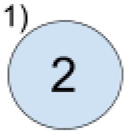
Nigel Decontie
V00853112

CSC 226 Assignment 1

1)



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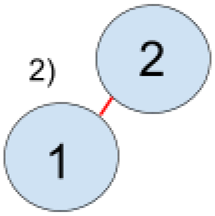

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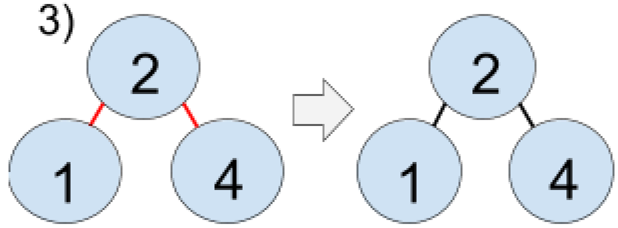

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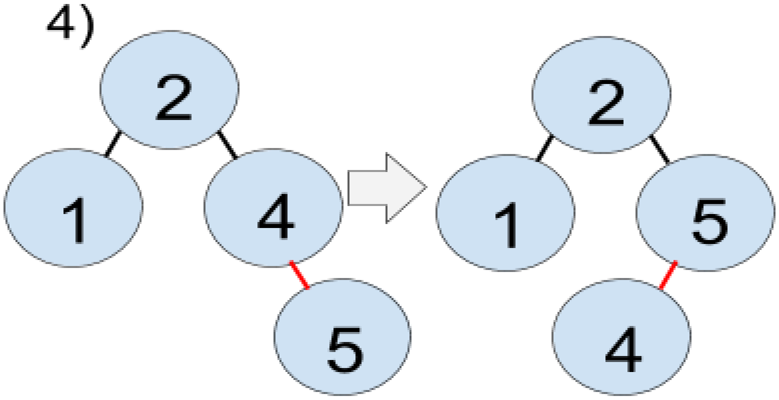

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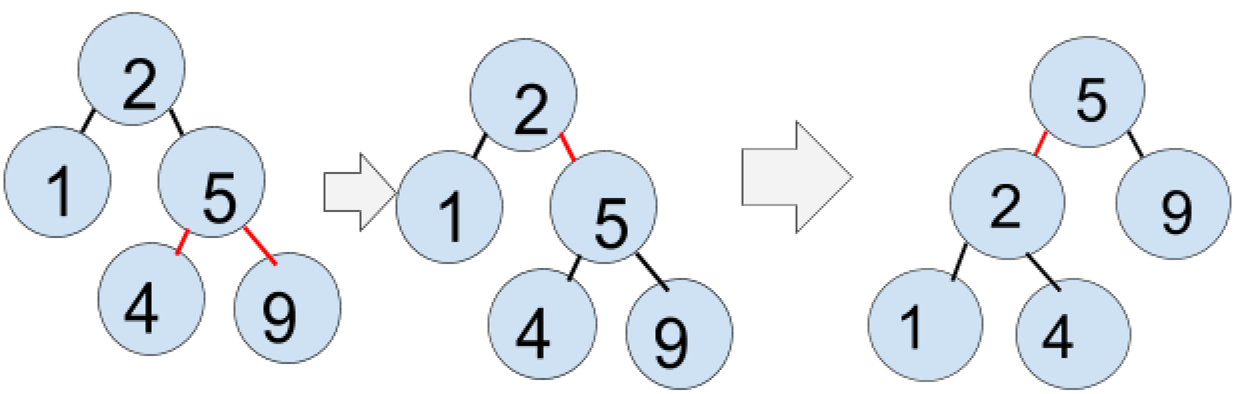

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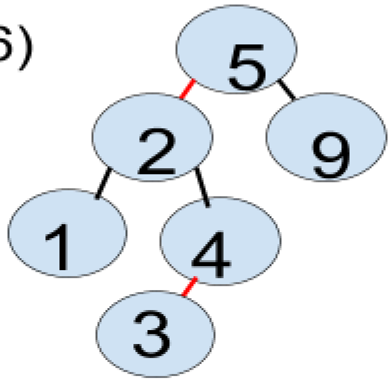

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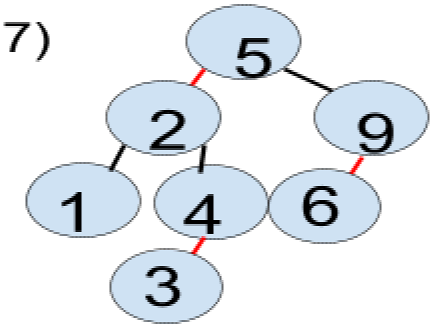

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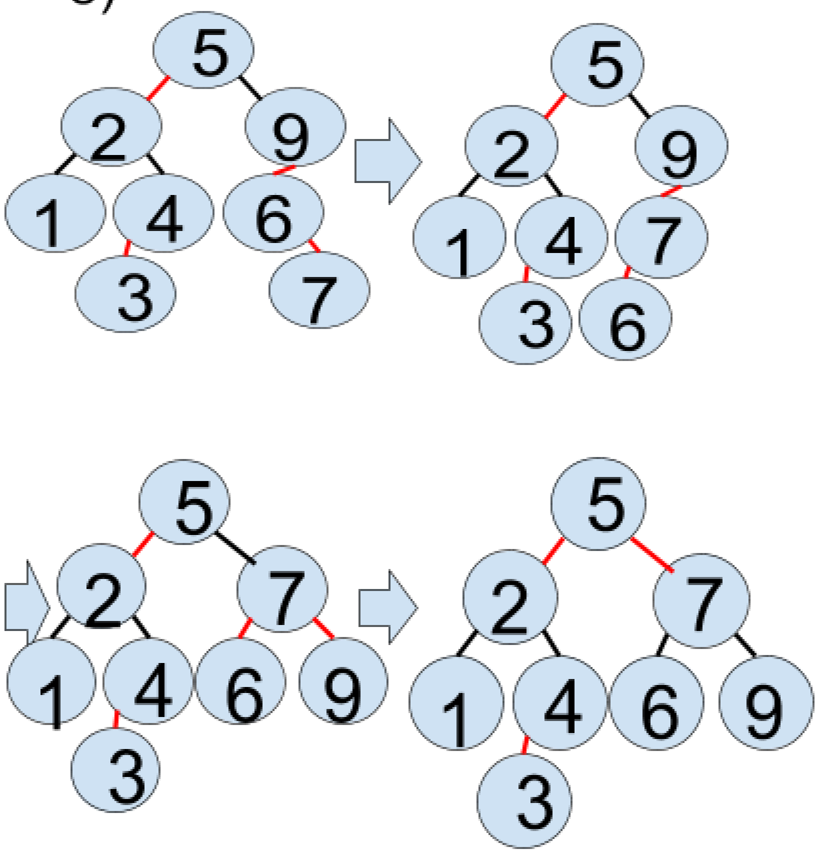

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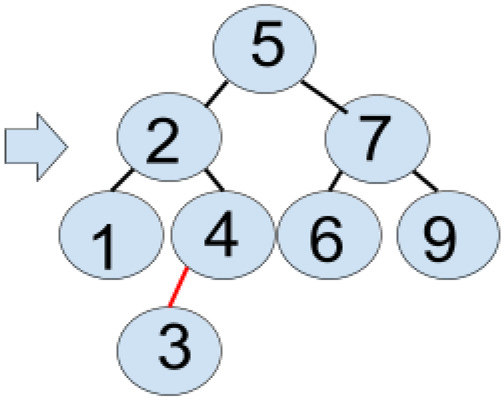

2. Prove: the height of a red-black tree with *n* nodes is at most 2log(*n*).
    - A subtree with root *x* has at least $2^{bh(x)}-1$ internal nodes, where bh(x) denotes the black height of the tree. When height = 0 (null leaf node), $2^0-1 = 0$ <- base case
    - Each child of *x, x1 and x2* have black height of either bh(x) or bh(x)-1. So the subtrees each have $2^{bh(x)-1}-1$ internal nodes.
    - Therefore, the subtree at *x* contains $(2^{bh(x)-1}-1) + (2^{bh(x)-1}-1) + 1 = 2 \cdot 2^{bh(x)-1}-1 = 2^{bh(x)}-1$ nodes.
    - Thus, at the root of the tree:
        - $n >= 2^{bh(root)-1}-1$
        - $n >= 2^{h/2}-1$ (since at most half the leaves on any path are red)
        - $n + 1 >= 2^{h/2}$
        - $\lg(n + 1) >= h/2$
        - $2\lg(n + 1) >= h$
    - Therefore the height is O(lg *n*)

3. Reverse delete algorithm (RDA). The graph produced by the RDA is not disconnected since the algorithm checks for disconnectedness at each deletion. It also cannot contain a cycle since it when checking edges, the max edge in a cycle would be encountered and deleted. Therefore, the resulting graph is a spanning tree. By nature of the algorithm, larger edges will be considered before smaller ones. As a result, the remaining paths between nodes will be the smallest possible.

4. Suppose there exist 2 MSTs for the graph G, A and B. Suppose we take an edge e from the nodes Q to R that exists in one of A and B and has the smallest cost. Suppose e exists in A but not in B. Then there must exist another path from Q to R that does not exist in A. Call this edge f. Adding e to B would create a cycle, so e cannot exist in B. Adding f to A would create a cycle as well, so f cannot exist in A. However since all edges have distinct weights and since, by definition, e has the smallest weight, then f has a greater weight than e. Replacing f with e would result in a smaller MST, which is a contradiction.

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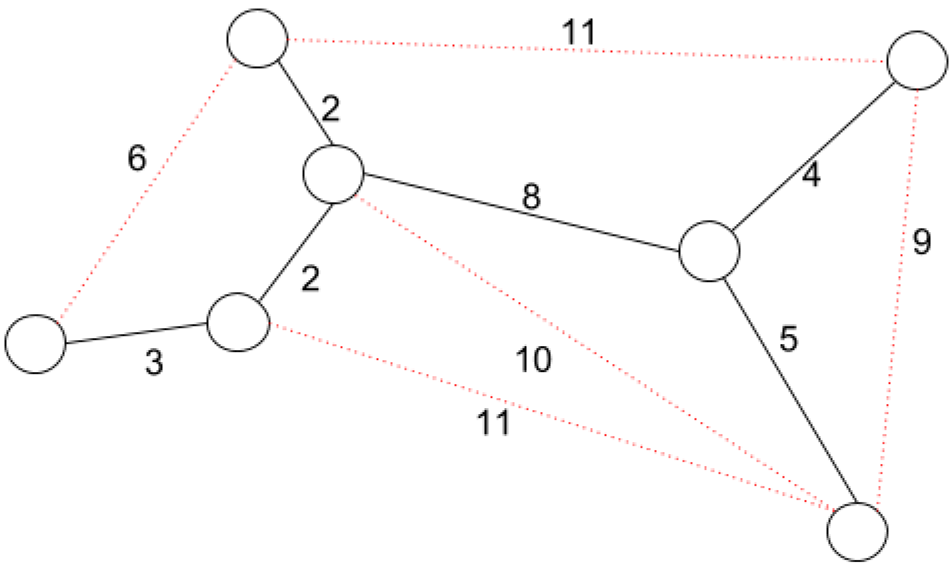
5. Minimum Bottleneck Spanning Tree

Possible MST of Graph G. Bottleneck B = 8
Total weight of 24



Possible MBST of Graph G. Bottleneck B = 8
Total weight of 28