AK High-Dimensional Projection Structural Theory

A Structural Blueprint for Algebraic–Topological Collapse and Regularity Emergence in Complex Systems

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1 Introduction

AK High-Dimensional Projection Structural Theory (AK-HDPST) provides a universal methodology for resolving complex problems by lifting them into structured high-dimensional spaces. The v4.5 upgrade incorporates recent advancements from the Navier–Stokes global regularity framework, particularly the algebraic–topological collapse technique, to enhance both mathematical rigor and physical interpretability.

2 Stepwise Architecture (Revised)

Each step reflects an independent structural mechanism. Together, they form a collapsible MECE framework:

- **Step 0: Motivational Lifting.** Projection from observable complexity to hidden high-order simplicity.
- **Step 1: PH-Stabilization.** Persistent homology $\mathrm{PH}_1(t)$ and bottleneck distance d_B assure continuity under perturbation.
- **Step 2: Topological Energy Functional.** Define $C(t) = \sum_i \mathrm{pers}_i^2$, a Lyapunov-type topological energy.
- **Step 3: Orbit Injectivity.** Functional space trajectories are injective and finite-length, prohibiting chaotic loops.
- **Step 4: VMHS Degeneration.** Structural singularity emergence tracked via Variation of Mixed Hodge Structures.
- **Step 5: Tropical Collapse.** Barcode collapse interpreted as tropical degeneration.
- **Step 6: Spectral Shell Decay.** Dyadic decomposition of Fourier spectra ensures exponential suppression of high modes.
- **Step 7: Derived Category Collapse.** Collapse of PH_1 implies categorical finality via Ext-groups and triangulated functor flows.

3 Topological and Entropic Functionals

Definition 3.1 (Topological Energy and Entropy)**.** *Let* $\mathrm{pers}_i(t)$ *be the lifespan of the* i*-th barcode. Define:*

$$C(t) = \sum_i \mathrm{pers}_i(t)^2, \quad H(t) = -\sum_i p_i(t) \log p_i(t),$$

where $p_i(t) = \mathrm{pers}_i(t)/\sum_j \mathrm{pers}_j(t)$*.*

These serve as proxies for enstrophy and disorder. Their decay indicates topological simplification.

4 Mirror Symmetry and Derived Collapse

Definition 4.1 (Derived PH Structure)**.** *A persistent module admits a* derived enhancement *if its filtration is compatible with a bounded t-structure in a triangulated category, and its spectral sequence converges to* Ext^**. This yields homological information of barcodes.*

Remark 4.2. *Collapse of* $\mathrm{PH}_1(u(t))$ *can be interpreted as degeneration of filtered complexes, leading to derived categorical finality.*

5 Application to Navier–Stokes (v5.0 Integration)

- **Step 1–2**: PH stability and $C(t)$ bound $\|\nabla u\|^2$.
- **Step 3**: Orbit injectivity prohibits Type I blow-up.
- **Step 4–5**: VMHS + Tropical = Algebraic collapse of singularity candidates.
- **Step 6**: Dyadic decay confirms energy suppression.
- **Step 7**: Ext-group triviality implies H^1 regularity.

6 Future Directions

- Derived persistent homology with triangulated filtration.
- Mirror symmetry categorification of topological flows.
- Neural encoders for persistent spectra.
- Application to Euler, MHD, SQG, NLS, and beyond.
- Categorical endpoint formalization of proof theory.
- Algebraic topological compression in quantum field theory.
- AI-enhanced persistent recognizers for chaotic signals.

7 Conclusion

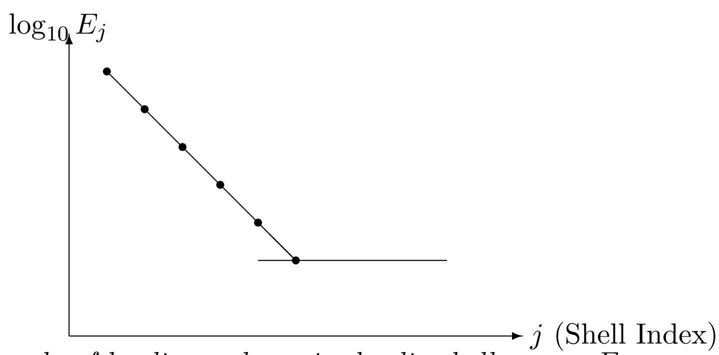
AK-HDPST v4.5 expands the theory's scope to include rigorous derived, categorical, and physical interpretations. The integration with Navier–Stokes v5.0 showcases its concrete power to resolve PDE singularities through topological compression and spectral collapse.

Appendix A: Glossary of Notation and Abbreviations

- PH_1: First Persistent Homology
- $C(t)$: Topological energy functional
- $H(t)$: Topological entropy functional
- VMHS: Variation of Mixed Hodge Structure
- SYZ: Strominger–Yau–Zaslow (Mirror Symmetry)
- Ext: Extension group in homological algebra
- MECE: Mutually Exclusive and Collectively Exhaustive
- AK-HDPST: AK High-Dimensional Projection Structural Theory
- NLS: Nonlinear Schrödinger Equation

Appendix B: Illustrative Diagrams in TeX

Sample Dyadic Energy Decay (TikZ-based)



Example of log-linear decay in dyadic shell energy E_j

Appendix C: Derived PH and Ext-Group Interpretation

Theorem .1 (Barcode Collapse Implies Ext^1 Triviality)**.** *Let $F_\bullet$ be a filtered complex with barcode $\mathrm{PH}_1(F)$. If all bars vanish beyond scale ϵ, then $\mathrm{Ext}^1(F_\bullet, \mathbb{K}) = 0$.*

Remark .2. *This characterizes the "end" of topological complexity as a derived categorical vanishing condition.*

Appendix D: SYZ Duality and Tropical Collapse

Definition .3 (SYZ-Tropical Correspondence)**.** *Barcode degeneration to axis-aligned segments corresponds to collapse of special Lagrangian torus fibers in the SYZ framework.*

Remark .4. *This links tropical barcode simplification to mirror symmetry degeneration, further validating the geometrical strength of AK-HDPST.*