HW 5.3 Concavity & Points of Inflection   Date_____   Period____

For each problem, find the x-coordinates of all points of inflection and find the open intervals where the function is concave up and concave down.

1) $f(x) = 2x^2 - 12x + 20$

2) $f(x) = -x^3 + 2x^2 + 1$

3) $f(x) = x^3 - 3x^2 + 3$

4) $f(x) = x^4 - x^3 - 3x^2 + 4$

5) $f(x) = \dfrac{3}{x+1}$

6) $f(x) = \dfrac{x^2}{2x+2}$

7) $f(x) = \dfrac{3x}{x+1}$

8) $f(x) = \dfrac{x}{x+1}$

For each problem, find the x-coordinates of all critical points and find the open intervals where the function is increasing and decreasing.

9) $f(x) = -2x^2 - 8x - 9$

10) $f(x) = x^3 - 4x^2 + 5$

11) $f(x) = -x^4 + 2x^2$

12) $f(x) = \dfrac{1}{x-1}$

For each problem, find the open intervals where the function is increasing and decreasing.

13) $f(x) = 2x^2 + 16x + 27$

14) $f(x) = x^3 - x^2 + 4$

15) $f(x) = x^4 + 2x^3 - 2x^2 + 2$

16) $f(x) = \dfrac{3}{x+2}$

For each problem, use implicit differentiation to find $\dfrac{dy}{dx}$ in terms of x and y.

17) $3y^2 + 2 = 5x^3$

18) $3x^3 + 5y^2 = 2$

19) $3y^3 + y^2 = 4x$

20) $3x^2 = 4y^2 + y$

21) $x^2 - 3y = 5x^3y^2$

22) $3x^2 - y^2 = xy^3$

23) $-3x^3y^3 + 1 = x$

24) $2 = 2x^2 + 3x^3y^3$

For each problem, find the indicated derivative with respect to x.

25) $f(x) = -5x^5 - 3x^3 + x^2$ Find f''

26) $f(x) = x^4 - 5x^2 + 5x$ Find f'''

27) $f(x) = 5x^3 + 5x^2 - 4x$ Find $f^{(4)}$

28) $f(x) = 5x^5 - 4x^4 - 4x^2$ Find f'''

Differentiate each function with respect to x.

29) $y = \dfrac{-2x - 3}{\left(-2x^2 + 5\right)^{-3}}$

30) $y = \left(3x^2 - 1\right)^{-3}\left(-x^3 - 2\right)$

Answers to HW 5.3 Concavity & Points of Inflection (ID: 1)

1) No inflection points exist.
   Concave up: $(-\infty, \infty)$ Concave down: No intervals exist.

2) Inflection point at: $x = \dfrac{2}{3}$

   Concave up: $\left(-\infty, \dfrac{2}{3}\right)$ Concave down: $\left(\dfrac{2}{3}, \infty\right)$

3) Inflection point at: $x = 1$
   Concave up: $(1, \infty)$ Concave down: $(-\infty, 1)$

4) Inflection points at: $x = -\dfrac{1}{2}, 1$

   Concave up: $\left(-\infty, -\dfrac{1}{2}\right), (1, \infty)$ Concave down: $\left(-\dfrac{1}{2}, 1\right)$

5) No inflection points exist.
   Concave up: $(-1, \infty)$ Concave down: $(-\infty, -1)$

6) No inflection points exist.
   Concave up: $(-1, \infty)$ Concave down: $(-\infty, -1)$

7) No inflection points exist.
   Concave up: $(-\infty, -1)$ Concave down: $(-1, \infty)$

8) No inflection points exist.
   Concave up: $(-\infty, -1)$ Concave down: $(-1, \infty)$

9) Critical point at: $x = -2$
   Increasing: $(-\infty, -2)$ Decreasing: $(-2, \infty)$

10) Critical points at: $x = 0, \dfrac{8}{3}$

   Increasing: $(-\infty, 0), \left(\dfrac{8}{3}, \infty\right)$ Decreasing: $\left(0, \dfrac{8}{3}\right)$

11) Critical points at: $x = -1, 0, 1$
   Increasing: $(-\infty, -1), (0, 1)$ Decreasing: $(-1, 0), (1, \infty)$

12) No critical points exist.
   Increasing: No intervals exist. Decreasing: $(-\infty, 1), (1, \infty)$

13) Increasing: $(-4, \infty)$ Decreasing: $(-\infty, -4)$

14) Increasing: $(-\infty, 0), \left(\dfrac{2}{3}, \infty\right)$ Decreasing: $\left(0, \dfrac{2}{3}\right)$

15) Increasing: $(-2, 0), \left(\dfrac{1}{2}, \infty\right)$ Decreasing: $(-\infty, -2), \left(0, \dfrac{1}{2}\right)$

16) Increasing: No intervals exist. Decreasing: $(-\infty, -2), (-2, \infty)$

17) $\dfrac{dy}{dx} = \dfrac{5x^2}{2y}$

18) $\dfrac{dy}{dx} = -\dfrac{9x^2}{10y}$

19) $\dfrac{dy}{dx} = \dfrac{4}{9y^2 + 2y}$

20) $\dfrac{dy}{dx} = \dfrac{6x}{8y + 1}$

21) $\dfrac{dy}{dx} = \dfrac{15x^2y^2 - 2x}{-3 - 10x^3y}$

22) $\dfrac{dy}{dx} = \dfrac{y^3 - 6x}{-2y - 3y^2x}$

23) $\dfrac{dy}{dx} = \dfrac{-1 - 9x^2y^3}{9x^3y^2}$

24) $\dfrac{dy}{dx} = \dfrac{-4 - 9xy^3}{9x^2y^2}$

25) $f''(x) = -100x^3 - 18x + 2$

26) $f'''(x) = 24x$

27) $f^{(4)}(x) = 0$

28) $f'''(x) = 300x^2 - 96x$

29) $\dfrac{dy}{dx} = \dfrac{(-2x^2 + 5)^{-3} \cdot -2 - (-2x - 3) \cdot -3(-2x^2 + 5)^{-4} \cdot -4x}{\left((-2x^2 + 5)^{-3}\right)^2}$

   $= 2(-2x^2 + 5)^2(14x^2 - 5 + 18x)$

30) $\dfrac{dy}{dx} = (3x^2 - 1)^{-3} \cdot -3x^2 + (-x^3 - 2) \cdot -3(3x^2 - 1)^{-4} \cdot 6x$

   $= \dfrac{3x(3x^3 + x + 12)}{(3x^2 - 1)^4}$

 Made with Infinite Calculus.