Section 9.6 – Alternating Series; Absolute & Conditional Convergence

Show that the series converges by confirming that it satisfies the hypotheses of the alternating series test.

1. $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{2k+1}$

Determine whether the alternating series converges; justify your answer

3. $\sum_{k=1}^{\infty} (-1)^{k+1} \frac{k+1}{3k+1}$

5. $\sum_{k=1}^{\infty} (-1)^{k+1} e^{-k}$

Use the ratio test for absolute convergence to determine whether the series converges or diverges.

7. $\sum_{k=1}^{\infty} \left(-\frac{3}{5}\right)^{k}$

9. $\sum_{k=1}^{\infty} (-1)^{k+1} \frac{3^k}{k^2}$

11. $\sum_{k=1}^{\infty} (-1)^{k} \frac{k^3}{e^k}$

Classify each series as absolutely convergent, conditionally convergent, or divergent.

13. $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{3k}$
(Limit Comparison Test)

15. $\sum_{k=1}^{\infty} \frac{(-4)^{k}}{k^2}$
(Divergence Test)

17. $\sum_{k=1}^{\infty} \frac{\cos(k\pi)}{k}$
(Alt. Series Test/p-series)

19. $\sum_{k=1}^{\infty} (-1)^{k+1} \frac{k+2}{k(k+3)}$
(Alt. Series Test/Limit Comp.)

21. $\sum_{k=1}^{\infty} \sin\left(\frac{k\pi}{2}\right)$
(Divergence Test)

23. $\sum_{k=1}^{\infty} \frac{(-1)^{k}}{k \ln k}$
(Alt. Series Test/Integral)

25. $\sum_{k=2}^{\infty} \left(-\frac{1}{\ln k}\right)^{k}$
(Root Test)

27. $\sum_{k=1}^{\infty} \frac{(-1)^{k+1} k!}{(2k-1)!}$
(Ratio Test)

Each series satisfies the hypotheses of the alternating series test. For the stated value of *n*, find an upper bound on the absolute error that results if the sum of the series is approximated by the *n*th partial sum.

33. $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k}; \ n = 7$

35. $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{\sqrt{k}}; \ n = 99$

Each series satisfies the hypotheses of the alternating series test. Find a value of *n* for which the *n*th partial sum is ensured to approximate the sum of the series to the stated accuracy.

37. $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k}$; $|error| < 0.0001$

39. $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{\sqrt{k}}$; *two decimal places*

Find an upper bound on the absolute error that results if s_{10} is used to approximate the sum of the given *geometric* series. Compute s_{10} rounded to four decimal places and compare this value with the exact sum of the series.

41. $\frac{3}{4} - \frac{3}{8} + \frac{3}{16} - \frac{3}{32} + \ldots$

Each series satisfies the hypotheses of the alternating series test. Approximate the sum of the series to two decimal-place accuracy.

43. $1 - \frac{1}{3!} + \frac{1}{5!} - \frac{1}{7!} + \ldots$