Section 4.8 – Rolle's Theorem & Mean-Value Theorem

Verify that the hypotheses of Rolle's Theorem are satisfied on the given interval and find all values of *c* in that interval that satisfy the conclusion of the theorem.

1. $f(x) = x^2 - 8x + 15;\ [3,5]$

3. $f(x) = \cos x\,;\ [\frac{\pi}{2}, \frac{3\pi}{2}]$

Verify that the hypotheses of the Mean-Value Theorem are satisfied on the given interval, and find all values of *c* in that interval that satisfy the conclusion of the theorem.

5. $f(x) = x^2 - x; [-3,5]$

7. $f(x) = \sqrt{25 - x^2};\ [-5,3]$

9. a. Find an interval [a,b] on which $f(x) = x^4 + x^3 - x^2 + x - 2$ satisfies the hypotheses of Rolle's Theorem
   b. Generate the graph of f '(x) and use it to make rough estimates of all values of *c* in the interval obtained in part (a) that satisfy the conclusion of Rolle's Theorem.

15. Let $f(x) = \tan x$.
   a. Show that there is no point c in the interval (0, π) such that f'(c)=0, even though f(0)=f(π)=0.
   b. Explain why the result in part (a) does not contradict Rolle's Theorem.

19. An automobile travels 4 mi along a straight road in 5 mins. Show that the speedometer reads exactly 48 mi/hr at least once during the trip. See example 5 on P306.

22. Use the fact that $\frac{d}{dx}[x\ln(2-x)] = \ln(2-x) - \frac{x}{2-x}$ to show that the equation $x = (2-x)\ln(2-x)$ has at least one solution in the interval (0,1).