《模式识别》

第二章 特征提取与表示

马锦华

https://cse.sysu.edu.cn/teacher/MaJinhua

SUN YAT-SEN University

中山大學 计算机学院

SCHOOL OF COMPUTER SCIENCE AND ENGINEERING, SYSU

课程目录（暂定）

第一章	课程简介与预备知识	6学时
第二章	特征提取与表示	6学时
第三章	主成分分析	3学时
第四章	归一化、判别分析、人脸识别	3学时
第五章	EM算法与聚类	3学时
第六章	贝叶斯决策理论	3学时
第七章	线性分类器与感知机	3学时
第八章	支持向量机	3学时
第九章	神经网络、正则项和优化方法	3学时
第十章	卷积神经网络及经典框架	3学时
第十一章	循环神经网络	3学时
第十二章	Transformer	3学时
第十三章	自监督与半监督学习	3学时
第十四章	开放世界模式识别	6学时

第一部分：模型拟合与局部特征

Most of our slides have been borrowed from the courses of Stanford CS131.

What we will learn today?

- A model fitting method for line detection
  – RANSAC
- Local invariant features
  – Motivation
  – Requirements, invariances
- Keypoint localization
  – Harris corner detector

What we will learn today?

- A model fitting method for line detection
  – RANSAC
- Local invariant features
  – Motivation
  – Requirements, invariances
- Keypoint localization
  – Harris corner detector

Fitting as Search in Parametric Space

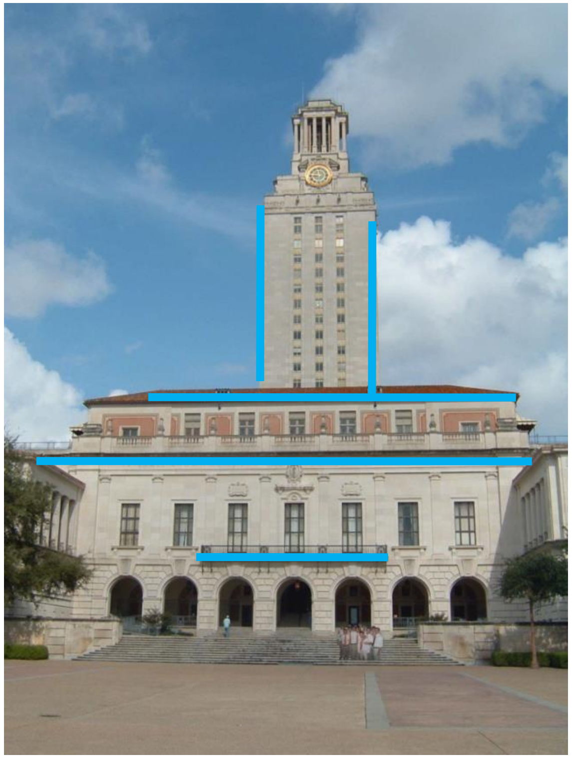
- **Let's say we have chosen a parametric model for a set of features:**
  - For example, we have a line equation that we want to fit to a set of edge points
- **We can 'search' in parameter space by trying many potential parameter values and see which set of parameters 'agree'/fit with our set of features**
- **Three main questions:**

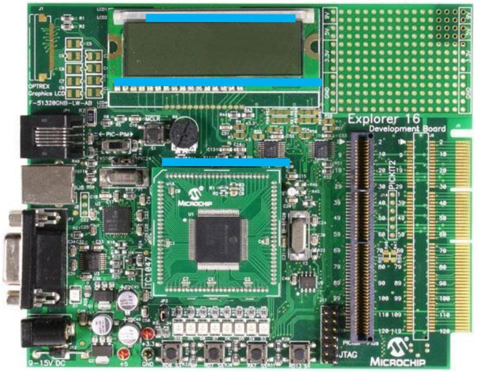
1. What model represents this set of features best?
2. Which of several model instances gets which feature?
3. How many model instances are there?

- **Computational complexity is important**
  - It is infeasible to examine every possible set of parameters and every possible combination of features

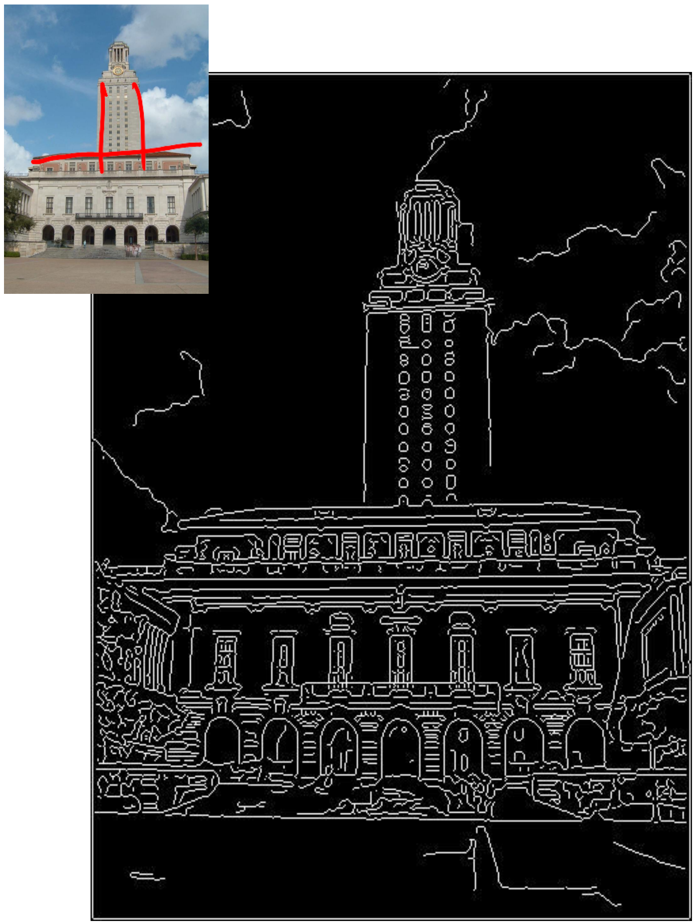
Example: Line Fitting

- Why fit lines? Many objects characterized by presence of straight lines:

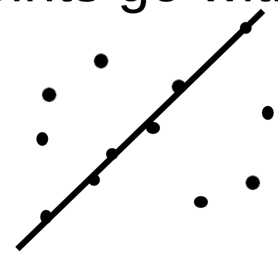




Difficulty of Line Fitting



- **Extra edge points (clutter), multiple models:**
  - Which points go with which line, if any?



- **Only some parts of each line detected, and some parts are missing:**
  - How to find a line that bridges missing evidence?
- **Noise in measured edge points, orientations:**
  - How to detect true underlying parameters?

Voting as a fitting technique

- **It's not feasible to check all combinations of features by fitting a model to each possible subset. For example, the naïve line fitting between every pair of two points is $O(N^2)$.**
- **Voting is a general technique where we let the features vote for all models that are compatible with it.**
  - Cycle through features, cast votes for model parameters.
  - Look for model parameters that receive a lot of votes.
- **Noise & clutter features will cast votes too, but typically their votes should be inconsistent with the majority of "good" features.**
- **Ok if some features not observed, as model can span multiple fragments.**

RANSAC (RANdom SAmple Consensus)

- **RANSAC [Fischler & Bolles 1981]**

  – Approach: we want to avoid the impact of outliers, so let's look for "inliers", and use only those.

- **Intuition: if an outlier is chosen to compute the current fit, then the resulting line won't have much support from rest of the points.**

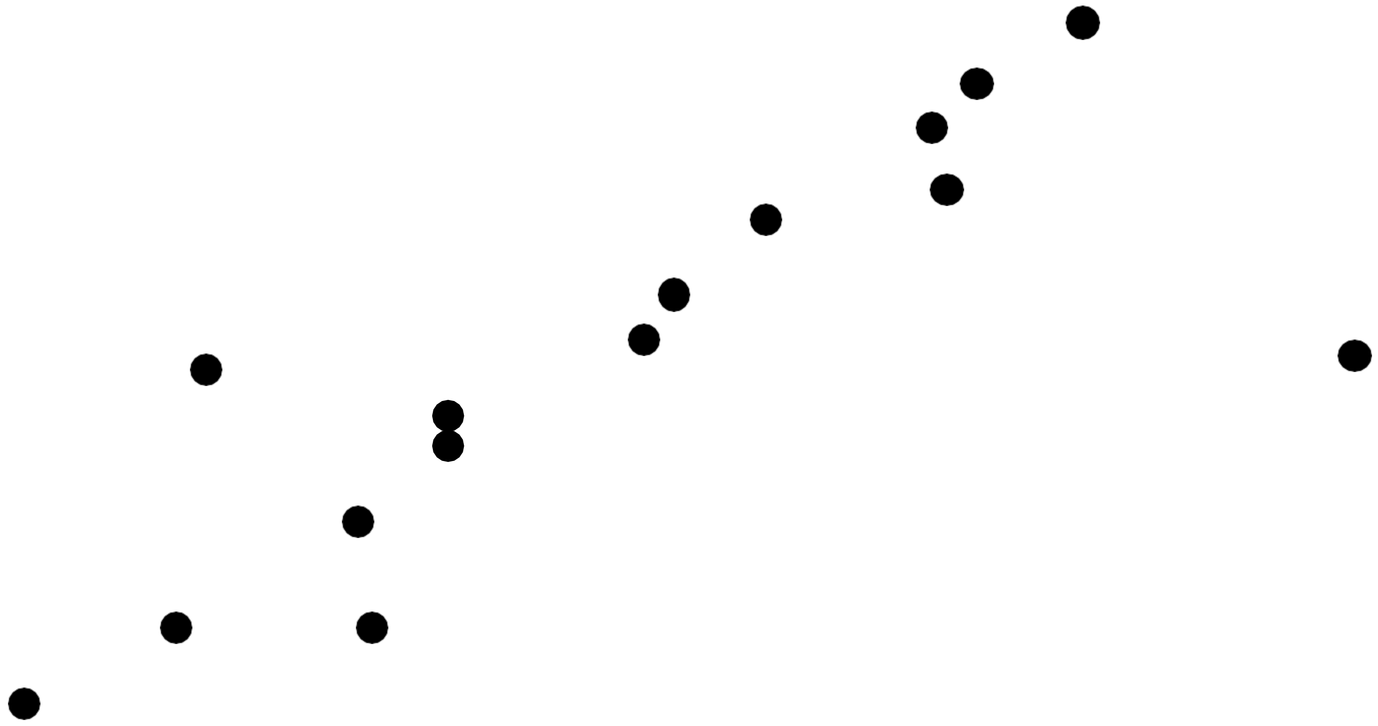
- **RANSAC loop:**

1. Randomly select a seed group of points on which to perform a model estimate (e.g., a group of good points)
2. Compute model parameters from seed group
3. Find inliers to this model
4. If the number of inliers is sufficiently large, re-compute least-squares estimate of model on all of the inliers

   – Keep the model with the largest number of inliers

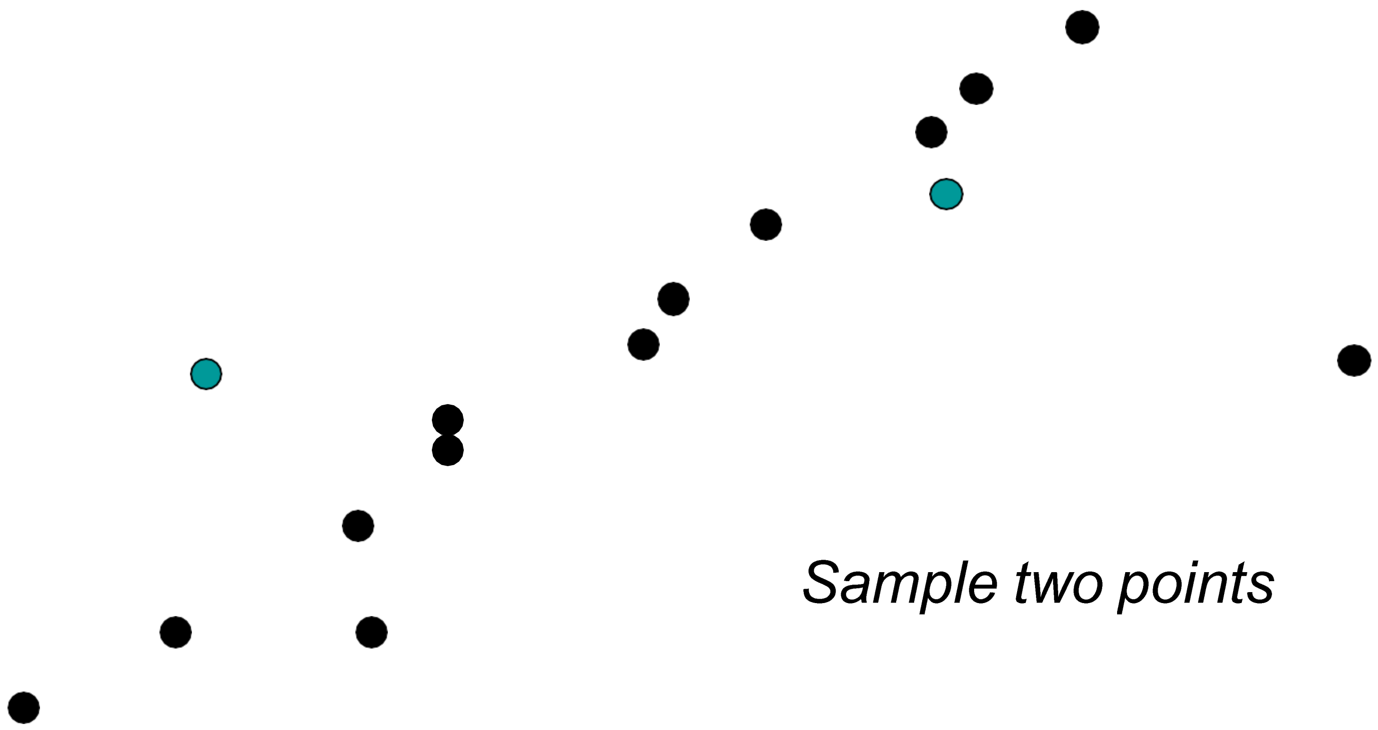
RANSAC Line Fitting Example

- **Task: Estimate the best line**
  - How many points do we need to estimate the line?



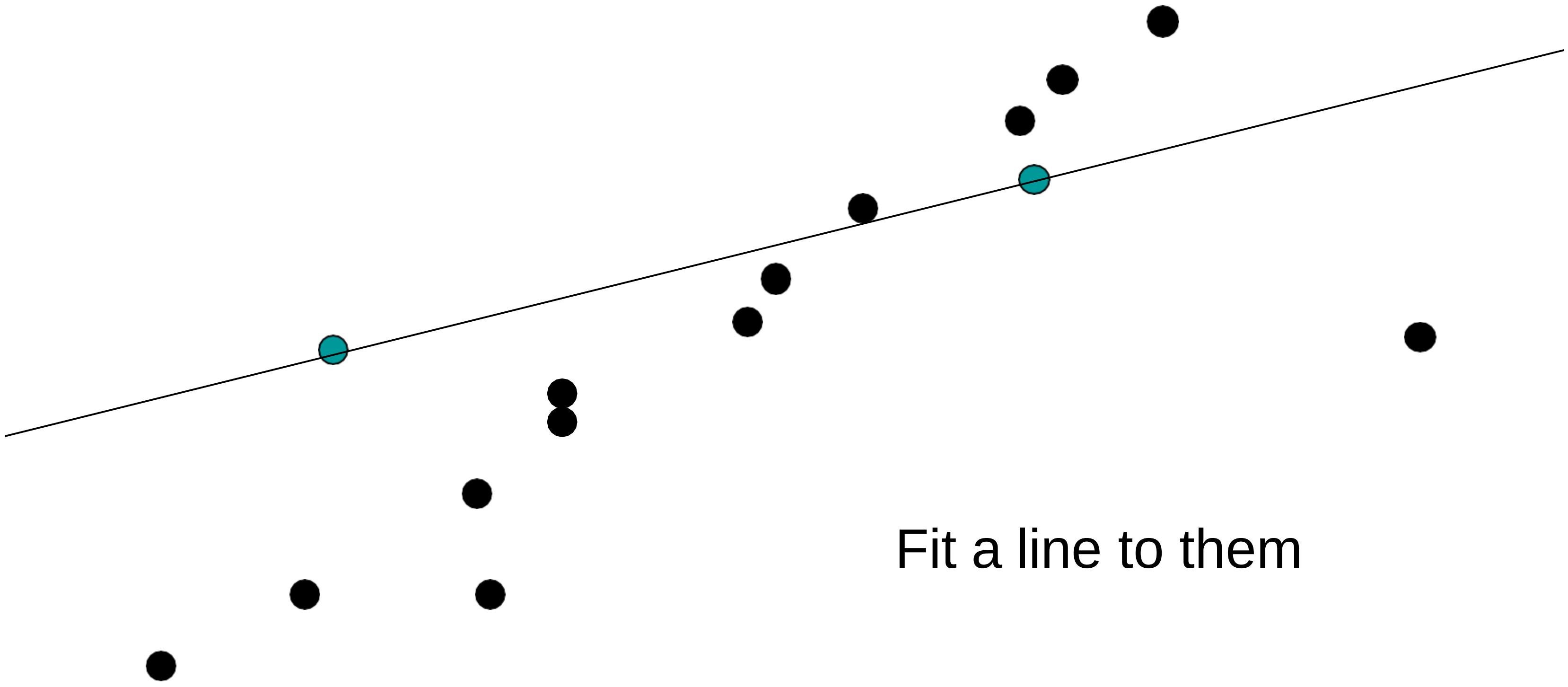
RANSAC Line Fitting Example

- **Task: Estimate the best line**



RANSAC Line Fitting Example

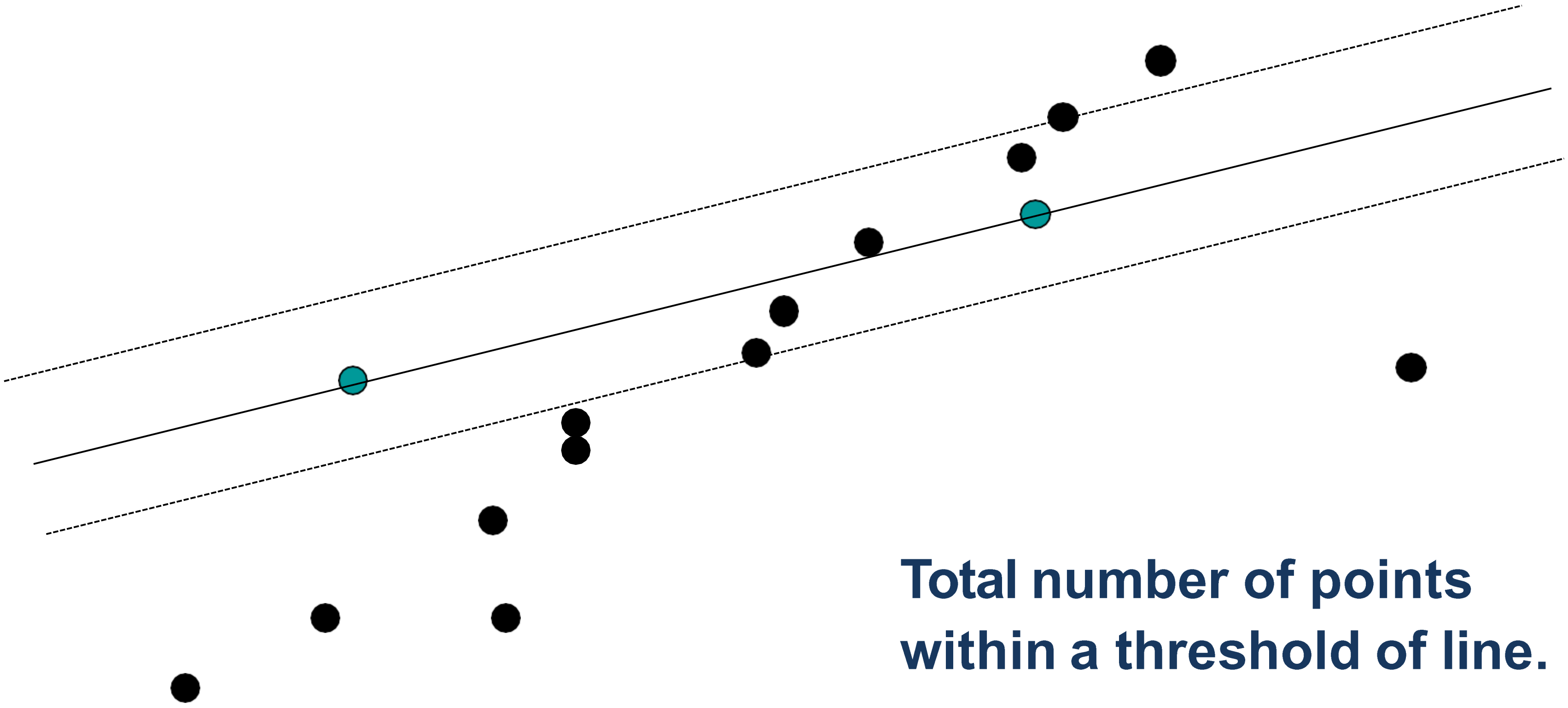
- **Task: Estimate the best line**



Fit a line to them

RANSAC Line Fitting Example

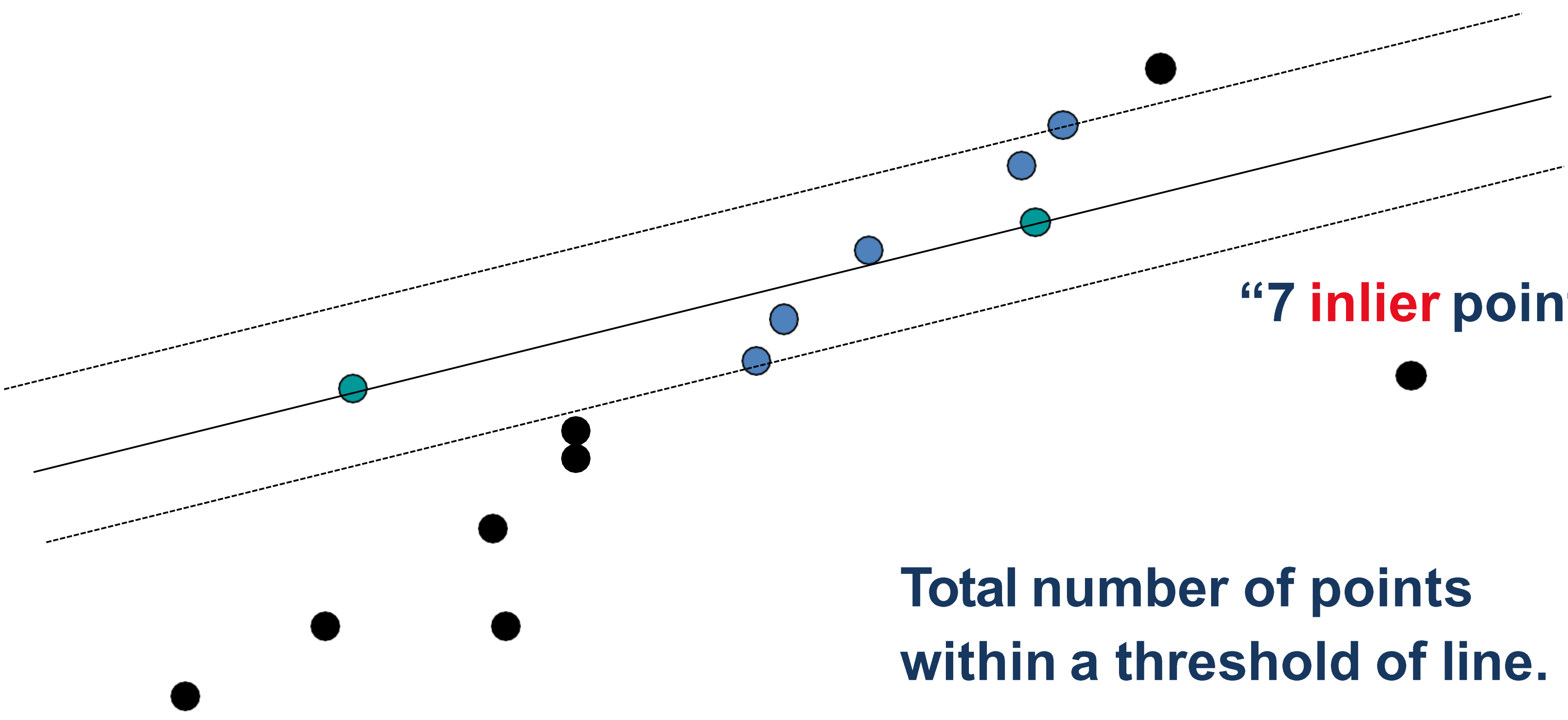
- **Task: Estimate the best line**



Total number of points within a threshold of line.

RANSAC Line Fitting Example

- **Task: Estimate the best line**

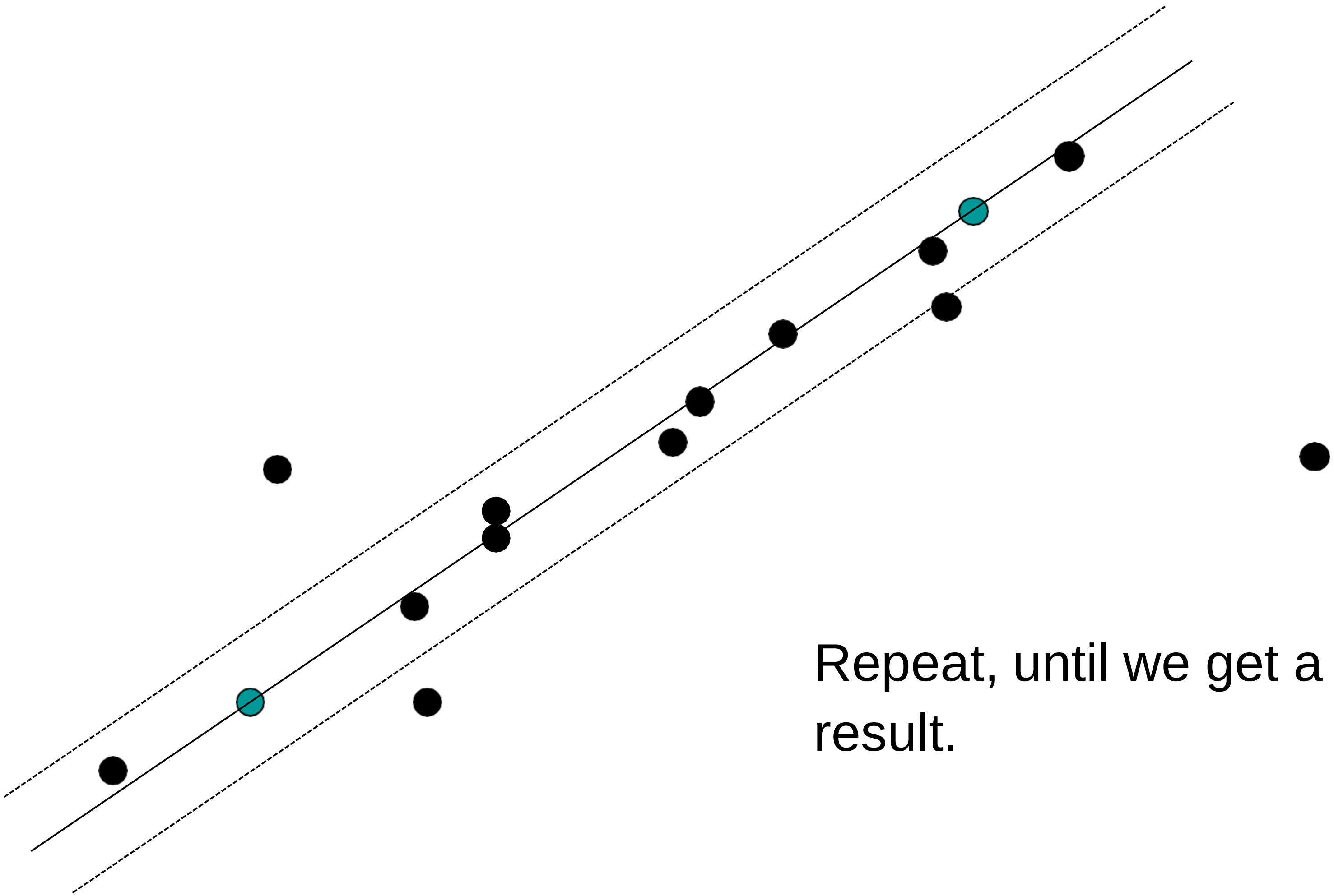


“7 inlier points”

Total number of points within a threshold of line.

RANSAC Line Fitting Example

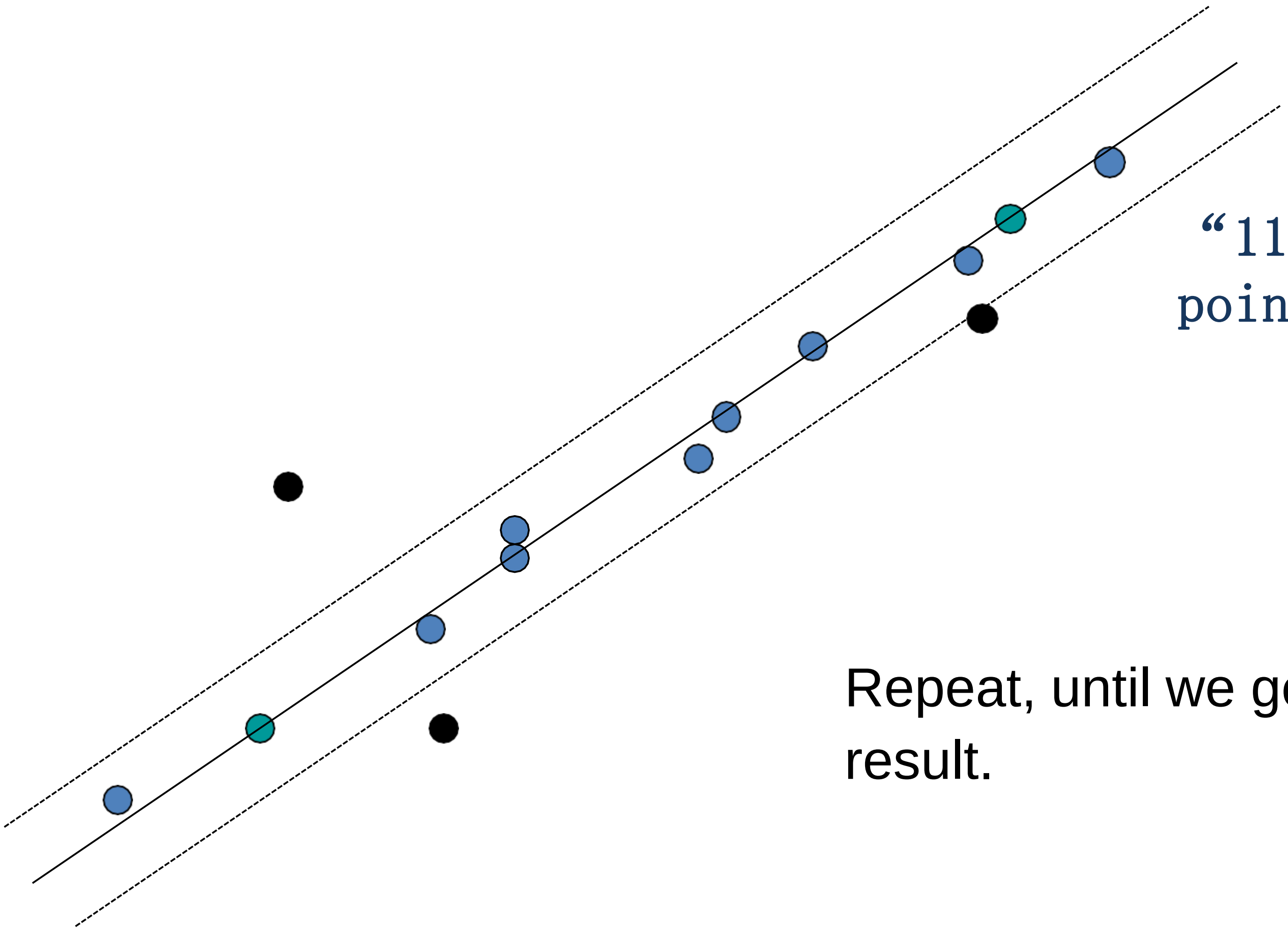
- **Task: Estimate the best line**



Repeat, until we get a good result.

RANSAC Line Fitting Example

- **Task: Estimate the best line**



"11 inlier points"

Repeat, until we get a good result.

Algorithm of RANSAC

Algorithm 15.4: RANSAC: fitting lines using random sample consensus

Determine:
- n — the smallest number of points required
- k — the number of iterations required
- t — the threshold used to identify a point that fits well
- d — the number of nearby points required to assert a model fits well

Until k iterations have occurred
- Draw a sample of n points from the data uniformly and at random
- Fit to that set of n points
- For each data point outside the sample
  - Test the distance from the point to the line against t; if the distance from the point to the line is less than t, the point is close
- end
- If there are d or more points close to the line then there is a good fit. Refit the line using all these points.

end

Use the best fit from this collection, using the fitting error as a criterion

RANSAC: How many iterations "k"?

- **How many samples (iterations) are needed?**
  - Suppose w is fraction of inliers (points from line).
  - n points needed to define hypothesis (e.g. 2 for lines)
  - k samples chosen.
- **Prob. that a single sample of n points is correct:** $\mathrm{w}^{\boldsymbol{n}}$
- **Prob. that a single sample of n points fails:** $\mathbf{1} - \mathrm{w}^{\boldsymbol{n}}$
- **Prob. that all k samples fail is:** $(\mathbf{1} - \mathrm{w}^{\boldsymbol{n}})^{\boldsymbol{k}}$
- **Prob. that at least one of the k samples is correct:**

$\mathbf{1} - (\mathbf{1} - \mathrm{w}^{\boldsymbol{n}})^{\boldsymbol{k}}$

- $\Rightarrow$ Choose k high enough to keep this below desired failure rate.

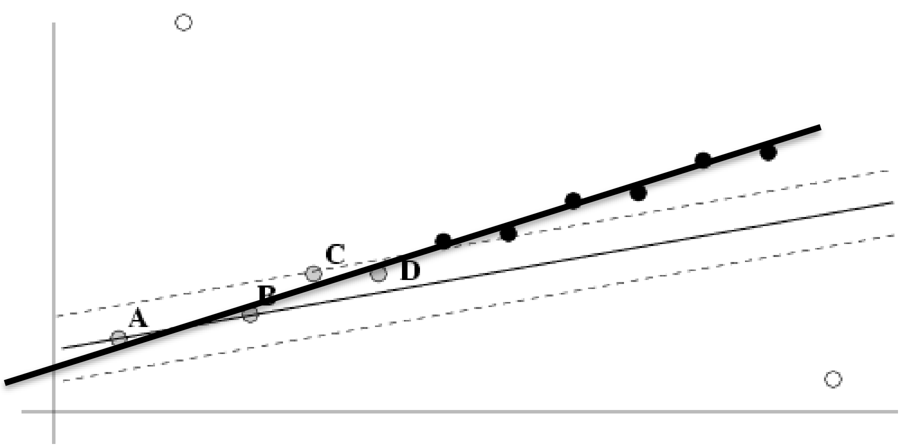
RANSAC: Computed k (p=0.99)

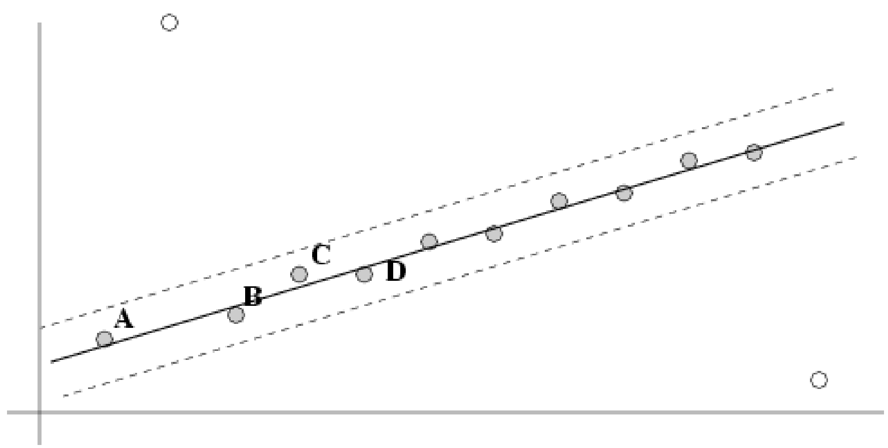
$$k = \frac{\log(z)}{\log(1 - w^n)} \qquad z = 1 - p$$

Sample size n	Proportion of outliers 5%	10%	20%	25%	30%	40%	50%
2	2	3	5	6	7	11	17
3	3	4	7	9	11	19	35
4	3	5	9	13	17	34	72
5	4	6	12	17	26	57	146
6	4	7	16	24	37	97	293
7	4	8	20	33	54	163	588
8	5	9	26	44	78	272	1177

Refining RANSAC estimate

- RANSAC computes its best estimate from a minimal sample of n points, and divides all data points into inliers and outliers using this estimate.
- We can improve this initial estimate by estimation over all inliers (e.g. with standard least-squares minimization).
- But this may change inliers, so alternate fitting with re-classification as inlier/outlier.





RANSAC: Pros and Cons

- **Pros:**

  – General method suited for a wide range of model fitting problems

  – Easy to implement and easy to calculate its failure rate (1-p)

- **Cons:**

  – Only handles a moderate percentage of outliers without cost blowing up

  – Many real problems have high rate of outliers (but sometimes selective choice of random subsets can help)

- **A voting strategy, The Hough transform [Hough, 1959], can handle high percentage of outliers**

  – Each point votes separately

  – But complexity of search time increases exponentially with the number of model parameters (e.g. 3 for circle)

Summary

- **RANSAC**
  - Algorithm
  - Analysis
    - Number of samples
    - Pros and cons

What we will learn today?

- A model fitting method for line detection
  - RANSAC
- Local invariant features
  - Motivation
  - Requirements, invariances
- Keypoint localization
  - Harris corner detector

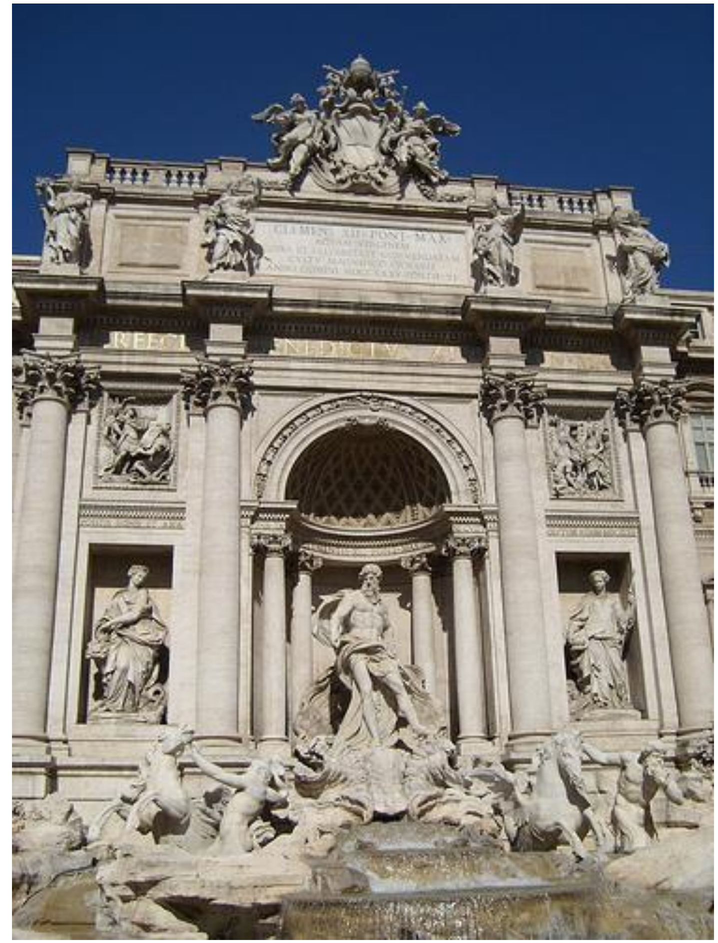
Some background reading:
Rick Szeliski, Chapter 4.1.1; David Lowe, IJCV 2004

Image matching: a challenging problem

Template

Query

Image matching: a challenging problem



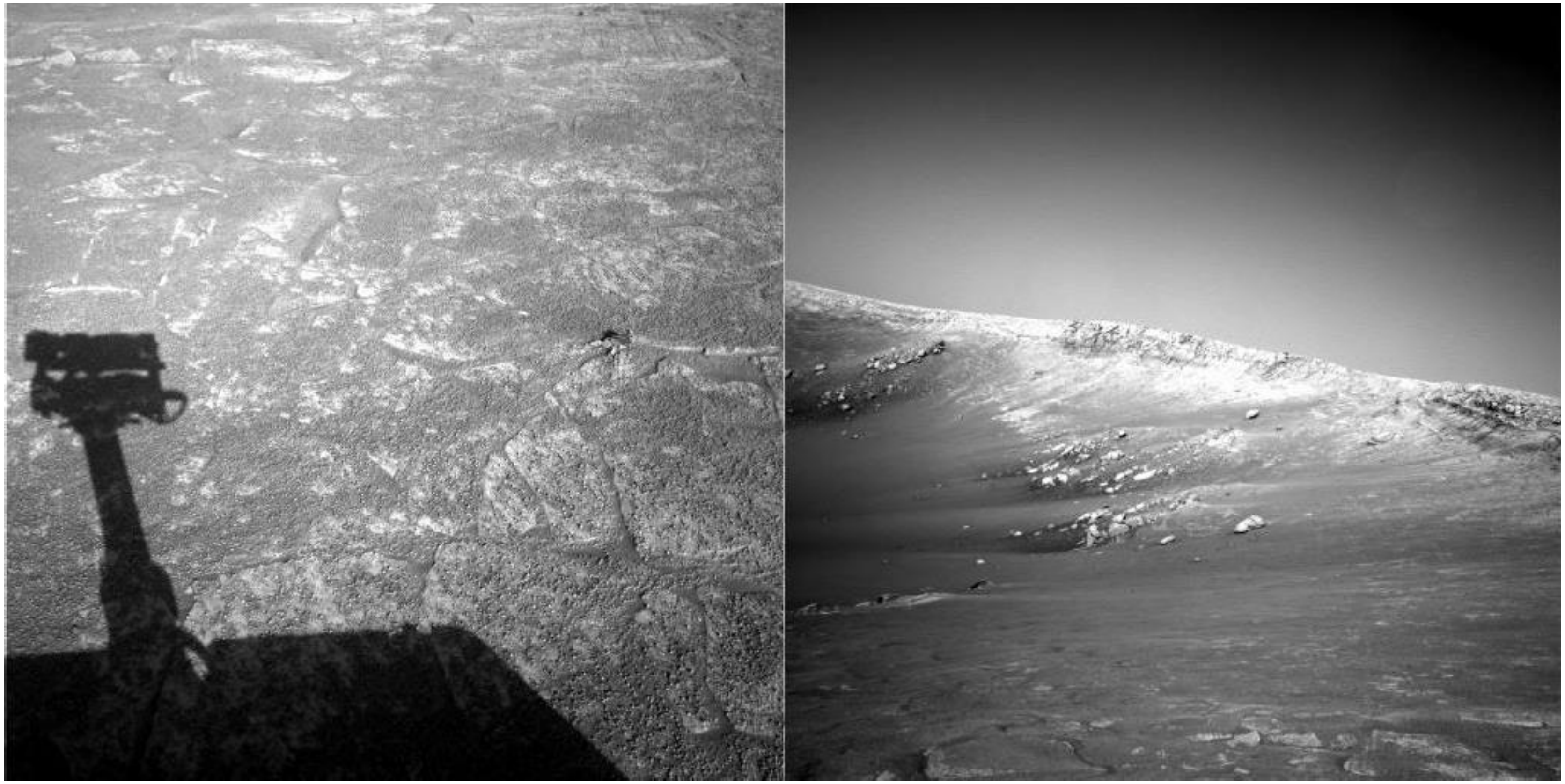
Different photos of the same location, Roma Trevi Fountain

Image matching: a challenging problem

- **Harder case:**

Image matching: a challenging problem

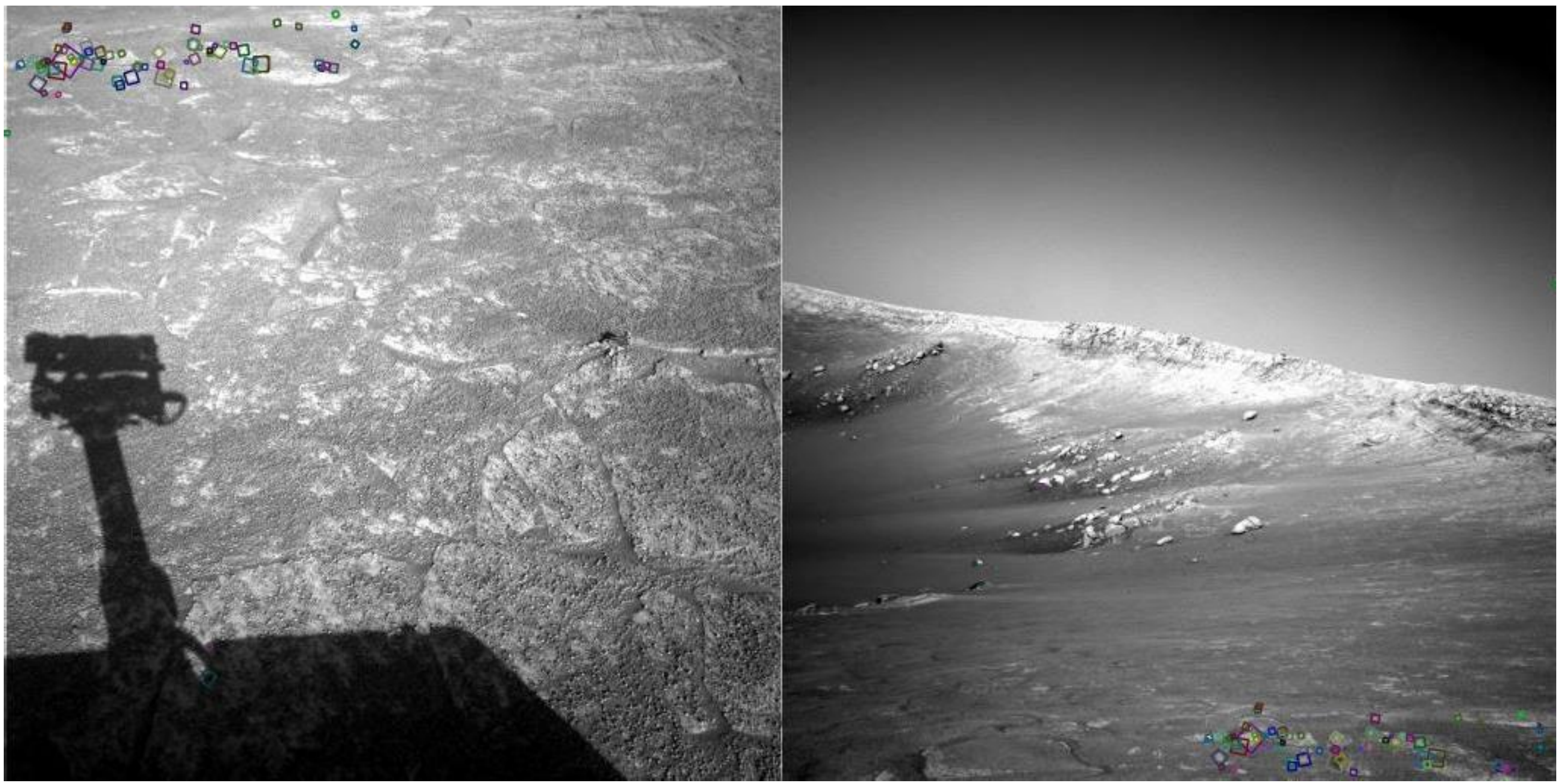
- **Harder Still?**



NASA Mars Rover images

Image matching: a challenging problem

- **Answer Below (Look for tiny colored squares)**

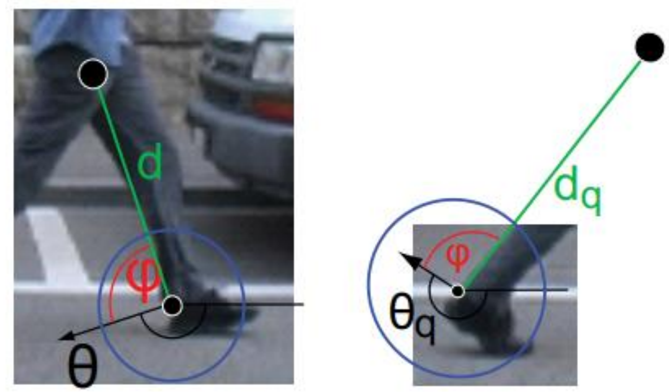


NASA Mars Rover images with SIFT feature matches

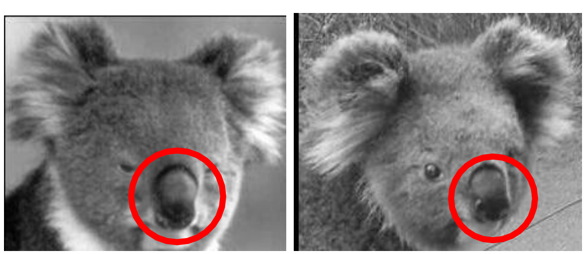
Motivation for using local features

- **Global representations have major limitations.**
- **Instead, describe and match only local regions**
- **Increased robustness to**
  - *Occlusions*

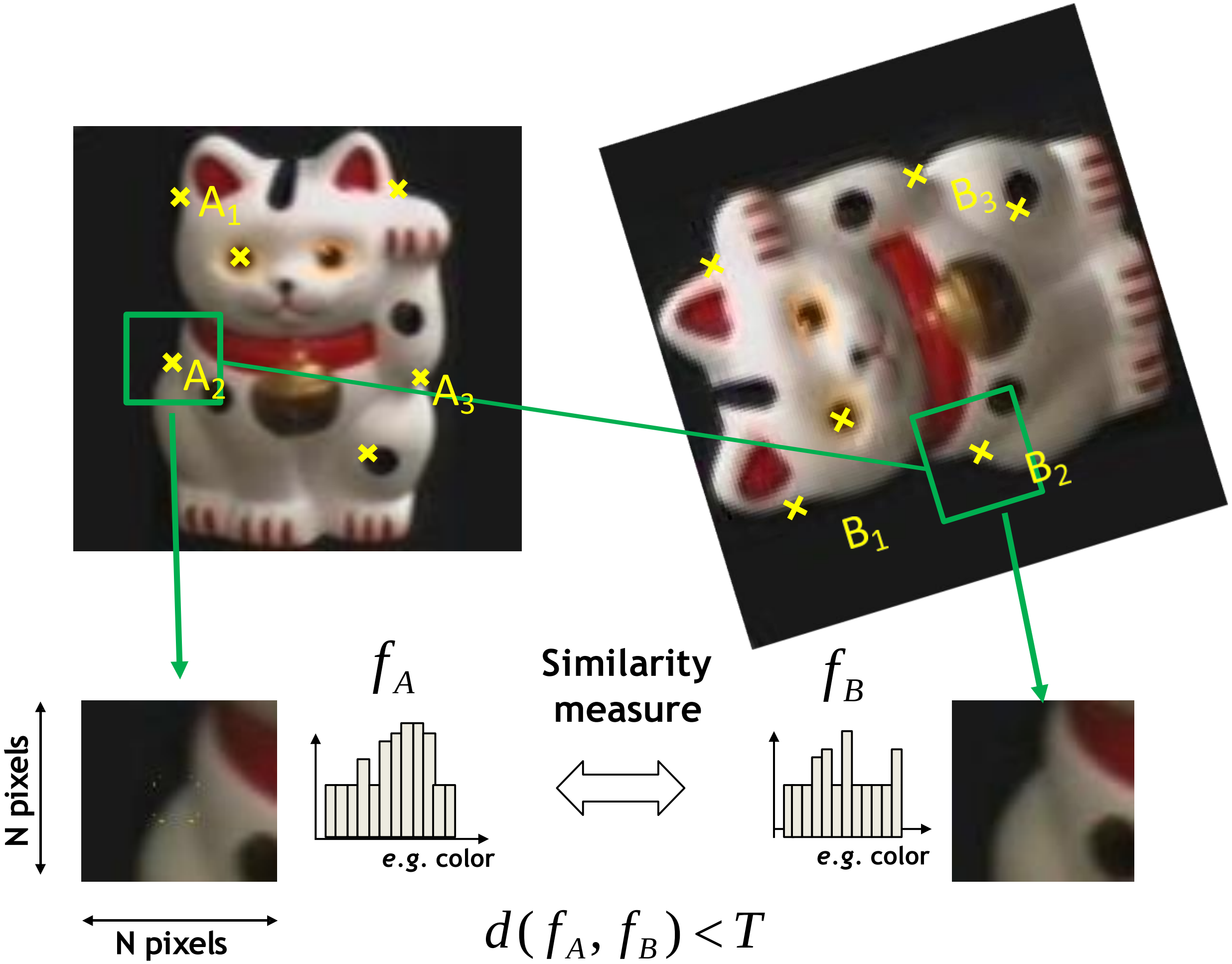
  - *Articulation*



  - *Intra-category variations*



General Approach



1. Find a set of distinctive key-points
2. Define a region around each keypoint
3. Extract and normalize the region content
4. Compute a local descriptor from the normalized region
5. Match local descriptors

Common Requirements

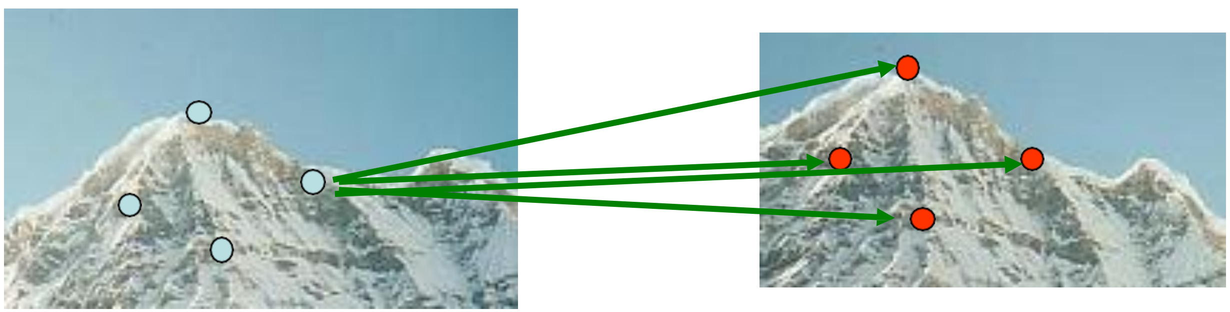
- Problem 1:
  – Detect the same point independently in both images

No chance to match!

We need a repeatable detector!

Common Requirements

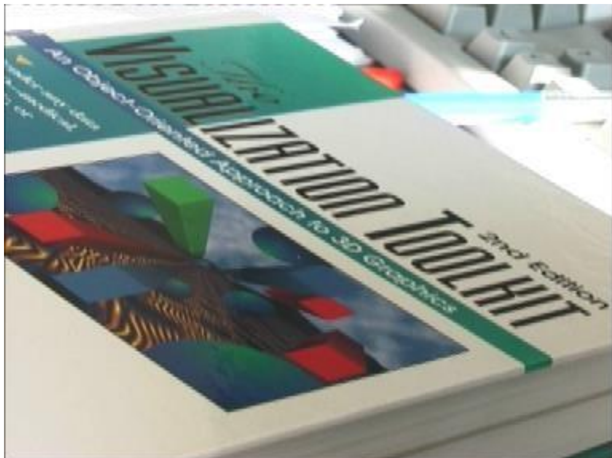
- Problem 1:
  – Detect the same point independently in both images
- Problem 2:
  – For each point correctly recognize the corresponding one

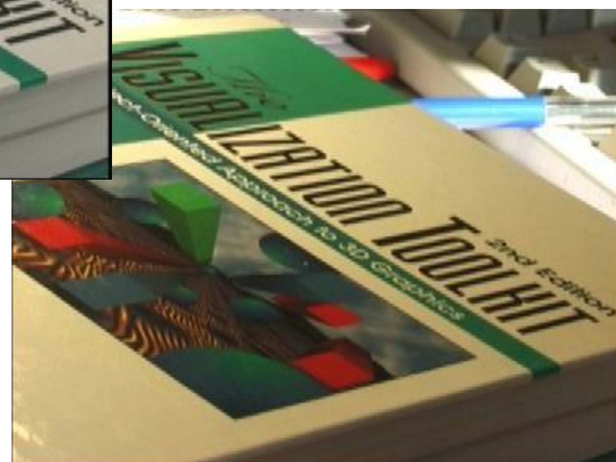


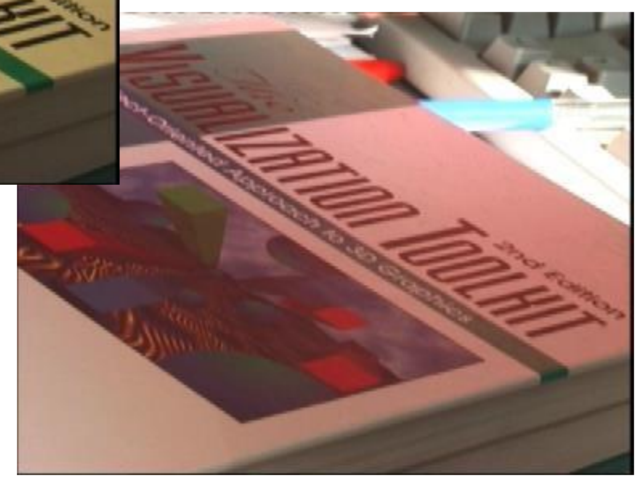
We need a repeatable detector!

Feature Invariances: Geometric Transformations

Feature Invariances: Photometric Transformations







- Often modeled as a linear transformation:
  – Scaling + Offset

Requirements for Local Features

- **Region extraction needs to be repeatable and accurate**
  - Invariant to translation, rotation, scale changes
  - Robust or covariant to out-of-plane (≈affine) transformations
  - Robust to lighting variations, noise, blur, quantization
- **Locality: Features are local, therefore robust to occlusion and clutter.**
- **Quantity: We need a sufficient number of regions to cover the object.**
- **Distinctiveness: The regions should contain "interesting" structure.**
- **Efficiency: Close to real-time performance.**

Many Existing Detectors Available

- Hessian & Harris [Beaudet ‘78], [Harris ‘88]
- Laplacian, DoG [Lindeberg ‘98], [Lowe ‘99]
- Harris-/Hessian-Laplace [Mikolajczyk & Schmid ‘01]
- Harris-/Hessian-Affine [Mikolajczyk & Schmid ‘04]
- EBR and IBR [Tuytelaars & Van Gool ‘04]
- MSER [Matas ‘02]
- Salient Regions [Kadir & Brady ‘01]
- Others…

- Those detectors have become a basic building block for many applications in Computer Vision

Summary

- **Region extraction needs to be repeatable and accurate**
  – Invariant to translation, rotation, scale changes
  – Robust or covariant to out-of-plane (≈affine) transformations
  – Robust to lighting variations, noise, blur, quantization
- **Local invariant features**
  – Motivation
  – General approach and requirements

What we will learn today?

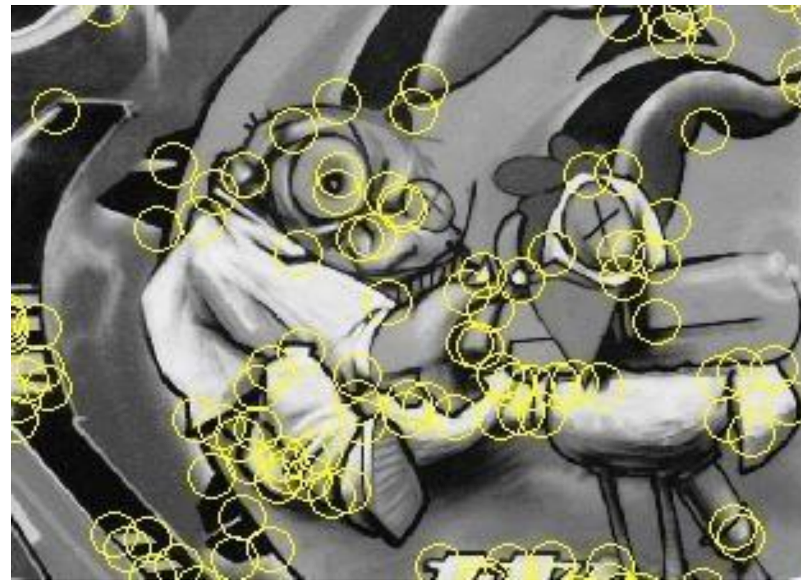
- A model fitting method for line detection
  - RANSAC
- Local invariant features
  - Motivation
  - Requirements, invariances
- Keypoint localization
  - Harris corner detector

Keypoint Localization

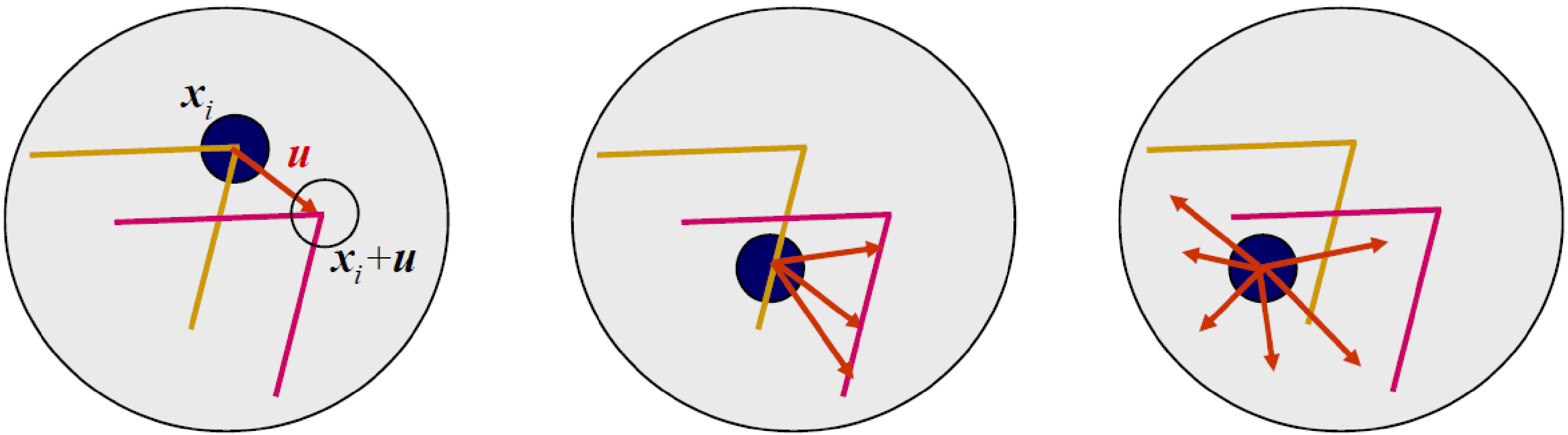
- **Goals:**

1. Repeatable detection
2. Precise localization
3. Interesting content

⇒ **Look for two-dimensional signal changes**

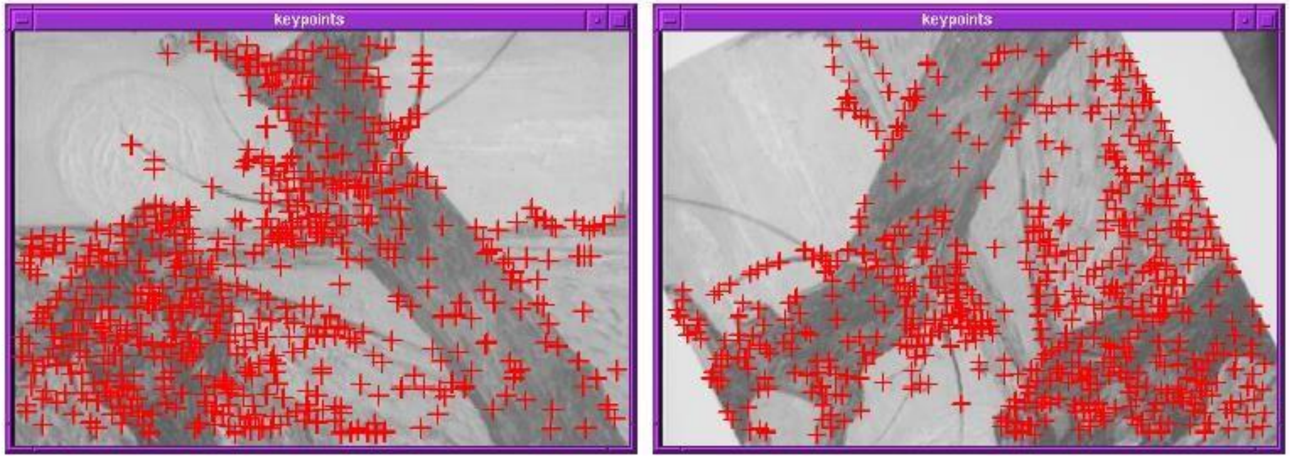


What are good keypoints?



(a) Corner (b) Edge (c) Textureless

Finding Corners



- **Key property:**

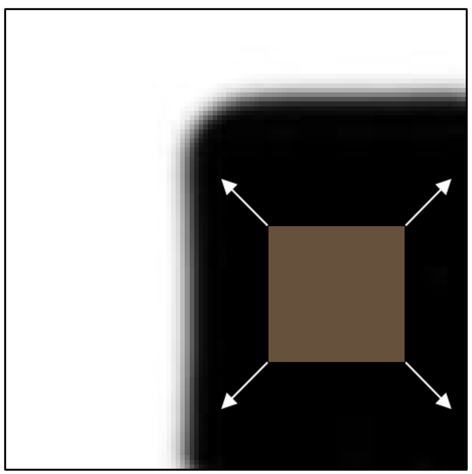
In the region around a corner, the image gradient has two or more dominant directions.

- **Corners are repeatable and distinctive**

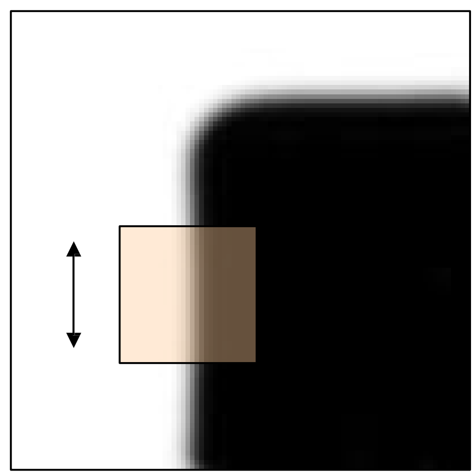
Corners as Distinctive Interest Points

- **Design criteria**

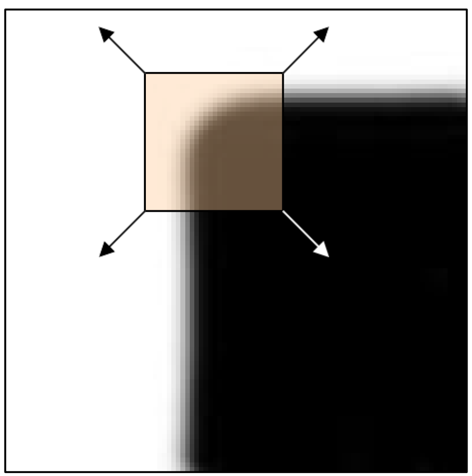
1. We should easily recognize the corner point by looking through a small window (**locality**).
2. Shifting the window in any direction should give a large change in intensity (**good localization**).



"flat" region: no change in all directions

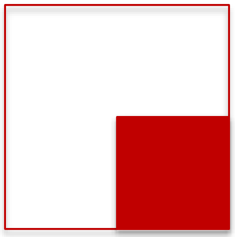


"edge": no change along the edge direction



"corner": significant change in all directions

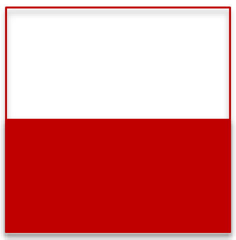
Corners versus edges



$\sum I_x^2 \longrightarrow$ Large

$\sum I_y^2 \longrightarrow$ Large

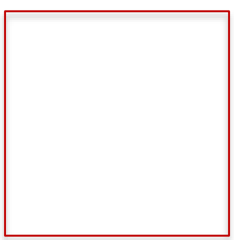
Corner



$\sum I_x^2 \longrightarrow$ Small

$\sum I_y^2 \longrightarrow$ Large

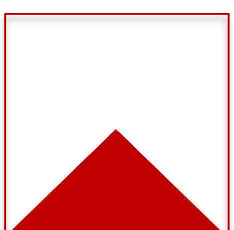
Edge



$\sum I_x^2 \longrightarrow$ Small

$\sum I_y^2 \longrightarrow$ Small

Flat



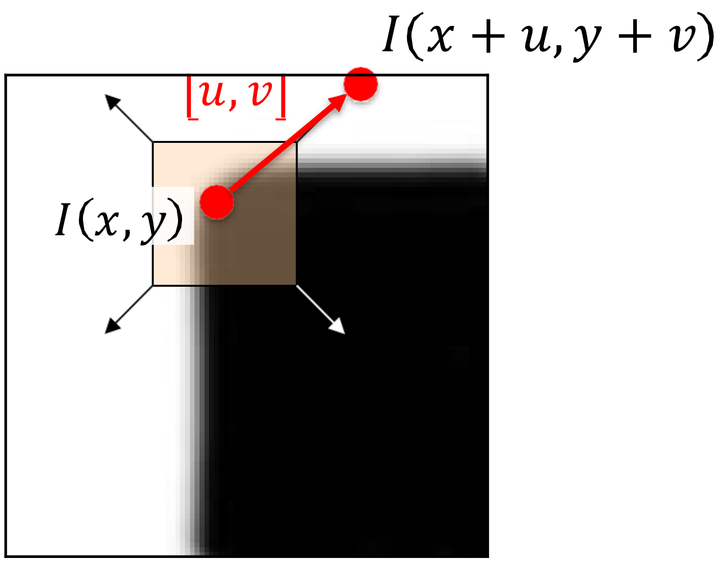
$\sum I_x^2 \longrightarrow$??

$\sum I_y^2 \longrightarrow$??

Corner

Harris Detector Formulation

- Localize patches that result in large change of intensity when shifted in any direction.
- When we shift by $[u, v]$, the intensity change at the center pixel is:



- *Measure change as intensity difference:*

$$(I(x+u, y+v) - I(x, y))$$

- *That's for a single point, but we have to accumulate over the patch or "small window" around that point…*

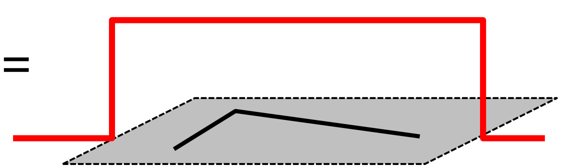
"corner": significant change in all directions

Harris Detector Formulation

- When we shift by [u, v], the change in intensity for the "small window" is:

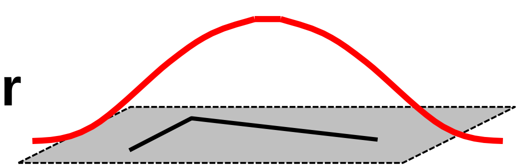
$$E(u,v) = \underbrace{\sum_{x,y}}_{\text{Sum over window}} \underbrace{w(x,y)}_{\text{Window function}} \underbrace{\big[\underbrace{I(x+u,y+v)}_{\text{Shifted intensity}} - \underbrace{I(x,y)}_{\text{Intensity}}\big]^2}_{\text{Intensity change}}$$

Window function $w(x,y) =$



1 in window, 0 outside

or



Gaussian

Harris Detector Formulation

- This measure of change can be approximated by (Taylor expansion):

$$E(u,v) \approx [u \;\; v] \; M \begin{bmatrix} u \\ v \end{bmatrix}$$

where M is a 2×2 matrix computed from image derivatives:

Auto-correlation matrix of gradients

$$M = \sum_{x,y} w(x,y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

Gradient with respect to x, times gradient with respect to y

Sum over image region – the area we are checking for corner

Harris Detector Formulation

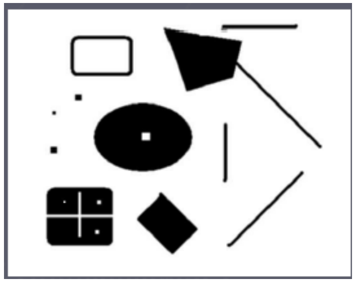
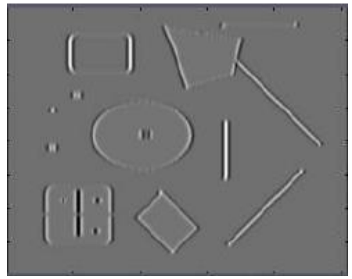
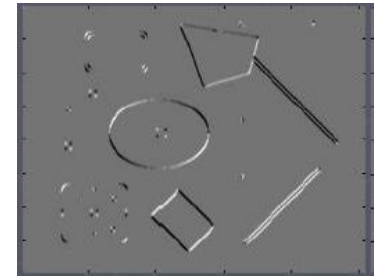


Image I



I_x

I_y



$I_x I_y$

- *where M is a 2×2 matrix computed from image derivatives:*

$$M = \sum_{x,y} w(x,y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

Gradient with respect to x, times gradient with respect to y

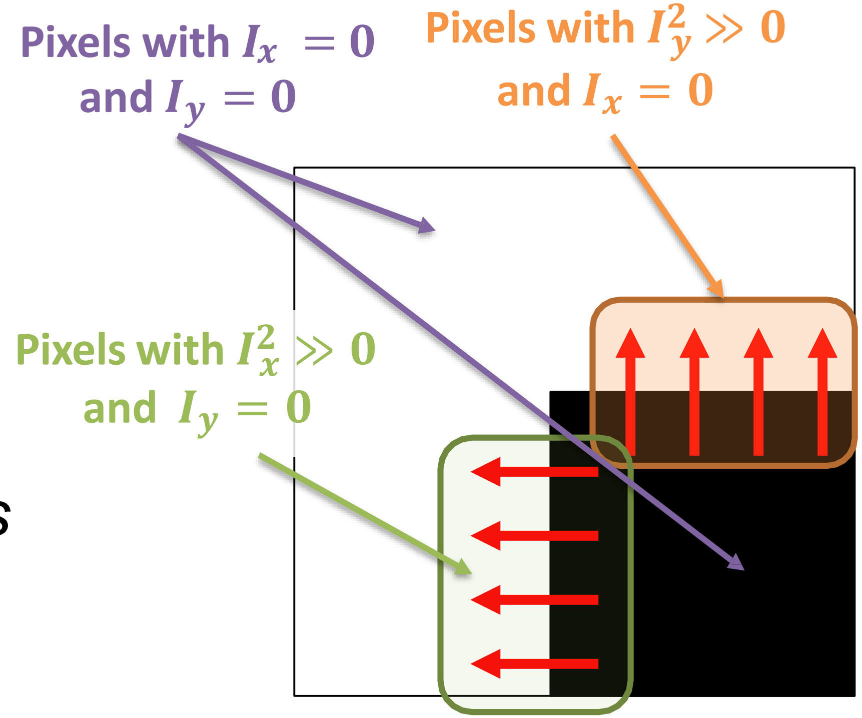
Sum over image region – the area we are checking for corner

$$M = \begin{bmatrix} \sum I_x I_x & \sum I_x I_y \\ \sum I_x I_y & \sum I_y I_y \end{bmatrix} = \sum \begin{bmatrix} I_x \\ I_y \end{bmatrix} [I_x \ I_y]$$

What Does This Matrix Reveal?

- *First, let's consider an axis-aligned corner.*
- *In that case, the dominant gradient directions align with the x or the y axis。*

$$M = \begin{bmatrix} \sum I_x^2 & \sum I_x I_y \\ \sum I_x I_y & \sum I_y^2 \end{bmatrix} = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$$



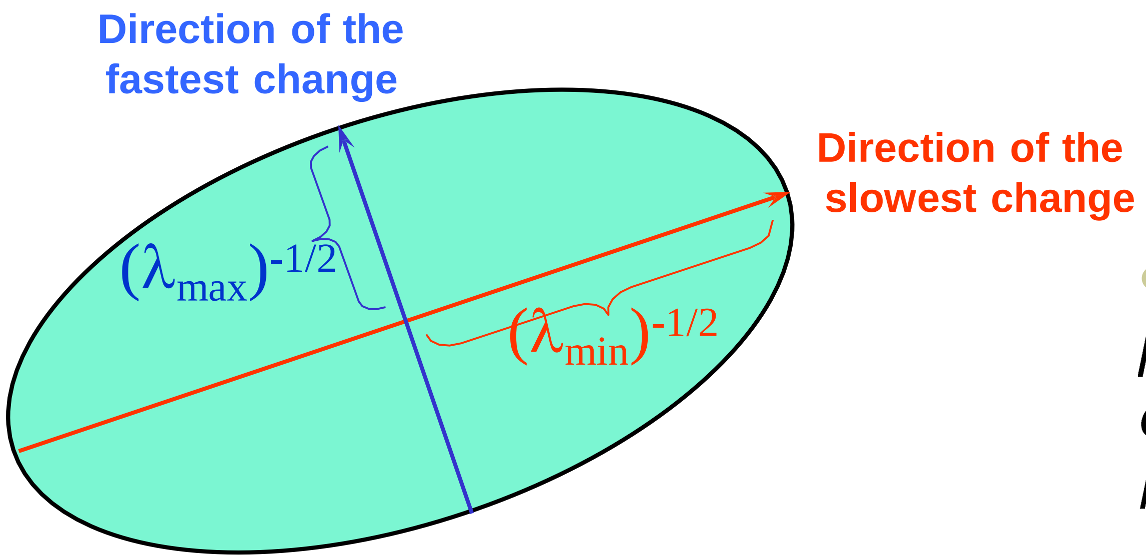
- *This means: if either λ is close to 0, then this is not a corner, so look for image windows where both λ are large.*
- *What if we have a corner that is not aligned with the image axes?*

General Case

- *Since* $M = \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$ *is symmetric, we can re-rewrite*

$$M = R^{-1} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} R$$. **(Eigenvalue decomposition)**

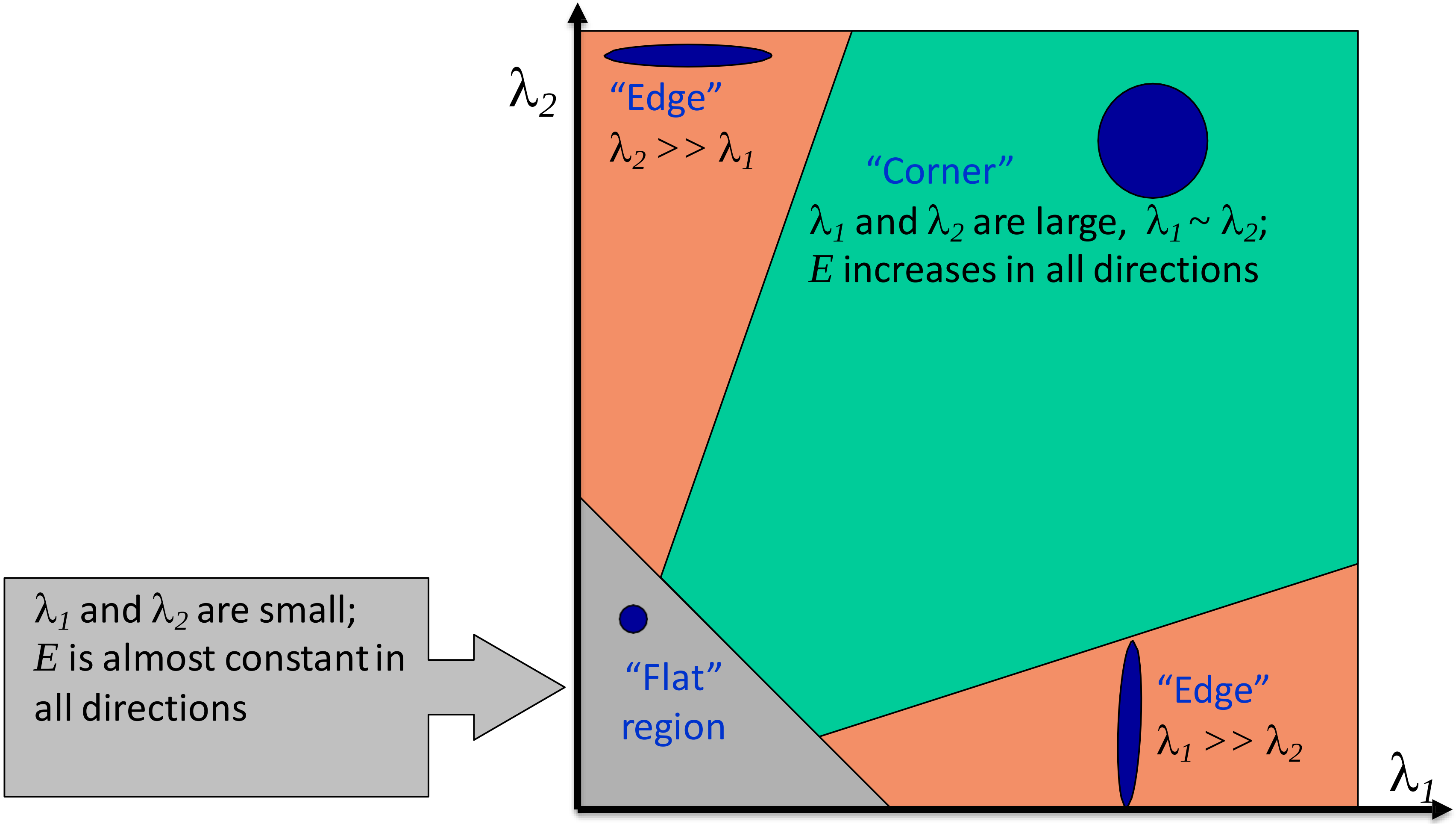
- We can think of M as an ellipse with its axis lengths determined by the eigenvalues λ_1 and λ_2; and its orientation determined by R



- *A rotated corner would produce the same eigenvalues as its non-rotated version.*

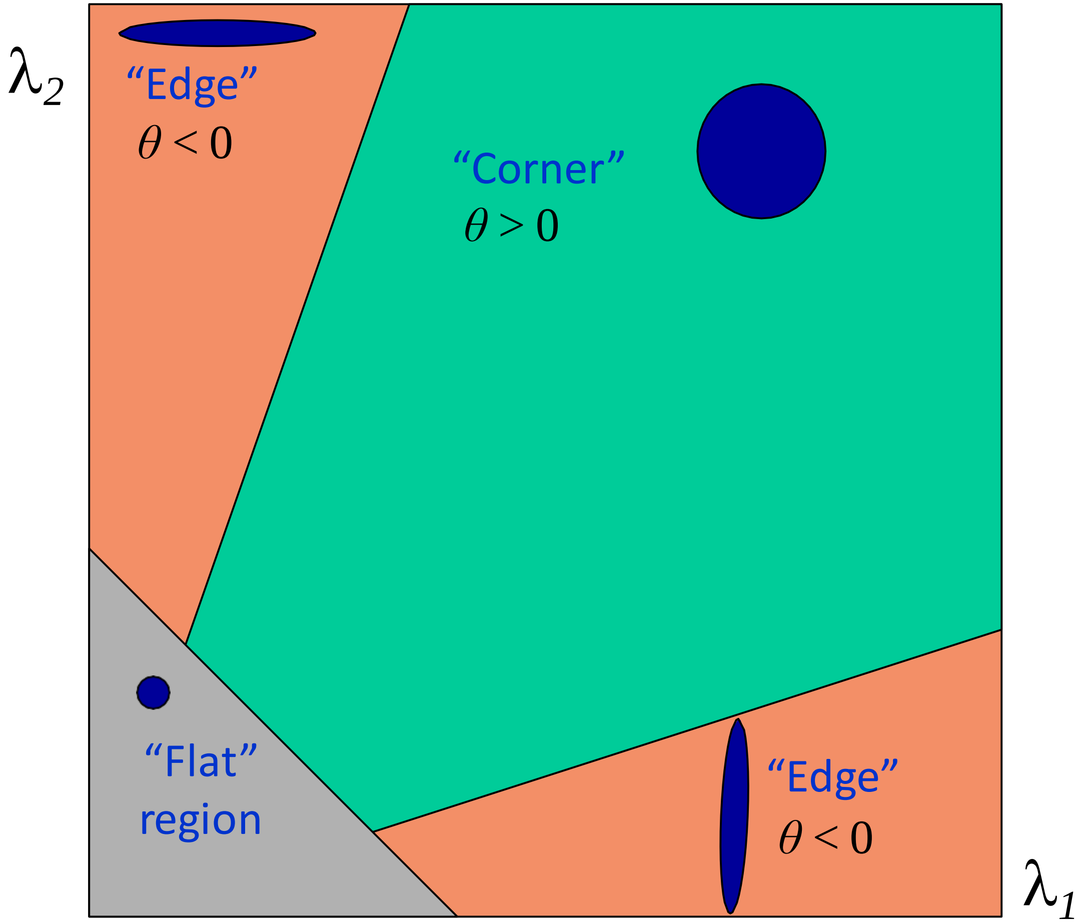
Interpreting the Eigenvalues

- *Classification of image points using eigenvalues of M:*



Corner Response Function

$$\theta = \det(M) - \alpha\,\mathrm{trace}(M)^2 = \lambda_1\lambda_2 - \alpha(\lambda_1+\lambda_2)^2$$



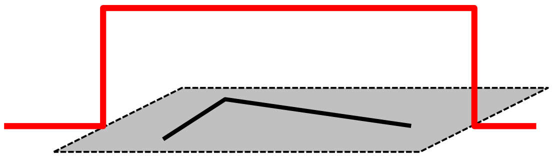
Fast approximation [Harris and Stephens, 1988]
– Avoid computing the eigenvalues
– α: constant
(0.04 to 0.06)

Window Function w(x,y)

$$M = \sum_{x,y} w(x,y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

- Option 1: uniform window
  - Sum over square window

$$M = \sum_{x,y} \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

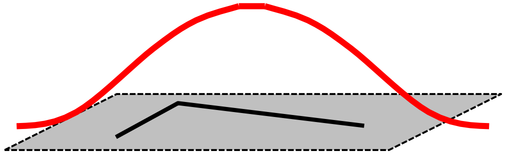


1 in window, 0 outside

  - Problem: not rotation invariant

- Option 2: Smooth with Gaussian
  - Gaussian already performs weighted sum

$$M = g(\sigma) * \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

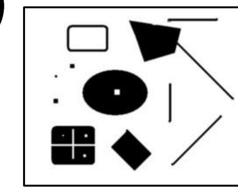


Gaussian

  - Result is rotation invariant

Summary: Harris Detector

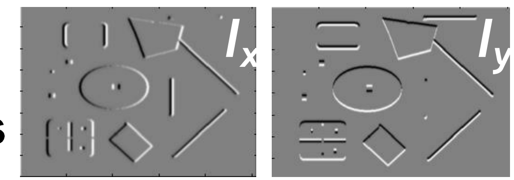
- *Compute second moment matrix (autocorrelation matrix)*



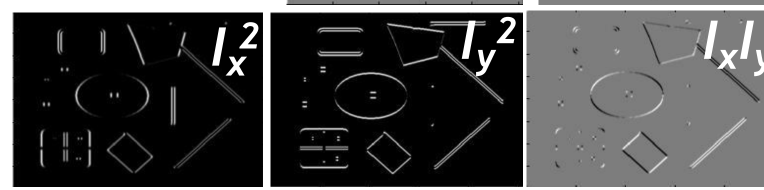
$$M(\sigma_I,\sigma_D)=g(\sigma_I)*\begin{bmatrix} I_x^2(\sigma_D) & I_xI_y(\sigma_D) \\ I_xI_y(\sigma_D) & I_y^2(\sigma_D) \end{bmatrix}$$

σ_D: for Gaussian in the derivative calculation
σ_I: for Gaussian in the windowing function

1. Image derivatives



2. Square of derivatives



3. Gaussian filter $g(\sigma_I)$

4. Cornerness function – two strong eigenvalues

$$\theta=\det[M(\sigma_I,\sigma_D)]-\alpha[\operatorname{trace}(M(\sigma_I,\sigma_D))]^2$$
$$=g(I_x^2)g(I_y^2)-[g(I_xI_y)]^2-\alpha[g(I_x^2)+g(I_y^2)]^2$$

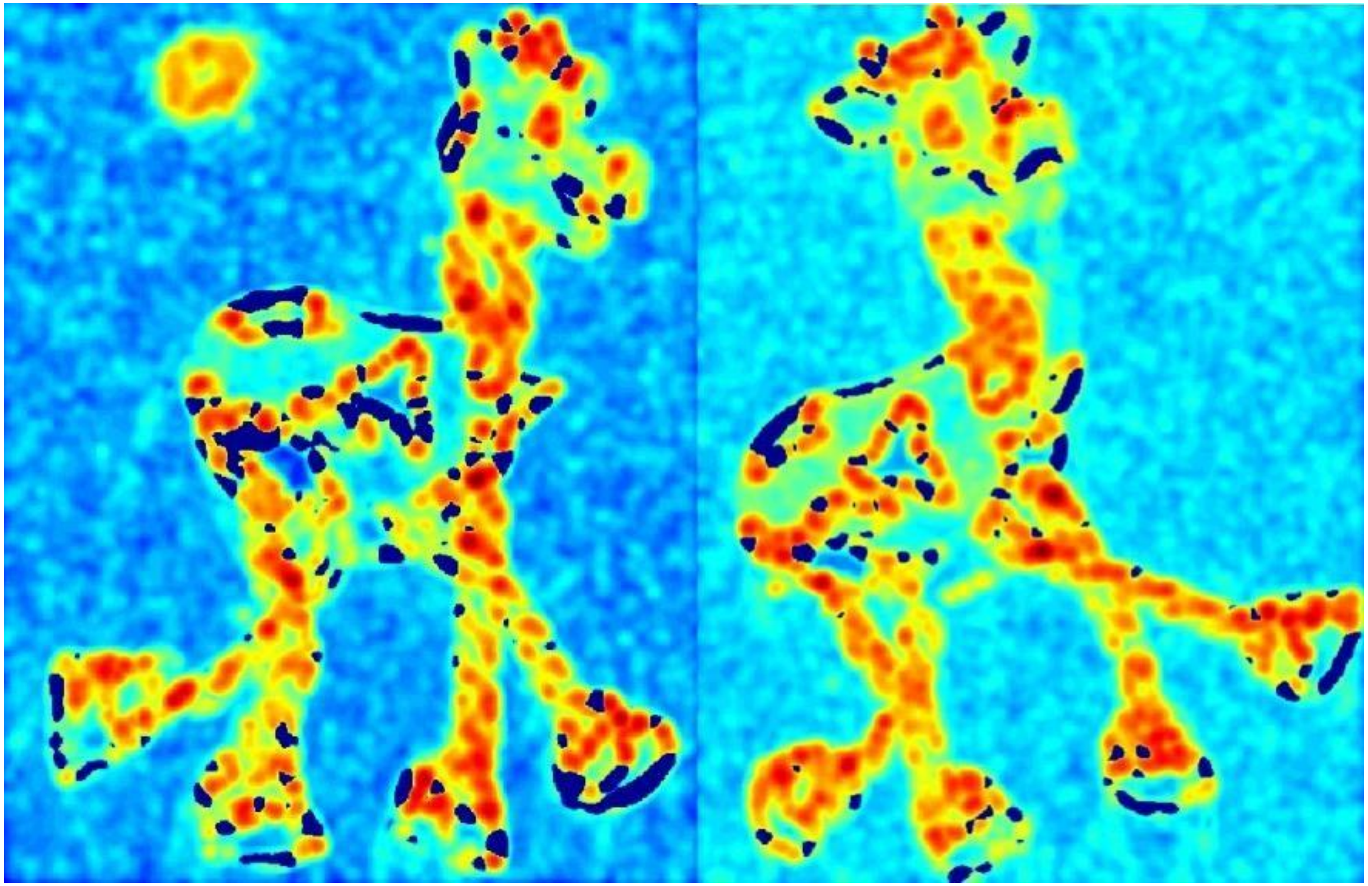
5. Perform non-maximum suppression

Harris Detector: Workflow

- Input image

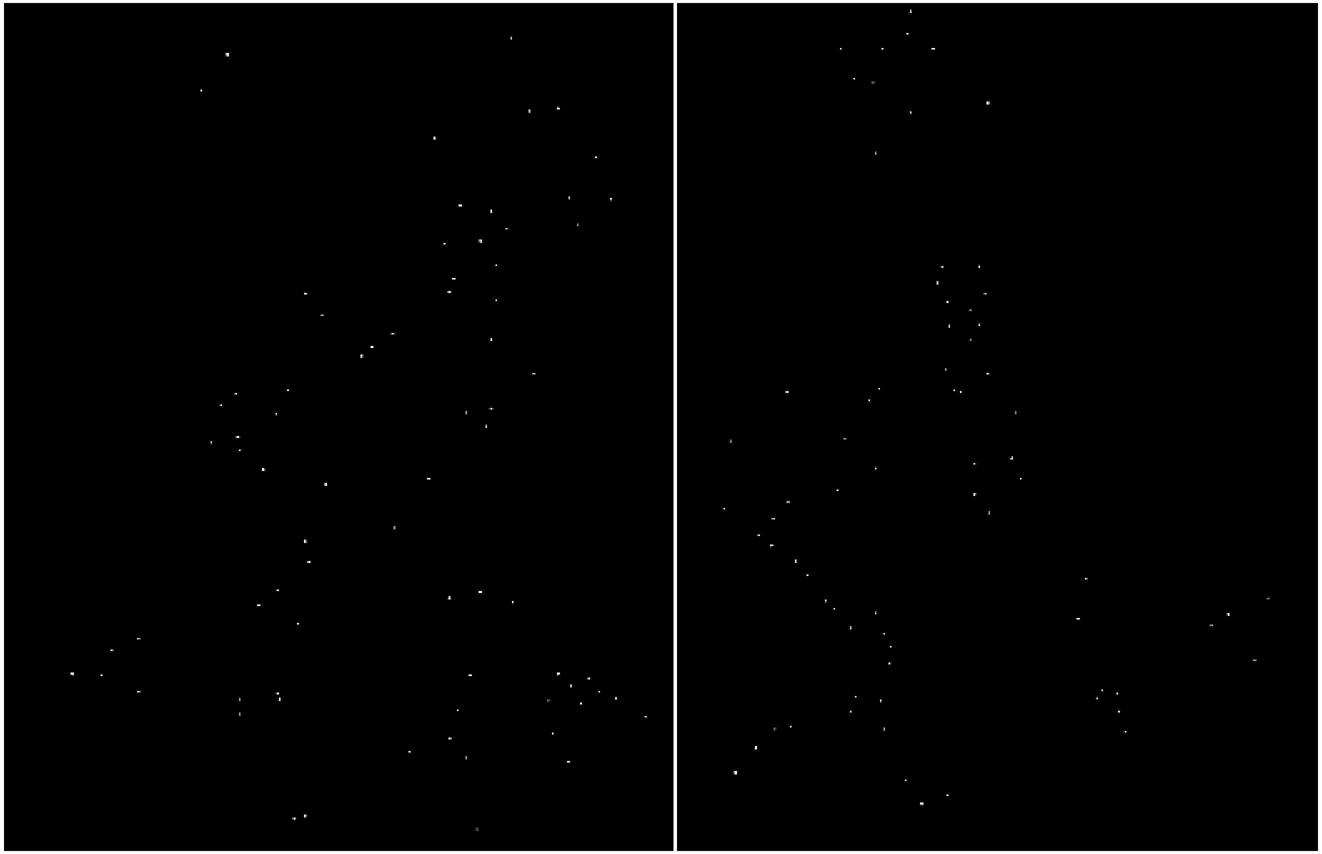
Harris Detector: Workflow

- Input Image
- Compute corner response function θ



Harris Detector: Workflow

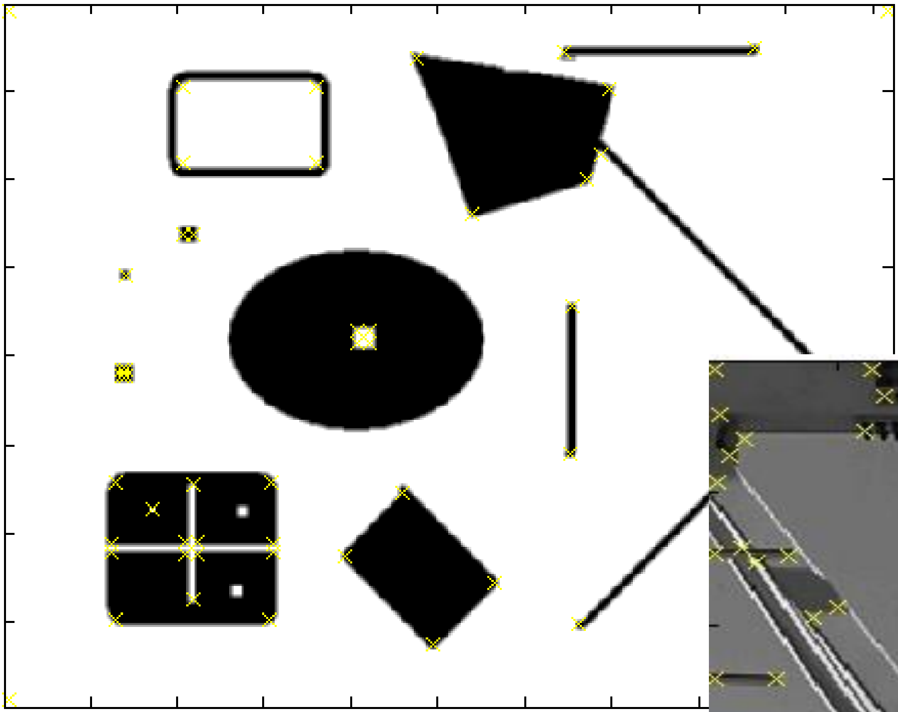
- Input Image
- Compute corner response function θ
- Take only the local maxima of θ, where θ > threshold



Harris Detector: Workflow

- Input Image
- Compute corner response function θ
- Take only the local maxima of θ, where θ > threshold

Harris Detector – Responses



Effect: A very precise corner detector.

Harris Detector – Responses

Harris Detector – Responses

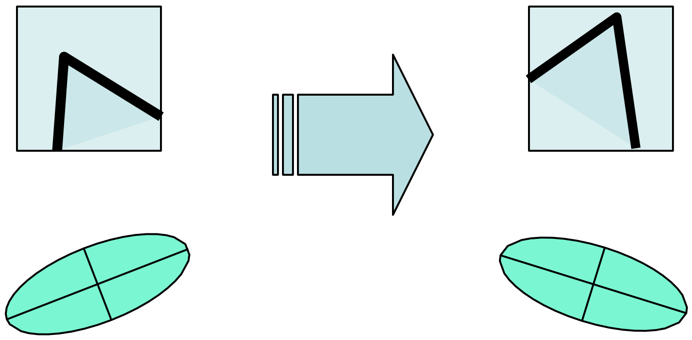
- Results are well suited for finding stereo correspondences

Harris Detector: Properties

- Translation invariance?

Harris Detector: Properties

- Translation invariance
- Rotation invariance?

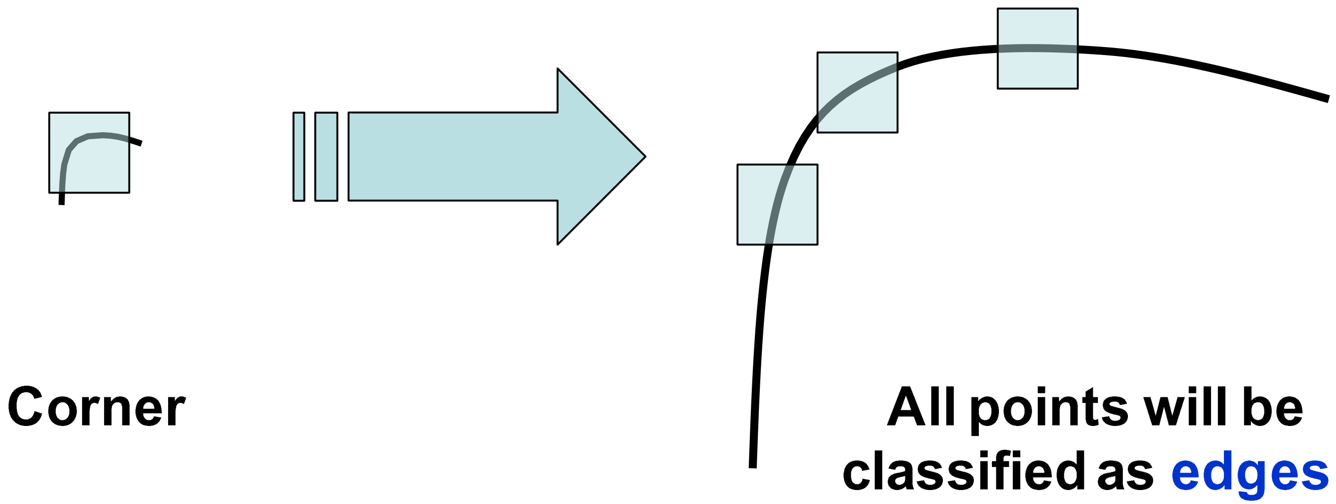


Ellipse rotates but its shape (i.e. eigenvalues) remains the same

Corner response θ **is invariant to image rotation**

Harris Detector: Properties

- Translation invariance
- Rotation invariance
- Scale invariance?



Not **invariant to image scale!**

Summary

- Harris corner detector
  - Formulation
  - Examples