《模式识别》

第四章 归一化、判别分析、人脸识别

马锦华

https://cse.sysu.edu.cn/teacher/MaJinhua

SUN YAT-SEN University

中山大學 计算机学院
SCHOOL OF COMPUTER SCIENCE AND ENGINEERING, SYSU

课程目录（暂定）

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第三章	主成分分析	3学时
第四章	归一化、判别分析、人脸识别	3学时
第五章	EM算法与聚类	3学时
第六章	贝叶斯决策理论	3学时
第七章	线性分类器与感知机	3学时
第八章	支持向量机	3学时
第九章	神经网络、正则项和优化方法	3学时
第十章	卷积神经网络及经典框架	3学时
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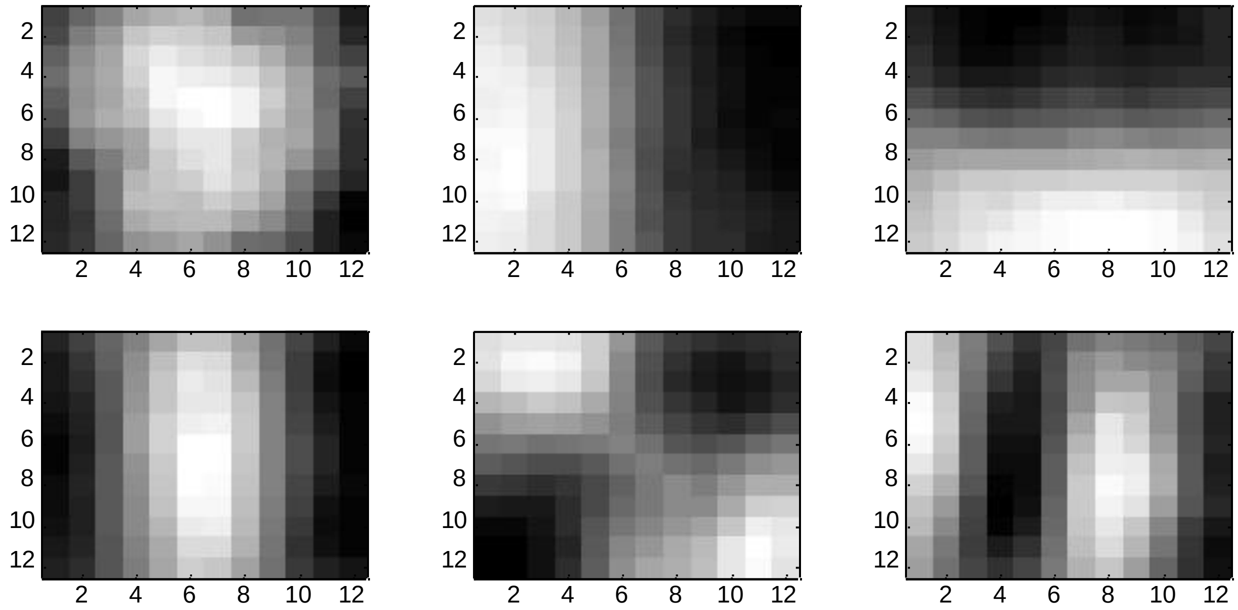
第十四章	开放世界模式识别	6学时

Let’s recap

- **PCA**

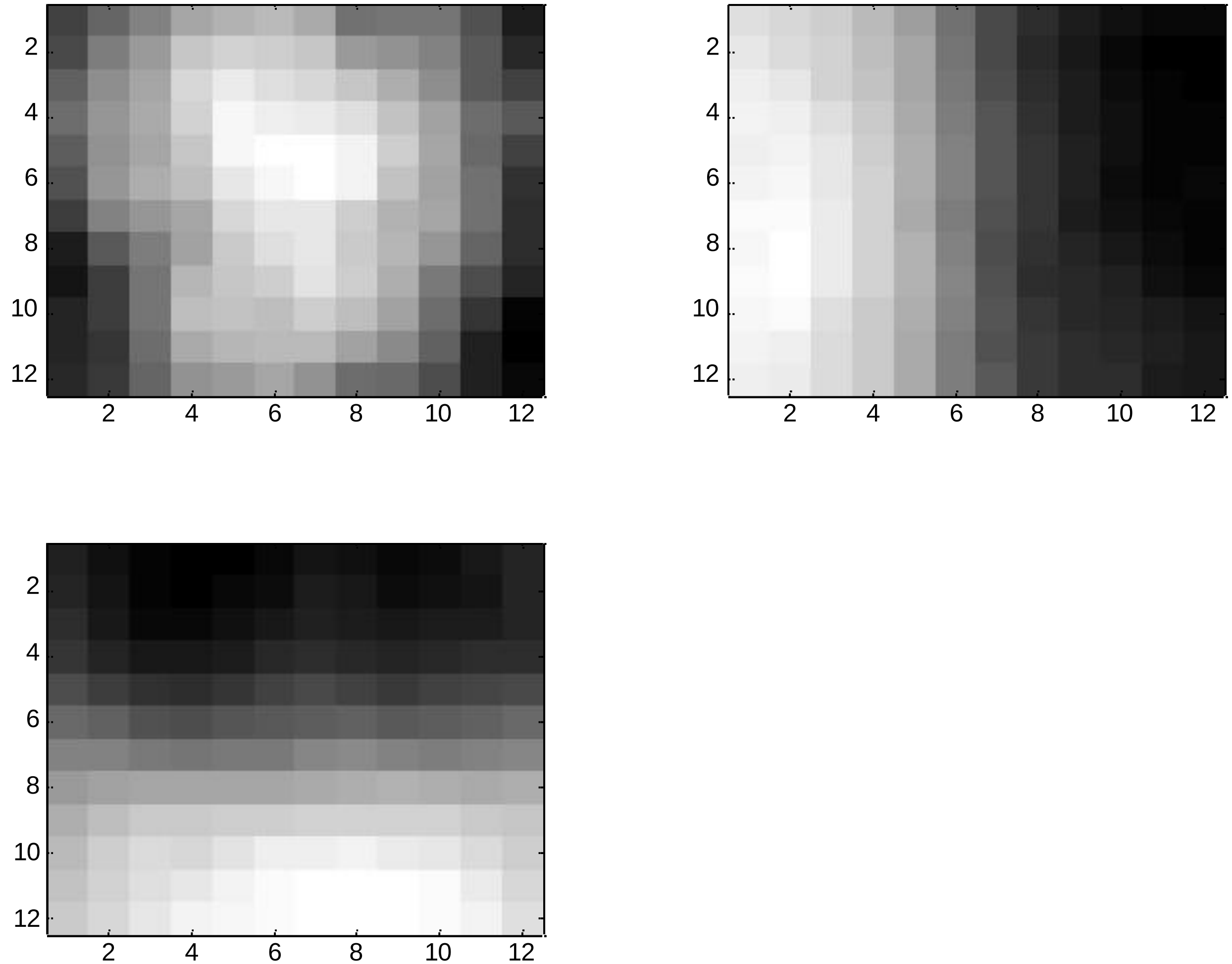
PCA compression: 144D -> 6D

6 most important eigenvectors



PCA compression: 144D) 3D

3 most important eigenvectors



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每维度规范化

- per-dimension normalization
  - 虚拟的例子（判别性别）
    - 假设用两个特征：身高和体重
    - 如果1. 身高单位毫米，体重单位吨，那么？
    - 如果2. 身高单位公里，体重单位克，那么？
  - 很多时候，不同的维度需要统一到同样的取值范围！
- 训练集：$\boldsymbol{x}_1, \dots, \boldsymbol{x}_n$, $\boldsymbol{x}_i = (x_{i1}, x_{i2}, \dots, x_{id})$
  - 对每一维j，其数据为$x_{1j}, x_{2j}, \dots, x_{nj}$
  - 取其最小值 $x_{min,j}$和最大值$x_{max,j}$
  - 对这一维的任何数据 $x_{ij} \leftarrow \frac{x_{ij}-x_{min,j}}{x_{max,j}-x_{min,j}}$

稀疏数据

- 新数据的范围是？各维度统一了吗？
  - [0 1] （训练集中的情况）
  - 若某一维$x_{max,j} = x_{min,j}$？
  - 也可以统一到[-1 1]

$$x_{ij} \leftarrow 2 \times \left(\frac{x_{ij} - x_{min,j}}{x_{max,j} - x_{min,j}} - 0.5 \right)$$

- 稀疏数据sparse data：数据中很多维度值为0
  - 如果所有数据≥ 0，在两种归一化中，原来是0的会变成什么？

ℓ_2或ℓ_1归一化

- 若各维度取值范围的不同是有意义的，但是不同数据点之间的大小（如向量长度norm）应保持一致
  - 对每个数据$\boldsymbol{x}_i = (x_{i1}, x_{i2}, \dots, x_{id})$

$$x_{ij} \leftarrow \frac{x_{ij}}{\|\boldsymbol{x}_i\|_{\ell_2}} \qquad \|\boldsymbol{x}_i\|_{\ell_2} = \sqrt{\boldsymbol{x}_i^T \boldsymbol{x}_i}$$

- ℓ_1归一化
  - 适用于非负的特征，即$x_{ij} \geq 0$总成立
  - 若数据$\boldsymbol{x}_i$是直方图(histogram)时，经常是最佳的

$$x_{ij} \leftarrow \frac{x_{ij}}{\|\boldsymbol{x}_i\|_{\ell_1}} \qquad \|\boldsymbol{x}_i\|_{\ell_1} = \sum_{j=1}^{d} |x_{ij}|$$

zero-mean, unit variance

- 有时候有理由相信每一个维度是服从高斯分布的
  - 希望每一个维度归一化到$N(0,1)$
- 对每一维j，其数据为$x_{1j}, x_{2j}, \ldots, x_{nj}$
  - 计算其均值$\hat{\mu}_j$和方差$\hat{\sigma}_j^2$
  - 对每一个特征值

$$x_{ij} \leftarrow \frac{x_{ij} - \hat{\mu}_j}{\hat{\sigma}_j}$$

规范化测试数据

- 怎样归一化测试数据？
  - 从测试集寻找最大值、最小值、均值？
- 除了在测试的时候，永远不要使用测试数据！
  - 测试集和训练集应该使用相同的归一化方法
    - 还记得吗？训练和测试集应该从相同的$p(\boldsymbol{x})$取样
    - 同样的归一化会保持这个限定！
  - 这个原则同样适用于交叉验证！
- 那么，怎样做？
  - 保存从训练集上取得的归一化参数(parameter)
  - 使用同样的公式和保存的参数来归一化测试集

小结

- 归一化的方法应该是根据数据的特点来选择的
  - 在做任何机器学习之前，先搞清你的数据的特点
    - 稀疏？
    - 每一维有没有含义？
    - 每一维里面值的分布情况？Gauss？
    - 看你的数据！Do visualization!
- 归一化可能对准确度有极大的影响！
  - 在有些例子里，正确的归一化能大幅度提高 accuracy
- 不同的归一化方法可以混合使用

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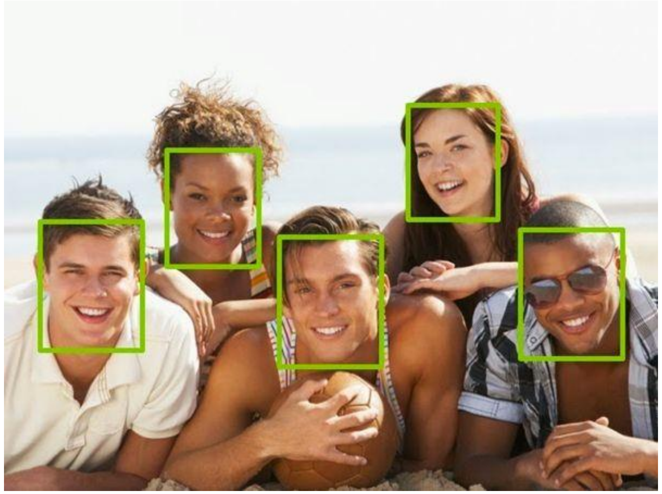
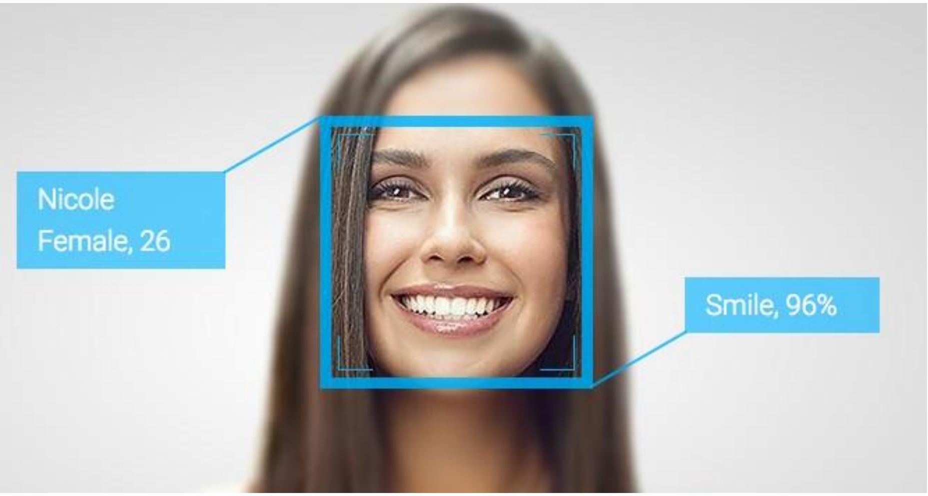
Detection versus Recognition



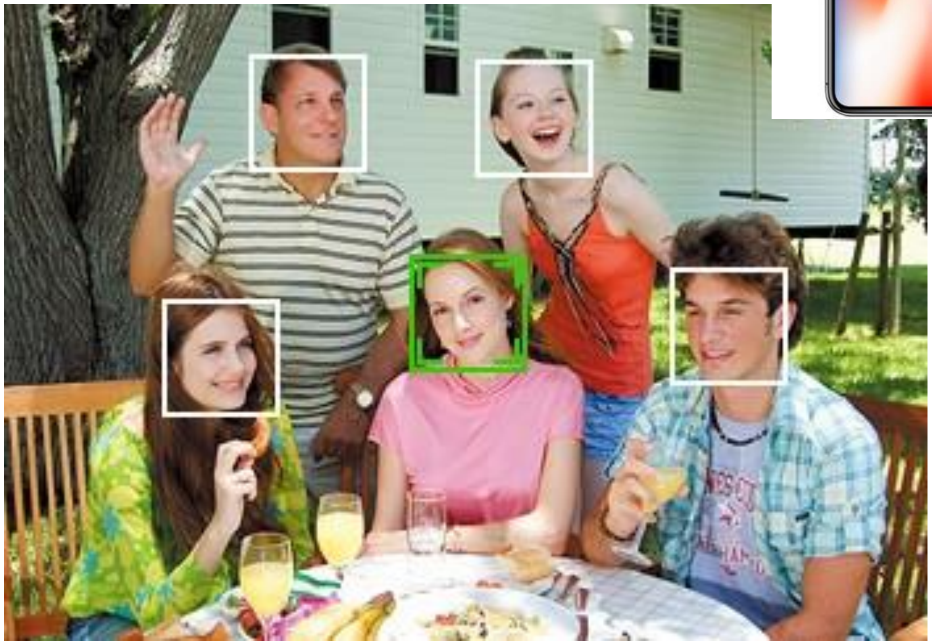


Detection finds the faces in images

Recognition recognizes WHO the person is

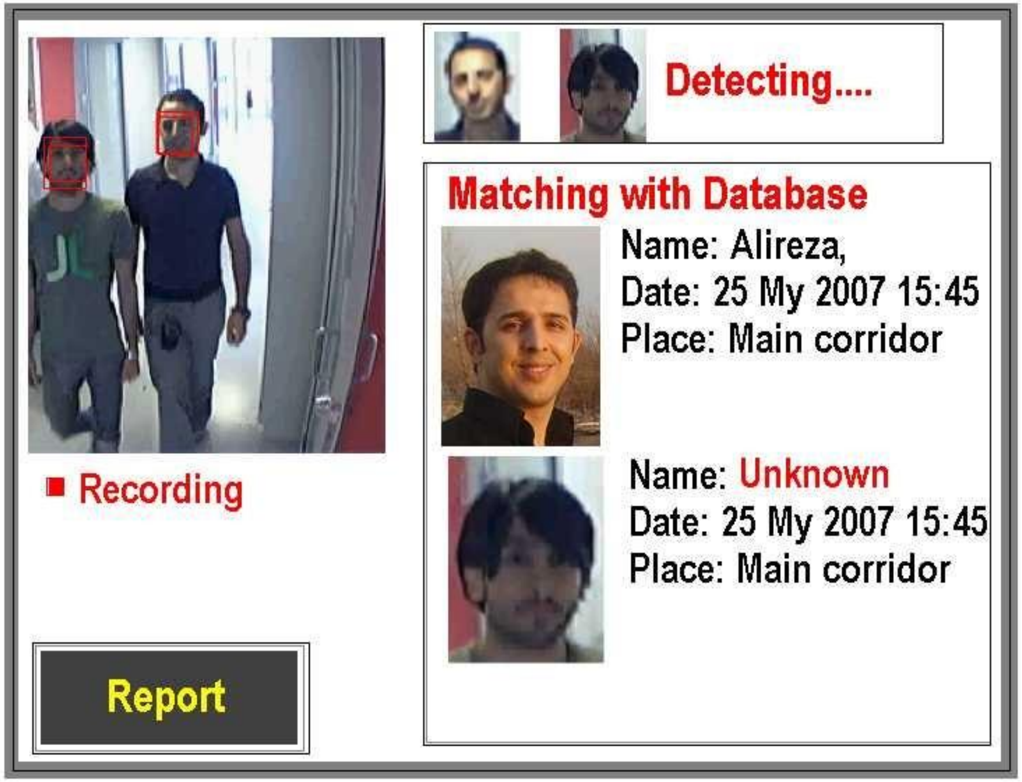
Face Recognition

- Digital photography



Face Recognition

- Digital photography
- Surveillance

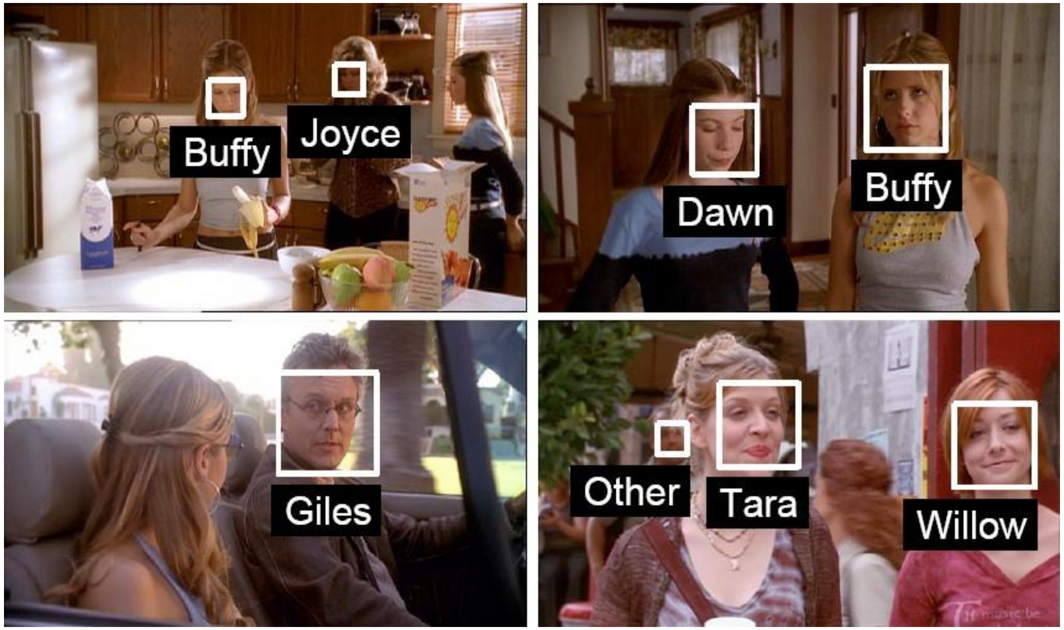


Face Recognition

- Digital photography
- Surveillance
- Album organization

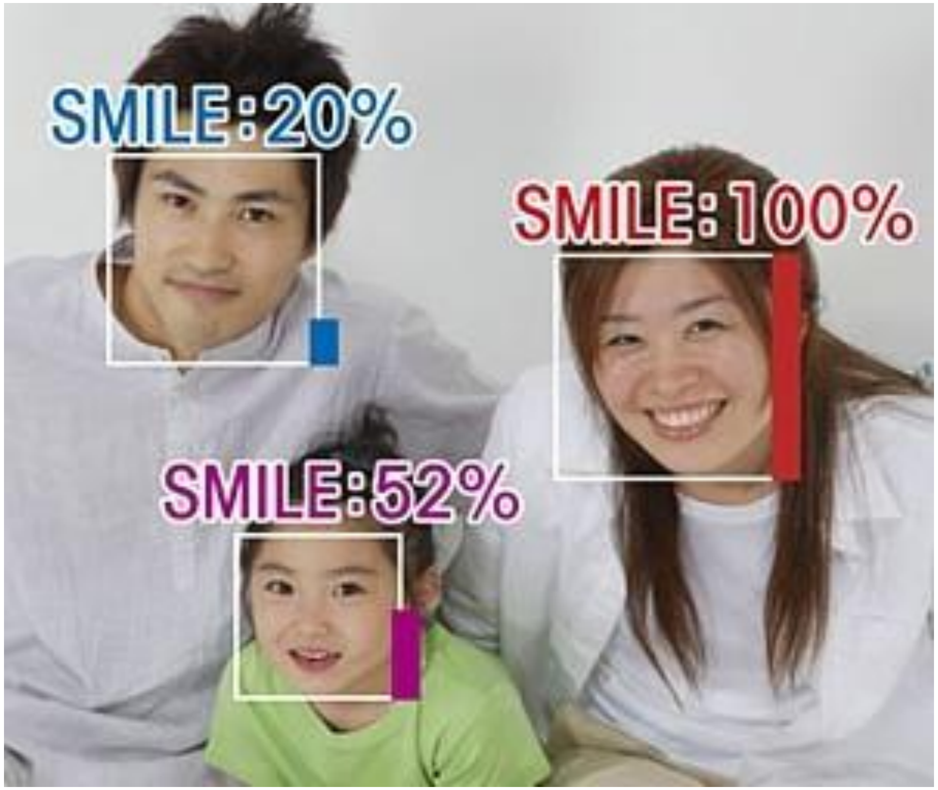
Face Recognition

- Digital photography
- Surveillance
- Album organization
- Person tracking/id.



Face Recognition

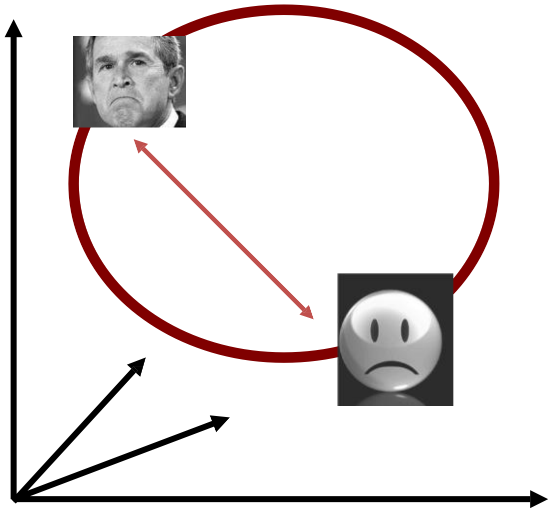
- Digital photography
- Surveillance
- Album organization
- Person tracking/id.
- Emotions and expressions



Face Recognition

- Digital photography
- Surveillance
- Album organization
- Person tracking/id.
- Emotions and expressions
- Security/warfare
- Tele-conferencing
- Etc.

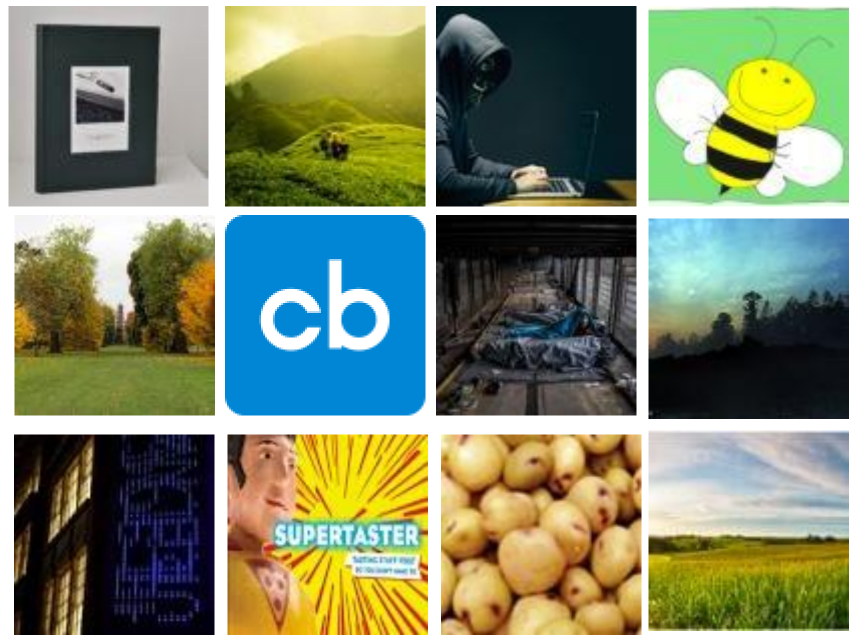
The Space of Faces



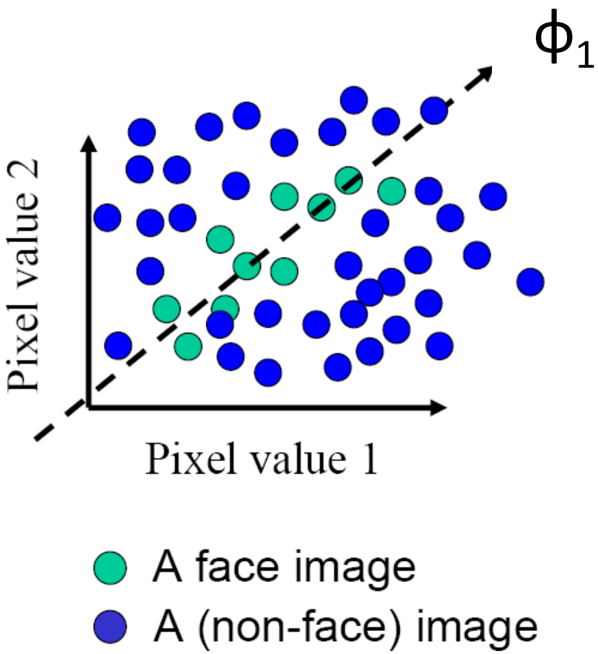
- An image is a point in a high dimensional space
  - If represented in grayscale intensity, an N x M image is a point in R^{NM}
  - E.g. 100x100 image = 10,000 dim

Slide credit: Chuck Dyer, Steve Seitz, Nishino

100x100 images can contain many things other than faces!



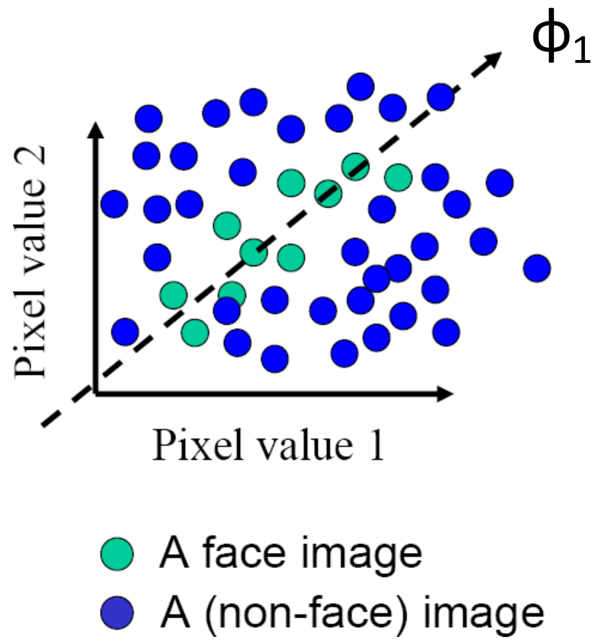
The Space of Faces

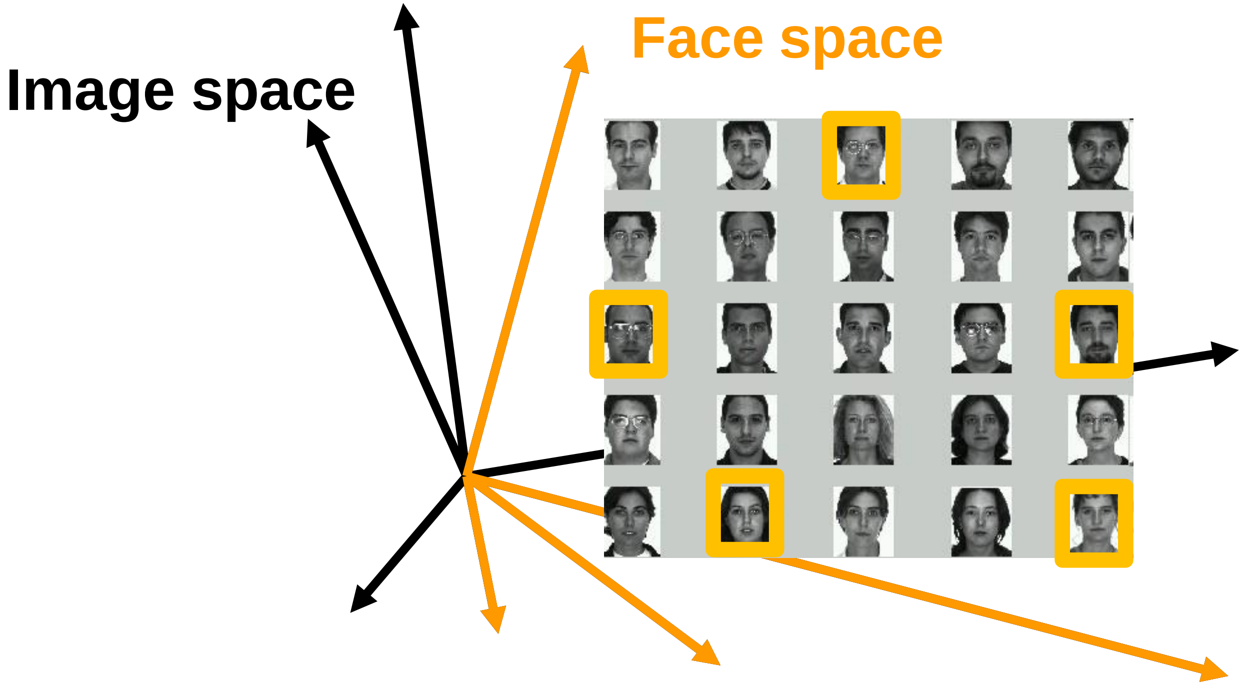


- An image is a point in a high dimensional space
  - If represented in grayscale intensity, an N x M image is a point in R^{NM}
  - E.g. 100x100 image = 10,000 dim
- However, relatively few high dimensional vectors correspond to valid face images
- We want to effectively model the subspace of face images

Slide credit: Chuck Dyer, Steve Seitz, Nishino

Where have we seen something like this before?





•Compute n-dim subspace such that the projection of the data points onto the subspace has the largest variance among all n-dim subspaces.

• Maximize the scatter of the training images in face space

Key Idea

- So, compress them to a low-dimensional subspace that captures key appearance characteristics of the visual DOFs.

- USE PCA for estimating the sub-space (dimensionality reduction)

- Compare two faces by projecting the images into the subspace and measuring the EUCLIDEAN distance between them.

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Object recognition:

a classification framework

- Apply a prediction function to a feature representation of the image to get the desired output:

f() = “apple”

f() = “tomato”

f() = “cow”

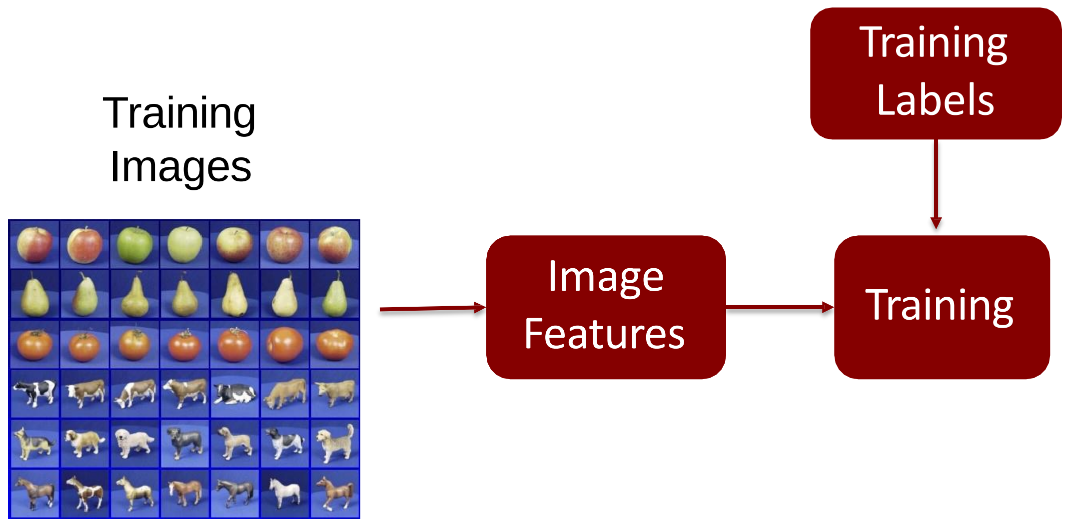
Dataset: ETH-80, by B. Leibe   Slide credit: L. Lazebnik

A simple pipeline - Training

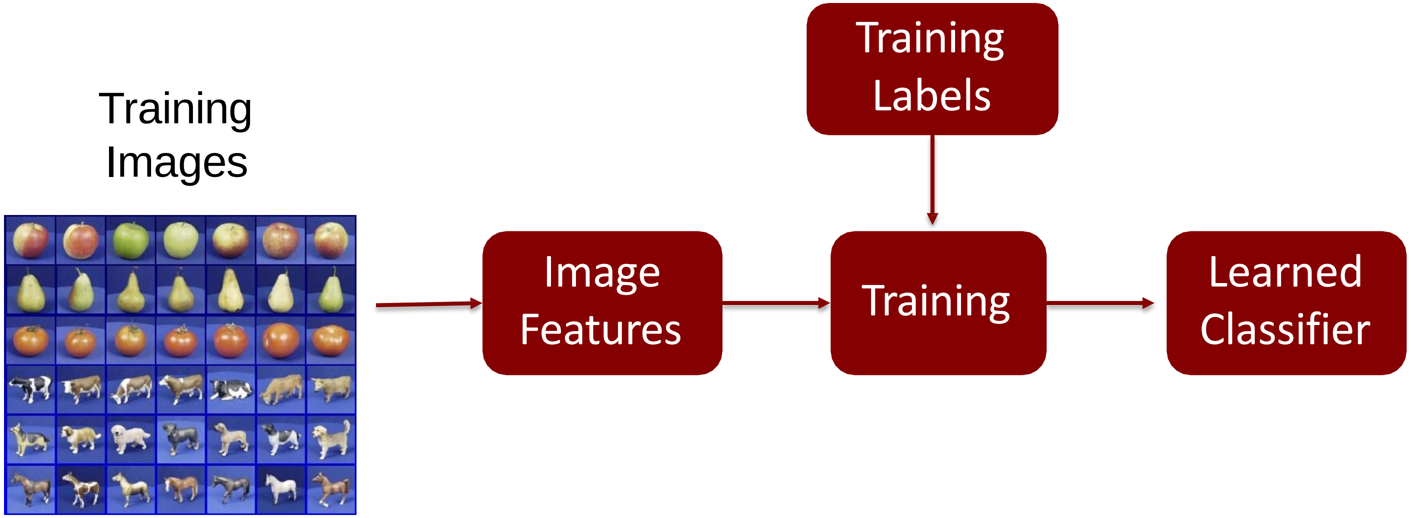
Training
Images

Image
Features

A simple pipeline - Training



A simple pipeline - Training



A simple pipeline - Testing

Training Images

Image Features

Training Labels

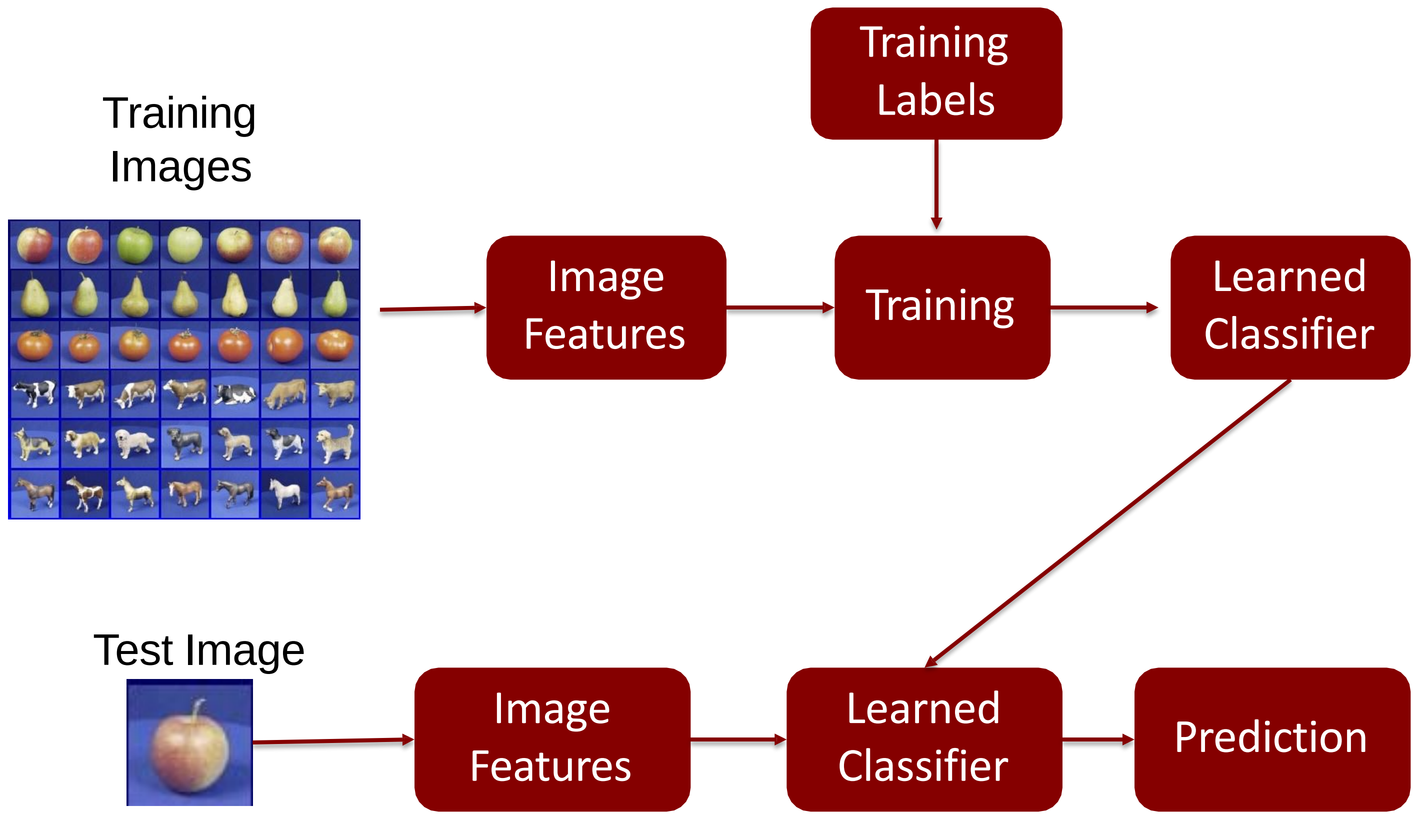
Training

Learned Classifier

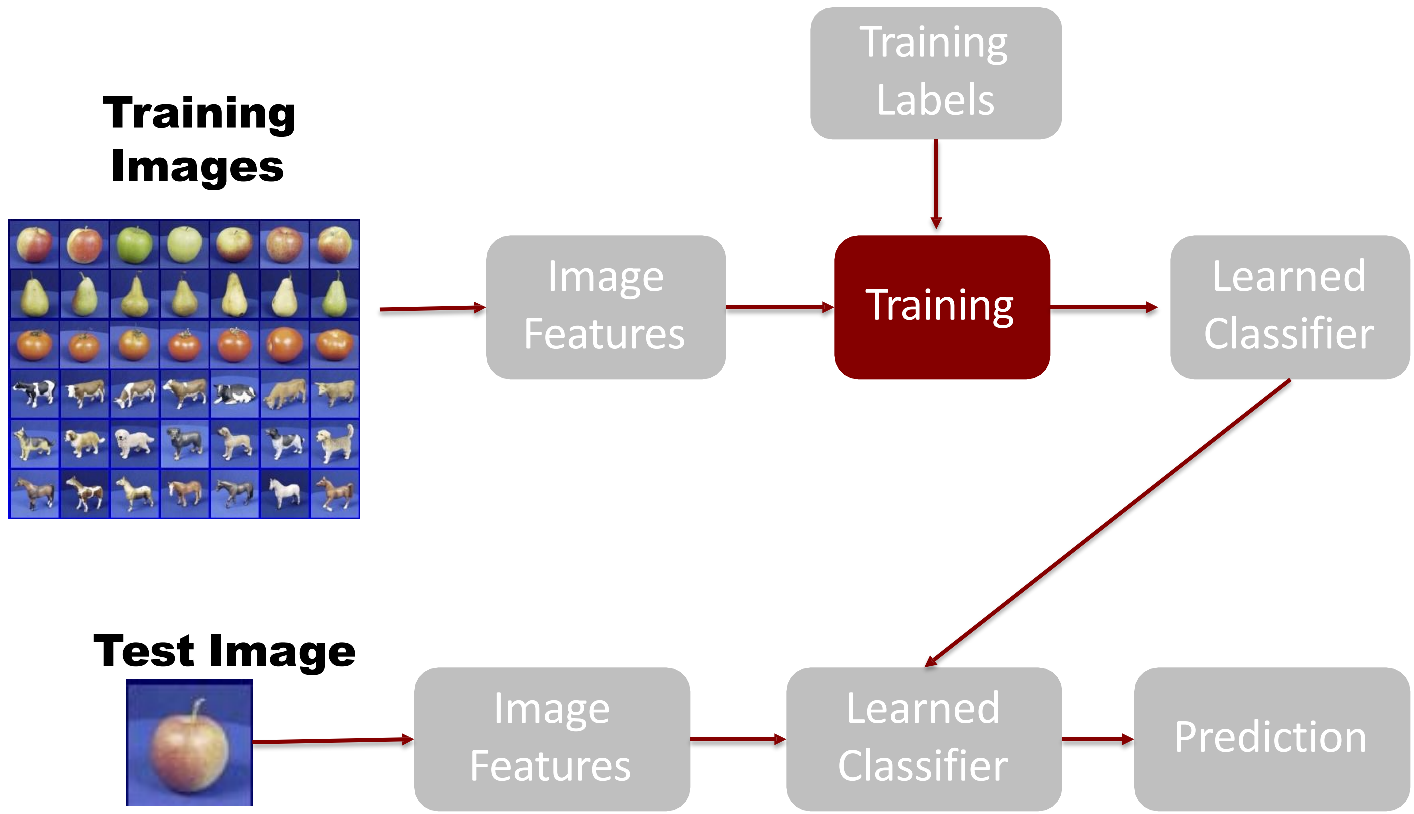
Test Image

Image Features

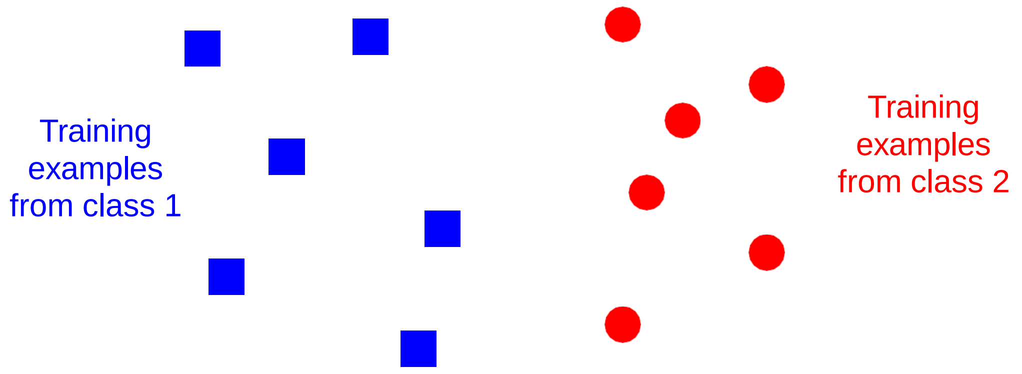
A simple pipeline - Testing



A simple pipeline - Training

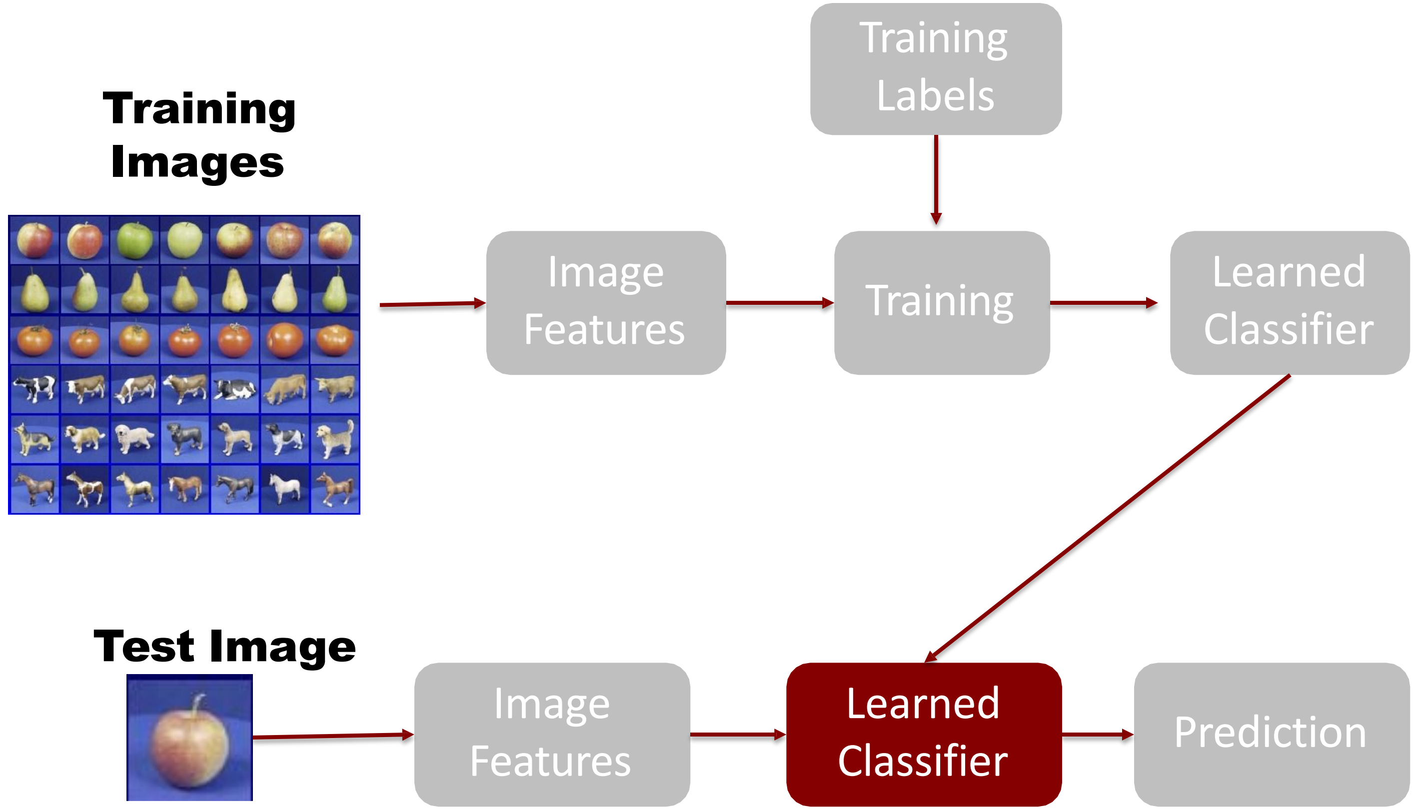


Classifiers: Nearest neighbor



Slide credit: L. Lazebnik

A simple pipeline - Testing



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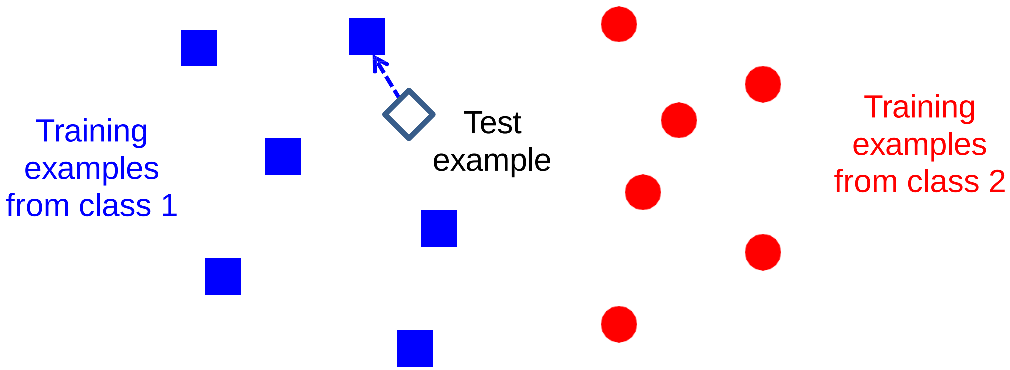

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Eigenfaces: key idea

- Assume that most face images lie on a low-dimensional subspace determined by the first k (k<<d) directions of maximum variance
- Use PCA to determine the vectors or “eigenfaces” that span that subspace
- Represent all face images in the dataset as linear combinations of eigenfaces

M. Turk and A. Pentland, Face Recognition using Eigenfaces, CVPR 1991

Training images: $\mathbf{x}_1,\ldots,\mathbf{x}_N$

Eigenface algorithm

- Training

1. Align training images $x_1, x_2, \ldots, x_N$

Note that each image is formulated into a long vector!

2. Compute average face

$$\mu = \frac{1}{N}\sum x_i$$

3. Compute the difference image (the <u>centered data matrix</u>)

$$X_c = \begin{bmatrix} | & & | \\ x_1 & \ldots & x_n \\ | & & | \end{bmatrix} - \begin{bmatrix} | & & | \\ \mu & \ldots & \mu \\ | & & | \end{bmatrix}$$

$$= X - \mu 1^T = X - \frac{1}{n} X 1 1^T = X\left(I - \frac{1}{n} 1 1^T\right)$$

Eigenface algorithm

4. Compute the covariance matrix

$$\boxed{\Sigma = \frac{1}{n}\begin{bmatrix} | & & | \\ x_1^c & \dots & x_n^c \\ | & & | \end{bmatrix}\begin{bmatrix} - & x_1^c & - \\ & \vdots & \\ - & x_n^c & - \end{bmatrix} = \frac{1}{n} X_c X_c^T}$$

5. Compute the eigenvectors of the covariance matrix Σ
6. Compute each training image x_i 's projections as

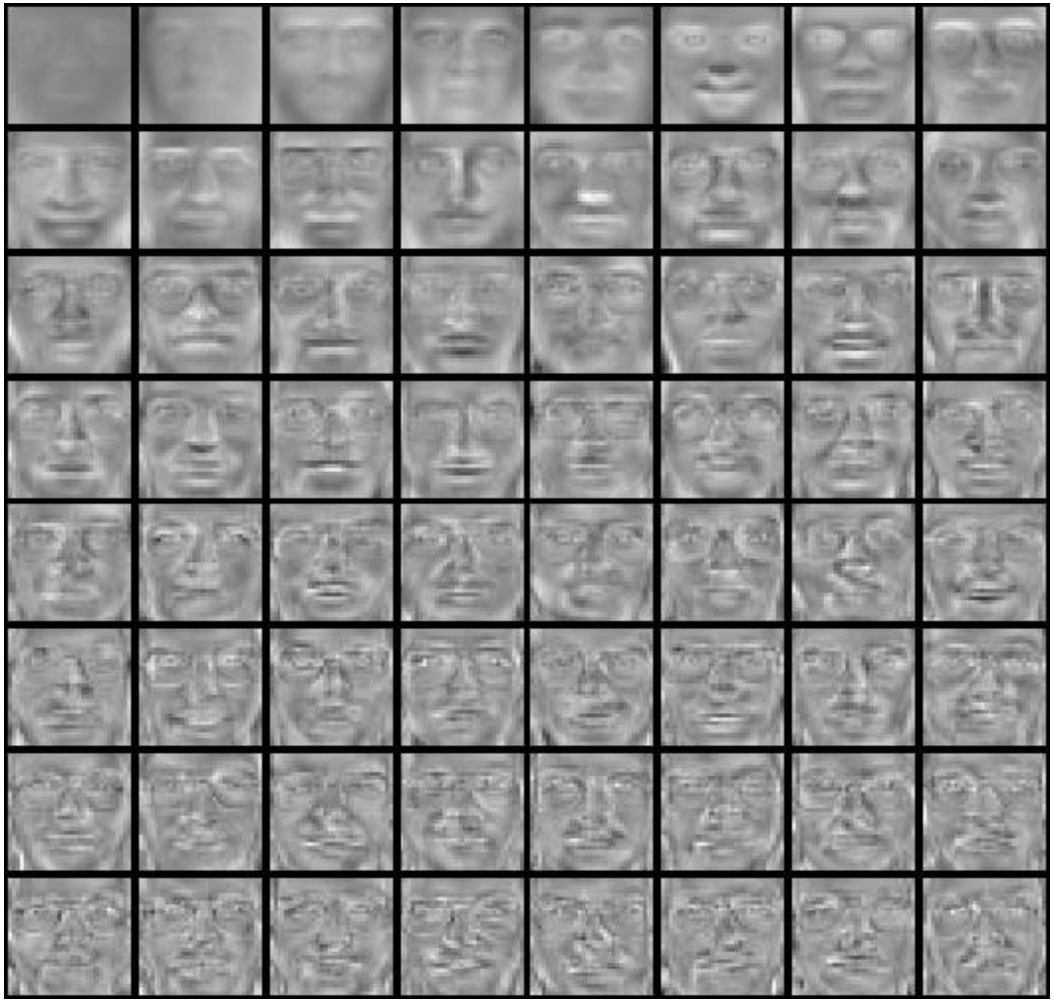
$$x_i \rightarrow \left(x_i^c \cdot \varphi_1, x_i^c \cdot \varphi_2, \ldots, x_i^c \cdot \varphi_K\right) \equiv \left(a_1, a_2, \ldots, a_K\right)$$

7. Visualize the estimated training face x_i

$$x_i \approx \mu + a_1\varphi_1 + a_2\varphi_2 + \ldots + a_K\varphi_K$$

Top eigenvectors: $\Phi_1, \ldots, \Phi_k$

Mean: μ



Eigenface algorithm

x_i

$a_1\varphi_1$ $a_2\varphi_2$ … $a_K\varphi_K$

Reconstructed training face

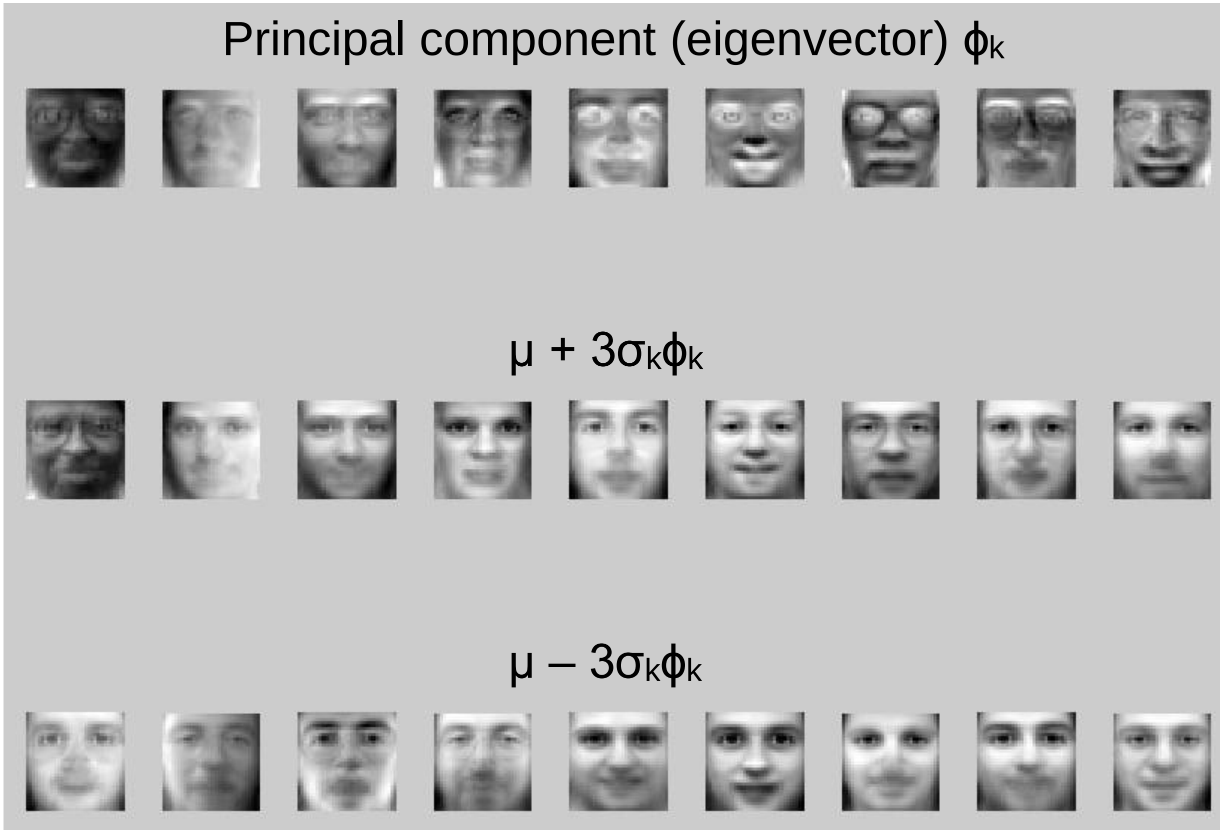
6. Compute each training image x_i 's projections as

$$x_i \rightarrow \left(x_i^c \cdot \varphi_1,\ x_i^c \cdot \varphi_2, \ldots,\ x_i^c \cdot \varphi_K\right) \equiv \left(a_1, a_2, \ldots, a_K\right)$$

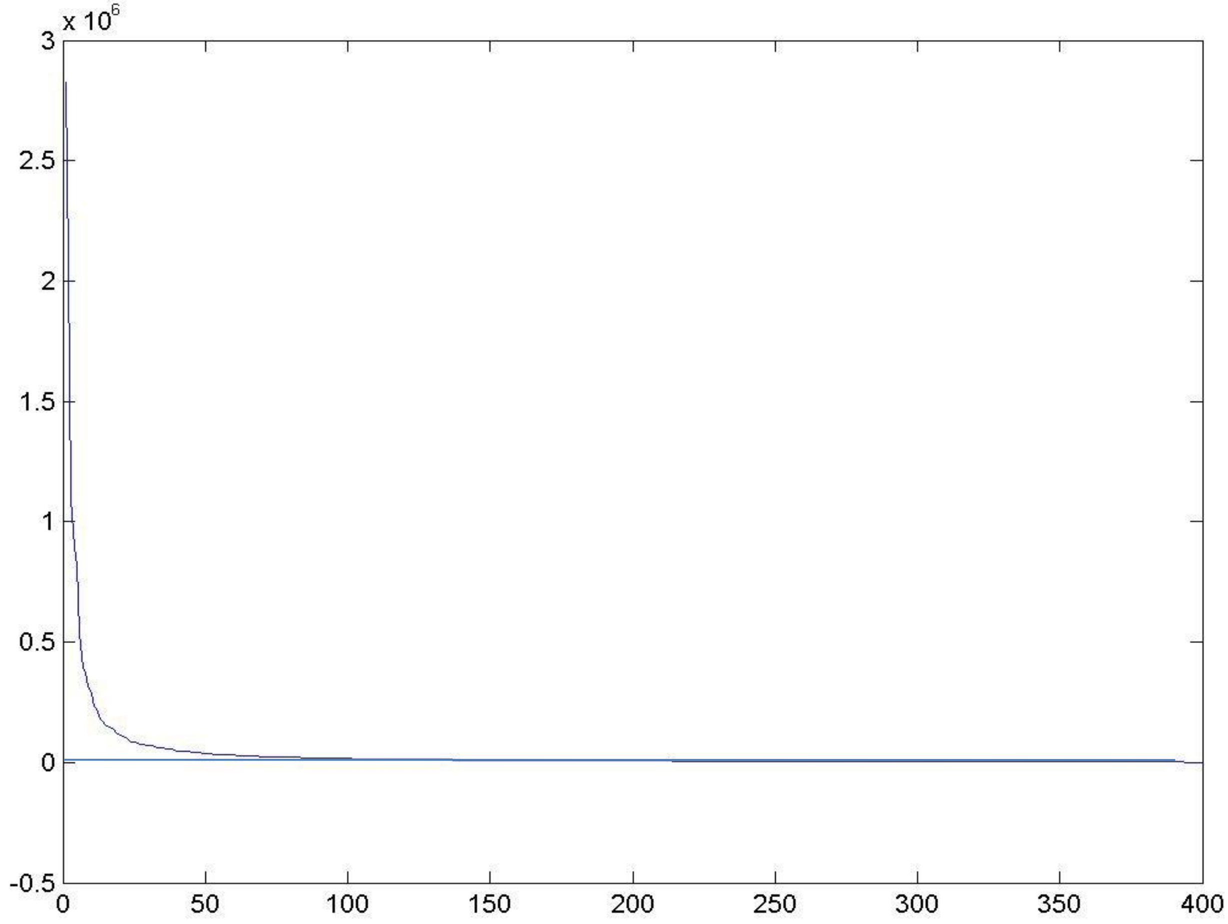
7. Visualize the reconstructed training face x_i

$$x_i \approx \mu + a_1\varphi_1 + a_2\varphi_2 + \ldots + a_K\varphi_K$$

Visualization of eigenfaces



Eigenvalues (variance along eigenvectors)



Reconstruction and Errors

K = 4

K = 200

K = 400

- Only selecting the top K eigenfaces, reduces the dimensionality.
- Fewer eigenfaces result in more information loss, and hence less discrimination between faces.

Eigenface algorithm

- Testing
  1. Take query image t
  2. Project into eigenface space and compute projection

$$t \rightarrow \left((t-\mu)\cdot\varphi_1, (t-\mu)\cdot\varphi_2, \ldots,\ (t-\mu)\cdot\varphi_K\right) \equiv \left(w_1, w_2, \ldots, w_K\right)$$

  3. Compare projection w with all N training projections
     - Simple comparison metric: Euclidean
     - Simple decision: K-Nearest Neighbor

       (note: this "K" refers to the k-NN algorithm, is different from the previous K's referring to the # of principal components)

Shortcomings

- Requires carefully controlled data:
  - All faces centered in frame
  - Same size
  - Some sensitivity to angle
- Alternative:
  - "Learn" one set of PCA vectors for each angle
  - Use the one with lowest error
- Method is completely knowledge free
  - (sometimes this is good!)
  - Doesn't know that faces are wrapped around 3D objects (heads)
  - Makes no effort to preserve class distinctions

Summary for Eigenface

Pros

- Non-iterative, globally optimal solution

Limitations

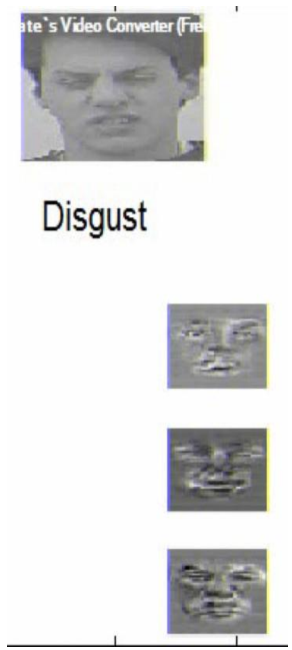
- PCA projection is **optimal for reconstruction** from a low dimensional basis, but **may NOT be optimal for discrimination…** Is there a better dimensionality reduction?

Besides face recognitions, we can also do Facial expression recognition

Happiness subspace (method A)

Disgust subspace (method A)

Facial Expression Recognition Movies (method A)



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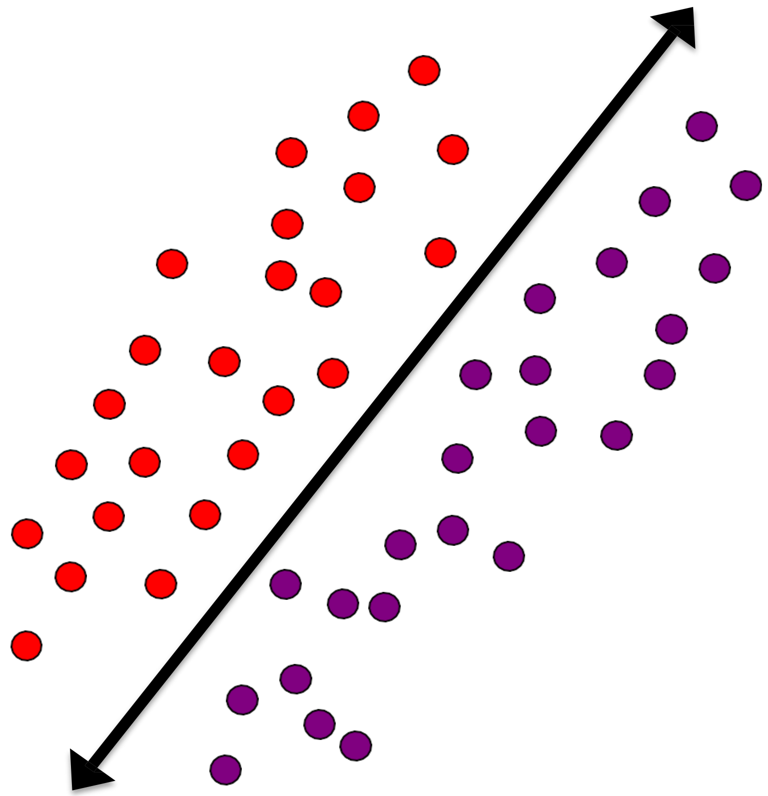
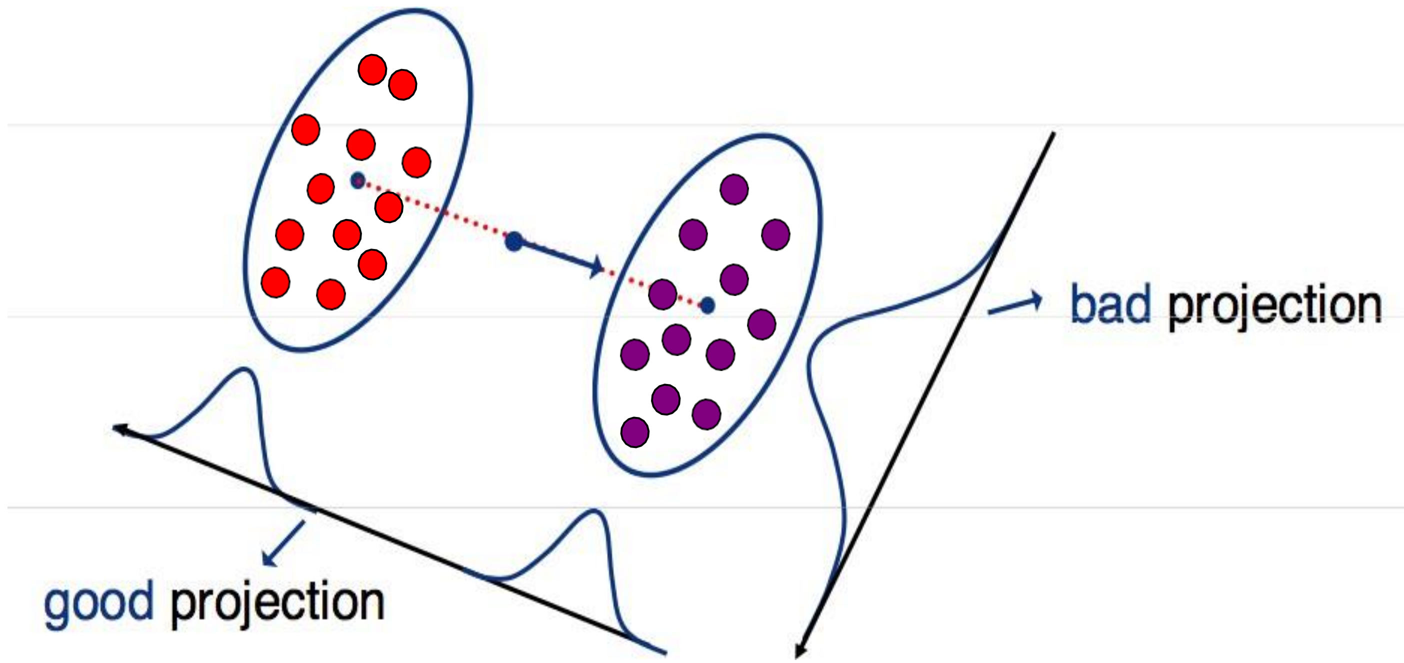
Which direction will be the first principle component?



Fischer’s Linear Discriminant Analysis

- Goal: find the best separation between two classes



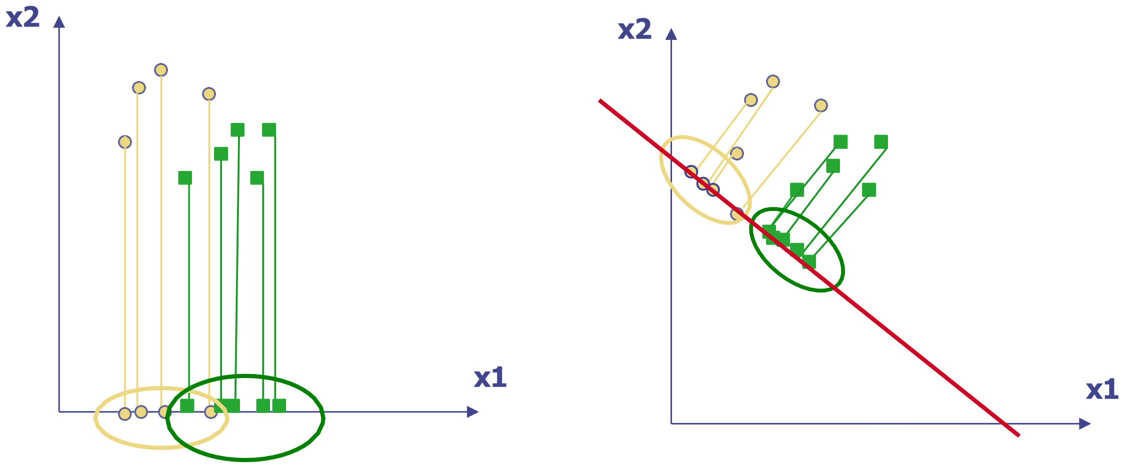
Slide inspired by N. Vasconcelos

Difference between PCA and LDA

- PCA preserves maximum variance
- LDA preserves discrimination
  - Find projection that maximizes scatter between classes and minimizes scatter within classes

Illustration of the Projection

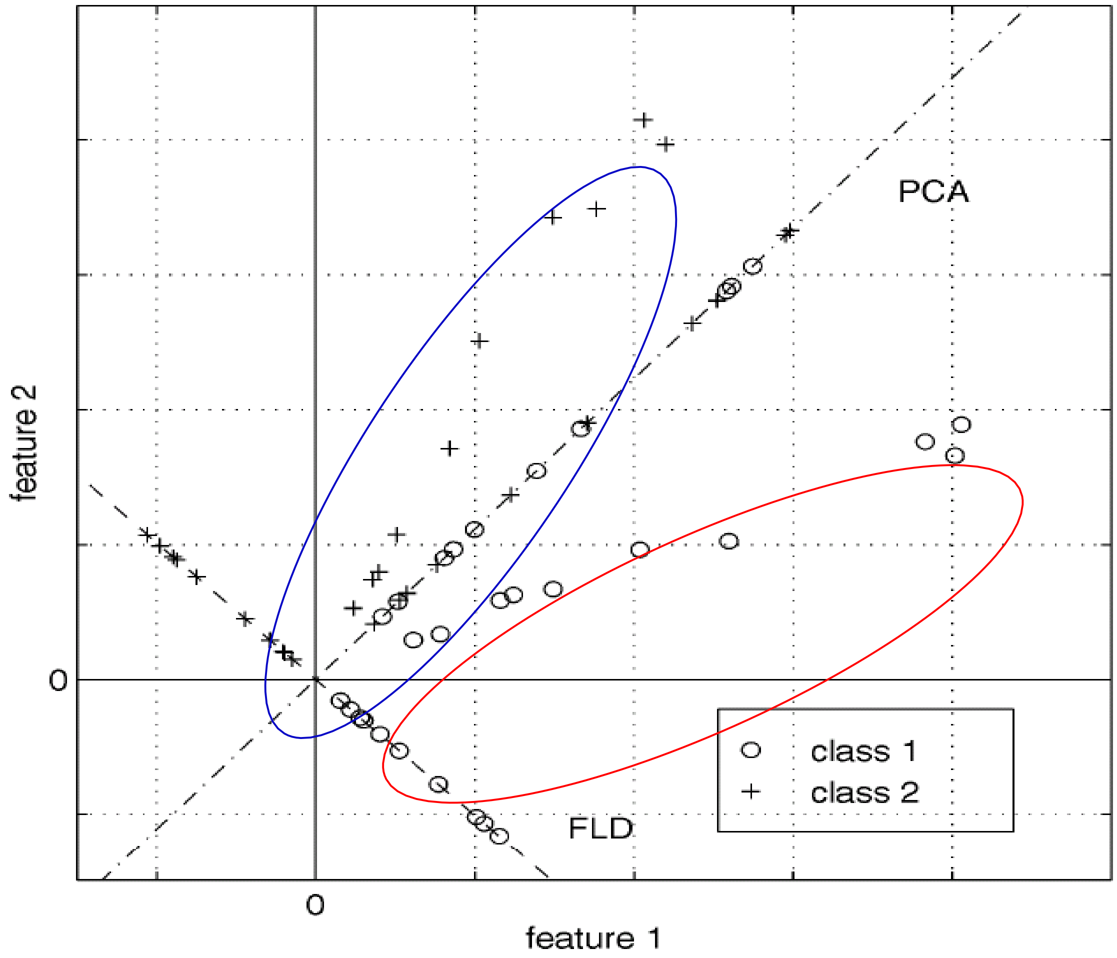
- Using two classes as example:



Poor Projection

Good

Basic intuition: PCA vs. LDA



LDA with 2 variables

- We want to learn a projection W such that the projection converts all the points from x to a new space (For this example, assume m == 1):

$$z = w^{\mathrm{T}} x \qquad z \in \mathbf{R}^m \quad x \in \mathbf{R}^n$$

- Let the **per class** means be:

$$E_{X|Y}[X \mid Y = i] = \mu_i$$

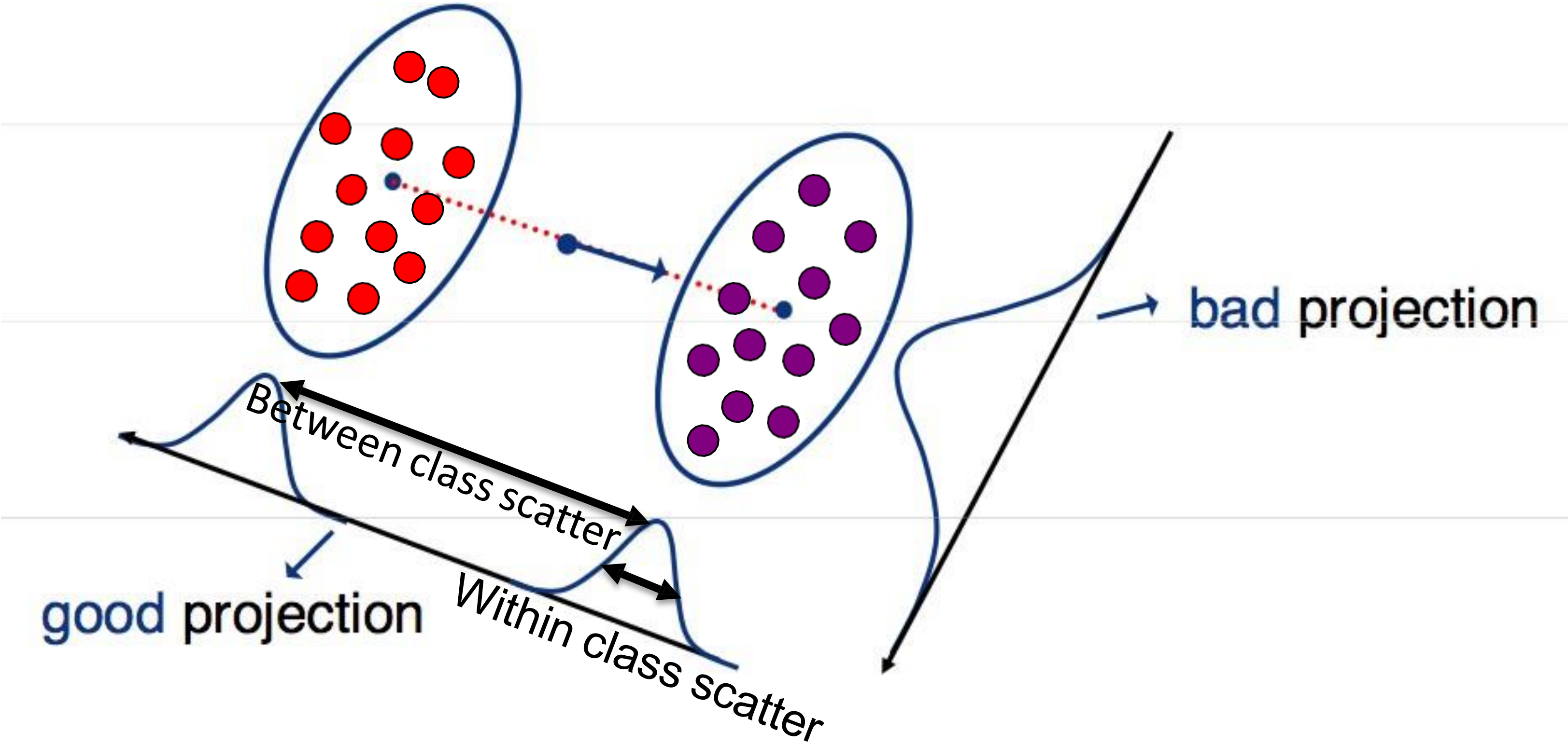
- And the **per class** covariance matrices be:

$$[(X - \mu_i)(X - \mu_i)^T \mid Y = i] = \Sigma_i$$

- We want a projection that maximizes:

$$J(w) = \max \frac{between\ \ class\ \ scatter}{within\ \ class\ \ scatter}$$

Fischer's Linear Discriminant Analysis



Slide inspired by N. Vasconcelos

LDA with 2 variables

The following objective function:

$$J(w) = \max \frac{between\ \ class\ \ scatter}{within\ \ class\ \ scatter}$$

Can be written as

$$J(w) = \frac{\left(E_{Z|Y}[Z \mid Y=1] - E_{Z|Y}[Z \mid Y=0]\right)^2}{\mathrm{var}[Z \mid Y=1] + \mathrm{var}[Z \mid Y=0]}$$

LDA with 2 variables

- We can write the between class scatter as:

$$\left(E_{Z|Y}[Z \mid Y=1] - E_{Z|Y}[Z \mid Y=0]\right)^2 = \left(w^T[\mu_1 - \mu_0]\right)^2$$
$$= w^T[\mu_1 - \mu_0][\mu_1 - \mu_0]^T w$$

- Also, the within class scatter becomes:

$$\mathrm{var}[Z \mid Y=i] = E_{Z|Y}\left\{\left(z - E_{Z|Y}[Z \mid Y=i]\right)^2 \mid Y=i\right\}$$
$$= E_{Z|Y}\left\{\left(w^T[x-\mu_i]\right)^2 \mid Y=i\right\}$$
$$= E_{Z|Y}\left\{w^T[x-\mu_i][x-\mu_i]^T w \mid Y=i\right\}$$
$$= w^T \Sigma_i w$$

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LDA with 2 variables

- We can plug in these scatter values to our objective function:

between class scatter

$$J(w) = \frac{w^T S_B w}{w^T S_W w} \qquad \begin{aligned} S_B &= (\mu_1 - \mu_0)(\mu_1 - \mu_0)^T \\ S_W &= (\Sigma_1 + \Sigma_0) \end{aligned}$$

within class scatter

- And our objective becomes:

$$J(w) = \frac{\left(E_{Z|Y}[Z \mid Y=1] - E_{Z|Y}[Z \mid Y=0]\right)^2}{\mathrm{var}[Z \mid Y=1] + \mathrm{var}[Z \mid Y=0]}$$

$$= \frac{w^T(\mu_1 - \mu_0)(\mu_1 - \mu_0)^T w}{w^T(\Sigma_1 + \Sigma_0)w}$$

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LDA with 2 variables

- The scatter variables

between class scatter

$$\boxed{\begin{aligned} S_B &= (\mu_1 - \mu_0)(\mu_1 - \mu_0)^T \\ S_W &= (\Sigma_1 + \Sigma_0) \end{aligned}}$$

within class scatter

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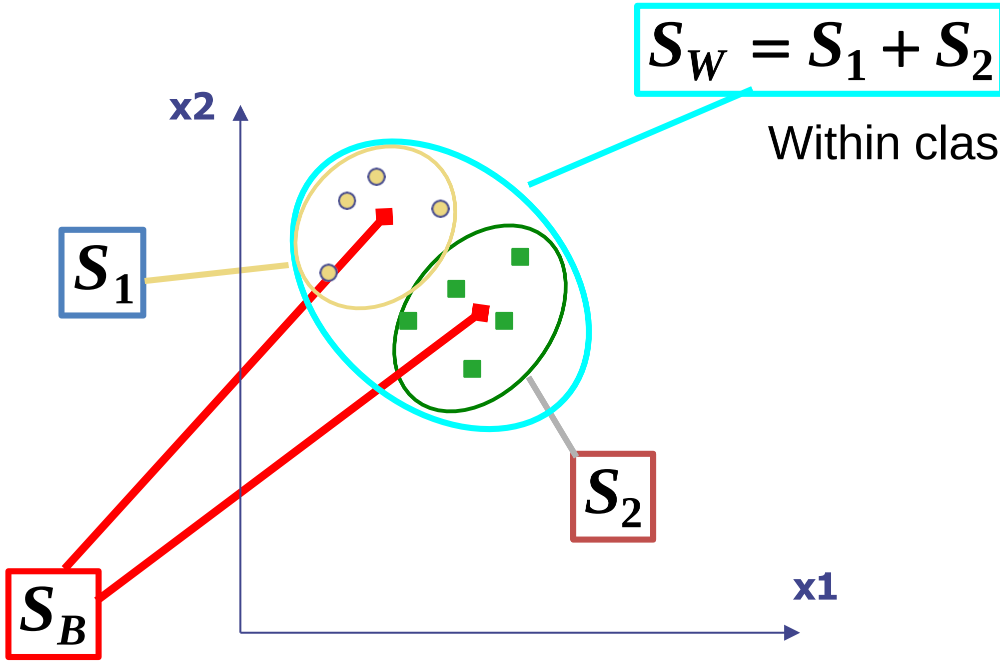
Visualization



Between class scatter

Linear Discriminant Analysis (LDA)

- Maximizing the ratio

$$J(w) = \frac{w^T S_B w}{w^T S_W w}$$

- Is equivalent to maximizing the numerator while keeping the denominator constant, i.e.

$$\boxed{\max_w w^T S_B w \quad \text{subject to} \quad w^T S_W w = K}$$

- And can be accomplished using Lagrange multipliers, where we define the Lagrangian as

$$\boxed{L = w^T S_B w - \lambda\left(w^T S_W w - K\right)}$$

- And maximize with respect to both w and λ

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Linear Discriminant Analysis (LDA)

- Setting the gradient of

$$L = w^T\left(S_B - \lambda S_W\right)w + \lambda K$$

With respect to w to zeros we get

$$\nabla_w L = 2\left(S_B - \lambda S_W\right)w = 0$$

or

$$\boxed{S_B w = \lambda S_W w}$$

- This is a generalized eigenvalue problem
- The solution is easy when $\boxed{S_w^{-1} = \left(\Sigma_1 + \Sigma_0\right)^{-1}}$

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Linear Discriminant Analysis (LDA)

- In this case

$$S_W^{-1} S_B w = \lambda w$$

- And using the definition of S_B

$$S_W^{-1} (\mu_1 - \mu_0)(\mu_1 - \mu_0)^T w = \lambda w$$

- Assuming that $(\mu_1 - \mu_0)^T w = \alpha$ is a scalar, this can be written as

$$S_W^{-1} (\mu_1 - \mu_0) = \frac{\lambda}{\alpha} w$$

- and since we don't care about the magnitude of w

$$\boxed{w^* = S_W^{-1} (\mu_1 - \mu_0) = (\Sigma_1 + \Sigma_0)^{-1} (\mu_1 - \mu_0)}$$

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LDA with n variables and C classes

Variables

- N Sample images: $\{\boldsymbol{x}_1, \cdots, \boldsymbol{x}_N\}$
- C classes: $\{Y_1, Y_2, \ldots, Y_c\}$
- Average of each class: $\boldsymbol{\mu}_i = \frac{1}{N_i} \sum_{x_k \in Y_i} \boldsymbol{x}_k$
- Average of all data: $\boldsymbol{\mu} = \frac{1}{N} \sum_{k=1}^{N} \boldsymbol{x}_k$

Scatter Matrices

- Scatter of class i: $S_i = \sum_{x_k \in Y_i} (x_k - \mu_i)(x_k - \mu_i)^T$
- Within class scatter: $S_W = \sum_{i=1}^{c} S_i$
- Between class scatter: $S_B = \sum_{i=1}^{C} \sum_{j \neq i} (\mu_i - \mu_j)(\mu_i - \mu_j)^T$ Time-consuming to compute
- Between class scatter (in practice):

$$S_B = S_T - S_W = \sum_{i=1}^{C} N_i (\mu_i - \mu)(\mu_i - \mu)^T$$

- Total scatter: $S_T = \sum_{\boldsymbol{x}} (\boldsymbol{x} - \boldsymbol{\mu})(\boldsymbol{x} - \boldsymbol{\mu})^T$

Mathematical Formulation

- Recall that we want to learn a projection W such that the projection converts all the points from x to a new space z:

$$z = w^{\mathrm{T}} x \qquad z \in \mathbf{R}^m \quad x \in \mathbf{R}^n$$

- After projection:
  - Between class scatter $\tilde{S}_B = W^T S_B W$
  - Within class scatter $\tilde{S}_W = W^T S_W W$
- So, the objective becomes:

$$W_{opt} = \arg\max_{W} \frac{|\tilde{S}_B|}{|\tilde{S}_W|} = \arg\max_{W} \frac{|W^T S_B W|}{|W^T S_W W|}$$

Mathematical Formulation

$$W_{opt} = \arg\max_{W} \frac{\left|W^T S_B W\right|}{\left|W^T S_W W\right|}$$

- Solve generalized eigenvector problem:

$$S_B w_i = \lambda_i S_W w_i \qquad i = 1, \ldots, m$$

Mathematical Formulation

- Solution: Generalized Eigenvectors

$$S_B w_i = \lambda_i S_W w_i \qquad\qquad i = 1, \ldots, m$$

- Rank of W_{opt} is limited
  - Rank(S_B) <= C-1
  - Rank(S_W) <= N-C

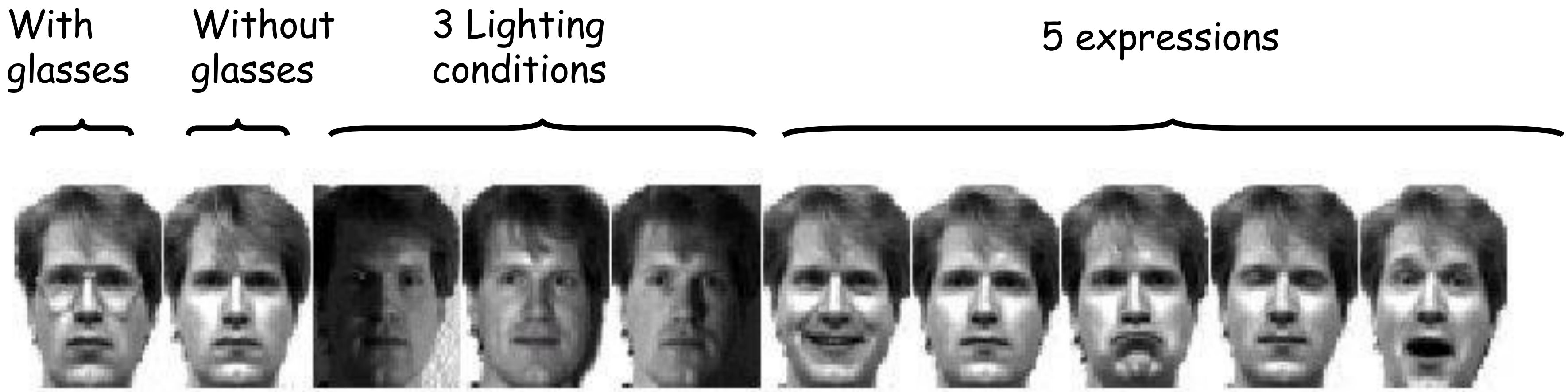
PCA vs. LDA

- Eigenfaces exploit the max scatter of the training images in face space
- Fisherfaces attempt to maximise the **between class scatter**, while minimising the **within class scatter**.

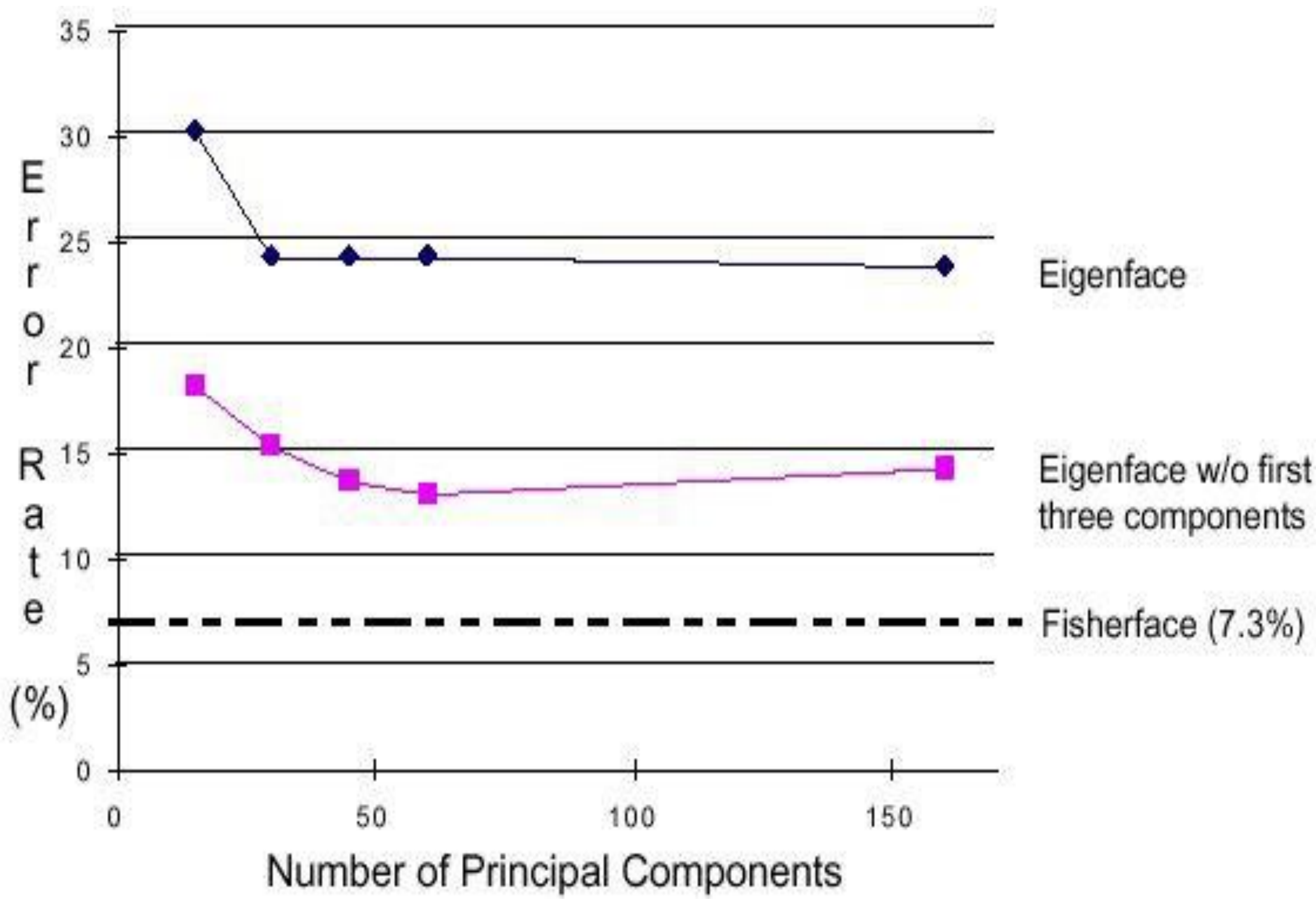
Results: Eigenface vs. Fisherface

- Input: 160 images of 16 people
- Train: 159 images
- Test: 1 image

- Variation in Facial Expression, Eyewear, and Lighting



Eigenface vs. Fisherface



What we have learned today

- Feature normalization
- Introduction to face recognition
- The Eigenfaces Algorithm
- Linear Discriminant Analysis (LDA)

Turk and Pentland, Eigenfaces for Recognition, *Journal of Cognitive Neuroscience* **3** (1): 71–86.

P. Belhumeur, J. Hespanha, and D. Kriegman. "Eigenfaces vs. Fisherfaces: Recognition Using Class Specific Linear Projection". *IEEE Transactions on pattern analysis and machine intelligence* **19** (7): 711. 1997.