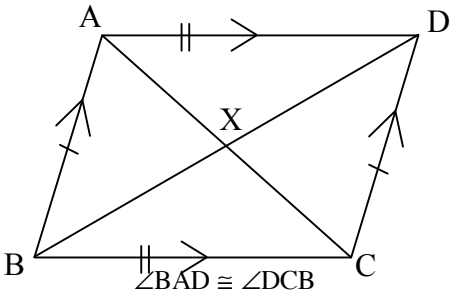
PROPERTIES OF QUADRILATERALS #13

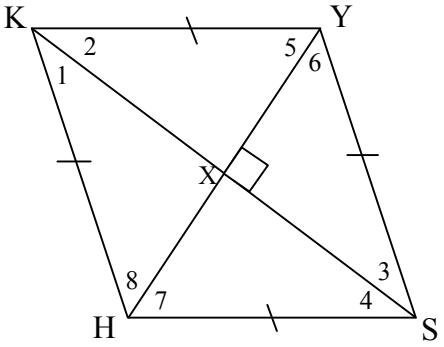
Polygons with four sides are called **QUADRILATERALS**. There are various properties associated with specific kinds of quadrilaterals. The names of quadrilaterals studied in the book are listed below along with their properties.

Quadrilateral ABCD is a **parallelogram**. Opposite sides are parallel and congruent. Consecutive angles are supplementary. The diagonals bisect.



$\angle BAD \cong \angle DCB$
$\angle ADC \cong \angle ABC$
$\overline{AX} \cong \overline{CX}$
$\overline{BX} \cong \overline{DX}$

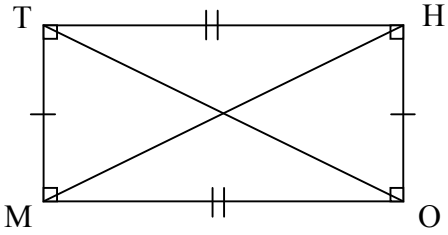
Quadrilateral KYSH is a **rhombus**. It is a parallelogram with four congruent sides. In addition to all the properties of a parallelogram, the diagonals are perpendicular and bisect the opposite angles.



$HY \perp KS$
$\angle 1 \cong \angle 2 \cong \angle 3 \cong \angle 4$
$\angle 5 \cong \angle 6 \cong \angle 7 \cong \angle 8$

Quadrilateral THOM is a **rectangle**. It is a parallelogram with four right angles. In addition to all the properties of a parallelogram, the diagonals are congruent.

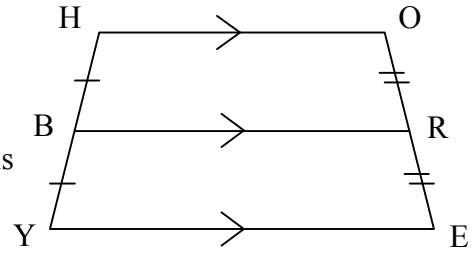
If a rectangle is also a rhombus, then it is called a **square**. A square would have all the properties listed previously.



$\overline{TO} \cong \overline{MH}$

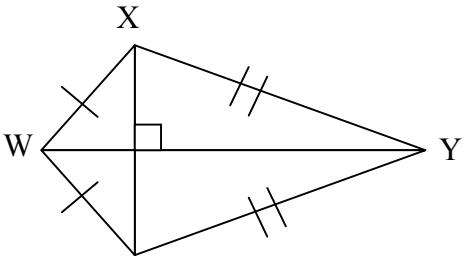
Two additional quadrilaterals are studied.

Quadrilateral HOEY is a **trapezoid**. It has exactly one set of parallel sides called bases. The segment connecting the midpoints of the non-parallel sides is parallel to the bases and has length equal to the average of the bases.



$BR = \frac{1}{2}(HO + EY)$

Quadrilateral WXYZ is a **kite**. It has one pair of opposite angles congruent and the diagonals are perpendicular.

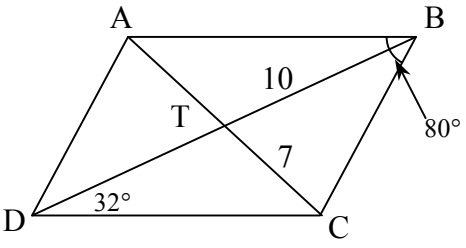


$XZ \perp WY$

$\angle WXY \cong \angle WZY$

Example 1

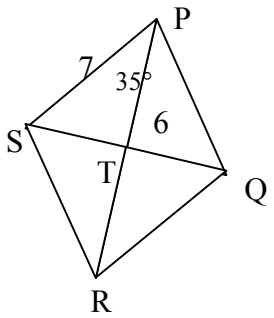
Use the information in the parallelogram at right to find $m\angle BAD$, $m\angle ADB$, AT, and BD.



Solution: $m\angle BAD = 100°$ since consecutive angles are supplementary. $m\angle ADB = 80° - 32° = 48° = 80° - 32° = 48°$ since opposite angles are congruent. AT = 7 and BD = 20 since diagonals bisect.

Example 2

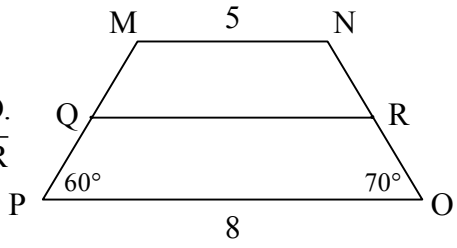
Use the information in the rhombus at right to find RQ, $m\angle SPQ$, $m\angle STR$, and RT.



Solution: RQ = 7 since all sides congruent. $m\angle SPQ = 70°$ since diagonals bisect opposite angles. $m\angle STR = 90°$ since diagonals are perpendicular. RT = 6 since diagonals bisect.

Example 3

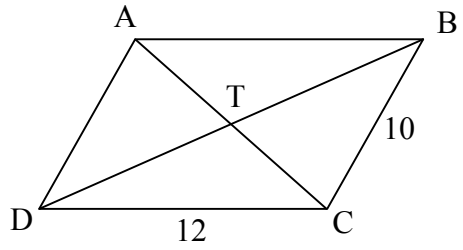
Given that Q and R are midpoints in the trapezoid at right to find $m\angle QMN$, $m\angle QRN$, and QR.



Solution: $m\angle QMN = 120°$ since MN is parallel to PO. $m\angle QRN = 70°$ since Q and R are midpoints, then $\overline{QR}$ is parallel to the bases and with parallel lines, corresponding angles are congruent.
$QR = \frac{1}{2}(5 + 8) = 6.5$.

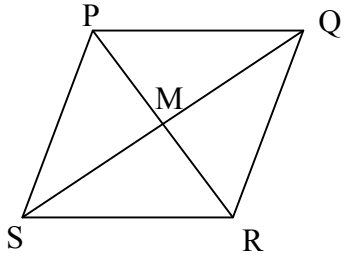
Find the required information and justify your answers.

For problems 1-4 use the parallelogram at right.



1. Find the perimeter.
2. If CT = 9, find AT.
3. If $m\angle CDA = 60°$, find $m\angle CBA$ and $m\angle BAD$.
4. If $AT = 4x - 7$ and $CT = -x + 13$, solve for x.

For problems 5-8 use the rhombus at right.

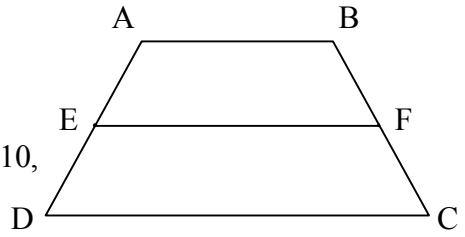


5. If $PS = \sqrt{6}$, what is the perimeter of PQRS?
6. If $PQ = 3x + 7$ and $QR = -x + 17$, solve for x.
7. If $m\angle PSM = 22°$, find $m\angle RSM$ and $m\angle SPQ$.
8. If $m\angle PMQ = 4x - 5$, solve for x.

For problems 9-12 use the quadrilateral at right.

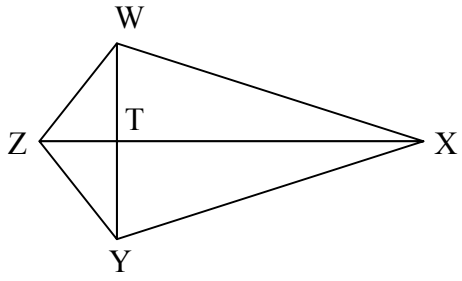
9. If WX = YZ and WZ = XY, must WXYZ be a rectangle?
10. If $m\angle WZY = 90°$, must WXYZ be a rectangle?
11. If the information in problems 9-10 are both true, must WXYZ be a rectangle?
12. If WY = 15 and WZ = 9, what is YZ and XZ?

For problems 13-16 use the trapezoid at right with midpoints E and F.



13. If $m\angle EDC = 60°$, find $m\angle AEF$.
14. If $m\angle DCB = 5x + 20$ and $m\angle ABC = 3x + 10$, solve for x.
15. If AB = 6 and DC = 10, find EF.
16. If EF = 9 and DC = 15, find AB.

For problems 17-20 use the kite at right.



17. If $m\angle XWZ = 95°$, find $m\angle XYZ$.
18. If $m\angle WZT = 110°$ and $m\angle WXY = 40°$, find $m\angle ZWX$.
19. If WZ = 5 and WT = 4, find ZT.
20. If WT = 4, TZ = 3, and TX = 10, find the perimeter of WXYZ.

Answers

1. 44 units	2. 9 units	3. 60°, 120°	4. 4
5. $4\sqrt{6}$	6. 2.5	7. 22°, 136°	8. 23.75
9. no	10. no	11. yes	12. 12, 15
13. 60°	14. 18.75	15. 8	16. 3
17. 95°	18. 105°	19. 3	20. $10 + 2\sqrt{116}$