

# SYSUCPC 2021 Online

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## A. Faulty Odometer II

You are given a car odometer which displays the miles traveled as an integer. The odometer has a defect, however: it proceeds from the digit 2 to the digit 4 and from the digit 7 to the digit 9, always skipping over the digit 3 and 8. This defect shows up in all positions (the one's, the ten's, the hundred's, etc.). For example, if the odometer displays 15229 and the car travels one mile, odometer reading changes to 15240 (instead of 15230).

### Input

Each line of input contains a positive integer in the range 1..999999999 which represents an odometer reading. (Leading zeros will not appear in the input.) The end of input is indicated by a line containing a single 0. You may assume that no odometer reading will contain the digit 3 and 8.

### Output

Each line of input will produce exactly one line of output, which will contain: the odometer reading from the input, a colon, one blank space, and the actual number of miles traveled by the car.

### Sample

| Input  | Output         |
|--------|----------------|
| 15     | 15: 12         |
| 2005   | 2005: 1028     |
| 250    | 250: 160       |
| 1500   | 1500: 768      |
| 999999 | 999999: 262143 |
| 0      |                |

## B. A very hard mathematic problem

Haoren is very good at solving mathematic problems. Today he is working a problem like this: Find three postive integers  $X$ ,  $Y$  and  $Z$  ( $X < Y, Z > 1$ ) that holds  $X^Z + Y^Z + XYZ = K$  where  $K$  is another given integer. Here the operator  $x^y$  means power, e.g.,  $2^3 = 2 * 2 * 2$ . Finding a solution is quite easy to Haoren. Now he wants to challenge more: What' s the total number of different solutions? Surprisingly, he is unable to solve this one. It seems that it' s really a very hard mathematic problem. Now, it' s your turn.

### Input

There are multiple test cases. For each case, there is only one integer  $K$  ( $0 < K < 2^{31}$ ) in a line.  $K = 0$  implies the end of input.

### Output

Output the total number of solutions in a line for each test case.

### Sample

| Input | Output |
|-------|--------|
| 9     | 1      |
| 53    | 1      |
| 6     | 0      |
| 0     |        |

### Explanation

$$9 = 1^2 + 2^2 + 1 * 2 * 2$$

$$53 = 2^3 + 3^3 + 2 * 3 * 3$$

## C. Collision Detection

Here comes an interesting problem, given a solid rectangle and a solid circle, all edges of the rectangle are parallel to coordinate axes, you need to detect whether they collide. We say that two objects collide if and only if they share at least one point.

### Input

The first line of input is the number of test case,  $T(T \leq 500)$ . Each test case contains two lines; the first line contains 4 integers  $X$ ,  $Y$ ,  $W$ , and  $H$ , representing the lower-left corner and the width and height of the given rectangle. The second line contains 3 integers  $X$ ,  $Y$ ,  $R$ , representing the center point of the ball and its radius. All integers given are non-negative and will be less than 10000.

### Output

For each cases output "Yes" Or "No" on a single line.

### Sample

| Input     | Output |
|-----------|--------|
| 3         | Yes    |
| 0 0 1 1   | Yes    |
| 2 0 1     | No     |
| 0 0 10 10 |        |
| 2 2 1     |        |
| 0 0 1 1   |        |
| 3 3 1     |        |

## D. Jump! Horse! Jump!

There is a chessboard with  $N$  rows and  $M$  columns, and there is a horse in position  $(r_0, c_0)$ . Your task is to write a program to calculate the minimum number of moves needed for a horse to reach another different position  $(r_1, c_1)$ , and calculate the number of all different ways to achieve it. Recall that a horse moves to any position that is on the opposite corner of a  $2 * 3$  rectangle from its current position.

### Input

The first line contains an integer  $T$ , representing the number of test cases. Each case contains one line with 6 integers  $N, M, r_0, c_0, r_1, c_1$  ( $1 \leq N, M \leq 1000$ ;  $1 \leq r_0, r_1 \leq N$ ;  $1 \leq c_0, c_1 \leq M$ ). The meanings of such integers are described above.

### Output

Output one line per test case, containing two integers. The first one is the minimum number of moves needed, and the second one is the remainder of the number of all different ways when divided by 10000. If there is no possible way, just output two zeros.

### Sample

| Input             | Output  |
|-------------------|---------|
| 3                 | 5 18    |
| 8 8 1 1 8 1       | 28 2562 |
| 100 100 1 1 31 51 | 0 0     |
| 1 10 1 4 1 9      |         |

## E. Maximum Module

There are  $N$  numbers. To form an equation, we could use operator  $+$ ,  $*$ ,  $($ ,  $)$  conjoining them and working out the result. Though these numbers are out of order, there may be many combinations. We must find out the result which mod by  $Q$  is maximal. Please output the module.

### Input

The first line of input is the number of test cases  $T$ . The following  $T$  lines each line contains  $N$  ( $N \leq 6$ ),  $Q$  ( $1 \leq Q \leq 32768$ ), followed by  $N$  integers within the range  $[0, 10000]$ .

### Output

For each test, output the maximum module on single line.

### Sample

| Input       | Output |
|-------------|--------|
| 2           | 6      |
| 3 7 2 2 2   | 4      |
| 4 5 1 2 3 4 |        |

### Explanation

$$(2 + 2 + 2) \% 7 = 6$$

$$((1 + 3) * (2 + 4)) \% 5 = 4$$

## F. Function

I believe that you have learned quadratic function, that is,  $f(x) = ax^2 + bx + c$ . Today, the teacher assigned you a lot of questions in class. Each question has a quadratic function, and it asks you whether it has  $x$  in  $[l, r]$ , making  $f(x) = 0$ .

### Input

The input file starts with an integer  $T$ , the number of testcases, in the first line. Then there are  $T$  lines, one for each testcase, each containing 5 integers  $a, b, c, l, r$ . It guarantees  $|a|, |b|, |c|, |l|, |r| \leq 100$ ,  $a = b = c = 0$  will not appear.

### Output

The output for each testcase consists of either one or two lines, depending on the result. The format is as follows. If exist  $x$  making  $f(x) = 0$ , print it from small to large, output the number to two decimal places. Otherwise, print “No solution!”

### Sample

| Input        | Output       |
|--------------|--------------|
| 2            | 3.00         |
| 1 -7 12 0 4  | 4.00         |
| 1 1 1 -10 10 | No solution! |

## G. Logarithm Joke

In mathematics, for given real numbers  $a$  and  $b$ , the logarithm  $\log_b a$  is a number  $x$  such that  $b^x = a$ . Analogously, in any group  $G$ , powers  $b^k$  can be defined for all integers  $k$ , and the discrete logarithm  $\log_b a$  is an integer  $k$  such that  $b^k = a$ . In number theory, the more commonly used term is index: we can write  $x = \text{ind}_r a(\text{mod } m)$  (read "the index of  $a$  to the base  $r$  modulo  $m$ ") for  $r^x \equiv a(\text{mod } m)$  if  $r$  is a **primitive root** (click url to know more information about primitive root) of  $m$  and  $\gcd(a, m) = 1$ .

Suppose  $a_0 = 1436138600$ ,  $m = 2013265921$ ,  $r = 31$ , and  $a_{n+1} = \text{ind}_r a_n(\text{mod } m)$ . Given  $n$ , you need to calculate the value of  $a_n$ .

### Input

The input file starts with an integer  $T$ , the number of testcases, in the first line. ( $1 \leq T \leq 10000$ )

Then there are  $T$  lines, one for each testcase, each containing one integer  $n$ . ( $1 \leq n \leq 100000$ )

### Output

The output for each testcase consists of one line, the value of  $a_n$ .

### Sample

| Input  | Output     |
|--------|------------|
| 5      | 1735960071 |
| 10     | 1774738167 |
| 100    | 782456885  |
| 1000   | 1521115966 |
| 10000  | 114514     |
| 100000 |            |