Section 10.1 – Parametric Equations; Tangent Lines & Arc Length

1. a. By eliminating the parameter, sketch the trajectory over the time interval 0≤t≤5 of the particle whose parametric equations of motion are: $x(t) = t - 1$ and $y(t) = t + 1$
   b. Indicate the direction of motion on your sketch.
   c. Make a table of x and y-coordinates of the particle at times t=0,1,2,3,4,5.
   d. Mark the position of the particle on the curve at the times in part (c), and label those positions with the vales of t.

11. Sketch the curve by eliminating the parameter, and indicate the direction of increasing t
    $x(t) = 2\sin^2 t$ and $y(t) = 3\cos^2 t$ (0≤t≤π/2)

Find parametric equations for the curve and check your work by generating the curve with a graphing utility.
13. A circle with radius 5, centered at the origin, oriented counterclockwise.
17. The portion of a parabola $x = y^2$ joining (1,-1) and (1,1), oriented down to up.

40. If a projectile is fired from ground level with an initial speed of v_0 meters per second at an angle α with the horizontal, and if air resistance is neglected, then its position after *t* seconds is $x(t) = (v_0 \cos \propto)t,\ y(t) = -\frac{1}{2}gt^2 + (v_0 \sin \propto)t + h_0$, where $g \approx 9.81\frac{m}{s^2}$ and h_0 is the initial height.

a. By eliminating the parameter, show that the trajectory lies on the graph of a quadratic polynomial.

b. Use a graphing utility to sketch the trajectory of $\propto = 30^o$ and $v_0 = 1000\frac{m}{s}$

c. Using the trajectory, how high does the shell rise?

d. Using the trajectory, how far does the shell travel horizontally?

47. Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at the given point without eliminating the parameter.
    $x = \sec t$ and $y = \tan t$; $t = \frac{\pi}{3}$

51. a. Find the equation of the tangent line to the curve $x = e^t$ and $y = e^{-t}$ at $t = 1$ without eliminating the parameter.
b. Find the equation of the tangent line in part (a) by eliminating the parameter.

62. Suppose that a bee follows the trajectory $x = t - 2\cos t,\ y = 2 - 2\sin t$ (0≤t<2π)
    a. At what times was the bee flying horizontally?
    b. At what times was the bee flying vertically?

Find the exact arc length of the curve over the stated interval.
65. $x = t^2, y = \frac{1}{3}t^3 \quad (0 \leq t \leq 1)$
67. $x = \cos 3t, y = \sin 3t \quad (0 \leq t \leq \pi)$
69. $x = e^{2t}(\sin t + \cos t), y = e^{2t}(\sin t - \cos t) \quad (-1 \leq t \leq 1)$