Section 9.2 – Monotone Sequences

Use the difference $a_{n+1} - a_n$ to show that the given sequence $\{a_n\}$ is strictly increasing or strictly decreasing.

1. $\left\{\frac{1}{n}\right\}_{n=1}^{\infty}$ 3. $\left\{\frac{n}{2n+1}\right\}_{n=1}^{\infty}$ 5. $\{n - 2^n\}_{n=1}^{\infty}$

Use the ratio $\frac{a_{n+1}}{a_n}$ to show that the given sequence $\{a_n\}$ is strictly increasing or strictly decreasing.

7. $\left\{\frac{n}{2n+1}\right\}_{n=1}^{\infty}$ 9. $\{ne^{-n}\}_{n=1}^{\infty}$ 11. $\left\{\frac{n^n}{n!}\right\}_{n=1}^{\infty}$

Use differentiation to show that the given sequence is strictly increasing or strictly decreasing.

17. $\left\{\frac{n}{2n+1}\right\}_{n=1}^{\infty}$ 19. $\{\tan^{-1} n\}_{n=1}^{\infty}$

Show that the given sequence is eventually strictly increasing or eventually strictly decreasing.

21. $\{2n^2 - 7n\}_{n=1}^{\infty}$ 23. $\left\{\frac{n!}{3^n}\right\}_{n=1}^{\infty}$

25. Suppose that $\{a_n\}$ is a monotone sequence such that $1 \leq a_n \leq 2$ for all *n*. Must the sequence converge? If so, what can you say about the limit?

28. Let $\{a_n\}$be the sequence defined by $a_1 = 1$ and $a_{n+1} = \frac{1}{2}\left[a_n + \frac{3}{a_n}\right]$ for $n \geq 1$.

a. Show that $a_n \geq \sqrt{3}$ for $n \geq 2$. [*Hint:* What is the minimum value of $\frac{1}{2}\left[x + \frac{3}{x}\right]$ for $x > 0$?]

b. Show that $\{a_n\}$ is eventually decreasing. [*Hint:* Examine $a_{n+1} - a_n$ or $\frac{a_{n+1}}{a_n}$ and use the result in part (a).]

c. Show that $\{a_n\}$ converges and find its limit L.

The Beverton-Holt model is used to describe changes in a population from one generation to the next under certain assumptions. If the population in generation *n* is given by x_n, the Beverton-Holt model predicts that the population in the next generation satisfies

$$x_{n+1} = \frac{RKx_n}{K + (R-1)x_n}$$

for some positive constants R and K with R>1. These exercises explore some properties of this population model.

29. Let $\{x_n\}$ be the sequence of population values defined recursively by $x_1 = 60$, and for $n \geq 1$, x_{n+1} is given by the Beverton-Holt model with $R = 10$ and $K = 300$.

a. List the first four terms of the sequence $\{x_n\}$.

b. If $0 < x_n < 300$, show that $0 < x_{n+1} < 300$. Conclude that $0 < x_n < 300$ for $n \geq 1$.

c. Show that $\{x_n\}$ is increasing.

d. Show that $\{x_n\}$ converges and find its limit L.