Section 9.10 – Differentiating & Integrating Power Series: Modeling with Taylor Series

1. In each part, obtain the Maclaurin series for the function by making an appropriate substitution in the Maclaurin series for $\frac{1}{1-x}$. Include the general term in your answer and state the radius of convergence of the series.

   a. $\frac{1}{1+x}$ b. $\frac{1}{1-x^2}$ c. $\frac{1}{1-2x}$ d. $\frac{1}{2-x}$

<u>5-8</u> Find the first four nonzero terms of the Maclaurin series for the function by making an appropriate substitution in a known Maclaurin series and performing any algebraic operations that are required. State the radius of convergence of the series.

5. a. $\sin 2x$ b. e^{-2x} c. e^{x^2} d. $x^2 \cos \pi x$
7. a. $\frac{x^2}{1+3x}$ c. $x_0 = x(1-x^2)^{3/2}$

Find the first four nonzero terms of the Maclaurin series for the function by using an appropriate trigonometric identity or property of logarithms and then substituting in a known Maclaurin series.

9. a. $\sin^2 x$ b. $\ln[(1+x^3)^{12}]$

Find the first four nonzero terms of the Maclaurin series for the function by multiplying the Maclaurin series of the factors.

13. a. $e^x \sin x$ b. $\sqrt{1+x}\ln(1+x)$

Find the first five nonzero terms of the Maclaurin series for the function by using partial fractions and a known Maclaurin series.

19. $\frac{4x-2}{x^2-1}$

Confirm the derivative formula by differentiating the appropriate Maclaurin series term by term.

21. a. $\frac{d}{dx}[\cos x] = -\sin x$ b. $\frac{d}{dx}[\ln(1+x)] = \frac{1}{1+x}$

25. Consider the series $\sum_{k=0}^{\infty} \frac{x^{k+1}}{(k+1)(k+2)}$. Determine the intervals of convergence for this series and for the series obtained by differentiating this series term by term.
27. a. Use the Maclaurin series for $\frac{1}{1-x}$ to find the Maclaurin series for $f(x) = \frac{x}{(1-x)^2}$
    b. Use the Maclaurin series obtained in part (a) to find $f^{(5)}(0)$ and $f^{(6)}(0)$
    c. What can you say about the value of $f^{(n)}(0)$?

31. Use Maclaurin series to approximate the integral to three decimal-place accuracy of $\int_0^1 \sin(x^2)\, dx$

35. a. Find the Maclaurin series for e^{x^4}. What is the radius of convergence?
    b. Explain two different ways to use the Maclaurin series for e^{x^4} find a sieres for $x^3 e^{x^4}$. Confirm that both methods produce the same series.