Sections 1.5-1.6: Continuity

5. Consider the fuctions

$$f(x)=\begin{cases}1, & x\neq 4\\ -1, & x=4\end{cases} \quad \text{and} \quad g(x)=\begin{cases}4x-10, & x\neq 4\\ -6, & x=4\end{cases}$$

In each part, is the given function continuous at x=4? *Justify*.

EXAMPLE: Since f(a)=$\lim\limits_{x\to a} f(x)$, then f(x) is continuous at x=a.

(a) f(x)
(b) g(x)
(c) -g(x)
(d) |f(x)|
(e) f(x)g(x)
(f) g(f(x))
(g) g(x)-6f(x)

Find values of *x*, if any, at which *f* is not continuous. State the reason why.

11. $f(x)=5x^4-3x+7$

15. $f(x)=\frac{x}{2x^2+x}$

21. $f(x)=\begin{cases}2x+3, & x\leq 4\\ 7+\frac{16}{x}, & x>4\end{cases}$

29. Find a value of the constant *k*, if possible, that will make the function continuous everywhere.

(a) $f(x)=\begin{cases}7x-2, & x\leq 1\\ kx^2, & x>1\end{cases}$

(b) $f(x)=\begin{cases}kx^2, & x\leq 2\\ 2x+k, & x>2\end{cases}$

36. Find the values of *x* (if any) at which *f* is not continuous and determine whether each such value is a **removable discontinuity**.

$$f(x)=\frac{x^2-4}{x^3-8}$$

Find the discontinuities, if any.

3. $f(x)=|\cot x|$

5. $f(x)=\csc x$

7. $f(x)=\frac{1}{1-2\sin x}$

9. Determine where $f(x)=\sin^{-1}2x$ is continuous.

15. Use a theorem in section 1.5 (continuity of composite functions) to show that the function is continuous everywhere.

(a) $\sin(x^3+7x+1)$

(b) $|\sin x|$

Find the limits.

17. $\lim\limits_{x\to\infty}\cos\left(\frac{1}{x}\right)$

21. $\lim\limits_{x\to 0} e^{\sin x}$

23. $\lim\limits_{\theta\to 0}\frac{\sin 3\theta}{\theta}$

25. $\lim\limits_{\theta\to 0^+}\frac{\sin\theta}{\theta^2}$

27. $\lim\limits_{x\to 0}\frac{\tan 7x}{\sin 3x}$

35. $\lim\limits_{\theta\to 0}\frac{\theta^2}{1-\cos\theta}$

61. Use the Squeeze Theorem to show that $\lim\limits_{x\to 0} x\cos\frac{50\pi}{x}=0$. Use a graphing utility to illustrate this problem using $y=|x|$, $y=-|x|$, and $y=x\cos\frac{50\pi}{x}$ and an appropriate window size.