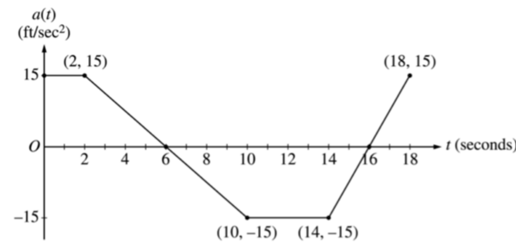
AP Calculus AB Practice: Accumulation Functions

AP® CALCULUS AB
2001 SCORING GUIDELINES

Question 3



A car is traveling on a straight road with velocity 55 ft/sec at time $t = 0$. For $0 \le t \le 18$ seconds, the car's acceleration $a(t)$, in ft/sec^2, is the piecewise linear function defined by the graph above.

(a) Is the velocity of the car increasing at $t = 2$ seconds? Why or why not?

(b) At what time in the interval $0 \le t \le 18$, other than $t = 0$, is the velocity of the car 55 ft/sec? Why?

(c) On the time interval $0 \le t \le 18$, what is the car's absolute maximum velocity, in ft/sec, and at what time does it occur? Justify your answer.

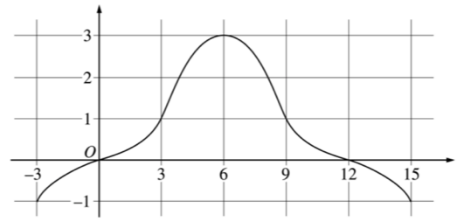
(d) At what times in the interval $0 \le t \le 18$, if any, is the car's velocity equal to zero? Justify your answer.

NO CALCULATOR

AP® CALCULUS AB
2002 SCORING GUIDELINES (Form B)

Question 4

NO CALCULATOR



Graph of f

The graph of a differentiable function f on the closed interval $[-3, 15]$ is shown in the figure above. The graph of f has a horizontal tangent line at $x = 6$. Let $g(x) = 5 + \int_6^x f(t)\,dt$ for $-3 \le x \le 15$.

(a) Find $g(6)$, $g'(6)$, and $g''(6)$.

(b) On what intervals is g decreasing? Justify your answer.

(c) On what intervals is the graph of g concave down? Justify your answer.

(d) Find a trapezoidal approximation of $\int_{-3}^{15} f(t)\,dt$ using six subintervals of length $\Delta t = 3$.

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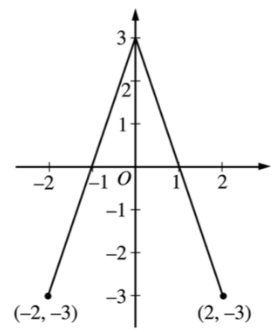
Question 4

NO CALCULATOR

The graph of the function f shown above consists of two line segments. Let g be the function given by $g(x) = \int_0^x f(t)\,dt$.



Graph of f

(a) Find $g(-1)$, $g'(-1)$, and $g''(-1)$.

(b) For what values of x in the open interval $(-2, 2)$ is g increasing? Explain your reasoning.

(c) For what values of x in the open interval $(-2, 2)$ is the graph of g concave down? Explain your reasoning.

(d) On the axes provided, sketch the graph of g on the closed interval $[-2, 2]$.

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2003 SCORING GUIDELINES (Form B)

Question 5

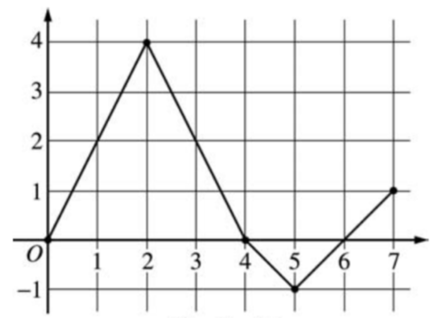
Let f be a function defined on the closed interval $[0,7]$. The graph of f, consisting of four line segments, is shown above. Let g be the function given by $g(x) = \int_2^x f(t)\,dt$.

(a) Find $g(3)$, $g'(3)$, and $g''(3)$.

(b) Find the average rate of change of g on the interval $0 \le x \le 3$.

(c) For how many values c, where $0 < c < 3$, is $g'(c)$ equal to the average rate found in part (b)? Explain your reasoning.

(d) Find the x-coordinate of each point of inflection of the graph of g on the interval $0 < x < 7$. Justify your answer.



Graph of f

NO CALCULATOR

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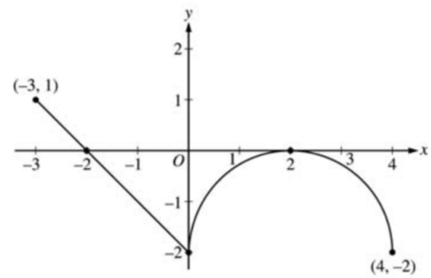
Question 4

Let f be a function defined on the closed interval $-3 \le x \le 4$ with $f(0) = 3$. The graph of f', the derivative of f, consists of one line segment and a semicircle, as shown above.



Graph of f'

NO CALCULATOR

(a) On what intervals, if any, is f increasing? Justify your answer.

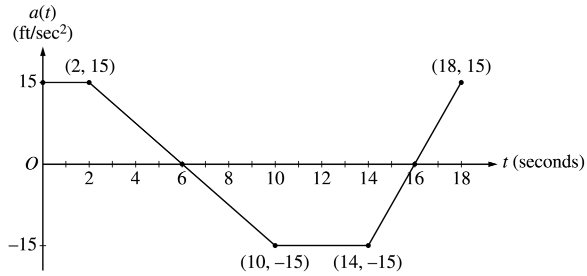
(b) Find the x-coordinate of each point of inflection of the graph of f on the open interval $-3 < x < 4$. Justify your answer.

(c) Find an equation for the line tangent to the graph of f at the point $(0,3)$.

(d) Find $f(-3)$ and $f(4)$. Show the work that leads to your answers.

Question 3

A car is traveling on a straight road with velocity 55 ft/sec at time $t = 0$. For $0 \le t \le 18$ seconds, the car's acceleration $a(t)$, in ft/sec^2, is the piecewise linear function defined by the graph above.



(a) Is the velocity of the car increasing at $t = 2$ seconds? Why or why not?

(b) At what time in the interval $0 \le t \le 18$, other than $t = 0$, is the velocity of the car 55 ft/sec? Why?

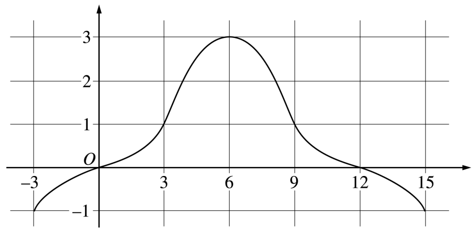
(c) On the time interval $0 \le t \le 18$, what is the car's absolute maximum velocity, in ft/sec, and at what time does it occur? Justify your answer.

(d) At what times in the interval $0 \le t \le 18$, if any, is the car's velocity equal to zero? Justify your answer.

(a) Since $v'(2) = a(2)$ and $a(2) = 15 > 0$, the velocity is increasing at $t = 2$.	1 : answer and reason
(b) At time $t = 12$ because $v(12) - v(0) = \int_0^{12} a(t)\,dt = 0$.	$2 : \begin{cases} 1 : t = 12 \\ 1 : \text{reason} \end{cases}$
(c) The absolute maximum velocity is 115 ft/sec at $t = 6$. The absolute maximum must occur at $t = 6$ or at an endpoint. $v(6) = 55 + \int_0^6 a(t)\,dt = 55 + 2(15) + \frac{1}{2}(4)(15) = 115 > v(0)$; $\int_6^{18} a(t)\,dt < 0$ so $v(18) < v(6)$	$4 : \begin{cases} 1 : t = 6 \\ 1 : \text{absolute maximum velocity} \\ 1 : \text{identifies } t = 6 \text{ and } t = 18 \text{ as candidates or indicates that } v \text{ increases, decreases, then increases} \\ 1 : \text{eliminates } t = 18 \end{cases}$
(d) The car's velocity is never equal to 0. The absolute minimum occurs at $t = 16$ where $v(16) = 115 + \int_6^{16} a(t)\,dt = 115 - 105 = 10 > 0$.	$2 : \begin{cases} 1 : \text{answer} \\ 1 : \text{reason} \end{cases}$

Question 4

The graph of a differentiable function f on the closed interval $[-3,15]$ is shown in the figure above. The graph of f has a horizontal tangent line at $x = 6$. Let $g(x) = 5 + \int_6^x f(t)\,dt$ for $-3 \le x \le 15$.



Graph of f

(a) Find $g(6)$, $g'(6)$, and $g''(6)$.

(b) On what intervals is g decreasing? Justify your answer.

(c) On what intervals is the graph of g concave down? Justify your answer.

(d) Find a trapezoidal approximation of $\int_{-3}^{15} f(t)\,dt$ using six subintervals of length $\Delta t = 3$.

(a) $g(6) = 5 + \int_6^6 f(t)\,dt = 5$ $g'(6) = f(6) = 3$ $g''(6) = f'(6) = 0$	$3 \begin{cases} 1 : g(6) \\ 1 : g'(6) \\ 1 : g''(6) \end{cases}$
(b) g is decreasing on $[-3,0]$ and $[12,15]$ since $g'(x) = f(x) < 0$ for $x < 0$ and $x > 12$.	$3 \begin{cases} 1 : [-3,0] \\ 1 : [12,15] \\ 1 : \text{justification} \end{cases}$
(c) The graph of g is concave down on $(6,15)$ since $g' = f$ is decreasing on this interval.	$2 \begin{cases} 1 : \text{interval} \\ 1 : \text{justification} \end{cases}$
(d) $\frac{3}{2}(-1 + 2(0 + 1 + 3 + 1 + 0) - 1)$ $= 12$	1 : trapezoidal method

Question 4

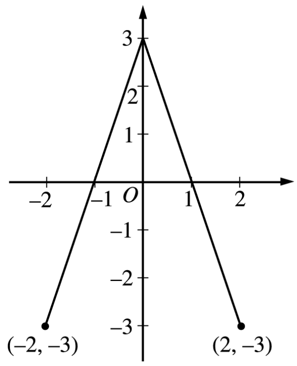
The graph of the function f shown above consists of two line segments. Let g be the function given by $g(x) = \int_0^x f(t)\,dt$.

(a) Find $g(-1)$, $g'(-1)$, and $g''(-1)$.

(b) For what values of x in the open interval $(-2,2)$ is g increasing? Explain your reasoning.

(c) For what values of x in the open interval $(-2,2)$ is the graph of g concave down? Explain your reasoning.

(d) On the axes provided, sketch the graph of g on the closed interval $[-2,2]$.



Graph of f

(a) $g(-1) = \int_0^{-1} f(t)\,dt = -\int_{-1}^0 f(t)\,dt = -\dfrac{3}{2}$

$g'(-1) = f(-1) = 0$

$g''(-1) = f'(-1) = 3$

$$3\begin{cases} 1:\ g(-1) \\ 1:\ g'(-1) \\ 1:\ g''(-1) \end{cases}$$

(b) g is increasing on $-1 < x < 1$ because $g'(x) = f(x) > 0$ on this interval.

$$2\begin{cases} 1:\ \text{interval} \\ 1:\ \text{reason} \end{cases}$$

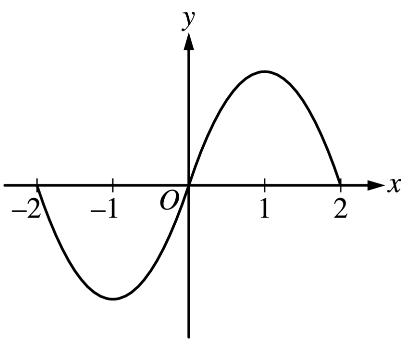
(c) The graph of g is concave down on $0 < x < 2$ because $g''(x) = f'(x) < 0$ on this interval.

or

because $g'(x) = f(x)$ is decreasing on this interval.

$$2\begin{cases} 1:\ \text{interval} \\ 1:\ \text{reason} \end{cases}$$

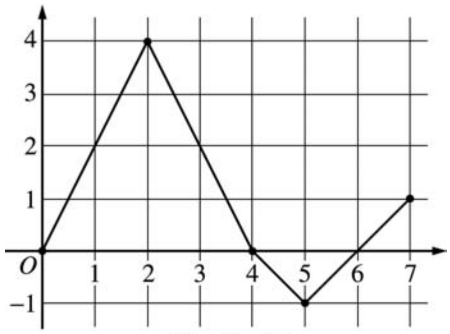
(d)



$$2\begin{cases} 1:\ g(-2) = g(0) = g(2) = 0 \\ 1:\ \text{appropriate increasing/decreasing} \\ \quad\ \ \text{and concavity behavior} \\ \quad\ \ < -1 > \text{vertical asymptote} \end{cases}$$

Question 5

Let f be a function defined on the closed interval $[0,7]$. The graph of f, consisting of four line segments, is shown above. Let g be the function given by $g(x) = \int_2^x f(t)\,dt$.



Graph of f

(a) Find $g(3)$, $g'(3)$, and $g''(3)$.

(b) Find the average rate of change of g on the interval $0 \le x \le 3$.

(c) For how many values c, where $0 < c < 3$, is $g'(c)$ equal to the average rate found in part (b)? Explain your reasoning.

(d) Find the x-coordinate of each point of inflection of the graph of g on the interval $0 < x < 7$. Justify your answer.

(a) $g(3) = \int_2^3 f(t)\,dt = \frac{1}{2}(4+2) = 3$

$g'(3) = f(3) = 2$

$g''(3) = f'(3) = \frac{0-4}{4-2} = -2$

$$3 : \begin{cases} 1 : g(3) \\ 1 : g'(3) \\ 1 : g''(3) \end{cases}$$

(b) $\frac{g(3)-g(0)}{3} = \frac{1}{3}\int_0^3 f(t)\,dt$

$= \frac{1}{3}\left(\frac{1}{2}(2)(4) + \frac{1}{2}(4+2)\right) = \frac{7}{3}$

$$2 : \begin{cases} 1 : g(3) - g(0) = \int_0^3 f(t)\,dt \\ 1 : \text{answer} \end{cases}$$

(c) There are two values of c.

We need $\frac{7}{3} = g'(c) = f(c)$

The graph of f intersects the line $y = \frac{7}{3}$ at two places between 0 and 3.

$$2 : \begin{cases} 1 : \text{answer of 2} \\ 1 : \text{reason} \end{cases}$$

Note: 1/2 if answer is 1 by MVT

(d) $x = 2$ and $x = 5$

because $g' = f$ changes from increasing to decreasing at $x = 2$, and from decreasing to increasing at $x = 5$.

$$2 : \begin{cases} 1 : x = 2 \text{ and } x = 5 \text{ only} \\ 1 : \text{justification} \\ \quad (\text{ignore discussion at } x = 4) \end{cases}$$

Question 4

Let f be a function defined on the closed interval $-3 \le x \le 4$ with $f(0) = 3$. The graph of f', the derivative of f, consists of one line segment and a semicircle, as shown above.

(a) On what intervals, if any, is f increasing? Justify your answer.

(b) Find the x-coordinate of each point of inflection of the graph of f on the open interval $-3 < x < 4$. Justify your answer.

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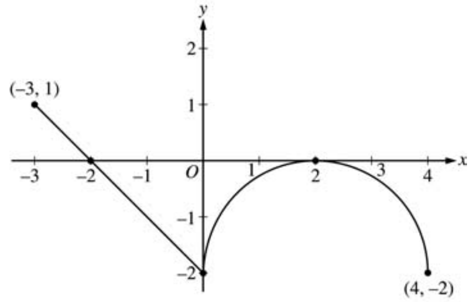

Graph of f'

(a) The function f is increasing on $[-3,-2]$ since $f' > 0$ for $-3 \le x < -2$.	$2: \begin{cases} 1: \text{interval} \\ 1: \text{reason} \end{cases}$
(b) $x = 0$ and $x = 2$ f' changes from decreasing to increasing at $x = 0$ and from increasing to decreasing at $x = 2$	$2: \begin{cases} 1: x = 0 \text{ and } x = 2 \text{ only} \\ 1: \text{justification} \end{cases}$
(c) $f'(0) = -2$ Tangent line is $y = -2x + 3$.	1 : equation
(d) $f(0) - f(-3) = \int_{-3}^{0} f'(t)\,dt = \frac{1}{2}(1)(1) - \frac{1}{2}(2)(2) = -\frac{3}{2}$ $f(-3) = f(0) + \frac{3}{2} = \frac{9}{2}$ $f(4) - f(0) = \int_{0}^{4} f'(t)\,dt = -\left(8 - \frac{1}{2}(2)^2\pi\right) = -8 + 2\pi$ $f(4) = f(0) - 8 + 2\pi = -5 + 2\pi$	$4: \begin{cases} 1: \pm\left(\frac{1}{2} - 2\right) \\ \quad \text{(difference of areas of triangles)} \\ 1: \text{answer for } f(-3) \text{ using FTC} \\ 1: \pm\left(8 - \frac{1}{2}(2)^2\pi\right) \\ \quad \text{(area of rectangle} - \text{area of semicircle)} \\ 1: \text{answer for } f(4) \text{ using FTC} \end{cases}$