Appendix C: AI-Based Recognition of Regularity via PH and Ext Collapse

Augmentation Strategy for Navier–Stokes Global Regularity Proof

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Motivation

This appendix outlines how recent upgrades in the **AK High-Dimensional Projection Structural Theory (v5.0)** can enhance the interpretation, detection, and numerical validation of global regularity for the 3D incompressible Navier–Stokes equations. We frame a bridge from *topological collapse* and *derived categorical degeneration* to *AI-based recognition systems*.

C.1 Persistent Barcode Embedding and Detection

We consider a family of velocity fields $\{u(t)\}$ evolving under NSE. Applying Isomap to the snapshots followed by persistent homology yields barcodes $\mathrm{PH}_1(u(t))$. These barcodes are then embedded as feature vectors:

$$\mathrm{PH}_1(u(t)) \mapsto \mathbf{v}_t \in \mathbb{R}^d$$

using geometric or neural encoders. This allows classification of topological collapse patterns indicative of smoothness.

C.2 Spectral Collapse as Ext-Degeneration

Dyadic shell energies $E_j(t)$ are computed as:

$$E_j(t) = \sum_{k \in \mathrm{Shell}_j} |\hat{u}_k(t)|^2$$

and interpreted as Ext-degeneration signatures:

$$\forall j,\ E_j(t) \downarrow 0 \iff \mathrm{Ext}^1(F_t, -) \to 0$$

where $F_t \in D^b(\mathcal{AK})$ models the derived categorical state. This collapse aligns with AK-HDPST's notion of *derived-finality*.

C.3 Learning Degeneration Phases via AI

Construct a training set labeled by smooth/singular evolution and feed it into a classifier over:

$$(\mathbf{v}_t,\ \vec{E}(t),\ \mathrm{Isomap}(u(t))) \in \mathbb{R}^{d+m+n}$$

where $\vec{E}(t)$ encodes dyadic shells. Supervised models (e.g., SVMs, neural nets) learn to recognize regularity patterns from topological and spectral cues.

C.4 Summary and GitHub Integration

This AI-augmented perspective allows us to numerically monitor regularity, quantify topological triviality, and validate theoretical predictions in AK-HDPST. This module is designed to accompany:

- **Main Repository:** `https://github.com/Kobayashi2501/Navier-Stokes-Global-Regularity`
- **AK-HDPST v5.0:** `https://github.com/Kobayashi2501/AK-High-Dimensional-Projection-Structur`