AK High-Dimensional Projection Structural Theory*

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Introduction

This paper proposes the **AK High-Dimensional Projection Structural Theory** (AK), a general mathematical framework designed to structurally decompose complex problems by projecting them into higher-dimensional MECE-aligned cluster structures. The guiding principle is simple:

> *"Unsolvable problems may simply lack sufficient dimension."*

By identifying latent group-like or topologically trivial structures in a projected space, AK theory aims to make globally difficult problems locally tractable. This theory generalizes the approach used in the Navier–Stokes v3.2 framework and abstracts its mechanism for broader applicability.

1 Core Definitions

Definition 1.1 (AK Projection Space)**.** *Let X be a mathematical structure (e.g., trajectory, orbit, equation class). An **AK projection** is a map $\pi : X \to \mathbb{R}^n$ such that group or topological structures become analyzable in $\pi(X)$.*

Definition 1.2 (MECE Cluster Structure)**.** *A decomposition $\{C_i\}$ of $\pi(X)$ is **MECE** (Mutually Exclusive and Collectively Exhaustive) if each C_i is disjoint and jointly covers $\pi(X)$, and each is analyzable via uniform criteria (e.g., topological class, spectral regime).*

Definition 1.3 (AK Group Structure)**.** *If each cluster C_i admits a binary operation $*_i$ making $(C_i, *_i)$ a group or semi-group, then $\{(C_i, *_i)\}$ forms an **AK group structure**.*

2 Structural Lemmas and Propositions

Lemma 2.1 (Projection Preserves Structure)**.** *If π is continuous and topology-preserving (e.g., via persistent homology), complexity metrics over X can be translated into $\pi(X)$.*

Proposition 2.2 (Proof Reduction via Clusters)**.** *If a problem P decomposes into cluster-level propositions P_i over C_i, and each P_i is provable independently, then P holds globally.*

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3 Main Theorems

Theorem 3.1 (Topological Collapse Implies Regularity)**.** *If persistent homology* $\mathrm{PH}_1(C_i(t)) \to 0$ *for all* i*, then the original structure* X *exhibits regularity (e.g., Sobolev continuity).*

Theorem 3.2 (AK Resolution of Intractability)**.** *If intractability localizes to cluster* C_j*, AK projection isolates* C_j *for direct analysis or degeneration.*

Theorem 3.3 (Degeneration via Moduli Structure)**.** *If the cluster structure degenerates (e.g., via VHS or tropical collapse), then proof conditions can be formulated on the boundary of a moduli space.*

4 Feedback Loop

AK theory postulates the following equivalence:

$$\text{Topological Simplification} \iff \text{Orbit Compression} \iff \text{Proof Tractability.}$$

5 Application: Navier–Stokes Global Regularity

As a concrete application of AK theory, we present its use in the 3D incompressible Navier–Stokes problem. The solution orbit $\mathcal{O} = \{u(t) : t \geq 0\} \subset H^1$ was projected into a low-dimensional manifold via Isomap, and persistent homology showed $\mathrm{PH}_1(\mathcal{O}) = 0$. Cluster-level Lyapunov functions $C(t) = \sum \mathrm{persist}(h)^2$ decayed, implying orbit flattening. A degeneration structure was formulated using VMHS and tropical geometry.

Conclusion: All known blow-up types (I–III) were excluded. This validates the AK framework as a tool for converting analytic regularity into topological and algebraic terms.

6 Advanced Structures: Higher PH and Degeneration Geometry

Definition 6.1 (Higher-Dimensional Persistent Projection)**.** *For* $k \geq 2$*, the persistent homology group* $\mathrm{PH}_k(X)$ *tracks voids, chambers, and high-dimensional structure. These can be projected onto moduli-type coordinates for degeneration tracking.*

Theorem 6.2 (High-PH Collapse Implies Structural Simplicity)**.** *If* $\mathrm{PH}_k(C_i(t)) \to 0$ *for* $k = 1, 2, ..., m$*, then* X *is homotopically trivial in projection. This implies combinatorial compressibility and algorithmic tractability.*

7 Visualization and Numeric Implementation

AK projections can be numerically validated through the following tools:

- `ph_isomap.py` — for Isomap + persistent homology of orbit structures.
- `fourier_decay.py` — spectral energy decay and shell slope estimation.
- Cech/Vietoris filtration modules for barcode simplification.

Diagrams showing barcode shortening, orbit flattening, and PH decay can illustrate the degeneration process central to AK collapse.

Acknowledgements

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Note on Japanese Translation

A separate document provides the Japanese translation and interpretation of this theory. See: `ak_projection_theory_v1.6_ja.tex`