

Hypothesis Test with Two Proportions Worksheet

MTH 112: Introduction to Statistics II Hypothesis Test of Two Proportions Worksheet

In clinical trials of Nasonex, 750 randomly selected pediatric patients (ages 3 to 11 years old) were randomly divided into two groups. Group 1 (experimental group) received 100 mcg of Nasonex, while the patients in Group 2 (control group) received a placebo. Of the 374 patients in the experimental group, 64 reported headaches as a side effect. Of the 376 patients in the control group, 68 reported headaches as a side effect. The researcher is trying to determine if there is enough pervasive evidence to conclude at $\alpha = 0.05$ that there is a difference in the proportion of headache sufferers taking Nasonex than those taking the placebo.

1. Find the proportion of headache sufferers in each group.

$$\hat{p}_N = \frac{64}{374} = 0.171 \qquad \hat{p}_P = \frac{68}{376} = 0.181$$

2. How can you verify that the sample sizes are large enough?

$$n_N \hat{p}_N \geq 10 \qquad n_N(1 - \hat{p}_N) \geq 10 \qquad n_P \hat{p}_P \geq 10 \qquad n_P(1 - \hat{p}_P) \geq 10$$

$$374(0.171) \geq 10 \quad 374(1 - 0.171) \geq 10 \quad 376(0.181) \geq 10 \quad 376(1 - 0.181) \geq 10$$

$$63 \geq 10 \qquad 310 \geq 10 \qquad 67 \geq 10 \qquad 307 \geq 10$$

All four criteria are true, therefore the sample size is large enough.

3. Construct a 90% confidence interval for the difference between the two population proportions.

$$CI = (\hat{p}_P - \hat{p}_N) \pm z^* \sqrt{\frac{\hat{p}_P(1 - \hat{p}_P)}{n_P} + \frac{\hat{p}_N(1 - \hat{p}_N)}{n_N}}$$

$$(0.181 - 0.171) - 1.645 \sqrt{\frac{0.181(1 - 0.181)}{376} + \frac{0.171(1 - 0.171)}{374}} \\ = -0.0357442355693$$

$$(0.181 - 0.171) + 1.645 \sqrt{\frac{0.181(1 - 0.181)}{376} + \frac{0.171(1 - 0.171)}{374}} \\ = 0.0557442355693$$

4. Interpret the confidence interval in terms of the two different groups.

We are 90% confident that the difference between the proportion of people suffering from

headaches is between -0.03574 to 0.05574 (-3.6% to 5.6%). NOTE: This interval includes 0. Think about what this means in terms of patients suffering from headaches as a side effect.

5. State the null and alternative hypothesis to test if there is a difference in the proportions of headache sufferers in each group.

$$H_0: \pi_N = \pi_P \quad \text{or} \quad \pi_N - \pi_P = 0$$

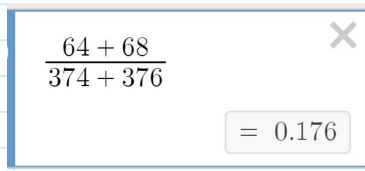
$$H_a: \pi_N \neq \pi_P \quad \text{or} \quad \pi_N - \pi_P \neq 0$$

6. Should the test be one or two tailed?

because $H_a: \pi_N \neq \pi_P$ is not equal to, therefore two-tailed

7. Calculate \hat{p}_{pooled} .

$$\hat{p}_{pooled} = \frac{x_P + x_N}{n_P + n_N}$$

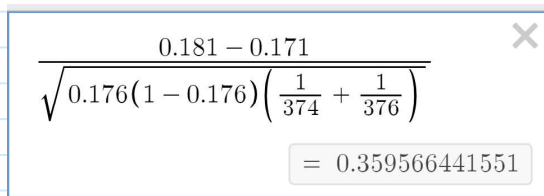


A calculator interface showing the calculation of the pooled proportion. The numerator is 64 + 68, and the denominator is 374 + 376. The result is 0.176.

$$\frac{64 + 68}{374 + 376} = 0.176$$

8. Calculate the test statistic.

$$z = \frac{\hat{p}_P - \hat{p}_N}{\sqrt{\hat{p}_{pooled}(1 - \hat{p}_{pooled})\left(\frac{1}{n_P} + \frac{1}{n_N}\right)}}$$



A calculator interface showing the calculation of the test statistic z. The numerator is 0.181 - 0.171. The denominator is the square root of 0.176(1 - 0.176) multiplied by (1/374 + 1/376). The result is 0.359566441551.

$$\frac{0.181 - 0.171}{\sqrt{0.176(1 - 0.176)\left(\frac{1}{374} + \frac{1}{376}\right)}} = 0.359566441551$$

9. Determine the p-value.

when $z = 0.36$ the probability is 0.6406.

two-tailed, upper tail, therefore,

$$p\text{-value} = 2(1\text{-prob}) = 2(1-0.6406) = 2(0.3594) = 0.7188$$

$$\begin{array}{ccc} p\text{-value} & & \text{significance level, } \alpha \\ 0.7188 & > & 0.05 \end{array}$$

Fail to reject H_0 .

10. Summarize your conclusions in terms of the two groups.

There is no significant difference between the proportion of headache sufferers taking Nasonex than those taking the placebo.