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Programming Languages — Technical Specification for C++ Extensions for Parallelism

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1 General

[parallel.general]

1.1 Scope

[parallel.general.scope]

¹ This Technical Specification describes requirements for implementations of an interface that computer programs written in the C++ programming language may use to invoke algorithms with parallel execution. The algorithms described by this Technical Specification are realizable across a broad class of computer architectures.

- ² This Technical Specification is non-normative. Some of the functionality described by this Technical Specification may be considered for standardization in a future version of C++, but it is not currently part of any C++ standard. Some of the functionality in this Technical Specification may never be standardized, and other functionality may be standardized in a substantially changed form.
- ³ The goal of this Technical Specification is to build widespread existing practice for parallelism in the C++ standard algorithms library. It gives advice on extensions to those vendors who wish to provide them.

1.2 Normative references

[parallel.general.references]

- The following referenced document is indispensable for the application of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.
 - ISO/IEC 14882: $-^1$, Programming Languages C++
- ² ISO/IEC 14882:— is herein called the *C++ Standard*. The library described in ISO/IEC 14882:— clauses 17-30 is herein called the *C++ Standard Library*. The C++ Standard Library components described in ISO/IEC 14882:— clauses 25, 26.7 and 20.7.2 are herein called the *C++ Standard Algorithms Library*.
- Unless otherwise specified, the whole of the C++ Standard's Library introduction (C++14 §17) is included into this Technical Specification by reference.

1.3 Namespaces and headers

[parallel.general.namespaces]

- ¹ Since the extensions described in this Technical Specification are experimental and not part of the C++ Standard Library, they should not be declared directly within namespace std. Unless otherwise specified, all components described in this Technical Specification are declared in namespace std::experimental::parallel::v1.
 - [Note: Once standardized, the components described by this Technical Specification are expected to be promoted to namespace std. $end\ note$]
- ² Unless otherwise specified, references to such entities described in this Technical Specification are assumed to be qualified with std::experimental::parallel::v1, and references to entities described in the C++ Standard Library are assumed to be qualified with std::.
- 3 Extensions that are expected to eventually be added to an existing header <meow> are provided inside the <experimental/meow> header, which shall include the standard contents of <meow> as if by
- 1. To be published. Section references are relative to N3937.

§ 1.3

#include <meow>

1.4 Terms and definitions

[parallel.general.defns]

¹ For the purposes of this document, the terms and definitions given in the C++ Standard and the following apply.

- ² A parallel algorithm is a function template described by this Technical Specification declared in namespace std::experimental::parallel::v1 with a formal template parameter named ExecutionPolicy.
- ³ Parallel algorithms access objects indirectly accessible via their arguments by invoking the following functions:
 - All operations of the categories of the iterators that the algorithm is instantiated with.
 - Functions on those sequence elements that are required by its specification.
 - User-provided function objects to be applied during the execution of the algorithm, if required by the specification.
 - Operations on those function objects required by the specification. [*Note:* See clause 25.1 of *C++ Standard Algorithms Library. end note*]

These functions are herein called *element access functions*.

[Example: The sort function may invoke the following element access functions:

- Methods of the random-access iterator of the actual template argument, as per 24.2.7, as implied by the name of the template parameters RandomAccessIterator.
- The swap function on the elements of the sequence (as per 25.4.1.1 [sort]/2).
- The user-provided Compare function object.

— end example]

1.5 Feature-testing recommendations

[parallel.general.features]

An implementation that provides support for this Technical Specification shall define the feature test macro(s) in Table 1.

Table 1 — Feature Test Macro(s)

Name	Value	Header
cpp_lib_experimental_parallel_algorithm	201505	<pre><experimental algorithm=""> <experimental exception_list=""> <experimental execution_policy=""> <experimental numeric=""></experimental></experimental></experimental></experimental></pre>

§ 1.5 5

2 Execution policies

[parallel.execpol]

2.1 In general

[parallel.execpol.general]

¹ This clause describes classes that are *execution policy* types. An object of an execution policy type indicates the kinds of parallelism allowed in the execution of an algorithm and expresses the consequent requirements on the element access functions.

```
[ Example:
```

```
std::vector<int> v = ...
 // standard sequential sort
 std::sort(v.begin(), v.end());
 using namespace std::experimental::parallel;
 // explicitly sequential sort
 sort(seq, v.begin(), v.end());
 // permitting parallel execution
 sort(par, v.begin(), v.end());
 // permitting vectorization as well
 sort(par vec, v.begin(), v.end());
 // sort with dynamically-selected execution
 size t threshold = ...
 execution policy exec = seq;
 if (v.size() > threshold)
 {
   exec = par;
 }
 sort(exec, v.begin(), v.end());
— end example ]
```

[*Note:* Because different parallel architectures may require idiosyncratic parameters for efficient execution, implementations of the Standard Library may provide additional execution policies to those described in this Technical Specification as extensions. — *end note*]

2.2 Header <experimental/execution_policy> synopsis [parallel.execpol.synopsis]

```
namespace std {
namespace experimental {
namespace parallel {
inline namespace v1 {
    // 2.3, Execution policy type trait
    template<class T> struct is_execution_policy;
    template<class T> constexpr bool is_execution_policy_v = is_execution_policy<T>::value;
```

§ 2.2

```
// 2.4, Sequential execution policy
class sequential_execution_policy;

// 2.5, Parallel execution policy
class parallel_execution_policy;

// 2.6, Parallel+Vector execution policy
class parallel_vector_execution_policy;

// 2.7, Dynamic execution policy
class execution_policy;
}
}
}
```

2.3 Execution policy type trait

[parallel.execpol.type]

template<class T> struct is_execution_policy { see below };

- ¹ is_execution_policy can be used to detect parallel execution policies for the purpose of excluding function signatures from otherwise ambiguous overload resolution participation.
- ² is_execution_policy<T> shall be a UnaryTypeTrait with a BaseCharacteristic of true_type if T is the type of a standard or implementation-defined execution policy, otherwise false_type.
 - [Note: This provision reserves the privilege of creating non-standard execution policies to the library implementation. $end\ note$]
- ³ The behavior of a program that adds specializations for is execution policy is undefined.

2.4 Sequential execution policy

[parallel.execpol.seq]

```
class sequential execution policy{ unspecified };
```

¹ The class sequential_execution_policy is an execution policy type used as a unique type to disambiguate parallel algorithm overloading and require that a parallel algorithm's execution may not be parallelized.

2.5 Parallel execution policy

[parallel.execpol.par]

```
class parallel execution policy{ unspecified };
```

¹ The class parallel_execution_policy is an execution policy type used as a unique type to disambiguate parallel algorithm overloading and indicate that a parallel algorithm's execution may be parallelized.

2.6 Parallel+Vector execution policy

[parallel.execpol.vec]

```
class parallel vector execution policy{ unspecified };
```

¹ The class class parallel_vector_execution_policy is an execution policy type used as a unique type to disambiguate parallel algorithm overloading and indicate that a parallel algorithm's execution may be vectorized and parallelized.

§ 2.6

2.7 Dynamic execution policy

[parallel.execpol.dynamic]

```
class execution policy
      public:
        // 2.7.1, execution policy construct/assign
        template<class T> execution policy(const T& exec);
        template<class T> execution policy& operator=(const T& exec);
        // 2.7.2, execution policy object access
        const type info& type() const noexcept;
        template<class T> T* get() noexcept;
        template<class T> const T* get() const noexcept;
   };
<sup>1</sup> The class execution policy is a container for execution policy objects. execution policy allows
  dynamic control over standard algorithm execution.
  [ Example:
    std::vector<float> sort me = ...
    using namespace std::experimental::parallel;
   execution policy exec = seq;
   if(sort me.size() > threshold)
    {
      exec = std::par;
   std::sort(exec, std::begin(sort me), std::end(sort me));
  — end example ]
 Objects of type execution policy shall be constructible and assignable from objects of type T for
```

which is execution_policy<T>::value is true.

2.7.1 execution policy construct/assign

[parallel.execpol.con]

- 1 template<class T> execution policy(const T& exec);
 - ² Effects: Constructs an execution policy object with a copy of exec's state.
 - 3 Remarks: This constructor shall not participate in overload resolution unless is execution policy<T>::value is true.
- 4 template<class T> execution policy& operator=(const T& exec);
 - ⁵ Effects: Assigns a copy of exec's state to *this.
 - 6 Returns: *this.

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2.7.2 execution_policy object access

[parallel.execpol.access]

```
1 const type info& type() const noexcept;
```

² Returns: typeid(T), such that T is the type of the execution policy object contained by *this.

```
3 template<class T> T* get() noexcept;
template<class T> const T* get() const noexcept;
```

- 4 Returns: If target_type() == typeid(T), a pointer to the stored execution policy object; otherwise a null pointer.
- ⁵ Requires: is execution policy<T>::value is true.

2.8 Execution policy objects

[parallel.execpol.objects]

```
constexpr sequential_execution_policy seq{};
constexpr parallel_execution_policy par{};
constexpr parallel_vector_execution_policy par_vec{};
```

¹ The header <experimental/execution_policy> declares a global object associated with each type of execution policy defined by this Technical Specification.

§ 2.8

3 Parallel exceptions

[parallel.exceptions]

3.1 Exception reporting behavior

[parallel.exceptions.behavior]

¹ During the execution of a standard parallel algorithm, if temporary memory resources are required and none are available, the algorithm throws a std::bad_alloc exception.

- During the execution of a standard parallel algorithm, if the invocation of an element access function exits via an uncaught exception, the behavior of the program is determined by the type of execution policy used to invoke the algorithm:
 - If the execution policy object is of type class parallel_vector_execution_policy, std::terminate shall be called.
 - If the execution policy object is of type sequential_execution_policy or parallel_execution_policy, the execution of the algorithm exits via an exception. The exception shall be an exception_list containing all uncaught exceptions thrown during the invocations of element access functions, or optionally the uncaught exception if there was only one.

[Note: For example, when for_each is executed sequentially, if an invocation of the user-provided function object throws an exception, for_each can exit via the uncaught exception, or throw an exception_list containing the original exception. — end note]

[Note: These guarantees imply that, unless the algorithm has failed to allocate memory and exits via std::bad_alloc, all exceptions thrown during the execution of the algorithm are communicated to the caller. It is unspecified whether an algorithm implementation will "forge ahead" after encountering and capturing a user exception. — end note]

[Note: The algorithm may exit via the std::bad_alloc exception even if one or more user-provided function objects have exited via an exception. For example, this can happen when an algorithm fails to allocate memory while creating or adding elements to the exception_list object. — end note]

 If the execution policy object is of any other type, the behavior is implementationdefined.

3

3.2 Header <experimental/exception list> synopsis

[parallel.exceptions.synopsis]

```
namespace std {
namespace experimental {
namespace parallel {
inline namespace v1 {

  class exception_list : public exception {
    public:
       typedef unspecified iterator;

    size_t size() const noexcept;
    iterator begin() const noexcept;
    iterator end() const noexcept;
```

§ 3.2

```
const char* what() const noexcept override;
};
}
}
}
```

¹ The class exception_list owns a sequence of exception_ptr objects. The parallel algorithms may use the exception_list to communicate uncaught exceptions encountered during parallel execution to the caller of the algorithm.

- ² The type exception_list::iterator shall fulfill the requirements of ForwardIterator.
- 3 size_t size() const noexcept;
 - ⁴ Returns: The number of exception_ptr objects contained within the exception_list.
 - ⁵ Complexity: Constant time.
- 6 iterator begin() const noexcept;
 - ⁷ Returns: An iterator referring to the first exception_ptr object contained within the exception_list.
- 8 iterator end() const noexcept;
 - ⁹ *Returns:* An iterator that is past the end of the owned sequence.
- 10 const char* what() const noexcept override;
 - 11 Returns: An implementation-defined NTBS.

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4 Parallel algorithms

[parallel.alg]

4.1 In general

[parallel.alg.general]

This clause describes components that C++ programs may use to perform operations on containers and other sequences in parallel.

4.1.1 Requirements on user-provided function objects

[parallel.alg.general.user]

¹ Function objects passed into parallel algorithms as objects of type BinaryPredicate, Compare, and BinaryOperation shall not directly or indirectly modify objects via their arguments.

4.1.2 Effect of execution policies on algorithm execution [parallel.alg.general.exec]

- Parallel algorithms have template parameters named ExecutionPolicy which describe the manner in which the execution of these algorithms may be parallelized and the manner in which they apply the element access functions.
- ² The invocations of element access functions in parallel algorithms invoked with an execution policy object of type sequential_execution_policy execute in sequential order in the calling thread.
- The invocations of element access functions in parallel algorithms invoked with an execution policy object of type parallel_execution_policy are permitted to execute in an unordered fashion in either the invoking thread or in a thread implicitly created by the library to support parallel algorithm execution. Any such invocations executing in the same thread are indeterminately sequenced with respect to each other. [*Note:* It is the caller's responsibility to ensure correctness, for example that the invocation does not introduce data races or deadlocks. end note]

```
[ Example:
```

```
using namespace std::experimental::parallel;
int a[] = {0,1};
std::vector<int> v;
for_each(par, std::begin(a), std::end(a), [&](int i) {
  v.push_back(i*2+1);
});
```

The program above has a data race because of the unsynchronized access to the container v. — $end\ example$]

[Example:

```
using namespace std::experimental::parallel;
std::atomic<int> x = 0;
int a[] = {1,2};
for_each(par, std::begin(a), std::end(a), [&](int n) {
   x.fetch_add(1, std::memory_order_relaxed);
   // spin wait for another iteration to change the value of x
   while (x.load(std::memory_order_relaxed) == 1) { }
});
```

The above example depends on the order of execution of the iterations, and is therefore undefined (may deadlock). - end example]

§ 4.1.2

[Example:

```
using namespace std::experimental::parallel;
int x=0;
std::mutex m;
int a[] = {1,2};
for_each(par, std::begin(a), std::end(a), [&](int) {
    m.lock();
    ++x;
    m.unlock();
});
```

The above example synchronizes access to object x ensuring that it is incremented correctly. — $end\ example$]

⁴ The invocations of element access functions in parallel algorithms invoked with an execution policy of type parallel_vector_execution_policy are permitted to execute in an unordered fashion in unspecified threads, and unsequenced with respect to one another within each thread. [*Note:* This means that multiple function object invocations may be interleaved on a single thread. — end note]

[*Note:* This overrides the usual guarantee from the C++ standard, Section 1.9 [intro.execution] that function executions do not interleave with one another. — *end note*]

Since parallel_vector_execution_policy allows the execution of element access functions to be interleaved on a single thread, synchronization, including the use of mutexes, risks deadlock. Thus the synchronization with parallel vector execution policy is restricted as follows:

A standard library function is *vectorization-unsafe* if it is specified to synchronize with another function invocation, or another function invocation is specified to synchronize with it, and if it is not a memory allocation or deallocation function. Vectorization-unsafe standard library functions may not be invoked by user code called from parallel_vector_execution_policy algorithms.

[*Note:* Implementations must ensure that internal synchronization inside standard library routines does not induce deadlock. — *end note*]

` [Example:

```
using namespace std::experimental::parallel;
int x=0;
std::mutex m;
int a[] = {1,2};
for_each(par_vec, std::begin(a), std::end(a), [&](int) {
    m.lock();
    ++x;
    m.unlock();
});
```

The above program is invalid because the applications of the function object are not guaranteed to run on different threads. — $end\ example$]

[Note: The application of the function object may result in two consecutive calls to m.lock on the same thread, which may deadlock. — end note]

[Note: The semantics of the parallel_execution_policy or the parallel_vector_execution_policy invocation allow the implementation to fall back to sequential execution if the system cannot parallelize an algorithm invocation due to lack of resources. — end note]

§ 4.1.2

⁶ Algorithms invoked with an execution policy object of type execution_policy execute internally as if invoked with the contained execution policy object.

⁷ The semantics of parallel algorithms invoked with an execution policy object of implementation-defined type are implementation-defined.

4.1.3 ExecutionPolicy algorithm overloads

[parallel.alg.overloads]

- ¹ The Parallel Algorithms Library provides overloads for each of the algorithms named in Table 1, corresponding to the algorithms with the same name in the C++ Standard Algorithms Library. For each algorithm in Table 2, if there are overloads for corresponding algorithms with the same name in the C++ Standard Algorithms Library, the overloads shall have an additional template type parameter named ExecutionPolicy, which shall be the first template parameter. In addition, each such overload shall have the new function parameter as the first function parameter of type ExecutionPolicy&&.
- ² Unless otherwise specified, the semantics of ExecutionPolicy algorithm overloads are identical to their overloads without.
- Parallel algorithms shall not participate in overload resolution unless is execution policy<decay t<ExecutionPolicy>>::value is true.

Table 2 — Table of parallel algorithms

Table 2 — Table of parallel algorithms								
adjacent_difference	adjacent_find	all_of	any_of					
сору	copy_if	copy_n	count					
count_if	equal	exclusive_scan	fill					
fill_n	find	find_end	find_first_of					
find_if	find_if_not	for_each	for_each_n					
generate	generate_n	includes	inclusive_scan					
inner_product	inplace_merge	is_heap	is_heap_until					
is_partitioned	is_sorted	is_sorted_until	lexicographical_compare					
max_element	merge	min_element	minmax_element					
mismatch	move	none_of	nth_element					
partial_sort	partial_sort_copy	partition	partition_copy					
reduce	remove	remove_copy	remove_copy_if					
remove_if	replace	replace_copy	replace_copy_if					
replace_if	reverse	reverse_copy	rotate					
rotate_copy	search	search_n	set_difference					
set_intersection	set_symmetric_difference	set_union	sort					
stable_partition	stable_sort	swap_ranges	transform					
transform_exclusive_scan	transform_inclusive_scan	transform_reduce	uninitialized_copy					
uninitialized_copy_n	uninitialized_fill	uninitialized_fill_n	unique					
unique_copy								

[Note: Not all algorithms in the Standard Library have counterparts in Table 2. — end note]

4.2 Definitions

[parallel.alg.defns]

```
Define GENERALIZED_SUM(op, a1, ..., aN) as follows:
```

- a1 when N is 1
- op(GENERALIZED_SUM(op, b1, ..., bK), GENERALIZED_SUM(op, bM, ..., bN)) where
 - b1, ..., bN may be any permutation of a1, ..., aN and
 - $-- 1 < K+1 = M \le N.$

§ 4.2

```
Define GENERALIZED_NONCOMMUTATIVE_SUM(op, a1, ..., aN) as follows:

— a1 when N is 1
— op(GENERALIZED_NONCOMMUTATIVE_SUM(op, a1, ..., aK), GENERALIZED_NONCOMMUTATIVE_SUM(op, aM, ..., aN) where 1 < K+1 = M ≤ N.
```

4.3 Non-Numeric Parallel Algorithms

[parallel.alg.ops]

4.3.1 Header <experimental/algorithm> synopsis

[parallel.alg.ops.synopsis]

```
namespace std {
namespace experimental {
namespace parallel {
inline namespace v1 {
  template<class ExecutionPolicy,
           class InputIterator, class Function>
    void for each(ExecutionPolicy&& exec,
                  InputIterator first, InputIterator last,
                  Function f):
  template<class InputIterator, class Size, class Function>
    InputIterator for each n(InputIterator first, Size n,
                             Function f);
  template<class ExecutionPolicy,
           class InputIterator, class Size, class Function>
    InputIterator for each n(ExecutionPolicy&& exec,
                             InputIterator first, Size n,
                             Function f);
```

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4.3.2 For each

[parallel.alg.foreach]

- ² Effects: Applies f to the result of dereferencing every iterator in the range [first,last). [Note: If the type of first satisfies the requirements of a mutable iterator, f may apply nonconstant functions through the dereferenced iterator. end note]
- ³ Complexity: Applies f exactly last first times.
- ⁴ *Remarks:* If f returns a result, the result is ignored.
- ⁵ *Notes:* Unlike its sequential form, the parallel overload of for_each does not return a copy of its Function parameter, since parallelization may not permit efficient state accumulation.
- ⁶ Requires: Unlike its sequential form, the parallel overload of for_each requires Function to meet the requirements of CopyConstructible.

- ⁸ Requires: Function shall meet the requirements of MoveConstructible [Note: Function need not meet the requirements of CopyConstructible. end note]
- ⁹ Effects: Applies f to the result of dereferencing every iterator in the range [first,first + n), starting from first and proceeding to first + n 1. [Note: If the type of first satisfies the requirements of a mutable iterator, f may apply nonconstant functions through the dereferenced iterator. end note]
- 10 Returns: first + n for non-negative values of n and first for negative values.
- 11 *Remarks:* If f returns a result, the result is ignored.

- 13 Effects: Applies f to the result of dereferencing every iterator in the range [first,first + n), starting from first and proceeding to first + n 1. [Note: If the type of first satisfies the requirements of a mutable iterator, f may apply nonconstant functions through the dereferenced iterator. end note]
- 14 Returns: first + n for non-negative values of n and first for negative values.
- 15 Remarks: If f returns a result, the result is ignored.
- Notes: Unlike its sequential form, the parallel overload of for_each_n requires Function to meet the requirements of CopyConstructible.

§ 4.3.2

4.4 Numeric Parallel Algorithms

[parallel.alg.numeric]

4.4.1 Header <experimental/numeric> synopsis

[parallel.alg.numeric.synopsis]

```
namespace std {
namespace experimental {
namespace parallel {
inline namespace v1 {
  template<class InputIterator>
    typename iterator traits<InputIterator>::value type
      reduce(InputIterator first, InputIterator last);
  template<class ExecutionPolicy,
           class InputIterator>
   typename iterator traits<InputIterator>::value type
      reduce(ExecutionPolicy&& exec,
             InputIterator first, InputIterator last);
  template<class InputIterator, class T>
   T reduce(InputIterator first, InputIterator last, T init);
  template<class ExecutionPolicy,
           class InputIterator, class T>
   T reduce(ExecutionPolicy&& exec,
             InputIterator first, InputIterator last, T init);
  template<class InputIterator, class T, class BinaryOperation>
   T reduce(InputIterator first, InputIterator last, T init,
             BinaryOperation binary op);
  template<class ExecutionPolicy, class InputIterator, class T, class BinaryOperation>
   T reduce(ExecutionPolicy&& exec,
             InputIterator first, InputIterator last, T init,
             BinaryOperation binary op);
  template<class InputIterator, class OutputIterator,
           class T>
   OutputIterator
      exclusive scan(InputIterator first, InputIterator last,
                     OutputIterator result,
                     T init):
  template<class ExecutionPolicy,
           class InputIterator, class OutputIterator,
           class T>
   OutputIterator
      exclusive scan(ExecutionPolicy&& exec,
                     InputIterator first, InputIterator last,
                     OutputIterator result,
                     T init);
  template<class InputIterator, class OutputIterator,
           class T, class BinaryOperation>
   OutputIterator
      exclusive scan(InputIterator first, InputIterator last,
                     OutputIterator result,
                     T init, BinaryOperation binary op);
  template<class ExecutionPolicy,
           class InputIterator, class OutputIterator,
```

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```
class T, class BinaryOperation>
  OutputIterator
    exclusive scan(ExecutionPolicy&& exec,
                   InputIterator first, InputIterator last,
                   OutputIterator result,
                   T init, BinaryOperation binary op);
template<class InputIterator, class OutputIterator>
  OutputIterator
    inclusive scan(InputIterator first, InputIterator last,
                   OutputIterator result);
template<class ExecutionPolicy,
         class InputIterator, class OutputIterator>
  OutputIterator
    inclusive scan(ExecutionPolicy&& exec,
                   InputIterator first, InputIterator last,
                   OutputIterator result);
template<class InputIterator, class OutputIterator,
         class BinaryOperation>
 OutputIterator
    inclusive scan(InputIterator first, InputIterator last,
                   OutputIterator result,
                   BinaryOperation binary op);
template<class ExecutionPolicy,
         class InputIterator, class OutputIterator,
         class BinaryOperation>
 OutputIterator
    inclusive scan(ExecutionPolicy&& exec,
                   InputIterator first, InputIterator last,
                   OutputIterator result,
                   BinaryOperation binary op);
template<class InputIterator, class OutputIterator,
         class BinaryOperation, class T>
 OutputIterator
    inclusive scan(InputIterator first, InputIterator last,
                   OutputIterator result,
                   BinaryOperation binary op, T init);
template<class ExecutionPolicy,
         class InputIterator, class OutputIterator,
         class BinaryOperation, class T>
 OutputIterator
    inclusive scan(ExecutionPolicy&& exec,
                   InputIterator first, InputIterator last,
                   OutputIterator result,
                   BinaryOperation binary op, T init);
template<class InputIterator, class UnaryOperation,
         class T, class BinaryOperation>
 T transform reduce(InputIterator first, InputIterator last,
                     UnaryOperation unary op,
                     T init, BinaryOperation binary op);
template<class ExecutionPolicy,</pre>
```

§ 4.4.1

```
class InputIterator, class UnaryOperation,
         class T, class BinaryOperation>
 T transform reduce(ExecutionPolicy&& exec,
                     InputIterator first, InputIterator last,
                     UnaryOperation unary op,
                     T init, BinaryOperation binary op);
template<class InputIterator, class OutputIterator,
         class UnaryOperation, class T, class BinaryOperation>
  OutputIterator
    transform exclusive scan(InputIterator first, InputIterator last,
                             OutputIterator result,
                             UnaryOperation unary op,
                             T init, BinaryOperation binary op);
template<class ExecutionPolicy,
         class InputIterator, class OutputIterator,
         class UnaryOperation, class T, class BinaryOperation>
  OutputIterator
    transform_exclusive_scan(ExecutionPolicy&& exec,
                             InputIterator first, InputIterator last,
                             OutputIterator result,
                             UnaryOperation unary op,
                             T init, BinaryOperation binary op);
template<class InputIterator, class OutputIterator,</pre>
         class UnaryOperation, class BinaryOperation>
 OutputIterator
    transform inclusive scan(InputIterator first, InputIterator last,
                             OutputIterator result,
                             UnaryOperation unary op,
                             BinaryOperation binary op);
template<class ExecutionPolicy,
         class InputIterator, class OutputIterator,
         class UnaryOperation, class BinaryOperation>
 OutputIterator
    transform inclusive scan(ExecutionPolicy&& exec,
                             InputIterator first, InputIterator last,
                             OutputIterator result,
                             UnaryOperation unary op,
                             BinaryOperation binary op);
template<class InputIterator, class OutputIterator,</pre>
         class UnaryOperation, class BinaryOperation, class T>
 OutputIterator
    transform inclusive scan(InputIterator first, InputIterator last,
                             OutputIterator result,
                             UnaryOperation unary op,
                             BinaryOperation binary op, T init);
template<class ExecutionPolicy,
         class InputIterator, class OutputIterator,
         class UnaryOperation, class BinaryOperation, class T>
  OutputIterator
```

§ 4.4.1

```
transform inclusive scan(ExecutionPolicy&& exec,
                                         InputIterator first, InputIterator last,
                                         OutputIterator result,
                                         UnaryOperation unary op,
                                         BinaryOperation binary op, T init);
  4.4.2 Reduce
                                                                             [parallel.alg.reduce]
1 template<class InputIterator>
     typename iterator traits<InputIterator>::value type
         reduce(InputIterator first, InputIterator last);
    2 Effects: Same as reduce(first, last, typename iterator_traits<InputIterator>::value_type{}).
3 template<class InputIterator, class T>
     T reduce(InputIterator first, InputIterator last, T init);
    4 Effects: Same as reduce(first, last, init, plus<>()).
5 template<class InputIterator, class T, class BinaryOperation>
     T reduce(InputIterator first, InputIterator last, T init,
              BinaryOperation binary op);
    6 Returns: GENERALIZED_SUM(binary_op, init, *first, ..., *(first + (last - first) - 1)).
    <sup>7</sup> Requires: binary op shall not invalidate iterators or subranges, nor modify elements in the
      range [first, last).
    8 Complexity: O(last - first) applications of binary op.
    9 Notes: The primary difference between reduce and accumulate is that the behavior of reduce
      may be non-deterministic for non-associative or non-commutative binary op.
  4.4.3 Exclusive scan
                                                                     [parallel.alg.exclusive.scan]
1 template<class InputIterator, class OutputIterator, class T>
     OutputIterator exclusive_scan(InputIterator first, InputIterator last,
                                  OutputIterator result,
                                  T init):
    2 Effects: Same as exclusive_scan(first, last, result, init, plus<>()).
```

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- 4 Effects: Assigns through each iterator i in [result, result + (last first)) the value of GENERALIZED_NONCOMMUTATIVE_SUM(binary_op, init, *first, ..., *(first + (i - result) - 1)).
- ⁵ Returns: The end of the resulting range beginning at result.
- ⁶ Requires: binary_op shall not invalidate iterators or subranges, nor modify elements in the ranges [first,last) or [result,result + (last first)).
- ⁷ Complexity: O(last first) applications of binary op.
- ⁸ Notes: The difference between exclusive_scan and inclusive_scan is that exclusive_scan excludes the ith input element from the ith sum. If binary_op is not mathematically associative, the behavior of exclusive scan may be non-deterministic.

4.4.4 Inclusive scan

[parallel.alg.inclusive.scan]

- - 2 Effects: Same as inclusive_scan(first, last, result, plus<>()).

- 4 Effects: Assigns through each iterator i in [result, result + (last first)) the value of GENERALIZED_NONCOMMUTATIVE_SUM(binary_op, *first, ..., *(first + (i - result))) or GENERALIZED_NONCOMMUTATIVE_SUM(binary_op, init, *first, ..., *(first + (i - result))) if init is provided.
- ⁵ Returns: The end of the resulting range beginning at result.
- ⁶ Requires: binary_op shall not invalidate iterators or subranges, nor modify elements in the ranges [first,last) or [result,result + (last first)).
- ⁷ Complexity: O(last first) applications of binary_op.
- ⁸ Notes: The difference between exclusive_scan and inclusive_scan is that inclusive_scan includes the ith input element in the ith sum. If binary_op is not mathematically associative, the behavior of inclusive_scan may be non-deterministic.

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4.4.5 Transform reduce

[parallel.alg.transform.reduce]

Property 2 Returns:
 GENERALIZED_SUM(binary_op, init, unary_op(*first), ..., unary_op(*(first + (last - first) - 1))).

- Requires: Neither unary_op nor binary_op shall invalidate subranges, or modify elements in the range [first,last)
- 4 Complexity: O(last first) applications each of unary_op and binary_op.
- ⁵ Notes: transform reduce does not apply unary op to init.

4.4.6 Transform exclusive scan

[parallel.alg.transform.exclusive.scan]

- 2 Effects: Assigns through each iterator i in [result, result + (last first)) the value of GENERALIZED_NONCOMMUTATIVE_SUM(binary_op, init, unary_op(*first), ..., unary_op(*(first + (i - result) - 1))).
- ³ Returns: The end of the resulting range beginning at result.
- ⁴ Requires: Neither unary_op nor binary_op shall invalidate iterators or subranges, or modify elements in the ranges [first,last) or [result,result + (last first)).
- ⁵ Complexity: O(last first) applications each of unary op and binary op.
- ⁶ Notes: The difference between transform_exclusive_scan and transform_inclusive_scan is that transform_exclusive_scan excludes the ith input element from the ith sum. If binary_op is not mathematically associative, the behavior of transform_exclusive_scan may be non-deterministic. transform_exclusive_scan does not apply unary_op to init.

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4.4.7 Transform inclusive scan

[parallel.alg.transform.inclusive.scan]

```
1 template<class InputIterator, class OutputIterator,</pre>
            class UnaryOperation,
            class BinaryOperation>
     OutputIterator transform inclusive scan(InputIterator first, InputIterator last,
                                               OutputIterator result,
                                               UnaryOperation unary op,
                                               BinaryOperation binary op);
  template<class InputIterator, class OutputIterator,
            class UnaryOperation,
            class BinaryOperation, class T>
     OutputIterator transform_inclusive_scan(InputIterator first, InputIterator last,
                                               OutputIterator result,
                                               UnaryOperation unary op,
                                               BinaryOperation binary op, T init);
    <sup>2</sup> Effects: Assigns through each iterator i in [result, result + (last - first)) the value of
      GENERALIZED NONCOMMUTATIVE_SUM(binary_op, unary_op(*first), ..., unary_op(*first + (i -
      result)))) or
      GENERALIZED NONCOMMUTATIVE SUM(binary op, init, unary_op(*first), ..., unary_op(*(first + (i
       - result)))) if init is provided.
```

- ³ Returns: The end of the resulting range beginning at result.
- 4 Requires: Neither unary_op nor binary_op shall invalidate iterators or subranges, or modify elements in the ranges [first,last) or [result,result + (last - first)).
- ⁵ Complexity: O(last first) applications each of unary_op and binary_op.
- ⁶ Notes: The difference between transform_exclusive_scan and transform_inclusive_scan is that transform_inclusive_scan includes the ith input element from the ith sum. If binary_op is not mathematically associative, the behavior of transform_inclusive_scan may be non-deterministic. transform inclusive scan does not apply unary op to init.

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