1 Defnition

Cartesian2D	$type = $ Cartesian2D
	structure
	rotation
	size
	zoom
	sizes
	method

2 Method

1. construct

CO | *structure*

$$\begin{bmatrix} this \blacktriangleright structure \\ structure \end{bmatrix}$$

$$\begin{bmatrix} this \blacktriangleright rotation \\ this \blacktriangleright structure[\text{rotation}] \end{bmatrix}$$

$$\begin{bmatrix} this \blacktriangleright size \\ this \blacktriangleright structure[\text{size}] \end{bmatrix}$$

$$\begin{bmatrix} this \blacktriangleright zoom \\ this \blacktriangleright structure[\text{zoom}] \end{bmatrix}$$

$$\begin{bmatrix} types\_object \\ \begin{bmatrix} \text{Angle2D} \mid \text{Curve2D} \mid \text{Face2D} \mid \text{Label2D} \mid \text{Point2D} \mid \text{Segment2D} \end{bmatrix} \end{bmatrix}$$

$$\begin{bmatrix} this \blacktriangleright objects \\ \textbf{index}(types\_object) \end{bmatrix}$$

$$\begin{bmatrix} this \blacktriangleright sizes \\ \textbf{index}(types\_object) \end{bmatrix}$$

sublevel

sublevel

$$\begin{bmatrix} types\_object\_nonLabel2D \\ \begin{bmatrix} \text{Angle2D} \mid \text{Curve2D} \mid \text{Face2D} \mid \text{Point2D} \mid \text{Segment2D} \end{bmatrix} \end{bmatrix}$$

sublevel

FO 1 | $types\_object$ | $type\_object$

$$this \blacktriangleright \begin{bmatrix} objects \\ type\_object \end{bmatrix} = \emptyset$$

FO 2 | $this \blacktriangleright structure[\text{elements}]$ | key | $object\_structure$

$$\begin{bmatrix} object \\ this \blacktriangleright \textbf{object}(object\_structure) \end{bmatrix}$$

$$object \blacktriangleright key\_object = \left| this \blacktriangleright \begin{bmatrix} objects \\ (object \blacktriangleright type) \end{bmatrix} \right|$$

$$\begin{bmatrix} object \blacktriangleright key \\ key \end{bmatrix}$$

$$this \blacktriangleright \begin{bmatrix} objects \\ (object \blacktriangleright type) \end{bmatrix} \overset{\pm}{=} object$$

3

FO | $types\_object\_nonLabel2D$ | $type$

  FO | $0 \leq i <$ | $this \blacktriangleright \begin{bmatrix} objects \\ type \end{bmatrix}$

    FO | $0 \leq j <$ | $this \blacktriangleright \begin{bmatrix} objects \\ Label2D \end{bmatrix}$

      IF | $\equiv$ | $this \blacktriangleright \begin{bmatrix} objects \\ types[i] \blacktriangleright key \end{bmatrix}$
$this \blacktriangleright \begin{bmatrix} objects \\ Label2D[j] \\ key\_labeled \end{bmatrix}$

$this \blacktriangleright \begin{bmatrix} objects \\ types[i] \\ key\_label \end{bmatrix} = this \blacktriangleright \begin{bmatrix} objects \\ Label2D[j] \\ key \end{bmatrix}$

$this \blacktriangleright \begin{bmatrix} objects \\ Label2D[j] \\ key\_labeled\_object \end{bmatrix} = this \blacktriangleright \begin{bmatrix} objects \\ type[i] \\ object \end{bmatrix}$

$this \blacktriangleright \begin{bmatrix} objects \\ Label2D[j] \\ type\_labeled \end{bmatrix} = type$

BR

2. | algorithm | apply algorithm

algorithm | $version$

  $\equiv$ | $\begin{bmatrix} index\_height \\ \begin{bmatrix} Angle2D \mid Measure2D \mid MeasureArc2D \end{bmatrix} \end{bmatrix}$

$heights = \mathbf{index}(index\_height)$

  SW | $version$

    KR | $\equiv$ | $\begin{bmatrix} heights \blacktriangleright Angle2D \\ 1 \end{bmatrix}$
$\equiv$ | $\begin{bmatrix} heights \blacktriangleright Measuer2D \\ 1 \end{bmatrix}$
$\equiv$ | $\begin{bmatrix} heights \blacktriangleright MeasureArc2D \\ 1 \end{bmatrix}$

    US | $\equiv$ | $\begin{bmatrix} heights \blacktriangleright Angle2D \\ 0 \end{bmatrix}$
$\equiv$ | $\begin{bmatrix} heights \blacktriangleright Measuer2D \\ 0 \end{bmatrix}$
$\equiv$ | $\begin{bmatrix} heights \blacktriangleright MeasureArc2D \\ 1 \end{bmatrix}$

$this\_updated = this$

$this\_updated = this\_updated \blacktriangleright \text{update}\left(\begin{array}{l} Angle2D \\ heights \blacktriangleright Angle2D \end{array} \right)$

$this\_updated = this\_updated \blacktriangleright \text{update}\left(\begin{array}{l} Arc2D \\ heights \blacktriangleright MeasureArc2D \end{array} \right)$

$\equiv$ | $\begin{bmatrix} this\_updated \\ this\_updated \blacktriangleright \text{update}(Face2D) \end{bmatrix}$

$this\_updated = this\_updated \blacktriangleright \text{update}\left(\begin{array}{l} Segment2D \\ heights \blacktriangleright Measure2D \end{array} \right)$

$\equiv$ | $\begin{bmatrix} this\_updated \\ this\_updated \blacktriangleright \text{update}(Point2D) \end{bmatrix}$

$\equiv$ | $\begin{bmatrix} this\_updated \\ this\_updated \blacktriangleright \text{update}(structure) \end{bmatrix}$

RE | $this\_updated$

3. | object | convert $object\_structure$ to object

object	object_structure		
	SW	object_structure[type]	
		angle-free	sublevel
		arc-free	sublevel
		curve-free	sublevel
		face-free	sublevel
		label-free	sublevel
		point-free	sublevel
		region-free	sublevel
		segment-free	sublevel

1	angle-free	$=$ $\begin{bmatrix} angle\_structure \\ object\_structure \end{bmatrix}$	
		$Angle2D = \mathbf{Angle2D}$	
		$Angle2D \blacktriangleright center = \mathbf{xy}$	$\left(angle\_structure \begin{bmatrix} struct \\ coords \\ center \\ x \end{bmatrix} \; angle\_structure \begin{bmatrix} struct \\ coords \\ center \\ y \end{bmatrix} \right)$
		$Angle2D \blacktriangleright start = \mathbf{xy}$	$\left(angle\_structure \begin{bmatrix} struct \\ coords \\ start \\ x \end{bmatrix} \; angle\_structure \begin{bmatrix} struct \\ coords \\ start \\ y \end{bmatrix} \right)$
		$Angle2D \blacktriangleright end = \mathbf{xy}$	$\left(angle\_structure \begin{bmatrix} struct \\ coords \\ end \\ x \end{bmatrix} \; angle\_structure \begin{bmatrix} struct \\ coords \\ end \\ y \end{bmatrix} \right)$
		$Angle2D \blacktriangleright height = angle\_structure \begin{bmatrix} struct \\ style \\ height \end{bmatrix}$	
		$Angle2D \blacktriangleright right = angle\_structure \begin{bmatrix} struct \\ style \\ rightAngle \end{bmatrix}$	
		RE $Angle2D$	

2 arc-free

$$\boxed{=}\begin{bmatrix} arc\_structure \\ object\_structure \end{bmatrix}$$

$Arc2D = \mathbf{Arc2D}$

$$Arc2D \blacktriangleright center = \mathbf{xy}\left(arc\_structure \begin{bmatrix} struct \\ coords \\ center \\ x \end{bmatrix}, \; arc\_structure \begin{bmatrix} struct \\ coords \\ center \\ y \end{bmatrix} \right)$$

$$Arc2D \blacktriangleright start = \mathbf{xy}\left(arc\_structure \begin{bmatrix} struct \\ coords \\ start \\ x \end{bmatrix}, \; arc\_structure \begin{bmatrix} struct \\ coords \\ start \\ y \end{bmatrix} \right)$$

$$angle\_sign = \mathrm{sign}\left(arc\_structure \begin{bmatrix} struct \\ angle \end{bmatrix} \right)$$

$$angle\_abs = \left| arc\_structure \begin{bmatrix} struct \\ angle \end{bmatrix} \right|$$

$$Arc2D \blacktriangleright angle = \boxed{\times}\begin{bmatrix} angle\_sign \\ angle\_abs \\ \frac{\pi}{180} \end{bmatrix}$$

$\boxed{\text{IF}}$ $arc\_structure \begin{bmatrix} struct \\ measure \\ type \end{bmatrix} \equiv \text{Blank}$ $\boxed{=}\begin{bmatrix} Arc2D \blacktriangleright measure \\ \emptyset \end{bmatrix}$

$$Arc2D \blacktriangleright measure = \mathbf{MeasureArc2D}\left(\begin{array}{l} Arc2D \blacktriangleright center \\ Arc2D \blacktriangleright start \\ Arc2D \blacktriangleright angle \\ arc\_structure \begin{bmatrix} struct \\ height \end{bmatrix} \end{array} \right)$$

$\boxed{\text{RE}}$ $Arc2D$

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3 curve-free

$$\boxed{=}\begin{bmatrix} curve\_structure \\ object\_structure \end{bmatrix}$$

$Curve2D = \mathbf{Curve2D}$

$\boxed{\text{RE}}$ $Curve2D$

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4 face-free

$$\boxed{=}\begin{bmatrix} face\_structure \\ object\_structure \end{bmatrix}$$

$Face2D = \mathbf{Face2D}$

$\boxed{\text{FO}}$ $face\_structure \begin{bmatrix} struct \\ coords \end{bmatrix}$ $coords$

$$Face2D \blacktriangleright vertices \stackrel{\pm}{=} \mathbf{xy}\left(\begin{array}{l} coords[\mathsf{x}] \\ coords[\mathsf{y}] \end{array} \right)$$

$\boxed{\text{RE}}$ $Face2D$

5 label-free

$$\boxed{=}\begin{bmatrix} label\_structure \\ object\_structure \end{bmatrix}$$

$Label2D = \mathbf{Label2D}$

$$Label2D \blacktriangleright center = \mathbf{xy}\left(\begin{array}{l} label\_structure \begin{bmatrix} struct \\ coords \\ x \end{bmatrix} \\ label\_structure \begin{bmatrix} struct \\ coords \\ y \end{bmatrix} \end{array}\right)$$

$$Label2D \blacktriangleright target = \mathbf{xy}\left(\begin{array}{l} label\_structure \begin{bmatrix} struct \\ target \\ x \end{bmatrix} \\ label\_structure \begin{bmatrix} struct \\ target \\ y \end{bmatrix} \end{array}\right)$$

$$\boxed{SW}\quad label\_structure \begin{bmatrix} struct \\ label \\ type \end{bmatrix}$$

Latex $\quad Label2D\_size = latex\_size\left(label\_structure \begin{bmatrix} struct \\ label \\ content \end{bmatrix}\right)$

Text $\quad Label2D\_size = text\_size\left(label\_structure \begin{bmatrix} struct \\ label \\ content \end{bmatrix}\right)$

$$Label2D \blacktriangleright size = \boxed{\div}\begin{bmatrix} Label2D\_size \\ \boxed{\times}\begin{bmatrix} this \blacktriangleright size \\ this \blacktriangleright zoom \end{bmatrix} \end{bmatrix}$$

$$\boxed{=}\begin{bmatrix} Label2D \blacktriangleright rotation \\ this \blacktriangleright rotation \end{bmatrix}$$

$$\boxed{FO}\quad \boxed{this \blacktriangleright struct[\text{elements}]}\quad \boxed{key\_object\_structure}\quad \boxed{object\_structure}$$

$$\boxed{IF}\ \boxed{\wedge}\begin{bmatrix} \boxed{\neq}\begin{bmatrix} object\_structure[\text{type}] \\ label\text{-}free \end{bmatrix} \\ \boxed{\equiv}\begin{bmatrix} label\_structure \begin{bmatrix} struct \\ dependency[0] \end{bmatrix} \\ object\_structure[\_id] \end{bmatrix} \end{bmatrix}\quad \boxed{=}\begin{bmatrix} Label2D \blacktriangleright key\_labeled \\ key\_object\_structure \end{bmatrix}$$

$$\boxed{RE}\quad Label2D$$

6 point-free

$$\boxed{=}\begin{bmatrix} point\_structure \\ object\_structure \end{bmatrix}$$

$$Point2D = \mathbf{Point2D}\left(\begin{array}{l} point\_structure \begin{bmatrix} struct \\ coord \\ x \end{bmatrix} \\ point\_structure \begin{bmatrix} struct \\ coord \\ y \end{bmatrix} \end{array}\right)$$

$$\boxed{RE}\quad Point2D$$

7 region-free

$$\boxed{=}\begin{bmatrix} region\_structure \\ object\_structure \end{bmatrix}$$

$Region2D = \mathbf{Region2D}$

$$\boxed{RE}\quad Region2D$$

8 segment-free

$=$ $\begin{bmatrix} segment\_structure \\ object\_structure \end{bmatrix}$

$Segment2D = \mathbf{Segment2D}$

$Segment2D \blacktriangleright start = \mathbf{xy}$ $\left(\begin{array}{l} segment\_structure \begin{bmatrix} \text{struct} \\ \text{coords} \\ \text{start} \\ \text{x} \end{bmatrix} \\ segment\_structure \begin{bmatrix} \text{struct} \\ \text{coords} \\ \text{start} \\ \text{y} \end{bmatrix} \end{array} \right)$

$Segment2D \blacktriangleright end = \mathbf{xy}$ $\left(\begin{array}{l} segment\_structure \begin{bmatrix} \text{struct} \\ \text{coords} \\ \text{end} \\ \text{x} \end{bmatrix} \\ segment\_structure \begin{bmatrix} \text{struct} \\ \text{coords} \\ \text{end} \\ \text{y} \end{bmatrix} \end{array} \right)$

IF $segment\_structure \begin{bmatrix} \text{struct} \\ \text{measure} \\ \text{type} \end{bmatrix} \neq \text{Blank}$ $=$ $\begin{bmatrix} Segment2D \blacktriangleright measure \\ \emptyset \end{bmatrix}$

$Segment2D \blacktriangleright measure = \mathbf{Measure2D}$ $\left(\begin{array}{l} Segment2D \blacktriangleright start \\ Segment2D \blacktriangleright end \\ segment\_structure \begin{bmatrix} \text{struct} \\ \text{measure} \\ \text{height} \end{bmatrix} \end{array} \right)$

RE $Segment2D$

---

4. | info | imformation of object

info	type		
SW	*type*		
	Angle2D	sublevel
	Arc2D	sublevel
	Curve2D	sublevel
	Point2D	sublevel
	Segment2D	sublevel

1 Angle2D

$index\_info = [\ objects\ |\ poset\ ]$

$info = \mathbf{index}(index\_info)$

$[=]\ \begin{bmatrix} info \blacktriangleright objects \\ \emptyset \end{bmatrix}$

$index\_object = [\ label\ |\ key\_object\ ]$

$index\_object\_label = [intervals\_angle]$

$relations = \emptyset$

FO $this \blacktriangleright \begin{bmatrix} objects \\ Angle2D \end{bmatrix}$ $Angle2D$

- $object = \mathbf{index}(index\_object)$

- $[=]\ \begin{bmatrix} object \blacktriangleright key\_object \\ Angle2D \blacktriangleright key\_object \end{bmatrix}$

- $[=]\ \begin{bmatrix} object \blacktriangleright label \\ \mathbf{index}(index\_object\_label) \end{bmatrix}$

- $object \blacktriangleright \begin{bmatrix} label \\ interval\_angle \end{bmatrix} = \emptyset$

- $[=]\ \begin{bmatrix} Angle2D\_size \\ Angle2D \blacktriangleright \text{size} \end{bmatrix}$

- $[=]\ \begin{bmatrix} relation \\ \mathbf{Relation}(Angle2D \blacktriangleright key\_object) \end{bmatrix}$

- **sublevel**

- $[=]\ \begin{bmatrix} interval\_angle \\ Angle2D \blacktriangleright \text{interval\_angle} \end{bmatrix}$

- $angles\_forbidden = \emptyset$

- $angle\_Angle2D\_start = Angle2D \blacktriangleright \begin{bmatrix} center \\ \text{vector\_angle}(Angle2D \blacktriangleright start) \end{bmatrix}$

- $[=]\ \begin{bmatrix} angles\_forbidden[0] \\ angle\_Angle2D\_start \end{bmatrix}$

- $angles\_forbidden = \emptyset$

- $angles\_forbidden\ [Angle2D \blacktriangleright \text{angle}] = Angle \blacktriangleright \begin{bmatrix} center \\ \text{vector\_angle}(Angle2D \blacktriangleright end) \end{bmatrix}$

- **sublevel**

- $angles\_forbidden = angles\_forbidden \blacktriangleright \text{ksort}$

- **sublevel**

- $object \blacktriangleright \begin{bmatrix} label \\ intervals\_angle \end{bmatrix} = \mathbf{Union\_interval\_angle}(intervals\_angle\_allowed)$

- $[=]\ \begin{bmatrix} info \blacktriangleright [Angle2D \blacktriangleright key\_object] \\ object \end{bmatrix}$

$[=]\ \begin{bmatrix} info \blacktriangleright poset \\ \mathbf{Poset}(relations) \end{bmatrix}$

RE $info$

---

1 1

FO $this \blacktriangleright \begin{bmatrix} objects \\ Angle2D \end{bmatrix}$ $Angle2D\_1$

- **sublevel**

- $[=]\ \begin{bmatrix} relations[relation \blacktriangleright element] \\ relation \end{bmatrix}$

1 | 1 | 1 IF $\neq \begin{bmatrix} Angle2D\_1 \blacktriangleright key\_object \\ Angle2D \blacktriangleright key\_object \end{bmatrix}$

IF $\wedge \begin{bmatrix} \neq \begin{bmatrix} Angle2D\_1 \blacktriangleright right \\ Angle2D \blacktriangleright right \end{bmatrix} \\ \cap \begin{bmatrix} Angle2D \\ Angle2D\_1 \end{bmatrix} \neq \emptyset \end{bmatrix}$

IF $Angle2D\_1 \blacktriangleright right$ $\pm \begin{bmatrix} relation \blacktriangleright lowers \\ Angle2D\_1 \blacktriangleright key\_object \end{bmatrix}$ $\pm \begin{bmatrix} relation \blacktriangleright uppers \\ Angle2D\_1 \blacktriangleright key\_object \end{bmatrix}$

$\wedge \begin{bmatrix} \equiv \begin{bmatrix} Angle2D \blacktriangleright right \\ \text{FA} \end{bmatrix} \\ \equiv \begin{bmatrix} Angle2D\_1 \blacktriangleright right \\ \text{FA} \end{bmatrix} \\ \cap \begin{bmatrix} Angle2D \\ Angle2D\_1 \end{bmatrix} \neq \emptyset \end{bmatrix}$

$\equiv \begin{bmatrix} Angle2D\_1\_size \\ Angle2D\_1 \blacktriangleright size \end{bmatrix}$

IF $\vee \begin{bmatrix} < \begin{bmatrix} Angle2D\_1\_size \\ Angle2D\_size \end{bmatrix} \\ \wedge \begin{bmatrix} \equiv \begin{bmatrix} Angle2D\_1\_size \\ Angle2D\_size \end{bmatrix} \\ < \begin{bmatrix} Angle2D\_1 \blacktriangleright key\_object \\ Angle2D \blacktriangleright key\_object \end{bmatrix} \end{bmatrix} \end{bmatrix}$ $\pm \begin{bmatrix} relation \blacktriangleright lowers \\ Angle2D\_1 \blacktriangleright key\_object \end{bmatrix}$

$\vee \begin{bmatrix} < \begin{bmatrix} Angle2D\_size \\ Angle2D\_1\_size \end{bmatrix} \\ \wedge \begin{bmatrix} \equiv \begin{bmatrix} Angle2D\_1\_size \\ Angle2D\_size \end{bmatrix} \\ < \begin{bmatrix} Angle2D \blacktriangleright key\_object \\ Angle2D\_1 \blacktriangleright key\_object \end{bmatrix} \end{bmatrix} \end{bmatrix}$ $\pm \begin{bmatrix} relation \blacktriangleright uppers \\ Angle2D\_1 \blacktriangleright key\_object \end{bmatrix}$

1 | 2 FO $this \blacktriangleright \begin{bmatrix} objects \\ Segment2D \end{bmatrix}$ $Segment2D$

IF $\wedge \begin{bmatrix} \subseteq \begin{bmatrix} Angle2D \blacktriangleright center \\ Segment2D \end{bmatrix} \\ \neq \begin{bmatrix} Angle2D \blacktriangleright center \\ Segment2D \blacktriangleright start \end{bmatrix} \end{bmatrix}$

$angle\_Segment2D\_start = Angle2D \blacktriangleright \begin{bmatrix} center \\ vector\_angle(Segment2D \blacktriangleright start) \end{bmatrix}$

IF $\in \begin{bmatrix} angle\_Segment2D\_start \\ (interval\_angle)^\circ \end{bmatrix}$ $angles\_forbidden \begin{bmatrix} - \begin{bmatrix} angle\_Segment2D\_start \\ angle\_Angle2D\_strict \end{bmatrix} \end{bmatrix} = angle\_Segment2D\_start$

$angle\_Segment2D\_end = Angle2D \blacktriangleright \begin{bmatrix} center \\ vector\_angle(Segment2D \blacktriangleright end) \end{bmatrix}$

IF $\in \begin{bmatrix} angle\_Segment2D\_end \\ (interval\_angle)^\circ \end{bmatrix}$ $angles\_forbidden \begin{bmatrix} - \begin{bmatrix} angle\_Segment2D\_end \\ angle\_Angle2D\_start \end{bmatrix} \end{bmatrix} = angle\_Segment2D\_end$

1 | 3 FO $0 \leq i < |angles\_forbidden|$

$intervals\_angle\_allowed \overset{\pm}{=} \textbf{Interval\_angle} \left(\begin{matrix} angles\_forbidden[i] \\ - \begin{bmatrix} angles\_forbidden[i+1] \\ angles\_forbidden[i] \end{bmatrix} \end{matrix} \right)$

2 | Arc2D

$index\_info = [\text{poset}]$

$info = \textbf{index}(index\_info)$

$\boxminus \begin{bmatrix} info \blacktriangleright objects \\ \emptyset \end{bmatrix}$

$relations = \emptyset$

FO | $this \blacktriangleright \begin{bmatrix} objects \\ Arc2D \end{bmatrix}$ | $Arc2D$

$relation = \textbf{Relation}(Arc2D \blacktriangleright key\_object)$

$\boxminus \begin{bmatrix} interval\_angle\_1 \\ Arc2D\_1 \blacktriangleright \text{interval\_angle} \end{bmatrix}$

FO | IF | $\subseteq \begin{bmatrix} interval\_angle\_1 \\ interval\_angle \end{bmatrix}$ | $\pm \begin{bmatrix} relation \blacktriangleright lowers \\ Arc2D\_1 \blacktriangleright key\_object \end{bmatrix}$

$\subseteq \begin{bmatrix} interval\_angle \\ interval\_angle\_1 \end{bmatrix}$ | $\pm \begin{bmatrix} relation \blacktriangleright uppers \\ Arc2D\_1 \blacktriangleright key\_object \end{bmatrix}$

$\boxminus \begin{bmatrix} relations[relation \blacktriangleright element] \\ Arc2D\_1 \blacktriangleright key\_object \end{bmatrix}$

$\boxminus \begin{bmatrix} info \blacktriangleright objects[Arc2D \blacktriangleright key\_object] \\ object \end{bmatrix}$

$\boxminus \begin{bmatrix} info \blacktriangleright poset \\ \textbf{Poset}(relation) \end{bmatrix}$

RE | $info$

3 | Curve2D

$index\_info = [\text{objects}]$

$info = \mathbf{index}(index\_info)$

[≡] $\begin{bmatrix} info \blacktriangleright objects \\ \emptyset \end{bmatrix}$

$index\_object = \begin{bmatrix} \text{Angle2Ds} \mid \text{label} \mid \text{key\_object} \end{bmatrix}$

[≡] $\begin{bmatrix} index\_object\_Angle2Ds \\ [\text{is\_surrounded}] \end{bmatrix}$

[≡] $\begin{bmatrix} index\_object\_Angle2Ds\_contained \\ \text{start} \mid \text{end} \end{bmatrix}$

[≡] $\begin{bmatrix} index\_object\_label \\ [\text{intervals\_angle}] \end{bmatrix}$

[4] Point2D

FO $this \blacktriangleright \begin{bmatrix} objects \\ Point2D \end{bmatrix}$ $Point2D$

IF [≡] $\begin{bmatrix} Point2D \blacktriangleright key\_label \\ \emptyset \end{bmatrix}$ CO

$object = \mathbf{index}(index\_object)$

[≡] $\begin{bmatrix} object \blacktriangleright key\_object \\ Point2D \blacktriangleright key\_object \end{bmatrix}$

[≡] $\begin{bmatrix} object \blacktriangleright Angle2Ds \\ \mathbf{index}(index\_object\_Angle2Ds) \end{bmatrix}$

$object \blacktriangleright \begin{bmatrix} Angle2Ds \\ contained \end{bmatrix} = \mathbf{index}(index\_object\_Angle2Ds\_contained)$

[≡] $\begin{bmatrix} object \blacktriangleright label \\ \mathbf{index}(index\_object\_label) \end{bmatrix}$

[≡] $\begin{bmatrix} intervals\_angle\_forbidden\_label \\ \emptyset \end{bmatrix}$

[≡] $\begin{bmatrix} intervals\_angle\_forbidden\_Angle2Ds \\ \emptyset \end{bmatrix}$

sublevel

[≡] $\begin{bmatrix} union\_forbidden\_Angle2Ds \\ \mathbf{Union\_interval\_angle}(intervals\_angle\_forbidden\_Angle2Ds) \end{bmatrix}$

IF $|union\_forbidden\_Angle2Ds| \equiv 2\pi$ $object \blacktriangleright \begin{bmatrix} Angle2Ds \\ is\_surrounded \end{bmatrix} =$ TR

$object \blacktriangleright \begin{bmatrix} Angle2Ds \\ is\_surrounded \end{bmatrix} =$ FA

sublevel

sublevel

sublevel

sublevel

sublevel

sublevel

[≡] $\begin{bmatrix} union\_forbidden\_label \\ \mathbf{Union\_interval\_angle}(intervals\_angle\_forbidden\_label) \end{bmatrix}$

$object \blacktriangleright \begin{bmatrix} label \\ intervals\_angle \end{bmatrix} = union\_forbidden\_label \blacktriangleright \begin{bmatrix} complement \\ max \end{bmatrix}$

[≡] $\begin{bmatrix} info \blacktriangleright objects\,[Point2D \blacktriangleright key\_object] \\ object \end{bmatrix}$

RE $info$

4 1

FO | $Point2D \blacktriangleright \begin{bmatrix} objects \\ Angle2D \end{bmatrix}$ | $Angle2D$

IF $\equiv \begin{bmatrix} Point2D \\ Angle2D \blacktriangleright center \end{bmatrix}$

$angle\_start = Angle2D \blacktriangleright \begin{bmatrix} center \\ vector\_angle(Angle2D \blacktriangleright center) \end{bmatrix}$

$size = \boxed{-} \begin{bmatrix} Angle2D \blacktriangleright \begin{bmatrix} center \\ vector\_angle(Angle2D \blacktriangleright end) \end{bmatrix} \\ angle\_start \end{bmatrix}$

$interval\_angle = \textbf{Interval\_angle} \begin{pmatrix} angle\_start \\ size \end{pmatrix}$

$\boxed{\pm} \begin{bmatrix} intervals\_angle\_forbidden\_label \\ interval\_angle \end{bmatrix}$

$\boxed{\pm} \begin{bmatrix} intervals\_angle\_forbidden\_Angle2Ds \\ interval\_angle \end{bmatrix}$

$Segment2D\_start = \textbf{Segment2D} \begin{pmatrix} Angle2D \blacktriangleright center \\ Angle2D \blacktriangleright start \end{pmatrix}$

IF $\subseteq \begin{bmatrix} Point2D \\ (Segment2D\_start)^\circ \end{bmatrix}$

$intervals\_angle\_forbidden\_label \stackrel{\pm}{=} \textbf{Interval\_angle} \begin{pmatrix} Segment2D\_start \blacktriangleright vector\_angle \\ \pi \end{pmatrix}$

$object \blacktriangleright \begin{bmatrix} Angle2Ds \\ contained \\ start \end{bmatrix} \stackrel{\pm}{=} Angle2D \blacktriangleright key\_object$

$Segment2D\_end = \textbf{Segment2D} \begin{pmatrix} Angle2D \blacktriangleright center \\ Angle2D \blacktriangleright end \end{pmatrix}$

IF $\in \begin{bmatrix} Point2D \\ (Segment2D\_end)^\circ \end{bmatrix}$

$intervals\_angle\_forbidden\_label \stackrel{\pm}{=} \textbf{Interval\_angle} \begin{pmatrix} Segment2D\_end \blacktriangleright vector\_angle \\ \pi \end{pmatrix}$

$object \blacktriangleright \begin{bmatrix} Angle2Ds \\ contained \\ end \end{bmatrix} \stackrel{\pm}{=} Angle2D \blacktriangleright key\_object$

4 2

FO | $this \blacktriangleright \begin{bmatrix} objects \\ Arc2D \end{bmatrix}$ | $Arc2D$

IF $Point2D \subseteq Arc2D$

$angle\_Point2D = Arc2D \blacktriangleright \begin{bmatrix} center \\ vector\_angle(Point2D) \end{bmatrix}$

$is\_measure = \boxed{\text{FA}}$

IF $\begin{bmatrix} Arc2D \blacktriangleright \begin{bmatrix} measure \\ height \end{bmatrix} \neq 0 \end{bmatrix}$ $is\_measure = \boxed{\text{TR}}$

$\boxed{=} \begin{bmatrix} interval\_angle\_Arc2D \\ Arc2D \blacktriangleright interval\_angle \end{bmatrix}$

$\boxed{\text{sublevel}}$

IF ∧ $\left[\neq \left[\begin{array}{l} Point2D \\ Arc2D \blacktriangleright start \end{array} \right] \right]$
$\left[\neq \left[\begin{array}{l} Point2D \\ Arc2D \blacktriangleright end \end{array} \right] \right]$

IF $is\_measure \equiv$ FA

$\boxed{\equiv} \left[\begin{array}{l} angle\_start \\ angle\_Point2D + \frac{\pi}{2} \end{array} \right]$

$size = \boxed{-} \left[\begin{array}{l} angle\_Point2D \\ interval\_angle\_Arc2D \blacktriangleright start \end{array} \right] \times \frac{1}{2}$

$intervals\_angle\_forbidden\_label \stackrel{\pm}{=} \textbf{Interval\_angle} \left(\begin{array}{l} angle\_start \\ size \end{array} \right)$

$\boxed{\equiv} \left[\begin{array}{l} angle\_start \\ angle\_Point2D - \frac{\pi}{2} \end{array} \right]$

$size = \boxed{-} \left[\begin{array}{l} angle\_Point2D \\ interval\_angle\_Arc2D \blacktriangleright start \end{array} \right] \times \frac{1}{2}$

$intervals\_angle\_forbidden\_label \stackrel{\pm}{=} \textbf{Interval\_angle} \left(\begin{array}{l} angle\_start \\ size \end{array} \right)$

$angle\_difference\_start = \boxed{-} \left[\begin{array}{l} angle\_Point2D \\ interval\_angle\_Arc2D \blacktriangleright start \end{array} \right]$

$angle\_difference\_end = \boxed{-} \left[\begin{array}{l} interval\_angle\_Arc2D \blacktriangleright \text{end} \\ angle\_Point2D \end{array} \right]$

$angle\_start = \boxed{+} \left[\begin{array}{l} angle\_Point2D \\ -\frac{\pi}{2} \\ \frac{1}{2} \times angle\_difference\_start \end{array} \right]$

$size = \boxed{+} \left[\begin{array}{l} \boxed{+} \left[\begin{array}{l} angle\_difference\_start \\ angle\_difference\_end \end{array} \right] \times \frac{1}{2} \\ \pi \end{array} \right]$

$intervals\_angle\_forbidden\_label \stackrel{\pm}{=} \textbf{Interval\_angle} \left(\begin{array}{l} angle\_start \\ size \end{array} \right)$

$sign = \text{sign}(Arc2D \blacktriangleright angle)$

IF $\equiv \left[\begin{array}{l} Point2D \\ Arc2D \blacktriangleright start \end{array} \right]$

IF $is\_measure \equiv$ FA

$angle\_start = \boxed{+} \left[\begin{array}{l} angle\_Point2D \\ sign \times \frac{\pi}{2} \end{array} \right]$

$\boxed{\equiv} \left[\begin{array}{l} size \\ \frac{1}{2} \times Arc2D \blacktriangleright angle \end{array} \right]$

$\boxed{\equiv} \left[\begin{array}{l} angle\_start \\ angle\_Point2D \end{array} \right]$

$size = \boxed{+} \left[\begin{array}{l} sign \times \frac{\pi}{2} \\ \frac{1}{2} \times Arc2D \blacktriangleright angle \end{array} \right]$

$intervals\_angle\_forbidden\_label \stackrel{\pm}{=} \textbf{Interval\_angle} \left(\begin{array}{l} angle\_start \\ size \\ sign \end{array} \right)$

IF $\equiv \left[\begin{array}{l} Point2D \\ Arc2D \blacktriangleright end \end{array} \right]$

IF $is\_measure \equiv$ FA

$angle\_end = \boxed{-} \left[\begin{array}{l} angle\_Point2D \\ sign \times \frac{\pi}{2} \end{array} \right]$

$angle\_start = \boxed{-} \left[\begin{array}{l} angle\_end \\ \frac{1}{2} \times Arc2D \blacktriangleright angle \end{array} \right]$

$\boxed{\equiv} \left[\begin{array}{l} angle\_end \\ angle\_Point2D \end{array} \right]$

$angle\_start = \boxed{+} \left[\begin{array}{l} angle\_end \\ -sign \times \frac{\pi}{2} \\ -\frac{1}{2} \times Arc2D \blacktriangleright angle \end{array} \right]$

$size = \boxed{-} \left[\begin{array}{l} angle\_end \\ angle\_start \end{array} \right]$

$intervals\_angle\_forbidden\_label \stackrel{\pm}{=} \textbf{Interval\_angle} \left(\begin{array}{l} angle\_start \\ size \\ sign \end{array} \right)$

4 3 FO $this \blacktriangleright \begin{bmatrix} objects \\ Curve2D \end{bmatrix}$ $Curve2D$

4 4 FO $this \blacktriangleright \begin{bmatrix} objects \\ Face2D \end{bmatrix}$ $Face2D$

$\boxed{=}$ $\begin{bmatrix} interior\_point \\ Face2D \blacktriangleright \text{centroid}(\mathbf{vertices}) \end{bmatrix}$

FO $1 \le i < |Face2D\_vertices|$

IF $Point2D \equiv vertices[i]$

$\boxed{=} \begin{bmatrix} angle\_start \\ Point2D \blacktriangleright \text{vector\_angle}\left(Face2D \blacktriangleright vertices[i+1]\right) \end{bmatrix}$

$\boxed{=} \begin{bmatrix} size \\ Point2D \blacktriangleright \text{vector\_angle}\left(Face2D \blacktriangleright vertices[i+1]\right) \end{bmatrix}$

$intervals\_angle\_forbidden\_label \overset{+}{=} \textbf{Interval\_angle}\begin{pmatrix} angle\_start \\ size \end{pmatrix}$

BR

$edge = \textbf{Segment2D}\begin{pmatrix} Face2D \blacktriangleright vertices[i] \\ Face2D \blacktriangleright vertices[i+1] \end{pmatrix}$

IF $Point2D \subseteq (edge)^\circ$

$\boxed{=} \begin{bmatrix} angle\_start \\ Point2D \blacktriangleright \text{vector\_angle}\left(Face2D \blacktriangleright vertices[i+1]\right) \end{bmatrix}$

$size = \boxed{-} \begin{bmatrix} Point2D \blacktriangleright \text{vector\_angle}\left(Face2D \blacktriangleright vertices[i]\right) \\ angle\_start \end{bmatrix}$

$intervals\_angle\_forbidden\_label \overset{+}{=} \textbf{Interval\_angle}\begin{pmatrix} angle\_start \\ size \end{pmatrix}$

RE

4 5 FO $this \blacktriangleright \begin{bmatrix} objects \\ Path2D \end{bmatrix}$ $Path2D$

$\boxed{=} \begin{bmatrix} vertices\_Path2D \\ Path2D \blacktriangleright \text{vertices} \end{bmatrix}$

FO $0 \le i \le |vertices\_Path2D| - 2$

$Segment2D = \textbf{Segment2D}\begin{pmatrix} vertices\_Path2D[i] \\ vertices\_Path2D[i+1] \end{pmatrix}$

$\boxed{=} \begin{bmatrix} angle\_Segment2D \\ Segment2D \blacktriangleright \text{vector\_angle} \end{bmatrix}$

IF $\boxed{\equiv} \begin{bmatrix} Point2D \\ Segment2D \blacktriangleright start \end{bmatrix}$ $intervals\_angle\_forbidden\_label \overset{+}{=} \textbf{Interval\_angle}\begin{pmatrix} angle\_Segment2D \\ 0 \end{pmatrix}$

BR

$\boxed{\subseteq} \begin{bmatrix} Point2D \\ (Segment2D)^\circ \end{bmatrix}$ $intervals\_angle\_forbidden\_label \overset{+}{=} \textbf{Interval\_angle}\begin{pmatrix} angle\_Segment2D \\ \pi \end{pmatrix}$

BR

$\boxed{\equiv} \begin{bmatrix} Point2D \\ Segment2D \blacktriangleright end \end{bmatrix}$ $intervals\_angle\_forbidden\_label \overset{+}{=} \textbf{Interval\_angle}\begin{pmatrix} angle\_Segment2D + \pi \\ 0 \end{pmatrix}$

BR

4 6 FO $this \blacktriangleright \begin{bmatrix} objects \\ Segment2D \end{bmatrix}$ $Segment2D$

sublevel

13

4 6 1 **IF** $Point2D \subseteq Segment2D$

$\boxed{\underline{=}}\begin{bmatrix} angle\_Segment2D \\ Segment2D \blacktriangleright \text{vector\_angle} \end{bmatrix}$

IF $\boxed{\equiv}\begin{bmatrix} Point2D \\ Segment2D \blacktriangleright start \end{bmatrix}$ `sublevel`

$\boxed{\equiv}\begin{bmatrix} Point2D \\ Segment2D \blacktriangleright end \end{bmatrix}$ `sublevel`

`sublevel`

---

4 6 1 1

$\boxed{\underline{=}}\begin{bmatrix} angle\_start \\ angle\_Segment2D \end{bmatrix}$

$\boxed{\underline{=}}\begin{bmatrix} angle\_end \\ angle\_start \end{bmatrix}$

IF $\boxed{\wedge}\begin{bmatrix} \boxed{\neq}\begin{bmatrix} Segment2D \blacktriangleright measure \\ \emptyset \end{bmatrix} \\ Segment2D \blacktriangleright \begin{bmatrix} measure \\ height \end{bmatrix} \neq 0 \end{bmatrix}$

$angle\_Segment2D\_measure = \text{vector\_angle}\left(\begin{array}{l} \dfrac{1}{2} \times |Segment2D| \\ 2 \times Segment2D \blacktriangleright \begin{bmatrix} measure \\ height \end{bmatrix} \end{array}\right)$

IF $\text{sign}\left(Segment2D \blacktriangleright \begin{bmatrix} measure \\ height \end{bmatrix}\right) \geq 0$

$\boxed{\underline{=}}\begin{bmatrix} angle\_start \\ angle\_Segment2D \end{bmatrix}$

$angle\_end = \boxed{+}\begin{bmatrix} angle\_Segment2D \\ angle\_Segment2D\_measure \end{bmatrix}$

$angle\_start = \boxed{-}\begin{bmatrix} angle\_Segment2D \\ angle\_Segment2D\_measure \end{bmatrix}$

$\boxed{\underline{=}}\begin{bmatrix} angle\_end \\ angle\_Segment2D \end{bmatrix}$

$intervals\_angle\_forbidden\_label \overset{+}{=} \textbf{Interval\_angle}\left(\boxed{-}\begin{bmatrix} angle\_start \\ angle\_end \\ angle\_start \end{bmatrix}\right)$

---

4 6 1 2

$\boxed{\underline{=}}\begin{bmatrix} angle\_start \\ angle\_Segment2D + \pi \end{bmatrix}$

$\boxed{\underline{=}}\begin{bmatrix} angle\_end \\ angle\_start \end{bmatrix}$

IF $\boxed{\wedge}\begin{bmatrix} \boxed{\neq}\begin{bmatrix} Segment2D \blacktriangleright measure \\ \emptyset \end{bmatrix} \\ Segment2D \blacktriangleright \begin{bmatrix} measure \\ height \end{bmatrix} \neq 0 \end{bmatrix}$

$angle\_Segment2D\_measure = \text{atan2}\left(\begin{array}{l} \dfrac{1}{2} \times |Segment2D| \\ 2 \times Segment2D \blacktriangleright \begin{bmatrix} measure \\ height \end{bmatrix} \end{array}\right)$

IF $\text{sign}\left(Segment2D \blacktriangleright \begin{bmatrix} measure \\ height \end{bmatrix}\right) \geq 0$

$angle\_start = \boxed{-}\begin{bmatrix} angle\_start \\ angle\_Segment2D\_measure \end{bmatrix}$

$\boxed{\underline{=}}\begin{bmatrix} angle\_end \\ angle\_Segment2D \end{bmatrix}$

$\boxed{\underline{=}}\begin{bmatrix} angle\_start \\ angle\_Segment2D \end{bmatrix}$

$angle\_end = \boxed{+}\begin{bmatrix} angle\_start \\ angle\_Segment2D\_measure \end{bmatrix}$

$intervals\_angle\_forbidden\_label \overset{+}{=} \textbf{Interval\_angle}\left(\boxed{-}\begin{bmatrix} angle\_start \\ angle\_end \\ angle\_start \end{bmatrix}\right)$

$$\boxed{=}\ \begin{bmatrix} angle\_start \\ angle\_Segment2D \end{bmatrix}$$

$$\boxed{=}\ \begin{bmatrix} angle\_end \\ angle\_start \end{bmatrix}$$

$$\boxed{4\ 6\ 1\ 3}\quad \boxed{\text{IF}}\ \boxed{\wedge}\ \begin{bmatrix} \boxed{\neq}\begin{bmatrix} Segment2D \blacktriangleright measure \\ \emptyset \end{bmatrix} \\ Segment2D \blacktriangleright \begin{bmatrix} measure \\ height \end{bmatrix} \neq 0 \end{bmatrix}$$

$$\boxed{=}\ \begin{bmatrix} sign\_Segment2D\_measure \\ \mathrm{sign}\left(Segment2D \blacktriangleright \begin{bmatrix} measure \\ height \end{bmatrix} \right) \end{bmatrix}$$

$$\boxed{\text{IF}}\ \mathrm{sign}\left(Segment2D \blacktriangleright \begin{bmatrix} measure \\ height \end{bmatrix} \right) \geq 0$$

$$\boxed{=}\ \begin{bmatrix} angle\_start \\ angle\_Segment2D \end{bmatrix}$$

$$angle\_end = \boxed{+}\begin{bmatrix} angle\_start \\ angle\_Segment2D\_measure \end{bmatrix}$$

$$angle\_start = \boxed{-}\begin{bmatrix} angle\_start \\ angle\_Segment2D\_measure \end{bmatrix}$$

$$\boxed{=}\ \begin{bmatrix} angle\_end \\ angle\_Segment2D \end{bmatrix}$$

$$intervals\_angle\_forbidden\_label \overset{+}{=} \textbf{Interval\_angle}\left(\begin{matrix} angle\_start \\ \boxed{-}\begin{bmatrix} angle\_end \\ angle\_start \end{bmatrix} \end{matrix} \right)$$

$$\boxed{5}\quad Segment2D$$

$$index\_info = \boxed{\ objects\ |\ poset\ }$$

$$info = \textbf{index}(index\_info)$$

$$\boxed{=}\ \begin{bmatrix} info \blacktriangleright objects \\ \emptyset \end{bmatrix}$$

$$index\_object = \boxed{\ angles\ |\ key\_object\ }$$

$$relations = \emptyset$$

$$\boxed{\text{sublevel}}$$

$$\boxed{=}\ \begin{bmatrix} info \blacktriangleright poset \\ \textbf{Poset}(relations) \end{bmatrix}$$

$$\boxed{\text{RE}}\ \boxed{\ info\ }$$

FO

$this \triangleright \begin{bmatrix} objects \\ Segment2D \end{bmatrix} \quad Segment2D$

5 1

IF $\quad Segment2D \triangleright \begin{bmatrix} measure \\ key\_label \end{bmatrix} \neq \emptyset \quad this \triangleright \begin{bmatrix} objects \\ Label2D \begin{bmatrix} Segment2D \triangleright \begin{bmatrix} measure \\ key\_label \end{bmatrix} \end{bmatrix} \\ center \end{bmatrix} = this \triangleright \begin{bmatrix} objects \\ Label2D \begin{bmatrix} Segment2D \triangleright \begin{bmatrix} measure \\ key\_label \end{bmatrix} \end{bmatrix} \\ center(Segment2D \triangleright measure) \end{bmatrix}$

$object = \mathbf{index}(index\_objct)$

$\boxed{=} \begin{bmatrix} object \triangleright key\_object \\ Segment2D \triangleright key\_object \end{bmatrix}$

$\boxed{=} \begin{bmatrix} objects \triangleright angles \\ \mathbf{index}(position) \end{bmatrix}$

$object \triangleright \begin{bmatrix} angles \\ start \end{bmatrix} = \mathbf{index}(sign)$

$object \triangleright \begin{bmatrix} angles \\ end \end{bmatrix} = \mathbf{index}(sign)$

$\boxed{=} \begin{bmatrix} relation \\ \mathbf{Relation}(Segment2D \triangleright key\_object) \end{bmatrix}$

$angles = \mathbf{index}(poisition)$

$\boxed{=} \begin{bmatrix} angles \triangleright start \\ \begin{bmatrix} -\frac{\pi}{2} \mid \frac{\pi}{2} \end{bmatrix} \end{bmatrix}$

$\boxed{=} \begin{bmatrix} angles \triangleright end \\ \begin{bmatrix} -\frac{\pi}{2} \mid \frac{\pi}{2} \end{bmatrix} \end{bmatrix}$

$signs = \mathbf{index}(position)$

$\boxed{=} \begin{bmatrix} signs \triangleright start \\ -1 \end{bmatrix}$

$\boxed{=} \begin{bmatrix} signs \triangleright end \\ 1 \end{bmatrix}$

$angle\_Segment2D = \mathbf{index}(poisition)$

$\boxed{=} \begin{bmatrix} angle\_Segment2D \triangleright end \\ Segment2D \triangleright vector\_angle \end{bmatrix}$

$\boxed{=} \begin{bmatrix} angle\_Segment2D \triangleright start \\ -angle\_Segment2D \triangleright end \end{bmatrix}$

`sublevel`

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`sublevel`

`sublevel`

`sublevel`

$object \triangleright \begin{bmatrix} angles \\ start \\ positive \end{bmatrix} = \pi - \max(angles \triangleright start)$

$object \triangleright \begin{bmatrix} angles \\ start \\ negative \end{bmatrix} = \pi - |\min(angles \triangleright start)|$

$object \triangleright \begin{bmatrix} angles \\ end \\ positive \end{bmatrix} = \pi - \max(angles \triangleright end)$

$object \triangleright \begin{bmatrix} angles \\ end \\ negative \end{bmatrix} = \pi - |\min(angles \triangleright end)|$

$\boxed{=} \begin{bmatrix} info \triangleright objects[Segment2D \triangleright key\_object] \\ object \end{bmatrix}$

FO

$this \blacktriangleright \begin{bmatrix} objects \\ Angle2D \end{bmatrix} \quad Angle2D$

$\boxed{5}\,\boxed{1}\,\boxed{1}$

IF $\quad \boxed{\subseteq} \begin{bmatrix} Angle2D \blacktriangleright center \\ Segment2D \end{bmatrix}$

$angles\_Angle2D = \textbf{index}(\text{position})$

$angles\_Angle2D \blacktriangleright start = Angle2D \blacktriangleright \begin{bmatrix} center \\ \text{vector\_angle}(Angle2D \blacktriangleright start) \end{bmatrix}$

$angles\_Angle2D \blacktriangleright end = Angle2D \blacktriangleright \begin{bmatrix} center \\ \text{vector\_angle}(Angle2D \blacktriangleright end) \end{bmatrix}$

$parities = \textbf{index}(\text{position})$

$parities \blacktriangleright start = \text{sign}\left(\left| \boxed{-} \begin{bmatrix} Angle2D \blacktriangleright center \\ Segment2D \blacktriangleright start \end{bmatrix} \right| \right)$

$parities \blacktriangleright end = \text{sign}\left(\left| \boxed{-} \begin{bmatrix} Angle2D \blacktriangleright center \\ Segment2D \blacktriangleright end \end{bmatrix} \right| \right)$

$angles\_start = \textbf{index}(\text{position})$

$angles\_start \blacktriangleright start = \boxed{\times} \begin{bmatrix} signs \blacktriangleright start \\ \boxed{+} \begin{bmatrix} angles\_Angle2D \blacktriangleright start \\ -angles\_Segment2D \blacktriangleright start \\ -\pi \end{bmatrix} \end{bmatrix}$

$angles\_start \blacktriangleright end = \boxed{\times} \begin{bmatrix} signs \blacktriangleright start \\ \boxed{+} \begin{bmatrix} angles\_Angle2D \blacktriangleright end \\ -angles\_Segment2D \blacktriangleright start \\ -\pi \end{bmatrix} \end{bmatrix}$

$angles\_end = \textbf{index}(\text{position})$

$angles\_end \blacktriangleright start = \boxed{\times} \begin{bmatrix} signs \blacktriangleright end \\ \boxed{+} \begin{bmatrix} angles\_Angle2D \blacktriangleright start \\ -angles\_Segment2D \blacktriangleright end \\ -\pi \end{bmatrix} \end{bmatrix}$

$angles\_end \blacktriangleright end = \boxed{\times} \begin{bmatrix} signs \blacktriangleright end \\ \boxed{+} \begin{bmatrix} angles\_Angle2D \blacktriangleright end \\ -angles\_Segment2D \blacktriangleright end \\ -\pi \end{bmatrix} \end{bmatrix}$

$angles \blacktriangleright start \overset{\pm}{=} \boxed{+} \begin{bmatrix} \boxed{\times} \begin{bmatrix} parities \blacktriangleright start \\ angles\_start \blacktriangleright start \end{bmatrix} \\ \boxed{\times} \begin{bmatrix} 1 - parities \blacktriangleright start \\ \pi \times \text{sign}(angles\_start \blacktriangleright start) \end{bmatrix} \end{bmatrix}$

$angles \blacktriangleright start \overset{\pm}{=} \boxed{+} \begin{bmatrix} \boxed{\times} \begin{bmatrix} parities \blacktriangleright start \\ angles\_start \blacktriangleright end \end{bmatrix} \\ \boxed{\times} \begin{bmatrix} 1 - parities \blacktriangleright start \\ \pi \times \text{sign}(angles\_start \blacktriangleright end) \end{bmatrix} \end{bmatrix}$

$angles \blacktriangleright end \overset{\pm}{=} \boxed{+} \begin{bmatrix} \boxed{\times} \begin{bmatrix} parities \blacktriangleright end \\ angles\_start \blacktriangleright start \end{bmatrix} \\ \boxed{\times} \begin{bmatrix} 1 - parities \blacktriangleright end \\ \pi \times \text{sign}(angles\_start \blacktriangleright start) \end{bmatrix} \end{bmatrix}$

$angles \blacktriangleright end \overset{\pm}{=} \boxed{+} \begin{bmatrix} \boxed{\times} \begin{bmatrix} parities \blacktriangleright end \\ angles\_start \blacktriangleright end \end{bmatrix} \\ \boxed{\times} \begin{bmatrix} 1 - parities \blacktriangleright end \\ \pi \times \text{sign}(angles\_start \blacktriangleright end) \end{bmatrix} \end{bmatrix}$

FO | $this \blacktriangleright \begin{bmatrix} objects \\ Arc2D \end{bmatrix}$ | $Arc2D$

$\boxed{=} \begin{bmatrix} interval\_angle\_Arc2D \\ Arc2D \blacktriangleright \text{interval\_angle} \end{bmatrix}$

$angles\_Arc2D = \textbf{index}(\text{position})$

$\boxed{=} \begin{bmatrix} angles\_Arc2D \blacktriangleright start \\ \dfrac{\pi}{2} + interval\_angle\_Arc2D \blacktriangleright start \end{bmatrix}$

$\boxed{=} \begin{bmatrix} angles\_Arc2D \blacktriangleright end \\ -\dfrac{\pi}{2} + interval\_angle\_Arc2D \blacktriangleright end \end{bmatrix}$

$\boxed{5\,|\,1\,|\,2}$

IF $\boxed{\vee} \begin{bmatrix} \boxed{\subseteq} \begin{bmatrix} Segment2D \blacktriangleright start \\ Arc2D \end{bmatrix} \\ \boxed{\subseteq} \begin{bmatrix} Segment2D \blacktriangleright end \\ Arc2D \end{bmatrix} \end{bmatrix}$

IF $\boxed{\subseteq} \begin{bmatrix} Segment2D \blacktriangleright start \\ Arc2D \end{bmatrix}$

$angle\_contact = Arc2D \blacktriangleright \begin{bmatrix} center \\ \text{vector\_angle}(Segment2D \blacktriangleright start) \end{bmatrix}$

$angles\_contact = \textbf{index}(\text{sign})$

$\boxed{=} \begin{bmatrix} angles\_contact \blacktriangleright negative \\ angle\_contact - \dfrac{\pi}{2} \end{bmatrix}$

$\boxed{=} \begin{bmatrix} angles\_contact \blacktriangleright positive \\ angle\_contact + \dfrac{\pi}{2} \end{bmatrix}$

$sign\_start = \left| \text{sign}\left(\boxed{-} \begin{bmatrix} interval\_angle\_Arc2D \blacktriangleright end \\ angle\_contact \end{bmatrix} \right) \right|$

$angles \blacktriangleright start \stackrel{\cdot}{=} \boxed{\times} \begin{bmatrix} sign\_start \\ signs \blacktriangleright start \\ \boxed{+} \begin{bmatrix} angles\_contact \blacktriangleright negative \\ -angles\_Segment2D \blacktriangleright start \\ -\pi \end{bmatrix} \end{bmatrix}$

$angles \blacktriangleright start \stackrel{\cdot}{=} \boxed{\times} \begin{bmatrix} sign\_start \\ signs \blacktriangleright start \\ \boxed{+} \begin{bmatrix} angles\_contact \blacktriangleright positive \\ -angles\_Segment2D \blacktriangleright start \\ -\pi \end{bmatrix} \end{bmatrix}$

IF $\boxed{\subseteq} \begin{bmatrix} Segment2D \blacktriangleright end \\ Arc2D \end{bmatrix}$

$angle\_contact = Arc2D \blacktriangleright \begin{bmatrix} center \\ \text{vector\_angle}(Segment2D \blacktriangleright end) \end{bmatrix}$

$angles\_contact = \textbf{index}(\text{sign})$

$\boxed{=} \begin{bmatrix} angles\_contact \blacktriangleright negative \\ angle\_contact - \dfrac{\pi}{2} \end{bmatrix}$

$\boxed{=} \begin{bmatrix} angles\_contact \blacktriangleright positive \\ angle\_contact + \dfrac{\pi}{2} \end{bmatrix}$

$sign\_end = \left| \text{sign}\left(\boxed{-} \begin{bmatrix} interval\_angle\_Arc2D \blacktriangleright start \\ angle\_contact \end{bmatrix} \right) \right|$

$angles \blacktriangleright end \stackrel{\cdot}{=} \boxed{\times} \begin{bmatrix} sign\_end \\ signs \blacktriangleright end \\ \boxed{+} \begin{bmatrix} angles\_contact \blacktriangleright negative \\ -angles\_Segment2D \blacktriangleright end \\ -\pi \end{bmatrix} \end{bmatrix}$

$angles \blacktriangleright end \stackrel{\cdot}{=} \boxed{\times} \begin{bmatrix} sign\_end \\ signs \blacktriangleright end \\ \boxed{+} \begin{bmatrix} angles\_contact \blacktriangleright positive \\ -angles\_Segment2D \blacktriangleright end \\ -\pi \end{bmatrix} \end{bmatrix}$

sublevel

$\boxed{5}\boxed{1}\boxed{2}\boxed{1}$

IF $\bigvee \begin{bmatrix} \subseteq \begin{bmatrix} Arc2D \blacktriangleright start \\ Segment2D \end{bmatrix} \\ \subseteq \begin{bmatrix} Arc2D \blacktriangleright end \\ Segment2D \end{bmatrix} \end{bmatrix}$

IF $\subseteq \begin{bmatrix} Arc2D \blacktriangleright start \\ Segment2D \end{bmatrix}$

$angles \blacktriangleright start \overset{+}{=} \pi \times \mathrm{sign}\left(\boxed{\times} \begin{bmatrix} \boxed{+} \begin{bmatrix} signs \blacktriangleright start \\ \begin{bmatrix} angles\_Arc2D \blacktriangleright start \\ -angles\_Segment2D \blacktriangleright start \\ -\pi \end{bmatrix} \end{bmatrix} \\ 1 \end{bmatrix} \right)$

$angles \blacktriangleright end \overset{+}{=} \pi \times \mathrm{sign}\left(\boxed{\times} \begin{bmatrix} \boxed{+} \begin{bmatrix} signs \blacktriangleright end \\ \begin{bmatrix} angles\_Arc2D \blacktriangleright start \\ -angles\_Segment2D \blacktriangleright end \\ -\pi \end{bmatrix} \end{bmatrix} \\ 1 \end{bmatrix} \right)$

IF $\subseteq \begin{bmatrix} Arc2D \blacktriangleright end \\ Segment2D \end{bmatrix}$

$angles \blacktriangleright start \overset{+}{=} \pi \times \mathrm{sign}\left(\boxed{\times} \begin{bmatrix} signs \blacktriangleright start \\ \boxed{+} \begin{bmatrix} angles\_Arc2D \blacktriangleright end \\ -angles\_Segment2D \blacktriangleright start \\ -\pi \end{bmatrix} \end{bmatrix} \right)$

$angles \blacktriangleright end \overset{+}{=} \pi \times \mathrm{sign}\left(\boxed{\times} \begin{bmatrix} signs \blacktriangleright end \\ \boxed{+} \begin{bmatrix} angles\_Arc2D \blacktriangleright end \\ -angles\_Segment2D \blacktriangleright end \\ -\pi \end{bmatrix} \end{bmatrix} \right)$

$Arc2D \blacktriangleright \mathrm{is\_tangent}(Segment2D)$

$\boxed{=} \begin{bmatrix} angles\_Arc2D\_Segment2D \\ \mathbf{index}(\text{position}) \end{bmatrix}$

$angles\_Arc2D\_Segment2D \blacktriangleright start = Arc2D \blacktriangleright \begin{bmatrix} center \\ \mathrm{vector\_angle}(Segment2D \blacktriangleright start) \end{bmatrix}$

$angles\_Arc2D\_Segment2D \blacktriangleright end = Arc2D \blacktriangleright \begin{bmatrix} center \\ \mathrm{vector\_angle}(Segment2D \blacktriangleright end) \end{bmatrix}$

$sign = \mathrm{sign}\left(\begin{bmatrix} \pi - \mathrm{polar}\left(\boxed{-} \begin{bmatrix} angles\_Arc2D\_Segment2D \blacktriangleright end \\ angles\_Arc2D\_Segment2D \blacktriangleright start \end{bmatrix} \right) \\ 1 \end{bmatrix} \right)$

$angles \blacktriangleright start \overset{+}{=} \boxed{\times} \begin{bmatrix} sign \\ angles\_Arc2D\_Segment2D \blacktriangleright start \end{bmatrix}$

$angles \blacktriangleright end \overset{+}{=} \boxed{\times} \begin{bmatrix} sign \\ angles\_Arc2D\_Segment2D \blacktriangleright end \end{bmatrix}$

FO

$this \blacktriangleright \begin{bmatrix} objects \\ Face2D \end{bmatrix} \quad Face2D$

FO $\quad 0 \le i < |Face2D \blacktriangleright vertices|$

$edge\_i = \mathbf{Segment2D} \begin{pmatrix} Face2D \blacktriangleright vertices[i] \\ Face2D \blacktriangleright vertices[i+1] \end{pmatrix}$

IF $\subseteq \begin{bmatrix} Segment2D \blacktriangleright start \\ edge\_i \end{bmatrix}$

$signs\_start = \mathbf{index}(\text{position})$

$signs\_start \blacktriangleright start = 1 - \text{sign}\left(\left| \boxed{-} \begin{bmatrix} Segment2D \blacktriangleright start \\ edge\_i \blacktriangleright start \end{bmatrix} \right| \right)$

$signs\_start \blacktriangleright start = 1 - \text{sign}\left(\left| \boxed{-} \begin{bmatrix} Segment2D \blacktriangleright start \\ edge\_i \blacktriangleright end \end{bmatrix} \right| \right)$

$angles\_edge\_i = \mathbf{index}(\text{position})$

$\boxed{=} \begin{bmatrix} angles\_edge\_i \blacktriangleright end \\ edge\_i \blacktriangleright \text{vector\_angle} \end{bmatrix}$

$\boxed{=} \begin{bmatrix} angles\_edge\_i \blacktriangleright start \\ 2\pi - angles\_edge\_i \blacktriangleright end \end{bmatrix}$

$angles \blacktriangleright start \overset{\pm}{=} \boxed{\times} \begin{bmatrix} signs\_start \blacktriangleright start \\ signs \blacktriangleright start \\ \boxed{+} \begin{bmatrix} angles\_edge\_i \blacktriangleright start \\ -angles\_Segment2D \blacktriangleright start \\ -\pi \end{bmatrix} \end{bmatrix}$

$angles \blacktriangleright start \overset{\pm}{=} \boxed{\times} \begin{bmatrix} signs\_start \blacktriangleright end \\ signs \blacktriangleright start \\ \boxed{+} \begin{bmatrix} angles\_edge\_i \blacktriangleright start \\ -angles\_Segment2D \blacktriangleright start \\ -\pi \end{bmatrix} \end{bmatrix}$

$\subseteq \begin{bmatrix} Segment2D \blacktriangleright end \\ edge\_i \end{bmatrix}$

$signs\_end = \mathbf{index}(\text{position})$

$signs\_end \blacktriangleright start = 1 - \text{sign}\left(\left| \boxed{-} \begin{bmatrix} Segment2D \blacktriangleright end \\ edge\_ \blacktriangleright start \end{bmatrix} \right| \right)$

$signs\_end \blacktriangleright end = 1 - \text{sign}\left(\left| \boxed{-} \begin{bmatrix} Segment2D \blacktriangleright end \\ edge\_ \blacktriangleright end \end{bmatrix} \right| \right)$

$angles\_edge\_i = \mathbf{index}(\text{position})$

$\boxed{=} \begin{bmatrix} angles\_edge\_i \blacktriangleright end \\ edge\_i \blacktriangleright \text{vector\_angle} \end{bmatrix}$

$\boxed{=} \begin{bmatrix} angles\_edge\_i \blacktriangleright start \\ 2\pi - angles\_edge\_i \blacktriangleright end \end{bmatrix}$

$angles \blacktriangleright end \overset{\pm}{=} \boxed{\times} \begin{bmatrix} signs\_end \blacktriangleright start \\ signs \blacktriangleright end \\ \boxed{+} \begin{bmatrix} angles\_edge\_i \blacktriangleright start \\ -angles\_Segment2D \blacktriangleright end \\ -\pi \end{bmatrix} \end{bmatrix}$

$angles \blacktriangleright end \overset{\pm}{=} \boxed{\times} \begin{bmatrix} signs\_end \blacktriangleright end \\ signs \blacktriangleright end \\ \boxed{+} \begin{bmatrix} angles\_edge\_i \blacktriangleright end \\ -angles\_Segment2D \blacktriangleright end \\ -\pi \end{bmatrix} \end{bmatrix}$

$\boxed{5\,|\,1\,|\,3}$

$this \blacktriangleright \begin{bmatrix} objects \\ Path2D \end{bmatrix}$ $Path2D$

$\boxed{=} \begin{bmatrix} vertices \\ Path2D \blacktriangleright vertices \end{bmatrix}$

FO $0 \leq i \leq |vertices| - 2$

$edge\_i = \textbf{Segment2D} \begin{pmatrix} vertices[i] \\ vertices[i+1] \end{pmatrix}$

IF $\boxed{\subseteq} \begin{bmatrix} Segment2D \blacktriangleright start \\ edge\_i \end{bmatrix}$

$signs\_start = \textbf{index}(\text{position})$

$signs\_start \blacktriangleright start = 1 - \text{sign}\left(\left|\boxed{-}\begin{bmatrix} Segment2D \blacktriangleright start \\ edge\_i \blacktriangleright start \end{bmatrix}\right|\right)$

$signs\_start \blacktriangleright end = 1 - \text{sign}\left(\left|\boxed{-}\begin{bmatrix} Segment2D \blacktriangleright start \\ edge\_i \blacktriangleright end \end{bmatrix}\right|\right)$

$signs\_start\_i = \textbf{index}(\text{position})$

$\boxed{=} \begin{bmatrix} angles\_edge\_i \blacktriangleright end \\ edge\_i \blacktriangleright \text{vector\_angle} \end{bmatrix}$

$\boxed{=} \begin{bmatrix} angles\_edge\_i \blacktriangleright start \\ 2\pi - angles\_edge\_i \blacktriangleright end \end{bmatrix}$

$angles \blacktriangleright start \overset{\pm}{=} \boxed{\times} \begin{bmatrix} signs\_start \blacktriangleright start \\ signs \blacktriangleright start \\ \boxed{+}\begin{bmatrix} angles\_edge\_i \blacktriangleright start \\ -angles\_Segment2D \blacktriangleright start \\ -\pi \end{bmatrix} \end{bmatrix}$

$angles \blacktriangleright start \overset{\pm}{=} \boxed{\times} \begin{bmatrix} signs\_start \blacktriangleright end \\ signs \blacktriangleright start \\ \boxed{+}\begin{bmatrix} angles\_edge\_i \blacktriangleright start \\ -angles\_Segment2D \blacktriangleright start \\ -\pi \end{bmatrix} \end{bmatrix}$

$\boxed{\subseteq} \begin{bmatrix} Segment2D \blacktriangleright end \\ edge\_i \end{bmatrix}$

$signs\_end = \textbf{index}(\text{position})$

$signs\_end \blacktriangleright start = 1 - \text{sign}\left(\left|\boxed{-}\begin{bmatrix} Segment2D \blacktriangleright end \\ edge\_i \blacktriangleright start \end{bmatrix}\right|\right)$

$signs\_end \blacktriangleright end = 1 - \text{sign}\left(\left|\boxed{-}\begin{bmatrix} egment2D \blacktriangleright end \\ edge\_i \blacktriangleright end \end{bmatrix}\right|\right)$

$angles\_edge\_i = \textbf{index}(\text{postion})$

$\boxed{=} \begin{bmatrix} angles\_edge\_i \blacktriangleright end \\ edge\_i \blacktriangleright \text{vector\_angle} \end{bmatrix}$

$\boxed{=} \begin{bmatrix} angles\_edge\_i \blacktriangleright start \\ 2\pi - angles\_edge\_i \blacktriangleright end \end{bmatrix}$

$angles \blacktriangleright end \overset{\pm}{=} \boxed{\times} \begin{bmatrix} signs\_end \blacktriangleright start \\ signs \blacktriangleright end \\ \boxed{+}\begin{bmatrix} angles\_edge\_i \blacktriangleright start \\ -angles\_Segment2D \blacktriangleright end \\ -\pi \end{bmatrix} \end{bmatrix}$

$angles \blacktriangleright end \overset{\pm}{=} \boxed{\times} \begin{bmatrix} signs\_end \blacktriangleright end \\ signs \blacktriangleright end \\ \boxed{+}\begin{pmatrix} angles\_edge\_i \blacktriangleright end \\ -angles\_Segment2D \blacktriangleright end \\ -\pi \end{pmatrix} \end{bmatrix}$

$\boxed{5\ 1\ 4}$

5 | 1 | 5

FO

$this \blacktriangleright \begin{bmatrix} objects \\ Segment2D \end{bmatrix}$ $Segment2D\_1$

IF $\neq \begin{bmatrix} Segment2D\_1 \blacktriangleright key\_object \\ Segment2D \blacktriangleright key\_object \end{bmatrix}$

IF

$Segment2D\_1 \subseteq Segment2D$ $\boxed{\pm} \begin{bmatrix} relation \blacktriangleright lowers \\ Segment2D\_1 \blacktriangleright key\_object \end{bmatrix}$

$Segment2D \subseteq Segment2D\_1$ $\boxed{\pm} \begin{bmatrix} relation \blacktriangleright uppers \\ Segment2D\_1 \blacktriangleright key\_object \end{bmatrix}$

$\boxed{=} \begin{bmatrix} angle\_Segment2D\_1\_end \\ Segment2D\_1 \blacktriangleright vector\_angle \end{bmatrix}$

$\boxed{=} \begin{bmatrix} angle\_Segment2D\_1\_start \\ 2\pi - angle\_Segment2D\_1\_end \end{bmatrix}$

$angles\_Segment2D\_1 = \textbf{index}(\text{position})$

$\boxed{=} \begin{bmatrix} angles\_Segment2D\_1 \blacktriangleright start \\ Segment2D\_1 \blacktriangleright vector\_angle \end{bmatrix}$

$\boxed{=} \begin{bmatrix} angles\_Segment2D\_1 \blacktriangleright end \\ 2\pi - angles\_Segment2D\_1 \blacktriangleright start \end{bmatrix}$

sublevel

$5\ 1\ 5\ 1$

IF $\in \begin{bmatrix} Segment2D \blacktriangleright start \\ Segment2D\_1 \end{bmatrix}$

- $signs\_start = \mathbf{index}(\text{position})$
- $signs\_start \blacktriangleright start = 1 - \mathrm{sign}\left(\left|\boxminus\begin{bmatrix} Segment2D \blacktriangleright start \\ Segment2D\_1 \blacktriangleright end \end{bmatrix}\right|\right)$
- $signs\_start \blacktriangleright end = 1 - \mathrm{sign}\left(\left|\boxminus\begin{bmatrix} Segment2D \blacktriangleright start \\ Segment2D\_1 \blacktriangleright start \end{bmatrix}\right|\right)$
- $angles \blacktriangleright start \stackrel{\pm}{\doteq} \boxtimes\begin{bmatrix} signs\_start \blacktriangleright start \\ signs \blacktriangleright start \\ \boxplus\begin{bmatrix} angles\_Segment2D\_1 \blacktriangleright start \\ -angles\_Segment2D \blacktriangleright start \\ -\pi \end{bmatrix} \end{bmatrix}$
- $angles \blacktriangleright start \stackrel{\pm}{\doteq} \boxtimes\begin{bmatrix} signs\_start \blacktriangleright end \\ signs \blacktriangleright start \\ \boxplus\begin{bmatrix} angles\_Segment2D\_1 \blacktriangleright end \\ -angles\_Segment2D \blacktriangleright start \\ -\pi \end{bmatrix} \end{bmatrix}$

$\subseteq \begin{bmatrix} Segment2D \blacktriangleright end \\ Segment2D\_1 \end{bmatrix}$

- $signs\_end = \mathbf{index}(\text{position})$
- $signs\_end \blacktriangleright start = 1 - \mathrm{sign}\left(\left|\boxminus\begin{bmatrix} Segment2D \blacktriangleright end \\ Segment2D\_ \blacktriangleright end \end{bmatrix}\right|\right)$
- $signs\_end \blacktriangleright end = 1 - \mathrm{sign}\left(\left|\boxminus\begin{bmatrix} Segment2D \blacktriangleright end \\ Segment2D\_ \blacktriangleright start \end{bmatrix}\right|\right)$
- $angles \blacktriangleright end \stackrel{\pm}{\doteq} \boxtimes\begin{bmatrix} sign\_end \blacktriangleright start \\ signs \blacktriangleright end \\ \boxplus\begin{bmatrix} angles\_Segment2D_1 \blacktriangleright start \\ -angles\_Segment2D \blacktriangleright end \\ -\pi \end{bmatrix} \end{bmatrix}$
- $angles \blacktriangleright end \stackrel{\pm}{\doteq} \boxtimes\begin{bmatrix} sign\_end \blacktriangleright end \\ signs \blacktriangleright end \\ \boxplus\begin{bmatrix} angles\_Segment2D_1 \blacktriangleright start \\ -angles\_Segment2D \blacktriangleright end \\ -\pi \end{bmatrix} \end{bmatrix}$

$\subseteq \begin{bmatrix} Segment2D\_1 \blacktriangleright start \\ Segment2D \end{bmatrix}$

- $signs\_end = \mathbf{index}(\text{position})$
- $signs\_end \blacktriangleright start = \mathrm{sign}\left(\boxplus\begin{bmatrix} angles\_Segment2D\_1 \blacktriangleright end \\ -angles\_Segment2D \blacktriangleright end \\ -\pi \\ 1 \end{bmatrix}\right)$
- $signs\_end \blacktriangleright end = \mathrm{sign}\left(\boxplus\begin{bmatrix} -angles\_Segment2D\_1 \blacktriangleright end \\ +angles\_Segment2D \blacktriangleright start \\ +\pi \\ 1 \end{bmatrix}\right)$
- $\stackrel{\pm}{\doteq}\begin{bmatrix} angles \blacktriangleright start \\ \pi \times sign\_end \blacktriangleright start \end{bmatrix}$
- $\stackrel{\pm}{\doteq}\begin{bmatrix} angles \blacktriangleright end \\ \pi \times sign\_end \blacktriangleright end \end{bmatrix}$

$\subseteq \begin{bmatrix} Segment2D\_1 \blacktriangleright end \\ Segment2D \end{bmatrix}$

- $signs\_start = \mathbf{index}(\text{position})$
- $signs\_start \blacktriangleright start = \mathrm{sign}\left(\boxplus\begin{bmatrix} angles\_Segment2D\_1 \blacktriangleright start \\ -angles\_Segment2D \blacktriangleright end \\ -\pi \\ 1 \end{bmatrix}\right)$
- $signs\_start \blacktriangleright end = \mathrm{sign}\left(\boxplus\begin{bmatrix} -angles\_Segment2D\_1 \blacktriangleright start \\ +angles\_Segment2D \blacktriangleright start \\ +\pi \\ 1 \end{bmatrix}\right)$
- $\stackrel{\pm}{\doteq}\begin{bmatrix} angles \blacktriangleright end \\ \pi \times signs\_start \blacktriangleright start \end{bmatrix}$
- $\stackrel{\pm}{\doteq}\begin{bmatrix} angles \blacktriangleright end \\ \pi \times signs\_start \blacktriangleright end \end{bmatrix}$

5. update | update status

update	*type*	*height_initial*	

	this_updated = this		
	SW	*type*	
		structure	sublevel
		Arc2D	sublevel
		Angle2D	sublevel
		Face2D	sublevel
		Point2D	sublevel
		Segment2D	sublevel
	RE	*this_updated*	

1 structure

FO $this\_updated \blacktriangleright \begin{bmatrix} objects \\ Angle2D \end{bmatrix}$ $Angle2D$

$this\_updated \blacktriangleright structure \begin{bmatrix} elements \\ Angle2D \blacktriangleright key \\ struct \\ styleheight \end{bmatrix} = Angle2D \blacktriangleright height$

FO $this\_updated \blacktriangleright \begin{bmatrix} objects \\ Arc2D \end{bmatrix}$ $Arc2D$

IF $this\_updated \blacktriangleright structure \begin{bmatrix} elements \\ Arc2D \blacktriangleright key \\ struct \\ measure \\ type \end{bmatrix} \neq \text{Blank}$ $this\_updated \blacktriangleright structure \begin{bmatrix} elements \\ Arc2D \blacktriangleright key \\ struct \\ measure \\ style \\ height \end{bmatrix} = Arc2D \blacktriangleright \begin{bmatrix} measure \\ height \end{bmatrix}$

FO $this\_updated \blacktriangleright \begin{bmatrix} objects \\ Label2D \end{bmatrix}$ $Label2D$

$this\_updated \blacktriangleright structure \begin{bmatrix} elements \\ Label2D \blacktriangleright key \\ struct \\ coords \\ center \\ x \end{bmatrix} = Label2D \blacktriangleright \begin{bmatrix} center \\ x \end{bmatrix}$

$this\_updated \blacktriangleright structure \begin{bmatrix} elements \\ Label2D \blacktriangleright key \\ struct \\ coords \\ center \\ y \end{bmatrix} = Label2D \blacktriangleright \begin{bmatrix} center \\ y \end{bmatrix}$

$this\_updated \blacktriangleright structure \begin{bmatrix} elements \\ Label2D \blacktriangleright key \\ struct \\ coords \\ target \\ x \end{bmatrix} = Label2D \blacktriangleright \begin{bmatrix} target \\ x \end{bmatrix}$

$this\_updated \blacktriangleright structure \begin{bmatrix} elements \\ Label2D \blacktriangleright key \\ struct \\ coords \\ target \\ y \end{bmatrix} = Label2D \blacktriangleright \begin{bmatrix} center \\ y \end{bmatrix}$

FO $this\_updated \blacktriangleright \begin{bmatrix} objects \\ Segment2D \end{bmatrix}$ $Segment2D$

IF $this\_updated \blacktriangleright structure \begin{bmatrix} elements \\ Segment2D \blacktriangleright key \\ struct \\ measure \\ type \end{bmatrix} \neq \text{Blank}$ $this\_updated \blacktriangleright structure \begin{bmatrix} elements \\ Segment2D \blacktriangleright key \\ struct \\ measure \\ style \\ height \end{bmatrix} = Segment2D \blacktriangleright \begin{bmatrix} measure \\ height \end{bmatrix}$

---

2 Arc2D

$\boxed{=} \begin{bmatrix} info \\ this\_updated \blacktriangleright \text{info}(type) \end{bmatrix}$

$height\_increment = \frac{1}{2}$

sublevel

25

2 1

FO $\quad 0 \le i \le info \triangleright \begin{bmatrix} poset \\ rank\_max \end{bmatrix}$

    FO $\quad info \triangleright poset[i] \quad | \quad relation$

      $Arc2D = this\_updated \triangleright \begin{bmatrix} objects \\ Arc2D[relation \triangleright element] \end{bmatrix}$

      $height\_max = height\_initial$

      FO $\quad relation \triangleright lowers \quad | \quad lower$

        $Arc2D\_lower = this\_updated \triangleright \begin{bmatrix} objects \\ Arc2D[lower] \end{bmatrix}$

        IF $\quad this\_updated \triangleright \begin{bmatrix} objects \\ Arc2D[lower] \\ measure \\ key\_label \end{bmatrix} \neq \emptyset$

          $this\_measure\_lower = this\_updated \triangleright \begin{bmatrix} objects \\ Label2D[Arc2D\_lower] \\ measure \\ key\_label \end{bmatrix}$

          $height\_max = \max \begin{pmatrix} height\_max \\ Arc2D \triangleright \begin{bmatrix} measure \\ height\_contact \begin{pmatrix} label\_measure\_lower \\ \mathsf{max} \end{pmatrix} \end{bmatrix} \end{pmatrix}$

        $height\_max = \max \begin{pmatrix} height\_max \\ \boxed{+} \begin{bmatrix} Arc2D \triangleright \begin{bmatrix} meausure \\ height \\ height\_increment \end{bmatrix} \end{bmatrix} \end{pmatrix}$

      $this\_updated \triangleright \begin{bmatrix} objects \\ Arc2D[relation \triangleright element] \\ measure \\ height \end{bmatrix} = height\_max$

      IF $\quad \boxed{\equiv} \begin{bmatrix} Arc2D \triangleright key\_label \\ \emptyset \end{bmatrix} \quad$ **CO**

      $label\_measure = this\_updated \triangleright \begin{bmatrix} objects \\ Label2D[Arc2D \triangleright key\_label] \end{bmatrix}$

      $this\_updated \triangleright \begin{bmatrix} objects \\ Label2D \begin{bmatrix} Arc2D \triangleright \begin{bmatrix} measure \\ key\_label \end{bmatrix} \end{bmatrix} \\ center \end{bmatrix} = label\_measure \triangleright \mathrm{center} \begin{pmatrix} Arc2D \\ Arc2D \triangleright interval\_angle \end{pmatrix}$

3 Angle2D

$$\boxminus \begin{bmatrix} info \\ this\_updated \blacktriangleright info(type) \end{bmatrix}$$

$$ratio\_right = \frac{1}{\sqrt{2}}$$

$$height\_increment = \frac{1}{2}$$

$$\boxminus \begin{bmatrix} intervals\_angle\_forbidden \\ \emptyset \end{bmatrix}$$

FO $\quad 0 \leq i \leq info \blacktriangleright \begin{bmatrix} poset \\ rank\_max \end{bmatrix}$

    FO $\quad \boxed{info \blacktriangleright poset[i]} \;\; \boxed{relation}$

$$Angle2D = this\_updated \blacktriangleright \begin{bmatrix} objects \\ Angle2D[relation \blacktriangleright element] \end{bmatrix}$$

$$parity = \mathrm{intval}\left(Angle2D \blacktriangleright \begin{bmatrix} measure \\ height \end{bmatrix} \right)$$

$$height\_max = \boxed{+} \begin{bmatrix} \boxed{\times} \begin{bmatrix} parity \\ ratio\_right \\ height\_initial \end{bmatrix} \\ \boxed{\times} \begin{bmatrix} 1 - parity \\ height\_initial \end{bmatrix} \end{bmatrix}$$

$$\boxminus \begin{bmatrix} intervals\_angle\_forbidden[relation \blacktriangleright element] \\ \boxed{\pm} \begin{bmatrix} Intervals\_angle\_forbidden \\ Angle2D\_lower \blacktriangleright interval\_angle \end{bmatrix} \end{bmatrix}$$

IF $\quad \boxed{\neq} \begin{bmatrix} Angle2D\_lower \blacktriangleright key\_label \\ \emptyset \end{bmatrix}$

$$height\_max = \max \left(\begin{array}{l} height\_max \\ Angle2D\_lower \blacktriangleright height\_contact \left(this\_updated \blacktriangleright \begin{bmatrix} objects \\ Label2D[Angle2D\_lower \blacktriangleright key\_label] \end{bmatrix} \right) \\ \min \end{array} \right)$$

$$\boxed{\cup} \begin{bmatrix} union\_forbidden \\ intervals\_angle\_forbidden[lower] \\ \boxed{\pm} \left(Angle2D \blacktriangleright \begin{bmatrix} center \\ interval\_angle\_forbidden \left(this\_updated \blacktriangleright \begin{bmatrix} objects \\ Label2D[Angle2D\_lower \blacktriangleright key\_label] \end{bmatrix} \right) \end{bmatrix} \right) \end{bmatrix}$$

$$height\_max = \max \left(\begin{array}{l} height\_max \\ \boxed{+} \begin{bmatrix} Angle2D\_lower \blacktriangleright height \\ height\_increment \end{bmatrix} \end{array} \right)$$

$$this\_updated \blacktriangleright \begin{bmatrix} objects \\ Angle2D[relation \blacktriangleright element] \\ height \end{bmatrix} = height\_max$$

IF $\quad \boxed{\equiv} \begin{bmatrix} Angle2D \blacktriangleright key\_label \\ \emptyset \end{bmatrix} \quad$ **CO**

$$info \blacktriangleright \begin{bmatrix} objects[relatino \blacktriangleright element] \\ label \\ intervals\_angle \end{bmatrix} = info \blacktriangleright \begin{bmatrix} objects[relation \blacktriangleright element] \\ label \\ intervals\_angle \\ difference(union\_forbidden) \end{bmatrix}$$

$$interval\_angle = info \blacktriangleright \begin{bmatrix} objects[relation \blacktriangleright element] \\ label \\ intervals\_angle[0] \end{bmatrix}$$

$$this\_update \blacktriangleright \begin{bmatrix} objects \\ Label2D[Angle2D \blacktriangleright key\_label] \\ center \end{bmatrix} = this\_updated \blacktriangleright \begin{bmatrix} objects \\ Label2D[Angle2D \blacktriangleright key\_label] \\ center \begin{pmatrix} Angle2D \\ interval\_angle \end{pmatrix} \end{bmatrix}$$

4 Face2D

FO

$$this\_updated \blacktriangleright \begin{bmatrix} objects \\ Face2D \end{bmatrix} \quad Face2D$$

IF $\neq \begin{bmatrix} Face2D \blacktriangleright key\_label \\ \emptyset \end{bmatrix}$ $\quad this\_updated \blacktriangleright \begin{bmatrix} objects \\ Label2D[Face2D \blacktriangleright key\_label] \\ center \end{bmatrix} = this \blacktriangleright \begin{bmatrix} objects \\ Label2D[Face2D \blacktriangleright key\_label] \\ center(Face2D) \end{bmatrix}$

---

5 Point2D

$\equiv \begin{bmatrix} info \\ this\_updated \blacktriangleright \mathrm{info}(type) \end{bmatrix}$

FO $\quad info \blacktriangleright \begin{bmatrix} objects \\ Point2D \end{bmatrix} \quad Point2D$

IF $\quad Point2D \blacktriangleright \begin{bmatrix} Angle2Ds \\ is\_surrounded \end{bmatrix} \equiv$ TR $\quad distance\_initial = 0$

$$interval\_angle = info \blacktriangleright \begin{bmatrix} objects[Point2D \blacktriangleright key\_object] \\ label \\ intervals\_angle[0] \end{bmatrix}$$

$$this\_updated \blacktriangleright \begin{bmatrix} objects \\ Label2D[Point2D \blacktriangleright key\_label] \\ center \end{bmatrix} = this\_updated \blacktriangleright \begin{bmatrix} objects \\ Label2D[Point2D \blacktriangleright key\_laebl] \\ center \begin{pmatrix} Point2D \\ interval\_angle \\ distance\_initial \end{pmatrix} \end{bmatrix}$$

---

6 Segment2D

$\equiv \begin{bmatrix} info \\ this\_updated \blacktriangleright \mathrm{info}(type) \end{bmatrix}$

$height\_increment = \dfrac{1}{2}$

sublevel

---

6 1

FO $\quad 0 \leq i \leq info \blacktriangleright \begin{bmatrix} poset \\ rank\_max \end{bmatrix}$

sublevel

FO | $info \blacktriangleright poset[i]$ | $relation$

6 | **1** | **1**

$heights = \mathbf{index}(\mathsf{sign})$

$angles = \mathbf{index}(\mathsf{sign})$

$Segment2D = this\_updated \blacktriangleright \begin{bmatrix} objects \\ Segment2D[relation \blacktriangleright element] \end{bmatrix}$

$label\_measure = this\_updated \blacktriangleright \begin{bmatrix} objects \\ Label2D \begin{bmatrix} Segment2D \blacktriangleright \begin{bmatrix} measure \\ key\_label \end{bmatrix} \end{bmatrix} \end{bmatrix}$

$angles \blacktriangleright positive = \min \left(\begin{matrix} Segment2D \blacktriangleright \begin{bmatrix} angles \\ start \\ positive \end{bmatrix} \\ Segment2D \blacktriangleright \begin{bmatrix} angles \\ end \\ positive \end{bmatrix} \end{matrix} \right)$

$angles \blacktriangleright negative = \min \left(\left| \begin{matrix} Segment2D \blacktriangleright \begin{bmatrix} angles \\ start \\ negative \end{bmatrix} \\ Segment2D \blacktriangleright \begin{bmatrix} angles \\ end \\ negative \end{bmatrix} \end{matrix} \right| \right)$

$\boxed{=} \begin{bmatrix} heights \blacktriangleright positive \\ height\_initial \end{bmatrix}$

$\boxed{=} \begin{bmatrix} heights \blacktriangleright negative \\ height\_initial \end{bmatrix}$

sublevel

$this\_updated \blacktriangleright \begin{bmatrix} objects \\ Label2D \begin{bmatrix} Segment2D \blacktriangleright \begin{bmatrix} measure \\ key\_label \end{bmatrix} \end{bmatrix} center \end{bmatrix} = label\_measure \blacktriangleright \mathrm{center}(Segment2D \blacktriangleright measure)$

$\boxed{6}\boxed{1}\boxed{1}\boxed{1}$ | **IF** | $\max\begin{pmatrix} angles \blacktriangleright positive \\ angles \blacktriangleright negative \end{pmatrix} \equiv \dfrac{\pi}{2}$

$\boxed{=}\begin{bmatrix} angle\_element \\ Segment2D \blacktriangleright \text{vector\_angle} \end{bmatrix}$

IF $\boxed{\equiv}\begin{bmatrix} relation \blacktriangleright lowers \\ \emptyset \end{bmatrix}$

$sign = \text{sign}\begin{pmatrix} \pi - \text{polar}\left(angle\_element - \dfrac{3\pi}{2}\right) \\ 1 \end{pmatrix}$

$this\_updated \blacktriangleright \begin{bmatrix} objects \\ Segment2D[relation \blacktriangleright element] \\ measure \blacktriangleright height \end{bmatrix} = \boxed{+}\begin{bmatrix} \boxed{\times}\begin{bmatrix} \max\begin{pmatrix} sign \\ 0 \end{pmatrix} \\ heights \blacktriangleright positive \end{bmatrix} \\ \boxed{\times}\begin{bmatrix} \min\begin{pmatrix} sign \\ 0 \end{pmatrix} \\ heights \blacktriangleright negative \end{bmatrix} \end{bmatrix}$

FO $info \blacktriangleright \begin{bmatrix} relations[relation \blacktriangleright element] \\ lowers \end{bmatrix}\begin{bmatrix} lower \end{bmatrix}$

$Segment2D\_lower = this\_updated \blacktriangleright \begin{bmatrix} objects \\ Segment2D[lower] \end{bmatrix}$

$height\_label\_max = Segment2D \blacktriangleright \begin{bmatrix} measure \\ \text{height\_contact}\begin{pmatrix} Segment2D\_lower \blacktriangleright measure \\ label\_measure \\ \max \end{pmatrix} \end{bmatrix}$

$\boxed{=}\begin{bmatrix} angle\_lower \\ measure\_lower \blacktriangleright \text{vector\_angle} \end{bmatrix}$

$sign\_direction\_lower = \text{sign}\begin{pmatrix} \dfrac{\pi}{2} - \text{polar}\left(\boxed{-}\begin{bmatrix} angle\_lower \\ angle\_element \end{bmatrix}\right) \\ 1 \end{pmatrix}$

$sign\_height\_lower = \text{sign}\begin{pmatrix} Segment2D\_lower \blacktriangleright \begin{bmatrix} measure \\ height \end{bmatrix} \\ 1 \end{pmatrix}$

$sign\_lower = \boxed{\times}\begin{bmatrix} sign\_direction\_lower \\ sign\_height \\ height\_lower \end{bmatrix}$

$heights \blacktriangleright positive = \max\begin{pmatrix} heights \blacktriangleright positive \\ \boxed{\times}\begin{bmatrix} \max\begin{pmatrix} sign\_lower \\ 0 \end{pmatrix} \\ height\_label\_max \end{bmatrix} \end{pmatrix}$

$heights \blacktriangleright negative = \max\begin{pmatrix} heights \blacktriangleright positive \\ \boxed{\times}\begin{bmatrix} -\min\begin{pmatrix} sign\_lower \\ 0 \end{pmatrix} \\ height\_label\_max \end{bmatrix} \end{pmatrix}$

$sign = \text{sign}\begin{pmatrix} \boxed{-}\begin{bmatrix} heights \blacktriangleright positive \\ heights \blacktriangleright negative \end{bmatrix} \\ 1 \end{pmatrix}$

$this\_updated \blacktriangleright \begin{bmatrix} objects \\ Segment2D[relation \blacktriangleright element] \\ measure \blacktriangleright height \end{bmatrix} = \boxed{+}\begin{bmatrix} \boxed{\times}\begin{bmatrix} \max\begin{pmatrix} sign \\ 0 \end{pmatrix} \\ heights \blacktriangleright positive \end{bmatrix} \\ \boxed{\times}\begin{bmatrix} \min\begin{pmatrix} sign \\ 0 \end{pmatrix} \\ heights \blacktriangleright negative \end{bmatrix} \end{bmatrix}$

sublevel

FO | $relation \blacktriangleright lowers$ | $lower$

$Segment2D\_lower = this\_updated \blacktriangleright \begin{bmatrix} objects \\ Segment2D[lower] \end{bmatrix}$

$height\_label\_max = Segment2D \blacktriangleright \begin{bmatrix} measure \\ \text{height\_contact} \begin{pmatrix} Segment2D\_lower \blacktriangleright measure \\ label\_measure \\ \mathsf{max} \end{pmatrix} \end{bmatrix}$

$sign\_lower = \text{sign} \begin{pmatrix} Segment2D\_lower \blacktriangleright \begin{bmatrix} measure \\ height \end{bmatrix} \\ 1 \end{pmatrix}$

$heights \blacktriangleright positive = \max \begin{pmatrix} heights \blacktriangleright positive \\ \boxed{\times} \begin{bmatrix} \max \begin{pmatrix} sign\_lower \\ 0 \end{pmatrix} \\ height\_label\_max \end{bmatrix} \end{pmatrix}$

$heights \blacktriangleright negative = \max \begin{pmatrix} heights \blacktriangleright positive \\ \boxed{\times} \begin{bmatrix} -\min \begin{pmatrix} sign\_lower \\ 0 \end{pmatrix} \\ height\_label\_max \end{bmatrix} \end{pmatrix}$

$heights \blacktriangleright positive = \min \begin{pmatrix} heights \blacktriangleright positive \\ Segment2D \blacktriangleright \begin{bmatrix} measure \\ \text{height}(angles \blacktriangleright positive) \end{bmatrix} \end{pmatrix}$

$heights \blacktriangleright negative = \min \begin{pmatrix} heights \blacktriangleright positive \\ Segment2D \blacktriangleright \begin{bmatrix} measure \\ \text{height}(angles \blacktriangleright negative) \end{bmatrix} \end{pmatrix}$

$sign = \text{sign} \begin{pmatrix} \boxed{-} \begin{bmatrix} angles \blacktriangleright positive \\ angles \blacktriangleright negative \end{bmatrix} \\ 1 \end{pmatrix}$

$this\_updated \blacktriangleright \begin{bmatrix} objects \\ Segment2D[relation \blacktriangleright element] \\ measure \\ height \end{bmatrix} = \boxed{+} \begin{bmatrix} \boxed{\times} \begin{bmatrix} \max \begin{pmatrix} sign \\ 0 \end{pmatrix} \\ heights \blacktriangleright positive \end{bmatrix} \\ \boxed{\times} \begin{bmatrix} \min \begin{pmatrix} sign \\ 0 \end{pmatrix} \\ heights \blacktriangleright negative \end{bmatrix} \end{bmatrix}$