CS-109

Date ______
Page ______

Counting : How many possible outcomes satisfy some event.

Step Rule of Counting (Product Rule)

If an experiment has two steps, where The first step's outcome are from set A, where $|A| = m$; and the second step's outcome are from step B, where $|B| = n$, and $|B|$ is unaffected by outcome of first exp.
Then the total no. of outcomes of the experiment is $|A||B| = mn$.

Sum Rule of Counting

If the outcome of an experiment can be either from,

set A, where $|A| = m$
or set B, where $|B| = n$

Then the total no. of outcomes $= |A| + |B|$
$= m + n$

The sets should be mutually ~~exept~~ exclusive.

$\# = |A| + |B| - |A \text{ and } B|$

$\hookrightarrow$ total no. of outcomes

Permutations

A permutation is an ordered arrangement of objects.

The # of unique orderings of n-distinct objects is

$$n! = n \times (n-1) \times (n-2) \times \cdots \times 2 \times 1$$

How do we find the # of permutations considering some objects are indistinct ?

By the product rule, permutation of distinct object is a two step process:

permutation of distinct objects = Permutation considering some objects are indistinct × Permutation of just the indistinct obj.

When there are n objects such that

n_1 are the same

n_2 are the same

⋮

n_r are the same

The # of unique orderings (permutations) is

$$\frac{n!}{n_1!\, n_2! \ldots n_r!}$$

Eg : MISSISSIPPI

# of letter orderings : $\dfrac{11!}{4!\,4!\,1!\,2!}$

Combinations :

A combination is an unordered selection of k objects from a set of n distinct objects.

The # of ways of Making this selection $= \dfrac{n!}{k!(n-k)!}$

$$= n! \times 1 \times \frac{1}{k!} \times \frac{1}{(n-k)!}$$

Order n distinct objects

Taken first k as chosen

Overcounted any ordering of chosen group is same choice

Overcounted Any ordering of unchosen grp is same choice

Put n objects in r buckets:

(i) When the n objects are distinct

$$\# \text{ of ways} = r^n$$

(ii) When the n objects are indistinct

Divider Method:
To divide n objects in r buckets make $r-1$ divides (indistinct)

Step 0: Make objects & divider distinct
Step 1: Order n distinct objects and $r-1$ distinct divides $(n+r-1)!$
Steps 2: Make n objects indistinct $(1/n!)$
Step 3: Make $r-1$ divider indistinct $(1/(r-1)!)$

$$\text{Total \# of ways} = \frac{(n+r-1)!}{n!\,(r-1)!}$$

Sample Space :

Sample Space, S, is set of all possible outcomes of an experiment.

Event Space :

Event, E, is some subset of S

$\{ E \subseteq S \}$

What is probability :
A number b/w 0 & 1 to which we ascribe meaning (our belief that an event E occurs)

Axioms of Probability :

- Axiom 1 : $0 \leq P(E) \leq 1$

- Axiom 2 : $P(S) = 1$

- Axiom 3 : If events E and F are mutually exclusive :

$$P(E \cup F) = P(E) + P(F)$$

$$P(E) = \frac{\#\text{ outcomes in } E}{\#\text{ outcomes in } S} = \frac{|E|}{|S|}$$

Make indistinct items distinct to get equally likely sample space outcomes.

Equally Likely Outcomes :

Each possible result of an experiment has the same chance of occuring.

When approaching an "Equally likely probability problem", start by defining sample space and event spaces.

Conditional Probability

The conditional probability of E given F is the probability that E occurs given that F has already occurred. This is known as conditioning of F.

Written as : $P(E|F)$

Means : $P(E,$ given F already observed$)$

$$P(E|F) = \frac{\text{\# of outcomes of E consistent with F}}{\text{\# of outcomes of S consistent with F}}$$

$$= \frac{|EF|}{|SF|} = \frac{|EF|}{|F|}$$

CHAIN RULE

$$P(E|F) = \frac{P(EF)}{P(F)} \Rightarrow P(EF) = P(F)\,P(E|F)$$

Law of Total Probability

Let F be an event where $P(F) > 0$. For any event E,

$$P(E) = P(E|F)\,P(F) + P(E|F^c)\,P(F^c)$$

Bayes' Theorem

Thm: For any event E and F where $P(E) > 0$ & $P(F) > 0$

$$P(F|E) = \frac{P(E|F)\,P(F)}{P(E)}$$

Exanded form:

$$P(F|E) = \frac{P(E|F)\,P(F)}{P(E|F)\,P(F) + P(E|F^c)\,P(F^c)}$$

Inclusion | Exclusion with Three Events

$$P(E \cup F \cup G) = P(E) + P(F) + P(G) - P(E \cap F) - P(E \cap G) - P(F \cap G) + P(E \cap F \cap G)$$

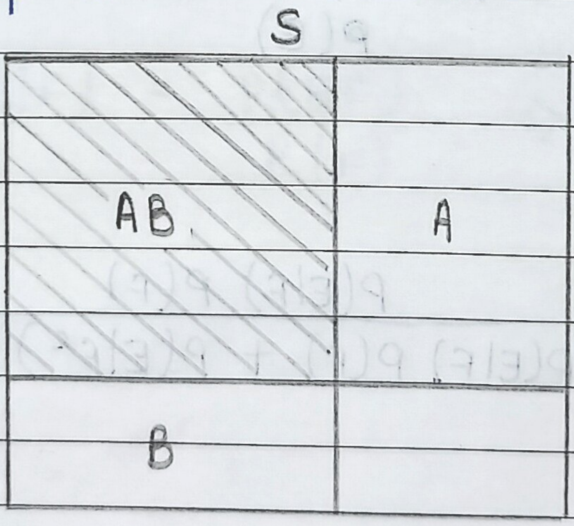
Independence

Two events A and B are called Independent if:

$$P(A) = P(A|B)$$

Knowing that B happend doesn't change our belief that A will happen.

Otherwise they are called dependent events.



$$P(A|B) = P(A)$$

$$\frac{|AB|}{|B|} = \frac{|A|}{|S|}$$

Conditional Independence

Two events E and F are called conditionally independent given G, if

$$P(EF|G) = P(E|G)\,P(F|G)$$

or, equivalently if :

$$P(E|FG) = P(E|G)$$