Vector Word Problem Review

A. Definition of Vectors
   1. Vectors are quantities that have both a magnitude and direction.
   2. By using vectors, real life examples can be analyzed and dissected into their components.
   3. Vectors can be represented in component form and direction-magnitude form (They can be manipulated in standard form and polar form respectively.)

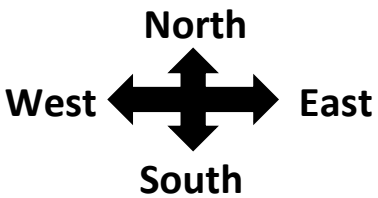
B. Vectors in Word Problems

When given 2 directions imagine the distance between them in degrees.

Take note of this, it is the maximum speed of the object at full power. This will be the Magnitude component when it is at full speed.

Burt and Ernie meet up to fly a toy helicopter. At full power the airplane can fly 100 km per hour in calm air. Burt has the controls and he makes the helicopter take off heading 45° North of East. After he feels comfortable with the controls he turns on full power. A steady wind begins to blow from north to south at a speed of 32 kilometers per hour. In what direction and what speed is the helicopter traveling now.

When given directions such as "North of East", "South of West" or "North to South" you must refer to a compass.



This is the speed of the wind that will act as the second vector in the problem.

This is the given direction of the helicopter.

C. Forms:

1. Component Form: <a,b> (vector)
2. Standard Form: a + bi (complex number)
3. Direction-Magnitude Form: |v| and Θ (vector)
4. Polar Form: rcisΘ (complex number)

D. Example:

Burt and Ernie meet up to fly a toy helicopter. At full power the airplane can fly 100 km per hour in calm air. Burt has the controls and he makes the helicopter take off heading 45^{0} North of East. After he feels comfortable with the controls he turns on full power. A steady wind begins to blow from north to south at a speed of 32 kilometers per hour. In what direction and what speed is the helicopter traveling now.

(1.) First you must identify what form your information is given in. We are given the Magnitude and Direction which can easily be put into Polar Form (rcisΘ, 100cis45 & 32cis270)

(2.) Now you must convert each Polar form to Component Form.

100cis45 → 100cos45,100sin45 → (70.71, 70.71)

32cis270 → 32cos270, 32sin270 → (0, -32)

(3.) Next we add the two vectors in Component Form to get the resultant (a+a, b+b)

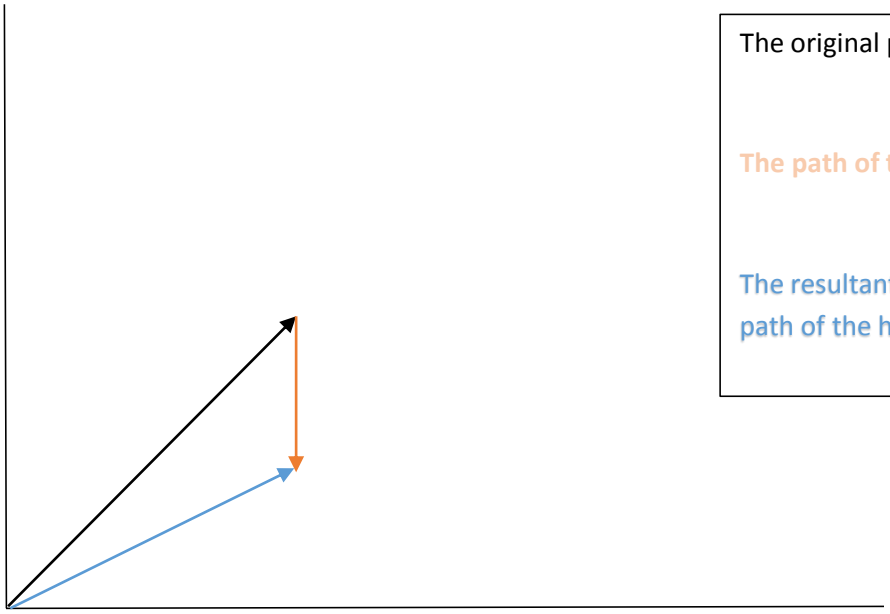
70.71+ 0, 70.71 + (-32) = <70.71, 38.71>

(4.) The question asks the new direction and speed of the helicopter so we must convert the resultant vector from Component Form to Direction-Magnitude Form.

$$\sqrt{70.71^2} + \sqrt{38.71^2} = |v| = 80.61\ Km/hr$$

$$\tan^{-1}\left(\frac{38.71}{70.71}\right) = \Theta = 28.69^{\,0}\ North\ of\ East$$

Since vector quantities call for magnitude AND direction be sure to include both units. This is displayed in the diagram as followed…



The original path of the helicopter.

The path of the wind.

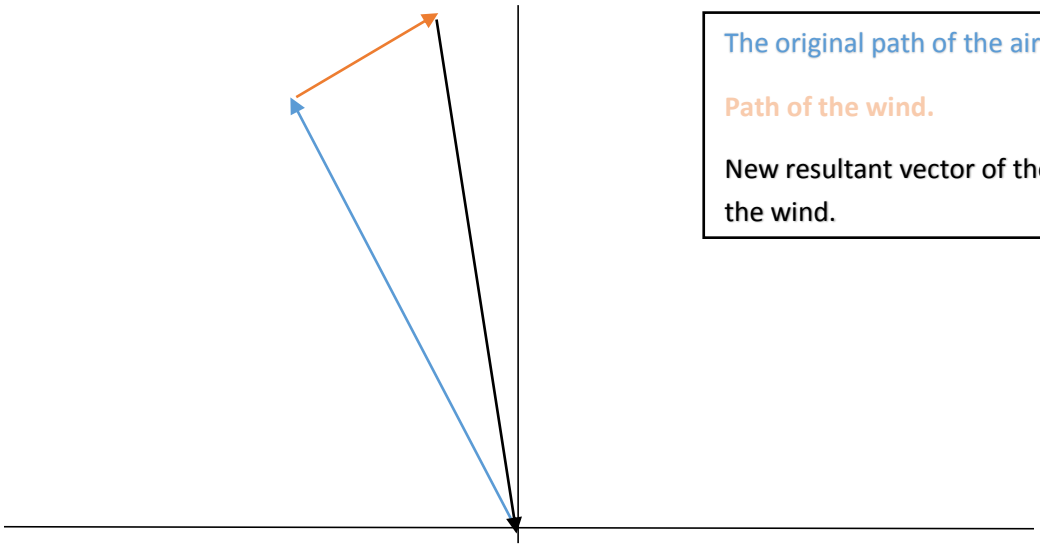
The resultant vector that is the new path of the helicopter

Our final answer is… 80.61 km/hr at 28.69^{o} North of East

E. Example(2):

A plane traveling 500 mph in the direction of 120 ^{o} from the horizontal, encounters a wind of 80 mph in the direction of 45^{o} from the horizontal. What is the resultant speed and direction of the plane?

(1) First it may help to draw the scenario to get a visual representation



The original path of the airplane.

Path of the wind.

New resultant vector of the plane affected by the wind.

(2.) Identify: we are given the information 500 mph at a direction of 120^o for vector one and 80 mph at a direction of 45^o from the horizontal for vector two.

(rcisΘ, 500cis120^o, 80cis45^o)

(3.) Convert to component form to make it possible to add

500cos120 → -250 80cos45 → 56.56

<-250, 433> & <56.56, 56.56>

500sin120 → 433 80sin45 → 56.56

(4.) Add the two vectors (a+a, b+b) to get the resultant vector.

<-250, 433> + <56.56, 56.56> = <-193.44, 489.56>

(5.) Find Direction-Magnitude form by finding |v|=$\sqrt{a^2} + \sqrt{b^2}$ and Θ= $\tan^{-1} b/a$

Speed=|v|= $\sqrt{-193.44^2 + 489.56^2}$ = 526.3 mph

Direction = Θ= $\tan^{-1}(489.56/-193.44)$ = -68.43^o + 180^o= 111.56^o

**because the vectors in the second quadrant we must add 180^o

Your final answer is 526.3 mph at 111.56^o from the horizontal.

Practice Problems:

(1.) The pilot of a plane points his airplane due South and flies with an airspeed of 120 m/s. While there is a steady wind blowing due West with a constant speed of 40 m/s.

a. What is the resultant speed of the airplane?

b. What is the angle (direction, Θ) of the plane referenced above?

(2.) A ship leaves port and travels 50 miles at a standard position of 45^o. The ship then travels for 90 miles in a standard position angle of 80^o. At that point the ship drops anchor. A helicopter, beginning from the same port, needs to join the ship as quickly as possible.

a. How far must the helicopter go (magnitude)?

b. And in what direction?

Answers:

(1.) A. 120cis270= <0, -120> 40cis180= <-40, 0>

$\sqrt{40^2 + 120^2}$ 126.49 meters/ second

B. $\tan^{-1}\left(\frac{120}{40}\right)$ = -71.56° + 360°= 288.43°

(2.) A. 50cis45° = <35.35, 35.35> + 90cis80°= <15.62, 88.63>

$\sqrt{}$50.97²+ 123.98² = 134.04 miles

B. $\tan^{-1}(123.98/50.97)$ = 67.65°

For extra practice or notes please visit:

https://www.khanacademy.org/math/precalculus/vectors-precalc/magnitude-direction/e/vector-word-problems