Relational Calculus
关系演算

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Relational Calculus

- *Query* has the form: $\{T \mid p(T)\}$
  - ***T*** is a tuple variable.
  - ***p(T)*** is a *formula* containing ***T*** .
- *Answer* = tuples T for which $p(T)$ = *true*.

（对比：集合的表示）

Formulae

- *Atomic formulae:*

  T $\in$ *Relation*

  T.a op T.b

  T.a op constant

  … *op* is one of $<, >, =, \leq, \geq, \neq$

- A *formula* can be*:*
  - an atomic formula
  - $\neg p, p \wedge q, p \vee q, p \Rightarrow q$
  - $\exists R(p(R))$
  - $\forall R(p(R))$

Free and Bound Variables自由与约束变量

- Quantifiers量词: ∃ and ∀
- Use of $\exists X$ or $\forall X$ *binds* X.
  - A variable that is not bound is free.
- Recall our definition of a query:
  - $\{T \mid p(T)\}$

- **Important restriction:**
  - T must be the *only* free variable in $p(T)$.
  - all other variables must be bound using a quantifier.

Simple Queries

- Find all sailors with rating above 7

$$\{S \mid S \in Sailors \wedge S.rating > 7\}$$

=RA: $\sigma_{rating>7}(Sailors)$

- Find names and ages of sailors with rating above 7.

$$\{S \mid \exists S1 \in Sailors(S1.rating > 7 \wedge S.sname = S1.sname \wedge S.age = S1.age)\}$$

=RA: $\pi_{sname,age}(\sigma_{rating>7}(Sailors))$

  - Note: *S* is a variable of 2 fields (i.e. S is a projection of *Sailors*)

Joins

Find sailors rated > 7 who've reserved boat #103

$\{ S \mid S \in Sailors \wedge S.rating > 7 \wedge$
$\exists R(R \in Reserves \wedge \mathbf{R.sid = S.sid}$
$\wedge R.bid = 103) \}$

=RA:

$$(\sigma_{rating>7}(Sailors)) \bowtie (\sigma_{bid=103}(Reserves))$$

Joins (continued)

Find sailors rated > 7 who've reserved a red boat

$$\{ S \mid S \in Sailors \wedge S.rating > 7 \wedge$$
$$\exists R(R \in Reserves \wedge R.sid = S.sid$$
$$\wedge \exists B(B \in Boats \wedge B.bid = R.bid$$
$$\wedge B.color = \text{'red'})) \}$$

$$(\sigma_{rating>7}(Sailors)) \bowtie Reserves \bowtie (\sigma_{color=red}(Boats))$$

This may look cumbersome, but it's not so different from SQL!

Universal Quantification

Find sailors who've reserved <u>all</u> boats

$$\{ S \mid S \in Sailors \wedge \forall B \in Boats \; (\exists R \in Reserves \; (S.sid = R.sid \wedge B.bid = R.bid)) \}$$

RA: (hint: use ÷)

A trickier example…

Find sailors who've reserved all Red boats

$$\{ S \mid S \in Sailors \wedge \\ \forall B \in Boats\ (B.color = \text{'red'} \Rightarrow \\ \exists R (R \in Reserves \wedge S.sid = R.sid \\ \wedge B.bid = R.bid)) \}$$

Alternatively…

$$\{ S \mid S \in Sailors \wedge \forall B \in Boats \, (B.color = \text{'red'} \Rightarrow \exists R (R \in Reserves \wedge S.sid = R.sid \wedge B.bid = R.bid)) \}$$

Sailors

sid	sname	rating	age
1	Fred	7	22
2	Jim	2	39
3	Nancy	8	27

Boats

bid	bname	color
101	Nina	red
102	Pinta	blue
103	Santa Maria	red

Reserves

sid	bid	day
2	102	9/13
1	101	10/12
3	101	11/30
1	103	8/12
2	101	7/1

$\mathbf{a \Rightarrow b}$ is the same as $\neg a \vee b$

			b	
			T	F
a	T		T	F
	F		T	T

A trickier example…

Find sailors who've reserved all Red boats

$$\{ S \mid S \in Sailors \wedge \forall B \in Boats\ (B.color = \textbf{`red'} \Rightarrow \exists R (R \in Reserves \wedge S.sid = R.sid \wedge B.bid = R.bid)) \}$$

Alternatively…

$$\{ S \mid S \in Sailors \wedge \forall B \in Boats\ (B.color \neq \textbf{`red'} \vee \exists R (R \in Reserves \wedge S.sid = R.sid \wedge B.bid = R.bid)) \}$$

A Remark: Unsafe Queries

- $\exists$ syntactically correct calculus queries that have an infinite number of answers! *Unsafe* queries.
  - e.g., $$\left\{ S \mid \neg \left(S \in Sailors \right) \right\}$$
  - Solution???? Don't do that!

Expressive Power

- Expressive Power (Theorem due to Codd):
  - Every query that can be expressed in relational algebra can be expressed as a safe query in relational calculus; the converse is also true.

- *Relational Completeness*:

  Query language (e.g., SQL) can express every query that is expressible in relational algebra/calculus.

  (actually, SQL is more powerful, as we will see…)

Summary

- Formal query languages — simple and powerful.
  - *Relational algebra* is <u>operational</u>
    - used as internal representation for query evaluation plans.
  - *Relational calculus* is "declarative"
    - query = "what you want", <u>not</u> "how to compute it"
  - *Same expressive power*

    --> *relational completeness.*

- Several ways of expressing a given query
  - a *query optimizer* should choose the most efficient version.

Your turn …

- Schema:

  Movie(<u>title</u>, year, studioName)

  ActsIn(<u>movieTitle</u>, <u>starName</u>)

  Star(<u>name</u>, gender, birthdate, salary)

- Queries to write in Relational Calculus:
  1. Find all movies by Paramount studio
  2. … movies whose stars are all women
  3. … movies starring 周润发
  4. Find stars who have been in a film w/ 周润发
  5. Stars within six degrees of 周润发*
  6. Stars connected to 周润发 via <u>any number</u> of films**

* Try *two* degrees for starters

** Good luck with this one!

Answers …

1. Find all movies by Paramount studio

$$\{M \mid M \in Movie \land M.studioName = \text{‘Paramount’}\}$$

Answers …

2. Movies whose stars are all women

$$\{M \mid M \in Movie \wedge \forall A \in ActsIn((A.movieTitle = M.title) \Rightarrow \exists S \in Star(S.name = A.starName \wedge S.gender = \text{'F'}))\}$$

Answers …

2. Movies whose stars are all women

$\{M \mid M \in Movie \wedge$
$\forall A \in ActsIn((A.movieTitle = M.title) \Rightarrow$
$\exists S \in Star(S.name = A.starName \wedge$
$S.gender = \text{'F'}))\}$

Movie

Title
阿凡达
变形金刚
星球大战

ActsIn

Title	Name
阿凡达	张
变形金刚	李
星球大战	李
阿凡达	周
变形金刚	周

Star

Name	Gender
陈	男
李	女
张	女
周	男

Answers …

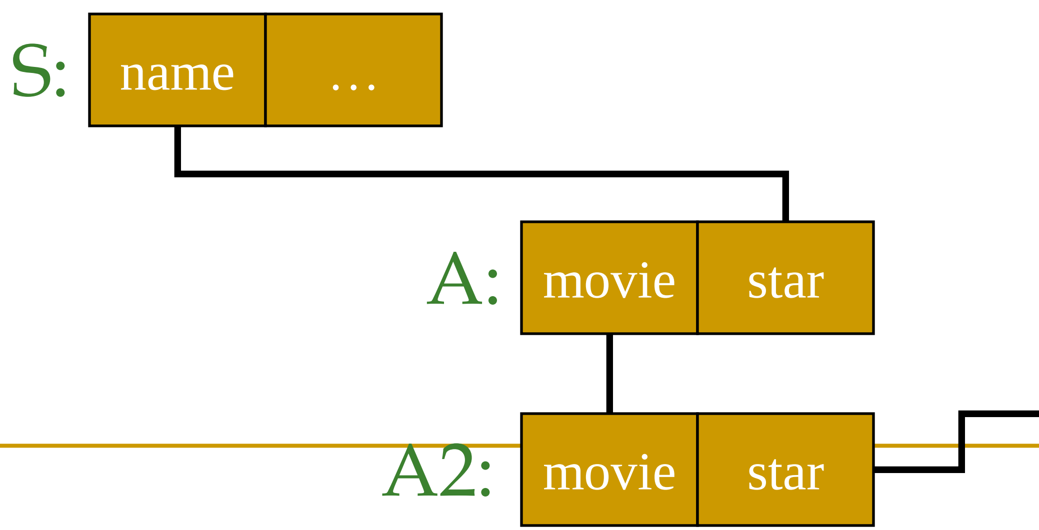
3. Movies starring 周润发

$$\{M \mid M \in Movie \wedge \exists A \in ActsIn(A.movieTitle = M.title \wedge A.starName = \text{‘周润发’}))\}$$

Answers …

4. Stars who have been in a film w/ 周润发

$$
\begin{aligned}
\{S \mid\ & S \in \mathit{Star} \wedge \\
& \exists A \in \mathit{ActsIn}(A.\mathit{starName} = S.\mathit{name} \wedge \\
& \quad \exists A2 \in \mathit{ActsIn}(A2.\mathit{movieTitle} = A.\mathit{movieTitle} \wedge \\
& \qquad A2.\mathit{starName} = \text{‘周润发’}))\}
\end{aligned}
$$

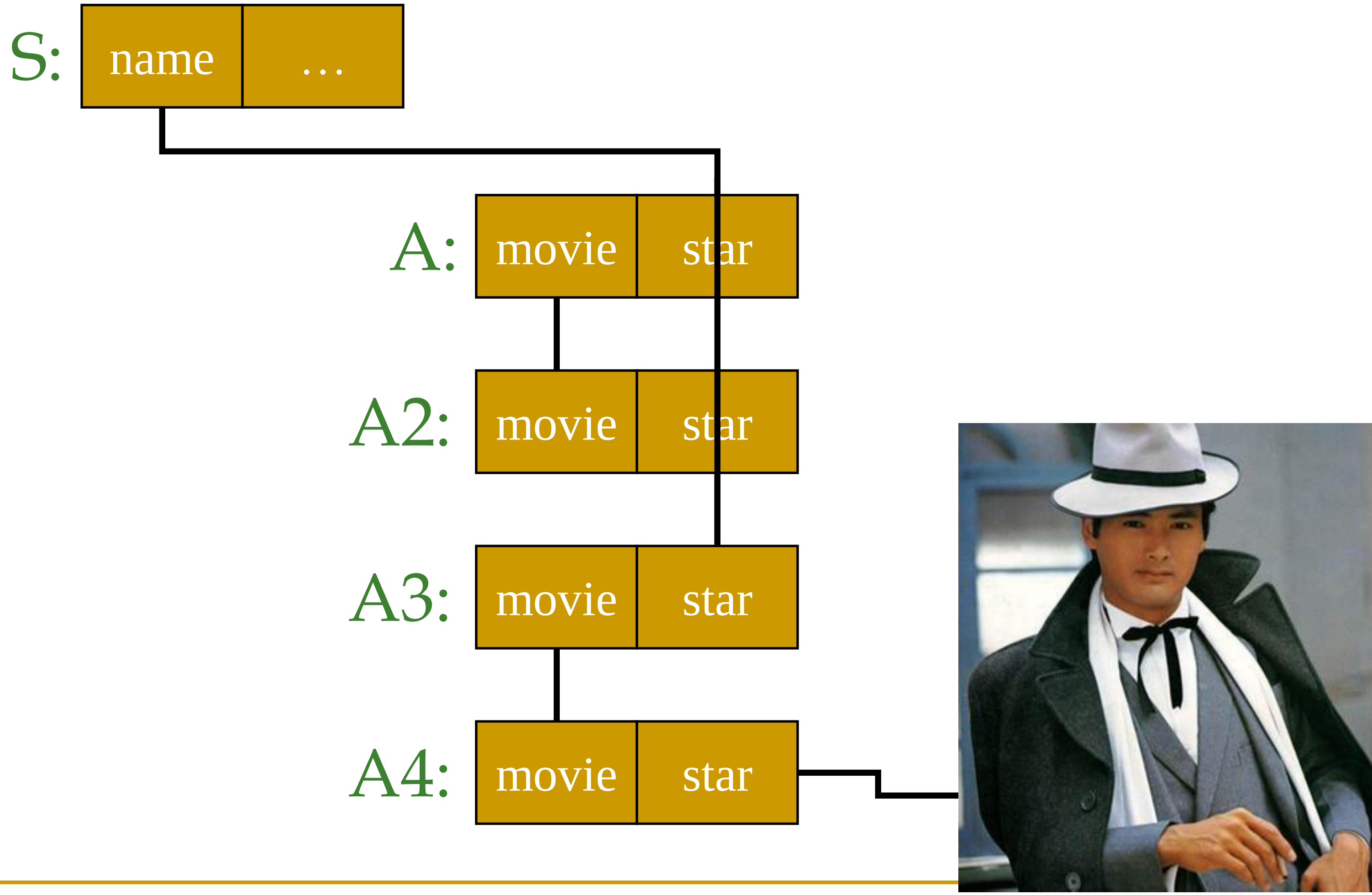


Answers …

5. Stars within ~~six~~ **two** degrees of 周润发

$\{S \mid S \in Star \wedge$
$\exists A \in ActsIn(A.starName = S.name \wedge$
$\exists A2 \in ActsIn(A2.movieTitle = A.movieTitle \wedge$
$\exists A3 \in ActsIn(A3.starName = A2.starName \wedge$
$\exists A4 \in ActsIn(A4.movieTitle = A3.movieTitle \wedge$
$A4.starName = \text{‘周润发’}))\}$

Two degrees:



Answers …

6. Stars connected to 周润发 via <u>any number</u> of films

- **Sorry … that was a trick question**
  - <u>Not expressible</u> in relational calculus!!
- **What about in relational algebra?**
  - We will be able to answer this question shortly …