AP Calculus AB Practice: Rate of Change

CALCULATOR

AP® CALCULUS AB 2002 SCORING GUIDELINES

Question 2

The rate at which people enter an amusement park on a given day is modeled by the function E defined by

$$E(t) = \frac{15600}{(t^2 - 24t + 160)}.$$

The rate at which people leave the same amusement park on the same day is modeled by the function L defined by

$$L(t) = \frac{9890}{(t^2 - 38t + 370)}.$$

Both $E(t)$ and $L(t)$ are measured in people per hour and time t is measured in hours after midnight. These functions are valid for $9 \le t \le 23$, the hours during which the park is open. At time $t = 9$, there are no people in the park.

(a) How many people have entered the park by 5:00 P.M. ($t = 17$)? Round answer to the nearest whole number.

(b) The price of admission to the park is \$15 until 5:00 P.M. ($t = 17$). After 5:00 P.M., the price of admission to the park is \$11. How many dollars are collected from admissions to the park on the given day? Round your answer to the nearest whole number.

(c) Let $H(t) = \int_9^t (E(x) - L(x))\,dx$ for $9 \le t \le 23$. The value of $H(17)$ to the nearest whole number is 3725. Find the value of $H'(17)$ and explain the meaning of $H(17)$ and $H'(17)$ in the context of the park.

(d) At what time t, for $9 \le t \le 23$, does the model predict that the number of people in the park is a maximum?

AP® CALCULUS AB
2003 SCORING GUIDELINES (Form B)

Question 2

CALCULATOR

A tank contains 125 gallons of heating oil at time $t = 0$. During the time interval $0 \le t \le 12$ hours, heating oil is pumped into the tank at the rate

$$H(t) = 2 + \frac{10}{(1 + \ln(t+1))} \text{ gallons per hour.}$$

During the same time interval, heating oil is removed from the tank at the rate

$$R(t) = 12\sin\left(\frac{t^2}{47}\right) \text{ gallons per hour.}$$

(a) How many gallons of heating oil are pumped into the tank during the time interval $0 \le t \le 12$ hours?

(b) Is the level of heating oil in the tank rising or falling at time $t = 6$ hours? Give a reason for your answer.

(c) How many gallons of heating oil are in the tank at time $t = 12$ hours?

(d) At what time t, for $0 \le t \le 12$, is the volume of heating oil in the tank the least? Show the analysis that leads to your conclusion.

AP® CALCULUS AB
2004 SCORING GUIDELINES (Form B)

Question 2

CALCULATOR

For $0 \le t \le 31,$ the rate of change of the number of mosquitoes on Tropical Island at time t days is modeled by $R(t) = 5\sqrt{t}\cos\left(\frac{t}{5}\right)$ mosquitoes per day. There are 1000 mosquitoes on Tropical Island at time $t = 0.$

(a) Show that the number of mosquitoes is increasing at time $t = 6.$

(b) At time $t = 6,$ is the number of mosquitoes increasing at an increasing rate, or is the number of mosquitoes increasing at a decreasing rate? Give a reason for your answer.

(c) According to the model, how many mosquitoes will be on the island at time $t = 31?$ Round your answer to the nearest whole number.

(d) To the nearest whole number, what is the maximum number of mosquitoes for $0 \le t \le 31?$ Show the analysis that leads to your conclusion.

AP® CALCULUS AB
2004 SCORING GUIDELINES

Question 1

CALCULATOR

Traffic flow is defined as the rate at which cars pass through an intersection, measured in cars per minute. The traffic flow at a particular intersection is modeled by the function F defined by

$$F(t) = 82 + 4\sin\left(\frac{t}{2}\right) \text{ for } 0 \le t \le 30,$$

where $F(t)$ is measured in cars per minute and t is measured in minutes.

(a) To the nearest whole number, how many cars pass through the intersection over the 30-minute period?

(b) Is the traffic flow increasing or decreasing at $t = 7$? Give a reason for your answer.

(c) What is the average value of the traffic flow over the time interval $10 \le t \le 15$? Indicate units of measure.

(d) What is the average rate of change of the traffic flow over the time interval $10 \le t \le 15$? Indicate units of measure.

AP® CALCULUS AB
2007 SCORING GUIDELINES

Question 2

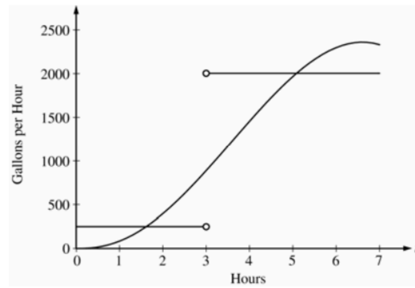
CALCULATOR

The amount of water in a storage tank, in gallons, is modeled by a continuous function on the time interval $0 \le t \le 7$, where t is measured in hours. In this model, rates are given as follows:

(i) The rate at which water enters the tank is $f(t) = 100t^2 \sin\left(\sqrt{t}\right)$ gallons per hour for $0 \le t \le 7$.

(ii) The rate at which water leaves the tank is

$$g(t) = \begin{cases} 250 & \text{for } 0 \le t < 3 \\ 2000 & \text{for } 3 < t \le 7 \end{cases} \text{ gallons per hour.}$$



The graphs of f and g, which intersect at $t = 1.617$ and $t = 5.076$, are shown in the figure above. At time $t = 0$, the amount of water in the tank is 5000 gallons.

(a) How many gallons of water enter the tank during the time interval $0 \le t \le 7$? Round your answer to the nearest gallon.

(b) For $0 \le t \le 7$, find the time intervals during which the amount of water in the tank is decreasing. Give a reason for each answer.

(c) For $0 \le t \le 7$, at what time t is the amount of water in the tank greatest? To the nearest gallon, compute the amount of water at this time. Justify your answer.

Question 2

The rate at which people enter an amusement park on a given day is modeled by the function E defined by

$$E(t) = \frac{15600}{\left(t^2 - 24t + 160\right)}.$$

The rate at which people leave the same amusement park on the same day is modeled by the function L defined by

$$L(t) = \frac{9890}{\left(t^2 - 38t + 370\right)}.$$

Both $E(t)$ and $L(t)$ are measured in people per hour and time t is measured in hours after midnight. These functions are valid for $9 \le t \le 23$, the hours during which the park is open. At time $t = 9$, there are no people in the park.

(a) How many people have entered the park by 5:00 P.M. ($t = 17$)? Round answer to the nearest whole number.

(b) The price of admission to the park is \$15 until 5:00 P.M. ($t = 17$). After 5:00 P.M., the price of admission to the park is \$11. How many dollars are collected from admissions to the park on the given day? Round your answer to the nearest whole number.

(c) Let $H(t) = \int_9^t (E(x) - L(x))\,dx$ for $9 \le t \le 23$. The value of $H(17)$ to the nearest whole number is 3725. Find the value of $H'(17)$ and explain the meaning of $H(17)$ and $H'(17)$ in the context of the park.

(d) At what time t, for $9 \le t \le 23$, does the model predict that the number of people in the park is a maximum?

(a) $\int_9^{17} E(t)\,dt = 6004.270$

6004 people entered the park by 5 pm.

$$3 \begin{cases} 1: \text{ limits} \\ 1: \text{ integrand} \\ 1: \text{ answer} \end{cases}$$

(b) $15\int_9^{17} E(t)\,dt + 11\int_{17}^{23} E(t)\,dt = 104048.165$

The amount collected was \$104,048.

or

$\int_{17}^{23} E(t)\,dt = 1271.283$

1271 people entered the park between 5 pm and 11 pm, so the amount collected was

$\$15 \cdot (6004) + \$11 \cdot (1271) = \$104,041.$

1 : setup

(c) $H'(17) = E(17) - L(17) = -380.281$

There were 3725 people in the park at $t = 17$.

The number of people in the park was decreasing at the rate of approximately 380 people/hr at time $t = 17$.

$$3 \begin{cases} 1: \text{ value of } H'(17) \\ 2: \text{ meanings} \\ \quad 1: \text{ meaning of } H(17) \\ \quad 1: \text{ meaning of } H'(17) \\ \quad < -1 > \text{ if no reference to } t = 17 \end{cases}$$

(d) $H'(t) = E(t) - L(t) = 0$

$t = 15.794$ or 15.795

$$2 \begin{cases} 1: \; E(t) - L(t) = 0 \\ 1: \text{ answer} \end{cases}$$

Question 2

A tank contains 125 gallons of heating oil at time $t = 0$. During the time interval $0 \le t \le 12$ hours, heating oil is pumped into the tank at the rate

$$H(t) = 2 + \frac{10}{(1 + \ln(t+1))} \text{ gallons per hour.}$$

During the same time interval, heating oil is removed from the tank at the rate

$$R(t) = 12\sin\left(\frac{t^2}{47}\right) \text{ gallons per hour.}$$

(a) How many gallons of heating oil are pumped into the tank during the time interval $0 \le t \le 12$ hours?

(b) Is the level of heating oil in the tank rising or falling at time $t = 6$ hours? Give a reason for your answer.

(c) How many gallons of heating oil are in the tank at time $t = 12$ hours?

(d) At what time t, for $0 \le t \le 12$, is the volume of heating oil in the tank the least? Show the analysis that leads to your conclusion.

(a) $\int_0^{12} H(t)\,dt = 70.570$ or 70.571	$2: \begin{cases} 1: \text{integral} \\ 1: \text{answer} \end{cases}$
(b) $H(6) - R(6) = -2.924$, so the level of heating oil is falling at $t = 6$.	1 : answer with reason
(c) $125 + \int_0^{12} (H(t) - R(t))\,dt = 122.025$ or 122.026	$3: \begin{cases} 1: \text{limits} \\ 1: \text{integrand} \\ 1: \text{answer} \end{cases}$
(d) The absolute minimum occurs at a critical point or an endpoint. $H(t) - R(t) = 0$ when $t = 4.790$ and $t = 11.318$. The volume increases until $t = 4.790$, then decreases until $t = 11.318$, then increases, so the absolute minimum will be at $t = 0$ or at $t = 11.318$. $125 + \int_0^{11.318} (H(t) - R(t))\,dt = 120.738$ Since the volume is 125 at $t = 0$, the volume is least at $t = 11.318$.	$3: \begin{cases} 1: \text{sets } H(t) - R(t) = 0 \\ 1: \text{volume is least at } t = 11.318 \\ 1: \text{analysis for absolute minimum} \end{cases}$

Question 2

For $0 \le t \le 31$, the rate of change of the number of mosquitoes on Tropical Island at time t days is modeled by $R(t) = 5\sqrt{t}\cos\left(\frac{t}{5}\right)$ mosquitoes per day. There are 1000 mosquitoes on Tropical Island at time $t = 0$.

(a) Show that the number of mosquitoes is increasing at time $t = 6$.

(b) At time $t = 6$, is the number of mosquitoes increasing at an increasing rate, or is the number of mosquitoes increasing at a decreasing rate? Give a reason for your answer.

(c) According to the model, how many mosquitoes will be on the island at time $t = 31$? Round your answer to the nearest whole number.

(d) To the nearest whole number, what is the maximum number of mosquitoes for $0 \le t \le 31$? Show the analysis that leads to your conclusion.

(a) Since $R(6) = 4.438 > 0$, the number of mosquitoes is increasing at $t = 6$.

1 : shows that $R(6) > 0$

(b) $R'(6) = -1.913$

Since $R'(6) < 0$, the number of mosquitoes is increasing at a decreasing rate at $t = 6$.

$2 : \begin{cases} 1 : \text{considers } R'(6) \\ 1 : \text{answer with reason} \end{cases}$

(c) $1000 + \int_0^{31} R(t)\,dt = 964.335$

To the nearest whole number, there are 964 mosquitoes.

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(d) $R(t) = 0$ when $t = 0$, $t = 2.5\pi$, or $t = 7.5\pi$

$R(t) > 0$ on $0 < t < 2.5\pi$

$R(t) < 0$ on $2.5\pi < t < 7.5\pi$

$R(t) > 0$ on $7.5\pi < t < 31$

The absolute maximum number of mosquitoes occurs at $t = 2.5\pi$ or at $t = 31$.

$1000 + \int_0^{2.5\pi} R(t)\,dt = 1039.357,$

There are 964 mosquitoes at $t = 31$, so the maximum number of mosquitoes is 1039, to the nearest whole number.

$4 : \begin{cases} 2 : \text{absolute maximum value} \\ \quad 1 : \text{integral} \\ \quad 1 : \text{answer} \\ 2 : \text{analysis} \\ \quad 1 : \text{computes interior critical points} \\ \quad 1 : \text{completes analysis} \end{cases}$

Question 1

Traffic flow is defined as the rate at which cars pass through an intersection, measured in cars per minute. The traffic flow at a particular intersection is modeled by the function F defined by

$$F(t) = 82 + 4\sin\left(\frac{t}{2}\right) \text{ for } 0 \le t \le 30,$$

where $F(t)$ is measured in cars per minute and t is measured in minutes.

(a) To the nearest whole number, how many cars pass through the intersection over the 30-minute period?

(b) Is the traffic flow increasing or decreasing at $t = 7$? Give a reason for your answer.

(c) What is the average value of the traffic flow over the time interval $10 \le t \le 15$? Indicate units of measure.

(d) What is the average rate of change of the traffic flow over the time interval $10 \le t \le 15$? Indicate units of measure.

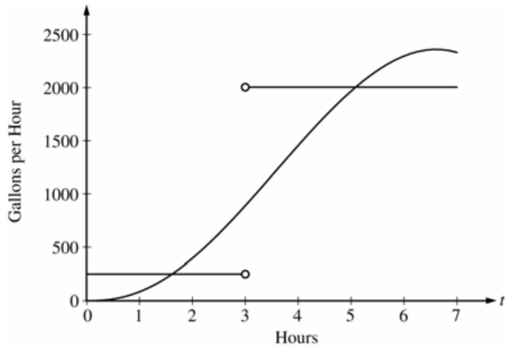
(a) $\int_0^{30} F(t)\,dt = 2474$ cars	$3 : \begin{cases} 1 : \text{limits} \\ 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$
(b) $F'(7) = -1.872$ or -1.873 Since $F'(7) < 0$, the traffic flow is decreasing at $t = 7$.	1 : answer with reason
(c) $\frac{1}{5}\int_{10}^{15} F(t)\,dt = 81.899$ cars/min	$3 : \begin{cases} 1 : \text{limits} \\ 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$
(d) $\frac{F(15) - F(10)}{15 - 10} = 1.517$ or 1.518 cars/min^2	1 : answer
Units of cars/min in (c) and cars/min^2 in (d)	1 : units in (c) and (d)

Question 2

The amount of water in a storage tank, in gallons, is modeled by a continuous function on the time interval $0 \le t \le 7$, where t is measured in hours. In this model, rates are given as follows:

(i) The rate at which water enters the tank is $f(t) = 100t^2 \sin\left(\sqrt{t}\right)$ gallons per hour for $0 \le t \le 7$.

(ii) The rate at which water leaves the tank is $g(t) = \begin{cases} 250 & \text{for } 0 \le t < 3 \\ 2000 & \text{for } 3 < t \le 7 \end{cases}$ gallons per hour.



The graphs of f and g, which intersect at $t = 1.617$ and $t = 5.076$, are shown in the figure above. At time $t = 0$, the amount of water in the tank is 5000 gallons.

(a) How many gallons of water enter the tank during the time interval $0 \le t \le 7$? Round your answer to the nearest gallon.

(b) For $0 \le t \le 7$, find the time intervals during which the amount of water in the tank is decreasing. Give a reason for each answer.

(c) For $0 \le t \le 7$, at what time t is the amount of water in the tank greatest? To the nearest gallon, compute the amount of water at this time. Justify your answer.

(a) $\int_0^7 f(t)\,dt \approx 8264$ gallons

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(b) The amount of water in the tank is decreasing on the intervals $0 \le t \le 1.617$ and $3 \le t \le 5.076$ because $f(t) < g(t)$ for $0 \le t < 1.617$ and $3 < t < 5.076$.

$2 : \begin{cases} 1 : \text{intervals} \\ 1 : \text{reason} \end{cases}$

(c) Since $f(t) - g(t)$ changes sign from positive to negative only at $t = 3$, the candidates for the absolute maximum are at $t = 0,\ 3,$ and 7.

t (hours)	gallons of water
0	5000
3	$5000 + \int_0^3 f(t)\,dt - 250(3) = 5126.591$
7	$5126.591 + \int_3^7 f(t)\,dt - 2000(4) = 4513.807$

The amount of water in the tank is greatest at 3 hours. At that time, the amount of water in the tank, rounded to the nearest gallon, is 5127 gallons.

$5 : \begin{cases} 1 : \text{identifies } t = 3 \text{ as a candidate} \\ 1 : \text{integrand} \\ 1 : \text{amount of water at } t = 3 \\ 1 : \text{amount of water at } t = 7 \\ 1 : \text{conclusion} \end{cases}$