Sections 4.2 & 4.3: Analysis of Functions II & III

3. a) Use both the first and second derivative tests to show that $f(x) = 3x^2 - 6x + 1$ has a relative minimum at x=1.

b) Use both the first and second derivative tests to show that $f(x) = x^3 - 3x + 3$ has a relative minimum at x=1 and a relative maximum at x=-1.

5. a) Show that both of the functions $f(x) = (x-1)^4$ and $g(x) = x^3 - 3x^2 + 3x - 2$ have stationary points at x=1.

b) What does the second derivative test tell you about the nature of these stationary points?

c) What does the first derivative test tell you about the nature of these stationary points?

Locate the critical points and identify which critical points are stationary points.

9. $f(x) = \frac{x+1}{x^2+3}$ 11. $f(x) = \sqrt[3]{x^2 - 25}$

Use the given derivative to find all critical points of *f*, and at each critical point determine whether a relative maximum, relative minimum, or neither occurs. Assum in each case that *f* is continuous everywhere.

27. $f'(x) = \frac{2-3x}{\sqrt[3]{x+2}}$ 29. $f'(x) = xe^{1-x^2}$

*Sketch* a graph of the polynomial and label the coordinates of the intercepts, stationary points, and inflection points. Check your work with a graphing utility.

55. $p(x) = (x+1)^2(2x - x^2)$ 57. $p(x) = x^4 - 2x^3 + 2x - 1$

17. Show that $y = x + 3$ is an oblique asymptote of the graph of $f(x) = \frac{x^2}{x-3}$. Sketch the graph of y=f(x) showing this asymptotic behavior.

Give a graph of the function and identify the locations of all critical points and inflection points. Check your work with a graphing utility.

33. $y = 2x + 3x^{2/3}$ 35. $y = 4x^{1/3} - x^{4/3}$

Give a graph of the function and identify the locations of all relative extrema and inflection points. Check your work with a graphing utility.

39. $y = x + \sin x$

Using L'Hospital's Rule one can verify that:

$$\lim_{x\to\infty} \frac{\ln x}{x^r} = 0 \qquad \lim_{x\to\infty} \frac{x^r}{\ln x} \to \infty \qquad \lim_{x\to 0^+} x^r \ln x = 0$$

For any positive real number *r*. (a) Use these results, as necessary, to find the limits of f(x) as $x \to \infty$ and as $x \to 0^+$. (b) Sketch a graph of f(x) and identify all relative extrema, inflection points, and asymptotes (as appropriate). Check your work with a graphing utility.

57. $f(x) = x^2 \ln(2x)$