Can the Longer-Term Volatility of U.S Gasoline Prices be Predicted?

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Abstract:

Gasoline retail markets have traditionally attracted a lot of attention from researchers and policy makers. This paper reviews 2 sets of questions studied by economists related to this market who seek to understand the behavior of their prices. The first is concerned with whether volatility can be predicted at longer horizons. To investigate volatility's predictability, a comparative study within the autoregressive conditional heteroskedasticity (ARCH) class of models is presented. For the period studied (2016-2022), it is found model rankings are insensitive to the forecast horizon with asymmetric volatility models providing the best forecasts.

The second focuses on whether the cyclical spike in gasoline prices can be predicted. In particular, it is widely documented gasoline prices generally start to rise in the spring, just before the start of the summer driving season, with a price spike often at the beginning of the rise. The goal here is to assess the value of a popular probabilistic framework, the Peaks over Threshold method, from Extreme Value theory in predicting these extreme price spikes.

Introduction

Ever since the start of the Covid-19 crisis, there has been renewed interest by practitioners and academics alike in reassessing the adequacy of gasoline price models. Soaring volatility especially in early 2022 has made it important to know how well our standard tools forecast volatility amid episodes of turmoil that pervade all corners of the economy. Volatility prediction is a critical task for the retail gasoline market as volatility often manifests itself in rampant price increases. Consumers are greatly concerned with gasoline price movements, as it occupies between 4.5% and 12.4% of households' disposable income, especially for poorer households. (Gicheva et al., 2007).

In this paper, it is explored the performance of volatility forecasting within the General Autoregressive Conditional Heteroskedastic (GARCH) class of models. In particular, the paper examines the GARCH(1,1), EGARCH(1,1), GJR-GARCH(1,1) and APARCH(1,1) to determine which volatility model provides the best longer-dated forecasts of gasoline price movement. While all considered models exhibit significant deterioration in volatility forecast accuracy as the forecasting horizons lengthens, it turns out, as this paper will report, the rate of deterioration among the models differ with the EGARCH (1,1) providing the least deterioration in accuracy and hence the best volatility forecasts.

The second half of the paper examines a related phenomenon that has gained a lot of attention in recent times – the increasing frequency of gasoline price hikes in 2022. As documented by Jean-Fancois Houde(2010), gasoline prices in many cities follow easily predictable asymmetric cycles akin to Edgeworth cycles (Edgeworth, 1925): price increases are fast and large (relenting phase), and are followed by a sequence of small decreases (undercutting phase). Literature, however, on predicting the extreme gasoline price behavior is lacking which suggests that the tail behavior of gasoline prices is not well understood.

The contribution of this part of the paper is to assess the accuracy of a popular probabilistic framework, the Peaks over Threshold method, from Extreme Value theory in predicting those tail behaviors. The analysis reveals this approach is not well-suited to predicting those large price increases. A suggestion is made, however, on improving the framework for further research.

The remainder of the paper is structured as follows, Literature Review, Part I - Volatility Forecasting, Part II - Extreme Gasoline Price-Change Forecasting, and Conclusion.[1] There are subsections under each part with each containing a Methodology, Empirical Evaluation, and Discussion

[1]The goal in Part II is to forecast the extremes of price movements not price itself.

section.

Literature Review

This paper can be placed into two main research areas in the field of predictive econometric analysis: (i) modeling of volatility and (ii) modeling of spike occurrences where there exists an extensive amount of research. Yet, the literature seems to be almost always be concerned with the financial markets and less so for the gasoline retail markets. As it relates to volatility forecasting, C. Brownlees et al.(2011) find across asset classes and volatility regimes, asymmetric GARCH models often are the best forecasters. More so, they document the threshold GARCH of Glosten et al. (1993) is often the best forecaster. While the threshold Garch is not considered, it is of concern in this paper to establish which asymmetric GARCH model performs best for gasoline price volatility forecasts among the GARCH(1,1), EGARCH(1,1), GJR-GARCH(1,1) and APARCH(1,1).

As it relates to extreme price behavior, different modeling techniques have been applied to capture the distribution of extreme price movements. Bystrom (2005) and Paraschiv et al. (2016) investigate the performance of Extreme Value Theory(EVT) on accurately modeling and forecasting the extreme tails of electricity price distributions in the Nord Pool Electricity Market.[2] Paraschiv (2016) in particular, obtains robust forecasts of extreme quantiles using the Generalized Pareto distribution (GPD), also known in literature as the Peaks-Over-Threshold method.[3] This paper attempts to confirm if this approach would work for modeling the spikes in the US retail gasoline markets.

Part I

Volatility Forecasting

[2] While a bit of a mouthful, what is meant is the price change distribution not prices in itself.

[3] While confusing, the quantile is really just the same thing as the percentile. For consistency with the econometric literature that deals with EVT, the paper uses quantiles.

Methodology:

Models Considered

The four models considered are chosen from the vast literature on GARCH modeling for their simplicity and demonstrated ability to forecast over alternatives. Of course, simplicity is taken to mean information about the volatility of the gasoline price change series is contained in the series' history and not dependent on exogenous variables.[4]

All four models describe the return/ price change time series as

$$r_t = \mu + \xi_t \tag{1}$$

where, r_t is the return time series, μ is the expected return, and ξ_t is a zero-mean white noise.[5] They also denote $\xi_t = \sigma_t z_t$ where z_t is standard Gaussian.[6]

The first, GARCH(1,1) by Bollerslev(1986) provides a natural starting point. The volatility process is described by the GARCH (1,1) model as:

$$\sigma_t^2 = w + \alpha\xi_{t-1}^2 + \beta\sigma_{t-1}^2 \tag{2}$$

The key feature of the GARCH volatility process is it says current of volatility is related to its immediate past value and a past squared random noise term. Since r_t is given by a constant, μ, plus a random noise term, the squared random noise term is often replaced with r_t. Another key feature is it captures shocks as low-persistent - which means that volatility shocks - both past and current - influence the future volatility ever so slightly.[7] Most importantly, however, it dictates a

[4] While plausible other indicator variables may help explain some of the dynamics of the volatility of gasoline price changes, the models used to represent such interactions are generally much more complicated.

[5] i.e, the series ξ_t is independent and identically distributed with a mean of zero.

[6] i.e. Standard Normal

[7] An easier way to think about persistence is how fast volatility mean-reverts. Low persistence means it takes a short time for volatility to return to its normal level. High persistence means it takes a long time for volatility to return to its normal level. Moderate persistence however, means the time taken to mean-reversion lies between the two extremes.

symmetric impact of past returns on future volatility. (i.e, the magnitude of past returns and not the algebraic sign influences future volatility).

The second, the exponential GARCH(1,1) or EGARCH(1,1) by Nelson (1991) describes the volatility process as:

$$ln(\sigma_t^2) = w + \alpha(|z_{t-1}| - E|z_{t-1}|) + \gamma|z_{t-1}| + \beta ln(\sigma_t^2) \tag{3}$$

Two key features of the EGARCH volatility process are that it captures shocks as moderately persistent and also captures the leverage effect with $\gamma < 0$. This leverage effect means the algebraic sign/direction of the return, not just its magnitude influences future volatility. Of course, moderate persistence means volatility shocks influence future volatility for a noticeable although not lengthy period of time.

The third, the asymmetric Power Arch or APARCH(1,1) by (Ding et al 1993) describes the volatility process as:

$$\sigma_t^\delta = w + \alpha(|\xi_{t-1}| - \gamma\xi_{t-1})^\delta + \beta\sigma_{t-1}^\delta \tag{4}$$

The key feature of the APARCH is it captures a greater persistence in shocks than the GARCH, E-GARCH or GJR-GARCH. As Ding et al. (1993) show, this persistence is related to the serial correlation of absolute returns.[8] It also incorporates the leverage effect via γ.

The fourth, Glosten-Jagannathan-Runkle GARCH(GJR-GARCH) by (Glosten et al 1990) describes the volatility process as:

$$GJR - GARCH : \sigma_t^2 = w + (\alpha + \gamma I_{t-1})\xi_{t-1}^2 + \beta\sigma_{t-1}^2, \begin{cases} I = 0, r_t \geq \mu \\ I = 1, r_t < \mu \end{cases} \tag{5}$$

[8] Remember serial correlation describes the relationship between a given variable and its lagged version over various time intervals.

The key feature of the GJR-GARCH is like the simple GARCH, shocks have low-persistence. The GJR-GARCH however, also incorporates the leverage effect where $\gamma > 0$.

Empirical Evaluation

Data

To implement the longer-term volatility forecasts, weekly data is obtained from the Energy Information Administration (EIA) where the observations run from to April 4, 2016 to May 16, 2022, on the all grades gasoline price series. For the purposes of fitting the models in an in-sample environment however, only a subset of the volatility series from April 4, 2016 to December 28, 2020 is used. The price series is log-differenced to get the price change series continuously compounded over time. Further, for the purposes of volatility modelling, the series is demeaned.[9] Some summary statistics of the in-sample data are presented in table 1.

Table 1: Central Moments of the Gasoline Price Change Series

	Summary Statistics
Mean	0.0000000
Variance	0.0002765
Skewness	0.0000027
Kurtosis	0.0000006

Modelling

Prior to fitting the model, it would be important to acknowledge the widely documented heavy tails for price change/returns. This means an adjustment is made to the term z_t in the error/innovation term ξ_t such that it is no more assumed to be Gaussian but rather follows a heavy-

[9]That is, the sample mean is subtracted from each price change observation so that the overall series is mean zero.

tailed distribution, like the Generalized Error Distribution (GED).[10] Forecasts are then produced and evaluated only on the out-of-sample 2021 volatility estimates. There are are 10 forecasts produced and a rolling 4-week basis. That is, the forecasts are made after every 4 weeks(from week 4 to week 40). This translates to forecasts from January 25,2021 to October 4, 2021. The results are annualized however.

Results

Table 2 reports the parameter estimates of all the GARCH class of models employed as well as their information criteria and the log-likelihood values.[11] The significance level of the coefficients are reported as well.

The AIC of the models are similar suggesting the models provide similar in-sample fits. Except for the GJR-GARCH, the asymmetric models have a significant leverage parameter γ which suggests the presence of leverage effects. The positivity of their leverage parameter however, suggests gasoline prices are more volatile during price increases than price decreases. Put simply for gasoline, price increases tend to be destabilizing.

Table 2: Parameter estimates , information criteria and log-likelihoods for GARCH models

MODEL:	**CONSTANT** $\boldsymbol{\omega}$	$\boldsymbol{\alpha}$	$\boldsymbol{\beta}$	$\boldsymbol{\gamma}$	$\boldsymbol{\delta}$	**AIC**	**LOG-LIKELIHOOD**
GARCH (1,1)	0.000083 (0.00003)***	0.553 (0.163) ***	0.289 (0.154)**	-	-	-5.578	**691.055**
EGARCH (1,1)	-2.421 (0.744)***	-0.226 (0.069) ***	0.706 (0.116) ***	0.553 (0.028)***	-	-5.586	**693.0418**
APARCH (1,1)	0	0.206 (0.076)***	0.130 (0.072)*	0.609 (0.118)***	3.759 (0.061)***	-5.598	**695.50**
GJR-GARCH (1,1)	**0.000114 (0.000054)**	**0.315 (0.132)****	**0.257 (0.231)****	**0.538 (0.325)**	**-**	**-5.603**	**695.12**

*90% Confidence Level **95% Confidence Level ***99% Confidence Level

[10] Econometric literature typically refers to heavy tailed distributions in terms of how it compares to the Normal distribution. Heavy usually means heavier than the normal distribution.

[11] The information criterion reported here is the Akaike information criterion.

Predictions

Table 3 provides a summary of the weekly volatility forecasts. Table 4 captures their root mean squared errors. It is important to mention the root mean squared error is the evaluation criterion for forecasting accuracy, where the model that produces the smallest error is said to have superior performance.

Lastly, as noted by Zivot (2008), it is standard practice to exclude standard errors for volatility forecasts as the errors would prove too noisy causing problems in interpreting the forecasts.[12] A plot of the in-sample volatility data, however, is provided in Figure 1.

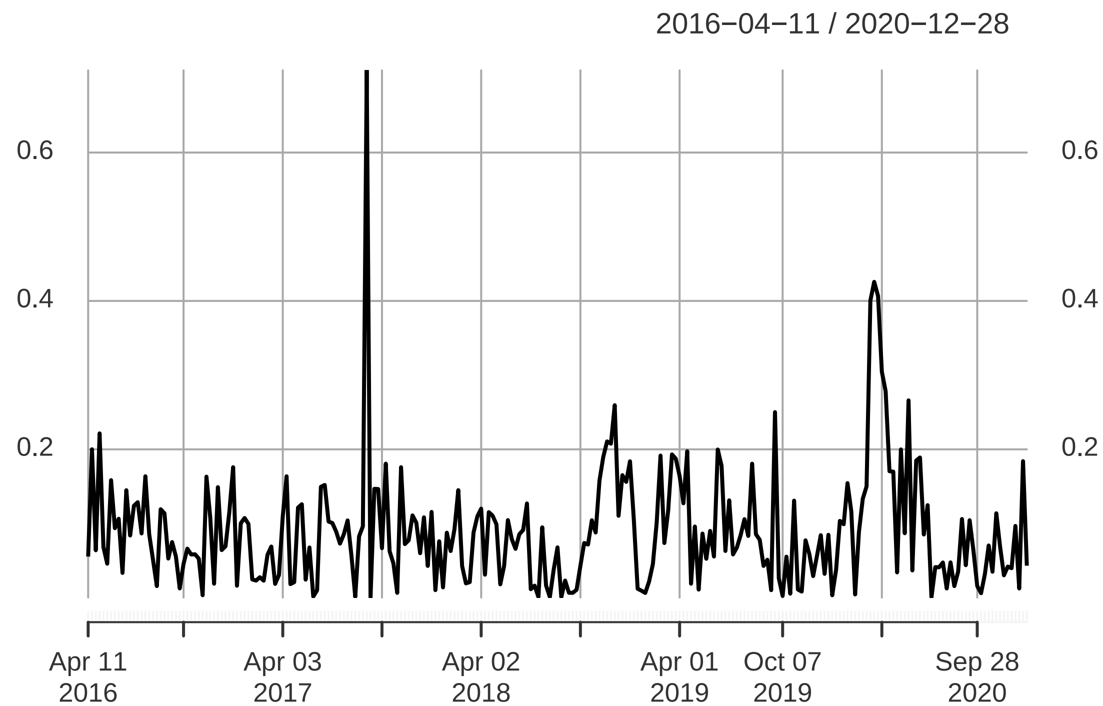

Figure 1: Realized/Actual Volatility of Gasoline Price-Changes (Volatility is Annualized)

Table 3: Volatility Forecasts

[12]This is true for two main reasons. First, standard errors measure the level of error variance - in this case, the so-called volatility of volatility (vol-of-vol). Put another way, it measures the standard deviation of residual volatility. This makes vol-of-vol a measure for assessing the performance of high volatilities since high volatilities tend to produce greater residuals, meaning the measure is limited in its ability to assess the value of a volatility prediction when volatility is low. What would be a better measure is if it measured the proportional errors over periods of both high and low volatility. Second, since volatility is assumed to be heteroskedastic, meaning it changes over time, standard errors would even be much more heteroskedastic leading to inaccurate assessments of the predicted range.

Index	GARCH	EGARCH	APARCH	GJR-GARCH	Actual/Realized Vol.
2021-01-25	0.1374	0.1085	0.1192	0.1148	0.0251
2021-02-22	0.1529	0.1146	0.1265	0.1183	0.3269
2021-03-22	0.1600	0.1161	0.1282	0.1187	0.0185
2021-04-19	0.1635	0.1165	0.1286	0.1187	0.0010
2021-05-17	0.1652	0.1166	0.1287	0.1187	0.1393
2021-06-14	0.1661	0.1166	0.1288	0.1187	0.0596
2021-07-12	0.1665	0.1166	0.1288	0.1187	0.0089
2021-08-09	0.1667	0.1167	0.1288	0.1187	0.0130
2021-09-06	0.1668	0.1167	0.1288	0.1187	0.0637
2021-10-04	0.1669	0.1167	0.1288	0.1187	0.0151

Table 4: Root Mean Squared Error

Index	GARCH	EGARCH	APARCH	GJR-GARCH
2021-01-25	0.1124	0.0834	0.0941	0.0897
2021-02-22	0.1740	0.2123	0.2004	0.2085
2021-03-22	0.1415	0.0976	0.1097	0.1002
2021-04-19	0.1626	0.1156	0.1277	0.1178
2021-05-17	0.0259	0.0227	0.0106	0.0206
2021-06-14	0.1064	0.0570	0.0691	0.0591
2021-07-12	0.1576	0.1077	0.1198	0.1098
2021-08-09	0.1537	0.1036	0.1157	0.1057
2021-09-06	0.1032	0.0530	0.0651	0.0550
2021-10-04	0.1518	0.1016	0.1137	0.1036

Discussion

The tabled results confirm the asymmetric models - the EGARCH, APARCH and GJR-GARCH - provide better volatility forecasts than the standard GARCH. Given these models typically differed from the standard GARCH with the inclusion of the leverage parameter, it can be seen the leverage effect is particularly important. As it applies to gasoline prices, this finding confirms gasoline prices tend to be more volatile as they rise.

It also becomes clear the EGARCH specification performs best for longer term forecasts, followed closely by the GJR-GARCH model. The worst performing of the four is the standard GARCH model. Expanding on the model ranking, the finding the EGARCH performs best also indicates that there is a persistence in gasoline price volatility although not as strong as the GJR-GARCH. This persistence then means volatility today in gasoline prices tends to continue for a considerable period into the future. Most importantly, this implies the volatility of gasoline price movement tends to be directionally persistent. Economically, this persistence implies gasoline price movements are driven by long-run dynamics, attributable to underlying structural market factors, with periodic bursts of increased price activity.

These results however, should not be overstated as the EGARCH does not significantly outperform the other asymmetric models. Additionally, all models including the standard GARCH also tend to overpredict volatility for lengthier prediction horizons.

Part II

Extreme Gasoline Price-Change Forecasting

As noted in the introduction, another goal of the paper is to assess the predictability of extreme gasoline price increases. Before then, it would be useful to describe what classifies a price increase as a spike. For this paper, spikes are price increases larger than the mean price change by 2 standard deviations also known as the 95% quantile.[13] Since this part of the paper is concerned with the tail

[13] Again, quantile is used for consistency with the literature.

behavior of gasoline price increases, this means it is concerned with the top 5% of price movement. This is important as it could be seen from Figure 2 things often go wrong in the gasoline market. Since these events occur with surprising regularity, it is important for the consumer who has no other choice than to bear the price increase to have the right reaction. Of course, the right reaction is dependent on their ability to predict them.

Using a similar approach to Paraschiv et. al, the Generalized Pareto distribution (GPD), also known as the Peaks-Over-Threshold(POT) approach, is assessed for its ability to forecast extreme gasoline price increases.

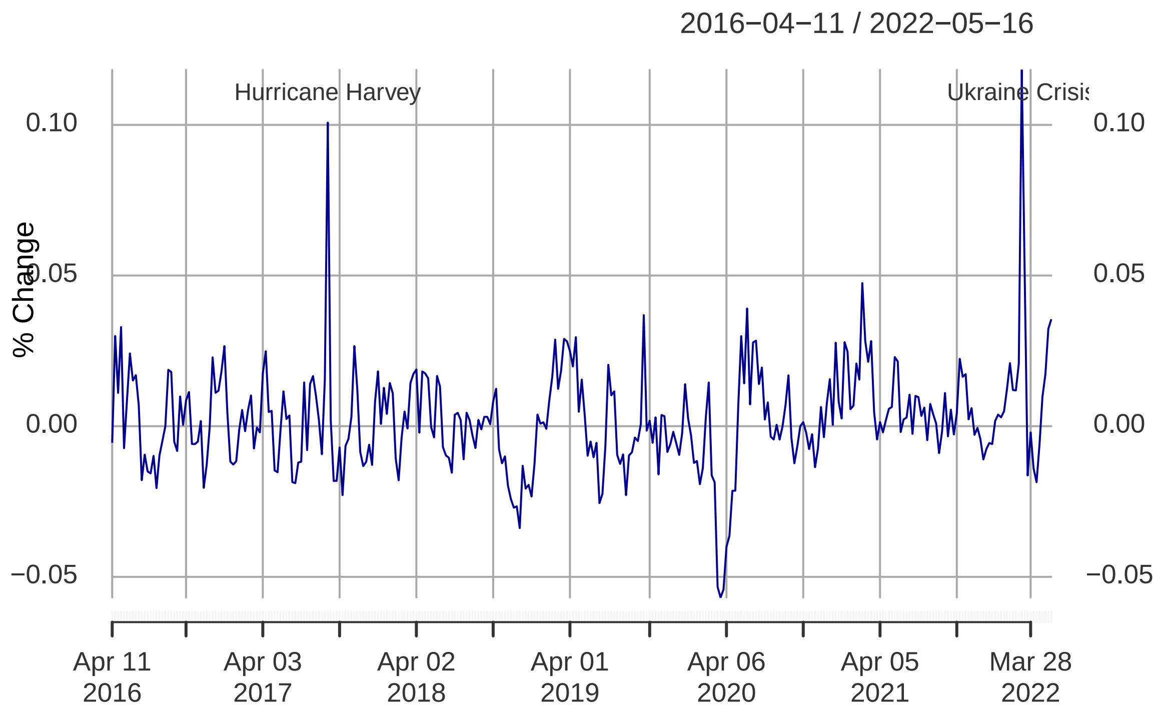

Figure 2: Weekly Percentage Change in Gasoline Prices from 2016-2022

Extreme Value Theory

Peaks over Threshold

Before making any predictions, it would be useful to briefly cover the POT approach. This is important as it provides some clarity on why this statistical tool is being used. To begin, the dominant POT approach, supposes when we are far out in the tails of a distribution, like the returns for an asset, we can obtain a probability distribution for extremely large observations. This probability distribution is often referred to as the Generalized Pareto Distribution(GPD). Following Kevin Dowd (2008), the GPD is given by

$$G_{\xi,\beta}(x) = \begin{cases} 1-(1-\xi\frac{x}{b})^{\frac{-1}{\xi}}, \xi \neq 0 \\ 1-(1-exp(\frac{x}{b})), \xi = 0 \end{cases} \tag{6}$$

where $x \geq= 0, \xi \geq -0$ and $0 \leq x \leq \frac{-\beta}{\xi}$ for $\xi \leq 0$.

x here is assumed to be an i.i.d random variable, β the positive scale parameter of the distribution and ξ the tail index, that can be positive, zero or negative. In plain language, β is the dispersion or spread of the distribution and ξ how heavy the tail of the distribution is.[14] For researchers, the case often of interest is where $\xi \geq 0$ since this is associated with the data being heavy tailed. Generally speaking, for price changes/returns data, it is always assumed the data we are working with is heavy-tailed.

The result that extremely large observations follow the GPD is useful because it tells us regardless of the underlying distribution of x, where x in this case is the gasoline price change series, the distribution of its tails stays the same. This then results in a simple analytic formula for V, the value we are fairly sure price increases will not exceed with a confidence level of $1-\alpha$, where alpha is our significance level. Like any statistical test, α is normally chosen to be 5%.

Before estimating V however, a predefined value c is chosen, which is often referred to in literature as the threshold. Often, c is chosen to be the mean, μ, of the data. Following Carol Alexander (2008), the analytic formula for V is given by:

$$V = \mu + \frac{\beta}{\xi}[(\frac{N}{n_{\mu}}(1-\alpha))^{-\xi} - 1], \tag{7}$$

where N is the number of sample data points, and n_{μ} the number of observations exceeding

[14] Again, heavy in this case means heavier than the normal distribution.

sample's average μ.

Simplifying the previous equation, the Peaks-Over-Threshold roughly speaking says for any data set, the most extreme value at the 95th quantile is given by its average plus some term relating the spread/dispersion of the data to its tailedness.

Empirical Evaluation

Data

As a reminder, the data being used is the weekly data obtained from the Energy Information Administration (EIA) where the observations run from to April 4, 2016 to May 16, 2022, on the all grades gasoline price series.

Modelling

Prior to modelling, it should be noted POT, like most extreme value methods, is structured to generate forecasts just a single time-step ahead. Its validity then is assessed by making rolling predictions over a specified period. This means its accuracy is determined by how well it predicts over a period and not just by assessing a single forecast.

The model is first fitted on the in-sample estimates of gasoline price changes prior to July 8, 2019 to produce an extreme price increase forecast. The in-sample estimates contain 150 weekly values. The model is then shifted forward by a single data point, each time, to make the next extreme forecast. This is done until the end of the data series(May 16, 2022). The forecasting window has a total of 168 forecasts.[15] A plot of the forecasts versus the underlying price change is supplied in Figure 3 with the forecasts as the dashed line above.

[15] While confusing, note a single forecast is produced using the previous 150 data points. We get to 168 forecasts because the part of the in-sample data we use for the prediction is constantly updated.

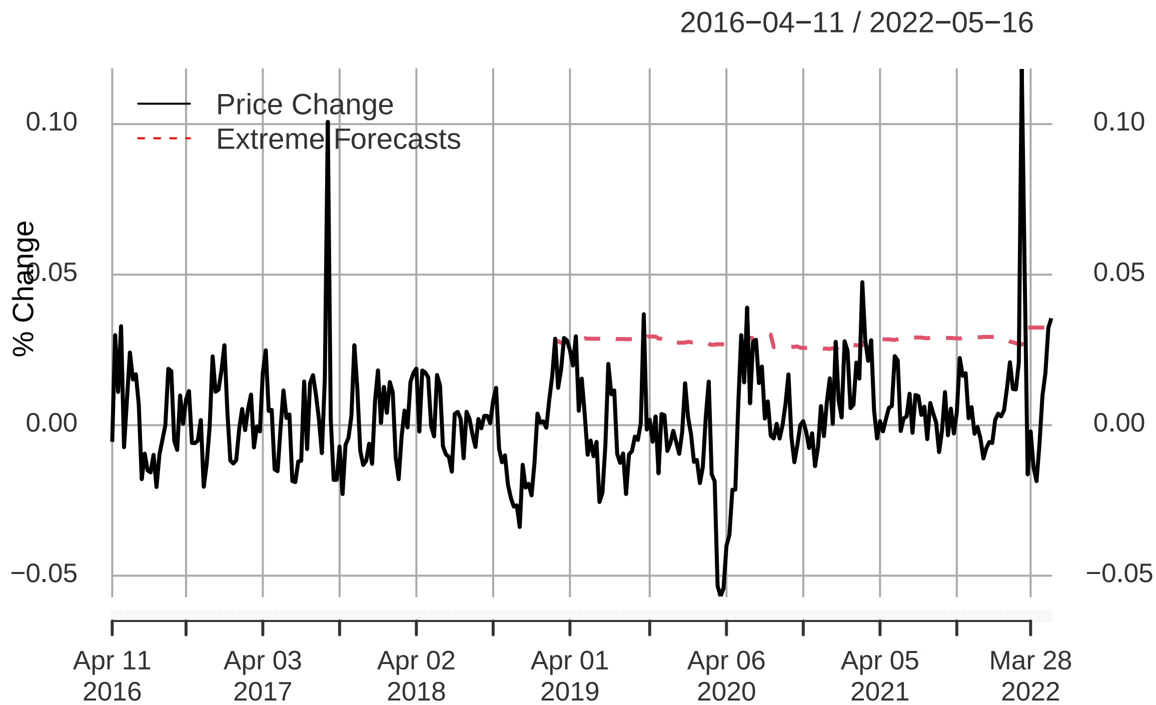

Figure 3: Extreme Price-Increase Forecasts versus Price Change/Return

Methodology

Goodness of fit

To assess how good these forecasts are, the formal unconditional coverage and independence statistical tests are usually employed. Hence it would be useful to cover briefly what these tests capture.

(i) Unconditional Coverage Test

Intuitively, the unconditional coverage test, also known in literature as the Kupiec test, seeks to identify whether the observed frequency of tail values is consistent with that predicted by the model. In particular, the null hypothesis supposes a model is 'good'(consistent with the data) if only 5% of the data exceeds our forecast.[16] As Kevin Dowd (2008) shows, it supposes given the predicted frequency of tail values is given by the following probability:

[16]Note 5% here is our chosen significance level, α.

$$Pr(x|n, p) = \binom{n}{x} p^x (1-p)^{n-x} \tag{8}$$

where p = 1 - α, our confidence level, n = the number positive or negative price changes/returns, and x = the number of extreme price changes/returns.[17]

Of course, the above equation is simply the binomial distribution. Thus, testing if a model gives us the expected number of observations that exceed the forecasts amounts to a binomial test.[18] Since the unconditional coverage test is effectively a binomial test, it assesses whether or not the true frequency of extreme price changes is equal to $1 - \alpha$.[19] The null hypothesis is given by $H_0 : p = \alpha$ which predicts only np observations - in the forecasting window - should exceed our forecasts. For this paper this means we expect only $0.05 * 168 = 8.4 \approx 8$ data points in the forecasting window to exceed our forecasts. The alternative hypothesis is however given by $H_a : p \neq \alpha$.

By this test, a model is bad if it does not generate the expected number of values that exceed our forecasts. For this paper, a model is bad if the actual number of observations - in the forecasting window - that exceeds the forecasts is statistically different from 8. On this basis, we reject the null hypothesis when $p < \alpha$.

(ii) Independence Test

It should be mentioned the unconditional coverage test takes into account all the forecasts made in an out-of-sample environment. The independence test differs in the sense that it assesses each forecast independently with independence defined in the following way: that the observations that exceed the forecasts are independent of each other. This test is essentially then a Chi-squared test which assesses whether two forecasts are related to each other or not. With this test, a model is

[17] For this paper, n is the number of positive changes.

[18] For reference, the binomial test is used whenever we want to assess whether the proportion of a binary variable is equal to some hypothesized value.

[19] Again, for this paper the extremes are just the extreme increases.

bad if the observations that exceed our forecasts show dependence which often means they cluster. Following the logic of the chi-squared test, a model is bad if the observations that exceed our forecasts are related to each other.

While not shown here for convenience, Kevin Dowd(2008) provides a full exposition of the test statistic. It should be noted the test statistic amounts to a likelihood ratio(LR) that follows a $\chi^2(1)$ distribution. And like the binomial test, a model is rejected if the p-value of the likelihood ratio(LR) is less than alpha. That is $H_0 : p = \alpha$ is rejected in favor of $H_a : p \neq \alpha$. Of course, this means the model is bad.

Results

In conducting the unconditional coverage aka the binomial test, a p-value of 2.568679e-07 is recorded. This p-value is negligible which means that $H_0 : p = 0.05$ is rejected in favor of $H_a : p \neq 0.05$. This is confirmed with only 5 observations exceeding our forecasts which according the test, differs from what was expected, 8. We get our first evidence then that the model is quite poor at predicting extreme gasoline price increases.

In conducting the independence aka the chi-squared test, a p-value of 1.338619e-07 is recorded. Similar to the unconditional coverage test, this p-value is negligible which means $H_0 : p = 0.05$ is rejected in favor of $H_a : p \neq 0.05$. The combination of both results thus show POT is not well suited to modeling extreme gasoline price increases.

Discussion

It is hard to say why the POT approach failed. Perhaps, as noted by Nair et al.(2022), its failure boils down to its assumption of having an i.i.d sample. Even more restricting, however, it assumes the distribution of the tails of the data always follow exactly the Generalized Pareto Distribution. Should the data have a highly dependent structure as has been documented for gasoline prices, any attempt to use this technique to predict extreme price movements would result in highly misleading results.[20]

Conclusion

The first part of the paper was concerned with modeling gasoline price volatility. It was found in the forecasting exercise that the EGARCH model tends to perform best among the examined GARCH models. More broadly, it seems all the models that allow for asymmetric effects (i.e, all models but the plain GARCH) tend to also perform better at longer-term forecasts. Indeed, the finding that the EGARCH is the best predictor also provides evidence that gasoline prices are driven mainly by long-run dynamics with the periodic burst of increased price activity.

The second part of the paper was concerned with modeling the extreme price spikes where it was found the Peaks-Over-Threshold method is not suitable for predicting those large spikes. Thus, a challenge still remains in finding a robust model to capture the extremes of gasoline price movements.

While both parts were examined separately, it is possible they are related with extreme price increases being strongly related to the volatility of gasoline price movements. In that case, conditional extreme value theory may provide better results as it would account for volatility affecting the extreme price movements. Then again, that would be an exercise for future research.

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[20] Of course, dependence here means price behavior in one period is determined by the values that came before it.

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