CMU 16-720 Computer Vision
3D Reconstruction Report

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Part 1 Basic Geometry

Part - 1.1

Suppose two cameras fixate on a point x (see Figure 1) in space such that their principal axes intersect at that point. If the image coordinates are normalized so that the coordinate origin (0; 0) coincides with the principal point, we are going to demonstrate the F33 element of fundamental matrix F33 will be 0

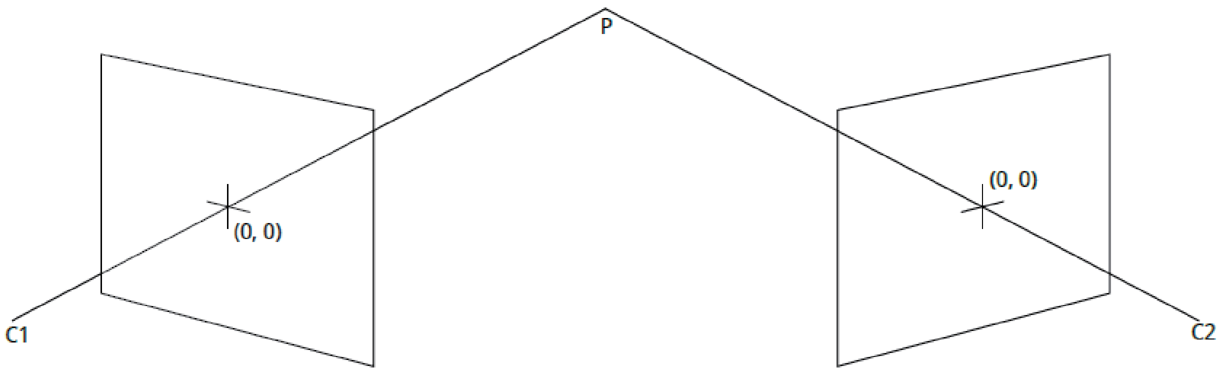


Figure 1: Figure for Q1.1. C1 and C2 are the optical centers. The principal axes intersect at point w (P in the figure).

We see that the an observed 2D point in left camerat view as x1 = [0;0], the corresponding point is x2 = [0;0] in right camera view. So according to the formula (1) we can write the formula(2):

$$\tilde{\mathbf{x}}_2^T \mathbf{F} \tilde{\mathbf{x}}_1 = 0, \qquad (1)$$

[0, 0, 1] F [0; 0; 1] = 0 (2)

(2) is correct if and only if F = [a, b, c; (a/b/c/d/e/f/g/h could be any number)
d, e, f;
g, h, 0]

In other word the F33 element of the fundamental matrix is always zero

Part - 1.2

Consider the case of two cameras viewing an object such that the second camera differs from the first by a pure translation that is parallel to the x-axis. We are going to demonstrate that the epipolar lines in the two cameras are also parallel to the x-axis

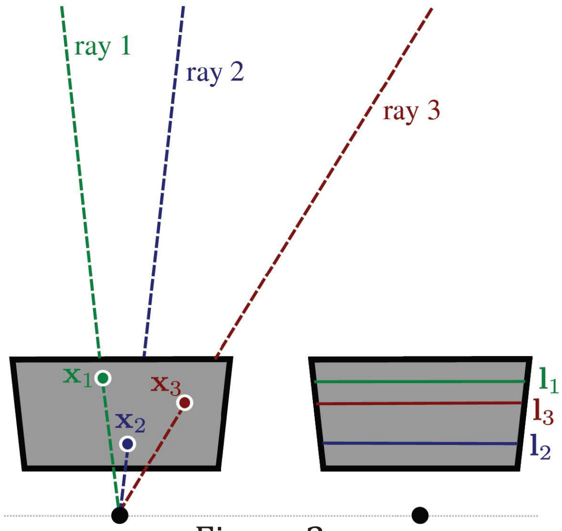


Figure 2

$$\tilde{\mathbf{x}}_2^T \mathbf{E} \tilde{\mathbf{x}}_1 = 0, \tag{1}$$

$$\mathbf{E} = \boldsymbol{\tau}_\times \boldsymbol{\Omega} \tag{2}$$

$$\boldsymbol{\tau}_\times = \begin{bmatrix} 0 & -\tau_z & \tau_y \\ \tau_z & 0 & -\tau_x \\ -\tau_y & \tau_x & 0 \end{bmatrix}. \tag{3}$$

In this case, $\Omega = I$ since there is no rotation. Since we only have translation in x direction, E = [0, 0, 0; (tx is the translation in x direction)
0, 0, -tx;
0, tx, 0]

from (1)(2)(3) we can write: $[x2, y2, 1] \begin{bmatrix} 0, & 0, & 0 \\ 0, & 0, & -tx \\ 0, & tx, & 0 \end{bmatrix} \begin{bmatrix} x1 \\ y2 \\ 1 \end{bmatrix} = 0$

$$[x2, y2, 1] \begin{bmatrix} 0 \\ tx \\ -tx * y1 \end{bmatrix} = 0$$

$$y2*tx - y1*tx = 0$$

$$y2 = y1 \tag{4}$$

from (4) we can see the constrains of epipolar line in right camera forms lines that parallel to the x-axis as shown in Figure(2) and vice versa

Part - 1.3

Suppose we have an inertial sensor which gives us the accurate positions (Ri and ti, the rotation matrix and translation vector) of the robot at time i. We are going to discuss what will be the effective rotation (Rrel) and translation (trel) between two frames at different time stamps and calculate the essential matrix (E) and the fundamental matrix (F) in terms of K, Rrel and trel while supposing the camera intrinsics (K) are known

The effective rotation matrix is $Rrel \neq I$ and $trel \neq [0\ 0\ 0]T$

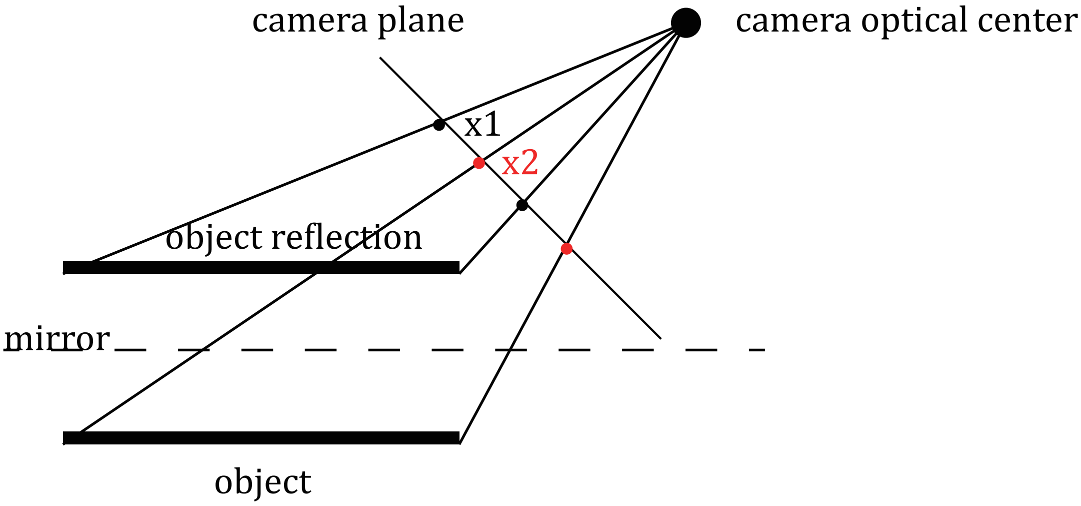
Suppose we set $$trel_X = \begin{bmatrix} 0, & -tz, & ty \\ tz, & 0, & -tx \\ -ty, & tx, & 0 \end{bmatrix}$$

then $$E = trel_X\ Rrel$$

Suppose K1 and K2 is the intrinsic matrix of camera1 and camera2

$$F = \left[K2^{-1}\right]^T E \left[K1^{-1}\right]$$

Part - 1.4
Suppose that a camera views an object and its rflection in a plane mirror. We are going to demonstrate that this situation is equivalent to having two images of the object which are related by a skew-symmetric fundamental matrix



We set the reflection matrix H as householder transformation matrix $H = I - 2vv^T$

if we set p as a point on object, then we get the x1 and x2 in the camera frame as

$$\lambda 1\ X1 = p$$
$$\lambda 2\ X2 = Hp + t \text{, where } t = \alpha v \text{ and } t_x(vv^T) = 0$$

So the essential matrix is

$$E = t_x H = t_x - 2t_x(vv^T) = t_x$$

$$t_x = \begin{bmatrix} 0, & -a, & b \\ a & 0, & -c \\ -b, & c, & 0 \end{bmatrix}$$

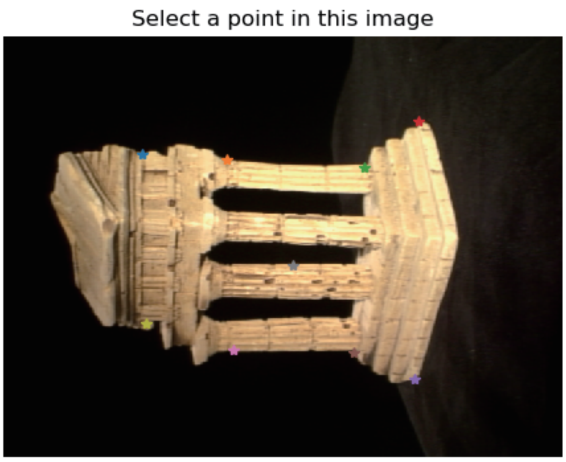
$$E^T = (t_x H)^T = H^T t_x^T = (I - 2vv^T)^T t_x^T = -t_x^T$$

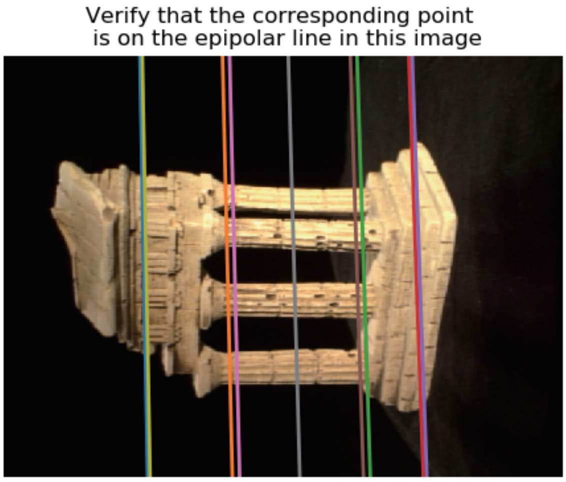
So we can demonstrate F is skew symmetric

$$F + F^T = [K^{-1}]\ (E + E^T)K^{-1} = 0$$

Part 2 Fundamental matrix estimation

Part - 2.1 We implement the 8-point algorithm to define the epipolar constrains from choosen points



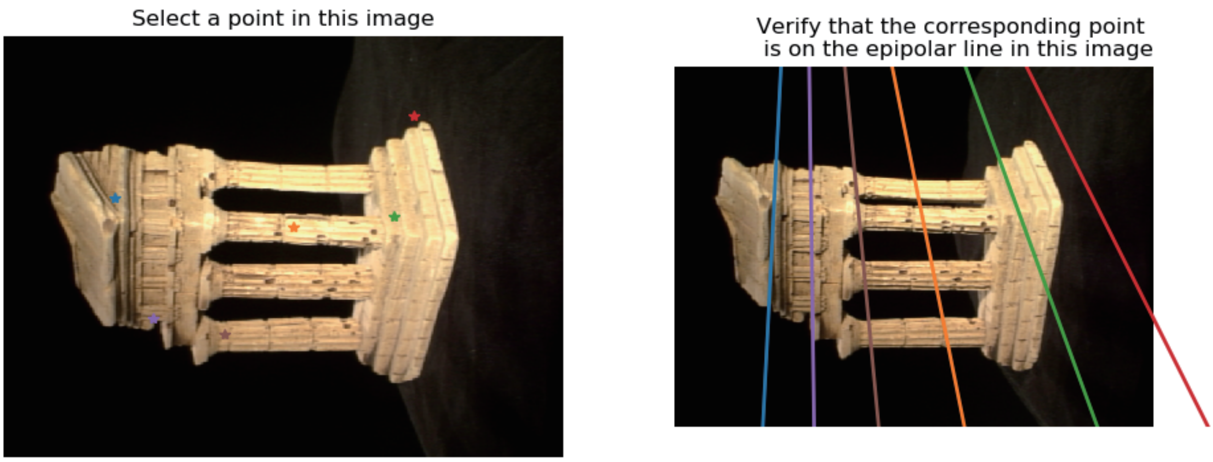


The F (fundamental matrix) of the above pictures is

```
[[ 9.80213861e-10 -1.32271663e-07  1.12586847e-03]
 [-5.72416248e-08  2.97011941e-09 -1.17899320e-05]
 [-1.08270296e-03  3.05098538e-05 -4.46974798e-03]]
```

Part 2 Fundamental matrix estimation

Part - 2.2 We implement the 7-point algorithm to define the epipolar constrains from choosen points



The F (fundamental matrix) of the above pictures is

[[-2.98179041e-08 -6.99672390e-07 -5.70389468e-04]
 [7.68098052e-07 2.01174921e-07 -2.68439554e-04]
 [5.83365641e-04 1.83268622e-04 2.48202593e-03]]

[[-2.98179041e-08 -6.99672390e-07 -5.70389468e-04]
 [7.68098052e-07 2.01174921e-07 -2.68439554e-04]
 [5.83365641e-04 1.83268622e-04 2.48202593e-03]]

[[-6.12763848e-07 4.81334834e-07 -6.95782569e-04]
 [-1.27449717e-06 1.51642606e-07 -8.08864913e-04]
 [1.25754731e-03 1.10820273e-03 -1.47017307e-01]]

Part 3 Metric Reconstruction

Part - 3.1 We reconstruct Fundamental matrix from Essential Matrix

```
      [[ 9.80213861e-10 -1.32271663e-07  1.12586847e-03]
F =    [-5.72416248e-08  2.97011941e-09 -1.17899320e-05]
       [-1.08270296e-03  3.05098538e-05 -4.46974798e-03]]
```

```
      [[ 2.26587820e-03   -3.06867395e-01    1.66257398e+00]
E=     [-1.32799331e-01    6.91553934e-03    -4.32775554e-02]
       [-1.66717617e+00  -1.33444257e-02    -6.72047195e-04]]
```

Part - 3.2 We triangulate a set of 2D coordinates in the image to a set of 3D points

Denote
p1 = [u1, v1, 1]T as the points in 2D coordinates in the first camera
p2 = [u2, v2, 1]T as the points in 2D coordinates in the second camera

P = [x, y, z, 1]T as the points in 3D coordinates

C1 = K1M1 as the projection matrices of the first camera
C2 = K2M2 as the projection matrices of the second camera

Since the p1 and P vector is in the same line
also p2 and P vector is in the same line, we can derive

$$p1 \times C1\ P = 0 \qquad \begin{pmatrix} p1_{\times} C1 \\ p2_{\times} C2 \end{pmatrix} P = 0$$
$$p2 \times C2\ P = 0$$

So

$$\begin{bmatrix} 0, & -1, & v1 \\ a & 0, & -u1 \\ -v1 & ,u1, & 0 \end{bmatrix} \begin{bmatrix} C1[1, :] \\ C1[2, :] \\ C1[3, :] \end{bmatrix} P = 0 \quad \begin{bmatrix} 0, & -1, & v2 \\ a & 0, & -u2 \\ -v2 & ,u2, & 0 \end{bmatrix} \begin{bmatrix} C2[1, :] \\ C2[2, :] \\ C2[3, :] \end{bmatrix} P = 0$$

This is the same as

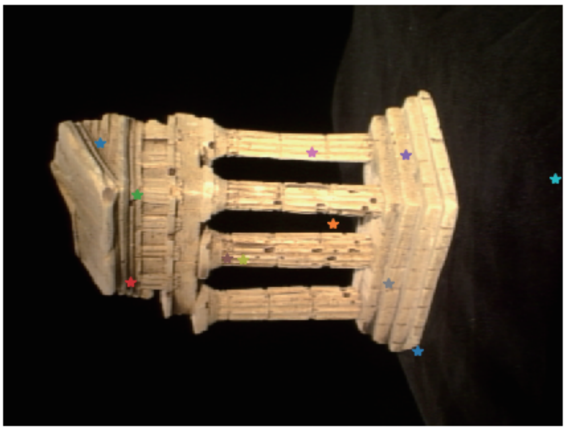
$$\begin{bmatrix} u1 * C1[3, :] - C1[1, :] \\ v1 * C1[3, :] - C1[2, :] \end{bmatrix} P = 0 \qquad \begin{bmatrix} u2 * C2[3, :] - C2[1, :] \\ v2 * C2[3, :] - C2[2, :] \end{bmatrix} P = 0$$

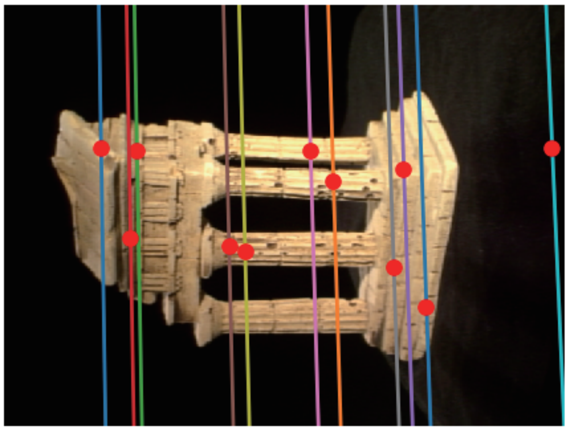
So

$$AP = \begin{bmatrix} u1 * C1[3, :] - C1[1, :] \\ v1 * C1[3, :] - C1[2, :] \\ u2 * C2[3, :] - C2[1, :] \\ v2 * C2[3, :] - C2[2, :] \end{bmatrix} P = 0$$

Part 3 Metric Reconstruction

Part - 4.1 Visualize the 3D points





Part - 4.3 3D visualization