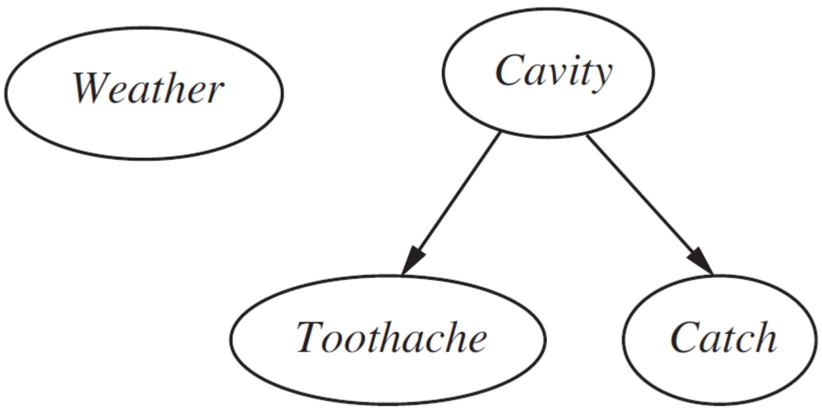
# Bayesian Networks

# Outline

- Concept of Bayesian Networks
- Constructing Bayesian Networks
- Types of Bayesian Networks
- Learning Bayesian Networks
- Using Bayesian Networks

# The Syntax of Bayesian Networks

Example:




- *Weather* is independent of the other variables
- *Toothache* and *Catch* are conditionally independent given *Cavity*

# The Syntax of Bayesian Networks

- A Bayesian network is a simple, graphical notation for conditional independence assertions and hence for compact specification of full joint distributions
- Syntax:
  - A set of nodes, one per variable
  - A directed, acyclic graph (link ≈ "directly influences")
  - A conditional distribution for each node given its parents:

$$\mathbf{P}(X_i \mid Parents(X_i))$$

- In the simplest case, conditional distribution represented as a conditional probability table (CPT) giving the distribution over $X_i$ for each combination of parent values
- Topology of network encodes conditional independence assertions

# The Syntax of Bayesian Networks

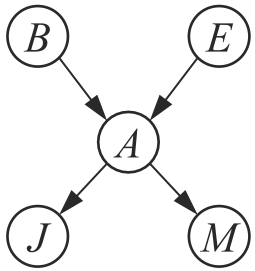
Example:

I'm at work. Neighbor John calls to say my alarm is ringing, but neighbor Mary doesn't call. Sometimes it's set off by minor earthquakes. Is there a burglar?

Variables: *Burglar*, *Earthquake*, *Alarm*, *JohnCalls*, *MaryCalls*

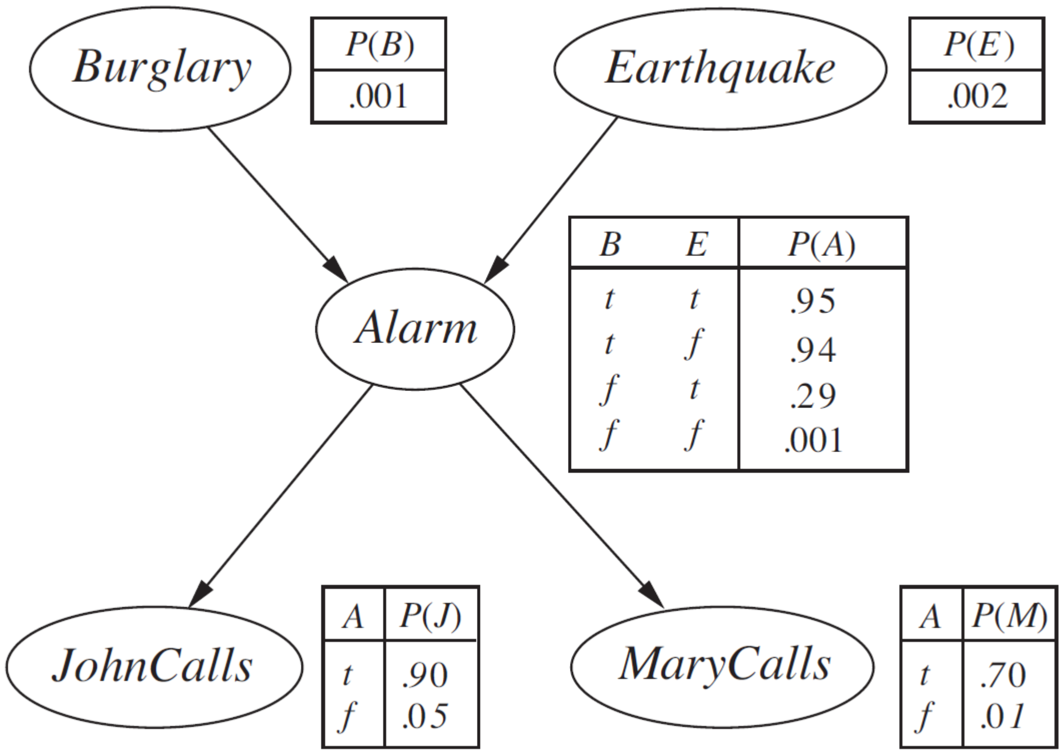
Network topology reflects "causal" knowledge:

- A burglar can set the alarm off
- An earthquake can set the alarm off
- The alarm can cause John to call
- The alarm can cause Mary to call



# The Syntax of Bayesian Networks

Example:




| $P(B)$ |
|---|
| .001 |

| $P(E)$ |
|---|
| .002 |

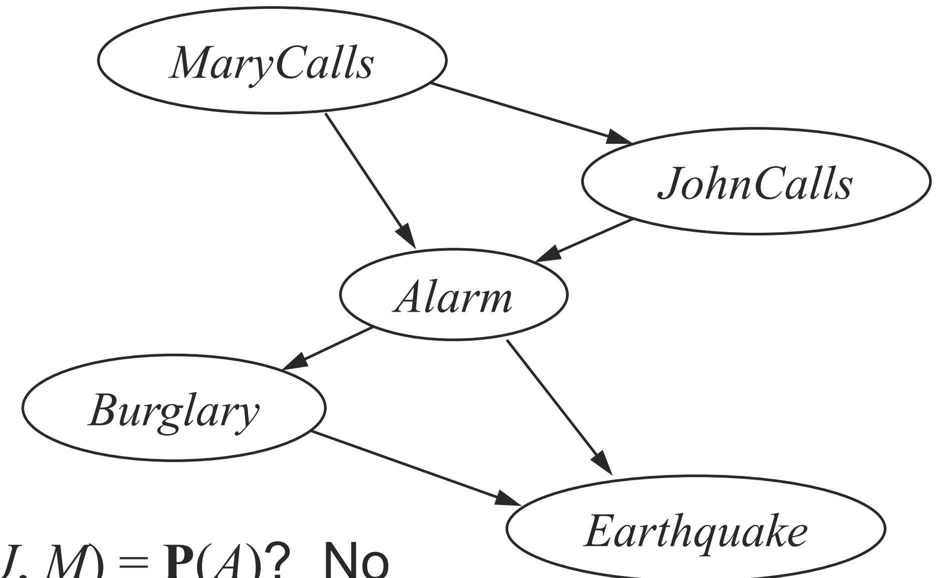
| $B$ | $E$ | $P(A)$ |
|---|---|---|
| $t$ | $t$ | .95 |
| $t$ | $f$ | .94 |
| $f$ | $t$ | .29 |
| $f$ | $f$ | .001 |

| $A$ | $P(J)$ |
|---|---|
| $t$ | .90 |
| $f$ | .05 |

| $A$ | $P(M)$ |
|---|---|
| $t$ | .70 |
| $f$ | .01 |

# Constructing Bayesian Networks

Example: Suppose we choose the ordering $M, J, A, B, E$




$\mathbf{P}(J \mid M) = \mathbf{P}(J)$? No

$\mathbf{P}(A \mid J, M) = \mathbf{P}(A \mid J)$? $\mathbf{P}(A \mid J, M) = \mathbf{P}(A)$? No

$\mathbf{P}(B \mid A, J, M) = \mathbf{P}(B \mid A)$? Yes

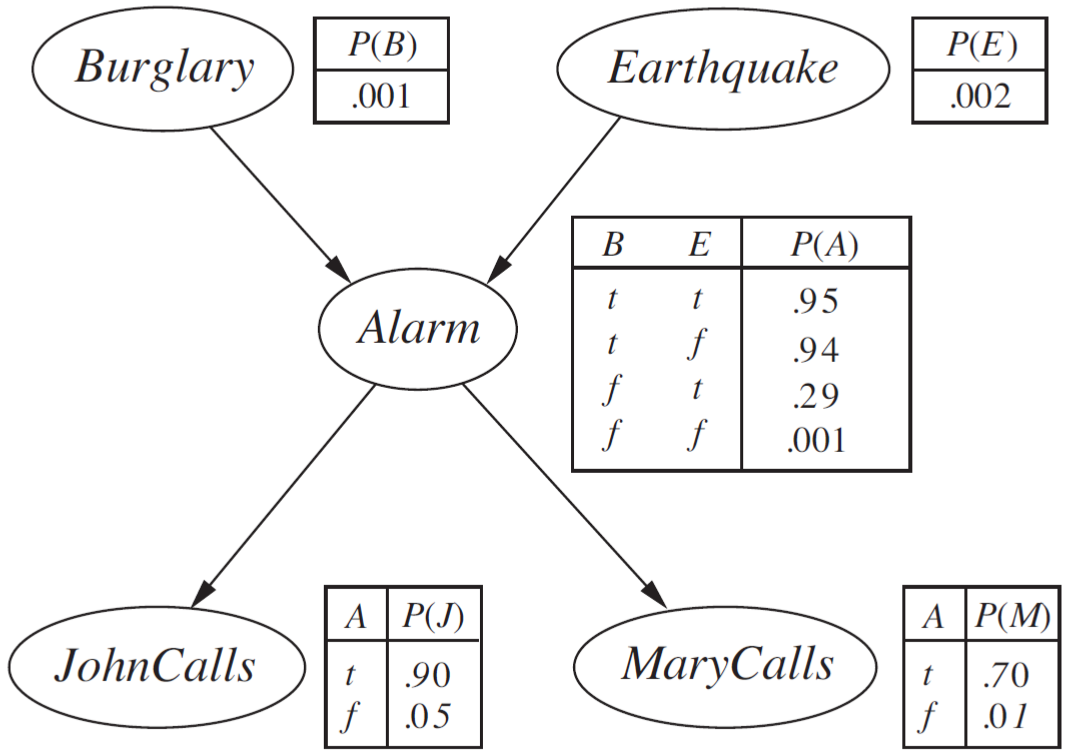
$\mathbf{P}(B \mid A, J, M) = \mathbf{P}(B)$? No

$\mathbf{P}(E \mid B, A, J, M) = \mathbf{P}(E \mid A)$? No

$\mathbf{P}(E \mid B, A, J, M) = \mathbf{P}(E \mid A, B)$? Yes

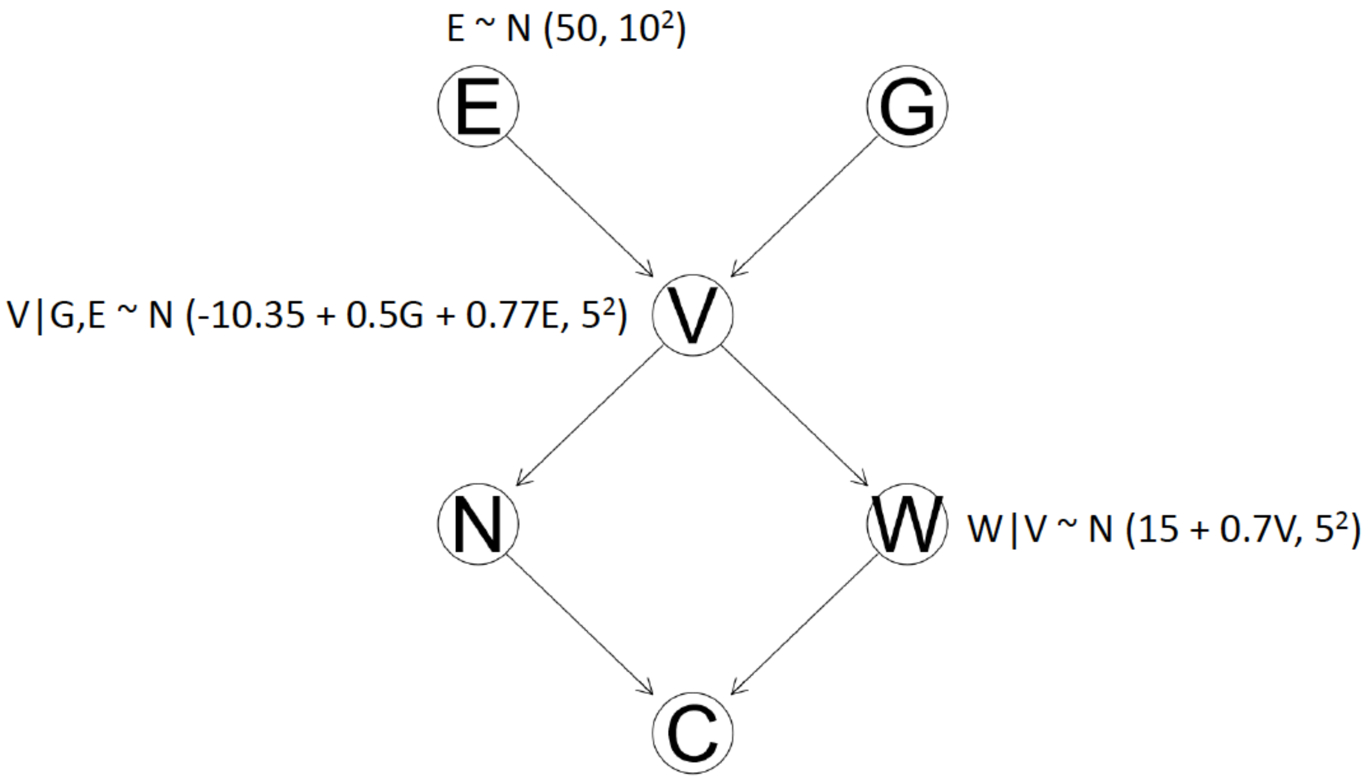
# Types of Bayesian Networks

- Discrete Bayesian Networks



# Types of Bayesian Networks

- Gaussian Bayesian Networks



# Types of Bayesian Networks

- Gaussian Bayesian Networks
  - Assumptions
    - Each node follows a normal (Gaussian) distribution
    - Root nodes are described by the respective marginal distributions (*i.e*. not conditional)
    - The conditioning effect of the parent nodes is given by an additive linear term in the mean, and does not affect the variance
  - Based on these assumptions, the joint distribution of all nodes ( global distribution) is multivariate normal

# Learning Bayesian Networks

- Structural Learning – determine the structure of a directed acyclic graph
  - Parents, the children and co-parents are learned
  - Set all direction of the arcs
  - e.g. Incremental Association
    - Two-phase forward selection
    - Remove false positives
    - Set the direction of undirected arcs using the topological ordering of the nodes or causal relationships from the experimental setting

# Learning Bayesian Networks

- Parameter Learning – determine the parameters of the graph
  - Estimate parameters based on the subset of data spanning the considered variable and its parents
  - Two common approaches
    - Maximum likelihood estimation – use observation
    - Bayesian estimation – find parameters to maximize the likelihood of given data

# Using Bayesian Networks

- Querying – ask questions
  - Conditional independence
  - Probabilistic reasoning or belief updating (*i.e.* most likely outcome)
- Inference
  - Re-evaluate the probability of the variables of interest