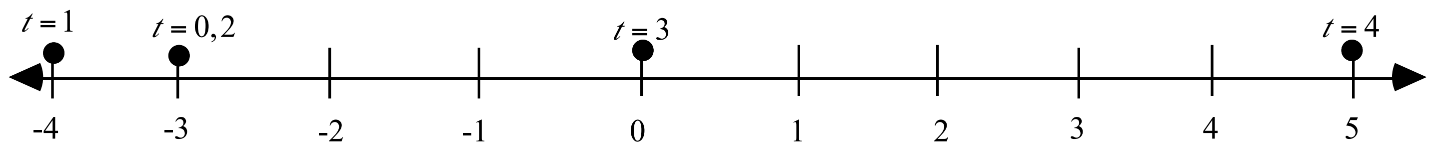
Straight Line Motion - Classwork

Consider an object moving along a straight line, either horizontally or vertically. There are many such objects, natural, and man-made. Write down several of them.

Horizontal	cars water	Vertical	rockets objects subjected to gravity

As an object moves, its position is a function of time. For its position function, we will denote the variable $s(t)$. For instance, when $s(t) = t^2 - 2t - 3$, t in seconds, $s(t)$, we are being told what position on the horizontal or vertical number line the particle occupies at different values of t.

Example 1) For $s(t) = t^2 - 2t - 3$, show its position on the number line for t = 0, 1, 2, 3, 4.



When an object moves, its position changes over time. So we can say that the velocity function, $v(t)$ is the change of the position function over time. We know this to be a derivative, and can thus say that $v(t) = s'(t)$.
For convenience sake, we will define $v(t)$ in the following way.

Motion	$v(t) > 0$	$v(t) < 0$	$v(t) = 0$
Horizontal Line	object moves to the right	object moves to the left	object stopped
Vertical Line	object moves up	object moves down	object stopped

Speed is not synonymous with velocity. Speed does not indicate direction. So we define the speed function: speed $= |\ v(t)\ |$. The speed of an object must either be positive or zero (meaning that the object is stopped).

The definition of acceleration is the change of velocity over time. We know this to be a derivative and can thus say that $a(t) = v'(t) = s''(t)$. So given the position function $s(t)$, we can now determine both the velocity and acceleration function. On your cars, you have two devices to change the velocity: accelerator, brake
Let us think as something accelerating the object to be some external force like wind or current. For convenience sake, let us define the acceleration function like this:

Motion	$a(t) > 0$	$a(t) < 0$	$a(t) = 0$
Horizontal Line	object accelerating to the right	object accelerating to the left	velocity not changing
Vertical Line	object accelerating upwards	object accelerating downwards	velocity not changing

Just because an object's acceleration is zero does not mean that the object is stopped. It means that the velocity is not changing. What device do you have on your cars that keeps the car's acceleration equal to zero? cruise control

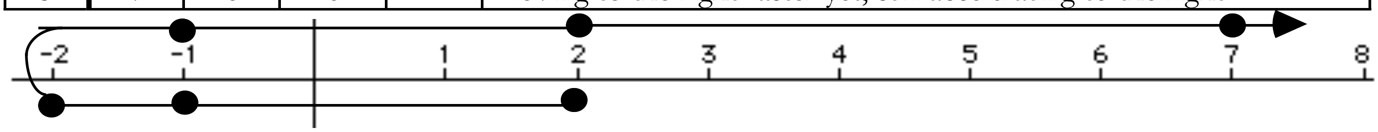
Also, just because you have a positive acceleration does not mean that you are moving to the right. For instance, suppose you were walking to the right $[v(t) > 0]$, when all of a sudden a large wind started to blow to the left $[a(t) < 0]$. What would that do to your velocity? slow you down.

Example 2) Given that a particle is moving along a horizontal line with position function $s(t) = t^2 - 4t + 2$.

The velocity function $v(t) = 2t - 4$ and the acceleration function $a(t) = 2$.

Let's complete the chart for the first 5 seconds and show where the object is on the number line.

t	$s(t)$	$v(t)$	$\lvert v(t) \rvert$	$a(t)$	Description of the particle's motion
0	2	-4	4	2	moving left, accelerating to the right
1	-1	-2	2	2	moving left but slower, still accelerating to the right
2	-2	0	0	2	stopped, still accelerating to the right
3	-1	2	2	2	moving to the right, accelerating to the right
4	2	4	4	2	moving to the right faster, still accelerating to the right
5	7	6	6	2	moving to the right faster yet, still accelerating to the right



It is too much work to do such work for complicated functions. We are generally interested when the particle is stopped or when it has no acceleration. We are also interested when the object is speeding up or slowing down. Realizing that an object's velocity is either, positive (moving right), negative (moving left) or zero (stopped) and an object's acceleration is either positive , negative, or zero (constant speed), we can now use a chart to determine all the possibilities of an object's motion as if you were looking at it from above.

	$a(t) > 0$	$a(t) < 0$	$a(t) = 0$
$v(t) > 0$	speeding up	slowing down	constant velocity right
$v(t) < 0$	slowing down	speeding up	constant velocity left
$v(t) = 0$	stopped, accelerating right	stopped, accelerating left	stopped, no acceleration

Example 3) A particle is moving along a horizontal line with position function $s(t) = t^2 - 6t + 5$. Do an analysis of the particle's direction (right, left), acceleration, motion (speeding up, slowing down), & position.

Step 1: $v(t) = 2t - 6$ So $v(t) = 0$ at $t = 3$

<--------------- -------------------->

Step 2: Make a number line of $v(t)$ showing when the object is stopped and the sign and direction of the object at times to the left and right of that. Assume $t > 0$.

$v(t)$ |---------------------0+++++++++++
0 3

Step 3: $a(t) = 2$. Does $a(t) = 0$? No

Step 4: Make a number line of $a(t)$ showing when the object has a positive and negative acceleration. Scale it exactly like the $v(t)$ number line.

$a(t)$ |++++++++++++++++++++++
0

slowing down speeding up

Step 5: Make a motion line directly below the last two putting all critical values, multiplying the signs and interpreting according to the chart above.

motion |----------------------+++++++++++
0 3

---------------------------------->

<------------------

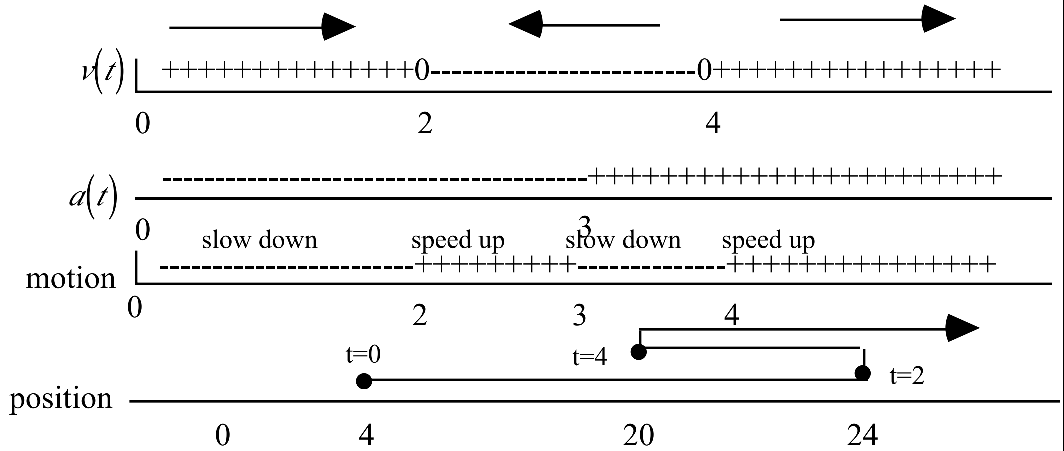
Step 6: Make a position graph to show where the object is at critical times and how it moves.

position ______t=3_____________t=0__________
-4 5

Example 3) A particle is moving along a horizontal line with position function $s(t) = t^3 - 9t^2 + 24t + 4$. Do an analysis of the particle's direction, acceleration, motion (speeding up or slowing down), and position.

$v(t) = 3t^2 - 18t + 24 = 0$
$v(t) = 3(t^2 - 6t + 8) = 0$
$v(t) = 3(t-2)(t-4) = 0$
$t = 2, t = 4$

$a(t) = 6t - 18 = 0$
$t = 3$



Note that the position graph is not like the other three graphs. It simply shows the position the object has with respect to the origin and critical times of its movement found by setting $v(t)$ and $a(t)$ = 0.

When an object is subjected to gravity, its position function is given by $s(t) = -16t^2 + v_0 t + s_0$, where t is measured in seconds, $s(t)$ is measured in feet, v_0 is the initial velocity (velocity at t = 0) and s_0 is the initial position (position at t = 0). The formula is given by $s(t) = -4.9t^2 + v_0 t + s_0$ if $s(t)$ is measured in meters.

From our original $s(t) = -16t^2 + v_0 t + s_0$, we can calculate the velocity function $v(t) = \boxed{-32t + v_o}$ and the acceleration function $a(t) = \boxed{-32}$. This is the acceleration due to gravity on earth.

When an object is thrown upward, it is subjected to gravity, We are usually interested how high the particle reaches and how fast it is going when it impacts the ground or water. Let us analyze what these mean:

When an object reaches its maximum height, what is its velocity? $\boxed{v = 0}$

So to find the maximum height of an object, set $v(t)$=0, solve for t, and find $s(t)$

When an object hits the ground, what is its final position? $\boxed{s = 0}$

So, to find the velocity of an object when it hits the ground, set $s(t)$ = 0, solve for t, and find $v(t)$

Example 4) . A projectile is launched vertically upward from ground level with an initial velocity of 112 ft/sec.

a. Find the velocity and speed at t = 3 and t = 5 seconds.

$s(t) = -16t^2 + 112t$
$v(t) = -32t + 112$
$v(3) = 16$ ft/sec speed = 16 ft/sec
$v(5) = -48$ ft/sec speed = 48 ft/sec

b. How high will the projectile rise?

$v(t) = -32t + 112 = 0$
$32t = 112$
$t = 3.5$
$s(3.5) = -16(3.5)^2 + 112(3.5)$
$s = 196$ ft

c. Find the speed of the projectile when it hits the ground.

$s(t) = -16t^2 + 112t = 0$
$-16(t-7) = 0$
$t = 7$
$v(7) = -32(7) + 112$
$v(7) = 112$ ft/sec

Example 5) The equations for free fall at the surfaces of Mars, Earth, and Jupiter (s in meters, t in seconds) are: Mars: $s(t)=1.86t^2$, Earth: $s(t)=4.9t^2$, Jupiter: $s(t)=11.44t^2$. How long would it take a rock, initially at rest in a space capsule over the planet, to reach a velocity of 16.6 m/sec?

Mars

$$\boxed{\begin{array}{l} v(t)=3.72t \\ 3.72t=16.6 \\ t=4.462\text{ sec}\end{array}}$$

Earth

$$\boxed{\begin{array}{l} v(t)=9.8t \\ 9.8t=16.6 \\ t=1.694\text{ sec}\end{array}}$$

Jupiter

$$\boxed{\begin{array}{l} v(t)=22.88t \\ 22.88t=16.6 \\ t=.726\text{ sec}\end{array}}$$

Example 6) A rock thrown vertically upward from the surface of the moon at a velocity of 24m/sec reaches a height of $s=24t-0.8t^2$ meters in t seconds.

a) Find the rock's velocity and acceleration as a function of time. (The acceleration in this case is the acceleration on the moon)

$$\boxed{v(t)=24-1.6t \quad a(t)=-1.6}$$

b) How long did it take the rock to reach its highest point?

$$\boxed{v(t)=24-1.6t=0 \Rightarrow t=15\text{ sec}}$$

c) How high did the rock go?

$$\boxed{\begin{array}{l} s(15)=24(15)-.8(15)^2 \\ s(15)=180\text{ m}\end{array}}$$

d) How long did it take the rock to reach half its maximum height?

$$\boxed{\begin{array}{l} 24t-0.8t^2=90 \\ .8t^2-24t+90=0 \Rightarrow t=4.393\text{ sec}\end{array}}$$

e) How long was the rock aloft?

$$\boxed{\begin{array}{l} s(t)=24t-0.8t^2=0 \\ 8t(3-.1t)=0 \\ t=0, t=30\text{ sec}\end{array}}$$

e) Find the rock's speed when hitting the moon.

$$\boxed{\begin{array}{l} v(30)=24-1.6(30) \\ v(t)=-24 \\ \text{speed}=24\text{ m/sec}\end{array}}$$

Example 7) A ball is dropped from the top of the Washington Monument which is 555 feet high.

a) How long will it take for the ball to hit the ground?

$$\boxed{\begin{array}{l} s(t)=-16t^2+555=0 \\ 16t^2=555 \Rightarrow t=5.89\text{ sec}\end{array}}$$

b) Find the ball's speed at impact.

$$\boxed{|v(5.89)|=|-32(5.89)|=188.48\text{ ft / sec}\approx 128.5\text{mph}}$$

Example 8) Paul has bought a ticket on a special roller coaster at an amusement park which moves in a straight line. The position $s(t)$ of the car in feet after t seconds is given by: $s(t)=-.01t^3+1.2t^2, \quad 0\le t\le 120$

a) Find the velocity and acceleration of the roller coaster after t seconds?

$$\boxed{v(t)=-.03t^2+2.4t \qquad a(t)=-.06t+2.4}$$

b) When is the roller coaster stopped?

$$\boxed{-.03t^2+2.4t=0 \rightarrow t=0, t=80\text{ sec}}$$

c) When is Paul speeding up and slowing down?

$$\boxed{\begin{array}{l} -.06t+2.4=0 \Rightarrow t=40 \\ \text{Speed up } (0,40),(80,120) \quad \text{Slow down } (40,80)\end{array}}$$

d) Where is Paul at critical times of his ride?

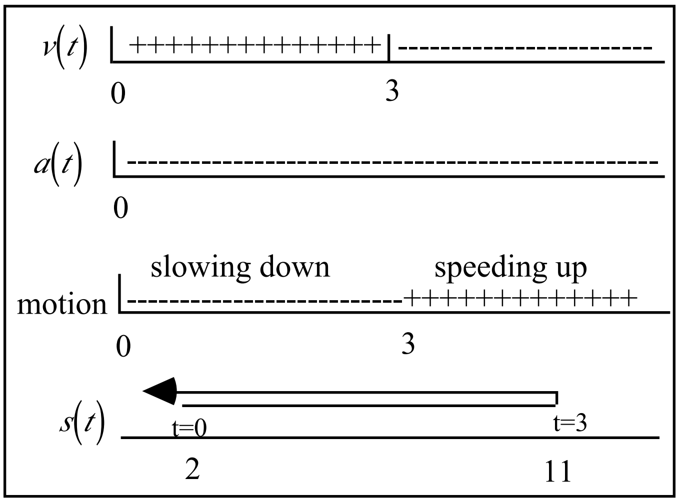
$$\boxed{t=0, s=0 \quad t=80, s=2560 \quad t=120, s=0}$$

Straight Line Motion - Homework

A particle is moving along a horizontal line with position function as given. Do an analysis of the particle's direction, acceleration, motion (speeding up or slowing down), and position.

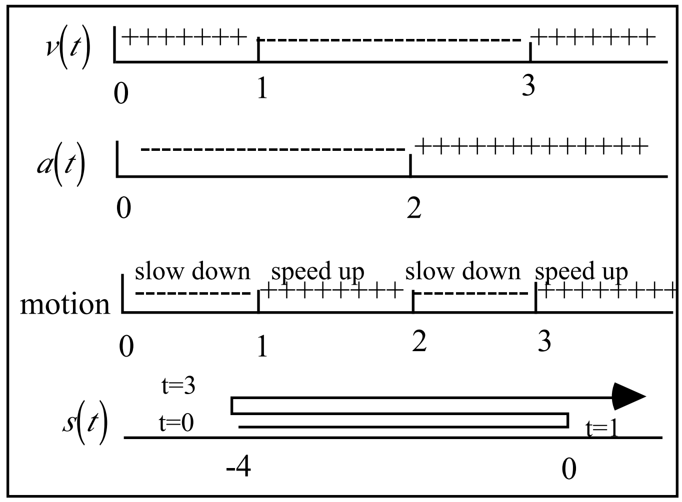
1. $s(t) = 2 + 6t - t^2$

$\boxed{v(t) = 6 - 2t = 0 \Rightarrow t = 3 \qquad a(t) = -2 \neq 0}$



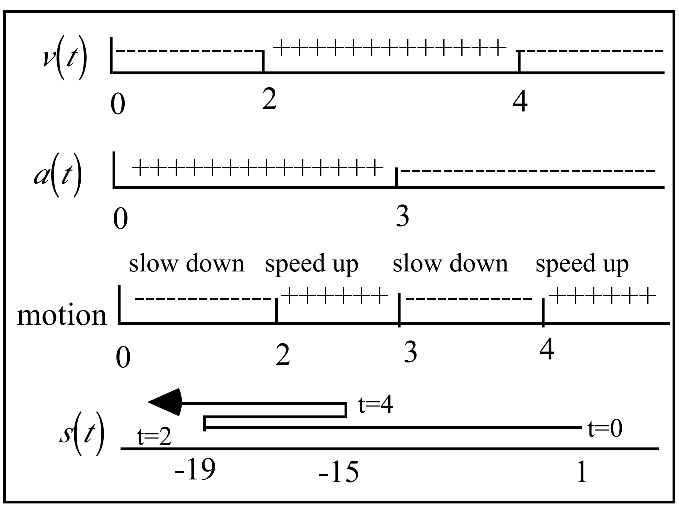
2. $s(t) = t^3 - 6t^2 + 9t - 4$

$\boxed{v(t) = 3t^2 - 12t + 9 \Rightarrow t = 1,3 \qquad a(t) = 6t - 12 \Rightarrow t = 2}$



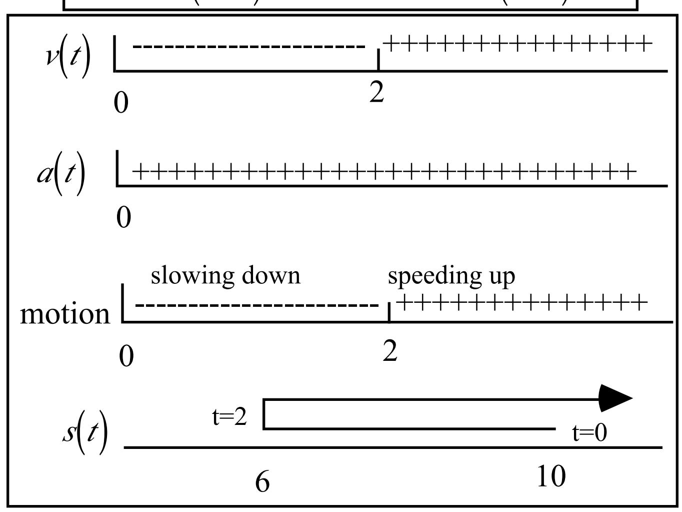
3. $s(t) = -t^3 + 9t^2 - 24t + 1$

$\boxed{v(t) = -3t^2 + 18t - 24 \Rightarrow t = 2,4 \qquad a(t) = -6t + 18 \Rightarrow t = 3}$



4. $s(t) = t + \dfrac{9}{t+1} + 1$

$\boxed{v(t) = 1 - \dfrac{9}{(t+1)^2} \Rightarrow t = 2 \qquad a(t) = \dfrac{-18}{(t+1)^3} \neq 0}$



5. A 45-caliber bullet fired straight up from the surface of the moon would reach a height of $s = 832t - 2.6t^2$ feet after t seconds. On Earth, in the absence of air, its height would be $s = 832t - 16t^2$ feet after t seconds. How long would it take the bullet to hit the ground in either case?

Earth

$$\boxed{\begin{array}{l} s(t) = t(832 - 16t) = 0 \\ 16t = 832 \Rightarrow t = 52 \text{ sec} \end{array}}$$

Moon

$$\boxed{\begin{array}{l} s(t) = t(832 - 2.6t) = 0 \\ 2.6t = 832 \Rightarrow t = 320 \text{ sec} \end{array}}$$

6. A ball fired downward from a height of 112 feet hits the ground in 2 seconds. Find its initial velocity.

$$\boxed{\begin{array}{l} s(t) = -16t^2 + v_o t + 112 = 0 \\ -64 + 2v_o + 112 = 0 \Rightarrow 2v_o = -48 \\ v_o = -24 \text{ ft/sec} \end{array}}$$

7. A projectile is fired vertically upward (earth) from ground level with an initial velocity of 16 ft/sec.

a. How long will it take for the projectile to hit the ground?

$$\boxed{\begin{array}{l} s(t) = -16t^2 + 16t = 0 \\ -16t(t-1) = 0 \\ t = 1 \text{ sec} \end{array}}$$

b. How high will the projectile get?

$$\boxed{\begin{array}{l} v(t) = -32t + 16 = 0 \\ 32t = 16 \Rightarrow t = .5 \\ s(.5) = -16(.5)^2 + 16(.5) = 4 \text{ ft} \end{array}}$$

8. A helicopter pilot drops a package when the helicopter is 200 ft. above the ground, rising at 20 ft/sec.

a. How long will it take for the package to hit the ground?

$$\boxed{\begin{array}{l} s(t) = -16t^2 + 20t + 200 = 0 \\ s(t) = -4(4t^2 - 5t - 50) \Rightarrow t = 4.215 \text{ sec} \end{array}}$$

b. What is the speed of the package at impact?

$$\boxed{\begin{array}{l} v(t) = -32t + 20 \\ |v(4.215)| = |-32(4.215) + 20| = 114.891 \text{ ft/sec} \end{array}}$$

9. A man drops a quarter from a bridge. How high is the bridge if the quarter hits the water 4 seconds later?

$$\boxed{\begin{array}{l} s(t) = -16t^2 + s_0 = 0 \\ s(t) = -16(4)^2 + s_0 = 0 \\ s_o = 256 \text{ ft} \end{array}}$$

10. A projectile fired upward from ground level is to reach a maximum height of 1,600 feet. What is its initial velocity?

$$\boxed{\begin{array}{l} s(t) = -16t^2 + v_0 t = 1600 \\ v(t) = -32t + v_0 = 0 \\ v_0 = 32t \\ -16t^2 + (32t)t = 1600 \\ 16t^2 = 1600 \\ t = 10 \text{ sec} \\ v_0 = 32(10) = 320 \text{ ft/sec} \end{array}}$$

11. A projectile is fired vertically upward with an initial velocity of 96 ft/sec from a tower 256 feet high.

a. How long will it take for the projectile to reach its maximum height.

$$\boxed{\begin{array}{l} s(t) = -16t^2 + 96t + 256 \\ v(t) = -32t + 96 = 0 \Rightarrow t = 3 \text{ sec} \end{array}}$$

b. What is its maximum height?

$$\boxed{\begin{array}{l} s(t) = -16t^2 + 96t + 256 \\ v(t) = -32t + 96 = 0 \Rightarrow t = 3 \text{ sec} \\ s(3) = 400 \text{ ft} \end{array}}$$

c. How long will it take the projectile to reach its starting height on the way down?

$$\boxed{\begin{array}{l} s(t) = -16t^2 + 96t + 256 = 256 \\ -16t(t-6) = 0 \Rightarrow t = 6 \text{ sec} \end{array}}$$

d. What is the velocity when it passes the starting point on the way down?

$\boxed{-96 \text{ ft/sec}}$

e. How long will it take to hit the ground?

$$\boxed{\begin{array}{l} s(t) = -16t^2 + 96t + 256 = 0 \\ -16(t^2 - 6t - 16) = 0 \Rightarrow t = 8 \text{ sec} \end{array}}$$

f. What will be its speed when it impacts the ground?

$\boxed{|v(8)| = |-32(8) + 96| = 160 \text{ ft / sec}}$

12. John's car runs out of gas as it goes up a hill. The car rolls to a stop then starts rolling backwards. As it rolls, its displacement $d(t)$ in feet from the bottom of the hill at t seconds since the car ran out of gas is given by: $d(t) = 125 + 31t - t^2$.

a. When is his velocity positive? What does this mean in real world terms?

$$\boxed{\begin{array}{l} v(t) = 31 - 2t > 0 \\ 0 \le t < 15.5 \text{ sec - going up the hill} \end{array}}$$

b. When did the car start to roll backwards? How far was it from the bottom of the hill at that time?

$$\boxed{\begin{array}{l} v(t) = 31 - 2t < 0 \\ 15.5 \text{ sec - } d(15.5) = 365.25 \text{ ft} \end{array}}$$

c. If John keeps his foot off the brake, when will he be at the bottom of the hill?

$\boxed{125 + 31t - t^2 = 0 \Rightarrow t \approx 34.612 \text{ sec}}$

d. How far was John from the bottom of the hill when he ran out of gas?

$\boxed{d(0) = 125 \text{ ft}}$

13. Ray is a sky-diver. When he free-falls, his downward velocity $v(t)$ feet per second is a function of t seconds from the time of the jump is given by: $v(t) = 251(1 - 0.88^t)$ measured in ft/sec. Plot $v(t)$ and $d(t)$ on your calculator for the first 30 seconds of his dive.

a. What is Ray's acceleration when he first jumps? Why does the acceleration decrease over time?

$\boxed{32.086 \text{ ft}^2\text{/sec - air resistance}}$

b. What appears to be the terminal velocity, $\lim_{t \to \infty} v(t)$?

$\boxed{251 \text{ ft/sec}}$