Section 9.4 – Convergence Tests

1. Use Theorem 9.4.3 to find the sum of each series.

   a. $\left(\frac{1}{2}+\frac{1}{4}\right)+\left(\frac{1}{2^2}+\frac{1}{4^2}\right)+\cdots+\left(\frac{1}{2^k}+\frac{1}{4^k}\right)+\cdots$

   b. $\sum_{k=1}^{\infty}\left(\frac{1}{5^k}-\frac{1}{k(k+1)}\right)$

For each given p-series, identify p and determine whether the series converges.

3. (a) $\sum_{k=1}^{\infty}\frac{1}{k^3}$ (b) $\sum_{k=1}^{\infty}\frac{1}{\sqrt{k}}$ (c) $\sum_{k=1}^{\infty}\frac{1}{k^{-1}}$ (d) $\sum_{k=1}^{\infty}\frac{1}{k^{-2/3}}$

Apply the divergence test and state what it tells you about the series.

5. (a) $\sum_{k=1}^{\infty}\frac{k^2+k+3}{2k^2+1}$ (b) $\sum_{k=1}^{\infty}\left(1+\frac{1}{k}\right)^k$ (c) $\sum_{k=1}^{\infty}\cos k\pi$ (d) $\sum_{k=1}^{\infty}\frac{1}{k!}$

Confirm that the integral test is applicable and use it to determine whether the series converges.

7. (a) $\sum_{k=1}^{\infty}\frac{1}{5k+2}$ (b) $\sum_{k=1}^{\infty}\frac{1}{1+9k^2}$

Determine whether the series converges.

9. $\sum_{k=1}^{\infty}\frac{1}{k+6}$ (p-series)

11. $\sum_{k=1}^{\infty}\frac{1}{\sqrt{k+5}}$ (p-series)

13. $\sum_{k=1}^{\infty}\frac{1}{\sqrt[3]{2k-1}}$ (integral test)

15. $\sum_{k=1}^{\infty}\frac{k}{\ln(k+1)}$ (divergence test)

17. $\sum_{k=1}^{\infty}\left(1+\frac{1}{k}\right)^{-k}$ (divergence test)

19. $\sum_{k=1}^{\infty}\frac{\tan^{-1}k}{1+k^2}$ (integral test)

21. $\sum_{k=1}^{\infty}k^2\sin^2\left(\frac{1}{k}\right)$ (divergence test)

23. $\sum_{k=1}^{\infty}7k^{-1.01}$ (p-series)

Use the integral test to investigate the relationship between the value of p and the convergence of the series.

25. $\sum_{k=1}^{\infty} \frac{1}{k(\ln k)^p}$

Use Theorem 9.4.3 to determine whether the series converges or diverges.

29. (a) $\sum_{k=1}^{\infty} \left[\left(\frac{2}{3} \right)^{k-1} + \frac{1}{k} \right]$ (b) $\sum_{k=1}^{\infty} \left[\frac{1}{3k+2} - \frac{1}{k^{3/2}} \right]$