AP Calculus AB Practice: Particle Motion

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CALCULATOR

Question 3

An object moves along the x-axis with initial position $x(0) = 2$. The velocity of the object at time $t \geq 0$ is given by $v(t) = \sin\left(\frac{\pi}{3}t\right)$.

(a) What is the acceleration of the object at time $t = 4$?

(b) Consider the following two statements.

Statement I: For $3 < t < 4.5$, the velocity of the object is decreasing.

Statement II: For $3 < t < 4.5$, the speed of the object is increasing.

Are either or both of these statements correct? For each statement provide a reason why it is correct or not correct.

(c) What is the total distance traveled by the object over the time interval $0 \leq t \leq 4$?

(d) What is the position of the object at time $t = 4$?

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Question 4

NO CALCULATOR

A particle moves along the x-axis with velocity at time $t \geq 0$ given by $v(t) = -1 + e^{1-t}$.

(a) Find the acceleration of the particle at time $t = 3$.

(b) Is the speed of the particle increasing at time $t = 3$? Give a reason for your answer.

(c) Find all values of t at which the particle changes direction. Justify your answer.

(d) Find the total distance traveled by the particle over the time interval $0 \leq t \leq 3$.

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Question 2

CALCULATOR

A particle moves along the x-axis so that its velocity at time t is given by

$$v(t) = -(t+1)\sin\left(\frac{t^2}{2}\right).$$

At time $t = 0$, the particle is at position $x = 1$.

(a) Find the acceleration of the particle at time $t = 2$. Is the speed of the particle increasing at $t = 2$? Why or why not?

(b) Find all times t in the open interval $0 < t < 3$ when the particle changes direction. Justify your answer.

(c) Find the total distance traveled by the particle from time $t = 0$ until time $t = 3$.

(d) During the time interval $0 \le t \le 3$, what is the greatest distance between the particle and the origin? Show the work that leads to your answer.

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Question 3

CALCULATOR

A particle moves along the y-axis so that its velocity v at time $t \geq 0$ is given by $v(t) = 1 - \tan^{-1}\left(e^t\right)$.

At time $t = 0$, the particle is at $y = -1$. (Note: $\tan^{-1} x = \arctan x$)

(a) Find the acceleration of the particle at time $t = 2$.

(b) Is the speed of the particle increasing or decreasing at time $t = 2$? Give a reason for your answer.

(c) Find the time $t \geq 0$ at which the particle reaches its highest point. Justify your answer.

(d) Find the position of the particle at time $t = 2$. Is the particle moving toward the origin or away from the origin at time $t = 2$? Justify your answer.

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Question 4

NO CALCULATOR

A particle moves along the x-axis with position at time t given by $x(t) = e^{-t}\sin t$ for $0 \le t \le 2\pi$.

(a) Find the time t at which the particle is farthest to the left. Justify your answer.

(b) Find the value of the constant A for which $x(t)$ satisfies the equation $Ax''(t) + x'(t) + x(t) = 0$ for $0 < t < 2\pi$.

Question 3

An object moves along the x-axis with initial position $x(0) = 2$. The velocity of the object at time $t \geq 0$ is given by $v(t) = \sin\left(\frac{\pi}{3}t\right)$.

(a) What is the acceleration of the object at time $t = 4$?

(b) Consider the following two statements.

Statement I: For $3 < t < 4.5$, the velocity of the object is decreasing.

Statement II: For $3 < t < 4.5$, the speed of the object is increasing.

Are either or both of these statements correct? For each statement provide a reason why it is correct or not correct.

(c) What is the total distance traveled by the object over the time interval $0 \leq t \leq 4$?

(d) What is the position of the object at time $t = 4$?

(a) $a(4) = v'(4) = \frac{\pi}{3}\cos\left(\frac{4\pi}{3}\right)$
$= -\frac{\pi}{6}$ or -0.523 or -0.524

1 : answer

(b) On $3 < t < 4.5$:
$a(t) = v'(t) = \frac{\pi}{3}\cos\left(\frac{\pi}{3}t\right) < 0$
Statement I is correct since $a(t) < 0$.
Statement II is correct since $v(t) < 0$ and $a(t) < 0$.

3 { 1 : I correct, with reason; 1 : II correct; 1 : reason for II }

(c) Distance $= \int_0^4 |v(t)|\,dt = 2.387$

OR

$x(t) = -\frac{3}{\pi}\cos\left(\frac{\pi}{3}t\right) + \frac{3}{\pi} + 2$
$x(0) = 2$
$x(4) = 2 + \frac{9}{2\pi} = 3.43239$
$v(t) = 0$ when $t = 3$
$x(3) = \frac{6}{\pi} + 2 = 3.90986$
$|x(3) - x(0)| + |x(4) - x(3)| = \frac{15}{2\pi} = 2.387$

3 { 1 : { limits of 0 and 4 on an integral of $v(t)$ or $|v(t)|$ or uses $x(0)$ and $x(4)$ to compute distance }; 1 : handles change of direction at student's turning point; 1 : answer; 0/1 if incorrect turning point or no turning point }

(d) $x(4) = x(0) + \int_0^4 v(t)\,dt = 3.432$

OR

$x(t) = -\frac{3}{\pi}\cos\left(\frac{\pi}{3}t\right) + \frac{3}{\pi} + 2$
$x(4) = 2 + \frac{9}{2\pi} = 3.432$

2 { 1 : integral; 1 : answer }

OR

2 { 1 : $x(t) = -\frac{3}{\pi}\cos\left(\frac{\pi}{3}t\right) + C$; 1 : answer; 0/1 if no constant of integration }

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Question 4

A particle moves along the x-axis with velocity at time $t \geq 0$ given by $v(t) = -1 + e^{1-t}$.

(a) Find the acceleration of the particle at time $t = 3$.

(b) Is the speed of the particle increasing at time $t = 3$? Give a reason for your answer.

(c) Find all values of t at which the particle changes direction. Justify your answer.

(d) Find the total distance traveled by the particle over the time interval $0 \leq t \leq 3$.

(a) $a(t) = v'(t) = -e^{1-t}$
$a(3) = -e^{-2}$

$2 : \begin{cases} 1 : v'(t) \\ 1 : a(3) \end{cases}$

(b) $a(3) < 0$
$v(3) = -1 + e^{-2} < 0$
Speed is increasing since $v(3) < 0$ and $a(3) < 0$.

1 : answer with reason

(c) $v(t) = 0$ when $1 = e^{1-t}$, so $t = 1$.

$v(t) > 0$ for $t < 1$ and $v(t) < 0$ for $t > 1$.
Therefore, the particle changes direction at $t = 1$.

$2 : \begin{cases} 1 : \text{solves } v(t) = 0 \text{ to get } t = 1 \\ 1 : \text{justifies change in direction at } t = 1 \end{cases}$

(d) $\text{Distance} = \displaystyle\int_0^3 |v(t)|\,dt$
$= \displaystyle\int_0^1 \left(-1 + e^{1-t}\right) dt + \int_1^3 \left(1 - e^{1-t}\right) dt$
$= \left(-t - e^{1-t}\Big|_0^1\right) + \left(t + e^{1-t}\Big|_1^3\right)$
$= (-1 - 1 + e) + \left(3 + e^{-2} - 1 - 1\right)$
$= e + e^{-2} - 1$

$4 : \begin{cases} 1 : \text{limits} \\ 1 : \text{integrand} \\ 1 : \text{antidifferentiation} \\ 1 : \text{evaluation} \end{cases}$

OR

OR

$x(t) = -t - e^{1-t}$
$x(0) = -e$
$x(1) = -2$
$x(3) = -e^{-2} - 3$
$\text{Distance} = (x(1) - x(0)) + (x(1) - x(3))$
$= (-2 + e) + \left(1 + e^{-2}\right)$
$= e + e^{-2} - 1$

$4 : \begin{cases} 1 : \text{any antiderivative} \\ 1 : \text{evaluates } x(t) \text{ when } t = 0, 1, 3 \\ 1 : \text{evaluates distance between points} \\ 1 : \text{evaluates total distance} \end{cases}$

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Question 2

A particle moves along the x-axis so that its velocity at time t is given by

$$v(t) = -(t+1)\sin\left(\frac{t^2}{2}\right).$$

At time $t = 0$, the particle is at position $x = 1$.

(a) Find the acceleration of the particle at time $t = 2$. Is the speed of the particle increasing at $t = 2$? Why or why not?

(b) Find all times t in the open interval $0 < t < 3$ when the particle changes direction. Justify your answer.

(c) Find the total distance traveled by the particle from time $t = 0$ until time $t = 3$.

(d) During the time interval $0 \le t \le 3$, what is the greatest distance between the particle and the origin? Show the work that leads to your answer.

(a) $a(2) = v'(2) = 1.587$ or 1.588
$v(2) = -3\sin(2) < 0$
Speed is decreasing since $a(2) > 0$ and $v(2) < 0$.

$2 :$ 1 : $a(2)$; 1 : speed decreasing with reason

(b) $v(t) = 0$ when $\dfrac{t^2}{2} = \pi$
$t = \sqrt{2\pi}$ or 2.506 or 2.507
Since $v(t) < 0$ for $0 < t < \sqrt{2\pi}$ and $v(t) > 0$ for $\sqrt{2\pi} < t < 3$, the particle changes directions at $t = \sqrt{2\pi}$.

$2 :$ 1 : $t = \sqrt{2\pi}$ only; 1 : justification

(c) Distance $= \displaystyle\int_0^3 |v(t)|\,dt = 4.333$ or 4.334

$3 :$ 1 : limits; 1 : integrand; 1 : answer

(d) $\displaystyle\int_0^{\sqrt{2\pi}} v(t)\,dt = -3.265$
$x\left(\sqrt{2\pi}\right) = x(0) + \displaystyle\int_0^{\sqrt{2\pi}} v(t)\,dt = -2.265$
Since the total distance from $t = 0$ to $t = 3$ is 4.334, the particle is still to the left of the origin at $t = 3$. Hence the greatest distance from the origin is 2.265.

$2 :$ 1 : $\pm$ (distance particle travels while velocity is negative); 1 : answer

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Question 3

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At time $t = 0$, the particle is at $y = -1$. (Note: $\tan^{-1} x = \arctan x$)

(a) Find the acceleration of the particle at time $t = 2$.

(b) Is the speed of the particle increasing or decreasing at time $t = 2$? Give a reason for your answer.

(c) Find the time $t \geq 0$ at which the particle reaches its highest point. Justify your answer.

(d) Find the position of the particle at time $t = 2$. Is the particle moving toward the origin or away from the origin at time $t = 2$? Justify your answer.

(a) $a(2) = v'(2) = -0.132$ or -0.133

1 : answer

(b) $v(2) = -0.436$
Speed is increasing since $a(2) < 0$ and $v(2) < 0$.

1 : answer with reason

(c) $v(t) = 0$ when $\tan^{-1}\left(e^t\right) = 1$
$t = \ln(\tan(1)) = 0.443$ is the only critical value for y.

$v(t) > 0$ for $0 < t < \ln(\tan(1))$
$v(t) < 0$ for $t > \ln(\tan(1))$

$y(t)$ has an absolute maximum at $t = 0.443$.

3 : { 1 : sets $v(t) = 0$; 1 : identifies $t = 0.443$ as a candidate; 1 : justifies absolute maximum }

(d) $y(2) = -1 + \int_0^2 v(t)\,dt = -1.360$ or -1.361
The particle is moving away from the origin since $v(2) < 0$ and $y(2) < 0$.

4 : { 1 : $\int_0^2 v(t)\,dt$; 1 : handles initial condition; 1 : value of $y(2)$; 1 : answer with reason }

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Question 4

A particle moves along the x-axis with position at time t given by $x(t) = e^{-t}\sin t$ for $0 \le t \le 2\pi$.

(a) Find the time t at which the particle is farthest to the left. Justify your answer.

(b) Find the value of the constant A for which $x(t)$ satisfies the equation $Ax''(t) + x'(t) + x(t) = 0$ for $0 < t < 2\pi$.

(a) $x'(t) = -e^{-t}\sin t + e^{-t}\cos t = e^{-t}(\cos t - \sin t)$

$x'(t) = 0$ when $\cos t = \sin t$. Therefore, $x'(t) = 0$ on $0 \le t \le 2\pi$ for $t = \frac{\pi}{4}$ and $t = \frac{5\pi}{4}$.

The candidates for the absolute minimum are at $t = 0, \frac{\pi}{4}, \frac{5\pi}{4}$, and 2π.

t	$x(t)$
0	$e^{0}\sin(0) = 0$
$\frac{\pi}{4}$	$e^{-\frac{\pi}{4}}\sin\left(\frac{\pi}{4}\right) > 0$
$\frac{5\pi}{4}$	$e^{-\frac{5\pi}{4}}\sin\left(\frac{5\pi}{4}\right) < 0$
2π	$e^{-2\pi}\sin(2\pi) = 0$

The particle is farthest to the left when $t = \frac{5\pi}{4}$.

$5:\begin{cases} 2: x'(t) \\ 1: \text{sets } x'(t) = 0 \\ 1: \text{answer} \\ 1: \text{justification} \end{cases}$

(b) $x''(t) = -e^{-t}(\cos t - \sin t) + e^{-t}(-\sin t - \cos t)$

$= -2e^{-t}\cos t$

$Ax''(t) + x'(t) + x(t)$

$= A\left(-2e^{-t}\cos t\right) + e^{-t}(\cos t - \sin t) + e^{-t}\sin t$

$= (-2A + 1)e^{-t}\cos t$

$= 0$

Therefore, $A = \frac{1}{2}$.

$4:\begin{cases} 2: x''(t) \\ 1: \text{substitutes } x''(t), x'(t), \text{ and } x(t) \text{ into } Ax''(t) + x'(t) + x(t) \\ 1: \text{answer} \end{cases}$