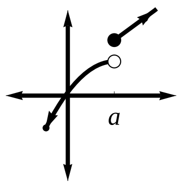
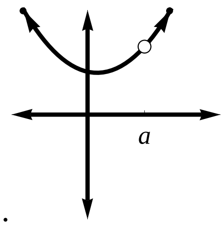
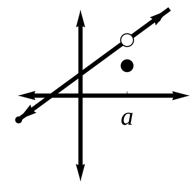
WORKSHEET ON CONTINUITY AND INTERMEDIATE VALUE THEOREM

Work the following on notebook paper. Use the definition of continuity to tell why each of the following graphs of $y = f(x)$ is not continuous at $x = a$.

1. 

2. 

3. 

Use the definition of continuity to justify. Sketch each problem.

4. $h(x)=\begin{cases}3, & x\le -1\\ 2ax+b, & -1<x<1\\ -3, & x\ge 1\end{cases}$ Is h continuous at $x=1$ if $a=3$ and $b=-3$? Justify.

5. $f(x)=\begin{cases}x+3, & x\ne 2\\ 3, & x=2\end{cases}$ Is f continuous at $x=2$? Justify your answer.

Find a and b so that the function is continuous for all real numbers.

6. $f(x)=\begin{cases}\dfrac{2\sin x}{x}, & x<0\\ a-4x, & x\ge 0\end{cases}$

8. $h(x)=\begin{cases}4, & x\le -1\\ ax+b, & -1<x<2\\ -4, & x\ge 2\end{cases}$

7. $g(x)=\begin{cases}\dfrac{x^2-a^2}{x-a}, & x\ne a\\ 6, & x=a\end{cases}$

In the following problems, a function f and a closed interval $[a, b]$ are given. Determine if the Intermediate Value Theorem holds for the given value of k. If the theorem holds, find a number c such that $f(c)=k$. If the theorem does not hold, give the reason. Sketch the curve and the line $y=k$.

9. $f(x)=2+x-x^2$

$[a, b]=[0, 3]$

$k=1$

10. $f(x)=\sqrt{9-x^2}$

$[a, b]=[-2.5, 3]$

$k=2$

11. $f(x)=\dfrac{1}{x-1}$

$[a, b]=[2, 5]$

$k=\dfrac{5}{6}$

Limits and Continuity Worksheet

1. $\lim\limits_{x\to 4}\dfrac{2-\sqrt{x}}{x-4}=$

2. $\lim\limits_{x\to 0}\left(\dfrac{1}{\sin x}-\dfrac{1}{x}\right)=$

3. $\lim\limits_{x\to 0}\dfrac{x\csc x+2}{x\csc x}=$

4. $\lim\limits_{x\to \frac{\pi}{2}^-}\dfrac{\tan x+2}{\tan^2 x}=$

5. $\lim\limits_{x\to 0}\dfrac{\sqrt{4+x}-2}{x}=$

6. $\lim\limits_{x\to a}\dfrac{\tan x-\tan a}{x-a}=$

7. $\lim\limits_{x\to \infty}x\sin\dfrac{1}{x}=$

8. $\lim\limits_{x\to 4}\dfrac{x^3-2x^2-8x}{x^2-4x}=$

9. $\lim\limits_{x\to \frac{\pi}{2}}\dfrac{\ln(\sin x)}{\frac{\pi}{2}-x}=$

10. $\lim\limits_{x\to 0}\dfrac{(1+x)^3-1}{x}=$

11. Find a value of k that makes $f(x)=\begin{cases}2x^2, & x\le \frac{1}{2}\\ \sin(kx), & x>\frac{1}{2}\end{cases}$ continuous at $x=\frac{1}{2}$.

12. If a function f is discontinuous at $x=2$, which of the following must be true?

I. $\lim\limits_{x\to 2} f(x)$ does not exist

II. $f(2)$ does not exist

III. $\lim\limits_{x\to 2^-} f(x)\ne \lim\limits_{x\to 2^+} f(x)$

IV. $\lim\limits_{x\to 2} f(x)\ne f(2)$

13. Find all horizontal and vertical asymptotes for $f(x)=\dfrac{e^{-x}}{x}$.

14. $\lim\limits_{x\to 0}\dfrac{3\cos x-3}{x}$ is equal to the derivative of what function at $x=0$?

15. If $f(x)=\begin{cases}x^2+3, & x>1\\ 5-x, & x\le 1\end{cases}$, then $\lim\limits_{x\to 1} f(x)=$

16. If $\lim\limits_{x\to 3} f(x)=11$ and $\lim\limits_{x\to 3} g(x)=3$, then $\lim\limits_{x\to 3}\dfrac{2\left(g(x)\right)^2}{f(x)-5}=$