Name ___________________________ Date _____ Per. ______ Ms. Williams/Mrs. Hertel

Day 7: Solving Exponential Word Problems involving Logarithms

Warm – Up

The growth of bacteria in a dish is modeled by the function $f(t) = 2^{\frac{t}{3}}$. For which value of t is $f(t) = 32$?

1) 8
2) 2
3) 15
4) 16

Exponential growth occurs when a quantity increases by the same rate *r* in each period *t*. When this happens, the value of the quantity at any given time can be calculated as a function of the rate and the original amount.

Exponential Growth

An exponential growth function has the form $y = a(1 + r)^t$, where $a > 0$.

- y represents the final amount.
- a represents the original amount.
- r represents the rate of growth expressed as a decimal.
- t represents time.

Exponential decay occurs when a quantity decreases by the same rate *r* in each time period *t*. Just like exponential growth, the value of the quantity at any given time can be calculated by using the rate and the original amount.

Exponential Decay

An exponential decay function has the form $y = a(1 - r)^t$, where $a > 0$.

- y represents the final amount.
- a represents the original amount.
- r represents the rate of decay as a decimal.
- t represents time.

In Summary, Annual growth or depreciation: $A = a(1 \pm r)^t$

growth +, depreciation -

Example 1: "Growth"

The original value of a painting is \$9,000 and the value increases by 7% each year.

Part a: Then find the painting's value in 15 years.

Part b: In what year, will the painting be worth \$50,000?

Example 2: "Decay"

The population of a town is decreasing at a rate of 3% per year. In 2000 there were 1700 people.

Part a: Find the population in 2012.

Part b: In what year, will the population be double?

3) Is the equation $A = 3200\,(0.70)^t$ a model of exponential growth or exponential decay, and what is the rate (percent) of change per time period?

Explain here!

1) exponential growth and 30%
2) exponential growth and 70%
3) exponential decay and 30%
4) exponential decay and 70%

---

4) Is the equation $A = 1756\,(1.17)^t$ a model of exponential growth or exponential decay, and what is the rate (percent) of change per time period?

Explain here!

1) exponential growth and 17%
2) exponential growth and 83%
3) exponential decay and 17%
4) exponential decay and 83%

---

5) Thi purchased a car for $22,900. The car depreciated at an annual rate of 16%. After 5 years Thi wants to sell her car. Which of the following equations models the value of Thi's car?

A $A = 22{,}900(1.16)^5$

B $A = 22{,}900(0.16)^5$

C $A = 16(22{,}900)^5$

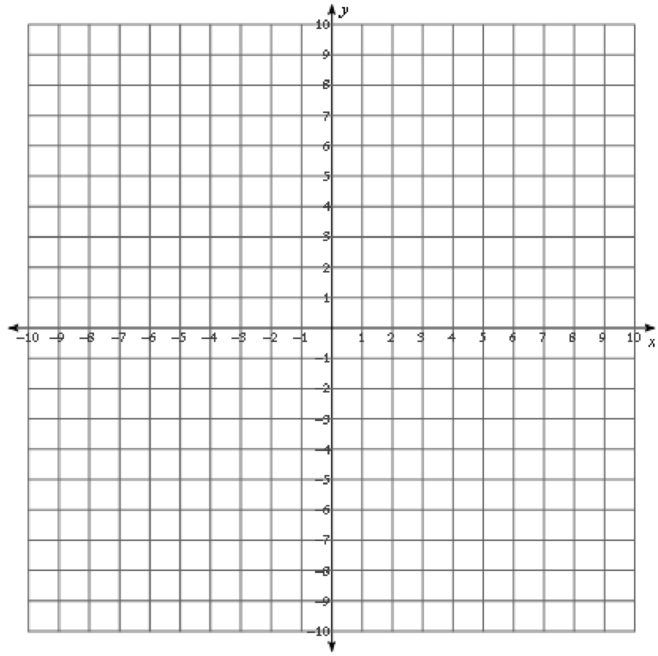
D $A = 22{,}900(0.84)^5$

Graphs of Logarithmic Functions

Using the table below: a) Complete the table of values for y= 2^x

b) sketch the graph of y= 2^x

x	y
-2	
-1	
0	
1	
2	



2) Recall:

How do we find the inverse of a function?

Find the inverse algebraically.

Properties of	**Properties of**
Domain:	Domain:
Range:	Range:
Asymptote:	Asymptote:
x-intercept:	x-intercept:
y-intercept:	y-intercept:

3) Graph the inverse of the function y = 2^x.

Graphing Logarithmic Functions

Steps:
1 - Find inverse
2 - Make xy chart
3 - Flip xy values to find values for log graph
4 - Graph

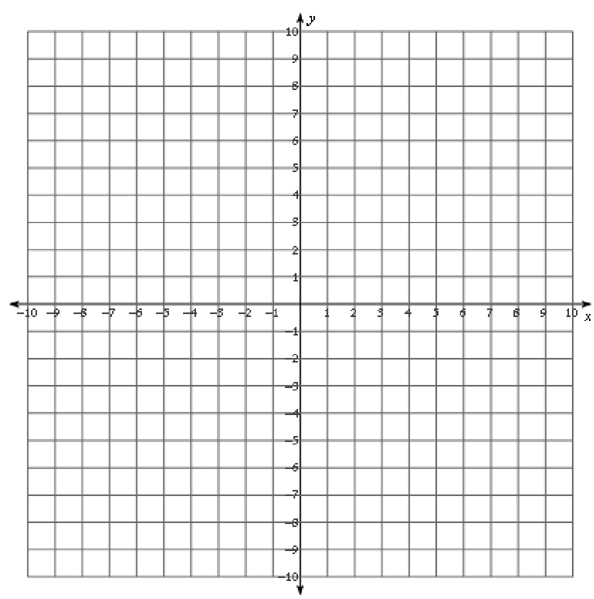
Rule for Graphing Exponential Functions

x	y
-1	$\frac{1}{b}$
0	1
1	b

Rule for Graphing Log Functions

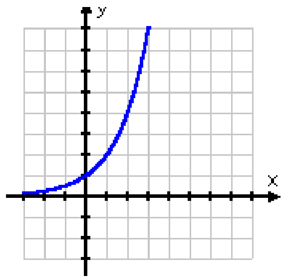
x	y
$\frac{1}{b}$	-1
1	0
b	1

Graph $y = \log_3 x$ and its inverse



Exponential Functions VS Logarithmic Functions

$y = 2^x$



All graphs intersect the point :

(0, 1)

Horizontal Asymptote :

y = 0

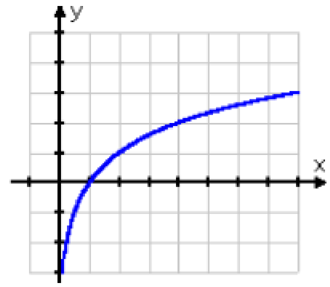
Domain : x is all real numbers

Range : y > 0

Vertical Shift -
changes Asymptote and Range

Horizontal Shift -
No change

$y = log_2(x)$



All graphs intersect the point :

(1, 0)

Vertical Asymptote :

x = 0

Domain : x > 0

Range : y is all real numbers

Vertical Shift -
No change

Horizontal Shift -
changes Asymptote and Range

Determine the shifts for the following:

1) y = ln x + 3

Asym :

Domain :
Range :

2) y = log(x-2) + 1

Asym :

Domain :
Range :

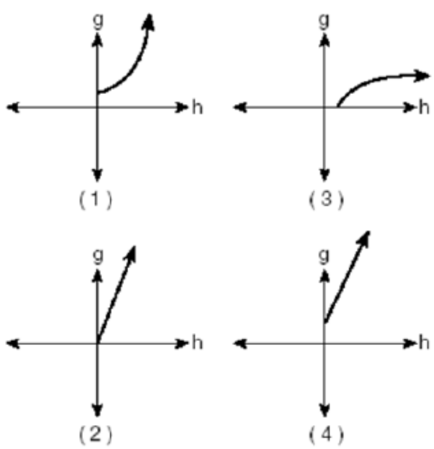
3) y = ln (x + 4)

Asym :

Domain :
Range :

PRACTICE

1. The cells of a particular organism increase logarithmically. If g represents cell growth and h represents time, in hours, which graph best represents the growth pattern of the cells of this organism?



2. What is the domain of $y = \log x$?

(1) all real numbers

(2) $x > 0$

(3) $x \geq 0$

(4) $x < 0$

3. What is the range of $y = \log(x) + 2$?

(1) $[1, \infty)$

(2) $[2, \infty)$

(3) $(2, \infty)$

(4) all real numbers

4. Which function is the inverse function of $y = \log_5 x$?

(1) $y = 5^x$

(2) $x = 5^y$

(3) $y = \log_5 x^{-1}$

(4) $y = \frac{1}{\log_5 x}$

5. The domain of $y = \log_3(x+5)$ in the real numbers is

(1) $\{x \mid x > 0\}$

(2) $\{x \mid x > -5\}$

(3) $\{x \mid x > 5\}$

(4) $\{x \mid x \geq -4\}$

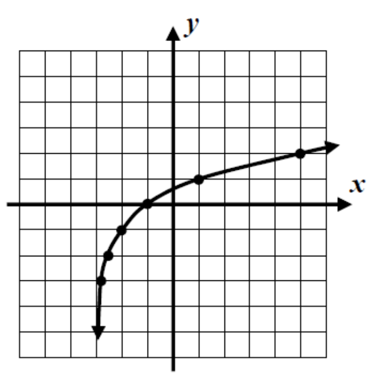
6. Which of the following equations describes the graph shown below? Show or explain how you made your choice.

(1) $y = \log_3(x+2) - 1$

(2) $y = \log_2(x-3) + 1$

(3) $y = \log_2(x+3) - 1$

(4) $y = \log_3(x+3) - 1$



______

SUMMARY

- ***What is the relationship between $y = 2^x$ and $y = \log_2 x$?***

- ***How can you determine this relationship without looking at a graph?***

Exit Ticket

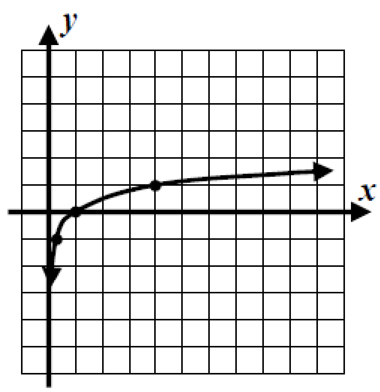
Which of the following equations describes the graph shown below?

(1) $y = \log_5 x$

(2) $y = \log_2 x$

(3) $y = \log_3 x$

(4) $y = \log_4 x$



Word Problems Homework Day 1

Write an <u>exponential growth/ decay</u> function to model each situation. Then find the value of the function after the given number of years.

1) The cost of tuition at a college is $12,000 and is increasing at a rate of 6% per year; 4 years.

2) The value of a car is $18,000 and is depreciating at a rate of 12% per year; 10 years.

3) The population of a city grows at a rate of 5% per year. The population in 1990 was 400,000. What would be the predicted current population? In what year would we predict the population to reach 1,000,000?

4) Is the equation $A = 10{,}000\,(0.45)^t$ a model of exponential growth or exponential decay, and what is the rate (percent) of change per time period?

1) exponential growth and 45%

2) exponential growth and 55%

3) exponential decay and 45%

4) exponential decay and 55%

Explain here!

5) Is the equation $A = 5400\,(1.07)^t$ a model of exponential growth or exponential decay, and what is the rate (percent) of change per time period?

1) exponential growth and 7%

2) exponential growth and 93%

3) exponential decay and 7%

4) exponential decay and 93%

Explain here!

6) Growth of a certain strain of bacteria is modeled by the equation $G = A(2.7)^{0.584t}$, where:

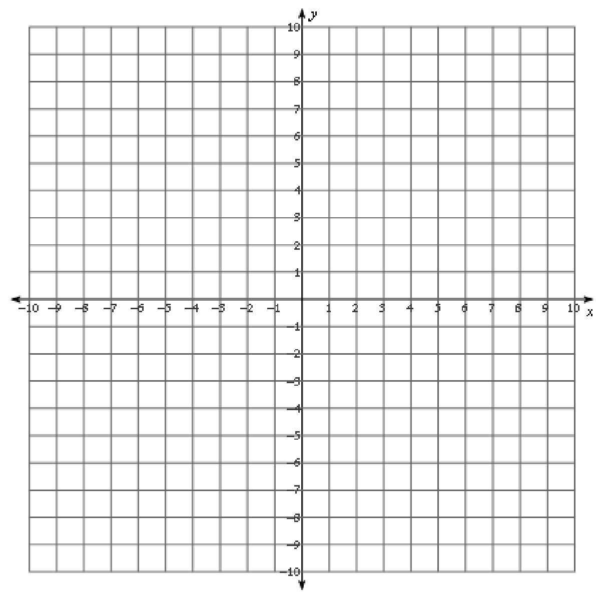
$G =$ final number of bacteria

$A =$ initial number of bacteria

$t =$ time (in hours)

In approximately how many hours will 4 bacteria first increase to 2,500 bacteria? Round your answer to the *nearest hour*.

7) Sketch below the graph of $y = \log_4 x$. Then, state the domain and range of the graph. Write the equation of the asymptote.



8) Determine the domains of each of the following logarithmic functions. State your answers using any accepted notation. Be sure to show the inequality that you are solving to find the domain and the work you use to solve the inequality.

(a) $y = \log_5(2x - 1)$ (b) $y = \log(6 - x)$

Name ____________________________ Date _____ Per. ______ Ms. Williams/Mrs. Hertel

<u>Day 8: Solving Exponential Word Problems involving Logarithms</u>

<u>Warm – Up</u>

1) In January 1995, the population of a small town was 8,000 people. Each year after 1995, the population decreased by 1%.

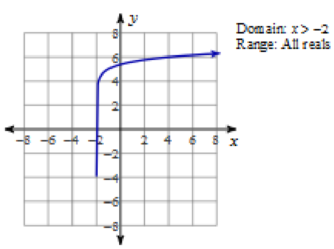
a. Find the population of the town in January 2000.

b. If this rate of decrease continues unchanged, what is the expected population of the town in January 2010?

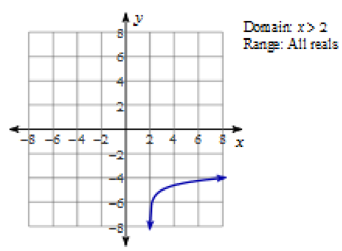
2) **Identify the domain and range of each. Then sketch the graph.**

1) $y = \log_6 (x - 2) + 5$

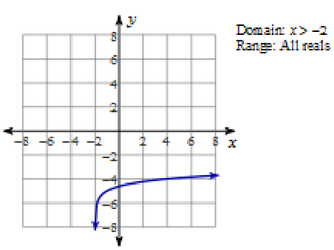
A)



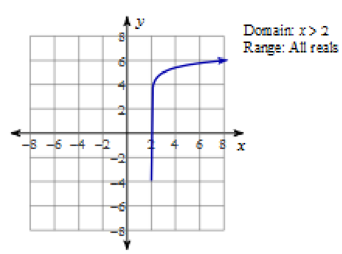
C)



B)



D)



Learning Goal: **How do we solve word problems involving compound interest?**

COMPOUND INTEREST

- ***What does interest mean when dealing with money?***

- ***What does principal mean when dealing with money?***

- ***What are some real-life examples where it is used?***

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

A = value of the investment after *t* years, *P* = principal invested, *r* = annual interest rate, and

n = number of times compounded per year

Compounded Annually	*Compounded Monthly*	*Compounded Quarterly*	*Compounded Semi-Annually*

Example 1: "Level A"

1. The Franklins inherited \$3,500, which they want to invest for their child's future college expenses. If they invest it at 8.25% with interest compounded monthly, determine the value of the account, to the nearest cent, after 5 years. Use the formula $A = P(1 + \frac{r}{n})^{nt}$, where *A* = value of the investment after *t* years, *P* = principal invested, *r* = annual interest rate, and *n* = number of times compounded per year.

Example 2: "Level B"

b. An amount of P dollars is deposited in an account paying an annual interest rate r (as a decimal) compounded n times per year. After t years, the amount of money in the account, in dollars, is given by the equation

$$A = P\left(1+\frac{r}{n}\right)^{nt}.$$

Rachel deposited \$1,000 at 2.8% annual interest, compounded quarterly. In how many years, to the *nearest tenth of a year*, will she have \$2,500 in the account?

COMPOUNDED CONTINUOUSLY

If compounding takes place without interruption (called compounded continuously), this formula becomes:

$$A = Pe^{rt}$$

A = value of the investment after *t* years, *P* = principal invested, *r* = annual interest rate, and

e = a special constant used when compounding continuously.

(e is an irrational number, approximately 2.71828183, named after the 18th century

Swiss mathematician, Leonhard Euler .)

Example 3: "Level A"

3. Matt places $1,500 in an investment account earning an annual rate of 6.5%, compounded continuously. Using the formula $V = Pe^{rt}$, where V is the value of the account in t years, P is the principal initially invested, and r is the rate of interest, determine the amount of money, to the *nearest cent*, that Matt will have in the account after 7 years.

Example 4: "Level B"

The world population reached 6 billion people on July 18, 1999, and was growing exponentially. The projected world population (in billions of people) t years after July 18, 1999, is given by $P(t) = 6e^{kt}$. On July 18, 2002, the population was 6.222 billion.

A) Find the value of k.
B) Find the world population on July 18, 2008.
C) In what year will the world population reach 7 billion?

Regents Question & Exit Ticket

The current population of Little Pond, New York, is 20,000. The population is *decreasing*, as represented by the formula $P = A(1.3)^{-0.234t}$, where P = final population, t = time, in years, and A = initial population. What will the population be 3 years from now? Round your answer to the *nearest hundred people*. To the *nearest tenth of a year*, how many years will it take for the population to reach half the present population? [The use of the grid is optional.]

Summary

Annual Interest/Compound Growth formula:

$$A = P(1+r)^t$$

Depreciation formula:

$$A = P(1 - r)^t$$

Continuous Growth formula:

$$A = Pe^{rt}$$

Compound n times per t Growth formula:

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

A = End Amount
P = Principle
r = rate as decimal
t = time

Exit Ticket

Susie invests \$500 in an account that is compounded continuously at an annual interest rate of 5%, according to the formula $A = Pe^{rt}$, where A is the amount accrued, P is the principal, r is the rate of interest, and t is the time, in years. Approximately how many years will it take for Susie's money to double?

A. 1.4 B. 6.0 C. 13.9 D. 14.7

Day 8 – Homework

1) The current population of Little Pond, New York, is 20,000. The population is *decreasing*, as represented by the formula $P = A(1.3)^{-0.234t}$, where P = final population, t = time, in years, and A = initial population.

What will the population be 3 years from now? Round your answer to the *nearest hundred people*.

To the *nearest tenth of a year*, how many years will it take for the population to reach half the present population?

2) Kristen invests $5,000 in a bank. The bank pays 6% interest compounded monthly. To the nearest tenth of a year, how long must she leave the money in the bank for it to double? (Use the formula $A = P(1+\frac{r}{n})^{nt}$, where A is the amount accrued, P is the principal, r is the interest rate, $n = 12$, and t is the length of time, in years.)

3) The number of bacteria present in a Petri dish can be modeled by the function $N = 50e^{3t}$, where N is the number of bacteria present in the Petri dish after t hours. Using this model, determine, to the *nearest hundredth*, the number of hours it will take for N to reach 30,700.

4) If \$5000 is invested at a rate of 3% interest compounded quarterly, what is the value of the investment in 5 years? (Use the formula $A = P\left(1 + \frac{r}{n}\right)^{nt}$, where A is the amount accrued, P is the principal, r is the interest rate, n is the number of times per year the money is compounded, and t is the length of time, in years.)

a) \$5190.33

b) \$5796.37

c) \$5805.92

d) \$5808.08

5) Estimate how long will it take \$4000 to triple if it is invested at 5% compounded continuously?

6) In 1984, the population of Greensboro, N.C. was 197,910. According to the U.S. Census Bureau, Greensboro has been growing at the rate of 6.9% annually since 1984. What equation models the population of Greensboro t years after 1984?

a) $y = 197,910(1 + 0.69)^t$

b) $y = 197,910(1 + 69)^t$

c) $y = 197,910(1 + 6.9)^t$

d) $y = 197,910(1 + 0.069)^t$

7)
You invest $3500 in a bank account that has a 4% annual interest rate. Calculate the amount you will have in 5 years if the interest is compounded:

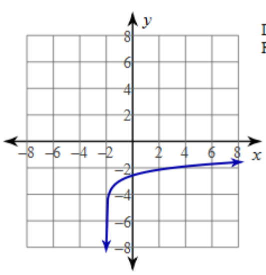
a. Annually
b. Quarterly
c. Monthly
d. Daily
e. Continuously

8)
You live near two banks. The first offers an account with an 8% interest rate compounded annually, while the second offers an account with a 7% interest rate, compounded continuously. You have five years to collect interest. Which plan is better? (Choose any amount you want to invest, it won't matter!)

9) Identify the domain and range of each. Then sketch the graph.

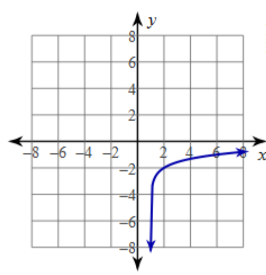
1) $y = \log_5 (x + 6)$

A)



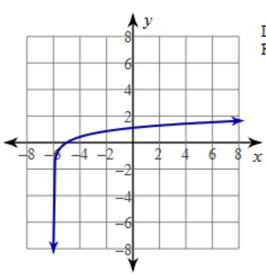
Domain: $x > -2$
Range: All reals

B)



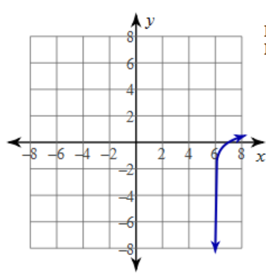
Domain: $x > 1$
Range: All reals

C)



Domain: $x > -6$
Range: All reals

D)



Domain: $x > 6$
Range: All reals

NAME:______________________________________ DATE:_________

Algebra 2/Trig – Unit 10: Logarithms REVIEW SHEET PERIOD:_______

Converting and Solving Logarithms

1. Solve for *x*: $\log_3(x - 1) = 2$

2. Find the value of ln 58.43 to four decimal places.

3. The relationship between the relative size of an earthquake, *S*, and the measure of the earthquake on the Richter scale, *R*, is given by the equation **log *S* = *R***. If an earthquake measured 6.2 on the Richter scale, what was its relative size to the *nearest tenth*?

4. The expression $\log_8 64$ is equivalent to

1) 8
2) 2
3) $\frac{1}{2}$
4) $\frac{1}{8}$

5. Solve for x to the *nearest hundredth*:

$$\ln(x - 1) = 2$$

Using the Power Law

6. Solve for x to the *nearest thousandth*:

$$2e^x = 7$$

7. Using logarithms, find *w* to the *nearest ten-thousandth*: $5^{2w} + 9 = 40$

Product and Quotient Laws

8. The expression $\frac{1}{2}\log a - 2\log b$ is equivalent to

1) $\log \frac{\sqrt{a}}{b^2}$
3) $\log \frac{a^2}{\sqrt{b}}$
2) $\log \sqrt{ab}$
4) $\log(\sqrt{a} - b^2)$

9. The expression $\ln\left(\frac{x^n}{\sqrt{y}}\right)$ is equivalent to

1) $n\ln x - \frac{1}{2}\ln y$
2) $n\ln x - 2\ln y$
3) $\ln(nx) - \ln(\frac{1}{2}y)$
4) $\ln(nx) - \ln(2y)$

Substitution with Logarithms

10. If $\ln a = x$ and $\ln b = y$, what is $\ln a\sqrt{b}$?

(1) $x+2y$ (3) $\frac{x+y}{2}$

(2) $2x+2y$ (4) $x+\frac{y}{2}$

11. Given: $\log_b 3 = p$ and $\log_b 5 = q$

Express in terms of *p* and *q*: $\log_b \frac{9}{5}$

Solving Logarithmic Equations

12. Solve algebraically for all values of *x*: $\log_{(x+4)}(17x-4) = 2$

13. Solve for *x*: $\log_4(x^2+3x)-\log_4(x+5)=1$

Undefined Logarithms

14. The expression $\log(x^2 - 4)$ is defined for all values of *x* such that

(1) $-2 \le x \le 2$ (3) $x \ge 2$ or $x \le -2$

(2) $-2 < x < 2$ (4) $x > 2$ or $x < -2$

Solving Logarithmic Word Problems

15. The scientists in a laboratory company raise amebas to sell to schools for use in biology classes. They know that one ameba divides into two amebas every hour and that the formula $t = \log_3 N$ can be used to determine how long in hours, *t*, it takes to produce a certain number of amebas, *N*. Determine, to the *nearest hundredth of an hour*, how long it takes to produce 5,000 amebas if they start with one ameba.

16. Sean invests \$10,000 at an annual rate of 5% compounded continuously, according to the formula $A = Pe^{rt}$, where A is the amount, P is the principal, r is the rate of interest, and t is time, in years.

Determine, to the *nearest dollar*, the amount of money he will have after 2 years.

Determine how many years, to the *nearest year,* it will take for his initial investment to double.

Inverse and Graphs of Logarithms

17. What is the inverse of the function $y = \log_3 x$

(1) $3^y = x$ (3) $x^3 = y$
(2) $3^x = y$ (4) $y = x^3$

18. Graph the equations $y = \log_2 x$ on the same set of axes. State the domain and range of $y = \log_2 x$. Write the equation of the asymptote of $y = \log_2 x$.