Section 9.1 – Sequences

1. In each part, find a formula for the general term of the sequence, starting with n=1.
   a. $1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \dots$
   b. $1, -\frac{1}{3}, \frac{1}{9}, -\frac{1}{27}, \dots$
   c. $\frac{1}{2}, \frac{3}{4}, \frac{5}{6}, \frac{7}{8}, \dots$
   d. $\frac{1}{\sqrt{\pi}}, \frac{4}{\sqrt[3]{\pi}}, \frac{9}{\sqrt[4]{\pi}}, \frac{16}{\sqrt[5]{\pi}}, \dots$

5. Let *f* be the function $f(x) = \cos\left(\frac{\pi}{2}x\right)$ and define sequence $\{a_n\}$ by $a_n = f(2n)$.
   a. Does $\lim\limits_{x\to\infty} f(x)$ exist? Explain.
   b. Evaluate $a_1, a_2, a_3, a_4, \text{and } a_5$
   c. Does $\{a_n\}$ converge? If so, find its limit.

Write out the first five terms of the sequence, determine whether the sequence converges, and if so find its limit.

7. $\left\{\frac{n}{n+2}\right\}_{n=1}^{\infty}$

9. $\left\{\frac{n^2}{2n+1}\right\}_{n=1}^{\infty}$

11. $\{2\}_{n=1}^{\infty}$

13. $\{1 + (-1)^n\}_{n=1}^{\infty}$

15. $\left\{(-1)^n \frac{2n^3}{n^3+1}\right\}_{n=1}^{\infty}$

17. $\left\{\frac{(n+1)(n+2)}{2n^2}\right\}_{n=1}^{\infty}$

19. $\{n^2 e^{-n}\}_{n=1}^{\infty}$

21. $\left\{\left(\frac{n+3}{n+1}\right)^n\right\}_{n=1}^{\infty}$

Find the general term of the sequence, starting with n=1, determine whether the sequence converges, if so find its limit.

23. $\frac{1}{2}, \frac{3}{4}, \frac{5}{6}, \frac{7}{8}, \dots$

27. $\left(1 - \frac{1}{2}\right), \left(\frac{1}{3} - \frac{1}{2}\right), \left(\frac{1}{3} - \frac{1}{4}\right), \left(\frac{1}{5} - \frac{1}{4}\right), \dots$

37. Give two examples of sequences, all of whose terms are between -10 and 10, that do not converge. Use graphs of your sequences to explain their properties.

41. Assuming that the recursive sequence $x_1 = 1, x_{n+1} = \frac{1}{2}\left(x_n + \frac{a}{x_n}\right)$ converges, use the method of Example 10 to show that the limit of this sequence is $\sqrt{a}$.

45. a. Use a graphing utility to generate the graph of the equation $y = (2^x + 3^x)^{1/x}$, and then use the graph to make a conjecture about the limit of the sequence $\left\{(2^n + 3^n)^{1/n}\right\}_{n=1}^{\infty}$
    b. Confirm your conjecture by calculating the limit.