to the limits of endurance its concentration is on the largest profit rather than maximum production. Thus the division of human family is done on the basis of economic circumstance. All this is injuries to division of human dignity. And when the harrowed poor turn to the founders of religion for succour, they rather offer a subtle defence of the established order. They promise future happiness for their present suffering and conjure up vision of paradise to redress the balance, the religion justifies it.

Q21. The passage indicates that the capitalist system is:

A. Fair
B. Ambitious
C. Prosperous
D. Dehumanizing

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Q22. The established order is supported by religion to:

A. Alleviate the suffering of the poor in the capitalist system.
B. Perpetuate the injustice imposed by the capitalist system.
C. Balance the suffering of the poor with hopes of future rewards.
D. Help the tortured men to seek redress.

Q23. In a capitalist system:

A. The means justify the ends.
B. The ends justify the means.
C. The means endorsed by religion are strictly followed.
D. Means which lead to exploitation are strictly prohibited.

Q24. Capitalism is injurious to human relations because it divides the society into two groups, i.e.

A. Working and non-working.
B. Exploiters and exploited.
C. Religious and irreligious.
D. Buyers and sellers.

Q25. In a capitalist system of society each man wishes:

A. To acquire maximum wealth.
B. To produce maximum wealth.
C. To have visions of paradise.
D. To soothe the sufferings or order.

Q26. Following question has a statement followed by few conclusions. Find which of them is definitely true.

Statement: Most of the policemen beat people to get a confession.

Conclusion:

A. Some of policemen beat people to make them confess.
B. There are no policemen who don't beat to get the confession.
C. There are some policemen who don't beat to get the confession.
D. All policemen beat.

Q27. **Assertion (A):** Whale is the largest animal known to have ever existed.

Reason (R): Whales have a two-chambered heart.

In the context of the two statements **(Assertion (A) and Reason (R))**, mark your answer as:

A. If both A and R are true and R is the correct explanation of A
B. If both A and R are true but R is not the correct explanation of A
C. If A is true but R is false
D. If A is false but R is true

Q28. Physical, aesthetics, signs and symbolic are the four categories of

A. Verbal communication
B. Non verbal communication.
C. Both Verbal communication and Non verbal communication
D. None of the above

Q29. Which one of the following is a barrier to effective listening?

A. knowing the answer
B. trying to be helpful
C. treating the discussion as a competition
D. All of the above

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Q30. Communication is

A. only verbal
B. only non verbal
C. both verbal and non-verbal
D. only gestures

Q31. Find the missing number in the following: 1/2; 3/4, 5/8, ?; 9/32

A. 11/9
B. 7/16
C. 6/16
D. 5/9

Q32. Identify the wrong number in the series: 5, 7, 11, 19, 36, 67, 131

A. 7
B. 36
C. 11
D. 67

Q33. Identify the letter series and find the missing letters: RECTANGLE, ECTANGL, CTANG, TAN, ?

A. B
B. TEN
C. NAT
D. A

Q34. In HUMIDITY is coded UHMIDTY, how will POLITICS be coded?

A. OPILITCS
B. OPLIITCS
C. OPLITISC
D. POILTISC

Q35. In a certain code "253" means "books are old", "946" means "man is old" and "378" means "boy good books". What stand for "are" in that code.

A. 2
B. 4
C. 5
D. 6

Q36. (1) P+Q means P is brother of Q
(2) P*Q means P is father of Q
(3) P-Q means P is sister of Q

Which of the following means S is the niece of T?

A. T*M+S-K
B. K-S*M+T
C. T+M*S-K
D. None of these

Q37. Ashok said, "This girl is the wife of grandson of my Mother". Who is Ashok to the girl?

A. Husband
B. Father
C. Grandfather
D. Father-in-law

Directions (Question 38-42): A and B are a married couple and have two daughters C and D. C is married to E who is the son of F. F is also the mother of G. G has two children H and I.

Q38. How is F related to I?

A. Grand mother
B. Son-in-law
C. Brother
D. Brother-in-law

Q39. How is E related to G?

A. Sister
B. Brother
C. Uncle
D. Grand mother

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Q40. The definite female members in the family are:

A. C, D, F
B. C, D
C. D, F
D. D, F, G

Q41. How is C related to F?

A. Daughter
B. Daughter-in-law
C. Son
D. Son-in-law

Q42. The definite male members in the family are:

A. E, G
B. E
C. G, F
D. E, I, H

Q43. There are five friends, Nidhi, Neha, Deepa, Shilpa and Geeta. They all live in a housing society where houses are numbered 211 through 216, from left to right. There is one empty house. The following conditions apply:

a. Nidhi lives in the house which is to immediate left of Neha.
b. Shilpa does not live in houses numbered 211, 213, or 215.
c. Geeta is the third occupied house from left.

In which of the following houses can Neha not be staying?

A. 212
B. 213
C. 214
D. 215

Q44. Accelerate is most dissimilar to

A. Delay
B. Risk
C. Monitor
D. Deny

Q45. ICSSR stands for

A. Indian Council for Survey and Research
B. Indian Council for strategic Research
C. Indian Council for Social Science Research
D. Inter National Council for Social Science Research

Q46. Soil pollution is caused by

A. Aerosol
B. Ozone
C. Acid rain
D. PAN

Directions (Questions 47-48): In the questions given below choose the pair which is least like the other pairs of words. Choose the odd pairs of words

Q47.

A. Watt : Power
B. Ampere : Current
C. Radian : degree
D. Joule : energy

Q48.

A. Sprinkle : pour
B. Happiness : Merriment
C. Mist : Fog
D. Sad : Unhappy

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Q49. Bank is related to "Money" in the same way as "transport" is related to

A. Goods
B. Road
C. Movement
D. Traffic

Q50. Choose the option which shows common feature in the relationship: **Theta : Phi : Omega**

A. These are Latin alphabets
B. These are signs of Algebra
C. These are Greek letters
D. These are used physical derivation

Q51. Spam is related

A. computer
B. art
C. music
D. game

Q52. Junk e-mail is also called

A. spoof
B. spool
C. sniffer script
D. spam

Q53. Consider the following statements

a. Multi-mode optical fibers are used for long distance communications links.
b. Single mode optical fibers are used for most communication links longer less than 500 meters

Which is correct?

A. Only a
B. Only b
C. Both a and b
D. Neither a nor b

Q54. Which one of the following mobile services in India launched the 3G services for the first time?

A. Idea cellular
B. Bharti airtel
C. reliance
D. BSNL

Q55. CDMA technology is used in mobiles phones for

A. Computer developed management application
B. Code division multiple application
C. Code division multiple access
D. Code division multiple application

Directions (Questions 56-58): Read the following instructions and solve the questions:

Amar, on his death bed, keeps half his property for his wife and divides the rest equality among his three sons Balwant, Chandra and Daljit. Some years later Balwant dies leaving half his property to his widow wife and half to his brothers Chandra and Daljit to be shared equality. When Chandra makes his will he keeps half his property for his widow wife and the rest he bequeaths to his young brother Daljit. When Daljit dies some years later, he keeps half his money for his widow wife and the remaining for his mother. The mother now has Rs. 15,75,000.

Q56. What was the worth of the total property?

A. Rs. 30 lakh
B. Rs. 8 lakh
C. Rs. 18 lakh
D. Rs. 24 lakh

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Q57. What was Chandra's original share?
A. Rs. 4 lakh
B. Rs. 12 lakh
C. Rs. 6 lakh
D. Rs. 5 lakh

Q58. What was the ratio of the property owned by the widows of three sons, in the end?
A. Rs. 8:10:15
B. Rs. 7:9:13
C. Rs. 5:7:9
D. Rs. 7:9:12

Q59. "Reasoning from general to particular" is called
A. Induction
B. Deduction
C. Observation
D. Experience

Q60. UGC Stands for
A. University Grants Commission
B. Union Government Commission
C. University Governance Council
D. Union government Council

Q61. The Academic purpose is to have .....
A. Information
B. firsthand knowledge
C. Knowledge and information
D. models

Q62. Which of the following is an example of primary data?
A. Book
B. Journal
C. News Paper
D. Census Report

Q63. The first page of the research report is
A. Appendix
B. Bibliography
C. Index
D. Title Page

Q64. The term Tsunami is coined from?
A. Chinese term
B. Indian term
C. German term
D. Japanese term

Q65. Tsunamis are waves generated by?
A. Earthquakes
B. Volcanic eruptions
C. Underwater landslides
D. All the above

Q66. National Institute of Disaster management is located at?
A. Pune
B. Midnapur
C. New Delhi
D. Calcutta

Q67. The size of a flood is measured by:
A. the rate of flow of water in a waterway or river
B. the level of water in a waterway or river
C. a river gauging station
D. All of the above.

Q68. Absence of normal pattern of rainfall in a region for a extended period of time is called_____
A. Deforestation
B. landslide
C. flood
D. drought

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Q69. The movement of earthy materials from higher region to lower region due to gravitational pull is called_____.

A. Earth quake
B. soil erosion
C. landslide
D. cyclone.

Q70. _____ is the device which is used to detect landslides.

A. Vibration sensor
B. piezoelectric crystal
C. strain gauge
D. Proximity sensor

Q71. All substances that are major causes of pollution are classified as

A. Compounds
B. Acids
C. Oxides
D. pollutants

Q72. Green Revolution in India was initiated by _____

A. M.S. Swaminathan
B. Sunderlal Bahuguna
C. Kurien
D. Medha Patkar

Q73. The ocean covers _____ percentage of Earth's surface.

A. 51%
B. 61%
C. 71%
D. 91%

Q74. 'Smog' is a mixture of

A. Smoke and Fog
B. Snow and Fog
C. Snow and Dust
D. Sulphur Dioxide and Fog

Q75. Moisture in the air is known as

A. Water
B. Fog
C. Snow
D. Humidity

Q76. Ganga Action Plan in India was launched in the year _____

A. 1988
B. 1985
C. 1980
D. 1978

Q77. The historical monument that is affected by acid rain is

A. Taj Mahal
B. Pyramid of Egypt
C. Pisa Tower
D. Golden Temple

Q78. Natural earthquakes are caused by _____

A. Earth's crust
B. Deep seas
C. Peak mountains
D. Forests

Q79. Infection of HIV is usually detected by which test

A. Elisa test
B. Hybridization
C. Gram staining
D. None

Q80. Modest most nearly means

A. attractive
B. clever
C. current
D. humble

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Q81. Fluid is most opposite to

A. solid
B. liquid
C. afraid
D. decent

Q82. A _____ teacher of English sometimes has to reach beyond the narrow confines of the textbook to _____ a linguist skill

A. competent, discuss
B. skilled, differentiate
C. qualified, inculcate
D. uncertified, teach

Directions (Questions 83-85): The underlined portion in each sentence given below contains the grammatical or syntactical error. Choose the option that corrects the error.

Q83. Archaeologist in Ireland believe that a recently discovered chalice, which dates from the eighth century, was probably buried to keep from being stolen by invaders

A. in order to keep from
B. in order that it would avoid
C. to keep it from
D. to avoid

Q84. Though accepting multiple points of view, the students were unable to either access or justify them

A. were unable either to access or justify them
B. were unable either to assess or justify them
C. were unable in ether assessing or justifying them
D. were unable either in accessing or at justifying them

Q85. If everyone would have contributed, we might have raised enough money to save the old landmark from developers

A. Had everyone contributed
B. If there was a contribution from everyone
C. Despite everyone contributing
D. If everyone contributes

Directions (Questions 86-89): Study the following table to answer these questions:

Plan-Wise Public Sector Outlays and Expenditure (1974-79 to 1985-90) (in Rs. Crores)

Five-year Plans	Period	Approved Outlay	Actual Expenditure
Fifth-Plan	1084-79	39,303	39,426
Sixth-Plan	1980-85	97,500	1,09,292
Seventh-Plan	1985-90	1,80,000	----

Q86. Actual expenditure in the Sixth Plan exceeded the outlay by about?

A. 10%
B. 17%
C. 23%
D. 30%

Q87. Sixth plan outlay exceeded the Fifth Plan outlay by almost

A. 50%
B. 100%
C. 150%
D. 200%

Q88. In the Fifth Plan the expenditure exceeded the outlay by:

A. 1%
B. 2%
C. Less than 0.5%
D. Did not exceed

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Q89. From the given table we can infer that the actual expenditure in the Seventh Plan will exceed the outlay by almost?

A. 40%
B. 60%
C. 80%
D. None of these

Q90. Spelling test - find correct spelling:

A. Acquainttance
B. Acqquaintance
C. Accquaintance
D. Acquaintance

Q91. Identify the letter-number pattern and find the missing letters-numbers in the following:

Q4R, R9S, S15T, T22U, ?

A. V29U
B. V30U
C. U30V
D. U29V

Directions (Questions 92-96): Choose the word or set of words that, when inserted in the sentence bet fits the meaning of the sentence as a whole

Q92. Great personal danger is one of the few things that will force a lion to relinquish its prey; carnivores will _____ a meal only at the point where self-preservation outweighs hunger.

A. Consume
B. Camouflage
C. Ravish
D. Forgo

Q93. The miser was so afraid of losing money that he was willing to pass up a _____ opportunity rather than _____ what he already possessed.

A. Lucrative, risk
B. Hazardous, ponder
C. Shrewd, squander
D. Preponderance, denigrate

Q94. One can argue that the scientific process is a constant, because despite numerous _____ in the instruments used, the step that scientists follow have been _____ for centuries.

A. Losses, converted
B. Refinements, a typical
C. Advances, unaltered
D. Improvements, reliable

Q95. Health experts have found that the use of antibiotics, while beneficial when used with _____, may actually be _____ in the long term.

A. On strain, advantageous
B. Moderation, detrimental
C. Dispatch, excruciating
D. Excess, pernicious

Q96. Some early American pioneers were so _____ to be beholden to others that they refused any favours or offers of help.

A. Fervent
B. Brazen
C. Loathe
D. loath

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Directions (Questions 97-100): Study the number figures carefully and find the answers of the following questions:

O	M	9
P	8	4
7	N	11

Q97. Which digit will replace the alphabet O?
A. 3
B. 5
C. 6
D. 9

Q98. Which number will replace the alphabet P?
A. 11
B. 8
C. 12
D. 14

Q99. Which number will replace the alphabet M?
A. 11
B. 8
C. 14
D. 0

Q100. Which number will replace the alphabet N?
A. 6
B. 14
C. 8
D. 11

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Question Booklet No. 184801

M.Phil/Ph. D. Entrance Examination-2018

THEORITICAL PHYSICS

(Part-II)

ROLL NO. P 5 3 3 9 0 0 3

Signature of Candidate

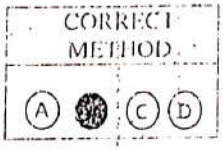
Signature of Invigilator

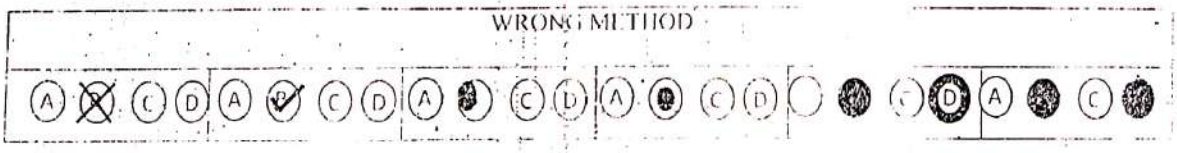
Timings: Two Hours

Total Marks: 100
(Part II(a) – 50 marks)
(Part II (b) – 50 marks)

Instructions to Candidates

1. Do not write your name or put any other mark of identification anywhere in the OMR Answer Sheet. **IF ANY MARK OF IDENTIFICATIONS IS DISCOVERED ANYWHERE IN OMR ANSWER SHEET, the OMR sheet will be cancelled, and will not be evaluated.**
2. This Question Booklet contains this cover page and a total of **50 Multiple Choice Questions of 1 mark each in Part II (a).** Space for rough work has been provided at the end. Available space on each page may also be used for rough work.
3. The candidate may carry this booklet.
4. There is no negative marking in Multiple Choice Questions.
5. USE OF CALCULATOR IS NOT PERMITTED.
6. USE/POSSESSION OF ELECTRONIC GADGETS LIKE MOBILE PHONE. iphone, iPad. pager ETC. is not permitted.
7. Candidate should check the serial order of questions at the beginning of the test. If any question is found missing in the serial order or any other abnormality.it should be immediately brought to the notice of the Invigilator. No page should be torn out from this question booklet.
8. Answers must be marked in the OMR answer sheet which is provided separately. OMR answer sheet must be handed over to the invigilator before you leave the seat.
9. The OMR answer sheet should not be folded or wrinkled. The folded or wrinkled OMR/Answer Sheet will not be evaluated.
10. Write your Roll Number in the appropriate space (above) and on the OMR Answer Sheet. Any other details. if asked for. should be written only in the space provided.
11. Use Black or Blue Ball Pen only for filling the ovals/circles in OMR Answer Sheet while answering the Questions.
12. There are four alternative answers to each question marked A. B. C and D. Select one of the answers you consider most appropriate and fill up the corresponding oval/circle in the OMR Answer Sheet provided to you. For your Choice of answers darken the correct oval/circle completely. If the correct answer is 'B'. the corresponding oval/circle should be completely filled and darkened as shown below.





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Note:- (i) The paper has two parts viz Part A and Part B. (ii) Part A has 50 Multiple Choice Questions (MCQ) of 1 Mark each and ***All questions are compulsory from Part A.*** *(iii) Part B has 20 Questions of 5 Marks each.* ***Attempt any 10 questions from Part B.***

PART-A

1. A particle is moving in X-Y plane under the action of a force such that its linear momentum is given by $p(\vec{t}) = \alpha\left[\vec{i}cos(kt) + \vec{j}sin(kt)\right]$, where α and k are constans. The angle between the force and linear momentum is

   (A) 90 deg.

   (B) 45 deg.

   (C) 0 deg.

   (D) 30 deg.

2. Two solid cylinders A and B of same mass, same radius and same material start rolling down a fixed inclined plane from the same height and at the same time. Cylinder A has most of its mass concentrated near its surface whereas the cylinder B has most its mass concentrated near the axis. Which statement is true?

   (A) Cylinder B reaches the ground with higher angular speed.

   (B) Cylinder A has higher linear acceleration than B.

   (C) Both the cylinder reaches ground at the same time.

   (D) Both the cylinders reach ground with same translational kinetic energy.

3. A ball is thrown across a cricket field and is at height h_1 and h_2 from the point of throw at time t_1 and t_2 respectively from the beginning. The ball is caught by another person at the same height as that of the throw. The total time of flight for the ball is

   (A) $\frac{h_1t_1^2-h_2t_2^2}{h_1t_1-h_2t_2}$.

   (B) $\frac{h_1t_2^2-h_2t_1^2}{h_1t_2-h_2t_1}$.

   (C) $\frac{h_1t_2^2+h-2t_1^2}{h_1t_2+h_2t_1}$.

   (D) $\frac{h_1t_2^2+h-2t_1^2}{h_1t_2+h_2t_1}$.

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4. Two masses m_1 and m_2 are connected in a vertical configuration by a massless spring with spring constant k. The mass m_1 is on the top. Initially m_1 is pushed down by a force F for sometime and then the force is removed. How big F should be so that in the subsequent motion, the mass m_2 is lifted from the ground?

(A) $F = (m_1 + m_2)g$.

(B) $F = (m_1 - m_2)g$.

(C) $F = \frac{m_1 m_2}{m_1 + m_2} g$.

(D) $F = \frac{m_1 m_2}{m_1 - m_2} g$.

5. A particle sits on the surface of a large smooth sphere of radius a. The particle begins to slide along the surface of the sphere. There is negligible frictional force between the surface of the sphere and the particle. Determine the angle θ with the vertical at which the particle will loose contact with the sphere.

(A) $\cos(\theta) = 2/3$.

(B) $\sin(\theta) = 2/3$.

(C) $\cos(\theta) = 1/3$.

(D) $\sin(\theta) = 1/3$.

6. If the total energy of a particle having mass m is equal to two times of its rest energy, the particle's relativistic momentum is

(A) $mc/2$.

(B) $mc/\sqrt{2}$.

(C) mc.

(D) $\sqrt{3}mc$.

$E = mc^2 = 2m_0c^2$ $m = 2m_0$

$\therefore p = mc$

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7. A satellite is circling around the earth in circular orbit with velocity v. What minimal velocity boost is required such that the satellie can leavs the earth completely (move to infinite distance from the earth)?

(A) $\Delta v = \sqrt{2}v$.

(B) $\Delta v = (\sqrt{2} - 2)v$.

(C) $\Delta v = (\sqrt{2} - 1)v$.

(D) $\Delta v = (\sqrt{2} + 1)v$.

$6a - 2x = 4a$

$\frac{a}{2b}$

$4bx^2 = 2a$

$x^2 = \frac{a}{2b}$

$x = \pm\sqrt{\frac{a}{2b}}$

8. In one inertial frame S, two events occur at the same instant in time and 3c minutes apart in space. In another intertial frame S', the same events occur at 5c minutes apart in space. Then what is the time interval between the events in frame S'?

(A) 8 mins.

(B) 2 mins.

(C) 4 mins.

(D) 6 mins.

$V' = -2ax + 4bx^3 = 0$

$V'' = -2a + 12bx^2 < 0$ for $x = 0$

$-2ax + 4bx^3 = 0$

$x = 0,$

$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{2a}{m}}$

$\omega = \sqrt{\frac{a}{2bm}}$

$\omega = \sqrt{\frac{-2a}{m}}$

$\sqrt{\frac{2a}{m}}$

$\therefore x = 2\pi\sqrt{\frac{2a}{m}}$

9. A particle of mass m moves in a one dimensional potential $V(x) = -ax^2 + bx^4$, where a and b are positive constants. The angular frequency of small oscillations about the minima of the potential is

(A) $\pi(a/2b)^{1/2}$.

(B) $\pi(a/m)^{1/2}$.

(C) $(a/mb)^{1/2}$.

(D) $2(a/m)^{1/2}$.

$\omega = \sqrt{\frac{k}{m}}$

$\frac{\partial V}{\partial x} = -2ax + 4bx^3 = 0$

$x(4bx^2 - 2a) = 0$

$\therefore \frac{\partial^2 V}{\partial x^2} = -2a + 12bx^2 < 0$

$x = 0, \quad x^2 = \frac{2a}{4b} = \frac{a}{2b}$

$x = \cancel{\frac{\pm a}{2b}} \pm\sqrt{\frac{a}{2b}}$

at $x = 0$, $-2a = k$

$-2a + 6a$

$\cancel{12 \times b \times \frac{a}{2b}}$

10. The motion of a particle in two dimensions is governed by the lagrangan $L = m\dot{q}_1\dot{q}_2$, where q_1 and q_2 are the two generalised coordinates. The hamilitonian of the system is given by

(A) $m\dot{q}_1\dot{q}_2$.

(B) $2\dot{q}_1\dot{q}_2/m$.

(C) $m(\dot{q}_1{}^2 + \dot{q}_2{}^2)$.

(D) $m(\dot{q}_1{}^2 - \dot{q}_2{}^2)$.

$p_1 = \frac{\partial L}{\partial \dot{q}_1} = m\dot{q}_2, \quad p_2 = m\dot{q}_1$

$\therefore p_1\dot{q}_1 + p_2\dot{q}_2 - L$

$\boxed{m\dot{q}_1\dot{q}_2} + m\dot{q}_1\dot{q}_2 - m\dot{q}_1\dot{q}_2$

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$(0,0,d)$

(x,y)

$V = \frac{1}{4\pi\epsilon_0}\int \frac{dq}{|r-r_0|}$

$V =$

$xi + yj - dx$

11. A disc of radius R and uniform surface charge density σ is placed in the x-y plabe centered at the origin. The electric potential along z-axis for $z \to \infty$ is given by

(A) $\frac{\sigma R^2}{4\epsilon_0 z}$.

(B) $\frac{\sigma R^2}{2\epsilon_0 z}$.

(C) $\frac{2\sigma R^2}{\epsilon_0 z}$.

(D) $\frac{\sigma R}{4\epsilon_0 z^2}$.

$= \frac{1}{4\pi\epsilon_0}\sigma\int\frac{ds}{\sqrt{r^2+d^2}}$

$\frac{\sigma}{4\pi\epsilon_0}\frac{\pi r^2}{r}$

12. A non conducting sphere of radius a carries a charge density $\rho(r) = \frac{k}{r^2}$ where k is a constant. The electric field at a distance r for $r \geq a$ is given by

(A) $\vec{E} = \frac{2ka^5}{5\epsilon_0 r^2}\hat{r}$.

(B) $\vec{E} = \frac{ka^5}{5\epsilon_0 r^2}\hat{r}$.

(C) $\vec{E} = \frac{ka^5}{\epsilon_0 r^2}\hat{r}$.

(D) $\vec{E} = \frac{ka^2}{\epsilon_0 r^2}\hat{r}$.

$E \cdot 4\pi r^2 = \frac{1}{\epsilon_0}\int\frac{k}{r^2}dV$

$E \cdot 4\pi r^2 = \frac{1}{\epsilon_0}\int\frac{k}{r^2}r^2\sin\theta\, d\theta\, d\phi\, dr$

$= \frac{1}{\epsilon_0}4\pi k\left[r\right]_0^a = \frac{4\pi ka}{\epsilon_0}$

$E = \frac{ka}{r^2\epsilon_0}$

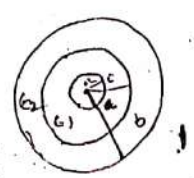
13. A electron is injected at right angles to a steady uniform magnetic field $\vec{B}$ and is undergoing a circular motion. Assuming the motion of the electron is non-relativistic, and its velocity and angular frequency of the motion is v and ω respectively, the power radiated by the electron is given by, (speed of light is c)

(A) $P = \frac{q^2(\omega v)^2}{6\pi\epsilon_0 c^3}$.

(B) $P = \frac{q^2(\omega v)^3}{6\pi\epsilon_0 c^3}$.

(C) $P = \frac{q(\omega v)^2}{6\pi\epsilon_0 c^3}$.

(D) $P = \frac{q^2(\omega v)^2}{6\pi\epsilon_0 c}$.



14. A spherical capacitor consists of two concentric spheres of radius a and b. The capacitor is filled up with dielecric material whose dielectric constants vary as

$$\epsilon(r) = \epsilon_1, \quad for\ a < r < c.$$

$$\epsilon(r) = \epsilon_2, \quad for\ c < r < b.$$

The charge in the inner conducting sphere is Q. What is the density of the bound charge on the boundary between the dielectric layers at $r = c$?

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(A) 0.

(B) $\frac{Q}{4\pi\epsilon_0 c^2}(\frac{1}{\epsilon_1} - \frac{1}{\epsilon_2})$.

(C) $\frac{Q}{4\pi\epsilon_0 c^2}(\frac{1}{\epsilon_1 \epsilon_2})$.

(D) $\frac{Q}{4\pi\epsilon_0 c^2}(\frac{\epsilon_1}{\epsilon_2})$.

15. A thin rod of length L is charged uniformly with charge density λ per unit length. The rod is rotated with angular velocity ω around an axis passing through one of its ends perpendicular to the rod. The magnetic dipole moment of the rod is

(A) $\frac{\lambda\omega L^2}{6}\hat{z}$.

(B) $\frac{\lambda\omega L^3}{2}\hat{z}$.

(C) $\frac{\lambda\omega L^4}{6}\hat{z}$.

(D) $\frac{\lambda\omega L^3}{6}\hat{z}$.

16. The Angle (θ) between two surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at the point $(2, -1, 2)$ is

(A) $\arccos\left(\frac{1}{2\sqrt{21}}\right)$.

(B) $\arccos\left(\frac{4}{3\sqrt{21}}\right)$.

(C) $\arccos\left(\frac{8}{3\sqrt{21}}\right)$.

(D) $\arccos\left(\frac{1}{\sqrt{21}}\right)$.

17. The work done by a force $\vec{F} = 2x^2y\hat{i} + 3xy\hat{j}$, which displace a particle in the xy plane from $(0, 0)$ to $(1, 4)$ along a curve $y = 4x^2$ is

(A) $\frac{104}{5}$.

(B). 10.

(C) $\frac{124}{5}$.

(D) 20.

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18. If λ is a eigenvalue of a matrix A, then the eigenvalue o A^n is

(A) λ^n.

(B) $1/\lambda^n$.

(C) $n\lambda$.

(D) n/λ.

19. If $f(z)$ is analytic and $f'(z)$ is continuous at all points inside and on a simple closed curve C, then

(A) $\oint_C f(z)dz = 0$.

(B) $\oint_C f(z)dz = 1$.

(C) $\oint_C f(z)dz \neq 0$.

(D) $\oint_C f(z)dz \neq 1$.

20. A singular point $z = z_0$ is said to be a ..... singular point of $f(z)$, if there is no other singular point in the neighborhood of z_0

(A) Pole.

(B) Isolated singularity.

(C) Removable singularity.

(D) Essential singularity.

21. If U and W are two subspace of a finite dimensional vector space then

(A) $\dim(U + W) = \dim U + \dim W$.

(B) $\dim(U + W) = \dim U \oplus \dim W$.

(C) $\dim(U + W) = \dim U + \dim W - \dim(U \cap W)$.

(D) $\dim(U + W) = \dim U + \dim W - \dim(U \cup W)$.

$\frac{(3-2i)}{1+4i} \times \frac{(1-4i)}{(1-4i)}$

$\frac{3-12i-2i+8i^2}{1+16}$

$\frac{-5-14i}{17}$

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22. Is an identity map onto

(A) Yes.

(B) No.

(C) Can be both.

(D) Not defined.

23. Let $T : U \to V$ be a linear map, then a one to one map is also onto if

(A) U is finite dimensional.

(B) V is finite dimensional.

(C) both U and V are infinite dimensional.

(D) both U and V are finite dimensional.

24. Let $T : U \to V$ be a linear map. The rank-nullity theorem suggest

(A) $\dim R(T) + \dim N(T) = \dim U$.

(B) $\dim R(T) + \dim N(T) = \dim V$.

(C) $\dim R(T) - \dim N(T) = \dim U$.

(D) $\dim R(T) - \dim N(T) = \dim V$.

25. Express $z = \frac{3-2i}{1+4i}$ in the form of $x + iy$

(A) $-\frac{11}{17} - \frac{10}{17}i$.

(B) $-\frac{11}{17} + \frac{10}{17}i$.

(C) $\frac{11}{17} + \frac{10}{17}i$.

(D) $\frac{10}{17} - \frac{11}{17}i$.

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26. If $X \sim B(n, p)$, that is, X is a binomially distributed random variable, n being the total number of experiments and p the probability of each experiment yielding a successful result, then the mean value of X is

(A) $np(1-p)$.

(B) $np(1+p)$.

(C) np.

(D) np^2.

$\boxed{A^\dagger = I}$

$(\quad)^\dagger = 1$

27. The object $|\phi\rangle\langle\phi|\psi\rangle\langle\psi|$ is a

(A) quantum state.

(B) operator.

(C) number.

(D) unit operator.

$(\quad)^\dagger$

$\langle\phi|\phi\rangle\langle\psi|\psi\rangle = 1$

28. If two operators commute with each other, then their uncertainties

(A) are always zero.

(B) can be made zero simultaneously in some state.

(C) can never be made zero simultaneously.

(D) cannot be defined simultaneously.

29. A state is given by $|\psi\rangle = (\frac{1}{\sqrt{3}}|\alpha\rangle - iC|\beta\rangle)$, where $|\alpha\rangle$ and $|\beta\rangle$ are orthonormal. The value of C for which $|\psi\rangle$ is normalized, is

(A) $1/\sqrt{3}$.

(B) $-1/\sqrt{3}$.

(C) 0.

(D) $2/\sqrt{3}$.

$\langle\psi|\psi\rangle = \frac{1}{3}\left(\frac{1}{\sqrt{3}}\langle\alpha| + iC\langle\beta|\right)\left(\frac{1}{\sqrt{3}}|\alpha\rangle - iC|\beta\rangle\right)$

$\frac{1}{3} + C^2 = 1$

$\Rightarrow C^2 = 1 - \frac{1}{3} = \frac{2}{3}$

$C = \pm$

$\frac{2}{3}$

$\frac{1}{3} + C^2 = 1$

$C^2 = 1 - \frac{1}{3} = \frac{2}{3}$

$+\frac{2}{3} + \frac{1}{3}$

$\boxed{C = \sqrt{\frac{2}{3}}}$

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30. An eigenfunction of the operator $A = \frac{1}{x}\frac{d}{dx}$ is

(A) e^{x}.

(B) e^{-x}.

(C) e^{-x^2}.

(D) e^{x^2}.

$\frac{1}{x}\frac{d}{dx}[e^{x^2}] = 2e^{x^2}$

$\frac{2x}{x}$

$\frac{2x}{x}e^{x^2}$

31. The commutator $[p, [x, p]]$, where x, p represent the position and momentum operator, respectively, is equal to

$[i\hbar]$

(A) 0.

(B) $i\hbar p$

(C) $i\hbar$.

(D) $-i\hbar$.

32. If an operator A is related to its adjoint $A^{\dagger}$ by the relation $A = A^{\dagger}$, the eigenvalues of A are...

(A) always imaginary.

(B) always complex.

(C) always real.

(D) always zero.

33. If, in the Heisenberg picture, an operator at time $t = 0$ is given by $A(0)$, at a later time $t = \tau$ it is given by

$A = U^{\dagger} A U$

(A) $\exp(iH\tau/\hbar)A(0)$.

(B) $\exp(-iH\tau/\hbar)A(0)$.

(C) $\exp(-iH\tau/\hbar)A(0)\exp(iH\tau/\hbar)$.

(D) $\exp(iH\tau/\hbar)A(0)\exp(-iH\tau/\hbar)$.

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34. If, in the Schrodinger picture, a wavefunction at time $t = 0$ is given by $\psi(0)$, at a later time $t = \tau$ it is given by

(A) $\exp(i\mathrm{H}\tau/\hbar)\psi(0)$.

(B) $\exp(-i\mathrm{H}\tau/\hbar)\psi(0)$.

(C) $\exp(-i\mathrm{H}\tau/\hbar)\psi(0)\exp(i\mathrm{H}\tau/\hbar)$.

(D) $\exp(i\mathrm{H}\tau/\hbar)\psi(0)\exp(-i\mathrm{H}\tau/\hbar)$.

35. For a spin-1/2, $|+\rangle$ and $|-\rangle$ are the eigenstates of S_z. If the spin is in the state $\frac{1}{\sqrt{3}}|+\rangle + i\frac{\sqrt{2}}{\sqrt{3}}|-\rangle$, and z-component of the spin is measured, what is the probability of the spin being found in the state $|-\rangle$?

(A) 2/3.

(B) $\sqrt{2/3}$

(C) $1/\sqrt{3}$.

(D) 0.

$|\psi\rangle = \frac{1}{\sqrt{3}}|+\rangle + i\sqrt{\frac{2}{3}}|-\rangle$

$\therefore \langle -|\psi\rangle = \left(i\sqrt{\frac{2}{3}}\right) = \frac{2}{3}$

36. Two states are given by $|\psi\rangle = (|\alpha\rangle + |\beta\rangle)/\sqrt{2}$ and $|\phi\rangle = (|\alpha\rangle - |\beta\rangle)/\sqrt{2}$, where $|\alpha\rangle$ and $|\beta\rangle$ are two orthonormal states. The inner product $\langle\psi|\phi\rangle$ is

(A) 1.

(B) 1/2.

(C) $1/\sqrt{2}$.

(D) 0.

$\left(\frac{\langle\alpha| + \langle\beta|}{\sqrt{2}}\right)\left(\frac{|\alpha\rangle - |\beta\rangle}{\sqrt{2}}\right)$

$= \frac{1}{2}(1 - 1) =$

37. If an operator $\mathrm{X} = \mathrm{ABC}$, what will be its adjoint, $\mathrm{X}^\dagger$?

(A) $\mathrm{A}^\dagger\mathrm{B}^\dagger\mathrm{C}^\dagger$.

(B) C B A.

(C) $\mathrm{B}^\dagger\mathrm{A}^\dagger\mathrm{C}^\dagger$

(D) $\mathrm{C}^\dagger\mathrm{A}^\dagger\mathrm{B}^\dagger$.

$C^\dagger B^\dagger A^\dagger = A^\dagger C^\dagger B^\dagger = B^\dagger A^\dagger C^\dagger$

38. Let $\{|n\rangle\}$ be a complete set of orthonormal states, and let functions $|\psi\rangle$ and $|\phi\rangle$ be defined as

$$|\psi\rangle = \sum_{n=0}^{\infty} e^{-2n}|2n\rangle \quad ; \quad |\phi\rangle = \sum_{n=0}^{\infty} e^{-(2n+1)}|2n+1\rangle$$

The value of $\langle\phi|\psi\rangle$ is

$\langle\phi|\psi\rangle$

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$\sum e^{(2n+1)} \langle 2n+1 | 2n \rangle = 0$

(A) e.

(B) $e^{-(4n+1)}$.

(C) 0.

(D) $1/e$.

39. Let $|a_i\rangle$ be a complete set of orthonormal states. What is the value of

$$\sum_{n=0}^{\infty} \langle a_n | a_j \rangle \langle a_k | a_n \rangle,$$

where $j \neq k$?

$\langle a_1 | a_j \rangle \langle a_k | a_1 \rangle$

(A) 0.

(B) 1.

(C) ∞.

(D) $1/\sqrt{2}$.

40. The canonical partition function of a two level atom with energies 0 and ϵ is given by

$\boxed{1 + e^{-\beta\epsilon}}$

(A) $1 + e^{-\beta\epsilon}$.

(B) $1/(1 + e^{-\beta\epsilon})$.

(C) $\cos(\beta\epsilon)$.

(D) $2\cos(\beta\epsilon)$.

41. Probability of a closed system to be found in a particular macrostate is proportional to

(A) the number of microstates in it.

(B) the energy of the state.

(C) inverse of the energy of the state.

(D) the temperature.

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42. In a grand canonical ensemble, a system A of fixed volume is in contact with a large reservoir B. Then

(A) A can exchange only energy with B.

(B) A can exchange only particles with B.

(C) A can exchange neither energy nor particles with B.

(D) A can exchange both energy and particles with B.

43. If Ω is the number of microstates of a closed system, which of the following is true for entropy?

(A) $S = -k\log(\Omega)$.

(B) $S = k\Omega$.

(C) $S = k\log(\Omega)$.

(D) none of the above.

44. In canonical ensemble, when the number of particles is increased, the relative energy fluctuation, $\sqrt{\langle E^2\rangle - \langle E\rangle^2}/\langle E\rangle$ $\propto \frac{1}{\sqrt{N}}$

(A) becomes larger.

(B) becomes smaller.

(C) remains same.

(D) becomes imaginary.

45. In canonical ensemble, average energy is given by

(A) $\langle E\rangle = \frac{\partial Z}{\partial T}$.

(B) $\langle E\rangle = \frac{\partial Z}{\partial \beta}$.

(C) $\langle E\rangle = \frac{\partial \log(Z)}{\partial T}$.

(D) $\langle E\rangle = -\frac{\partial \log(Z)}{\partial \beta}$.

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46. A factor of $1/N!$ is multiplied to classical partition function

(A) to average over number of particles.

(B) to equally distribute the energy.

(C) to make the expression converge.

(D) to account for indistinguishability of particles.

47. N noninteracting spin-1 particles are sitting on fixed lattice sites. The entropy of this system is given by

$\Omega = (2s+1)^N$
$= (3)^N$

(A) $S = Nk\log(2)$.

(B) $S = 2k\log(N!)$.

(C) $S = 3k\log(N)$.

(D) $S = Nk\log(3)$.

48. If two systems have their number of microstates given by Ω_1 and Ω_2, respectively, the number of microstates of the combined system is given by

(A) $\Omega_1 + \Omega_2$.

(B) $\log(\Omega_1 + \Omega_2)$.

(C) $\sqrt{\Omega_1^2 + \Omega_2^2}$.

(D) $\Omega_1\Omega_2$.

49. In grand-canonical ensemble, the Helmholtz free energy is given by

(A) $F = k\log E$.

(B) $F = -kT\log E$.

(C) $F = -kT\log Z$.

(D) $F = -k\log Z + \mu\langle N\rangle$.

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50. Consider a quantum mechanical rotator with a Hamiltonian $\hat{H} = \frac{\hat{L}^2}{2I}$. Assuming that the rotator can take only two angular momentum values $l = 0$ and $l = 1$, it's canonical partition function is

(A) $1 + 3\exp(-\beta\hbar^2/I)$.

(B) $1 + \exp(-\beta\hbar^2/I)$.

(C) $1 - \exp(-\beta\hbar^2/I)$.

(D) $1 - 3\exp(-\beta\hbar^2/I)$.

$H = \frac{L^2}{2I}$

$\therefore$ ~~$Z = \exp(-\beta(\beta/2I))$~~

$L^2 = \hbar^2 l(l+1)$

$\therefore H = \frac{\hbar^2 l(l+1)}{2I}$

$\therefore Z = \exp(0) + \exp-\left(\beta \frac{\hbar^2}{I}\right)$

$= 1 -$

$(1-\lambda)[(1-\lambda)(3-\lambda) - 9] - 1[3-\lambda + 9]$

$+ 3[-3 - 3(1-\lambda)]$.

$(1-\lambda)(1-\lambda)(3-\lambda) - 9(1-\lambda) - (3-\lambda) - 9$

$-9 - 9(1-\lambda)$

$(1-\lambda)(1-\lambda)(3-\lambda) - 18(1-\lambda) - 18 - (3-\lambda) =$

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PART-B

1. A particle of mass m moves along x-axis such that it has the lagrangian

$$L = \frac{m^2\dot{x}^4}{12} + m\dot{x}^2 V(x) - V^2(x),$$

where V is some differentiable function of x. Find the equation of motion for $x(t)$ and describe the physical nature of the system on the basis of this equation.

2. Write down the Hamilton's equation for the Hamiltonian

$$H(t, q, p) = \frac{p^2}{2m} - mCtq,$$

where C is a constant and m is the mass of the particle. Solve these equations for the initial conditions $q = 0, p = p_0$ at $t = 0$.

3. Given a Hamiltonian $H = \frac{1}{2}p^2 + \frac{1}{2}q^2$, consider the following transformation $Q = p/(2q)$ and $P = -q^2$. Write down the Hamilton's equations for both the original and transformed system. Solve whichever system of Hamilton's equations look nicer to you.

4. Show that the following transformation is canonical:

$$P = p + q^2 + pq^2, \quad Q = \tan^{-1} q$$

5. Consider a cylinder of radius a and length l that is uniformly charged with chage density ρ. The cylinder rotates with a uniform angular velocity ω around the z-axis which is also the center axis of the cylinder. Compute the magnetic induction $vecB$ along the z-axis at $\vec{r} = (0, 0, h)$.

6. The surface of sphere of radius a contains charge with charge density varying as $\sigma = k\cos(\theta)$, where k is a constant. Use the multipole expansion to find the electric field inside the sphere.

7. Find all eigenvalues and only one eigenvector of the matrix A

$$A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 1 & -3 \\ 3 & -3 & 3 \end{bmatrix}$$

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8. Show that when $f(z) = x^3 + i(1-y)^3$, it is legitimate to write

$$f'(z) = u_x + iv_x = 3x^2$$

only when $z = i$.

9. Solve the differential equation $\frac{dy}{dx} + \frac{y}{x} = 2x^3y^4$.

10. Use Cauchy-Residue theorem to evaluate the integral

$$\int_C \frac{5z-2}{z(z-1)}dz.$$

11. If $P_n(x)$ is Legendre's polynomial then prove

$$(2n+1)xP_n(x) = (n+1)P_{n+1}(x) + nP_{n-1}(x). \qquad n = 1, 2, 3, \ldots.$$

Hence find $P_3(x)$.

12. Find the Fourier series of

$$f(x) = \begin{cases} 0 & -\pi < x < 0, \\ h & 0 < x < \pi. \end{cases}$$

13. If $\psi(x) = A\exp(-x^4)$ is the eigenfunction of a 1-D Hamiltonian with eigenvalue $E = 0$, then find the potential $V(x)$ (in units where $\hbar = 2m = 1$).

14. Obtain the probability of finding the particle in infinite square well in the range $0 < x < a/2$ with width a where the potential is given by

$$V(x) = \begin{cases} 0, & \text{for } 0 < x < a, \\ \infty, & \text{otherwise,} \end{cases}$$

15. A quantum harmonic oscillator is in the state $|n>$ and a time independent perturbation $\lambda(a^\dagger a)^2$ acts on the particle where λ is a constant of suitable dimension with a and $a^\dagger$ as lowering and raising operator respectively. Find the first order energy shift.

16. Find the energy of the first excited quantum state of a particle in the 2-D potential

$$V(x, y) = \frac{1}{2}m\omega^2(x^2 + 4y^2).$$

17. A two-level atom has a ground state with energy 0 and degeneracy 1, and an excited state with energy ϵ and degenracy 3. Assuming that the atom is in thermal equilibrium, find the temperature at which the probabilities of the atom to have the energies 0 and ϵ are equal.

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18. Magnetization m of a ferromagnet is given by $m = \tanh(\beta J m)$, where $\beta = 1/kT$ and J is a constant exchange coupling. Show that just below T_c, the magnetization behave like $m \sim (T_c - T)^{1/2}$.

19. Consider a single harmonic oscillator of frequency ω and mass m. Using classical canonical ensemble, calculate the average energy.

20. Consider N particles of spin 1/2, in a magnetic field B, with the Hamiltonian

$$\hat{H} = -\frac{2\mu_B B}{\hbar} \mathbf{S}_z$$

Use quantum canonical partition function to calculate the average energy.

$\mu = \mu_0 M$

$\mu_0 B \, S_z \, \sigma_j^c$

$\frac{2\mu B}{\hbar}$

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