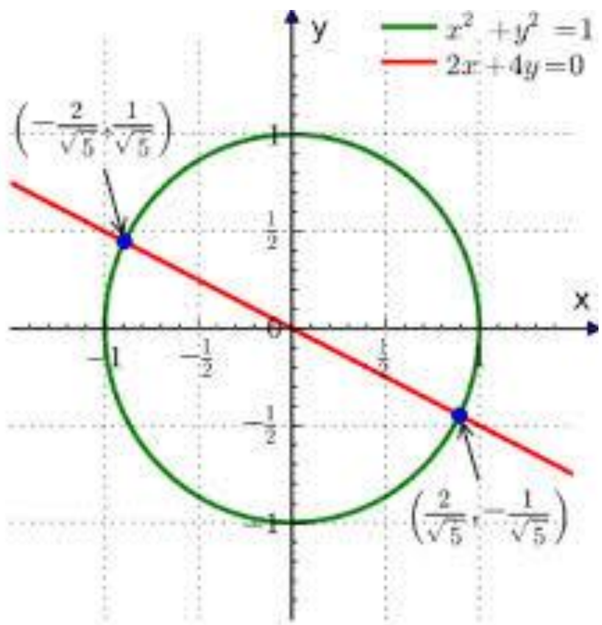
Algebra 2 and Trigonometry

Chapter 5:
Graphing Quadratics
Systems of Equations



Name:________________________________

Teacher:______________________________

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Algebra 2/Trig: Chapter 5 – Graphing Quadratics Packet

In this unit we will:

- Determine the properties (vertex) of the quadratic from looking at the coefficients in vertex form $\boldsymbol{y = a(x - h)^2 + k}$
- Solve a quadratic-linear system of equations
- Solve a Non-Linear system of equations
- Graph and Solve Quadratic Inequalities in Two-Variables

Table of Contents

Day 1 - Quadratic Linear Systems

SWBAT: Solve a quadratic-linear system of equations

Warm - Up:

Determine the value that would make each of the following a perfect square.

a) $x^2 + 6x +$ _______ is a perfect square trinomial because it is = ($\qquad$)^2

b) $x^2 - 20x +$ ______ is a perfect square trinomial because it is = ($\qquad$)^2

What is the "magic number" that completes the square?

Concept 1: Writing a Quadratic Function in Vertex Form

The vertex form for a quadratic equation in the form of $y = ax^2 + bx + c$ is

$$\boldsymbol{y = a(x - h)^2 + k}$$

where (h, k) are the vertex of the quadratic equation.

Example 1:

___ 1) Write the equation of the parabola that opens upwards, has a vertex V(2,-3), and is congruent to $y = x^2$. [*Place the answer in the form* $y = a(x - h)^2 + k$.]

A) $y = (x + 2)^2 + 3$

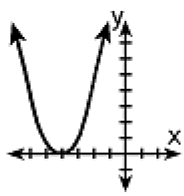
B) $y = (x - 2)^2 + 3$

C) $y = (x + 2)^2 - 3$

D) $y = (x - 2)^2 - 3$

___ **2)** Which of the following could be the graph of $y = (x + 4)^2$?

A)



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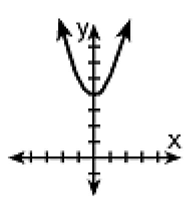

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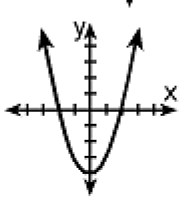

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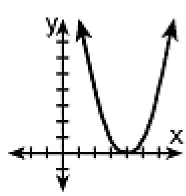


Example 2: **Write each function in vertex form, and identify its vertex.**

Teacher Modeled

$f(x) = x^2 + 10x - 13$

Step 1: $(x^2 + 10x + \_\_\_\_) - 13 - \_\_\_\_$

Step 2: $[\underbrace{x^2 + 10x + (\quad)^2}] - 13 - (\quad)^2$

Step 3: $(\qquad)^2 -$

$\boxed{f(x) = \qquad\qquad}$

Student Try it!

$f(x) = x^2 - 6x + 7$

Example 3: **Write each function in vertex form, and identify its vertex.**

Teacher Modeled

$\boldsymbol{f(x) = 2x^2 - 8x + 3}$

Step 1: $(2x^2 - 8x) + 3$

Step 2: $2[x^2 - 4x + (\quad)^2] + 3 - 2(\quad)^2$

Step 3: $2[\underbrace{x^2 - 4x + (\quad)^2}] + 3 - 2(\quad)^2$

$2(\qquad)^2 -$

$\boxed{f(x) = \qquad\qquad}$

1. ***Group ax^2 and bx term***
2. ***Set up to complete the square.*** *Because $\left(\frac{b}{2}\right)^2$ is multiplied by 2, we must subtract $2 \cdot \left(\frac{b}{2}\right)^2$.*
3. *Find magic number that completes the square.*

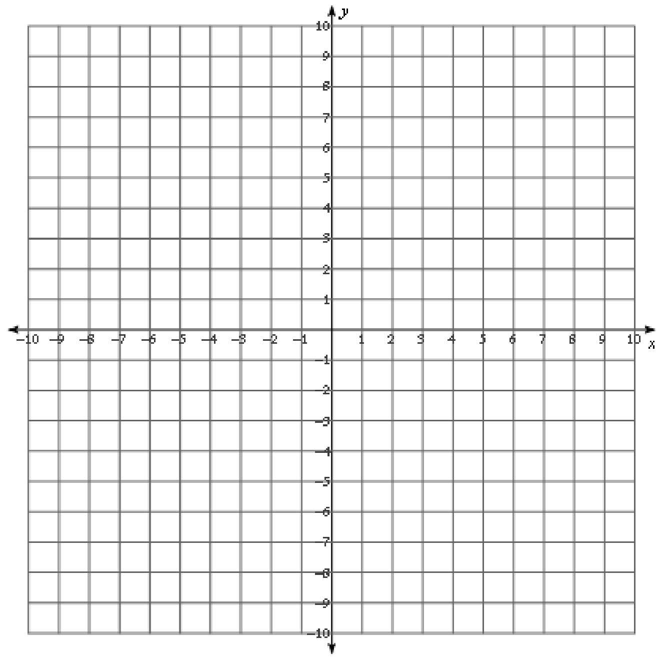
Student Try it!

$f(x) = -5x^2 + 15x + 9$

Concept 2: Solving a Quadratic – Linear System of Equations by Graphing

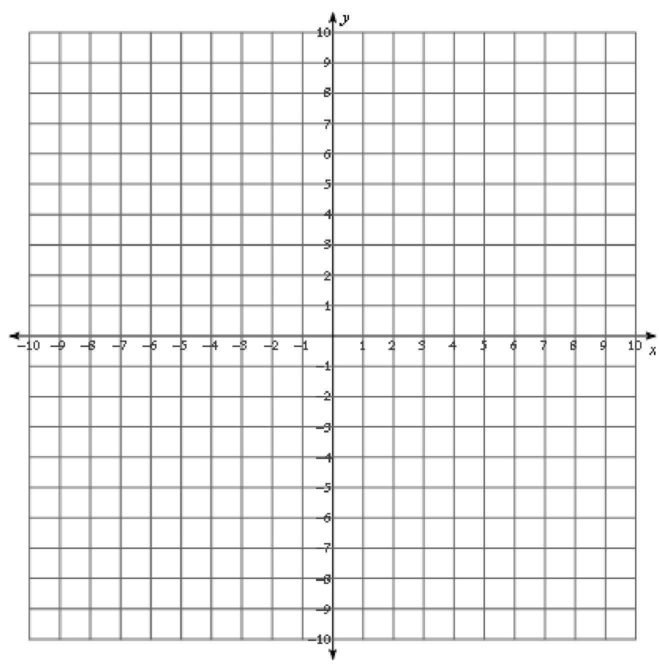
Example 4: On the accompanying grid, solve the following system of equations graphically:

$$\begin{aligned} y &= (x+4)^2 - 3 \\ y &= 2x + 5 \end{aligned}$$



Example 5: On the accompanying grid, solve the following system of equations graphically:

$$\begin{aligned} y &= x^2 - 4x + 6 \\ y &= x + 2 \end{aligned}$$



Concept 3: Solving a Quadratic – Linear System of Equations Algebraically

Teacher Modeled	***Student Try It!***
Example: $y = -x^2 + 2x + 4$ $x + y = 4 \;\rightarrow\;$ y = Substitute the y's Solve for x. The result will also be quadratic. You might need to factor or use the quadratic equation to solve for x. Find a corresponding y coordinate for each x value. Write the solutions as a set as a set of ordered pairs.	Example: $y = x^2 + 4x + 3$ $y - 2x = 6$ Substitute the y's Solve for x. The result will also be quadratic. You might need to factor or use the quadratic equation to solve for x. Find a corresponding y coordinate for each x value. Write the solutions as a set as a set of ordered pairs.

Challenge

Solve the system of equations:

$$\begin{aligned} x &= 3 \\ 5x+4y &= -9 \\ -x+4y-2z &= -25 \end{aligned}$$

SUMMARY

Example: Find the solution(s) to the following system of equations using the substitution method:

$y - 6 = x^2$
$y - 10 = 0$

Step 1: Solve each equation for y:

$y = x^2 + 6$
$y = 10$

Step 2: Substitute y = 10 into $y = x^2 + 6$

$y = x^2 + 6$
$10 = x^2 + 6$
$4 = x^2$

Step 3: Solve for x. Don't forget ±!

$x^2 = 4$
$\sqrt{x^2} = \pm\sqrt{4}$
$x = \pm 2$

Use square roots to "undo" squaring.

Step 4: Write your solutions:

(2, 10) and (-2, 10)

EXIT TICKET

Which ordered pair is a solution of the system of equations $y = x^2 - x - 20$ and $y = 3x - 15$?

1) $(-5,-30)$
2) $(-1,-18)$
3) $(0,5)$
4) $(5,-1)$

Day 1 - Homework

Writing a Quadratic Function in Vertex Form

1. Write the equation of the parabola in vertex form.

$$y = x^2 + 2x - 2$$

- $y = (x + 1)^2 + 3$
- $y = (x + 1)^2 - 3$
- $y = (x - 1)^2 - 3$
- $y = (x - 1)^2 + 3$

2. Rewrite $y = x^2 + 4x + 5$ in vertex form. Then find the vertex.

- $y = (x - 2)^2 + 9$; $(2, -9)$
- $y = (x - 2)^2 - 21$; $(2, 21)$
- $y = (x + 2)^2 + 9$; $(-2, 9)$
- $y = (x + 2)^2 + 1$; $(-2, 1)$

3. Write $y = -4x^2 - 64x - 265$ in vertex form.

- $y = -4(x + 8)^2 - 9$
- $y = -4(x - 8)^2 + 9$
- $y = -4(x - 8)^2 - 9$
- $y = -4(x + 8)^2 + 9$

4. Write $y = -3x^2 + 12x - 21$ in vertex form.

- $y = -3(x - 2)^2 - 9$
- $y = -3(x + 2)^2 - 9$
- $y = -3(x + 2)^2 + 9$
- $y = -3(x - 2)^2 + 9$

Quadratic-Linear Systems

5.

The graphs of the equations $y = x^2 - 5x + 6$ and $x + y = 6$ are drawn on the same set of axes. At what point do the graphs intersect?

A) (2,4)
C) (3,3)
B) (5,1)
D) (4,2)

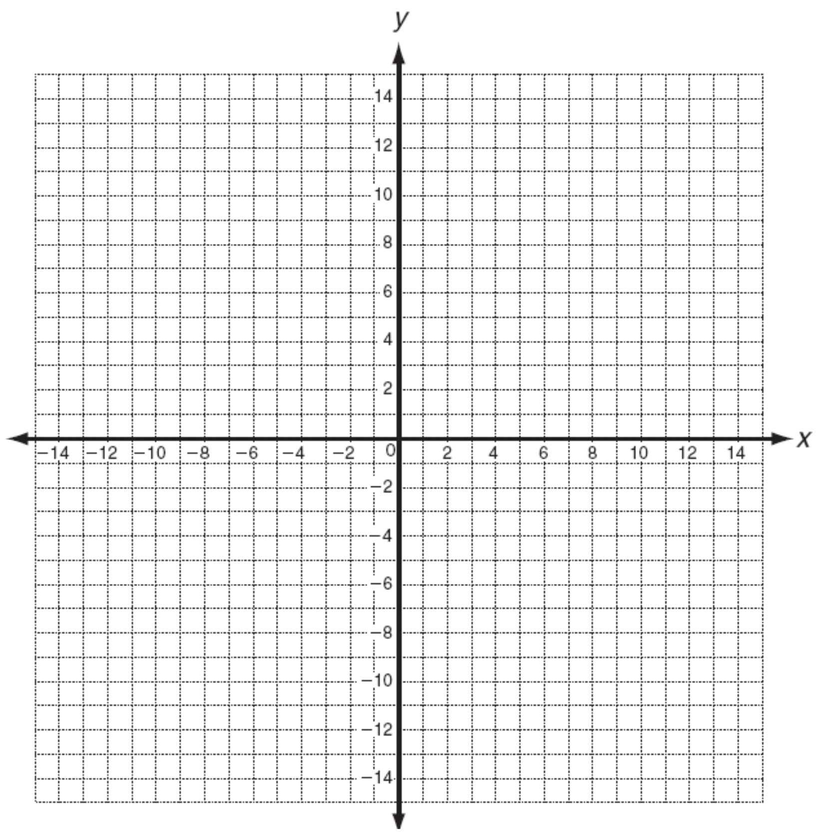
6.

The graphs of the equations $y = x^2 + 4x - 1$ and $y + 3 = x$ are drawn on the same set of axes. At which point do the graphs intersect?

A) (1,-2)
C) (-2,1)
B) (1,4)
D) (-2,-5)

7\. Solve the following system of equations:

$y = x^2 - 4x + 9$
$y - x = 5$



8) $y = x^2 + 4x - 1$
$y = 7x + 9$

x = ____	y =

x = ____	y =

9) $y = x^2 + 2x + 7$
$y = 6x + 3$

x = ____	y =

x = ____	y =

10) $y = x^2 + 2x - 6$
$3x + y = -12$

x = ____	y =

x = ____	y =

11) $y - 10x = 5$
$y = x^2 + 7x + 5$

x = ____	y =

x = ____	y =

Day 2 - More Non-Linear Systems

Warm – Up

Rewrite $y = x^2 - 10x + 3$ in vertex form. Then find the vertex.

A) $y = (x + 5)^2 + 28; (-5, -28)$

B) $y = (x - 5)^2 - 22; (5, -22)$

C) $y = (x + 5)^2 - 103; (-5, 103)$

D) $y = (x - 5)^2 + 28; (5, 28)$

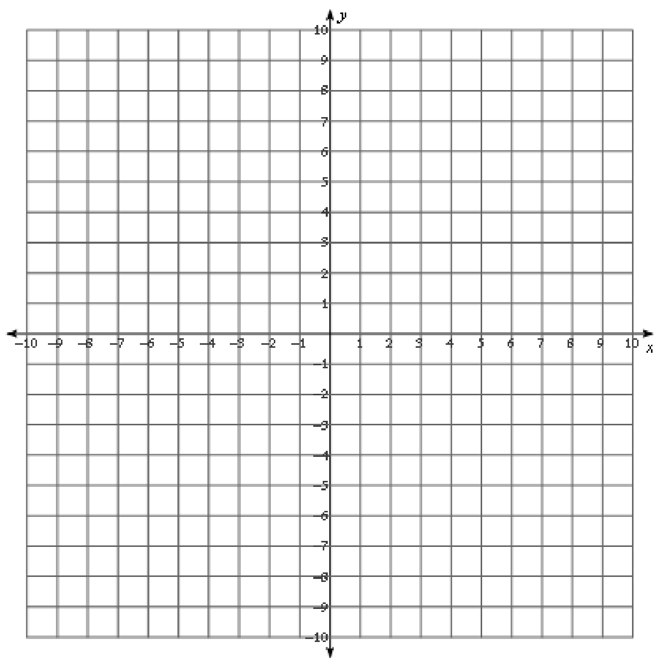
Some non-linear Systems contain two variables. They are solved in the same way (substitution), but your resulting equation will have a binomial to be FOILed in the problem.

Example 1:

Part a: On the set of axes provided below, graph both equations.

$x^2 + y^2 = 4$

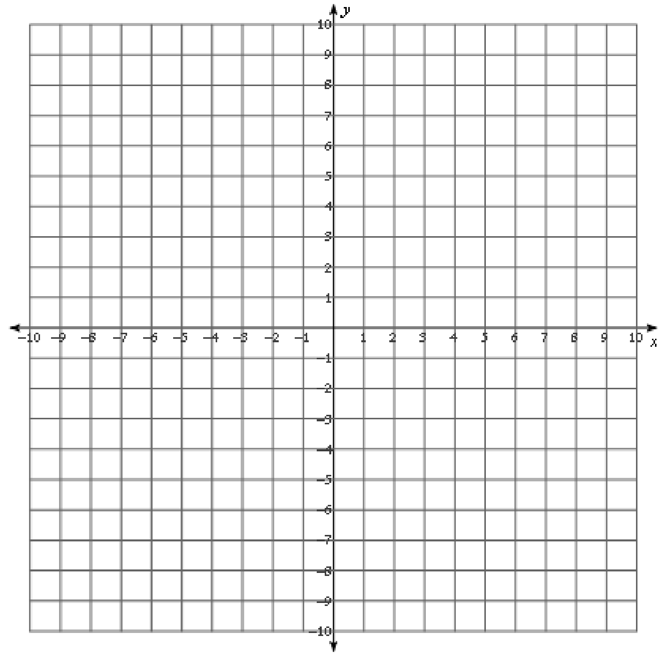
$y - x = 0$



Part b: What is the total number of points of intersection of the two graphs?

Part c: Find the exact coordinates of the points of intersection.

Example 2: On the set of axes provided below, sketch a circle with a radius of 3 and center at (2,1) and also sketch the graph of the line $2x + y = 8$.



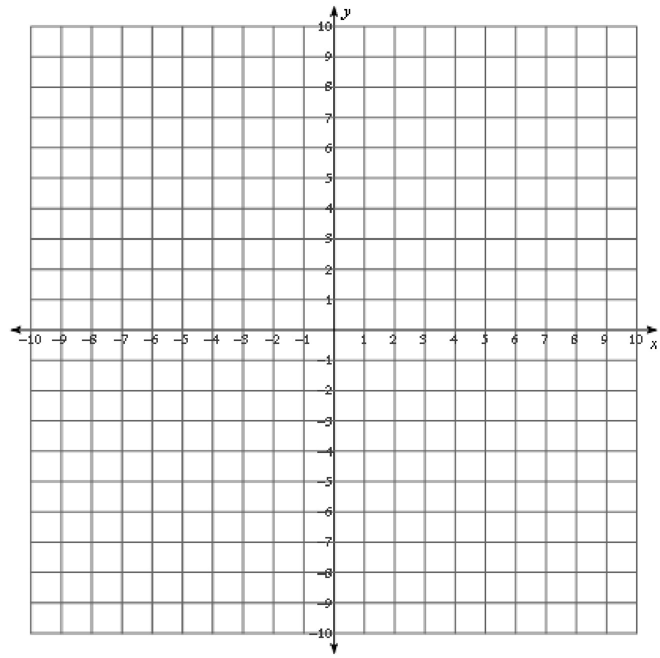
b What is the total number of points of intersection of the two graphs?

c Find the exact coordinates of the points of intersection.

Example 3: Solve the following system of equations algebraically:

$$9x^2 + y^2 = 9$$
$$3x - y = 3$$

Example 4: Two circles whose equations are $(x-3)^2+(y-5)^2=25$ and $(x-7)^2+(y-5)^2=9$ intersect in two points. Find the exact coordinates of the points of intersection.



Challenge

A number is composed of two digits the difference of whose squares is 20. If the digits are interchanged the resulting number is 18 less than the original number. Find the number.

SUMMARY

Solving a Nonlinear System by Substitution

Solve $\begin{cases} x^2 + y^2 = 25 \\ y + 5 = \frac{1}{2}x^2 \end{cases}$ by using the substitution method.

The graph of the first equation is a circle, and the graph of the second equation is a parabola. There may be as many as four points of intersection.

Step 1 It is simplest to solve for x^2 because both equations have x^2 terms.

$x^2 = 2y + 10$ *Solve for x^2 in the second equation.*

Step 2 Use substitution.

$(2y + 10) + y^2 = 25$ *Substitute this value into the first equation.*

$y^2 + 2y - 15 = 0$ *Simplify, and set equal to 0.*

$(y - 3)(y + 5) = 0$ *Factor.*

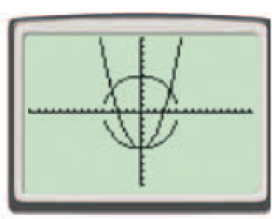
$y = 3$ or $y = -5$

Step 3 Substitute 3 and -5 into $x^2 = 2y + 10$ to find values for x.

$x^2 = 2(3) + 10$	$x^2 = 2(-5) + 10$
$x^2 = 16$	$x^2 = 0$
$x = \pm 4$	$x = 0$
$(4, 3)$ and $(-4, 3)$ are solutions.	$(0, -5)$ is a solution.

The solution set of the system is $\{(4, 3), (-4, 3), (0, -5)\}$.

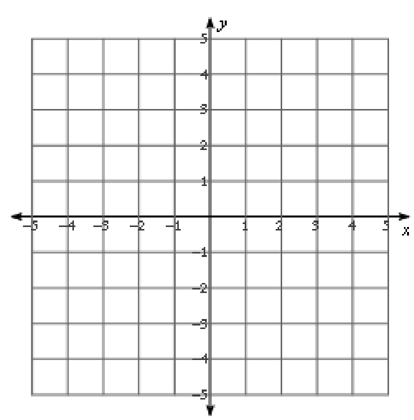
Check Use a graphing calculator. The graph supports that there are three points of intersection.



Exit Ticket

What is the total number of points of intersection in the graphs of the equations $x^2 + y^2 = 16$ and $y = 4$?

1) 1
2) 2
3) 3
4) 0

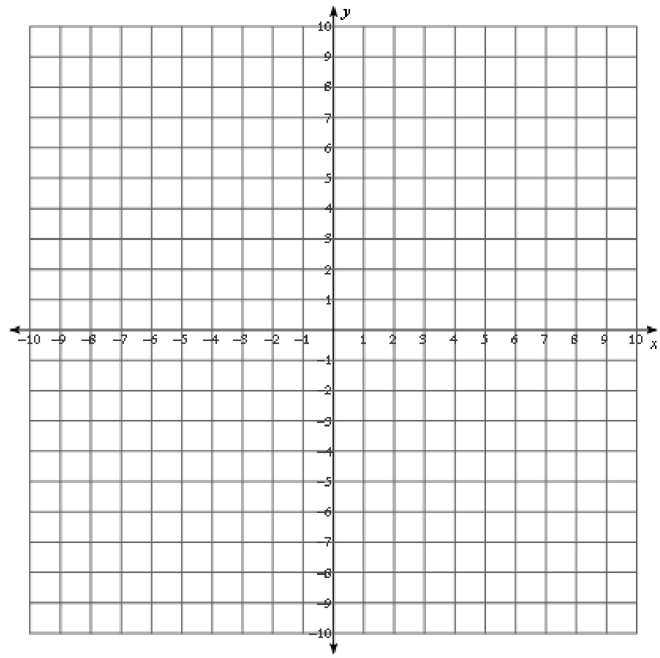


Day 2 – Homework

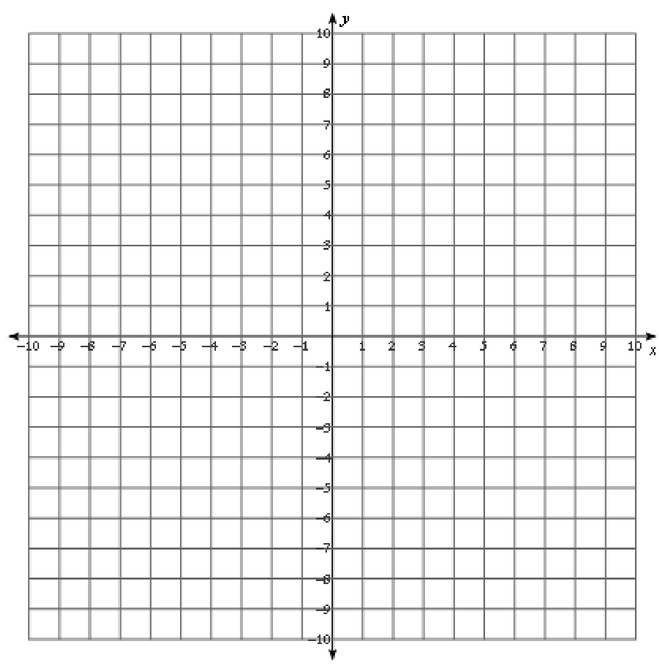
1. Solve the following system of equations algebraically or graphically:

$$x^2 + y^2 = 25$$
$$3y - 4x = 0$$

[The use of the accompanying grid is optional.]



2. Two circles whose equations are $(\boldsymbol{x-2})^{\boldsymbol{2}} + (\boldsymbol{y-3})^{\boldsymbol{2}} = \boldsymbol{25}$ and $(\boldsymbol{x-2})^{\boldsymbol{2}} + (\boldsymbol{y-1})^{\boldsymbol{2}} = \boldsymbol{9}$ intersect in two points. Find the exact coordinates of the points of intersection.



Solve the following systems algebraically.

3. $\begin{cases} y + x = 17 \\ x^2 + y^2 = 169 \end{cases}$	*4.* $\begin{cases} x^2 + y^2 = 36 \\ x + 2y = 16 \end{cases}$
5. $\begin{cases} x^2 + y^2 = 36 \\ y + 6 = \frac{1}{3}x^2 \end{cases}$	*6.* $\begin{Bmatrix} (\boldsymbol{x} + \mathbf{1})^2 + (\boldsymbol{y} - \mathbf{2})^2 = \mathbf{9} \\ \boldsymbol{y} + \mathbf{2} = \mathbf{2}\boldsymbol{x} \end{Bmatrix}$

Day 3 – Graphing Quadratic Inequalities

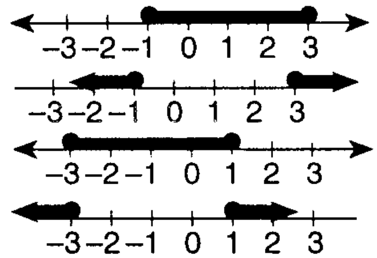
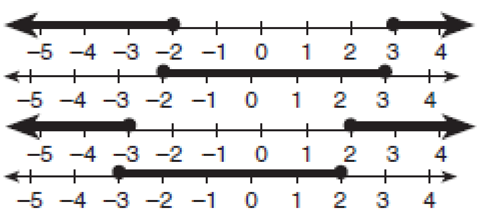
Warm - Up:

Solve the following systems algebraically.

$$\begin{Bmatrix} (x-3)^2 + (y-3)^2 = 64 \\ y + 3 = 2x \end{Bmatrix}$$

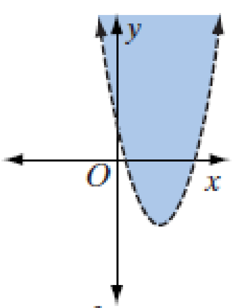
Quadratic inequalities can be solved graphically or algebraically.

Concept 1: The quadratic inequality in <u>ONE VARIABLE</u> $ax^2 + bx + c$ with roots $\{r_1, r_2\}$:

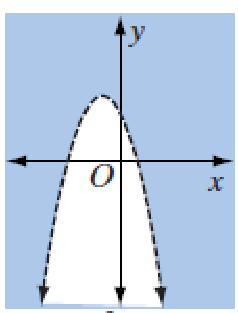
1. What is the solution set of the inequality $x^2 - 3x - 10 > 0$? A) $\{x \mid -2 < x < 5\}$ B) $\{x \mid x < -2 \text{ or } x > 5\}$ C) $\{x \mid x < -5 \text{ or } x > 2\}$ D) $\{x \mid -5 < x < 2\}$	2. What is the solution set for the inequality $x^2 - 2x - 3 \le 0$? 1) 2) 3) 4) 
3. Which graph represents the solution of the inequality $x^2 - x - 6 \ge 0$? 1) 2) 3) 4) 	4. What is the solution of the inequality $x^2 + 2x - 15 < 0$? A) $-5 < x < 3$ B) $-3 < x < 5$ C) $x < -3$ or $x > 5$ D) $x < -5$ or $x > 3$

We can use the same techniques from above to graph quadratic inequalities in ***<u>TWO VARIABLES</u>*** on the coordinate plane!

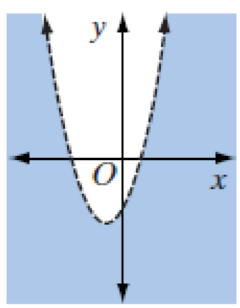
The following figures demonstrate some of these graphing rules:



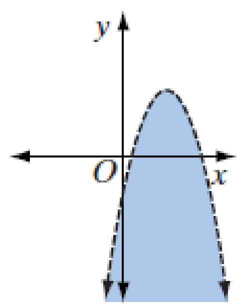
$y > ax^2 + bx + c$



$y > -ax^2 + bx + c$



$y < ax^2 + bx + c$



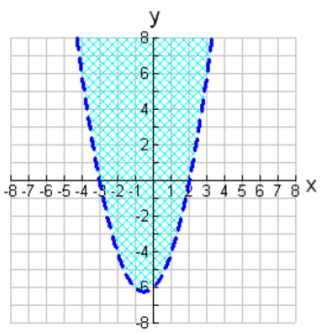
$y < -ax^2 + bx + c$

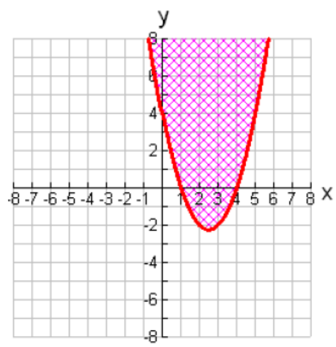
Graphing Quadratic Inequalities	
Step 1:	Solve the inequality for y ($y = ax^2 + bx + c$).
Step 2:	Graph the boundary line. Use a <u>**solid line**</u> for $\le$ or $\ge$. Use a <u>**dashed line**</u> for < or >.
Step 3:	Pick a point and plug it into the inequality to determine what area needs to be shaded. • Shade the region <u>above</u> the parabola for y $>$ or $\ge$. • Shade the region <u>below</u> the parabola for y $<$ or $\le$.

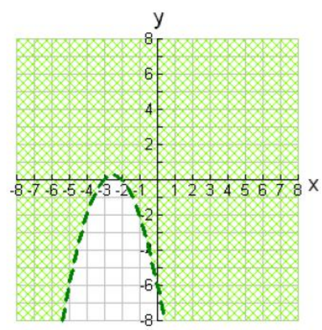
Concept 2: Matching Graphs to Inequalities

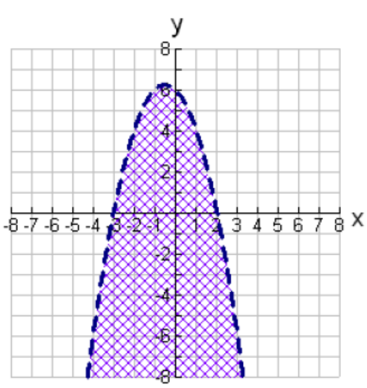
Match each graph with the appropriate inequality.

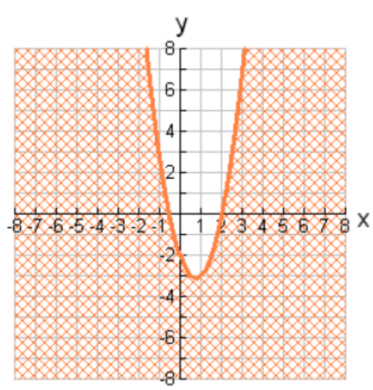
A. $y > -x^2 - 5x - 6$	**B.** $y > x^2 + x - 6$	**C.** $y \le 2x^2 - 3x - 2$
D. $y \ge x^2 - 5x + 4$	**E.** $y < -x^2 - x + 6$	











Concept 3: Graphing Quadratic Inequalities in Two Variables

Steps:

1) Solve the inequality for y. It's nice to have y on the left hand side!
2) Graph the corresponding quadratic function.

- Use the appropriate curve:

  $< \ or \ >$ dashed curve

  $\leq \ or \ \geq$ solid curve

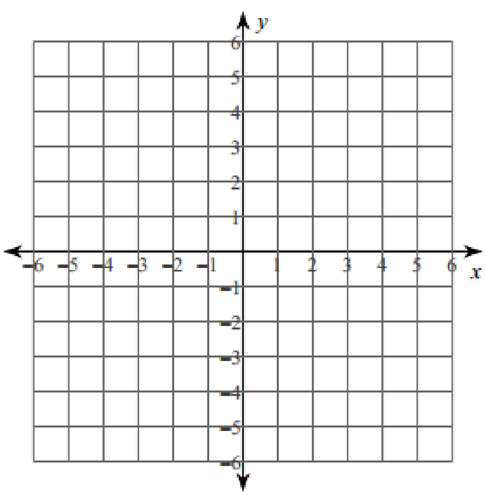
- Shade according to your inequality symbol.

  $< \ or \ \leq$ shade down
  $> \ or \ \geq$ shade up

3) Use test points to verify where to shade!

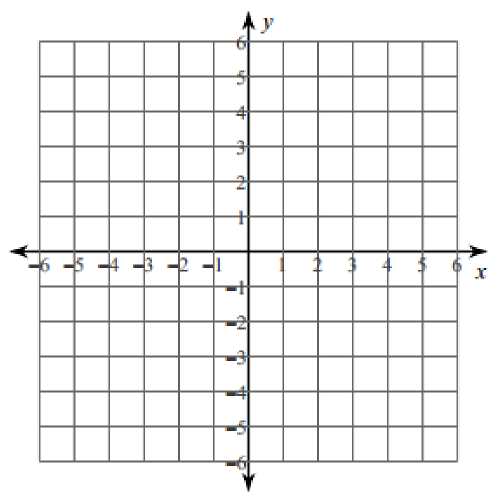
Ex 1: $y - 3 \geq x^2 + 4x$

x	y



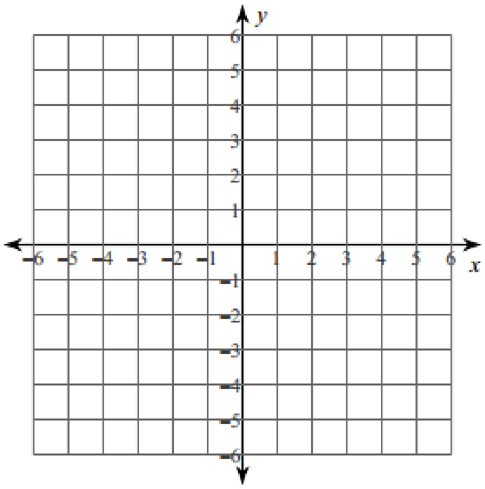
Ex 2: $y - 4x - 1 < -x^2$

x	y



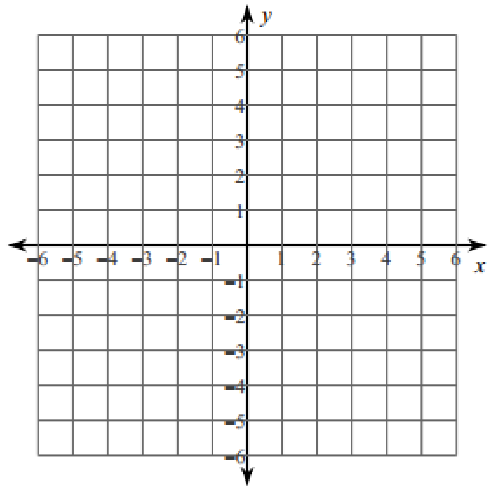
Ex 3: $y + 2 > (x + 3)^2$

a = ___; Vertex: (   )



Ex 4: $y - 5 \le -2(x - 1)^2$

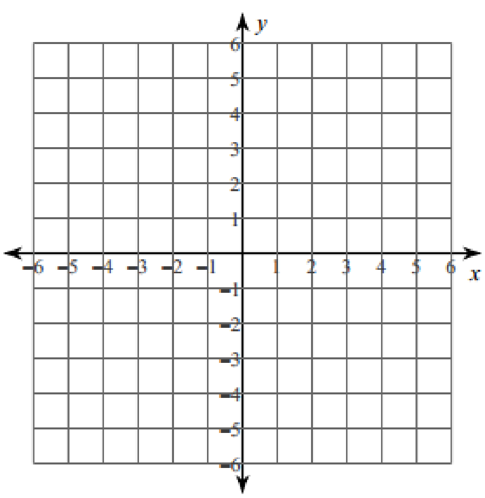
a = ___; Vertex: (   )



Concept 4: Solving a Quadratic Inequality by Graphing

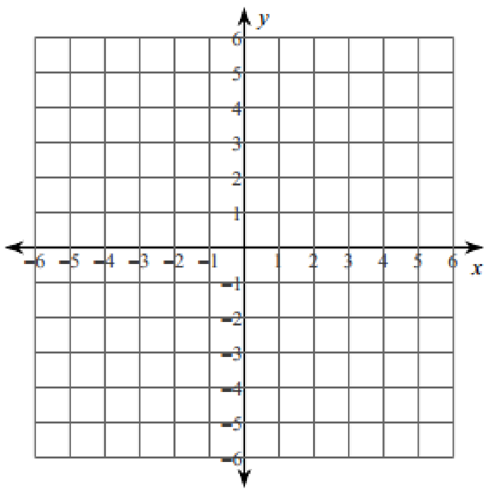
Example 5: Solve $2x^2 + 3x - 3 \le 0$

x	y



Example 6: Solve $-2x^2 + 12x - 15 > 0$

x	y

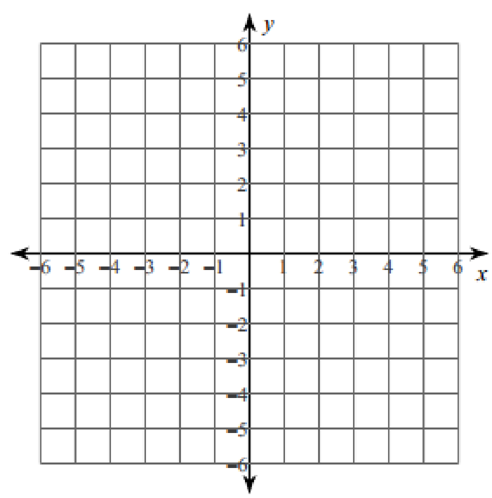


Challenge

Graph the system of quadratic inequalities.

$$y \geq x^2 - 4$$
$$y < -x^2 - x + 2$$

Identify a point in the solution region.



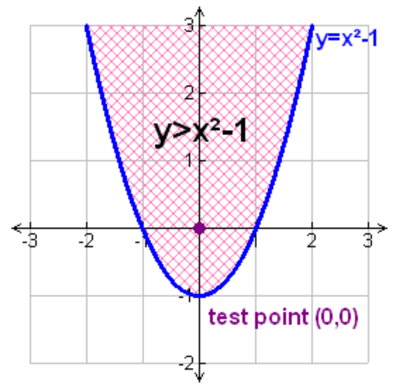
Summary/Closure

$Solve: \; y \geq x^2 - 1$

Begin by graphing the corresponding equation $y = x^2 - 1$.

(Use a dashed line for < or > and a solid line for $\leq$ or $\geq$.)

Test a point above the parabola and a point below the parabola into the original inequality. Shade the entire region where the test point yields a true result.



The parabola graph was drawn using a solid line since the inequality was "greater than or equal to".

The point (0,0) was tested into the inequality and found to be **true.**

$$0 \geq 0^2 - 1$$

The point (0,-2) was tested into the inequality and found to be **false.**

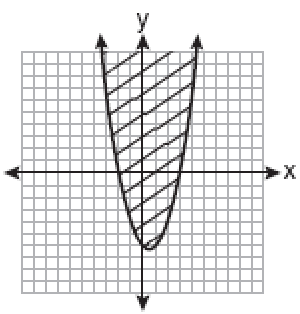
$$-2 \not\geq 0^2 - 1$$

The graph was shaded in the region where the true test point was located.

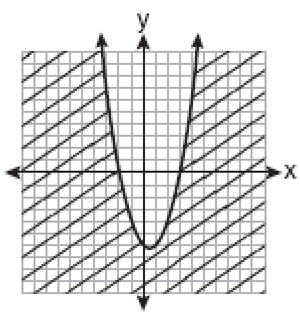
ANSWER: The shaded area (including the solid line of the parabola) contains all of the points that make this inequality true.

Exit Ticket:

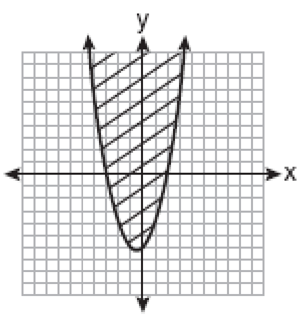
Which graph best represents the inequality $y + 6 \geq x^2 - x$?



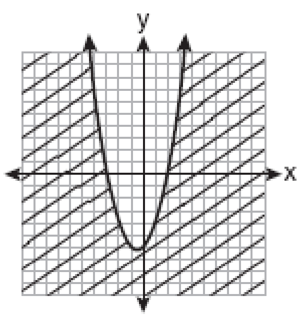
(1)



(3)



(2)



(4)

Day 3 – Homework

#1-3 Determine whether the ordered pair is a solution of the inequality. Show your work then answer **yes** or **no**.

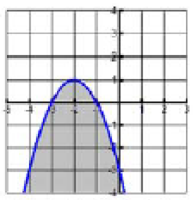
1. $y < x^2 - 2x + 4$, $(1,2)$ 2. $y > 2x^2 + x - 5$, $(-2,1)$ 3. $y \le -2x^2 + 5x + 6$, $(4,-4)$

#4-9 Match the inequality with its graph.

_____ 4. $y \ge -x^2 + 4x - 3$ _____ 5. $y \le -x^2 - 4x - 3$ _____ 6. $y \le x^2 + 2x - 3$

_____ 7. $y < x^2 - 4x + 3$ _____ 8. $y > -x^2 + 4x - 3$ _____ 9. $y > x^2 - 2x - 3$

Use A-F to match with quadratic inequalities #4-9.

A.


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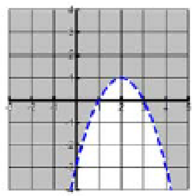

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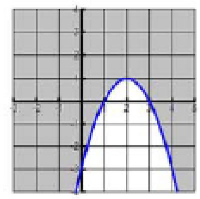

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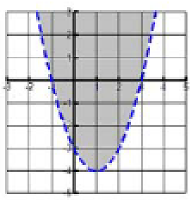
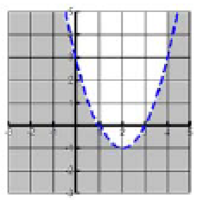
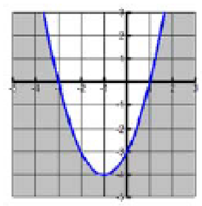


E.


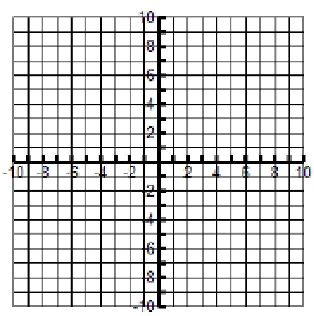
F.


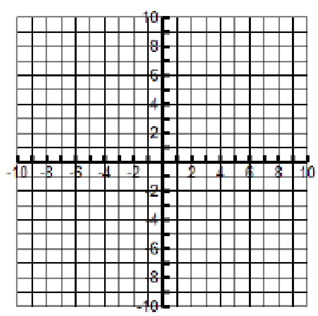
#10-12 Solve each quadratic inequality algebraically, then graph the solution on a number line.

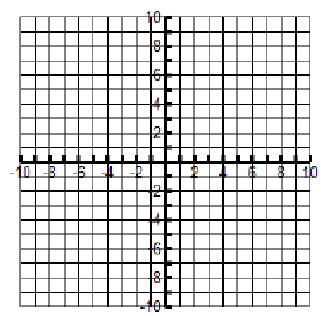
10. $x^2 - 2x - 15 < 0$ 11. $x^2 + 7x + 12 \ge 0$ 12. $3x^2 + 4 \le 7x$

#13-15 Graph each quadratic inequality.

13. $y \le x^2 - 6x + 8$ 14. $y - 2 \le -(x-3)^2$ 15. $y > 2x^2 - 4x - 6$







Day 1 – Answers

Writing a Quadratic Function in Vertex Form

1. Write the equation of the parabola in vertex form.

- $y = (x + 1)^2 + 3$
- $\boxed{y = (x + 1)^2 - 3}$
- $y = (x - 1)^2 - 3$
- $y = (x - 1)^2 + 3$

$$y = x^2 + 2x - 2$$

$(x^2 + 2x) - 2$

$[x^2 + 2x + (1)^2] - 2 - (1)^2$

$y = (x+1)^2 - 3$

2. Rewrite $y = x^2 + 4x + 5$ in vertex form. Then find the vertex.

- $y = (x - 2)^2 + 9;\ (2, -9)$
- $y = (x - 2)^2 - 21;\ (2, 21)$
- $y = (x + 2)^2 + 9;\ (-2, 9)$
- $\boxed{y = (x + 2)^2 + 1;\ (-2, 1)}$

$y = (x^2 + 4x) + 5$

$y = [x^2 + 4x + (2)^2] + 5 - (2)^2$

$y = (x+2)^2 + 1;\ V = (-2, 1)$

3. Write $y = -4x^2 - 64x - 265$ in vertex form.

- $\boxed{y = -4(x + 8)^2 - 9}$
- $y = -4(x - 8)^2 + 9$
- $y = -4(x - 8)^2 - 9$
- $y = -4(x + 8)^2 + 9$

$y = (-4x^2 - 64x) - 265$

$y = -4[x^2 + 16x + (8)^2] - 265 - (-4)(8)^2$

$y = -4(x+8)^2 - 265 + 256$

$y = -4(x+8)^2 - 9$

4. Write $y = -3x^2 + 12x - 21$ in vertex form.

- $\boxed{y = -3(x - 2)^2 - 9}$
- $y = -3(x + 2)^2 - 9$
- $y = -3(x + 2)^2 + 9$
- $y = -3(x - 2)^2 + 9$

$y = -3(x^2 + -4x) - 21$

$y = -3[x^2 - 4x + (-2)^2] - 21 - (-3)(-2)^2$

$y = -3(x-2)^2 - 21 + 12$

$y = -3(x-2)^2 - 9$

Quadratic-Linear Systems

5.

The graphs of the equations $y = x^2 - 5x + 6$ and $x + y = 6$ are drawn on the same set of axes. At what point do the graphs intersect?

A) (2,4) C) (3,3)
B) (5,1) D) (4,2)

X	Y1	Y2
0	6	6
1	2	5
2	0	4
3	0	3
4	2	2
5	6	1
6	12	0

6.

The graphs of the equations $y = x^2 + 4x - 1$ and $y + 3 = x$ are drawn on the same set of axes. At which point do the graphs intersect?

A) (1,-2) C) (-2,1)
B) (1,4) D) (-2,-5)

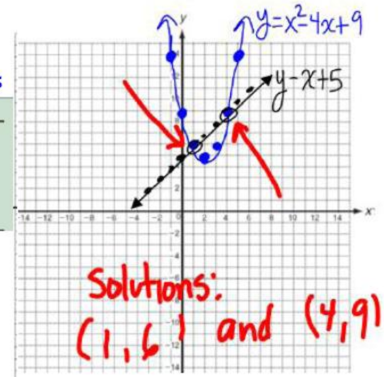
X	Y1	Y2
-3	-4	-6
-2	-5	-5
-1	-4	-4
0	-1	-3
1	4	-2
2	11	-1
3	20	0

7. Solve the following system of equations:

$y = x^2 - 4x + 9$
$y - x = 5$

$y = x^2 - 4x + 9$ $y = x + 5$

X	Y1	Y2
0	9	5
1	6	6
2	5	7
3	6	8
4	9	9
5	14	10
6	21	11



8) $y = x^2 + 4x - 1$
$y = 7x + 9$

$$x^2 + 4x - 1 = 7x + \cancel{9}$$
$$-7x - 9 = -7\cancel{x} - 9$$
$$x^2 - 3x - 10 = 0$$
$$(x+2)(x-5) = 0$$

-10, 2, -5, -3

$(x+2) = 0$ | $(x-5) = 0$
$x = -2$ | $x = 5$

$y = 7x + 9$ | $y = 7x + 9$
$y = 7(-2) + 9$ | $y = 7(5) + 9$
$y = -5$ | $y = 44$

Solution: (-2, -5) and (5, 44)

9) $y = x^2 + 2x + 7$
$y = 6x + 3$

$$x^2 + 2x + 7 = 6x + \cancel{3}$$
$$-6x - 3 = -6\cancel{x} - 3$$
$$x^2 - 4x + 4 = 0$$
$$(x-2)(x-2) = 0$$

4, -2, -2, -4

$(x-2) = 0$ | $(x-2) = 0$
$x = 2$ | $x = 2$

$y = 6x + 3$
$y = 6(2) + 3$
$y = 15$

Solution: (2, 15)

10) $y = x^2 + 2x - 6$
$3x + y = -12$

$$x^2 + 2x - 6 = -3x \cancel{-} 12$$
$$+3x + 12 = \cancel{3x} + 12$$
$$x^2 + 5x + 6 = 0$$
$$(x+2)(x+3) = 0$$

6, 2, 3, 5

$(x+2) = 0$ | $(x+3) = 0$
$x = -2$ | $x = -3$

$y = -3x - 12$ | $y = -3x - 12$
$y = -3(-2) - 12$ | $y = -3(-3) - 12$
$y = -6$ | $y = -3$

Solutions: (-2, -6) and (-3, -3)

11) $y - 10x = 5$
$y = x^2 + 7x + 5$

$$x^2 + 7x + 5 = 10x \cancel{+} 5$$
$$-10x - 5 = -\cancel{10x} - 5$$
$$x^2 - 3x = 0$$
$$x(x-3) = 0$$

0, 0, -3, -3

$x = 0$ | $(x-3) = 0$
 | $x = 3$

$y = 10x + 5$ | $y = 10x + 5$
$y = 10(0) + 5$ | $y = 10(3) + 5$
$y = 5$ | $y = 35$

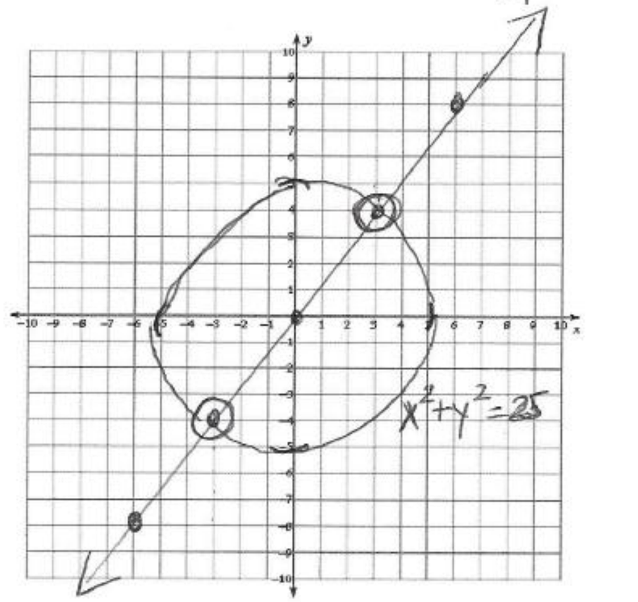
Solutions: (0, 5) and (3, 35)

Day 2 – Answers

1. Solve the following system of equations algebraically or graphically:

$$x^2 + y^2 = 25$$
$$3y - 4x = 0$$

[The use of the accompanying grid is optional.]



$3y - 4x = 0$

$3y = 4x$

$y = \frac{4}{3}x$

$x^2 + \left(\frac{4}{3}x\right)^2 = 25$

$x^2 + \frac{16}{9}x^2 = 25$

$9\left(x^2 + \frac{16}{9}x^2\right) = 25 \cdot 9$

$9x^2 + 16x^2 = 225$

$25x^2 = 225$

$\sqrt{x^2} = \sqrt{9} \rightarrow x = \pm 3$

when $x = 3$,

$y = \frac{4}{3}(3) = 4$

when $x = -3$

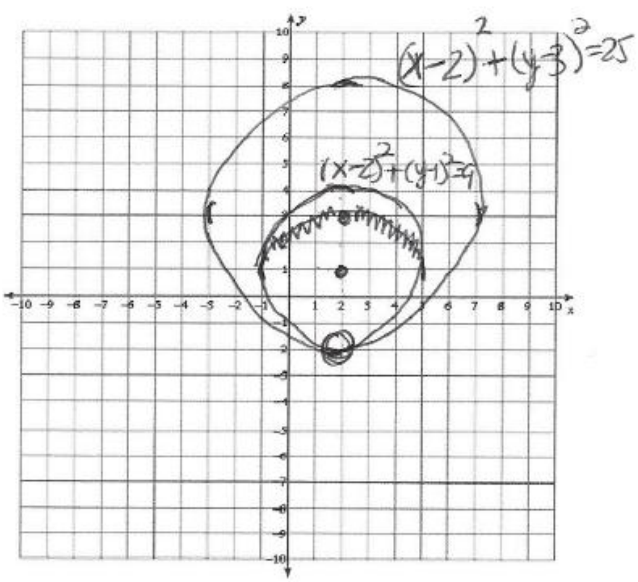
$y = \frac{4}{3}(-3) = -4$

$\boxed{\text{sol'n: } (3,4) \quad (-3,-4)}$

2. Two circles whose equations are $(x-2)^2 + (y-3)^2 = 25$ and $(x-2)^2 + (y-1)^2 = 9$ intersect in two points. Find the exact coordinates of the points of intersection.

$(x-2)^2 + (y-3)^2 = 25 \rightarrow C = (2,3) \quad r = 5$

$(x-2)^2 + (y-1)^2 = 9 \rightarrow C = (2,1) \quad r = 3$



$(x-2)^2 = 25 - (y-3)^2$

$(x-2)^2 = 9 - (y-1)^2$

$25 - (y-3)^2 = 9 - (y-1)^2$

$25 - (y^2 - 6y + 9) = 9 - (y^2 - 2y + 1)$

$25 - \cancel{y^2} + 6y - 9 = 9 - \cancel{y^2} + 2y - 1$

$6y + 16 = 2y + 8$

$-2y \qquad -2y$

$4y + 16 = 8$

$4y = -8$

$y = -2$

$(x-2)^2 = 25 - (y-3)^2$

$(x-2)^2 = 25 - (-2-3)^2$

$(x-2)^2 = 25 - (-5)^2$

$(x-2)^2 = 25 - 25$

$\sqrt{(x-2)^2} = \sqrt{0}$

$\rightarrow x - 2 = 0$

$x = 2$

$\boxed{\text{sol'n: } (2,-2)}$

Solve the following systems algebraically.

3.

$\begin{cases} y + x = 17 \rightarrow y = -x+17 \\ x^2 + y^2 = 169 \end{cases}$

$x^2 + (-x+17)^2 = 169$

$x^2 + (-x+17)(-x+17) = 169$

$x^2 + x^2 - 17x - 17x + 289 = 169$

$\frac{2x^2 - 34x + 120}{2} = \frac{0}{2}$

$x^2 - 17x + 60 = 0$

$(x-12)(x-5) = 0$

$x - 12 = 0$, $x = 12$ | $x - 5 = 0$, $x = 5$

$y = -(12)+17$, $y = 5$ | $y = -5+17$, $y = 12$

$\boxed{\text{sol'n}: (12,5)(5,12)}$

4.

$\begin{cases} x^2 + y^2 = 36 \\ x + 2y = 16 \rightarrow x = -2y+16 \end{cases}$

$(-2y+16)^2 + y^2 = 36$

$(-2y+16)(-2y+16) + y^2 = 36$

$4y^2 - 32y - 32y + 256 + y^2 = 36$

$5y^2 - 64y + 220 = 0$

$a = 5$, $b = -64$, $c = 220$

$y = \frac{-(-64) \pm \sqrt{(-64)^2 - 4(5)(220)}}{2(5)}$

$y = \frac{64 \pm \sqrt{-304}}{10}$

* imaginary when the discriminant is simplified → $\boxed{\text{No solution}}$

5.

$\begin{cases} x^2 + y^2 = 36 \\ y + 6 = \frac{1}{3}x^2 \rightarrow 3y + 18 = x^2 \end{cases}$

$\cdot 3$

$x^2 + y^2 = 36$

$3y + 18 + y^2 = 36$

$y^2 + 3y - 18 = 0$

$(y+6)(y-3) = 0$

$y + 6 = 0$, $y = -6$, $3(-6) + 18 = x^2$, $0 = x^2$, $0 = x$ | $y - 3 = 0$, $y = 3$, $3(3) + 18 = x^2$, $27 = x^2$, $\sqrt{27} = \sqrt{x^2}$, $\pm\sqrt{9}\sqrt{3} = x$, $\pm 3\sqrt{3} = x$

$\boxed{\text{sol'n}: (0,-6),\ (3\sqrt{3}, 3),\ (-3\sqrt{3}, 3)}$

6.

$\begin{cases} (x+1)^2 + (y-2)^2 = 9 \\ y + 2 = 2x \end{cases}$

$y = 2x - 2$

$\boxed{\text{Sol'n}: (2,2),\ \left(\frac{4}{5}, -\frac{2}{5}\right)}$

$(x+1)^2 + (2x-2-2)^2 = 9$

$(x+1)^2 + (2x-4)^2 = 9$

$(x+1)(x+1) + (2x-4)(2x-4) = 9$

$x^2 + 1x + 1x + 1 + 4x^2 - 8x - 8x + 16 = 9$

$x^2 + 2x + 1 + 4x^2 - 16x + 16 - 9 = 0$

$x^2 + 2x + 1 + 4x^2 - 16x + 7$

$5x^2 - 14x + 8 = 0$

$a = 5$, $b = -14$, $c = 8$

$x = \frac{-(-14) \pm \sqrt{(-14)^2 - 4(5)(8)}}{2(5)}$

$x = \frac{14 \pm \sqrt{36}}{10}$

$x = \frac{14 \pm 6}{10} \rightarrow x_1 = \frac{14+6}{10} = 2$, $x_2 = \frac{14-6}{10} = \frac{4}{5}$

$x_1 = 2$, $y = 2(2) - 2 = 2$

$x_1 = \frac{4}{5}$, $y = 2\left(\frac{4}{5}\right) - 2$, $y = -\frac{2}{5}$

Day 3 – Homework

#1-3 Determine whether the ordered pair is a solution of the inequality. Show your work then answer yes or no.

1. $y < x^2 - 2x + 4$, $(1, 2)$ x y

$2 < (1)^2 - 2(1) + 4$

$2 < 3$ ✓ $\boxed{\text{yes}}$

2. $y > 2x^2 + x - 5$, $(-2, 1)$

$1 > 2(-2)^2 + (-2) - 5$

$1 > 1$ x $\boxed{\text{NO}}$

3. $y \le -2x^2 + 5x + 6$, $(4, -4)$

$-4 \le -2(4)^2 + 5(4) + 6$

$-4 \le -6$ x

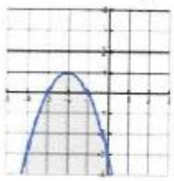
$\boxed{\text{NO}}$

#4-9 Match the inequality with its graph.

C 4. $y \ge -x^2 + 4x - 3$ A 5. $y \le -x^2 - 4x - 3$ F 6. $y \le x^2 + 2x - 3$

E 7. $y < x^2 - 4x + 3$ B 8. $y > -x^2 + 4x - 3$ D 9. $y > x^2 - 2x - 3$

Use A-F to match with quadratic inequalities #4-9.

A.


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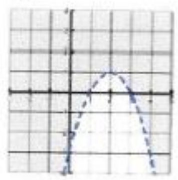

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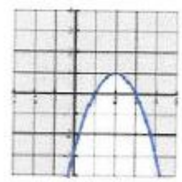

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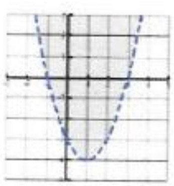

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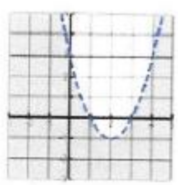

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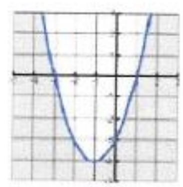


#10-12 Solve each quadratic inequality algebraically, then graph the solution on a number line. shade in between

10. $x^2 - 2x - 15 < 0$ $\boxed{-3 < x < 5}$

Shade in between

$(x-5)(x+3) = 0$

$x - 5 = 0$ $x + 3 = 0$

$x = 5$ $x = -3$

-3   5

11. $x^2 + 7x + 12 \ge 0$

"wings"

$(x+3)(x+4) = 0$

$x + 3 = 0$ $x + 4 = 0$

$x = -3$ $x = -4$

-4   -3

$\boxed{x \le -4 \text{ or } x \ge -3}$

12. $3x^2 + 4 \le 7x$

$3x^2 - 7x + 4 \le 0$

$a = 3$, $b = -7$, $c = 4$

$x = \dfrac{-(-7) \pm \sqrt{(-7)^2 - 4(3)(4)}}{2(3)}$

$x = \dfrac{7 \pm \sqrt{1}}{6}$

$x = \dfrac{7 \pm 1}{6}$

$x_1 = \frac{8}{6} = \frac{4}{3}$

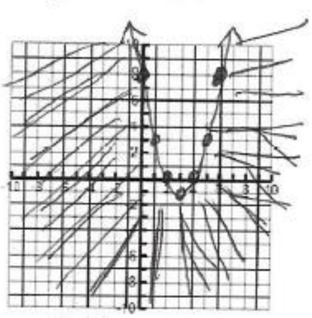
$x_2 = \frac{6}{6} = 1$

1    $\frac{4}{3}$

$\boxed{1 \le x \le \frac{4}{3}}$

#13-15 Graph each quadratic inequality.

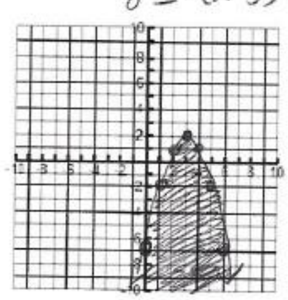
13. $y \le x^2 - 6x + 8$



x	y
0	8
1	3
2	0
3	-1
4	0
5	3
6	8

14. $y - 2 \le -(x-3)^2$

$y \le -(x-3)^2 + 2$

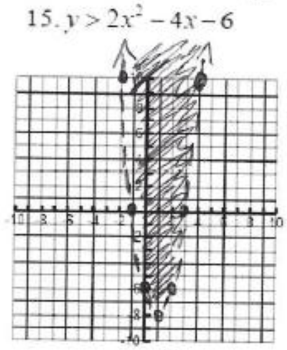


vertex: (3, 2)

$a = -1$

Rule $\frac{-1}{1}, \frac{-3}{1}, \frac{-5}{1}, \frac{-7}{1}$

15. $y > 2x^2 - 4x - 6$



x	y
-3	24
-2	10
-1	0
0	-6
1	-8
2	-6
3	0
4	10
5	24