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Mathematical models of tidal disruption events in the centers of galaxies

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What are tidal disruption events?

- A tidal disruption event (TDE) is a bright flare in the center of an inactive galaxy. Such flares are usually explained by the tidal disruption of stars by strongly gravitating objects at the centers of galaxies.
- In fact, these events are very rare (once in about ten thousand years in a galaxy). To date, astronomers have observed slightly less than two hundreds of TDEs.
- Relevance: by comparing bolometric light curves with theoretical models, we can study the properties of the central strongly gravitating objects.

Black holes vs naked singularities – I

- The main issue is whether the central objects are black holes or they have other nature; they could be, e.g., naked singularities or wormholes.
- We focus on static, asymptotically flat, spherically symmetric black holes and naked singularities supported by a real self-gravitating scalar field minimally coupled to gravity.
- Our choice is made partly because these configurations can be treated in one and the same manner, and largely because the behavior of matter in the central regions of scalar field naked singularities provides an alternative explanation of TDEs.

Dark matter is modeled by a nonlinear scalar field

The action for our model has the form

$$S = \frac{1}{8\pi} \int \left(-\frac{1}{2} R + \langle d\phi, d\phi \rangle - 2V(\phi) \right) \sqrt{|g|} \, d^4 x \,, \tag{1}$$

where R is the scalar curvature, $V(\phi)$ is a self-interaction potential of a nonlinear scalar field ϕ, and the angle brackets denote the pointwise scalar product induced by the spacetime metric g.

In order to obtain the required structure of spacetime geometry in the center of a galaxy, we have the considerable degree of freedom in the choice of the potential $V(\phi)$.

Spacetime metric and field equations

The metric is (ϕ, A, F, f depend only on the radial coordinate r)

$$ds^2 = Adt^2 - \frac{dr^2}{f} - r^2(d\theta^2 + \sin^2\theta\, d\varphi^2), \quad A = \mathrm{e}^{2F} f. \tag{2}$$

Asymptotic conditions ($\alpha > 0$ and M is the Schwarzschild mass):

$$\phi = O\big(r^{-1/2-\,\alpha}\big),\ \mathrm{e}^F = 1 + o\big(r^{-1}\big),\ A = 1 - \frac{2M}{r} + o\big(r^{-1}\big),\ r \to \infty. \tag{3}$$

The Einstein-Klein-Gordon equations are (a prime is d/dr)

$$\frac{f'}{r} = \frac{1-f}{r^2} - \phi'^2 f - 2V, \qquad F' = \phi'^2 f, \tag{4}$$

$$f\phi'' + \frac{\phi'}{2} f' + \phi' f \left(F' + \frac{1}{2}\frac{f'}{f} + \frac{2}{r} \right) = \frac{dV}{d\phi}\,. \tag{5}$$

The restored potential method and quadratures

$$F(r) = -\int_r^\infty \phi'^2 r dr\,, \quad \xi(r) = r + \int_r^\infty \left(1 - \mathrm{e}^F\right) dr\,, \tag{6}$$

$$A(r) = 2r^2 \int_r^\infty \frac{\xi - 3M}{r^4}\, \mathrm{e}^F dr\,, \qquad f(r) = \mathrm{e}^{-2F} A\,, \tag{7}$$

$$\widetilde{V}(r) = \frac{1}{2r^2}\left(1 - 3f + r^2 \phi'^2 f + 2\,\mathrm{e}^{-F}\,\frac{\xi - 3M}{r}\right) = V(\phi(r)). \tag{8}$$

It is required that one of the function ϕ, F or ξ is given. For all $r > 0$

$$F \leq 0, \quad \mathrm{e}^F \leq 1, \quad \xi > 0, \quad \xi' = \mathrm{e}^F > 0, \quad \xi'' = r\phi'^2 \mathrm{e}^F \geq 0. \tag{9}$$

We choose a strictly increasing, convex downwards function $\xi(r)$:

$$\xi \to \mathrm{e}^F = \xi' \to A \to f\,, \quad \mathrm{e}^F \to F \to \phi' = \sqrt{F'/r} \to \phi \to \widetilde{V}(r) \to V(\phi)\,.$$

Black holes vs naked singularities – II

$$A(r) = 2r^2 \int_r^\infty \frac{\xi - 3M}{r^4}\,\mathrm{e}^F dr\,, \quad f(r) = \mathrm{e}^{-2F} A\,.$$

The type of solution is determined only by the parameters $\xi(0)$ and M.

- Black holes are obtained if $\xi(0) < 3M$, since the integrand becomes negative near the origin ($A \to -\infty \quad r \to \infty$).
- The parameter domain $\xi(0) > 3M$ corresponds to naked singularities ($A \to +\infty \quad r \to \infty$).
- A solution with $\xi(0) = 3M$ is either a regular solution, a black hole, or a naked singularity, depending on the behavior of $F(r)$.

An example

The boundary conditions for ξ are $(0 \leq \alpha \leq 1)$

$$\xi = \xi(0) + \alpha r + O\left(r^2\right) \quad r \to 0, \qquad \xi = r + o(1), \quad r \to \infty, \tag{10}$$

Ansatz: $\xi(r) = \sqrt{r^2 + ar + a^2} - a/2$ ($a > 0$).

The metric functions:

$$A(r) = 1 - \frac{2M}{r} + \frac{3a^3 + 18Ma^2}{40r^3} + O(r^{-4}), \quad r \to \infty, \tag{11}$$

$$A(r) = \frac{a - 6M}{6r} + \frac{5a - 18M}{8a} + + \frac{9a + 54M}{16a^2} r + O(r^2), \quad r \to 0. \tag{12}$$

Black holes vs naked singularities – III

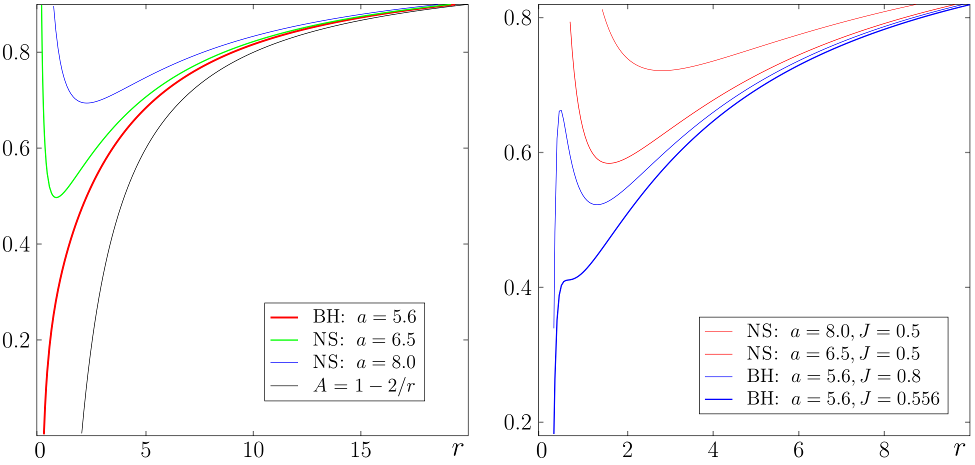

Figure: 1. Left: the metric functions $A(r)$ with the same mass $(M = 1)$. Right: the effective potentials of freely falling particles.

Tidal forces

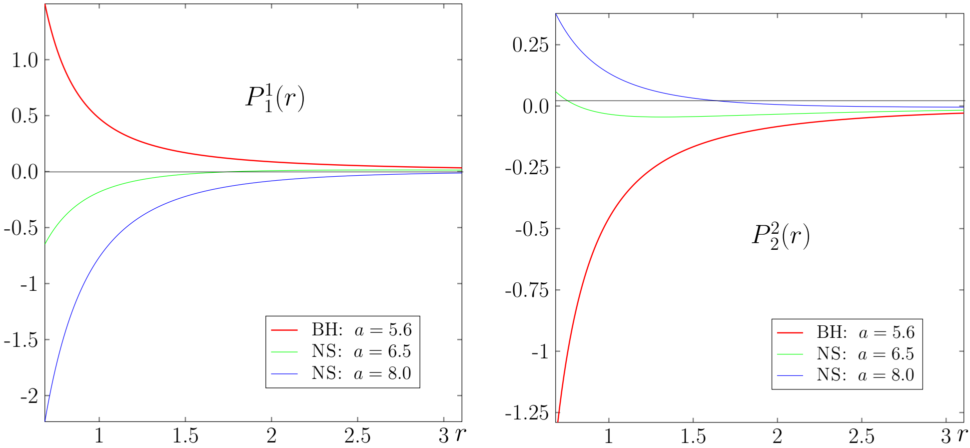

Figure: 2. Tidal forces: $F_t^i \equiv \dfrac{D^2\eta^i}{ds^2} = R^i_{jkl}U^jU^k\eta^l = P^i_l\eta^l$.

What happens when a star is destroyed by a supermassive black hole?

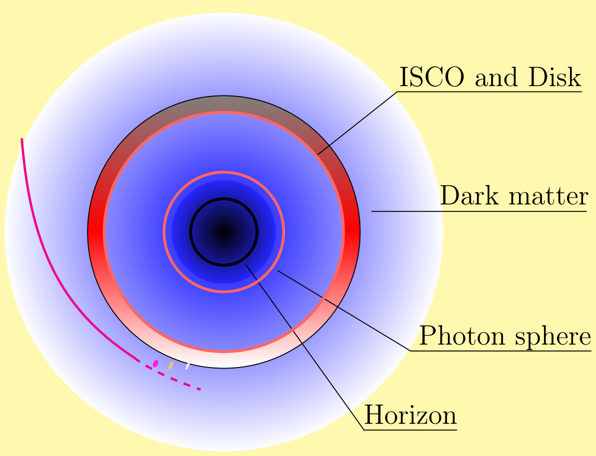

Figure: 3. The tidal disruption of a star near a BH. One part of the debris falls onto the accretion disk, while the other escapes from the pericenter.

What happens when a star collides with a shell of gray matter?

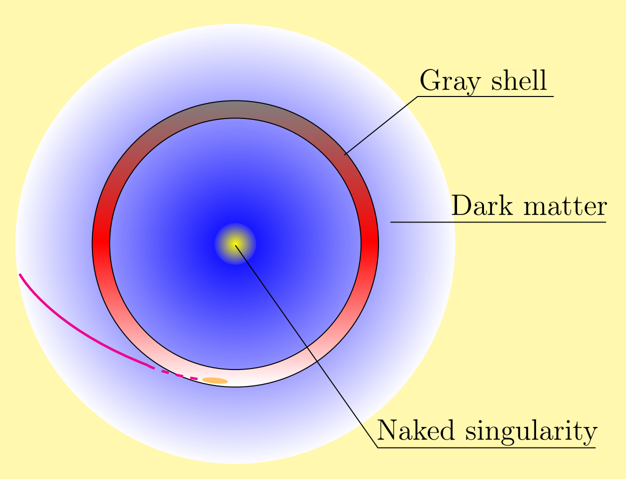

Figure: 4. The collision of a star having a small angular momentum with the gray

Conclusion

Further observations of tidal disruption events will help us to distinguish between supermassive black holes and naked singularities in the centers of galaxies.

Thank you for your attention!