

Sections 5.9-5.10: Evaluating Integrals with Substitution & Functions with Integrals

Evaluate the definite integral two ways: first by finding a u -substitution in the definite integral and then by a u -substitution in the corresponding indefinite integral.

13. $\int_{-2}^{-1} \frac{x}{(x^2+2)^3} dx$

17. $\int_1^3 \frac{dx}{\sqrt{x}(x+1)}$

Evaluate the definite integral by expressing it in terms of u and evaluating the resulting integral using a formula from geometry.

19. $\int_{-5/3}^{5/3} \sqrt{25 - 9x^2} dx$; $u = 3x$

Evaluate the integral by any method.

39. $\int_0^{\sqrt{\pi}} 5x \cos(x^2) dx$

43. $\int_0^1 \frac{y^2 dy}{\sqrt{4-3y}}$

47. $\int_0^1 \frac{x}{\sqrt{4-3x^4}} dx$

1. $\int_1^{1/e} \frac{1}{t} dt = \underline{\hspace{2cm}}$

5. $\frac{d}{dx} \int_0^{e^{-x}} \frac{1}{1+t^4} dt = \underline{\hspace{2cm}}$

Find the derivative using Formula (18), and check your answer by evaluating the integral and then differentiating the result.

15. a. $\frac{d}{dx} \int_1^{x^3} \frac{1}{t} dt$

b. $\frac{d}{dx} \int_1^{\ln x} e^t dt$

24. Show that

a. $\frac{d}{dx} \int_x^a f(t) dt = -f(x)$

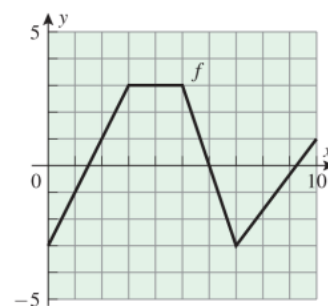
b. $\frac{d}{dx} \int_{g(x)}^a f(t) dt = -f(g(x))g'(x)$

27. Find $\frac{d}{dx} \int_{3x}^{x^2} \frac{t-1}{t^2+1} dt$

30. Prove that the function $F(x) = \int_x^{5x} \frac{1}{t} dt$ is constant on the interval $(0, \infty)$ by finding $F'(x)$. What is the constant?

31. Let $F(x) = \int_0^x f(t) dt$, where f is the function whose graph is shown in the accompanying figure.

- Find $F(0)$, $F(3)$, $F(5)$, $F(7)$, and $F(10)$.
- On what subintervals of the interval $[0, 10]$ is F increasing? Decreasing?
- Where does F have its maximum value? Its minimum value?
- Sketch the graph of F .



33. Express $F(x) = \int_{-1}^x |t| dt$ in a piecewise form that does not involve an integral.

41. The accompanying figure shows the graphs of $y = f(x)$ and $y = \int_0^x f(t) dt$. Determine which graph is which and explain your reasoning.

