Sections 3.5-3.6: Local Linear Approximation & L'Hospital's Rule

3. a. Find the local linear approximation of the function $f(x) = \sqrt{1+x}$ at $x_0 = 0$, and use it to approximate $\sqrt{0.9}$ and $\sqrt{1.1}$.
   b. Graph f and it tangent line at x_0 together, and use the graphs to illustrate the relationship between the exact values and the approximations of $\sqrt{0.9}$ and $\sqrt{1.1}$.

6. Confirm that the stated formula is the local linear approximation at $x_0 = 0$,

$$\frac{1}{\sqrt{1-x}} \approx 1 + \frac{1}{2}x$$

11. Confirm that the stated formula is the local linear approximation at $x_0 = 1$, where $\Delta x = x - 1$

$$f(x) = x^4; \;\; (1+\Delta x)^4 \approx 1 + 4\Delta x$$

17. Confirm that the stated formula is the local linear approximation at $x_0 = 0$, and use a graphing calculator to estimate an interval of *x*-values on which the error is at most ± 0.1.

$$\sqrt{x+3} \approx \sqrt{3} + \frac{1}{2\sqrt{3}}x$$

21. a. Use the local linear approximation of sin x at $x_0 = 0$ obtained in example 2 to approximate $\sin\ 1°$, and compare the approximation to the result produced directly by your calculator.
    b. How would you choose x_0 to approximate $\sin\ 44°$?
    c. Approximate $\sin\ 44°$; compare the approximation to the result produced directly on your calculator.

31. Use an appropriate local linear approximation to estimate the value $\cos\ 31°$.

37. a. Let y=1/x. Find dy and Δy at x=1 with $dx = \Delta x = -0.5$.
    b. Sketch the graph of y=1/x,, showing dy and Δy in the picture.

Find the formulas for dy and Δy.

39. $y = x^3$ 43. $y = x\cos x$

56. The side of a cube is measured to be 25 cm, with a possible error of ±1 cm.
    a. Use differentials to estimate the error in the calculated volume.
    b. Estimate the percentage errors in the side and volume.

62. The side of a square is measured with a possible percentage error of ±1%. Use differentials to estimate the percentage error in the area.

1. Evaluate the given limit without using L'Hospital's Rule, and then check that your answer is correct using L'Hospital's Rule.
    a. $\lim_{x \to 2} \frac{x^2-4}{x^2+2x-8}$
    b. $\lim_{x \to \infty} \frac{2x-5}{3x+7}$

Find the limits.

7. $\lim_{x \to 0} \frac{e^x-1}{\sin x}$

9. $\lim_{x \to 0} \frac{\tan \theta}{\theta}$

11. $\lim_{x \to \pi^+} \frac{\sin x}{x-\pi}$

13. $\lim_{x \to \infty} \frac{\ln x}{x}$

15. $\lim_{x \to 0^+} \frac{\cot x}{\ln x}$

17. $\lim_{x \to \infty} \frac{x^{100}}{e^x}$

23. $\lim_{x \to \infty} x \sin \frac{\pi}{x}$

33. $\lim_{x \to 0} (\csc x - \frac{1}{x})$

39. $\lim_{x \to 0^+} x^{\sin x}$

Make a conjecture about the limit by graphing the function involved with a graphing utility, then check your conjecture using L'Hospital's Rule.

49. $\lim_{x \to \infty} \frac{\ln(\ln(x))}{\sqrt{x}}$

57. Limits of the type $\frac{0}{\infty}, \frac{\infty}{0}, 0^\infty, \infty \cdot \infty, \infty + \infty, \infty - (-\infty), -\infty + (-\infty), -\infty - \infty$ are NOT indeterminate forms. Find the following limits by inspection.

a. $\lim_{x \to 0^+} \frac{x}{\ln x}$
b. $\lim_{x \to \infty} \frac{x^3}{e^{-x}}$
c. $\lim_{x \to \frac{\pi}{2}^-} (\cos x)^{\tan x}$
d. $\lim_{x \to 0^+} (\ln x) \cot x$
e. $\lim_{x \to 0^+} \left(\frac{1}{x} - \ln x\right)$
f. $\lim_{x \to -\infty} (x + x^3)$