AP Calculus AB Practice: Slope Fields & Differential Equations

AP® CALCULUS AB
2004 SCORING GUIDELINES (Form B)

Question 5

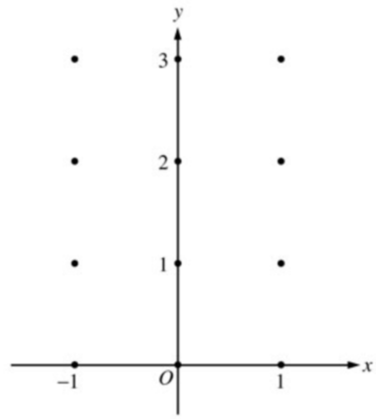
NO CALCULATOR

Consider the differential equation $\frac{dy}{dx} = x^4(y-2)$.

(a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated.
(Note: Use the axes provided in the test booklet.)

(b) While the slope field in part (a) is drawn at only twelve points, it is defined at every point in the xy-plane. Describe all points in the xy-plane for which the slopes are negative.

(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = 0$.



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2004 SCORING GUIDELINES

Question 6

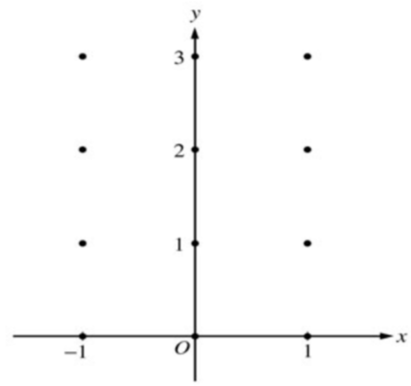
NO CALCULATOR

Consider the differential equation $\frac{dy}{dx} = x^2(y-1)$.

(a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated. (**Note: Use the axes provided in the pink test booklet.**)

(b) While the slope field in part (a) is drawn at only twelve points, it is defined at every point in the xy-plane. Describe all points in the xy-plane for which the slopes are positive.

(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = 3$.

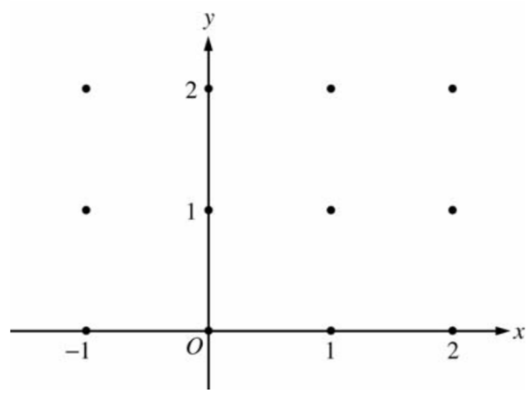


AP® CALCULUS AB
2005 SCORING GUIDELINES (Form B)

Question 6

NO CALCULATOR

Consider the differential equation $\frac{dy}{dx} = \frac{-xy^2}{2}$. Let $y = f(x)$ be the particular solution to this differential equation with the initial condition $f(-1) = 2$.



(a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated.
(Note: Use the axes provided in the test booklet.)

(b) Write an equation for the line tangent to the graph of f at $x = -1$.

(c) Find the solution $y = f(x)$ to the given differential equation with the initial condition $f(-1) = 2$.

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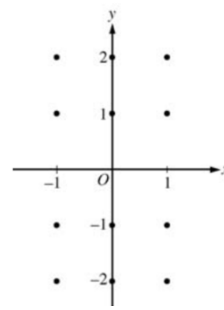
Question 6

Consider the differential equation $\frac{dy}{dx} = -\frac{2x}{y}$.

(a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated.
(Note: Use the axes provided in the pink test booklet.)

(b) Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(1) = -1$. Write an equation for the line tangent to the graph of f at $(1, -1)$ and use it to approximate $f(1.1)$.

(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(1) = -1$.



NO CALCULATOR

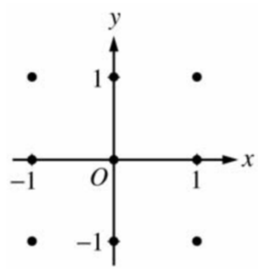
AP® CALCULUS AB
2006 SCORING GUIDELINES (Form B)

Question 5

NO CALCULATOR

Consider the differential equation $\dfrac{dy}{dx} = (y-1)^2 \cos(\pi x)$.

(a) On the axes provided, sketch a slope field for the given differential equation at the nine points indicated. **(Note: Use the axes provided in the exam booklet.)**



(b) There is a horizontal line with equation $y = c$ that satisfies this differential equation. Find the value of c.

(c) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(1) = 0$.

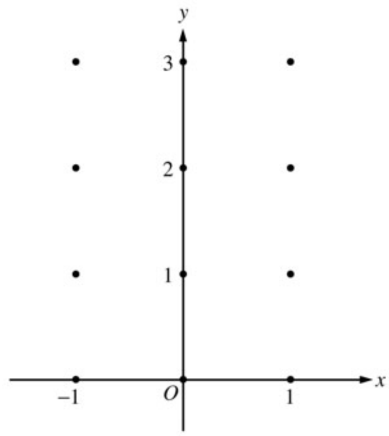
Question 5

Consider the differential equation $\frac{dy}{dx} = x^4(y-2)$.

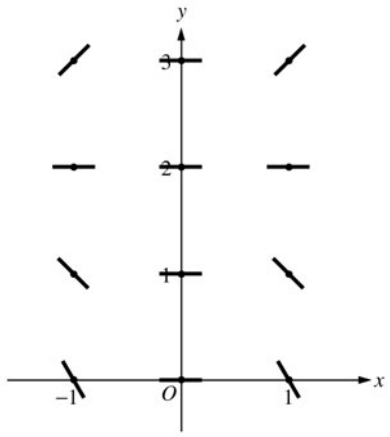
(a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated.
(Note: Use the axes provided in the test booklet.)

(b) While the slope field in part (a) is drawn at only twelve points, it is defined at every point in the xy-plane. Describe all points in the xy-plane for which the slopes are negative.

(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = 0$.



(a)



2 :
- 1 : zero slope at each point (x, y) where $x = 0$ or $y = 2$
- 1 :
  - positive slope at each point (x, y) where $x \neq 0$ and $y > 2$
  - negative slope at each point (x, y) where $x \neq 0$ and $y < 2$

(b) Slopes are negative at points (x, y) where $x \neq 0$ and $y < 2$.

1 : description

(c) $\frac{1}{y-2}dy = x^4 dx$

$\ln|y-2| = \frac{1}{5}x^5 + C$

$|y-2| = e^C e^{\frac{1}{5}x^5}$

$y - 2 = Ke^{\frac{1}{5}x^5}, \ K = \pm e^C$

$-2 = Ke^0 = K$

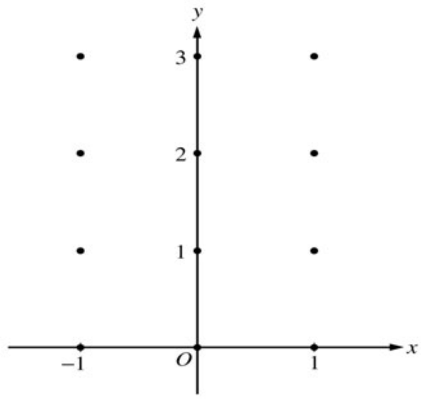
$y = 2 - 2e^{\frac{1}{5}x^5}$

6 :
- 1 : separates variables
- 2 : antiderivatives
- 1 : constant of integration
- 1 : uses initial condition
- 1 : solves for y
  - $0/1$ if y is not exponential

Note: max $3/6$ [1-2-0-0-0] if no constant of integration
Note: $0/6$ if no separation of variables

Question 6

Consider the differential equation $\frac{dy}{dx} = x^2(y-1)$.

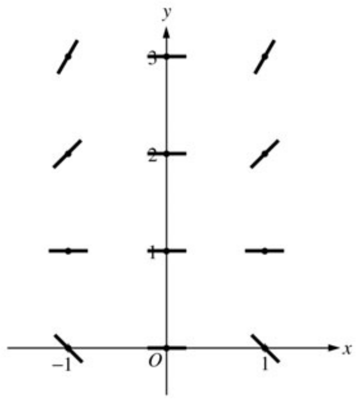


(a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated. (**Note: Use the axes provided in the pink test booklet.**)

(b) While the slope field in part (a) is drawn at only twelve points, it is defined at every point in the xy-plane. Describe all points in the xy-plane for which the slopes are positive.

(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = 3$.

(a)



2 :
- 1 : zero slope at each point (x, y) where $x = 0$ or $y = 1$
- 1 :
  - positive slope at each point (x, y) where $x \neq 0$ and $y > 1$
  - negative slope at each point (x, y) where $x \neq 0$ and $y < 1$

(b) Slopes are positive at points (x, y) where $x \neq 0$ and $y > 1$.

1 : description

(c) $\frac{1}{y-1}dy = x^2 dx$

$\ln|y-1| = \frac{1}{3}x^3 + C$

$|y-1| = e^C e^{\frac{1}{3}x^3}$

$y - 1 = Ke^{\frac{1}{3}x^3}, K = \pm e^C$

$2 = Ke^0 = K$

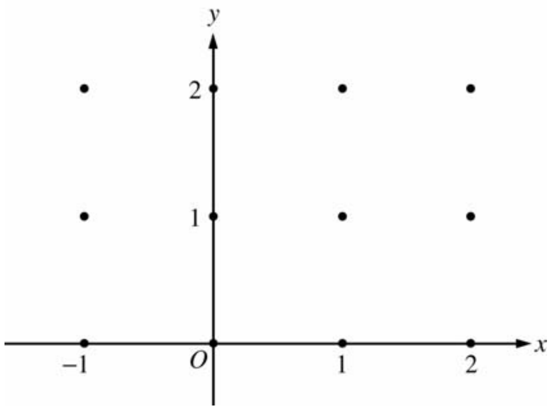
$y = 1 + 2e^{\frac{1}{3}x^3}$

6 :
- 1 : separates variables
- 2 : antiderivatives
- 1 : constant of integration
- 1 : uses initial condition
- 1 : solves for y
  - $0/1$ if y is not exponential

Note: max $3/6$ [1-2-0-0-0] if no constant of integration
Note: $0/6$ if no separation of variables

Question 6

Consider the differential equation $\frac{dy}{dx} = \frac{-xy^2}{2}$. Let $y = f(x)$ be the particular solution to this differential equation with the initial condition $f(-1) = 2$.

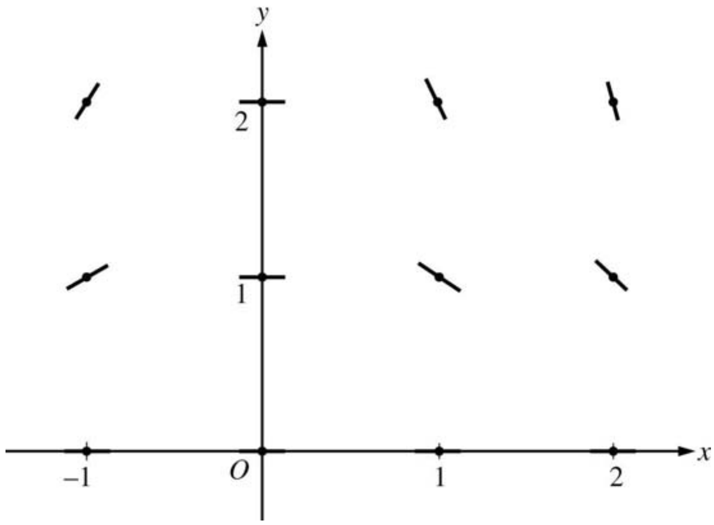


(a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated.
(Note: Use the axes provided in the test booklet.)

(b) Write an equation for the line tangent to the graph of f at $x = -1$.

(c) Find the solution $y = f(x)$ to the given differential equation with the initial condition $f(-1) = 2$.

(a)



$2 : \begin{cases} 1 : \text{zero slopes} \\ 1 : \text{nonzero slopes} \end{cases}$

(b) Slope $= \frac{-(-1)4}{2} = 2$

$y - 2 = 2(x + 1)$

1 : equation

(c) $\frac{1}{y^2}\,dy = -\frac{x}{2}\,dx$

$-\frac{1}{y} = -\frac{x^2}{4} + C$

$-\frac{1}{2} = -\frac{1}{4} + C;\ C = -\frac{1}{4}$

$y = \dfrac{1}{\frac{x^2}{4} + \frac{1}{4}} = \dfrac{4}{x^2 + 1}$

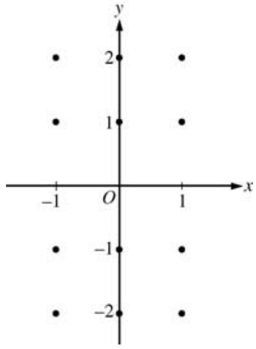
$6 : \begin{cases} 1 : \text{separates variables} \\ 2 : \text{antiderivatives} \\ 1 : \text{constant of integration} \\ 1 : \text{uses initial condition} \\ 1 : \text{solves for } y \end{cases}$

Note: max $3/6$ [1-2-0-0-0] if no constant of integration

Note: $0/6$ if no separation of variables

Question 6

Consider the differential equation $\frac{dy}{dx} = -\frac{2x}{y}$.

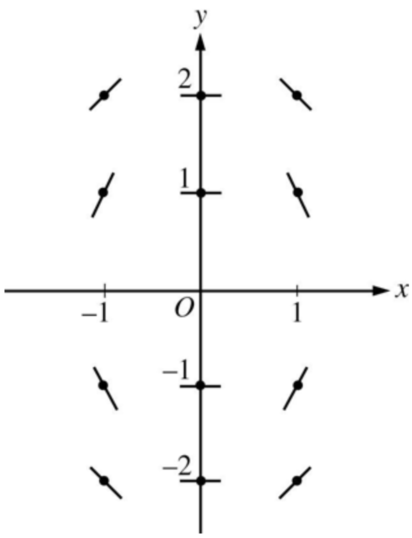


(a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated.
(**Note: Use the axes provided in the pink test booklet.**)

(b) Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(1) = -1$. Write an equation for the line tangent to the graph of f at $(1, -1)$ and use it to approximate $f(1.1)$.

(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(1) = -1$.

(a)



$$2 : \begin{cases} 1 : \text{zero slopes} \\ 1 : \text{nonzero slopes} \end{cases}$$

(b) The line tangent to f at $(1, -1)$ is $y + 1 = 2(x - 1)$.
Thus, $f(1.1)$ is approximately -0.8.

$$2 : \begin{cases} 1 : \text{equation of the tangent line} \\ 1 : \text{approximation for } f(1.1) \end{cases}$$

(c) $\frac{dy}{dx} = -\frac{2x}{y}$

$y\,dy = -2x\,dx$

$\frac{y^2}{2} = -x^2 + C$

$\frac{1}{2} = -1 + C;\ C = \frac{3}{2}$

$y^2 = -2x^2 + 3$

Since the particular solution goes through $(1, -1)$, y must be negative.

Thus the particular solution is $y = -\sqrt{3 - 2x^2}$.

$$5 : \begin{cases} 1 : \text{separates variables} \\ 1 : \text{antiderivatives} \\ 1 : \text{constant of integration} \\ 1 : \text{uses initial condition} \\ 1 : \text{solves for } y \end{cases}$$

Note: max $2/5$ [1-1-0-0-0] if no constant of integration
Note: $0/5$ if no separation of variables

Question 5

Consider the differential equation $\frac{dy}{dx} = (y-1)^2 \cos(\pi x)$.

(a) On the axes provided, sketch a slope field for the given differential equation at the nine points indicated.
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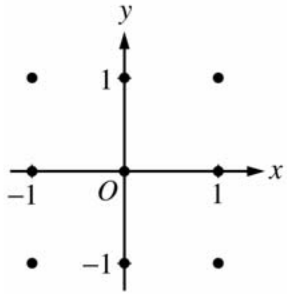
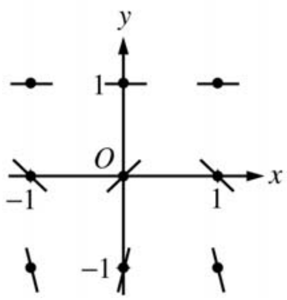


(b) There is a horizontal line with equation $y = c$ that satisfies this differential equation. Find the value of c.

(c) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(1) = 0$.

(a)



$2 : \begin{cases} 1 : \text{zero slopes} \\ 1 : \text{all other slopes} \end{cases}$

(b) The line $y = 1$ satisfies the differential equation, so $c = 1$.

$1 : c = 1$

(c)
$$\frac{1}{(y-1)^2}\, dy = \cos(\pi x)\, dx$$
$$-(y-1)^{-1} = \frac{1}{\pi}\sin(\pi x) + C$$
$$\frac{1}{1-y} = \frac{1}{\pi}\sin(\pi x) + C$$
$$1 = \frac{1}{\pi}\sin(\pi) + C = C$$
$$\frac{1}{1-y} = \frac{1}{\pi}\sin(\pi x) + 1$$
$$\frac{\pi}{1-y} = \sin(\pi x) + \pi$$
$$y = 1 - \frac{\pi}{\sin(\pi x) + \pi} \text{ for } -\infty < x < \infty$$

$6 : \begin{cases} 1 : \text{separates variables} \\ 2 : \text{antiderivatives} \\ 1 : \text{constant of integration} \\ 1 : \text{uses initial condition} \\ 1 : \text{answer} \end{cases}$

Note: max $3/6$ [1-2-0-0-0] if no constant of integration
Note: $0/6$ if no separation of variables