A.APR.C.4: Polynomial Identities

1 Emmeline is working on one side of a polynomial identity proof used to form Pythagorean triples. Her work is shown below:

$$(5x)^2 + (5x^2 - 5)^2$$

Step 1: $25x^2 + (5x^2 - 5)^2$

Step 2: $25x^2 + 25x^2 + 25$

Step 3: $50x^2 + 25$

Step 4: $75x^2$

What statement is true regarding Emmeline's work?

1) Emmeline's work is entirely correct.
2) There is a mistake in step 2, only.
3) There are mistakes in step 2 and step 4.
4) There is a mistake in step 4, only.

2 The expression $(x+a)(x+b)$ can *not* be written as

1) $a(x+b) + x(x+b)$
2) $x^2 + abx + ab$
3) $x^2 + (a+b)x + ab$
4) $x(x+a) + b(x+a)$

3 Which expression can be rewritten as $(x+7)(x-1)$?

1) $(x+3)^2 - 16$
2) $(x+3)^2 - 10(x+3) - 2(x+3) + 20$
3) $\dfrac{(x-1)(x^2-6x-7)}{(x+1)}$
4) $\dfrac{(x+7)(x^2+4x+3)}{(x+3)}$

4 Given the polynomial identity $x^6 + y^6 = (x^2 + y^2)(x^4 - x^2y^2 + y^4)$, which equation must also be true for all values of x and y?

1) $x^6 + y^6 = x^2(x^4 - x^2y^2 + y^4) + y^2(x^4 - x^2y^2 + y^4)$
2) $x^6 + y^6 = (x^2 + y^2)(x^2 - y^2)(x^2 - y^2)$
3) $(x^3 + y^3)^2 = (x^2 + y^2)(x^4 - x^2y^2 + y^4)$
4) $(x^6 + y^6) - (x^2 + y^2) = x^4 - x^2y^2 + y^4$

5 Mr. Farison gave his class the three mathematical rules shown below to either prove or disprove. Which rules can be proved for all real numbers?

I $(m+p)^2 = m^2 + 2mp + p^2$

II $(x+y)^3 = x^3 + 3xy + y^3$

III $(a^2 + b^2)^2 = (a^2 - b^2)^2 + (2ab)^2$

1) I, only
2) I and II
3) II and III
4) I and III

6 Which statement(s) are true for all real numbers?

I $(x-y)^2 = x^2 + y^2$

II $(x+y)^3 = x^3 + 3xy + y^3$

1) I, only
2) II, only
3) I and II
4) neither I nor II

7 Given the following polynomials

$$x = (a+b+c)^2$$
$$y = a^2 + b^2 + c^2$$
$$z = ab + bc + ac$$

Which identity is true?

1) $x = y - z$
2) $x = y + z$
3) $x = y - 2z$
4) $x = y + 2z$

8 Algebraically prove that the difference of the squares of any two consecutive integers is an odd integer.

9 Verify the following Pythagorean identity for all values of x and y:

$$(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$$

10 Erin and Christa were working on cubing binomials for math homework. Erin believed they could save time with a shortcut. She wrote down the rule below for Christa to follow.

$$(a+b)^3 = a^3 + b^3$$

Does Erin's shortcut always work? Justify your result algebraically.

11 Algebraically prove that $\dfrac{x^3+9}{x^3+8} = 1 + \dfrac{1}{x^3+8}$, where $x \neq -2$.

12 Algebraically determine the values of h and k to correctly complete the identity stated below.

$$2x^3 - 10x^2 + 11x - 7 = (x-4)(2x^2 + hx + 3) + k$$

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Answer Section

1 ANS: 3 REF: 012003aii
2 ANS: 2 REF: 011806aii
3 ANS: 1
$(x+7)(x-1)=x^2+6x-7=x^2+6x+9-7-9=(x+3)^2-16$

REF: 061808aii
4 ANS: 1
2) $(x^4-x^2y^2+y^4)\neq(x^2-y^2)(x^2-y^2)$; 3) $x^6+y^6\neq(x^3+y^3)^2$; 4) $\dfrac{x^6+y^6}{x^2+y^2}\neq x^6+y^6-(x^2+y^2)$

REF: 082219aii
5 ANS: 4
$(x+y)^3=x^3+3x^2y+3xy^2+y^3\neq x^3+3xy+y^3$

REF: 081620aii
6 ANS: 4
$(x-y)^2=x^2-2xy+y^2$ $(x+y)^3=x^3+3x^2y+3xy^2+y^3$

REF: 061902aii
7 ANS: 4
$(a+b+c)^2=a^2+ab+ac+ab+b^2+bc+ac+ab+c^2$

$$x=a^2+b^2+c^2+2(ab+bc+ac)$$
$$x=y+2z$$

REF: 061822aii
8 ANS:
Let x equal the first integer and $x+1$ equal the next. $(x+1)^2-x^2=x^2+2x+1-x^2=2x+1$. $2x+1$ is an odd integer.

REF: fall1511aii
9 ANS:

$$(x^2+y^2)^2=(x^2-y^2)^2+(2xy)^2$$
$$x^4+2x^2y^2+y^4=x^4-2x^2y^2+y^4+4x^2y^2$$
$$x^4+2x^2y^2+y^4=x^4+2x^2y^2+y^4$$

REF: 081727aii

10 ANS:

$$(a+b)^3 = a^3 + b^3$$

No. Erin's shortcut only works if $a = 0$, $b = 0$ or $a = -b$.

$$a^3 + 3a^2b + 3ab^2 + b^3 = a^3 + b^3$$

$$3ab^2 + 3a^2b = 0$$

$$3ab(b+a) = 0$$

$$a = 0, b = 0, a = -b$$

REF: 011927aii

11 ANS:

$$\frac{x^3+9}{x^3+8} = \frac{x^3+8}{x^3+8} + \frac{1}{x^3+8}$$

$$\frac{x^3+9}{x^3+8} = \frac{x^3+9}{x^3+8}$$

REF: 061631aii

12 ANS:

$$2x^3 - 10x^2 + 11x - 7 = 2x^3 + hx^2 + 3x - 8x^2 - 4hx - 12 + k \quad h = -2$$

$$-2x^2 + 8x + 5 = hx^2 - 4hx + k \quad k = 5$$

REF: 011733aii