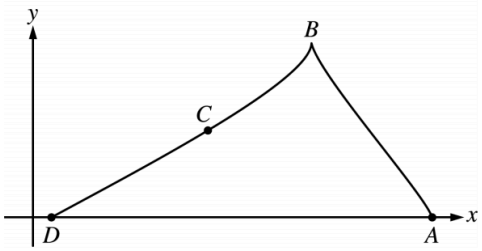
AP Calculus BC - Parametric Free Response Practice

2003 AP® CALCULUS BC FREE-RESPONSE QUESTIONS



CALCULATOR PROBLEM

2. A particle starts at point A on the positive x-axis at time $t = 0$ and travels along the curve from A to B to C to D, as shown above. The coordinates of the particle's position $(x(t), y(t))$ are differentiable functions of t, where $x'(t) = \frac{dx}{dt} = -9\cos\left(\frac{\pi t}{6}\right)\sin\left(\frac{\pi\sqrt{t+1}}{2}\right)$ and $y'(t) = \frac{dy}{dt}$ is not explicitly given. At time $t = 9$, the particle reaches its final position at point D on the positive x-axis.

   (a) At point C, is $\frac{dy}{dt}$ positive? At point C, is $\frac{dx}{dt}$ positive? Give a reason for each answer.

   (b) The slope of the curve is undefined at point B. At what time t is the particle at point B?

   (c) The line tangent to the curve at the point $(x(8), y(8))$ has equation $y = \frac{5}{9}x - 2$. Find the velocity vector and the speed of the particle at this point.

   (d) How far apart are points A and D, the initial and final positions, respectively, of the particle?

CALCULATOR PROBLEM

1. A particle moving along a curve in the plane has position $(x(t), y(t))$ at time t, where

$$\frac{dx}{dt} = \sqrt{t^4 + 9} \text{ and } \frac{dy}{dt} = 2e^t + 5e^{-t}$$

for all real values of t. At time $t = 0$, the particle is at the point $(4, 1)$.

(a) Find the speed of the particle and its acceleration vector at time $t = 0$.

(b) Find an equation of the line tangent to the path of the particle at time $t = 0$.

(c) Find the total distance traveled by the particle over the time interval $0 \le t \le 3$.

(d) Find the x-coordinate of the position of the particle at time $t = 3$.

CALCULATOR PROBLEM

1. An object moving along a curve in the xy-plane has position $(x(t), y(t))$ at time $t \geq 0$ with

$$\frac{dx}{dt} = 12t - 3t^2 \text{ and } \frac{dy}{dt} = \ln\left(1 + (t-4)^4\right).$$

At time $t = 0$, the object is at position $(-13, 5)$. At time $t = 2$, the object is at point P with x-coordinate 3.

(a) Find the acceleration vector at time $t = 2$ and the speed at time $t = 2$.

(b) Find the y-coordinate of P.

(c) Write an equation for the line tangent to the curve at P.

(d) For what value of t, if any, is the object at rest? Explain your reasoning.

NO CALCULATOR

4. A particle moves in the xy-plane so that the position of the particle at any time t is given by

$$x(t) = 2e^{3t} + e^{-7t} \text{ and } y(t) = 3e^{3t} - e^{-2t}.$$

(a) Find the velocity vector for the particle in terms of t, and find the speed of the particle at time $t = 0$.

(b) Find $\frac{dy}{dx}$ in terms of t, and find $\lim_{t \to \infty} \frac{dy}{dx}$.

(c) Find each value t at which the line tangent to the path of the particle is horizontal, or explain why none exists.

(d) Find each value t at which the line tangent to the path of the particle is vertical, or explain why none exists.

NO CALCULATOR

3. An object moving along a curve in the xy-plane has position $(x(t), y(t))$ at time $t \geq 0$ with $\frac{dx}{dt} = 3 + \cos\left(t^2\right)$. The derivative $\frac{dy}{dt}$ is not explicitly given. At time $t = 2$, the object is at position $(1, 8)$.

   (a) Find the x-coordinate of the position of the object at time $t = 4$.

   (b) At time $t = 2$, the value of $\frac{dy}{dt}$ is -7. Write an equation for the line tangent to the curve at the point $(x(2), y(2))$.

   (c) Find the speed of the object at time $t = 2$.

   (d) For $t \geq 3$, the line tangent to the curve at $(x(t), y(t))$ has a slope of $2t + 1$. Find the acceleration vector of the object at time $t = 4$.

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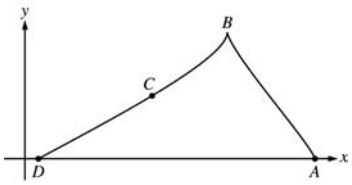
Question 2

A particle starts at point A on the positive x-axis at time $t = 0$ and travels along the curve from A to B to C to D, as shown above. The coordinates of the particle's position $\big(x(t),\, y(t)\big)$ are differentiable functions of t, where $x'(t) = \dfrac{dx}{dt} = -9\cos\left(\dfrac{\pi t}{6}\right)\sin\left(\dfrac{\pi\sqrt{t+1}}{2}\right)$ and $y'(t) = \dfrac{dy}{dt}$ is not explicitly given.



At time $t = 9$, the particle reaches its final position at point D on the positive x-axis.

(a) At point C, is $\dfrac{dy}{dt}$ positive? At point C, is $\dfrac{dx}{dt}$ positive? Give a reason for each answer.

(b) The slope of the curve is undefined at point B. At what time t is the particle at point B?

(c) The line tangent to the curve at the point $\big(x(8),\, y(8)\big)$ has equation $y = \dfrac{5}{9}x - 2$. Find the velocity vector and the speed of the particle at this point.

(d) How far apart are points A and D, the initial and final positions, respectively, of the particle?

(a) At point C, $\dfrac{dy}{dt}$ is not positive because $y(t)$ is decreasing along the arc BD as t increases.
At point C, $\dfrac{dx}{dt}$ is not positive because $x(t)$ is decreasing along the arc BD as t increases.

$2 :$ 1 : $\dfrac{dy}{dt}$ not positive with reason; 1 : $\dfrac{dx}{dt}$ not positive with reason

(b) $\dfrac{dx}{dt} = 0$; $\cos\left(\dfrac{\pi t}{6}\right) = 0$ or $\sin\left(\dfrac{\pi\sqrt{t+1}}{2}\right) = 0$
$\dfrac{\pi t}{6} = \dfrac{\pi}{2}$ or $\dfrac{\pi\sqrt{t+1}}{2} = \pi$; $t = 3$ for both.
Particle is at point B at $t = 3$.

$2 :$ 1 : sets $\dfrac{dx}{dt} = 0$; 1 : $t = 3$

(c) $x'(8) = -9\cos\left(\dfrac{4\pi}{3}\right)\sin\left(\dfrac{3\pi}{2}\right) = -\dfrac{9}{2}$
$\dfrac{y'(8)}{x'(8)} = \dfrac{dy}{dx} = \dfrac{5}{9}$
$y'(8) = \dfrac{5}{9}x'(8) = -\dfrac{5}{2}$
The velocity vector is $< -4.5, -2.5 >$.
Speed $= \sqrt{4.5^2 + 2.5^2} = 5.147$ or 5.148

$3 :$ 1 : $x'(8)$; 1 : $y'(8)$; 1 : speed

(d) $x(9) - x(0) = \displaystyle\int_0^9 x'(t)\,dt$
$= -39.255$
The initial and final positions are 39.255 apart.

$2 :$ 1 : integral; 1 : answer

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2004 SCORING GUIDELINES (Form B)

Question 1

A particle moving along a curve in the plane has position $(x(t), y(t))$ at time t, where

$$\frac{dx}{dt} = \sqrt{t^4 + 9} \text{ and } \frac{dy}{dt} = 2e^t + 5e^{-t}$$

for all real values of t. At time $t = 0$, the particle is at the point $(4, 1)$.

(a) Find the speed of the particle and its acceleration vector at time $t = 0$.
(b) Find an equation of the line tangent to the path of the particle at time $t = 0$.
(c) Find the total distance traveled by the particle over the time interval $0 \le t \le 3$.
(d) Find the x-coordinate of the position of the particle at time $t = 3$.

(a) At time $t = 0$:

Speed $= \sqrt{x'(0)^2 + y'(0)^2} = \sqrt{3^2 + 7^2} = \sqrt{58}$

Acceleration vector $= \langle x''(0), y''(0) \rangle = \langle 0, -3 \rangle$

$2 : \begin{cases} 1 : \text{speed} \\ 1 : \text{acceleration vector} \end{cases}$

(b) $\frac{dy}{dx} = \frac{y'(0)}{x'(0)} = \frac{7}{3}$

Tangent line is $y = \frac{7}{3}(x - 4) + 1$

$2 : \begin{cases} 1 : \text{slope} \\ 1 : \text{tangent line} \end{cases}$

(c) Distance $= \int_0^3 \sqrt{\left(\sqrt{t^4 + 9}\right)^2 + \left(2e^t + 5e^{-t}\right)^2}\, dt$
$= 45.226$ or 45.227

$3 : \begin{cases} 2 : \text{distance integral} \\ \quad \langle -1 \rangle \text{ each integrand error} \\ \quad \langle -1 \rangle \text{ error in limits} \\ 1 : \text{answer} \end{cases}$

(d) $x(3) = 4 + \int_0^3 \sqrt{t^4 + 9}\, dt$
$= 17.930$ or 17.931

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

AP® CALCULUS BC
2005 SCORING GUIDELINES (Form B)

Question 1

An object moving along a curve in the xy-plane has position $(x(t), y(t))$ at time $t \geq 0$ with

$$\frac{dx}{dt} = 12t - 3t^2 \text{ and } \frac{dy}{dt} = \ln\left(1 + (t-4)^4\right).$$

At time $t = 0$, the object is at position $(-13, 5)$. At time $t = 2$, the object is at point P with x-coordinate 3.

(a) Find the acceleration vector at time $t = 2$ and the speed at time $t = 2$.

(b) Find the y-coordinate of P.

(c) Write an equation for the line tangent to the curve at P.

(d) For what value of t, if any, is the object at rest? Explain your reasoning.

(a) $x''(2) = 0,\ y''(2) = -\dfrac{32}{17} = -1.882$

$a(2) = \langle 0, -1.882 \rangle$

Speed $= \sqrt{12^2 + (\ln(17))^2} = 12.329$ or 12.330

$2 : \begin{cases} 1 : \text{acceleration vector} \\ 1 : \text{speed} \end{cases}$

(b) $y(t) = y(0) + \displaystyle\int_0^t \ln\left(1 + (u-4)^4\right) du$

$y(2) = 5 + \displaystyle\int_0^2 \ln\left(1 + (u-4)^4\right) du = 13.671$

$3 : \begin{cases} 1 : \displaystyle\int_0^2 \ln\left(1 + (u-4)^4\right) du \\ 1 : \text{handles initial condition} \\ 1 : \text{answer} \end{cases}$

(c) At $t = 2$, slope $= \dfrac{\frac{dy}{dt}}{\frac{dx}{dt}} = \dfrac{\ln(17)}{12} = 0.236$

$y - 13.671 = 0.236(x - 3)$

$2 : \begin{cases} 1 : \text{slope} \\ 1 : \text{equation} \end{cases}$

(d) $x'(t) = 0$ if $t = 0, 4$

$y'(t) = 0$ if $t = 4$

$t = 4$

$2 : \begin{cases} 1 : \text{reason} \\ 1 : \text{answer} \end{cases}$

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2003 SCORING GUIDELINES (Form B)

Question 4

A particle moves in the xy-plane so that the position of the particle at any time t is given by

$$x(t) = 2e^{3t} + e^{-7t} \text{ and } y(t) = 3e^{3t} - e^{-2t}.$$

(a) Find the velocity vector for the particle in terms of t, and find the speed of the particle at time $t = 0$.

(b) Find $\frac{dy}{dx}$ in terms of t, and find $\lim_{t\to\infty} \frac{dy}{dx}$.

(c) Find each value t at which the line tangent to the path of the particle is horizontal, or explain why none exists.

(d) Find each value t at which the line tangent to the path of the particle is vertical, or explain why none exists.

(a) $x'(t) = 6e^{3t} - 7e^{-7t}$
$y'(t) = 9e^{3t} + 2e^{-2t}$
Velocity vector is $< 6e^{3t} - 7e^{-7t},\ 9e^{3t} + 2e^{-2t} >$

$3 : \begin{cases} 1 : x'(t) \\ 1 : y'(t) \\ 1 : \text{speed} \end{cases}$

$\text{Speed} = \sqrt{x'(0)^2 + y'(0)^2} = \sqrt{(-1)^2 + 11^2}$
$= \sqrt{122}$

(b) $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{9e^{3t} + 2e^{-2t}}{6e^{3t} - 7e^{-7t}}$

$2 : \begin{cases} 1 : \frac{dy}{dx} \text{ in terms of } t \\ 1 : \text{limit} \end{cases}$

$\lim_{t\to\infty} \frac{dy}{dx} = \lim_{t\to\infty} \frac{9e^{3t} + 2e^{-2t}}{6e^{3t} - 7e^{-7t}} = \frac{9}{6} = \frac{3}{2}$

(c) Need $y'(t) = 0$, but $9e^{3t} + 2e^{-2t} > 0$ for all t, so none exists.

$2 : \begin{cases} 1 : \text{considers } y'(t) = 0 \\ 1 : \text{explains why none exists} \end{cases}$

(d) Need $x'(t) = 0$ and $y'(t) \neq 0$.
$6e^{3t} = 7e^{-7t}$
$e^{10t} = \frac{7}{6}$
$t = \frac{1}{10}\ln\left(\frac{7}{6}\right)$

$2 : \begin{cases} 1 : \text{considers } x'(t) = 0 \\ 1 : \text{solution} \end{cases}$

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Question 3

An object moving along a curve in the xy-plane has position $(x(t), y(t))$ at time $t \geq 0$ with $\frac{dx}{dt} = 3 + \cos\left(t^2\right)$. The derivative $\frac{dy}{dt}$ is not explicitly given. At time $t = 2$, the object is at position $(1, 8)$.

(a) Find the x-coordinate of the position of the object at time $t = 4$.

(b) At time $t = 2$, the value of $\frac{dy}{dt}$ is -7. Write an equation for the line tangent to the curve at the point $(x(2), y(2))$.

(c) Find the speed of the object at time $t = 2$.

(d) For $t \geq 3$, the line tangent to the curve at $(x(t), y(t))$ has a slope of $2t + 1$. Find the acceleration vector of the object at time $t = 4$.

(a) $x(4) = x(2) + \int_2^4 \left(3 + \cos\left(t^2\right)\right) dt$

$= 1 + \int_2^4 \left(3 + \cos\left(t^2\right)\right) dt = 7.132$ or 7.133

3 : { 1 : $\int_2^4 \left(3 + \cos\left(t^2\right)\right) dt$; 1 : handles initial condition; 1 : answer }

(b) $\left.\frac{dy}{dx}\right|_{t=2} = \left.\frac{\frac{dy}{dt}}{\frac{dx}{dt}}\right|_{t=2} = \frac{-7}{3 + \cos 4} = -2.983$

$y - 8 = -2.983(x - 1)$

2 : { 1 : finds $\left.\frac{dy}{dx}\right|_{t=2}$; 1 : equation }

(c) The speed of the object at time $t = 2$ is $\sqrt{(x'(2))^2 + (y'(2))^2} = 7.382$ or 7.383.

1 : answer

(d) $x''(4) = 2.303$

$y'(t) = \frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt} = (2t + 1)\left(3 + \cos\left(t^2\right)\right)$

$y''(4) = 24.813$ or 24.814

The acceleration vector at $t = 4$ is $\langle 2.303, 24.813 \rangle$ or $\langle 2.303, 24.814 \rangle$.

3 : { 1 : $x''(4)$; 1 : $\frac{dy}{dt}$; 1 : answer }