Sections 8.1 & 8.3 – Differential Equations & Euler's Method

1. Confirm that $y = 3e^{x^3}$ is a solution of the initial-value problem $y' = 3x^2 y, y(0) = 3$.

Verify that the functions are solutions of the differential equation by substituting the functions into the equation.

13. $y'' + 4y = 0$
    a. $\sin 2x$ and $\cos 2x$
    b. $c_1 \sin 2x + c_2 \cos 2x$ (c_1 and c_2 are constants)

Use the result in #13 to find a solution to the initial-value problem.

19. $y'' + 4y = 0,\ y(0) = 1,\ y'(0) = 2$

Find a solution to the initial-value problem.

21. $y' + 4x = 2,\ y(0) = 3$
25. $x^2 y' + 2xy = 0,\ y(1) = 2$ [Hint: Interpret the left-hand side of the equation as the derivative of a product of two functions]

A. Verify that $y = \frac{8}{1+e^{-2t}}$ satisfies the logistic differential equation $\frac{dy}{dt} = ky\left(1 - \frac{y}{L}\right)$.
B. The logistic equation $y = \frac{2100}{1+29e^{-0.75t}}$ models the growth of a population.
    i. Find the value of *k*
    ii. Find the carrying capacity
    iii. Find the initial population
    iv. Determine when the population will reach 50% of its carrying capacity
    v. Write the logistic differential equation that has the solution P(t)
C. A conservation organization releases 25 Florida panthers into a game preserve. After 2 years, there are 39 panthers. The Florida preserve has a carrying capacity of 200 panthers.
    a. Write a logistic equation that models the population of panthers in the preserve.
    b. Find the population after 5 years.
    c. When will the population reach 100?
    d. Write a logistic differential equation that models the growth rate of the panther population. Then repeat part (b) using Euler's Method with a step size of 1. Compare the approximation with the exact answer.
    e. After how many years is the panther population growing the most rapidly? Explain.

9. Use Euler's Method with the given step size to approximate the solution of the initial-value problem over the stated interval. Present your answer as a table.

$$\frac{dy}{dt} = \cos y\,, y(0) = 1, 0 \le t \le 2, \Delta t = 0.5$$

11. Consider the initial value problem $y' = \sin \pi t\,, y(0) = 0$. Use Euler's Method with five steps to approximate y(1).