Precalculus Worksheet Sequences, Series, Binomial Theorem

General

1. Write the first 5 terms of the sequence whose general term is given below. Assume the sequence begins with $n=1$.

   a) $a_n = \dfrac{2n+1}{n+3}$ b) $a_n = \dfrac{n!}{2^n}$

2. Write the first 5 terms of the sequence defined recursively.

   a) $a_1 = 12,\ a_{n+1} = \dfrac{a_n}{2} + 1$ b) $a_1 = 2,\ a_2 = 6,\ a_{n+2} = a_{n+1} + 2a_n$

3. Write a non-recursive formula for the general term, a_n, for each of these sequences. The first term should correspond to $n=1$.

   a) $1, 4, 7, 10, 13, \ldots$ b) $\dfrac{1}{4}, -\dfrac{2}{9}, \dfrac{3}{16}, -\dfrac{4}{25}, \dfrac{5}{36}, \ldots$ c) $1, \dfrac{3}{2}, \dfrac{5}{6}, \dfrac{7}{24}, \dfrac{9}{120}, \ldots$

4. Rewrite each of these sums using sigma notation.

   a) $5+9+13+17+\ldots+85$ b) $\dfrac{1}{4}+\dfrac{2}{9}+\dfrac{3}{16}+\dfrac{4}{25}+\dfrac{5}{36}+\ldots+\dfrac{12}{169}$

   c) $\dfrac{2}{3}+\dfrac{4}{5}+\dfrac{6}{7}+\dfrac{8}{9}+\dfrac{10}{11}+\ldots+\dfrac{20}{21}$ d) $\dfrac{1}{2}+\dfrac{2}{4}+\dfrac{6}{8}+\dfrac{24}{16}+\dfrac{120}{32}+\dfrac{720}{64}$

5. Evaluate each of the following summations.

a) $\sum_{n=0}^{3}\left(\frac{1}{n^2+1}\right)$ b) $\sum_{i=1}^{4} 2^{3-i}$ c) $\sum_{k=1}^{\infty} 3\left(\frac{2}{5}\right)^k$

d) $\sum_{i=1}^{8}\left(i^2-3i+2\right)$ e) $\sum_{n=1}^{\infty}\left(\frac{1}{n(n+1)}\right)$

<u>Arithmetic Sequences and Series</u>

6. For each of these sequences, determine if it is arithmetic. If it is, find the common difference. In each case, find a formula for the general term, a_n .

a) $10, 8, 6, 4, 2, \ldots$ b) $\frac{2}{3}, \frac{3}{6}, \frac{4}{9}, \frac{5}{12}, \frac{6}{15}, \ldots$

c) $-24, -16, -8, 0, 8, \ldots$

7. For an arithmetic sequence, $a_3 = 6$ and $a_5 = 20$. Find a_{19}.

8. For the arithmetic sequence described in #7, find S_{19}, the 19$^{\text{th}}$ partial sum.

9. Evaluate each of these sums.

a) $\sum_{n=1}^{25} 5n - 2$

b) $\sum_{k=7}^{32} 2k + 3$

Geometric Sequences and Series

10. For a geometric sequence with $a_1 = 3$ and $r = \sqrt{5}$, find the 6$^{\text{th}}$ term.

11. For a geometric sequence with $a_2 = 24$ and $a_5 = 3$, find:

a) a_{12}

b) find S_5, the 5$^{\text{th}}$ partial sum,.

12. Find:

a) $\sum_{k=1}^{10} 2^{k-1}$ b) $\sum_{k=0}^{\infty} 5\left(\frac{1}{4}\right)^k$ c) $\sum_{n=1}^{\infty} 4(0.2)^n$ e) $9+6+4+\frac{8}{3}+\ldots$

13. Rewrite the series $192-96+48-\ldots-\frac{3}{8}$ in summation notation.

14. An infinite geometric series converges to 12 and $a_1 = 3$. Find a_3.

15. Express $6.434343\ldots$ as a ratio of integers.

<u>Binomial Theorem</u>

16. What is the 3rd term of $(2x+y^2)^6$?

17. Find the term containing x^6 in the expansion of $(5x^2 - y^{-3})^8$.

<u>General</u>

18. Does the series $\sum_{i=1}^{\infty}(-1)^{i+1} = 1-1+1-1+1-1+\ldots$ converge? If so, to what does it converge; if not, why not?

Answers

1. a) $\frac{3}{4}, \frac{5}{5}, \frac{7}{6}, \frac{9}{7}, \frac{11}{8}$ b) $\frac{1}{2}, \frac{2}{4}, \frac{6}{8}, \frac{24}{16}, \frac{120}{32}$

2. a) $12, 7, \frac{9}{2}, \frac{13}{4}, \frac{21}{8}$ b) $2, 6, 10, 22, 42$

3. a) $a_n = 3n-2,\ n=1,2,3,\ldots$ b) $a_n = (-1)^{n+1}\frac{n}{(n+1)^2},\ n=1,2,3,\ldots$ c) $a_n = \frac{2n-1}{n!},\ n=1,2,3,\ldots$

4. a) $\sum_{n=1}^{21} 4n+1$ b) $\sum_{n=1}^{12}\frac{n}{(n+1)^2}$ c) $\sum_{n=1}^{10}\frac{2n}{2n+1}$ d) $\sum_{n=1}^{6}\frac{n!}{2^n}$

5. a) $\frac{9}{5}$ b) $\frac{15}{2}$ c) 2 d) 112 e) 1

6. a) arithmetic, $d=-2,\ a_n = 12-2n,\ n=1,2,3\ldots$ b) not arithmetic, $a_n = \frac{n+1}{3n},\ n=1,2,3,\ldots$
   c) arithmetic, $d=8,\ a_n = 8n-32,\ n=1,2,3\ldots$

7. $a_5 = a_3 + 2d \Rightarrow d = 7$. So, $a_{19} = a_5 + 14d = 118$.

8. For the sequence in #7, $a_1 = -8$. $S_{19} = \frac{19(-8+118)}{2} = 1045$.

9. a) $\sum_{n=1}^{25} 5n-2 = \frac{25(3+123)}{2} = 1575$ b) $\sum_{k=7}^{32} 2k+3 = \frac{26(17+67)}{2} = 1092$

10. $a_6 = 3\left(\sqrt{5}\right)^5 = 75\sqrt{5}$

11. a) $a_5 = a_2 r^3 \Rightarrow r = \frac{1}{2}$. So, $a_{12} = a_5\left(\frac{1}{2}\right)^7 = \frac{3}{128}$
    b) $a_1 = 48$. $S_5 = \frac{48\left(1-\left(\frac{1}{2}\right)^5\right)}{1-\left(\frac{1}{2}\right)} = 93$

12. a) $\sum_{k=1}^{10} 2^{k-1} = \frac{1\left(1-2^{10}\right)}{1-2} = 1023$ b) $\sum_{k=0}^{\infty} 5\left(\frac{1}{4}\right)^k = \frac{5}{1-\frac{1}{4}} = \frac{20}{3}$

    c) $\sum_{n=1}^{\infty} 4(0.2)^n = \frac{0.8}{1-0.2} = 1$ d) $9+6+4+\frac{8}{3}+\ldots = \frac{9}{1-\frac{2}{3}} = 27$

13. $\sum_{n=0}^{9} 192\left(-\frac{1}{2}\right)^n$

14. $12 = \frac{3}{1-r} \Rightarrow r = \frac{3}{4}$. So, $a_3 = 3\left(\frac{3}{4}\right)^2 = \frac{27}{16}$.

15. $6.434343\ldots = 6 + .43 + .0043 + .000043 + \ldots = 6 + \frac{.43}{1-.01} = \frac{637}{99}$

16. $\binom{6}{2}(2x)^4\left(y^2\right)^2 = 240x^4y^4$

17. $\binom{8}{5}\left(5x^2\right)^3\left(-y^{-3}\right)^5 = -7000x^6y^{-15}$

18. The infinite series diverges because the sequence of partial sums 1, 0, 1, 0, 1, … diverges.