# Today's objectives:

1. Define segments related to triangles - medians, altitudes, perpendicular bisectors and angle bisectors.

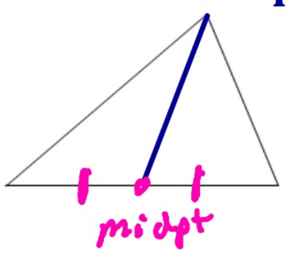
2. Identify all of these segments in triangles.

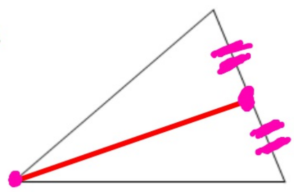
Geometry/Trigonometry II Name ___________________________________

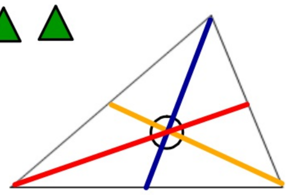
4.7 Medians, Altitudes, & Perpendicular Bisectors

***New Terminology***

Median – **A segment drawn from a vertex to the midpoint of the opposite side.**



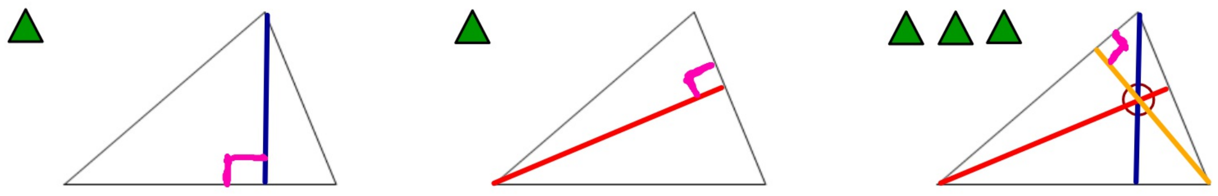





**Centroid**

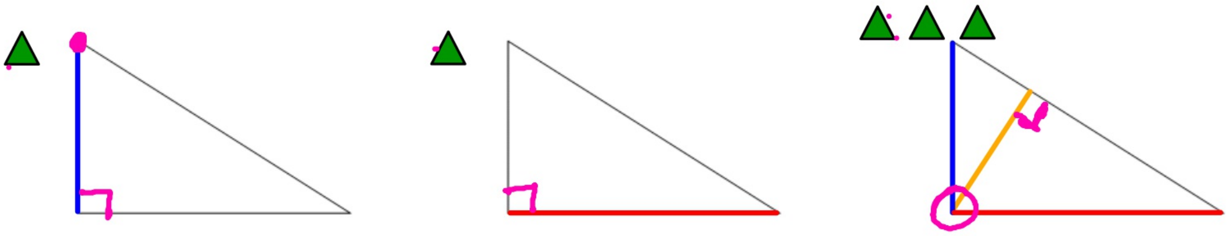
Altitude – **A perpendicular segment drawn from a vertex to a line containing the opposite side.**

*Altitudes in acute triangles:*



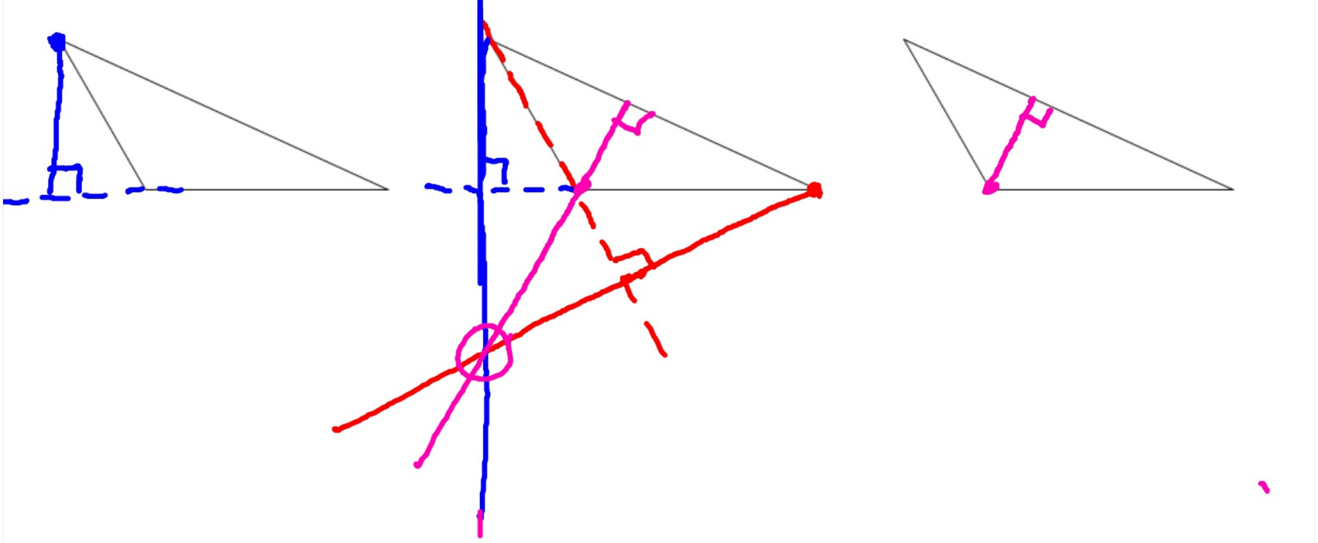
**Orthocenter**

*Altitudes in right triangles:*



**The legs of a right triangle are altitudes.**

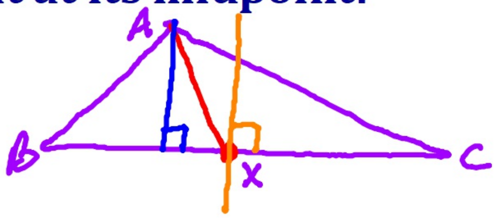
*Altitudes in obtuse triangles:*



**Altitudes drawn from the acute angles of an obtuse triangle are found outside of the triangle.**

Perpendicular Bisector – **A line, ray, or segment that is perpendicular to a segment at its midpoint.**

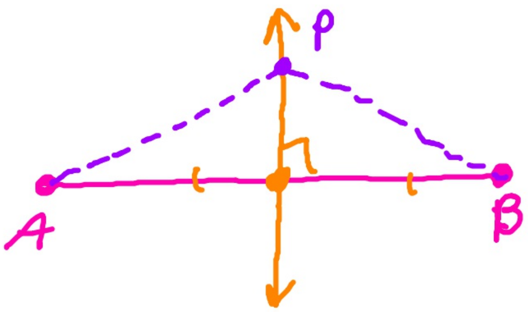
median
altitude
⊥ bisector




** notes about a perpendicular bisector*

- **In equilateral triangles, medians and altitudes are the same segments - in other words, they are <u>perpendicular bisectors</u>.**
- **The median to the base of an isosceles triangle is also an altitude - this is also a <u>perpendicular bisector</u>.**
- **Perpendicular bisectors of scalene triangles do not intersect any of the vertices.**

| |
|---|
| **Theorem 4-5** If a point lies on the perpendicular bisector of a segment, then the point is equidistant from the endpoints of the segment. |



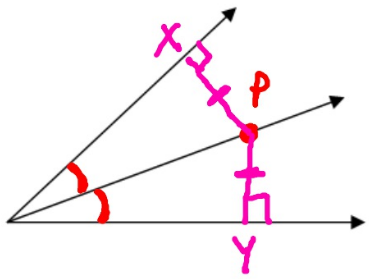

$PA = PB$

| |
|---|
| **Theorem 4-6** If a point is equidistant from the endpoints of a segment, then the point lies on the perpendicular bisector of the segment. |

Distance from a point to a line (or plane) – **The length of the perpendicular segment drawn from the point to the line.**

(the shortest possible distance)

**Theorem 4-7** If a point lies on the bisector of an angle, then the point is equidistant from the sides of the angle.




$\overline{PX} = \overline{PY}$

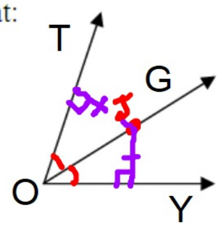
**Theorem 4-8** If a point is equidistant from the sides of an angle, then the point lies on the bisector of the angle.

**Fill in the blanks with sometimes, always, or never:**

1. An altitude is **always** perpendicular to the line containing the opposite side.
2. A median is **sometimes** perpendicular to the opposite side.
3. An altitude is **sometimes** an angle bisector.
4. An angle bisector is **sometimes** perpendicular to the opposite side.
5. A point that lies on the perpendicular bisector of a segment is **always** equidistant from the endpoints of a segment.

**What conclusion can you draw from the following information:**

Suppose $\overrightarrow{OG}$ bisects $\angle TOY$. What can you deduce if you also know that:




a. Point J lies on $\overrightarrow{OG}$ $\overrightarrow{OT}$ & $\overrightarrow{OY}$ are equidistant from J.

b. A point K is such that the distance from K to $\overrightarrow{OT}$ and to $\overrightarrow{OY}$ is 13 cm.

- K is on the bisector of $\angle TOY$.
- K is on $\overrightarrow{OG}$.

## Classroom Exercises

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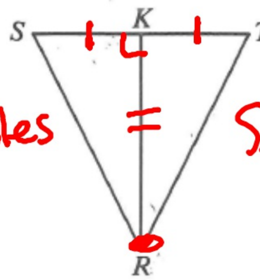
**Complete.**

**1.** If $K$ is the midpoint of $\overline{ST}$, then $\overline{RK}$ is called a(n) _?_ of $\triangle RST$. median

**2.** If $\overline{RK} \perp \overline{ST}$, then $\overline{RK}$ is called a(n) _?_ of $\triangle RST$. altitude

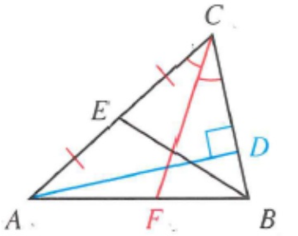
**3.** If $K$ is the midpoint of $\overline{ST}$ and $\overline{RK} \perp \overline{ST}$, then $\overline{RK}$ is called a(n) _?_ of $\overline{ST}$. ⊥ bisector




**4.** If $\overline{RK}$ is both an altitude and a median of $\triangle RST$, then:
**a.** $\triangle RSK \cong \triangle RTK$ by SAS **b.** $\triangle RST$ is a(n) _?_ triangle. isosceles

**5.** If $R$ is on the perpendicular bisector of $\overline{ST}$, then $R$ is equidistant from S and T. Thus $\overline{SR}$ = $\overline{TR}$.

**6.** Refer to $\triangle ABC$ and name each of the following.
**a.** a median of $\triangle ABC$ $\overline{EB}$
**b.** an altitude of $\triangle ABC$ $\overline{AD}$
**c.** a bisector of an angle of $\triangle ABC$ $\overline{FC}$

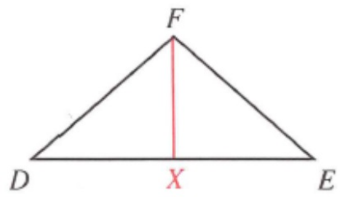



**7.** Draw $\overline{XY}$. Label its midpoint $Q$.
**a.** Select a point $P$ equidistant from $X$ and $Y$. Draw $\overline{PX}$, $\overline{PY}$, and $\overline{PQ}$.
**b.** What postulate justifies the statement $\triangle PQX \cong \triangle PQY$?
**c.** What reason justifies the statement $\angle PQX \cong \angle PQY$?
**d.** What reason justifies the statement $\overleftrightarrow{PQ} \perp \overline{XY}$?
**e.** What name for $\overleftrightarrow{PQ}$ best describes the relationship between $\overleftrightarrow{PQ}$ and $\overline{XY}$?

**8.** Given: $\triangle DEF$ is isosceles with $DF = EF$;
$\overline{FX}$ bisects $\angle DFE$.

**a.** Would the median drawn from $F$ to $\overline{DE}$ be the same segment as $\overline{FX}$?

**b.** Would the altitude drawn from $F$ to $\overline{DE}$ be the same segment as $\overline{FX}$?




**9.** What kind of triangle has three angle bisectors that are also altitudes and medians?

**10.** Given: $\overrightarrow{NO}$ bisects $\angle N$.
What can you conclude from each of the following additional statements?

**a.** $P$ lies on $\overrightarrow{NO}$.

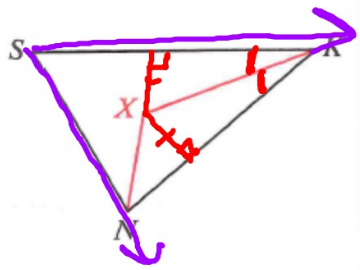
**b.** The distance from a point $Q$ to each side of $\angle N$ is 13.

**Complete each statement.**

**7.** If $X$ is on the bisector of $\boxed{\angle SKN}$, then $X$ is equidistant from $\overline{KS}$ and $\overline{KN}$.

**8.** If $X$ is on the bisector of $\angle SNK$, then $X$ is equidistant from $\overline{NK}$ and $\overline{NS}$.

**9.** If $X$ is equidistant from $\overline{SK}$ and $\overline{SN}$, then $X$ lies on the bisector $\angle NSK$

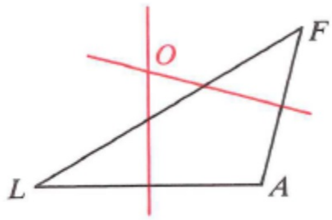



**10.** If $O$ is on the perpendicular bisector of $\overline{LA}$, then $O$ is equidistant from L and A.

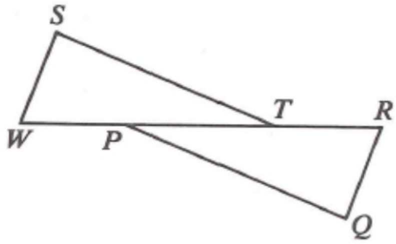
**11.** If $O$ is on the perpendicular bisector of $\overline{AF}$, then $O$ is equidistant from A and F.

**12.** If $O$ is equidistant from $L$ and $F$, then $O$ lies on the _?_.

$\perp$ bisector of $\overline{LF}$



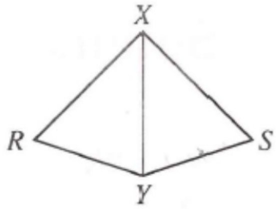
**The two triangles shown are congruent. Complete.**




4–1

1. $\triangle STW \cong$ __?__
2. $\triangle PQR \cong$ __?__
3. $\angle R \cong$ __?__
4. __?__ $= RP$

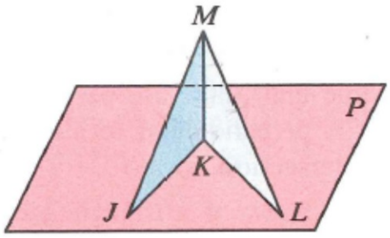
**Can you deduce from the given information that $\triangle RXY \cong \triangle SXY$? If so, what postulate can you use?**




4–2

5. Given: $\overline{RX} \cong \overline{SX}$; $\overline{RY} \cong \overline{SY}$
6. Given: $\overline{RY} \cong \overline{SY}$; $\angle R \cong \angle S$
7. Given: $\overline{XY}$ bisects $\angle RXS$ and $\angle RYS$.
8. Given: $\angle RXY \cong \angle SXY$; $\overline{RX} \cong \overline{SX}$

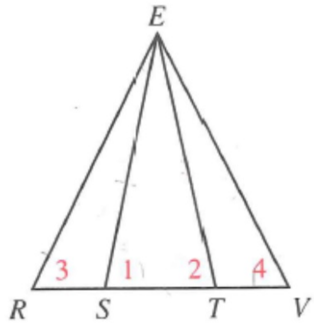
**Write proofs in two-column form.**




4–3

9. Given: $\overline{JM} \cong \overline{LM}$; $\overline{JK} \cong \overline{LK}$
   Prove: $\angle MJK \cong \angle MLK$
10. Given: $\angle JMK \cong \angle LMK$; $\overline{MK} \perp$ plane $P$
    Prove: $\overline{JK} \cong \overline{LK}$

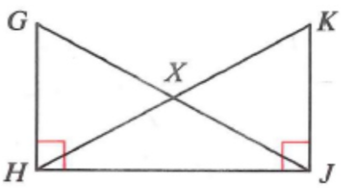
**Complete.**




4–4

11. If $\angle 3 \cong \angle 4$, then which segments must be congruent?
12. If $\triangle REV$ is an equiangular triangle, then $\triangle REV$ is also a(n) __?__ triangle.
13. If $\overline{ES} \cong \overline{ET}$, $m\angle 1 = 75$, and $m\angle 2 = 3x$, then $x =$ __?__.
14. If $\angle 1 \cong \angle 2$, $ES = 3y + 5$, and $ET = 25 - y$, then $y =$ __?__.

**Write proofs in two-column form.**




4–5

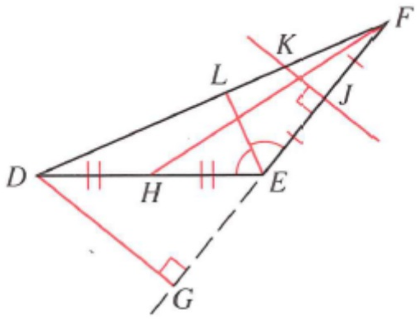
15. Given: $\overline{GH} \perp \overline{HJ}$; $\overline{KJ} \perp \overline{HJ}$;
    $\angle G \cong \angle K$
    Prove: $\triangle GHJ \cong \triangle KJH$
16. Given: $\overline{GH} \perp \overline{HJ}$; $\overline{KJ} \perp \overline{HJ}$;
    $\overline{GJ} \cong \overline{KH}$
    Prove: $\overline{GH} \cong \overline{KJ}$

**18.** Refer to $\triangle DEF$ and name each of the following: 4–7

**a.** an altitude

**b.** a median

**c.** the perpendicular bisector of a side of the triangle




**11.** In $\triangle TOP$, if $OT > OP$, then $m\angle P >$ _?_. 6–4

**12.** In $\triangle RED$, if $m\angle D < m\angle E$, then $RD >$ _?_.

**13.** Points $X$ and $Y$ are in plane $M$. If $\overline{PX} \perp$ plane $M$, then $PX$ _?_ $PY$.

**14.** Two sides of a triangle have lengths 6 and 8. The length of the third side must be greater than _?_ and less than _?_.