Section 9.3 – Infinite Series

In each part, find exact values for the first four partial sums, find a closed form for the *n*th partial sum, and determine whether the series converges by calculating the limit of the *n*th partial sum. If the series converges, then state its sum.

1. (a) $2+\frac{2}{5}+\frac{2}{5^2}+\cdots+\frac{2}{5^{k-1}}+\cdots$

   (b) $\frac{1}{4}+\frac{2}{4}+\frac{2^2}{4}+\cdots+\frac{2^{k-1}}{4}+\cdots$

   (c) $\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}+\frac{1}{4\cdot 5}+\cdots+\frac{1}{(k+1)(k+2)}+\cdots$

Determine whether the series converges, and if so find its sum.

3. $\sum_{k=1}^{\infty}\left(-\frac{3}{4}\right)^{k-1}$

5. $\sum_{k=1}^{\infty}(-1)^{k-1}\frac{7}{6^{k-1}}$

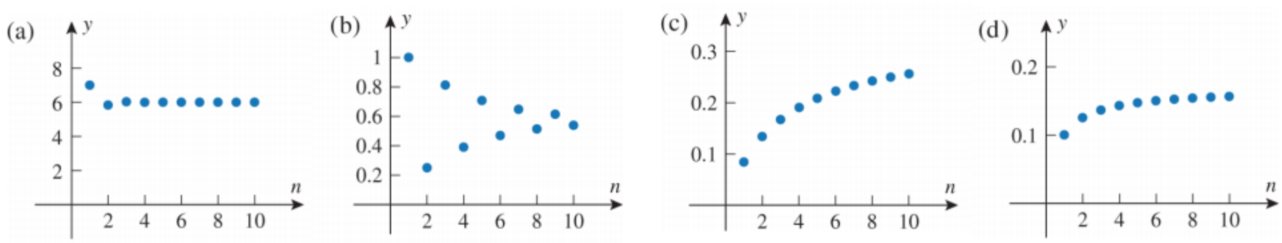
7. $\sum_{k=1}^{\infty}\frac{1}{(k+2)(k+3)}$

9. $\sum_{k=1}^{\infty}\frac{1}{9k^2+3k-2}$

11. $\sum_{k=1}^{\infty}\frac{1}{k-2}$

13. $\sum_{k=1}^{\infty}\frac{4^{k+2}}{7^{k-1}}$

15. Match the series from one of exercises 3, 5, 7, or 9 with the graph of it sequence of partial sums.



Express the repeating decimal as a fraction.

22. 0.4444…..

23. 5.373737…

29. In each part, find a closed form for the *n*th partial sum of the series, and determine whether the series converges. If so, find its sum.

   (a) $\ln\frac{1}{2}+\ln\frac{2}{3}+\ln\frac{3}{4}+\cdots+\ln\frac{k}{k+1}+\cdots$

   (b) $\ln\left(1-\frac{1}{4}\right)+\ln\left(1-\frac{1}{9}\right)+\ln\left(1-\frac{1}{16}\right)+\cdots+\ln\left(1-\frac{1}{(k+1)^2}\right)+\cdots$

31. In each part, find all values of *x* for which the series converges, and find the sum of the series for those values of *x*.

   (a) $x-x^3+x^5-x^7+\cdots$

   (b) $\frac{1}{x^2}+\frac{2}{x^3}+\frac{4}{x^4}+\frac{8}{x^5}+\frac{16}{x^6}+\ldots$

   (c) $e^{-x}+e^{-2x}+e^{-3x}+e^{-4x}+\cdots$