Sections 1.1-1.3: Limits

9. For the function f graphed in the accompanying figure, find
(a) $\lim_{x\to -2} f(x)$ (b) $\lim_{x\to 0^-} f(x)$
(c) $\lim_{x\to 0^+} f(x)$ (d) $\lim_{x\to 2^-} f(x)$
(e) $\lim_{x\to 2^+} f(x)$
(f) the vertical asymptotes of the graph of f.

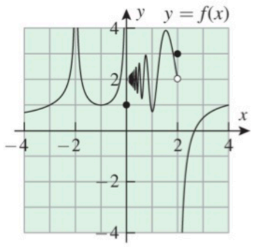

Figure Ex-9

10. For the function f graphed in the accompanying figure, find
(a) $\lim_{x\to -2^-} f(x)$ (b) $\lim_{x\to -2^+} f(x)$ (c) $\lim_{x\to 0^-} f(x)$
(d) $\lim_{x\to 0^+} f(x)$ (e) $\lim_{x\to 2^-} f(x)$ (f) $\lim_{x\to 2^+} f(x)$
(g) the vertical asymptotes of the graph of f.

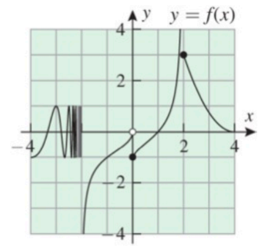

Figure Ex-10

Find the limits.

5. $\lim_{x\to 3} \frac{x^2-2x}{x+1}$

7. $\lim_{x\to 1^+} \frac{x^4-1}{x-1}$

9. $\lim_{x\to -1} \frac{x^2+6x+5}{x^2-3x-4}$

13. $\lim_{t\to 2} \frac{t^3+3t^2-12t+4}{t^3-4t}$

15. $\lim_{x\to 3^+} \frac{x}{x-3}$

16. $\lim_{x\to 3^-} \frac{x}{x-3}$

17. $\lim_{x\to 3} \frac{x}{x-3}$

20. $\lim_{x\to 2} \frac{x}{x^2-4}$

25. $\lim_{x\to 4^-} \frac{3-x}{x^2-2x-8}$

27. $\lim_{x\to 2^+} \frac{1}{|2-x|}$

29. $\lim_{x\to 9} \frac{x-9}{\sqrt{x}-3}$

31. Let $f(x) = \begin{cases} x-1, & x \le 3 \\ 3x-7, & x > 3 \end{cases}$, find $\lim_{x\to 3} f(x)$. *JUSTIFY.*

EXAMPLE: Since $\lim_{x\to a^-} f(x) = b$ and $\lim_{x\to a^+} f(x) = b$, then $\lim_{x\to a} f(x) = b$.

35. **TRUE/FALSE:** If $\lim_{x\to a} f(x)$ and $\lim_{x\to a} g(x)$ both exist and are equal, then $\lim_{x\to a} \frac{f(x)}{g(x)} = 1$. *Justify.*

37. Rationalize the numerator and then find the limit of $\lim_{x\to 0} \frac{\sqrt{x+4}-2}{x}$.

4. For the function G graphed in the accompanying figure, find
(a) $\lim_{x\to -\infty} G(x)$ (b) $\lim_{x\to +\infty} G(x)$.

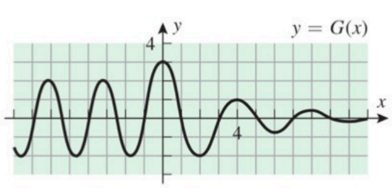

Figure Ex-4

5. Given that:
$\lim_{x\to +\infty} f(x) = 3$, $\lim_{x\to +\infty} g(x) = -5$, and $\lim_{x\to +\infty} h(x) = 0$

Find the limits that exist. If the limit does not exist, explain.
(a) $\lim_{x\to +\infty} [f(x) + 3g(x)]$
(b) $\lim_{x\to +\infty} [h(x) - 4g(x) + 1]$
(c) $\lim_{x\to +\infty} [f(x)g(x)]$
(d) $\lim_{x\to +\infty} [g(x)]^2$

Find the limits.

11. $\lim\limits_{x\to+\infty} \sqrt{x}$

13. $\lim\limits_{x\to+\infty} \frac{3x+1}{2x-5}$

15. $\lim\limits_{y\to-\infty} \frac{3}{y+4}$

20. $\lim\limits_{t\to-\infty} \frac{5-2t^3}{t^2+1}$

23. $\lim\limits_{x\to+\infty} \sqrt[3]{\frac{2+3x-5x^2}{1+8x^2}}$

25. $\lim\limits_{x\to-\infty} \frac{\sqrt{5x^2-2}}{x+3}$

31*. $\lim\limits_{x\to+\infty} \sqrt{x^2+3}-x$

35. $\lim\limits_{x\to+\infty} \frac{e^x+e^{-x}}{e^x-e^{-x}}$

Evaluate the limit using an appropriate substitution.

59. $\lim\limits_{x\to+\infty} \frac{\ln(2x)}{\ln(3x)}$ (Hint: t=lnx)

62. $\lim\limits_{x\to+\infty} \left(1+\frac{2}{x}\right)^x$ (Hint: t=x/2)

66. The population *p* of the United States (in millions) in year *t* can be modeled by

$$p(t) = \frac{525}{1+1.1e^{-0.0222(t-1990)}}$$

(a) Based on this model, what was the US population in 1990?
(b) Plot *p* versus *t* for the 200-year period from 1950 to 2150.
(c) By evaluating an appropriate limit, show that the graph of *p* versus *t* has a horizontal asymptote *p=c* for an appropriate *c*.
(d) What is the significance of the constant *c* in part (c) for the population predicted by this model?