CALCULUS BC
WORKSHEET 2 ON LOGISTIC GROWTH

Work the following on **notebook paper**. Use your calculator on 3(c), 5(b), and 5(c) only.

1. Suppose a rumor is spreading through a dance at a rate modeled by the logistic differential equation $\frac{dP}{dt} = P\left(3 - \frac{P}{2000}\right)$. What is $\lim_{t\to\infty} P(t)$? What does this number represent in the context of this problem?

2. Suppose you are in charge of stocking a fish pond with fish for which the rate of population growth is modeled by the differential equation $\frac{dP}{dt} = 8P - 0.02P^2$.

(a) If $P(0) = 50$, find $\lim_{t\to\infty} P(t)$. Justify your answer. Sketch the graph of $P(t)$.

(b) If $P(0) = 300$, find $\lim_{t\to\infty} P(t)$. Justify your answer. Sketch the graph of $P(t)$.

(c) If $P(0) = 500$, find $\lim_{t\to\infty} P(t)$. Justify your answer. Sketch the graph of $P(t)$.

(d) Which of these graphs, a, b, or c, has an inflection point? Which are increasing? Which are decreasing? Justify your answers.

3. The rate at which a rumor spreads through a high school of 2000 students can be modeled by the differential equation $\frac{dP}{dt} = 0.003P(2000 - P)$, where P is the number of students who have heard the rumor t hours after 9AM.

(a) How many students have heard the rumor when it is spreading the fastest? Justify your answer.

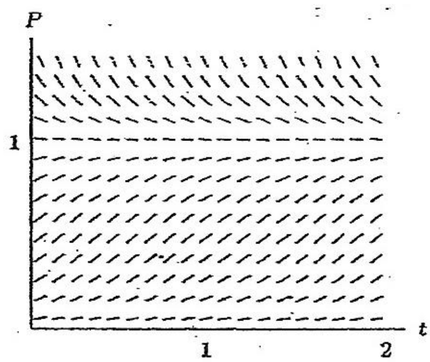
(b) If $P(0) = 5$, solve for P as a function of t.

(c) Use your answer to (b) to determine how many hours have passed when half the student body has heard the rumor.

(d) How many students have heard the rumor after 2 hours?

4. (a) On the slope field shown on the right for $\frac{dP}{dt} = 3P - 3P^2$, sketch three solution curves showing different types of behavior for the population P.

(b) Describe the meaning of the shape of the solution curves for the population.
Where is P increasing?
Decreasing?
What happens in the long run?
Are there any inflection points?
Where?
What do they mean for the population?



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5. A certain national park is known to be capable of supporting no more than 100 grizzly bears. Ten bears are in the park at present. The population growth of bears can be modeled by the logistic differential equation $\frac{dP}{dt} = 0.1P - 0.001P^2$, where t is measured in years.

(a) Solve for P as a function of t.

(b) Use your solution to (a) to find the number of bears in the park when $t = 3$ years.

(c) Use your solution to (a) to find how many years it will take for the bear population to reach 50 bears.

Answers to Worksheet 1 on Logistic Growth

1. (a) 2500; increasing
   (b) 2500; increasing
   (c) 2500; decreasing
   (d) 1250
2. C
3. $\dfrac{dP}{dt} = \dfrac{1}{1280} P\left(2000 - P\right)$
4. (a) $P = \dfrac{100e^t}{e^t + 4}$ or $P = \dfrac{100}{1 + 4e^{-t}}$
   (b) 83.393 animals
   (c) 2.773 years
5. (a) 100
   (b) Close to 0? $P = 0$ and $P = 100$
   Largest? $P = 50$
   Increasing? $P\left(0\right) < 100$
   Decreasing? $P\left(0\right) > 100$
   (c) In common? All have a limit of 100.
   Differ? Two are increasing; one is decreasing.
   Inflection points? The one with initial condition of 20.
   At what pop. level does the inflection point occur? When $P = 50$.

Answers to Worksheet 2 on Logistic Growth

1. 6000; the number of people at the dance.
2. (a) 400
   (b) 400
   (c) 400
   (d) Only (a) has an inflection point. (a) and (b) are increasing; (c) is decreasing.
3. (a) 1000 students
   (b) $P = \dfrac{2000e^{6t}}{e^{6t} + 399}$ or $P = \dfrac{2000}{1 + 399e^{-6t}}$
   (c) 0.998 hours
   (d) 1995.1089… so 1995 people
4. (b) Increasing? $P\left(0\right) < 1$
   Decreasing? $P\left(0\right) > 1$
   In the long run? $\lim_{t \to \infty} P\left(t\right) = 1$
   Any inflection points? Yes
   Where? When $P\left(0\right) = 0.5$
   What do they mean for the population? The population is growing the fastest when $P\left(0\right) = 0.5$.
5. (a) $P = \dfrac{100e^{0.1t}}{e^{0.1t} + 9}$ or $P = \dfrac{100}{1 + 9e^{-0.1t}}$
   (b) 13.042… or 13 bears
   (c) 21.972 years