Pre Calc

POLAR COORDINATES

(CHAPTER 6)

REVIEW OF COMPLEX NUMBERS

1) Solve: $x^2 + 16 = 0$

2) Multiply: $\sqrt{-25} \cdot \sqrt{-16}$

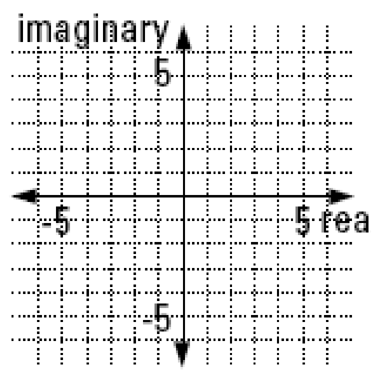
3) Complex Number: Can be written in the form a + bi where a is the ____________part,

b is the _________________ part and i = ________.

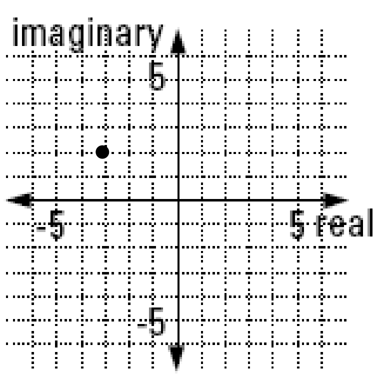
4) Subtract –5 + 2i from 4 – 7i.

5) Express $\dfrac{6+2i}{8+5i}$ in simplest a + bi form.

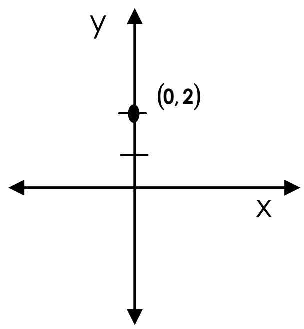
6) On the grid at the right, graph EFGH where E = 3 + i, F = -1 – i, G = -2 – 3i, and H = 2 – i.



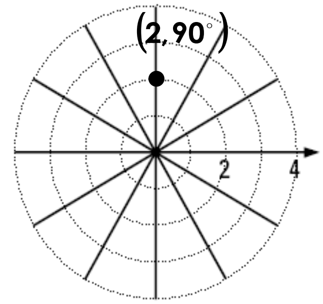
7) The point (-3, 2) graphed below represents what complex number?



POLAR COORDINATES

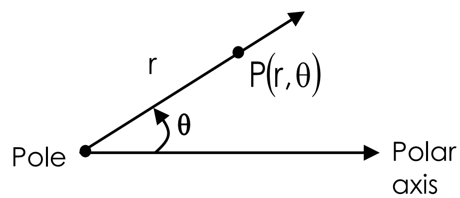


Rectangular Coordinates



Polar Coordinates

Polar coordinates = [r, θ]

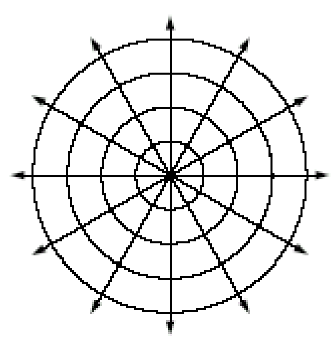


- r is the distance from the pole to point P
- θ is the measure of the angle from the polar axis to ray $\overrightarrow{OP}$
- If θ > 0, the polar angle is obtained by rotating ray $\overrightarrow{OP}$ ____________________ from the polar axis, if θ < 0, the rotation is ____________________.

<u>Examples</u>:

1) Plot each point [r, θ], where θ is in degrees using the grid below.

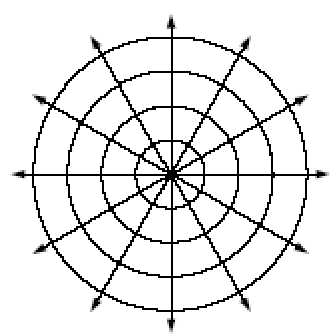
a. [2,60°] b. [4,–30°] c. [–4,210°]



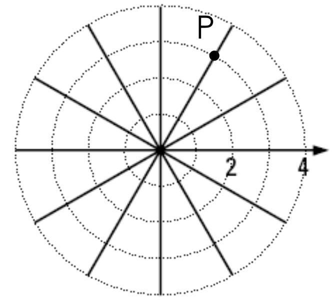
<u>Examples</u>:

2) Plot each point [r, θ], where θ is in radians using the grid below.

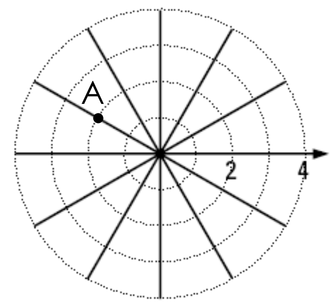
a. $\left[2, \frac{\pi}{3}\right]$ b. $\left[1.4, -\frac{\pi}{2}\right]$ c. $\left[-2, \frac{5\pi}{3}\right]$



3) Consider point P, find 4 different ways to name point P.

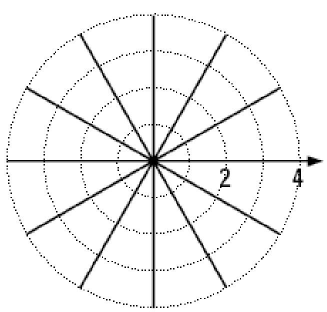


4) Consider point A, find 4 different ways to name point A.

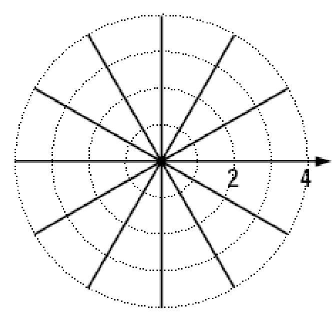


Each polar coordinate has an r and a θ. What if you wanted to represent all of the points that have an r of 4 or all the coordinates that have a θ of 120°

5) Graph r = 4



6) $\theta = 120^\circ$



Recall Exact Values
Evaluate to the nearest ten thousandth.

$\frac{\sqrt{3}}{2}$

$\frac{\sqrt{2}}{2}$

$\sqrt{3}$

Find the exact value for each:

1) $\cos 30^\circ$

2) $\sin 45^\circ$

3) $\cos 60^\circ$

CONVERTING: POLAR COORDINATES $\leftrightarrow$ RECTANGULAR COORDINATES

Polar to Rectangular $(r,\theta) \to (x, y)$

cos θ =   therefore x =

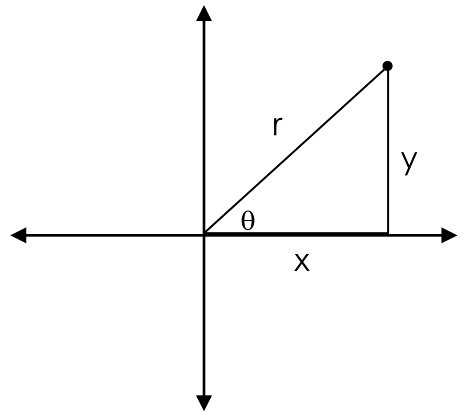
sin θ =   therefore y =

Rectangular to Polar $(x, y) \to (r,\theta)$

r^2 =   therefore r =

$\tan\theta$ =   therefore θ =

When x < 0; θ =



<u>Examples</u>:

1) Find the rectangular coordinates for each of the given polar coordinates.

a) $[4, 300°]$ b) $\left[-2, \frac{\pi}{6}\right]$

c) $\left[4, \frac{\pi}{2}\right]$ d) $\left[-3, \frac{2\pi}{3}\right]$

e) $\left[2, \frac{\pi}{3}\right]$ f) $\left[-3, \frac{-\pi}{3}\right]$

2) Find the polar coordinates for each of the given rectangular coordinates.

a) (-2,-5)

b) $(-\sqrt{3},1)$

c) (3,-3)

d) (-5, 0)

e) (0, 5) What happens when x is 0?

3) Convert the rectangular equations to polar equations:

a) $x = 4$

b) $x + y = 2$

c) $x^2 + y^2 = 9$

4) Convert the polar equations to rectangular equations:

a) $r = 5\csc\theta$

b) $r = 3\sec\theta$

Converting Complex Numbers:

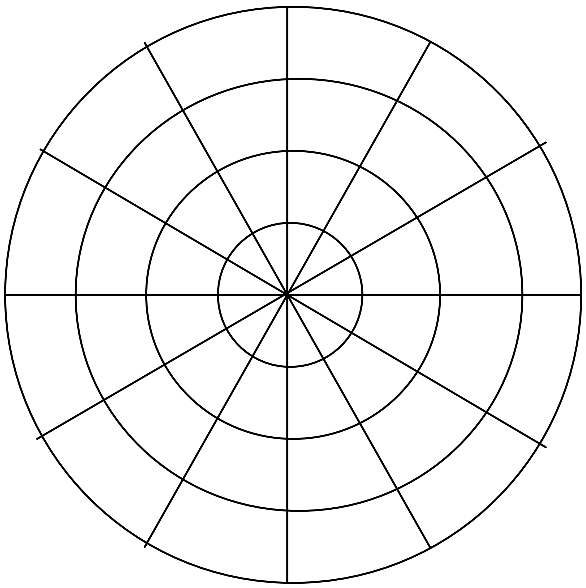
- Write the complex number as a rectangular coordinate and then convert to polar.

5) Write the complex number –4 + 2i in polar form.

MIXED PRACTICE

Label the graph in degrees and then plot each pair of polar coordinates.

1) [2,50] 2) [-1.5,120] 3) $\left[3, \frac{\pi}{4}\right]$ 4) $\left[-2, -\frac{5\pi}{6}\right]$



5) Find three other representations of the point (6, 45°) according to the following restrictions.

a) $r > 0,\ \theta < 0$

b) $r < 0,\ \theta > 0$

c) $r < 0,\ \theta < 0$

Convert to polar coordinates in the form of $[r, \theta]$, where θ is in degrees, for each point.

6) (-2,2) 7) (5,0) 8) -1 – i

Convert to rectangular coordinates (x, y) for each point:

9) [4,120]

10) [-3,90]

11) [6,-180]

Convert to polar coordinates in the form of [r, θ], where θ is in radians, for each point:

12) $(-1, -\sqrt{3})$

13) (-2,0)

14) $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

Convert to rectangular coordinates (x, y) for each point:

15) $\left[-2, \frac{\pi}{6}\right]$

16) $\left[-3, -\frac{\pi}{3}\right]$

17) $\left[1, \frac{5\pi}{6}\right]$

Write the rectangular equation in **polar** form:

18) $x^2 + y^2 = 49$

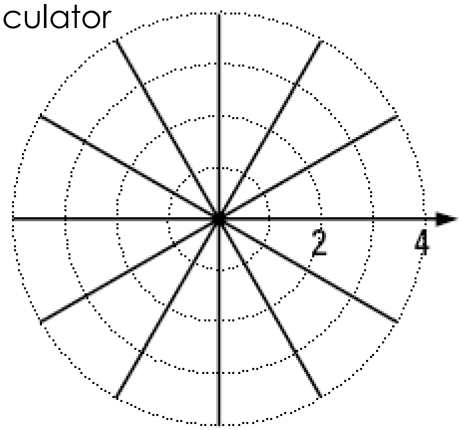
19) x = 8

Write the polar equations in **rectangular** form:

20) $r = 3\csc\theta$

GRAPHING POLAR EQUATIONS

Graph 1 and 2 on the same graph without using your calculator

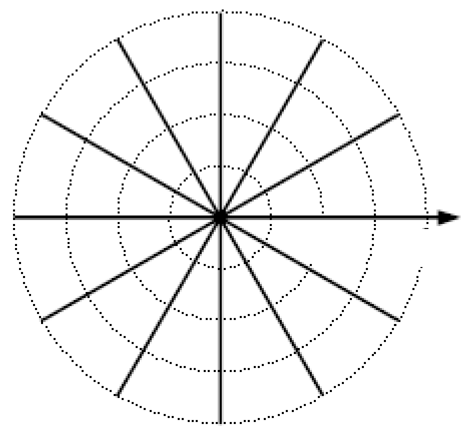


1. $\theta = -60^\circ$ shape____________

2. $r = \frac{3}{2}$ shape____________

Using your calculator follow the directions below.

Step 1: Change the mode from Function to Polar in your calculator.
(Now you have r = instead of y =)



Step 2: Put $r = 4\cos(\theta) - 2$ in r_1. (push x for θ)

Step 3: Go to zoom 6: standard, then zoom 5.

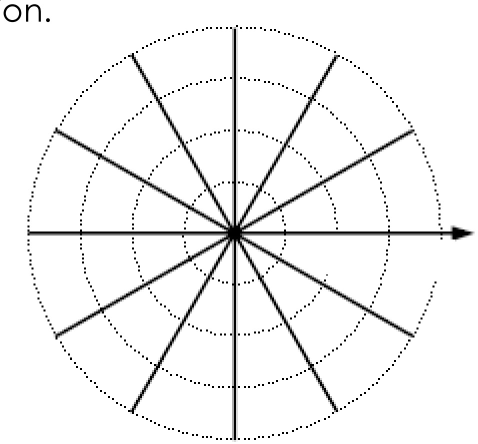
Step 4: Zoom in if necessary to investigate
the shape of the graph.

Sketch a graph of the curve. (Pay attention to intercepts)
This curve is called a **limacon**.

How can you relate the equation $r = 4\cos\theta - 2$ to the curve?

Follow steps 1 – 4 in order to graph the following polar equation.

$r = 4 + 4\cos\theta$

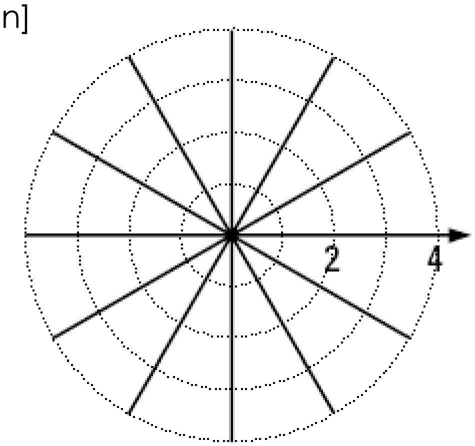


How can you relate the equation $r = 4 + 4\cos\theta$ to the curve?

This curve is a special kind of limacon, called a cardioid.

MORE GRAPHING POLAR EQUATIONS

1. $r = 3\cos(2\theta)$ [zoom standard $\rightarrow$ zoom square $\rightarrow$ zoom in]

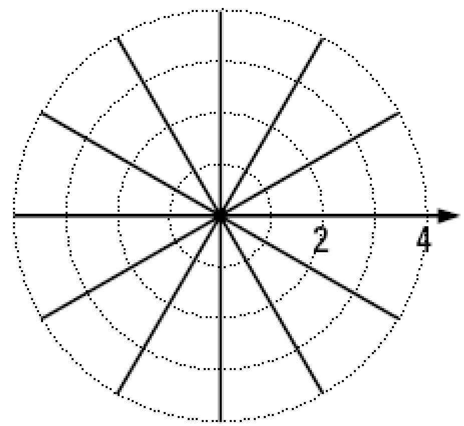


This is called a rose curve. Sketch the curve on the graph to the right.

How many petals does $r = 3\cos(2\theta)$ have?

What is the length of each petal?

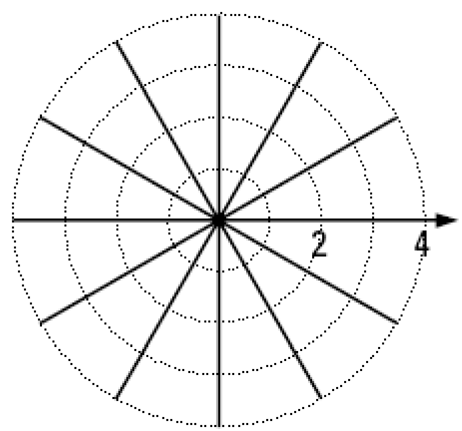
2. Graph $r = 3\cos(4\theta)$ and sketch the curve on the graph to the right.



How many petals does $r = 3\cos(4\theta)$ have?

What is the length of each petal?

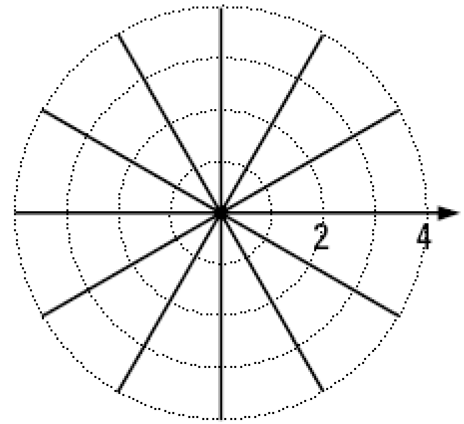
3. Graph $r = 4\sin(3\theta)$ and sketch the curve on the graph to the right.



How many petals does $r = 4\sin(3\theta)$ have?

What is the length of each petal?

4. Graph $r = 4\cos(5\theta)$ and sketch the curve on the graph to the right.



How many petals does $r = 4\cos(5\theta)$ have?

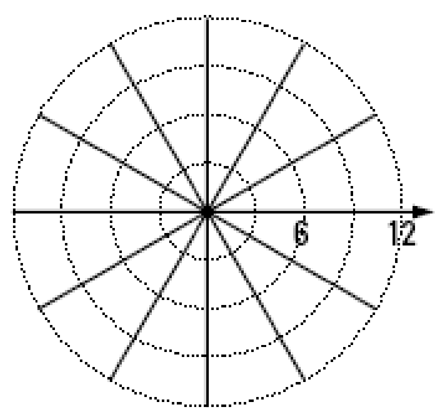
What is the length of each petal?

A rose curve is of the form $r = a\cos(n\theta)$ and $r = a\sin(n\theta)$ where $a > 0$ and $n \in \mathbf{Z}^{+}$.

Making Observations
How do a and n relate to the graph? (If you are not sure, graph more in your calculator with different numbers for a and n)

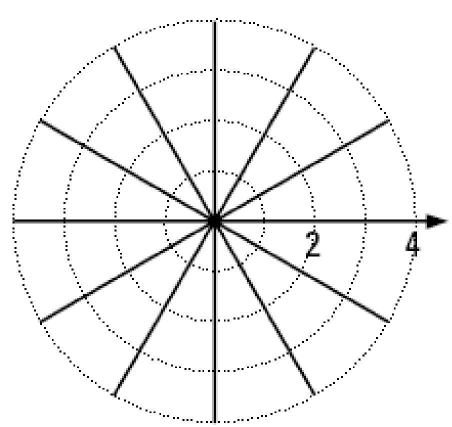
Put calculator in radian mode such that $0 \le \theta \le 2\pi$ by $\frac{\pi}{2}$

5. Graph $r = 1\theta + 2$



Polar graphs of the form $r = a\theta + b$ where $a > 1$ and $b \ge 0$ are called ***spirals of Archimedes***.

6. Graph $r = (1.25)^{\theta}$



Polar graphs of the form $r = ab^{\theta}$ where $a > 0$ and $b > 1$ are called ***logarithmic spirals***.

TRIGONOMETRIC FORM

Warm Up: Convert 2 – 4i to rectangular form and polar form.

The answers to the warm up do not convey that we started out with a complex number. Therefore we need a form that shows us we are working with complex numbers. This is called trigonometric form: $r(\cos\theta + i\sin\theta)$

The process for writing a complex number in trigonometric form is as follows:

complex # → rectangular → polar → trigonometric

$$a + bi \rightarrow (a,b) \rightarrow [r,\theta] \rightarrow r(\cos\theta + i\sin\theta)$$

2 – 4i written in trigonometric form would look like:

Convert the following complex numbers to trigonometric form and round the nearest hundredth when necessary:

a) 6 + 3i

b) $-2 - 2i\sqrt{3}$

c) $5 + i\sqrt{2}$

DEMOIVRE'S THEOREM

Warm Up: If $z = -1 - i\sqrt{3}$ find z^2 and z^3

If you were required to find z^9 you would continue to multiply out the polynomials for a while. There is another way to find powers of complex numbers.

DeMoivre's Theorem (dee mwavs'):

$$z = r(\cos\theta + i\sin\theta) \text{ then } z^n = r^n(\cos n\theta + i\sin n\theta)$$

Example 1: Using z from the warm up find z^9.

Example 2: Find the exact value of $\left(2 + 2i\sqrt{3}\right)^5$

Example 3: Find the exact value of $\left(2 - 2i\sqrt{3}\right)^{11}$

Example 4: If $5(\cos 30° + i\sin 30°)$ is the square root of z, find z.

ROOTS OF COMPLEX NUMBERS

To find roots of complex numbers, we have to use DeMoivre's Theorem.

Let's try an example. Find all third roots of 64*i*. Write your answers in trigonometric form.

64i	Steps
	Write in rectangular form
	Convert to polar form. After θ , you must put + 360°k since there are infinitely many values of θ.
	Use DeMoivre's Theorem to raise the polar coordinates to the $\frac{1}{3}$ power.
	There must be 3 cube roots, so sub in 0, 1 and 2 for k. If we were looking for fourth roots, there would be 4 of them so we would sub 0, 1, 2, 3 (and so on).
	Write the answers in trigonometric form.

Find all fourth roots of 81i. Write your answers in trigonometric form.

FINDING ZEROS

Find all zeros for the polynomial $k(x) = 4x^4 - 8x^3 + 9x^2 - 8x + 5$ given that i is a zero of $k(x)$.

Remember, if i is a zero of the polynomial, what else must be a zero?________________

If i and _________ are zeros of the polynomial, then what are the factors of the polynomial?

Can we multiply the factors together to make one factor?

Notice the i's are gone. When we multiply a complex number by its conjugate, the result is always a real number.

Now that we know a factor of the polynomial, we can divide it out in order to solve the remaining polynomial.

Let's try another one….
Find all zeros for the polynomial $k(x) = x^4 + 2x^3 + 4x^2 + 6x + 3$ given that $i\sqrt{3}$ is a zero of $k(x)$.

Multiplying polar coordinates is easy. Just multiply the r's and add the θ's.

Example 1: If g = [4, 80°] and p = [7, 40°], what is gp?

Example 2: If p = [- 2, 30°] and u = [5, 70°], what is pu?

Example 3: If h = [-7, $\frac{\pi}{2}$] and d = [-4, π], what is hd?

Example 4: If q = 5(cos 60° + i sin 60°) and t = 2(cos 30° + i sin 30°), what is qt?