Section 5.4 – Definition of Area as a Limit; Sigma Notation

Evaluate without a calculator.

1. (a) $\sum_{k=1}^{3} k^3$ (b) $\sum_{j=2}^{6} (3j-1)$ (c) $\sum_{i=-4}^{1} (i^2-i)$

   (d) $\sum_{n=0}^{5} 1$ (e) $\sum_{k=0}^{4} (-2)^k$ (f) $\sum_{n=1}^{6} \sin n\pi$

Write each expression in sigma notation but do not evaluate.

4. $3 \cdot 1 + 3 \cdot 2 + 3 \cdot 3 + \cdots + 3 \cdot 20$
6. $1 + 3 + 5 + 7 + \cdots + 15$

10. Express in sigma notation.
   a. $a_1 - a_2 + a_3 - a_4 + a_5$
   b. $-b_0 + b_1 - b_2 + b_3 - b_4 + b_5$
   c. $a_0 + a_1 x + a_2 x^2 + \cdots + a_n x^n$
   d. $a^5 + a^4 b + a^3 b^2 + a^2 b^3 + ab^4 + b^5$

Evaluate the sums using formulas. Then check your work using a calculator.

11. $\sum_{k=1}^{100} k$ 12. $\sum_{k=1}^{100} (7k+1)$ 16. $\sum_{k=1}^{6} (k-k^3)$

18. Express $\sum_{k=1}^{n-1} \frac{k^2}{n}$ in closed form.

Use a calculating utility to obtain an approximate value for the area between the curve y=f(x) and the specified interval with *n*=10, 20, and 50 subintervals using the (a) LHS, (b) RHS, (c) midpoint sum, *AND (d) TRAPEZOID SUM.*

31. $f(x) = \frac{1}{x}; [1,2]$ 32. $f(x) = \frac{1}{x^2};\ [1,3]$ 33. $f(x) = \sqrt{x};\ [0,4]$

Use the definition $\lim_{n\to\infty} \sum_{k=1}^{n} f\left(x_n^*\right)\Delta x$ using the specified point of each subinterval to find the area under the curve y=f(x) over the specified interval.

35. $f(x) = \frac{x}{2}; [1,4]$ RIGHT ENDPOINT
40. $f(x) = 1 - x^3; [-3,-1]$ LEFT ENDPOINT
43. $f(x) = 9 - x^2;\ [0,3]$ LEFT ENDPOINT
48. $f(x) = x^2;\ [-1,1]$ MIDPOINT