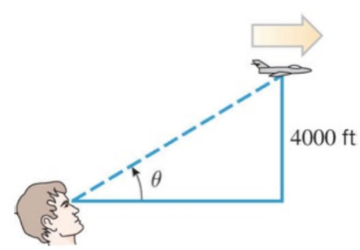
Section 3.4 – Related Rates

1. Let $y = 3x + 5$
   a. Given that dx/dt=2, find dy/dt when x=1
   b. Given that dy/dt=-1, find dx/dt when x=0

3. Equation $4x^2 + 9y^2 = 1$
   a. Given that dx/dt=3, find dy/dt when $(x, y) = \left(\frac{1}{2\sqrt{2}}, \frac{1}{3\sqrt{2}}\right)$
   b. Given that dy/dt=8, find dx/dt when $(x, y) = \left(\frac{1}{3}, -\frac{\sqrt{5}}{9}\right)$

5. Let A be the area of a square whose sides have length x, and assume that x varies with time t.
   a. Draw a picture of the square with the labels A and x placed appropriately.
   b. Write an equation that relates A to x.
   c. Use the equation in part (b) to find an equation that relates dA/dt and dx/dt.
   d. At a certain instant the sides are 3 ft long and increasing at a rate of 2 ft/min. How fast is the area increasing at that instant?
9. Let θ (in radians) be an acute angle in a right triangle, and let x and y, respectively, be the lengths of the sides adjacent to and opposite θ. Suppose that x and y vary with time.
   a. How are dθ/dt, dx/dt, and dy/dt related?
   b. At a certain instant, x=2 units and is increasing at 1 unit/s, while y=2 units and is decreasing at ¼ unit/s. How fast is θ changing at that instant? Is θ increasing or decreasing?
13. Oil spilled from a ruptured tanker spreads in a circle whose area increases at a constant rate of 6 mi^2/hr. How fast is the radius of the spill increasing when the area is 9 mi^2?
17. A 13 ft ladder is leaning against a wall. If the top of the ladder slips down the wall at a rate of 2 ft/s, how fast will the foot be moving away from the wall when the top is 5 ft from the ground?
24. An aircraft is flying horizontally at a constant height of 4000 ft above a fixed observation point (see figure). At a certain instant the angle of elevation is 30° and decreasing, and the speed of the aircraft is 300 mi/h.



   a. How fast is θ decreasing at this instant? Express the result in deg/sec.
   b. How fast is the distance between the aircraft and the observation point changing at this instant? Express the result in units ft/s.
25. A conical water tank with vertex down has a radius of 10 ft at the top nand is 24 ft high. If water flows into the tank at a rate of 20 cubic feet per minute, how fast is the depth of the water increasing when the water is 16 ft deep?
37. A particle is moving along the curve whose equation is $\frac{xy^3}{1+y^2} = \frac{8}{5}$. Assume that the x-coordinate is increasing at the rate of 6 units/s when the particle is at the point (1,2).
   a. At what rate is the y-coordinate of the point changing at that instant?
   b. Is the particle rising or falling at that instant?
40. A point P is moving along the curve whose equation is $y = \sqrt{x}$. Suppose that x is increasing at the rate of 4 units/s when x=3.
   a. How fast is the distance between P and the point (2,0) changing at this instant?
   b. How fast is the angle of inclination of the line segment from P to (2,0) changing at this instant?