Predicting Longer-Term Volatility

Kobby Amoah

Abstract:

Gasoline retail markets have traditionally attracted a lot of attention from researchers and policy makers. This entry reviews 2 sets of questions studied by economists related to this market who seek to understand the behavior of their prices. The first is concerned with whether volatility can be predicted at longer horizons. To investigate volatility's predictability, a comparative study within the autoregressive conditional heteroskedasticity (ARCH) class of models is presented. For the period studied (2016-2022), it is found model rankings are insensitive to the forecast horizon with asymmetric volatility models providing the best forecasts.

The second focuses on whether the cyclical spike in gasoline prices can be predicted. In particular, it is widely documented gasoline prices generally start to rise in the spring, just before the start of the summer driving season, with a price spike often at the beginning of the rise. The goal here is to assess the value of a popular Extreme Value approach in predicting these extreme price spikes.

Introduction

Ever since the start of the Covid-19 crisis, there has been renewed interest by practitioners and academics alike in reassessing the adequacy of gasoline price models. Soaring volatility especially in early 2022 has made it important to know how well our standard tools forecast volatility amid episodes of turmoil that pervade all corners of the economy. Volatility prediction is a critical task for the retail gasoline market as volatility often manifests itself in rampant price increases. Consumers are greatly concerned with gasoline price movements, as it occupies between 4.5% and 12.4% of households' disposable income, especially for poorer households. (Gicheva et al., 2007).

In this paper, it is explored the performance of volatility forecasting within the General Autoregressive Conditional Heteroskedastic (GARCH) class of models. In particular, the paper examines the GARCH(1,1), EGARCH(1,1), GJR-GARCH(1,1) and APARCH(1,1) to determine which volatility model provides the best longer-dated forecasts of gasoline price movement. While all the models exhibit significant deterioration in volatility forecast accuracy as the forecasting horizons lengthens, it turns out, as this paper will report, the rate of deterioration among the models differ with the EGARCH (1,1) providing the least deterioration in accuracy and hence the best volatility forecasts.

The second half of the paper examines a related phenomenon that has gained a lot of attention in recent times – the increasing frequency of gasoline price hikes in 2022. As documented by Jean-Fancois Houde(2010), gasoline prices in many cities follow easily predictable asymmetric cycles akin to Edgeworth cycles (Edgeworth, 1925): price increases are fast and large (relenting phase), and are followed by a sequence of small decreases (undercutting phase). Literature, however, on predicting the extreme gasoline price behavior is lacking which suggests that the tail behavior of gasoline prices is not well understood.

The contribution of this part of the paper is to assess the accuracy of a popular probabilistic framework, the Peaks over Threshold method, from Extreme Value theory in predicting those tail behaviors. The analysis reveals this approach is not well-suited to predicting those large price increases. A suggestion is made, however, on improving the framework for further research.

The remainder of the paper is structured as follows, Literature Review, Part I - Volatility Forecasting, Part II - Extreme Gasoline Price-Change Forecasting, Conclusion, and Appendix(Tables and Figures).[1] There are sub-sections under each part with each containing a Methodology, Data, Results and Discussion section.

Literature Review

This paper can be placed into two main research areas in the field of predictive econometric analysis: (*i*) modeling of volatility and (*ii*) modeling of spike occurrences where there exists an extensive amount of research. Yet, the literature seems to be almost always be concerned with the financial markets and less so for the gasoline retail markets. As it relates to volatility forecasting, C. Brownlees et al.(2011) find across asset classes and volatility regimes, asymmetric GARCH models often are the best forecasters. More so, they document the threshold GARCH of Glosten et al. (1993) is often the best forecaster. While the threshold Garch is not considered, it is of concern in this paper to establish which asymmetric GARCH model performs best for gasoline price volatility forecasts among the GARCH(1,1), EGARCH(1,1), GJR-GARCH(1,1) and APARCH(1,1).

As it relates to extreme price behavior, different modeling techniques have been applied to capture the distribution of extreme price movements. Bystrom (2005) and Paraschiv et al. (2016) investigate the performance of Extreme Value Theory(EVT) on accurately modeling and forecasting the extreme tails of electricity price distributions in the Nord Pool Electricity Market.[2] Paraschiv (2016) in particular, obtains robust forecasts of extreme quantiles using the Generalized Pareto distribution (GPD), also known in literature as the Peaks-Over-Threshold method.[3] This paper attempts to confirm if this approach would work for modeling the spikes in the US retail gasoline markets.

Part I

Volatility Forecasting

Methodology:

Models Considered

The four models considered to forecast are chosen from the vast literature on GARCH modeling for their simplicity and demonstrated ability to forecast over alternatives. They all describe a return/ price change time series as

[1]The goal in Part II is to forecast the extremes of price movements not price itself.

[2]While a bit of a mouthful, what is meant is the price change distribution not prices in itself.

[3]While confusing, the quantile is really just the same thing as the percentile. For consistency with the econometric literature that deals with EVT, the paper uses quantiles.

$$r_t = \mu_t + \xi_t \tag{1}$$

where,r_t is the return time series, μ is the expected return, and ξ_t is a zero-mean white noise.[4] They also denote $\xi_t = \sigma_t z_t$ where z_t is standard Gaussian.[5]

The first, GARCH(1,1) by Bollerslev(1986) provides a natural starting point. The volatility process is described by the GARCH (1,1) model as:

$$\sigma_t^2 = w + \alpha\xi_{t-1}^2 + \beta\sigma_{t-1}^2 \tag{2}$$

The key feature of the GARCH volatility process is it says current of volatility is related to its immediate past value and a past squared random noise term. Since r_t is given by a constant, μ, plus a random noise term, the squared random noise term is often replaced with r_t. Another key feature is it captures shocks as non-persistent. Most importantly, however, it dictates a symmetric impact of past returns on future volatility. (i.e, the magnitude of past returns and not the algebraic sign influences future volatility).

The second, the exponential GARCH(1,1) or EGARCH(1,1) by Nelson (1991) describes the volatility process as:

$$ln(\sigma_t^2) = w + \alpha(|z_{t-1}| - E|z_{t-1}|) + \gamma|z_{t-1}| + \beta ln(\sigma_t^2) \tag{3}$$

Two key features of the EGARCH volatility process are that it captures shocks as persistent and also captures the leverage effect with $\gamma < 0$. This leverage effect means the algebraic sign/direction of the return, not just its magnitude influences future volatility.

The third, the asymmetric Power Arch or APARCH(1,1) by (Ding et al 1993) describes the volatility process as:

$$\sigma_t^\delta = w + \alpha(|\xi_{t-1}| - \gamma\xi_{t-1})^\delta + \beta\sigma_{t-1}^\delta \tag{4}$$

The key feature of the APARCH is it captures a greater persistence in shocks than the GARCH, E-GARCH or GJR-GARCH. As Ding et al. (1993) show, this persistence is related to the serial correlation of absolute returns.[6] It also incorporates the leverage effect via γ.

The fourth, Glosten-Jagannathan-Runkle GARCH(GJR-GARCH) by (Glosten et al 1990) describes the volatility process as:

$$GJR - GARCH : \sigma_t^2 = w + (\alpha + \gamma I_{t-1})\xi_{t-1}^2 + \beta\sigma_{t-1}^2, \begin{cases} I = 0, r_t \geq \mu \\ I = 1, r_t < \mu \end{cases} \tag{5}$$

The key feature of the GJR-GARCH is like the simple GARCH, shocks are non-persistent. The GJR-GARCH however, also incorporates the leverage effect where $\gamma > 0$.

[4] i.e, the series ξ_t is independent and identically distributed with a mean of zero.

[5] i.e. Standard Normal

[6] Remember serial correlation describes the relationship between a given variable and its lagged version over various time intervals.

Data

To implement the longer-term volatility forecasts, weekly data is obtained from the Energy Information Administration (EIA) where the observations run from to April 4, 2016 to May 16, 2022, on the all grades gasoline price series. The models are fitted on an in-sample subset of the volatility series from April 4, 2016 to December 28, 2020. Additionally, since it has been widely documented the tails for price change/returns data follow heavy-tailed distributions, like the Generalized Error Distribution (GED), the GED is chosen to fit the error terms.[7] Forecasts are then produced and evaluated only on the out-of-sample 2021 volatility estimates. The are are 10 forecasts produced and these made after every 4 weeks(from week 4 to week 40). This translates to foreasts from January 25,2021 to October 4, 2021. The results are annualized however.

Results

Table 1(Appendix) reports the parameter estimates of all the GARCH class of models employed in the analysis as well as their information criteria and the log-likelihoods. The significance level of the coefficients are reported as well.

The AIC of the models are similar suggesting the models provide similar in-sample fits. Except for the GJR-GARCH, the asymmetric models have a significant leverage parameter γ which suggests the presence of leverage effects. The positivity of their leverage parameter however, suggests gasoline prices are more volatile during price increases than price decreases. Put simply for gasoline, price increases tend to be destabilizing.

Model Predictions

Table 2(Appendix) provides a summary of the results of the weekly volatility forecasts. Table 3(Appendix) captures their subsequent root mean squared errors. It is important to mention the root mean squared error is the evaluation criterion for forecasting accuracy, where the model that produces the smallest error is said to have superior performance.

Lastly, as noted by Zivot (2008), it is standard practice to exclude standard errors for volatility forecasts as the errors would prove too noisy causing problems in interpreting the forecasts. A plot of the in-sample volatility data, however, is provided in Figure 1 in the appendix.

Discussion

The tabled results (Appendix) confirm the asymmetric models, the EGARCH, APARCH and GJR-GARCH provide better volatility forecasts than the standard GARCH. Given these models typically differed from the standard GARCH with the inclusion of the leverage parameter, it can be seen the leverage effect is particularly important. As it applies to gasoline prices, this confirms gasoline prices tend to be more volatile as they rise and continue to be volatile for prolonged periods.

It also becomes clear the EGARCH specification performs best for longer term forecasts, followed closely by the GJR-GARCH model. The worst performing of the four is the standard GARCH model. Expanding on the model ranking, the finding the EGARCH performs best also indicates that there is a persistence in gasoline price volatility although not as strong as the GJR-GARCH.

[7]Econometric literature typically refers to heavy tailed distributions in terms of how it compares to the Normal distribution. Heavy usually means heavier than the normal distribution.

This persistence then means that gasoline prices are driven by long-run dynamics, attributable to structural factors affecting the market, with the occasional bursts of increased price activity.

These results should not be overstated as the EGARCH does not significantly outperform the other asymmetric models. Additionally, all models including the standard GARCH also tend to overpredict volatility at the longer forecasts.

Part II

Extreme Gasoline Price-Change Forecasting

As noted in the introduction, another goal of the paper is to assess the predictability of extreme gasoline price increases. Before then, it would be useful to describe what classifies a price increase as a spike. For this paper, spikes are price increases larger than the mean price change by 2 standard deviations also known as the 95% quantile.[8] Since this part of the paper is concerned with the tail behavior of gasoline price increases, this means it is concerned with the top 5% of highest price increases. This is important as it could be seen from Figure 2(Appendix) things often go wrong in the gasoline market. Since these events occur with surprising regularity, it is important for the consumer who has no other choice than to bear the price increase to have the right reaction. Of course, the right reaction is dependent on one's ability to predict them.

Using a similar approach to Paraschiv et. al, the Generalized Pareto distribution (GPD), also known as the Peaks-Over-Threshold(POT) approach, is assessed for its ability to forecast extreme gasoline price increases.

Extreme Value Theory

Peaks over Threshold

Before making any predictions, it would be useful to briefly cover the POT approach. This is important as it provides some clarity on why this statistical tool is being used. To begin, the dominant POT approach, supposes when we are far out in the tails of a distribution, like returns for an asset, we can obtain a probability distribution for extremely large observations. This probability distribution is often referred to as the Generalized Pareto Distribution(GPD). Following Kevin Dowd (2008), the GPD is given by

$$G_{\xi,\beta}(x) = \begin{cases} 1 - (1 - \xi \frac{x}{b})^{\frac{-1}{\xi}}, \xi \neq 0 \\ 1 - (1 - exp(\frac{x}{b})), \xi = 0 \end{cases} \tag{6}$$

where $x \geq= 0$, $\xi \geq -0$ and $0 \leq x \leq \frac{-\beta}{\xi}$ for $\xi \leq 0$.

x here is assumed to be an i.i.d random variable, β the positive scale parameter of the distribution and ξ the tail index, that can be positive, zero or negative. In plain language, β is the dispersion or spread of the distribution and ξ how heavy the tail of the distribution is.[9] For researchers, the case often of interest is where $\xi \geq 0$ since this is associated with the data being heavy tailed. Generally speaking, for price changes/returns data, it is always assumed the data we are working with is heavy-tailed.

[8] Again, quantile is used for consistency with the literature.

[9] Again, heavy in this case means heavier than the normal distribution.

The result that extremely large observations follow the GPD is useful because it tells us regardless of the underlying distribution of x, where x in this case is the gasoline price change series, the distribution of its tails stays the same. This then results in a simple analytic formula for V, the value we are fairly sure price increases will not exceed with a confidence level of $1-\alpha$, where alpha is our significance level. Like any statistical test, α is normally chosen to be 5%.

Before estimating V however, a predefined value c is chosen, which is often referred to in literature as the threshold. Often, c is chosen to be the mean, μ, of the data. Following Carol Alexander (2008), the analytic formula for V is given by:

$$V = \mu + \frac{\beta}{\xi}[(\frac{N}{n_\mu}(1-\alpha))^{-\xi} - 1], \tag{7}$$

where N is the number of sample data points, and n_μ the number of observations exceeding sample's average μ.

Simplifying the above equation, the Peaks-Over-Threshold roughly speaking says for any data set, the most extreme value at the 95th quantile is given by its average plus some term relating how spread/dispersed the data is to how heavy tailed it is.

Data

As a reminder, the data being used is the weekly data obtained from the Energy Information Administration (EIA) where the observations run from to April 4, 2016 to May 16, 2022, on the all grades gasoline price series. It should be noted POT, like most extreme value methods, is structured to generate forecasts just a single time-step ahead. Its validity then is normally assessed by making rolling predictions over a specified period. This means its accuracy is determined by how well it predicts over a period and not just by assessing a single forecast.

The model is first fitted on the in-sample estimates of gasoline price changes prior to July 8, 2019 to produce an extreme price increase forecast. The in-sample estimates contain 150 weekly values. The model is then shifted forward by a single data point, each time, to make the next extreme forecast. This is done until the end of the data series(May 16, 2022). The forecasting window has a total of 168 forecasts.[10] A plot of the forecasts versus the underlying price change is supplied in Figure 3 (Appendix) with the forecasts as the dashed line above.

Methodology

Goodness of fit

To assess how good these forecasts are, the formal unconditional coverage and independence statistical tests are usually employed. Hence it would be useful to cover briefly what these tests capture.

(i) Unconditional Coverage Test

Intuitively, the unconditional coverage test, also known in literature as the Kupiec test, seeks to identify whether the observed frequency of tail values is consistent with that predicted by the

[10]While confusing, note a single forecast is produced using the previous 150 data points. We get to 168 forecasts because the part of the in-sample data we use for the prediction is constantly updated.

model. In particular, under the null hypothesis, a model is 'good'(consistent with the data) if only 5 of the data exceeds our forecast.[11] As Kevin Dowd (2008) shows, it supposes given 'n', where n is the number positive or negative price changes/returns in the data, the predicted frequency of tail values is given by the following probability:

$$Pr(x|n,p) = \binom{n}{x} p^x (1-p)^{n-x} \tag{8}$$

where p = 1 - α, our confidence level, and x = the number of extreme price changes/returns.[12]

The above equation is simply the binomial distribution. Thus, testing if a model gives us the expected number of observations that exceed the forecasts amounts to a binomial test. Since the unconditional coverage test is effectively a binomial test, it assesses whether or not the true frequency of extreme price changes is equal to $1-\alpha$.[13] The null hypothesis is given by $H_0 : p = \alpha$ which predicts only np observations - in the forecasting window - should exceed our forecasts. For this paper this means we expect only $0.05 * 168 = 8.4 \approx 8$ data points in the forecasting window to exceed our forecasts. The alternative hypothesis is however given by $H_a : p \neq \alpha$.

By this test, a model is bad if it does not generate the expected number of values that exceed our forecast. For this paper, a model is bad if the actual number of observations - in the forecasting window - that exceeds the forecasts is statistically different from 8. On this basis, we reject the null hypothesis when $p < \alpha$.

(ii) Independence Test

It should be mentioned the unconditional coverage test takes into account all the forecasts made in an out-of-sample environment. The independence test differs in the sense that it assesses each forecast independently with independence defined in the following way: that the observations that exceed the forecasts are independent of each other. This test is essentially then a Chi-squared test which assesses whether two forecasts are related to each other or not. With this test, a model is bad if the observations that exceed our forecast show dependence which often means they cluster. Following the logic of the chi-squared test, a model is bad if the forecasts are related to each other.

While not shown here for convenience, Kevin Dowd(2008) provides a full exposition of the test statistic. It should be noted the test statistic amounts to a likelihood ratio(LR) that follows a $\chi^2(1)$ distribution. And like the binomial test, a model is rejected if the p-value of the likelihood ratio(LR) is less than alpha. That is $H_0 : p = \alpha$ is rejected in favor of $H_a : p \neq \alpha$.

Results

In conducting the unconditional coverage aka the binomial test, a p-value of 2.568679e-07 is recorded. This p-value is negligible which means that $H_0 : p = 0.05$ is rejected in favor of $H_a : p \neq 0.05$. This is confirmed with only 5 observations exceeding our forecasts which according the test, differs from what was expected, 8. We get our first evidence then that the model is very bad at predicting extreme gasoline price increases.

In conducting the independence aka the chi-squared test, a p-value of 1.338619e-07 is recorded.

[11] Note 5% here is our chosen significance level, α

[12] For this paper, n is the number of positive changes

[13] Again, for this paper the extremes are just the extreme increases.

Similar to the unconditional coverage test, this p-value is negligible which means $H_0 : p = 0.05$ is rejected in favor of $H_a : p \neq 0.05$. The combination of both results thus show POT is not well suited to modeling extreme gasoline price increases.

Discussion

It is hard to say why the POT approach failed. Perhaps, as noted by Nair et al.(2022), its failure boils down to its assumption of having an i.i.d sample. Even more restricting, however, is the assumption the distribution of the tails of the data always exactly follow the Generalized Pareto Distribution. Should the data have a highly dependent structure as has been documented for gasoline prices, any attempt to use this technique to predict extreme price movements would lead to highly misleading results.

Conclusion

The first part of the paper was concerned with modeling gasoline price volatility. It was found in the forecasting exercise that the EGARCH model tends to perform best among the examined GARCH models. More broadly, it seems all the models that allow for asymmetric effects(i.e, all models but the plain GARCH) tend to also perform better at longer-term forecasts. Indeed, the result of that EGARCH is the best predictor also provides evidence that gasoline prices are driven by main long-run dynamics with the occasions bursts of increased price activity.

The second part of the paper was concerned with modeling the extreme price spikes where it was found the Peaks-Over-Threshold method is not suitable for predicting those large spikes. Thus a challenge still remains in finding a robust model to capture the extremes of gasoline price movements.

While both parts were examined distinctly, it is possible they are related with extreme price increases being strongly related to the volatility of gasoline price movements. In that case, conditional extreme value theory may provide better results as it would account for volatility affecting the extreme price movements. Then again, that would be an exercise for future research.

References

Fan, J. and Zhang, W. (1999): Additive and varying coefficient models - statistical estimation in varying coefficient model. The Annals of Statistics 27, 1491- 1518.

He, C., Terasvirta, T. and Malmsten, H. (2002): Moment Structure of a Family of First-Order Exponential Garch Models. Econometric Theory, 18, 868-885

Glosten, L. R., R. Jagannathan, and D. E. Runkle, 1993. On The Relation between The Expected Value and The Volatility of Nominal Excess Return on stocks. Journal of Finance 48: 1779-1801. https://www.jstor.org/stable/2329067

Nelson, D. B., 1991. Conditional Heteroskedasticity in Asset Returns: A New Approach. Econometrica 59: 347-370. https://www.jstor.org/stable/2938260

Ding, Z., Granger C.W., Engle R.F., 1991. A Long Memory Property of Stock Market Returns and a New Model. Journal of Empirical Finance 1 (1993) 83-106. https://doi.org/10.1016/S0304-4076(97)00072-9

Bollerslev, T., 1986. Generalized Autoregressive Conditional Heteroskedasticity. Journal of Econometrics 31: 5- 59. https://www.sciencedirect.com/science/article/pii/0304407686900631

Alexander, Carol. Market Risk Analysis. Chichester, England.; Hoboken, NJ: Wiley, 2008.

Dowd, Kevin. Measuring Market Risk. 2. ed. repr. Chichester: Wiley, 2008.

Glosten, L. R., R. Jagannathan, and D. E. Runkle, 1993. On The Relation between The Expected Value and The Volatility of Nominal Excess Return on stocks. Journal of Finance 48: 1779-1801. https://www.jstor.org/stable/2329067

Nelson, D. B., 1991. Conditional Heteroskedasticity in Asset Returns: A New Approach. Econometrica 59: 347-370. https://www.jstor.org/stable/2938260

Bollerslev, T., 1986. Generalized Autoregressive Conditional Heteroskedasticity. Journal of Econometrics 31: 5- 59. https://www.sciencedirect.com/science/article/pii/0304407686900631

Ding, Z., Granger C.W., Engle R.F., 1991. A Long Memory Property of Stock Market Returns and a New Model. Journal of Empirical Finance 1 (1993) 83-106. https://doi.org/10.1016/S0304-4076(97)00072-9

Edgeworth, F. 1925. The pure theory of monopoly. Papers Relating to Political Economy 1, 111–142.

Gicheva, D. Hastings, J. and Villas-Boas, S.B. 2007. Revisiting the income effect: Gasoline prices and grocery purchases. Working Paper 13614, NBER.

Jean-François Houde. “Gasoline markets”, The New Palgrave Dictionary of Economics, Steven N. Durlauf and Lawrence E. Blume, Eds, Palgrave Macmillan, 2010.

Byström, H. N. (2005). Extreme value theory and extremely large electricity price changes. International Review of Economics & Finance, 14(1):41–55.

Paraschiv, F., Hadzi-Mishev, R., and Keles, D. (Forthcoming 2016). Extreme value theory for heavy-tails in elec- tricity prices. Journal of Energy Markets.

Computational Statistics and Data Analysis 51(4),. 2232–2245. Brownlees, C., Engle, R., and Kelly, B. (2011). A practical guide to volatility forecasting.

Eric Zivot, 2008. “Practical Issues in the Analysis of Univariate GARCH Models,” Working Papers UWEC-2008-03-FC, University of Washington, Department of Economics.

Nair, Jayakrishnan, Adam Wierman, and Bert Zwart. The Fundamentals of Heavy Tails: Properties, Emergence, and Estimation. Cambridge, United Kingdom: Cambridge University Press, 2022.

Appendix(Tables and Figures)

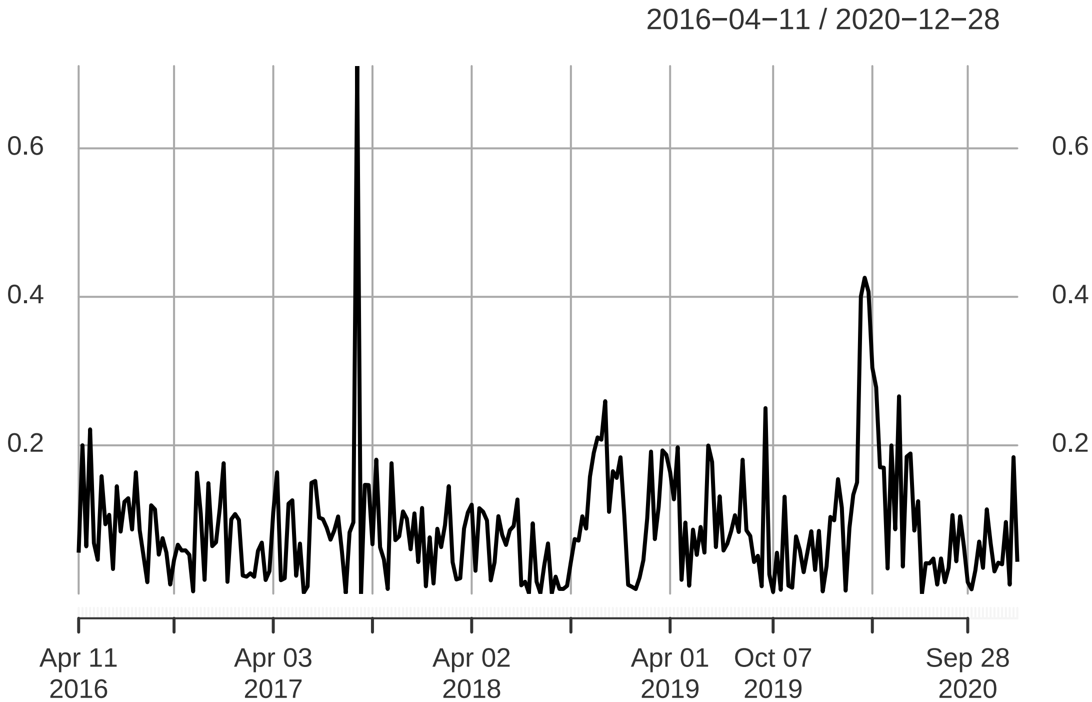

Figure 1: Realized/Actual Volatility of Gasoline Price-Changes (Volatility is Annualized)

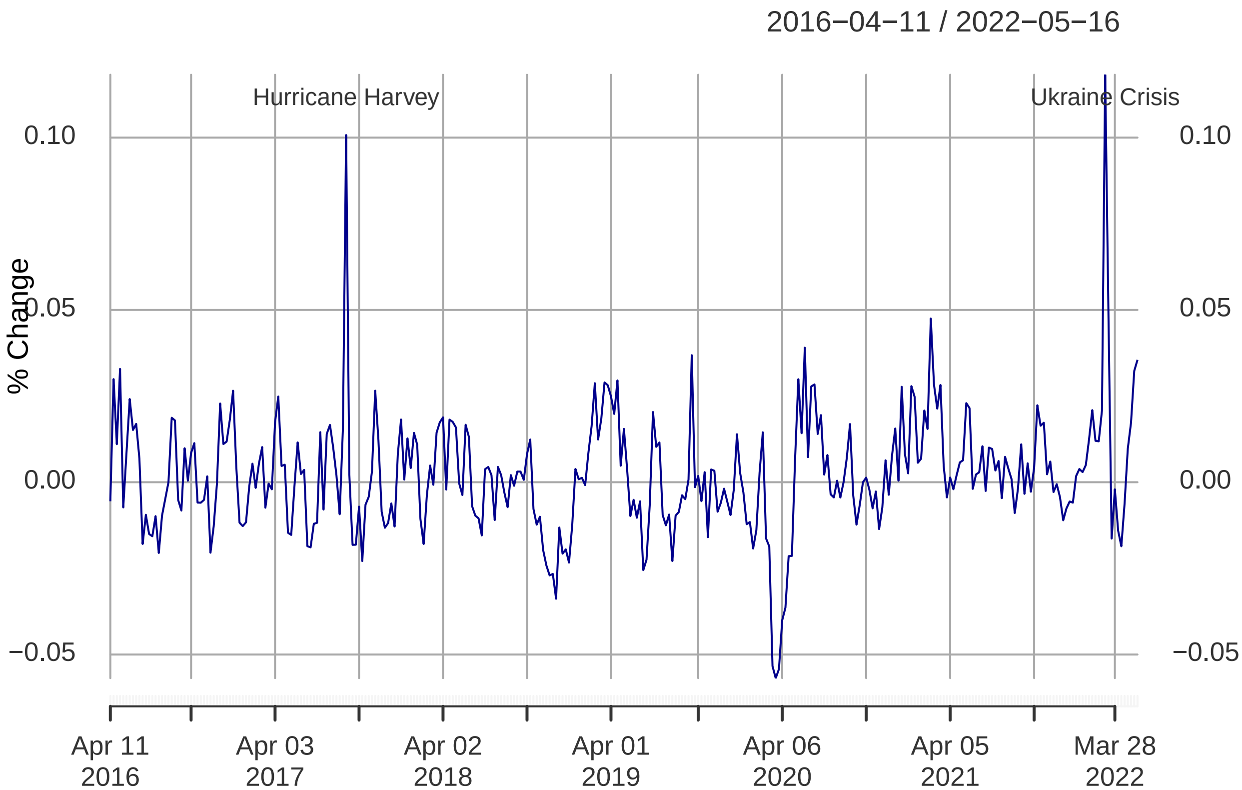

Figure 2: Weekly Percentage Change in Gasoline Prices from 2016-2022

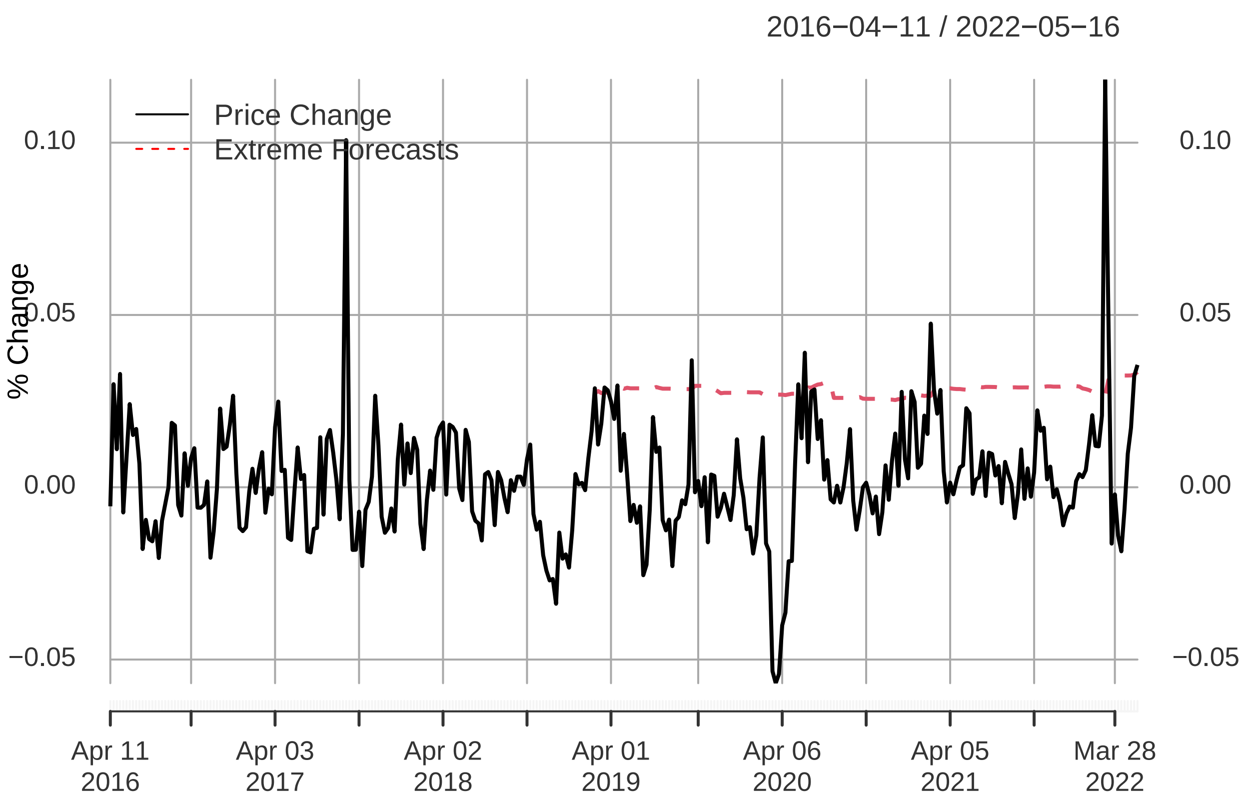

Figure 3: Extreme Price-Increase Forecasts versus Price Change/Returns

Table 1: Parameter estimates , information criteria and log-likelihoods for GARCH models

MODEL:	CONSTANT ω	α	β	γ	δ	AIC	LOG-LIKELIHOOD
GARCH (1,1)	0.000083 (0.00003)***	0.553 (0.163) ***	0.289 (0.154)**	-	-	-5.578	**691.055**
EGARCH (1,1)	-2.421 (0.744)***	-0.226 (0.069) ***	0.706 (0.116) ***	0.553 (0.028)***	-	-5.586	**693.0418**
APARCH (1,1)	0	0.206 (0.076)***	0.130 (0.072)*	0.609 (0.118)***	3.759 (0.061)***	-5.598	**695.50**
GJR-GARCH (1,1)	**0.000114 (0.000054)**	**0.315 (0.132)****	**0.257 (0.231)****	**0.538 (0.325)**	-	**-5.603**	**695.12**

*90% Confidence Level **95% Confidence Level ***99% Confidence Level

Table 2: Volatility Forecasts

Index	GARCH	EGARCH	APARCH	GJR-GARCH	Actual/Realized Vol.
2021-01-25	0.1374	0.1085	0.1192	0.1148	0.0251
2021-02-22	0.1529	0.1146	0.1265	0.1183	0.3269
2021-03-22	0.1600	0.1161	0.1282	0.1187	0.0185
2021-04-19	0.1635	0.1165	0.1286	0.1187	0.0010
2021-05-17	0.1652	0.1166	0.1287	0.1187	0.1393
2021-06-14	0.1661	0.1166	0.1288	0.1187	0.0596
2021-07-12	0.1665	0.1166	0.1288	0.1187	0.0089
2021-08-09	0.1667	0.1167	0.1288	0.1187	0.0130
2021-09-06	0.1668	0.1167	0.1288	0.1187	0.0637
2021-10-04	0.1669	0.1167	0.1288	0.1187	0.0151

Table 3: Root Mean Squared Error

Index	GARCH	EGARCH	APARCH	GJR-GARCH
2021-01-25	0.1124	0.0834	0.0941	0.0897
2021-02-22	0.1740	0.2123	0.2004	0.2085
2021-03-22	0.1415	0.0976	0.1097	0.1002
2021-04-19	0.1626	0.1156	0.1277	0.1178
2021-05-17	0.0259	0.0227	0.0106	0.0206
2021-06-14	0.1064	0.0570	0.0691	0.0591
2021-07-12	0.1576	0.1077	0.1198	0.1098
2021-08-09	0.1537	0.1036	0.1157	0.1057
2021-09-06	0.1032	0.0530	0.0651	0.0550
2021-10-04	0.1518	0.1016	0.1137	0.1036