AK High-Dimensional Projection Structural Theory (v4.2) Categorial, Mirror-Symmetric, Entropic, and Derived Extensions

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Abstract

We present version 4.2 of the AK High-Dimensional Projection Structural Theory (AK-HDPST), a comprehensive geometric-topological-analytic framework for decomposing complex PDE systems via high-dimensional projections and MECE structures. This release incorporates: (1) a formal functorial framework and fibered MECE categories; (2) SYZ mirror symmetry and tropical degeneration for persistent homology; (3) entropy-energy coupling via a topological thermostat model; (4) a derived categorical extension to persistent barcodes; and (5) connections to information complexity and fractal dimensionality reduction. These enhancements reinforce AK-HDPST as a unifying architecture for certifiable smoothness across analytic, geometric, and algebraic regimes.

Contents

1 Introduction

1.1 Motivation and Scope

The AK-HDPST seeks to resolve the analytic intractability of singular PDEs (e.g., Navier–Stokes, MHD, SQG) by projecting their solutions into higher-dimensional structured spaces where topological simplification and analytic regularity can be exposed. This version introduces a derived, entropic, and mirror-symmetric extension, supporting both theoretical formulation and empirical verification.

1.2 Core Philosophy and Workflow

> *"If the solution cannot be found, the dimension may be insufficient."*

We organize the methodology as:

1. Projection: Map analytic orbit to topological/feature space;
2. Decomposition: Extract MECE clusters via persistent homology (PH);
3. Collapse: Use entropy, energy, and geometric flows to enforce simplification;
4. Reconstruction: Infer regularity through topological invariants and degeneration.

2 Categorial and Fibered Structure of AK Projections

2.1 Projection Functor and Structured Categories

Define categories:

- $\mathcal{C}$: analytic objects (e.g., H^s orbits);
- $\mathcal{D}$: topological invariants (e.g., barcode spaces);
- $\Phi : \mathcal{C} \to \mathcal{D}$, a functor: $u \mapsto \mathrm{PH}_1(u)$.

For morphisms:

$$\Phi(f : u_1 \to u_2) := d_B(\mathrm{PH}_1(u_1), \mathrm{PH}_1(u_2))$$

This functor preserves persistent topological structure.

2.2 Fibered MECE Category and Triviality Class

We define $\mathcal{F}$ as a fibered category over $\mathcal{D}$:

$$p : \mathcal{F} \to \mathcal{D} \quad \text{with fibers } \mathcal{F}_d := \Phi^{-1}(d)$$

This reflects MECE decomposability of the analytic space and allows tracking structural simplification across fibers.

3 Mirror Symmetry, Tropicalization, and Moduli Collapse

3.1 SYZ Projection of PH Coordinates

Given barcode diagram $B(t)$:

$$T(B(t)) := \{\log \mathrm{persist}(h)\} \subset \mathbb{T}^n$$

The degeneration $T(B(t)) \to 0$ reflects contraction toward a Lagrangian torus fiber in SYZ mirror symmetry.

3.2 Mixed Hodge Degeneration and Mirror Duality

Variation of mixed Hodge structures (VMHS) $\{F^p(t)\}$ degenerate to boundary strata in moduli space:

$$\text{Degenerate: } B(t) \rightsquigarrow \text{Limit MHS} \leftrightarrow \text{Mirror boundary under SYZ}$$

This supports regularity via collapse of tropical and Hodge-theoretic structure.

4 Derived Category Perspective on Persistent Homology

4.1 Barcodes as Objects in $D^b(\mathcal{F})$

View PH barcodes as filtered complexes:

$$\mathrm{PH}_1(u) \in \mathrm{Ob}(D^b(\mathcal{F}))$$

This permits interpretation of barcode death/birth as morphisms in a derived triangulated category.

4.2 Distinguished Triangles and Degeneration

Persistent simplification corresponds to a sequence:

$$A \to B \to C \rightsquigarrow A[1]$$

interpreting topological degeneration as categorical collapse.

5 Entropy–Energy–Geometry Coupled Model

5.1 Threefold Coupled Evolution

Define:

$$\begin{aligned}
C(t) &:= \sum_h \mathrm{persist}(h)^2, \\
H(t) &:= -\sum_h \frac{\mathrm{persist}(h)^2}{C(t)} \log \left(\frac{\mathrm{persist}(h)^2}{C(t)} \right), \\
D(t) &:= \dim_B(A_t) \text{ (box-counting dimension).}
\end{aligned}$$

Evolution equations:

$$\frac{dC}{dt} = -\gamma_1 \|\nabla u\|^2 + \epsilon_1,$$
$$\frac{dH}{dt} = -\gamma_2 H + \epsilon_2,$$
$$\frac{dD}{dt} = -\gamma_3 C(t) + \epsilon_3.$$

5.2 Topological Thermostat Principle

When $C(t), H(t), D(t) \to 0$, the system reaches a state of maximal compressibility and minimal complexity: i.e., structural regularity emerges from topological and entropic decay.

6 Kolmogorov Complexity and Information Collapse

6.1 Entropy and Algorithmic Compressibility

The decay $H(t) \downarrow$ implies a decrease in Kolmogorov complexity $K(u(t))$:

$$H(t) \to 0 \Rightarrow K(u(t)) \to \text{low}$$

This supports flow field simplification and learnability.

7 Application to Navier–Stokes Regularity (v3.2)

- Steps 1–6 correspond to projection, fiber contraction, and entropy flattening.
- Step 7 aligns with VMHS and tropical degeneration.
- Empirical simulation modules confirm orbit injectivity, PH decay, and spectral collapse.

8 Conclusion and Future Development

Version 4.2 completes a multi-layered framework unifying:

- Projection theory (categorical and MECE fibered);
- Mirror symmetry and Hodge-theoretic degeneration;
- Entropic Lyapunov models and complexity bounds;
- Derived category interpretation of topological collapse.

Next extensions include:

- Incorporation of persistent sheaf cohomology and interleaving metrics;
- Real-time numerical entropy predictors for active flow control;
- Topological descriptors for deep learning on PDE attractors.