ASYMPTOTES AND HOLES

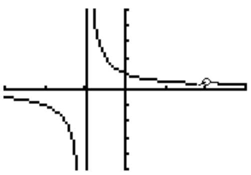
Given a rational function if a number causes the denominator and the numerator to be 0 then both the numerator and denominator can be factored and the common zero can be cancelled out. This means there is a hole in the function at this point.

Example: Find the holes in the following function $f(x) = \frac{x-2}{x^2-x-2}$

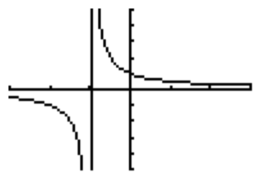
Solution: When x=2 is substituted into the function the denominator and numerator both are 0.

Factoring and canceling: $f(x) = \frac{\cancel{x-2}}{(x+1)\cancel{(x-2)}}$

$f(x) = \frac{1}{(x+1)}$ but (x≠2) this restriction is from the original function before canceling. The graph of the function f(x) will look identical to $y = \frac{1}{(x+1)}$ except for the hole at x=2.



$f(x) = \frac{x-2}{x^2-x-2}$ note the hole at x=2



$y = \frac{1}{(x+1)}$

Given a rational function if a number causes the denominator to be 0 but not the numerator to be 0 then there is a vertical asymptote at that x value.

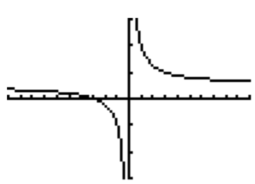
Example: Find the vertical asymptotes for the function $f(x) = \frac{x-2}{x^2-x-2}$

Solution: When x=-1 is substituted into f(x) then the numerator is -1 and the denominator is 0 therefore there is an asymptote at x=1. See the graphs above.

Given a rational function if a number causes the numerator to be 0 but not the denominator to be 0 then the value is an x-intercept for the rational function.

Example: Discuss the zeroes in the numerator and denominator $f(x) = \frac{x+3}{2x}$

Solution: When x=-3 is substituted into the function the numerator is 0 and the denominator is -6 so the value of the function is f(-3)=0 and the graph crosses the x-axis at x=-3. Also note that for x=0 the numerator is 3 and the denominator is 0 so there is a vertical asymptote at x=0. The graph is below.



Example: Find the holes, vertical asymptotes and x-intercepts for the given function:

$$f(x)=\frac{x^2-3x}{3x^2+6x}$$

Solution: First we must factor to find all the zeroes for both the numerator and denominator:

$$f(x)=\frac{x(x-3)}{3x(x+2)}$$

Numerator has zeroes x=0 and x=3
Denominator has zeroes x=0 and x=-2.

x=0 is a hole
x=-2 is a vertical asymptote
x=3 is a x-intercept

Problem Set V

For each function below list all holes, vertical asymptotes and x-intercepts

1. $f(x)=\frac{(x-3)(x+2)}{(x-3)(2x+1)}$
2. $y=\frac{x^2-1}{2x^2+x-1}$
3. $f(x)=\frac{x^3-12x^2+32x}{x^2-2x-8}$
4. $g(x)=\frac{x^2-9x+14}{x^2+3x+2}$