Section 5.3 – Integration by Substitution

Evaluate the integrals using the indicated substitutions.

11. a. $\int \frac{x^2}{1+x^6} dx$; $u = x^3$ b. $\int \frac{dx}{x\sqrt{1-(\ln x)^2}}$; $u = \ln x$

Evaluate the integrals using appropriate substitutions.

15. $\int (4x-3)^9 dx$

20. $\int \sec^2 5x \, dx$

28. $\int \frac{x^2+1}{\sqrt{x^3+3x}} dx$

31. $\int e^{\sin x} \cos x \, dx$

38. $\int \frac{\sec^2(\sqrt{x})}{\sqrt{x}} dx$

46. $\int \frac{\sin\theta}{\cos^2\theta+1} d\theta$

47. $\int \sec^3 2x \tan 2x \, dx$

50. $\int \sqrt{e^x} dx$

67. a. Evaluate the integral $\int \sin x \cos x \, dx$ by two methods: first by letting $u = \sin x$, and then by letting $u = \cos x$.
    b. Explain why the two apparently different answers obtained in part (a) are really equivalent.

Solve the initial-value problems.

69. $\frac{dy}{dx} = \sqrt{5x+1}$, $y(3) = -2$

71. $\frac{dy}{dt} = -e^{2t}$, $y(0) = 6$

73. a. Evaluate $\int \frac{x}{\sqrt{x^2+1}} dx$.
    b. Use a graphing utility to generate some typical integral curves of $f(x) = \frac{x}{\sqrt{x^2+1}}$ over the interval (-5,5).

76. A population of i'a in a lake is estimated to be 100,000 at the beginning of the year 2010. Suppose that *t* years after the beginning of 2010 the rate of growth of the population *p*(*t*) (in thousands) is given by $p'(t) = (3 + 0.12t)^{3/2}$. Estimate the projected population at the beginning of the year 2015.

77. Let *y*(*t*) denote the number of *E.coli* cells in a container of nutrient solution *t* minutes after the start of an experiment. Assume that *y*(*t*) is modeled by the initial-value problem $\frac{dy}{dt} = (\ln 2)2^{t/20}, y(0) = 20$. Use this model to estimate the number of *E.coli* cells in the container 2 hours after the start of the experiment.