使用C++实现ROBDD算法

本文简介

笔者使用C++实现了对CTL表达式的求解，允许嵌套且支持多种表达式。本文的主要目的是阐述算法的主要实现流程。作品已分享至知乎：https://zhuanlan.zhihu.com/p/535182189

CTL相关知识

可以参考https://blog.csdn.net/qq_37400312/article/details/112330068，本文不再给出。

基本算法

决策树

决策树是一颗二进制树，满足：

- 每个节点被布尔变量标记；
- 每个叶子被赋予0或1。

通过从树根开始，如果对应节点变量是true则向左走，否则向右走，直到抵达叶子节点。如果叶子节点是1说明结果是true，否则是false。很显然，任意一个布尔函数都可以用决策树表达。然而，决策树的空间可能非常大。如果节点数量较多，可以考虑合并或消除。

```
    p
0       0
```

如图，P的两个叶子节点均为0，则p可以直接用0替换。

ROBDD

如果忽略树每个节点入度至多为1这一特点，将所有相同的子节点进行合并，则得到ROBDD，它比决策树更节省空间。

```
            a
    p               s
q       r       r       t
```

如图，a的子节点p和s可以共享r。

ROBDD的运算：

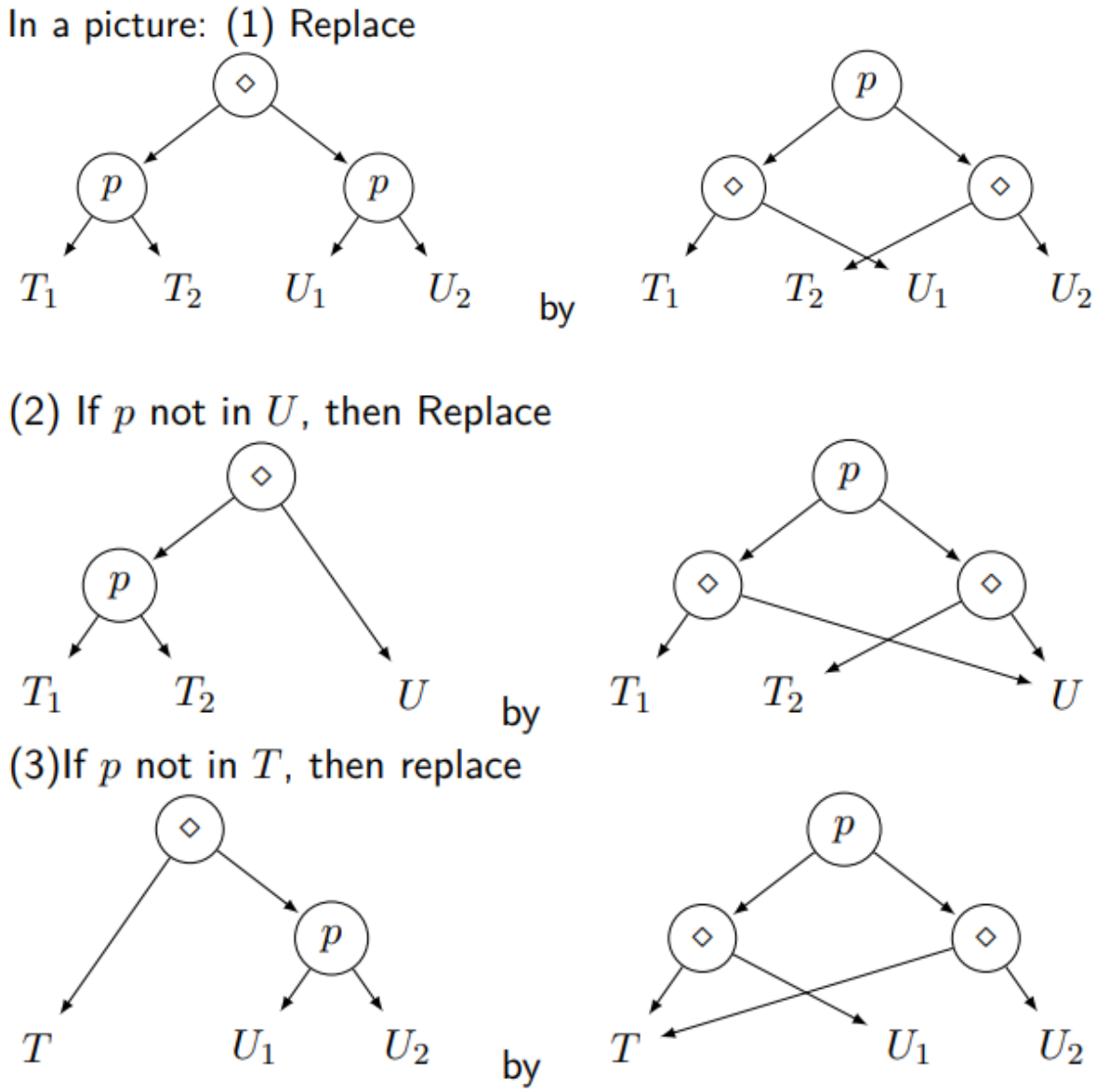
- ROBDD(F)和ROBDD(T)是trival的；
- ROBDD(~p)=ROBDD(p->F)；
- ROBDD(p R q)=apply(ROBDD(p),ROBDD(q),R)，其中R是合取、析取、蕴含、等值的一种。

现在最主要的问题变为计算apply(ROBDD(p),ROBDD(q),R)。事实上，通过以下逻辑关系可以递归化简（这里假设p的左右子树分别为p1和p2，q的左右子树分别为q1和q2。当p或q是叶子节点时，直接可以计算消去运算符号）：

- R(p(p1,p2),p(q1,q2))=p(R(p1,p2),R(q1,q2));

- 当p不在q出现时，R(p(p1,p2),q)=p(R(p1,q),R(p2,q))(p加括号蕴含p是最上层的一个节点，q也一样);
- 当q不在p出现时，R(p,q(q1,q2))=q(R(p,q1),R(p,q2)).

可以通过下图直观理解:



对EX,EG,EU的求解

对它们的求解归根结底在形式化求解$\{s \in T | \exists t \in U : s \to t\}$的ROBDD。
例如，对于EG，可以按如下步骤求解:

- $T_0 = S_\phi; n = 0$
- repeat $T_{n+1} = T_n \cap \{s \in S_\phi | \exists t \in T_n : s \to t\}; n = n + 1$ until $T_n = T_{n-1}$。
- $ROBDD(T_n)$即为所求。

$s \to t$可以分解为:

- $P_1 = (a_1, \cdots, a_n) \to (a'_1, \cdots, a'_n)$
- $P_2 = (a'_1, \cdots, a'_n) \in U$
- $P = P_1 \wedge P_2$

这里P_1可以理解成a表示的点在图上连到a'表示的点。
存在性可以通过尝试得出。假设$\{s \in T | \exists t \in U : s \to t\}$计算结果为V，则$V = T \cap S_{P_e}$，其中$S_{P_e} = \{(a_1, \cdots, a_n) | \exists a'_1, \cdots, a'_n : P(a_1, \cdots, a_n, a'_1, \cdots, a'_n)\}$。

代码详解

首先定义基本图类，仅包括增边、增加节点操作：

```
//Graph.h
#pragma once
#include<vector>
#include<cstdlib>
using namespace std;
struct GraphNode
{
    int value;
    vector<GraphNode*> next;
    vector<int> nextidx; //corresponding index
};
class Graph
{
public:
    int num_nodes;
    vector<GraphNode*> nodes;
    Graph();
    void AddNode(int value);
    void AddEdge(int nodesrc, int nodedst); //Add an edge from nodesrc to nodedst
};
```

```
//Graph.cpp
#include "Graph.h"

Graph::Graph()
{
    num_nodes = 0;
}

void Graph::AddNode(int value)
{
    num_nodes++;
    GraphNode* NewNode = new GraphNode;
    nodes.push_back(NewNode);
    nodes[num_nodes - 1]->value = value;
}

void Graph::AddEdge(int nodesrc, int nodedst)
{
    if (nodesrc < 0 || nodedst < 0 || nodesrc >= num_nodes || nodedst >= num_nodes)
throw "Added an illegal edge!";
    nodes[nodesrc]->next.push_back(nodes[nodedst]);
    nodes[nodesrc]->nextidx.push_back(nodedst);
}
```

对于ROBDD，建立了一个ROBDD类，支持由图转化（已废弃但是还能用）、由真值表转化、化简、打印、克隆（保持拓扑结构）、查找结果、拓扑等价判定(Equal)、逻辑运算、三种CTL运算。ROBDD类的nodes保存了节点指针，方便遍历。

```
//ROBDD.h
#pragma once
#include<vector>
#include"Graph.h"
using namespace std;
struct ROBDDNode
{
    int label; //-1 for leaf
    union
    {
        struct
        {
            ROBDDNode* true_branch;
            ROBDDNode* false_branch;
        }successor;
        int value;
    }value;
};
ROBDDNode* Clone(ROBDDNode* src);
bool Contain(ROBDDNode* node, int label);
class ROBDD
{
public:
    ROBDDNode* root;
    vector<ROBDDNode*> nodes;
    void ConvertFromGraph(Graph graph);
    void FromTrueValueVector(vector<int> TrueValues);
    void Simplify();
    void Print();
    ROBDD CloneROBDD();
    bool Walk(int path, int pathlen); //walk down the path, see if it ends.
};
vector<ROBDDNode*> NodeVector(ROBDDNode* StartVector);
bool Equal(ROBDDNode* node1, ROBDDNode* node2);
ROBDD AND(ROBDD robdd1, ROBDD robdd2);
ROBDD OR(ROBDD robdd1, ROBDD robdd2);
ROBDD IMPLY(ROBDD robdd1, ROBDD robdd2);
ROBDD NOT(ROBDD robdd);
ROBDD EX(Graph G, ROBDD robdd);
ROBDD EG(Graph G, ROBDD robdd);
ROBDD EU(Graph G, ROBDD robdd1, ROBDD robdd2);
```

label的说明：为执行CTL，需要把整数分解为二进制数，label标志着二进制数从高到低的位。进行结果判定时，应该检测当前label的值，根据是否为true进行下一个节点label的检测。比如，4->100，节点将有三个label值。检测时label0对应1，label1和label2对应0。检测结果是ROBDD顺着这条路径走到的叶子节点。

首先识别相等判定：递归检查label值，如果全部对应上则说明相等。
如未特殊说明，下面的代码均在ROBDD.cpp中。

```
bool Equal(ROBDDNode* node1, ROBDDNode* node2) //label cannot be less than -1
{
    if (node1->label == -1 && node2->label == -1)
    {
        if (node1->value.value == node2->value.value) return true;
        else return false;
    }
    else if (node1->label == -1 || node2->label == -1) return false;
    else return Equal(node1->value.successor.true_branch, node2-
>value.successor.true_branch) & Equal(node1->value.successor.false_branch, node2-
>value.successor.false_branch);
}
```

从图转ROBDD：
对于每一个值为true的节点，将节点标号翻译为二进制数，在已有ROBDD中按二进制数对应的路往下走。如果发现走到头，需要创建一个ROBDD节点，直到走完整个二进制数。最后一步到达的为叶子节点，需要修改节点属性。到最后，需要将ROBDD化简。

```
void ROBDD::ConvertFromGraph(Graph graph)
{
    if (!nodes.empty()) nodes.clear();
    root = new ROBDDNode; //root
    root->value.successor.true_branch = root->value.successor.false_branch = NULL;
    root->label = 0;
    nodes.push_back(root);
    int depth = ceil(log2(graph.num_nodes));
    ROBDDNode* current;
    for (int i = 0; i < graph.num_nodes; i++)
    {
        if (graph.nodes[i]->value == 0) continue;
        vector<bool> path = IntToBinVec(i, depth);
        current = root;
        for (int j = 0; j < path.size(); j++)
        {
            if (path[j] == true)  //walk to true branch
            {
                if (current->value.successor.true_branch == NULL) //create a new
node
                {
                    ROBDDNode* NewNode = new ROBDDNode;
                    current->label = j;
                    current->value.successor.true_branch = NewNode;
                    NewNode->label = j + 1;
                    NewNode->value.successor.true_branch = NULL;
                    NewNode->value.successor.false_branch = NULL;
                    nodes.push_back(NewNode);
                    current = NewNode;
                }
                else //move down current
                {
                    current = current->value.successor.true_branch;
                }
            }
            else //walk to false branch
```

```
            {
                if (current->value.successor.false_branch == NULL) //create a new
node
                {
                    ROBDDNode* NewNode = new ROBDDNode;
                    current->label = j;
                    current->value.successor.false_branch = NewNode;
                    NewNode->value.successor.true_branch = NULL;
                    NewNode->value.successor.false_branch = NULL;
                    NewNode->label = j + 1;
                    nodes.push_back(NewNode);
                    current = NewNode;
                }
                else //move down current
                {
                    current = current->value.successor.false_branch;
                }
            }
        }
        current->label = -1; //convert the last node to leaf
        current->value.value = 1;
    }
    vector<ROBDDNode*> NodeToAdd;
    for (int i = 0; i < nodes.size(); i++) //assign false leaves
    {
        if (nodes[i]->label != -1 && nodes[i]->value.successor.true_branch == NULL)
        {
            ROBDDNode* NewNode = new ROBDDNode;
            NewNode->label = -1;
            NewNode->value.value = 0;
            nodes[i]->value.successor.true_branch = NewNode;
            NodeToAdd.push_back(NewNode);
        }
        if (nodes[i]->label != -1 && nodes[i]->value.successor.false_branch ==
NULL)
        {
            ROBDDNode* NewNode = new ROBDDNode;
            NewNode->label = -1;
            NewNode->value.value = 0;
            nodes[i]->value.successor.false_branch = NewNode;
            NodeToAdd.push_back(NewNode);
        }
    }
    for (int i = 0; i < NodeToAdd.size(); i++) nodes.push_back(NodeToAdd[i]);
    Simplify();
}
```

ROBDD的化简：当某节点的左右子树相同时，可以消除这个节点把它用左右子树代替：

```
if (nodes[i]->label != -1 && Equal(nodes[i]->value.successor.true_branch, nodes[i]-
>value.successor.false_branch))
{
    nodes[i]->label = nodes[i]->value.successor.true_branch->label;
    if (nodes[i]->value.successor.true_branch->label == -1)
        nodes[i]->value.value = nodes[i]->value.successor.true_branch->value.value;
    else
    {
        nodes[i]->value.successor.true_branch = nodes[i]-
>value.successor.false_branch->value.successor.true_branch;
        nodes[i]->value.successor.false_branch = nodes[i]-
>value.successor.false_branch->value.successor.false_branch;
    }
    flag = 1;
}
```

当两个节点中一个节点的子树和另一个节点的子树拓扑等价时，可以考虑合并，删除一个多余的子树（如下代码仅展示一种情况）。

```
if (nodes[j]->value.successor.true_branch != nodes[i]-
>value.successor.false_branch&&Equal(nodes[j]->value.successor.true_branch,
nodes[i]->value.successor.false_branch))
{
    flag = 1;
    for (vector<ROBDDNode*>::iterator ite = nodes.begin(); ite != nodes.end();
ite++)
    {
        if (*ite == nodes[j]->value.successor.true_branch)
        {
            nodes[j]->value.successor.true_branch = nodes[i]-
>value.successor.false_branch;
            goto erase;
        }
    }
}
```

完整代码:

```
void ROBDD::Simplify() //label nannot be less than -1
{
    int flag;
    do
    {
        flag = 0;
        for (int i = 0; i < nodes.size(); i++)
        {
            if (nodes[i]->label != -1 && Equal(nodes[i]-
>value.successor.true_branch, nodes[i]->value.successor.false_branch))
            {
                nodes[i]->label = nodes[i]->value.successor.true_branch->label;
                if (nodes[i]->value.successor.true_branch->label == -1)
                    nodes[i]->value.value = nodes[i]->value.successor.true_branch-
>value.value;
```

```
                else
                {
                    nodes[i]->value.successor.true_branch = nodes[i]-
>value.successor.false_branch->value.successor.true_branch;
                    nodes[i]->value.successor.false_branch = nodes[i]-
>value.successor.false_branch->value.successor.false_branch;
                }
                flag = 1;
            }
        }
        nodes = NodeVector(root);
        for (int i = 0; i < nodes.size(); i++) //merge
        {
            for (int j = i + 1; j < nodes.size(); j++)
            {
                if (nodes[i]->label == -1 || nodes[j]->label == -1) continue;
                if (nodes[j]->value.successor.false_branch != nodes[i]-
>value.successor.false_branch&&Equal(nodes[j]->value.successor.false_branch,
nodes[i]->value.successor.false_branch))
                {
                    flag = 1;
                    for (vector<ROBDDNode*>::iterator ite = nodes.begin(); ite !=
nodes.end(); ite++)
                    {
                        if (*ite == nodes[j]->value.successor.false_branch)
                        {
                            nodes[j]->value.successor.false_branch = nodes[i]-
>value.successor.false_branch;
                            goto erase;
                        }
                    }
                }
                if (nodes[j]->value.successor.false_branch != nodes[i]-
>value.successor.true_branch&&Equal(nodes[j]->value.successor.false_branch,
nodes[i]->value.successor.true_branch))
                {
                    flag = 1;
                    for (vector<ROBDDNode*>::iterator ite = nodes.begin(); ite !=
nodes.end(); ite++)
                    {
                        if (*ite == nodes[j]->value.successor.false_branch)
                        {
                            nodes[j]->value.successor.false_branch = nodes[i]-
>value.successor.true_branch;
                            goto erase;
                        }
                    }
                }
                if (nodes[j]->value.successor.true_branch != nodes[i]-
>value.successor.false_branch&&Equal(nodes[j]->value.successor.true_branch,
nodes[i]->value.successor.false_branch))
                {
                    flag = 1;
                    for (vector<ROBDDNode*>::iterator ite = nodes.begin(); ite !=
nodes.end(); ite++)
```

```
                    {
                        if (*ite == nodes[j]->value.successor.true_branch)
                        {
                            nodes[j]->value.successor.true_branch = nodes[i]-
>value.successor.false_branch;
                            goto erase;
                        }
                    }
                }
                if (nodes[j]->value.successor.true_branch != nodes[i]-
>value.successor.true_branch&&Equal(nodes[j]->value.successor.true_branch,
nodes[i]->value.successor.true_branch))
                {
                    flag = 1;
                    for (vector<ROBDDNode*>::iterator ite = nodes.begin(); ite !=
nodes.end(); ite++)
                    {
                        if (*ite == nodes[j]->value.successor.true_branch)
                        {
                            nodes[j]->value.successor.true_branch = nodes[i]-
>value.successor.true_branch;
                            goto erase;
                        }
                    }
                }
            }
        }
    erase:;
        nodes = NodeVector(root);
    } while (flag);
}
```

根据真值表生成ROBDD其实和根据图很像，只是把对应true的节点标号换成了真值表向量中给定的数据：

```
void ROBDD::FromTrueValueVector(vector<int> TrueValues)
{
    if (!nodes.empty()) nodes.clear();
    root = new ROBDDNode; //root
    root->value.successor.true_branch = root->value.successor.false_branch = NULL;
    root->label = 0;
    nodes.push_back(root);
    int max = 0;
    for (int i = 0; i < TrueValues.size(); i++)
    {
        if (TrueValues[i] > max) max = TrueValues[i];
    }
    int depth = ceil(log2(max + 1));
    ROBDDNode* current;
    for (int i = 0; i < TrueValues.size(); i++)
    {
        vector<bool> path = IntToBinVec(TrueValues[i], depth);
        current = root;
        for (int j = 0; j < path.size(); j++)
        {
            if (path[j] == true)  //walk to true branch
```

```
            {
                if (current->value.successor.true_branch == NULL) //create a new
node
                {
                    ROBDDNode* NewNode = new ROBDDNode;
                    current->label = j;
                    current->value.successor.true_branch = NewNode;
                    NewNode->label = j + 1;
                    NewNode->value.successor.true_branch = NULL;
                    NewNode->value.successor.false_branch = NULL;
                    nodes.push_back(NewNode);
                    current = NewNode;
                }
                else //move down current
                {
                    current = current->value.successor.true_branch;
                }
            }
            else //walk to false branch
            {
                if (current->value.successor.false_branch == NULL) //create a new
node
                {
                    ROBDDNode* NewNode = new ROBDDNode;
                    current->label = j;
                    current->value.successor.false_branch = NewNode;
                    NewNode->value.successor.true_branch = NULL;
                    NewNode->value.successor.false_branch = NULL;
                    NewNode->label = j + 1;
                    nodes.push_back(NewNode);
                    current = NewNode;
                }
                else //move down current
                {
                    current = current->value.successor.false_branch;
                }
            }
        }
        current->label = -1; //convert the last node to leaf
        current->value.value = 1;
    }
    vector<ROBDDNode*> NodeToAdd;
    for (int i = 0; i < nodes.size(); i++) //assign false leaves
    {
        if (nodes[i]->label != -1 && nodes[i]->value.successor.true_branch == NULL)
        {
            ROBDDNode* NewNode = new ROBDDNode;
            NewNode->label = -1;
            NewNode->value.value = 0;
            nodes[i]->value.successor.true_branch = NewNode;
            NodeToAdd.push_back(NewNode);
        }
        if (nodes[i]->label != -1 && nodes[i]->value.successor.false_branch ==
NULL)
        {
```

```
            ROBDDNode* NewNode = new ROBDDNode;
            NewNode->label = -1;
            NewNode->value.value = 0;
            nodes[i]->value.successor.false_branch = NewNode;
            NodeToAdd.push_back(NewNode);
        }
    }
    for (int i = 0; i < NodeToAdd.size(); i++) nodes.push_back(NodeToAdd[i]);
    Simplify();
}
```

打印函数很容易实现。这里把节点地址和节点标号做了一个哈希映射，这样可以方便显示出节点标号，有利于调试。输出括号中(test xxx)是指节点对应的label是xxx，经过这个节点时需要对xxx做测试以决定下一步要到的节点。

```
void ROBDD::Print() //label cannot be less than -1
{
    map<ROBDDNode*, int> ID;
    for (int i = 0; i < nodes.size(); i++) ID[nodes[i]] = i;
    for (int i = 0; i < nodes.size(); i++)
    {
        if (nodes[i]->label != -1)
        {
            if (nodes[i]->value.successor.false_branch->label != -1)
                cout << ID[nodes[i]] << "(tests x" << nodes[i]->label << ")" << "
----False---->  " << ID[nodes[i]->value.successor.false_branch] << endl;
            else
                if (nodes[i]->value.successor.false_branch->value.value == 1)
                    cout << ID[nodes[i]] << "(tests x" << nodes[i]->label << ")" <<
"  ----False---->  " << ID[nodes[i]->value.successor.false_branch] << "(True)" <<
endl;
                else
                    cout << ID[nodes[i]] << "(tests x" << nodes[i]->label << ")" <<
"  ----False---->  " << ID[nodes[i]->value.successor.false_branch] << "(False)" <<
endl;
            if (nodes[i]->value.successor.true_branch->label != -1)
                cout << ID[nodes[i]] << "(tests x" << nodes[i]->label << ")" << "
----True---->  " << ID[nodes[i]->value.successor.true_branch] << endl;
            else
                if (nodes[i]->value.successor.true_branch->value.value == 1)
                    cout << ID[nodes[i]] << "(tests x" << nodes[i]->label << ")" <<
"  ----True---->  " << ID[nodes[i]->value.successor.true_branch] << "(True)" <<
endl;
                else
                    cout << ID[nodes[i]] << "(tests x" << nodes[i]->label << ")" <<
"  ----True---->  " << ID[nodes[i]->value.successor.true_branch] << "(False)" <<
endl;
        }
        else
        {
            if (nodes[i]->value.value == 1)
                cout << ID[nodes[i]] << "  stands for True" << endl;
            else
                cout << ID[nodes[i]] << "  stands for False" << endl;
```

```
        }
    }
}
```

遍历结果：

```
bool ROBDD::Walk(int path, int pathlen)
{
    vector<bool> _path = IntToBinVec(path, pathlen);
    ROBDDNode* current = root;
    for (int i = 0; i < _path.size(); i++)
    {
        if (i != current->label) continue;
        if (_path[i]) current = current->value.successor.true_branch;
        else current = current->value.successor.false_branch;
    }
    if (current->value.value == 0) return false;
    return true;
}
```

这里需要在参数指定二进制路径的长度。

由于nodes不好实时管理，写了一个可以自动从根节点遍历并生成节点列表的函数：

```
vector<ROBDDNode*> NodeVector(ROBDDNode * StartVector)
{
    vector<ROBDDNode*> ret, TrueBranch, FalseBranch;
    ret.push_back(StartVector);
    if (StartVector->label >= 0)
    {
        TrueBranch = NodeVector(StartVector->value.successor.true_branch);
        FalseBranch = NodeVector(StartVector->value.successor.false_branch);
        for (int i = 0; i < TrueBranch.size(); i++)
        {
            vector<ROBDDNode*>::iterator it;
            it = find(ret.begin(), ret.end(), TrueBranch[i]);
            if (it != ret.end()) continue;
            ret.push_back(TrueBranch[i]);
        }
        for (int i = 0; i < FalseBranch.size(); i++)
        {
            vector<ROBDDNode*>::iterator it;
            it = find(ret.begin(), ret.end(), FalseBranch[i]);
            if (it != ret.end()) continue;
            ret.push_back(FalseBranch[i]);
        }
    }
    return ret;
}
```

节点的克隆由函数Clone实现：

```
```

```
ROBDDNode * Clone(ROBDDNode* src)
{
    ROBDDNode* ret = new ROBDDNode;
    if (src->label < 0)
    {
        ret->label = src->label;
        ret->value.value = src->value.value;
        return ret;
    }
    ret->label = src->label;
    ret->value.successor.false_branch = Clone(src->value.successor.false_branch);
    ret->value.successor.true_branch = Clone(src->value.successor.true_branch);
    return ret;
}
```

先复制节点值，再递归复刻左右子树的结构。

ROBDD类的整体克隆：

```
ROBDD ROBDD::CloneROBDD()
{
    ROBDD ret;
    ret.root = Clone(root);
    ret.nodes = NodeVector(ret.root);
    ret.Simplify();
    return ret;
}
```

逻辑运算：按照前述知识提到的几种方法递归地将运算符号逐层下移，直到有一个操作数是True或False。以AND为例：

```
ROBDD AND(ROBDD robdd1, ROBDD robdd2)
{
    ROBDD cloned_left = robdd1.CloneROBDD();
    ROBDD cloned_right = robdd2.CloneROBDD();
    ROBDD robdd_false;
    ROBDDNode* NewNode = new ROBDDNode;
    NewNode->label = -1;
    NewNode->value.value = 0;
    robdd_false.nodes.push_back(NewNode);
    robdd_false.root = NewNode;
    if (cloned_left.nodes.size() == 1)
    {
        if (cloned_left.root->value.value == 1) return cloned_right;
        else return robdd_false;
    }
    if (cloned_right.nodes.size() == 1)
    {
        if (cloned_right.root->value.value == 1) return cloned_left;
        else return robdd_false;
    }
    if (robdd1.root->label == robdd2.root->label)
    {
        ROBDD TrueROBDD, FalseROBDD;
```

```
        ROBDD cloned_left_left, cloned_left_right, cloned_right_left,
cloned_right_right;
        NewNode = Clone(cloned_left.root->value.successor.true_branch);
        cloned_left_left.root = NewNode;
        cloned_left_left.nodes = NodeVector(cloned_left_left.root);
        NewNode = Clone(cloned_left.root->value.successor.false_branch);
        cloned_left_right.root = NewNode;
        cloned_left_right.nodes = NodeVector(cloned_left_right.root);
        NewNode = Clone(cloned_right.root->value.successor.true_branch);
        cloned_right_left.root = NewNode;
        cloned_right_left.nodes = NodeVector(cloned_right_left.root);
        NewNode = Clone(cloned_right.root->value.successor.false_branch);
        cloned_right_right.root = NewNode;
        cloned_right_right.nodes = NodeVector(cloned_right_right.root);
        TrueROBDD = AND(cloned_left_left, cloned_right_left);
        FalseROBDD = AND(cloned_left_right, cloned_right_right);
        ROBDD ret;
        ret.root = new ROBDDNode;
        ret.root->label = robdd1.root->label;
        ret.root->value.successor.true_branch = TrueROBDD.root;
        ret.root->value.successor.false_branch = FalseROBDD.root;
        ret.nodes = NodeVector(ret.root);
        ret.Simplify();
        return ret;
    }
    ROBDD ret;
    if (!Contain(cloned_right.root, cloned_left.root->label))
    {
        ROBDD TrueROBDD, FalseROBDD;
        ROBDD cloned_left_left, cloned_left_right;
        NewNode = Clone(cloned_left.root->value.successor.true_branch);
        cloned_left_left.root = NewNode;
        cloned_left_left.nodes = NodeVector(cloned_left_left.root);
        NewNode = Clone(cloned_left.root->value.successor.false_branch);
        cloned_left_right.root = NewNode;
        cloned_left_right.nodes = NodeVector(cloned_left_right.root);
        TrueROBDD = AND(cloned_left_left, cloned_right);
        FalseROBDD = AND(cloned_left_right, cloned_right);
        ret.root = new ROBDDNode;
        ret.root->label = robdd1.root->label;
        ret.root->value.successor.true_branch = TrueROBDD.root;
        ret.root->value.successor.false_branch = FalseROBDD.root;
    }
    else
    {
        ROBDD TrueROBDD, FalseROBDD;
        ROBDD cloned_right_left, cloned_right_right;
        NewNode = Clone(cloned_right.root->value.successor.true_branch);
        cloned_right_left.root = NewNode;
        cloned_right_left.nodes = NodeVector(cloned_right_left.root);
        NewNode = Clone(cloned_right.root->value.successor.false_branch);
        cloned_right_right.root = NewNode;
        cloned_right_right.nodes = NodeVector(cloned_right_right.root);
        TrueROBDD = AND(cloned_right_left, cloned_left);
        FalseROBDD = AND(cloned_right_right, cloned_left);
```

```
        ret.root = new ROBDDNode;
        ret.root->label = robdd2.root->label;
        ret.root->value.successor.true_branch = TrueROBDD.root;
        ret.root->value.successor.false_branch = FalseROBDD.root;
    }
    ret.nodes = NodeVector(ret.root);
    for (int i = 0; i < ret.nodes.size(); i++)
    {
        if (ret.nodes[i]->label == 0)
        {
            ret.root = ret.nodes[i];
            break;
        }
    }
    ret.nodes = NodeVector(ret.root);
    ret.Simplify();
    return ret;
}
```

函数Contain表示包含关系，接受第一个参数作为带搜索树的根节点，第二个参数作为标签。如果子树存在该标签，则返回1，否则返回0。

```
bool Contain(ROBDDNode * node, int label)
{
    vector<ROBDDNode*> nodes;
    nodes = NodeVector(node);
    for (int i = 0; i < nodes.size(); i++)
    {
        if (nodes[i]->label == label) return true;
    }
    return false;
}
```

其余逻辑函数实现方法大同小异。

```
ROBDD OR(ROBDD robdd1, ROBDD robdd2)
{
    ROBDD cloned_left = robdd1.CloneROBDD();
    ROBDD cloned_right = robdd2.CloneROBDD();
    ROBDD robdd_true;
    ROBDDNode* NewNode = new ROBDDNode;
    NewNode->label = -1;
    NewNode->value.value = 1;
    robdd_true.nodes.push_back(NewNode);
    robdd_true.root = NewNode;
    if (cloned_left.nodes.size() == 1)
    {
        if (cloned_left.root->value.value == 1) return robdd_true;
        else return cloned_right;
    }
    if (cloned_right.nodes.size() == 1)
    {
        if (cloned_right.root->value.value == 1) return robdd_true;
        else return cloned_left;
```

```
    }
    if (robdd1.root->label == robdd2.root->label)
    {
        ROBDD TrueROBDD, FalseROBDD;
        ROBDD cloned_left_left, cloned_left_right, cloned_right_left,
cloned_right_right;
        NewNode = Clone(cloned_left.root->value.successor.true_branch);
        cloned_left_left.root = NewNode;
        cloned_left_left.nodes = NodeVector(cloned_left_left.root);
        NewNode = Clone(cloned_left.root->value.successor.false_branch);
        cloned_left_right.root = NewNode;
        cloned_left_right.nodes = NodeVector(cloned_left_right.root);
        NewNode = Clone(cloned_right.root->value.successor.true_branch);
        cloned_right_left.root = NewNode;
        cloned_right_left.nodes = NodeVector(cloned_right_left.root);
        NewNode = Clone(cloned_right.root->value.successor.false_branch);
        cloned_right_right.root = NewNode;
        cloned_right_right.nodes = NodeVector(cloned_right_right.root);
        TrueROBDD = OR(cloned_left_left, cloned_right_left);
        FalseROBDD = OR(cloned_left_right, cloned_right_right);
        ROBDD ret;
        ret.root = new ROBDDNode;
        ret.root->label = robdd1.root->label;
        ret.root->value.successor.true_branch = TrueROBDD.root;
        ret.root->value.successor.false_branch = FalseROBDD.root;
        ret.nodes = NodeVector(ret.root);
        ret.Simplify();
        return ret;
    }
    ROBDD ret;
    if (!Contain(cloned_right.root, cloned_left.root->label))
    {
        ROBDD TrueROBDD, FalseROBDD;
        ROBDD cloned_left_left, cloned_left_right;
        NewNode = Clone(cloned_left.root->value.successor.true_branch);
        cloned_left_left.root = NewNode;
        cloned_left_left.nodes = NodeVector(cloned_left_left.root);
        NewNode = Clone(cloned_left.root->value.successor.false_branch);
        cloned_left_right.root = NewNode;
        cloned_left_right.nodes = NodeVector(cloned_left_right.root);
        TrueROBDD = OR(cloned_left_left, cloned_right);
        FalseROBDD = OR(cloned_left_right, cloned_right);
        ret.root = new ROBDDNode;
        ret.root->label = robdd1.root->label;
        ret.root->value.successor.true_branch = TrueROBDD.root;
        ret.root->value.successor.false_branch = FalseROBDD.root;
    }
    else
    {
        ROBDD TrueROBDD, FalseROBDD;
        ROBDD cloned_right_left, cloned_right_right;
        NewNode = Clone(cloned_right.root->value.successor.true_branch);
        cloned_right_left.root = NewNode;
        cloned_right_left.nodes = NodeVector(cloned_right_left.root);
        NewNode = Clone(cloned_right.root->value.successor.false_branch);
```

```
        cloned_right_right.root = NewNode;
        cloned_right_right.nodes = NodeVector(cloned_right_right.root);
        TrueROBDD = OR(cloned_right_left, cloned_left);
        FalseROBDD = OR(cloned_right_right, cloned_left);
        ret.root = new ROBDDNode;
        ret.root->label = robdd2.root->label;
        ret.root->value.successor.true_branch = TrueROBDD.root;
        ret.root->value.successor.false_branch = FalseROBDD.root;
    }
    ret.nodes = NodeVector(ret.root);
    for (int i = 0; i < ret.nodes.size(); i++)
    {
        if (ret.nodes[i]->label == 0)
        {
            ret.root = ret.nodes[i];
            break;
        }
    }
    ret.nodes = NodeVector(ret.root);
    ret.Simplify();
    return ret;
}

ROBDD IMPLY(ROBDD robdd1, ROBDD robdd2)
{
    ROBDD cloned_left = robdd1.CloneROBDD();
    ROBDD cloned_right = robdd2.CloneROBDD();
    ROBDD robdd_true;
    ROBDDNode* NewNode = new ROBDDNode;
    NewNode->label = -1;
    NewNode->value.value = 1;
    robdd_true.nodes.push_back(NewNode);
    robdd_true.root = NewNode;
    if (cloned_left.nodes.size() == 1)
    {
        if (cloned_left.root->value.value == 1) return cloned_right;
        else return robdd_true;
    }
    if (cloned_right.nodes.size() == 1)
    {
        if (cloned_right.root->value.value == 1) return robdd_true;
        else return cloned_left;
    }
    if (robdd1.root->label == robdd2.root->label)
    {
        ROBDD TrueROBDD, FalseROBDD;
        ROBDD cloned_left_left, cloned_left_right, cloned_right_left,
cloned_right_right;
        NewNode = Clone(cloned_left.root->value.successor.true_branch);
        cloned_left_left.root = NewNode;
        cloned_left_left.nodes = NodeVector(cloned_left_left.root);
        NewNode = Clone(cloned_left.root->value.successor.false_branch);
        cloned_left_right.root = NewNode;
        cloned_left_right.nodes = NodeVector(cloned_left_right.root);
        NewNode = Clone(cloned_right.root->value.successor.true_branch);
```

```
        cloned_right_left.root = NewNode;
        cloned_right_left.nodes = NodeVector(cloned_right_left.root);
        NewNode = Clone(cloned_right.root->value.successor.false_branch);
        cloned_right_right.root = NewNode;
        cloned_right_right.nodes = NodeVector(cloned_right_right.root);
        TrueROBDD = IMPLY(cloned_left_left, cloned_right_left);
        FalseROBDD = IMPLY(cloned_left_right, cloned_right_right);
        ROBDD ret;
        ret.root = new ROBDDNode;
        ret.root->label = robdd1.root->label;
        ret.root->value.successor.true_branch = TrueROBDD.root;
        ret.root->value.successor.false_branch = FalseROBDD.root;
        ret.nodes = NodeVector(ret.root);
        for (int i = 0; i < ret.nodes.size(); i++)
        {
            if (ret.nodes[i]->label == 0)
            {
                ret.root = ret.nodes[i];
                break;
            }
        }
        ret.nodes = NodeVector(ret.root);
        ret.Simplify();
        return ret;
    }
    ROBDD ret;
    if (!Contain(cloned_right.root, cloned_left.root->label))
    {
        ROBDD TrueROBDD, FalseROBDD;
        ROBDD cloned_left_left, cloned_left_right;
        NewNode = Clone(cloned_left.root->value.successor.true_branch);
        cloned_left_left.root = NewNode;
        cloned_left_left.nodes = NodeVector(cloned_left_left.root);
        NewNode = Clone(cloned_left.root->value.successor.false_branch);
        cloned_left_right.root = NewNode;
        cloned_left_right.nodes = NodeVector(cloned_left_right.root);
        TrueROBDD = IMPLY(cloned_left_left, cloned_right);
        FalseROBDD = IMPLY(cloned_left_right, cloned_right);
        ret.root = new ROBDDNode;
        ret.root->label = robdd1.root->label;
        ret.root->value.successor.true_branch = TrueROBDD.root;
        ret.root->value.successor.false_branch = FalseROBDD.root;
    }
    else
    {
        ROBDD TrueROBDD, FalseROBDD;
        ROBDD cloned_right_left, cloned_right_right;
        NewNode = Clone(cloned_right.root->value.successor.true_branch);
        cloned_right_left.root = NewNode;
        cloned_right_left.nodes = NodeVector(cloned_right_left.root);
        NewNode = Clone(cloned_right.root->value.successor.false_branch);
        cloned_right_right.root = NewNode;
        cloned_right_right.nodes = NodeVector(cloned_right_right.root);
        TrueROBDD = IMPLY(cloned_right_left, cloned_left);
        FalseROBDD = IMPLY(cloned_right_right, cloned_left);
```

```
        ret.root = new ROBDDNode;
        ret.root->label = robdd2.root->label;
        ret.root->value.successor.true_branch = TrueROBDD.root;
        ret.root->value.successor.false_branch = FalseROBDD.root;
    }
    ret.nodes = NodeVector(ret.root);
    ret.Simplify();
    return ret;
}

ROBDD NOT(ROBDD robdd)
{
    ROBDD ret = robdd.CloneROBDD();
    ret.nodes = NodeVector(ret.root);
    for (int i = 0; i < ret.nodes.size(); i++)
    {
        if (ret.nodes[i]->label == -1) ret.nodes[i]->value.value = 1 -
ret.nodes[i]->value.value;
    }
    ret.Simplify();
    return ret;
}
```

EG，EU和EX仿照算法逐步计算即可。

```
ROBDD EG(Graph G, ROBDD robdd)
{ //V = {s ∈ T | ∃t ∈ U : s → t}
    cout << "\nImplementing EG..." << endl;
    int finished = 0;
    ROBDD T = robdd.CloneROBDD();
    ROBDD t0 = robdd.CloneROBDD();
    ROBDD tn = t0.CloneROBDD();
    int epoch = 0;
    while (!finished)
    {
        cout << "\nEpoch " << epoch << endl;
        ROBDD U = tn.CloneROBDD();
        cout << "\nt" << epoch << ":" << endl;
        tn.Print();
        vector<int> P1_table;
        int depth = ceil(log2(G.num_nodes));
        for (int i = 0; i < G.num_nodes; i++)
        {
            for (int j = 0; j < G.nodes[i]->next.size(); j++)
            {
                P1_table.push_back((i << depth) + G.nodes[i]->nextidx[j]);
            }
        }
        ROBDD P1, P2 = U.CloneROBDD();
        P1.FromTrueValueVector(P1_table);
        for (int i = 0; i < P2.nodes.size(); i++)
        {
            if (P2.nodes[i]->label >= 0) P2.nodes[i]->label += depth;
        }
        cout << "\nP1:" << endl;
```

```
        P1.Print();
        cout << "\nP2:" << endl;
        P2.Print();
        ROBDD P = AND(P1, P2);
        cout << "\nP:" << endl;
        P.Print();
        vector<int> SPe_table;
        for (int i = 0; i < pow(2,depth*2); i++)
        {
            if (P.Walk(i, depth * 2)) SPe_table.push_back(i >> depth);
        }
        ROBDD SPe;
        SPe.FromTrueValueVector(SPe_table);
        cout << "\nSPe:" << endl;
        SPe.Print();
        cout << "\nT:" << endl;
        T.Print();
        ROBDD V = AND(T, SPe);
        cout << "\nV:" << endl;
        V.Print();
        ROBDD last = tn.CloneROBDD();
        tn = AND(tn, V);
        if (Equal(tn.root, last.root))
        {
            cout << "\ntn=tn-1" << endl;
            finished = 1;
        }
        epoch++;
    }
    return tn;
}

ROBDD EX(Graph G, ROBDD robdd)
{ //V = {s ∈ T | ∃t ∈ U : s → t}
    cout << "\nImplementing EX..." << endl;
    int finished = 0;
    int depth = ceil(log2(G.num_nodes));
    ROBDD U = robdd.CloneROBDD();
    ROBDD T;
    vector<int> T_tables;
    for (int i = 0; i < G.num_nodes; i++)
    {
        T_tables.push_back(i);
    }
    T.FromTrueValueVector(T_tables);
    ROBDD tn = U.CloneROBDD();
    cout << "\nT:" << endl;
    T.Print();
    vector<int> P1_table;
    for (int i = 0; i < G.num_nodes; i++)
    {
        for (int j = 0; j < G.nodes[i]->next.size(); j++)
        {
            P1_table.push_back((i << depth) + G.nodes[i]->nextidx[j]);
        }
```

```
    }
    ROBDD P1, P2 = U.CloneROBDD();
    P1.FromTrueValueVector(P1_table);
    for (int i = 0; i < P2.nodes.size(); i++)
    {
        if (P2.nodes[i]->label >= 0) P2.nodes[i]->label += depth;
    }
    cout << "\nP1:" << endl;
    P1.Print();
    cout << "\nP2:" << endl;
    P2.Print();
    ROBDD P = AND(P1, P2);
    cout << "\nP:" << endl;
    P.Print();
    vector<int> SPe_table;
    for (int i = 0; i < pow(2, depth * 2); i++)
    {
        if (P.Walk(i, depth * 2)) SPe_table.push_back(i >> depth);
    }
    ROBDD SPe;
    SPe.FromTrueValueVector(SPe_table);
    cout << "\nSPe:" << endl;
    SPe.Print();
    ROBDD V = AND(T, SPe);
    cout << "\nV:" << endl;
    V.Print();
    tn = AND(tn, V);
    return tn;
}

ROBDD EU(Graph G, ROBDD robdd1, ROBDD robdd2)
{ //V = {s ∈ T | ∃t ∈ U : s → t}
    cout << "\nImplementing EU..." << endl;
    int finished = 0;
    ROBDD T = robdd1.CloneROBDD();
    ROBDD u0 = robdd2.CloneROBDD();
    ROBDD un = u0.CloneROBDD();
    int epoch = 0;
    while (!finished)
    {
        cout << "\nEpoch " << epoch << endl;
        ROBDD U = un.CloneROBDD();
        cout << "\nu" << epoch << ":" << endl;
        un.Print();
        vector<int> P1_table;
        int depth = ceil(log2(G.num_nodes));
        for (int i = 0; i < G.num_nodes; i++)
        {
            for (int j = 0; j < G.nodes[i]->next.size(); j++)
            {
                P1_table.push_back((i << depth) + G.nodes[i]->nextidx[j]);
            }
        }
        ROBDD P1, P2 = U.CloneROBDD();
        P1.FromTrueValueVector(P1_table);
```

```
        for (int i = 0; i < P2.nodes.size(); i++)
        {
            if (P2.nodes[i]->label >= 0) P2.nodes[i]->label += depth;
        }
        cout << "\nP1:" << endl;
        P1.Print();
        cout << "\nP2:" << endl;
        P2.Print();
        ROBDD P = AND(P1, P2);
        cout << "\nP:" << endl;
        P.Print();
        vector<int> SPe_table;
        for (int i = 0; i < pow(2, depth * 2); i++)
        {
            if (P.Walk(i, depth * 2)) SPe_table.push_back(i >> depth);
        }
        ROBDD SPe;
        SPe.FromTrueValueVector(SPe_table);
        cout << "\nSPe:" << endl;
        SPe.Print();
        cout << "\nT:" << endl;
        SPe.Print();
        ROBDD V = AND(T, SPe);
        cout << "\nV:" << endl;
        V.Print();
        ROBDD last = un.CloneROBDD();
        un = OR(un, V);
        if (Equal(un.root, last.root))
        {
            cout << "\nun=un-1" << endl;
            finished = 1;
        }
        epoch++;
    }
    return un;
}
```

主函数：读入图和符号表以及表达式，进行ROBDD的计算。表达式使用函数式写法，相邻参数用逗号隔开，允许空格。支持的函数如下：

- AND,OR,NOT
- IMPLY：蕴含
- EX,EG,EU
- AF,AX,EF,AG

```
#include <iostream>
#include <string>
#include <map>
#include "ROBDD.h"

using namespace std;
ROBDD parse(string expression);
map<string, int> sym_to_graph;
map<int, string> graph_to_sym;
```

```
vector<ROBDD> robdds;
Graph total_graph;
int main()
{
    int n;
    cout << "Input number of symbols:";
    cin >> n;
    cout << "Input these symbols,separated by blank:\n";
    for (int i = 0; i < n; i++)
    {
        string sym;
        cin >> sym;
        sym_to_graph[sym] = i;
        graph_to_sym[i] = sym;
    }
    int num_vert, num_edge;
    cout << "Input total number of vertices:";
    cin >> num_vert;
    cout << "Input total number of edges:";
    cin >> num_edge;
    for (int i = 0; i < num_vert; i++)
    {
        total_graph.AddNode(0);
    }
    cout << "Input the source node and destination node of each edge respectively:"
<< endl;
    for (int i = 0; i < num_edge; i++)
    {
        int src, dst;
        cin >> src >> dst;
        total_graph.AddEdge(src, dst);
    }
    for (int i = 0; i < n; i++)
    {
        cout << "Input true vertices for symbol " << graph_to_sym[i] << ", -1
indicates end";
        int vert;
        vector<int> table;
        while(1)
        {
            cin >> vert;
            if (vert == -1) break;
            table.push_back(vert);
        }
        ROBDD robdd;
        robdd.FromTrueValueVector(table);
        robdds.push_back(robdd);
    }
    while (1)
    {
        cout << "Input your expression(input exit to quit):\n";
        string expression;
        cin >> expression;
        if (expression == "exit") break;
        ROBDD result = parse(expression);
```

```
        cout << "\nResult:" << endl;
        result.Print();
    }
    return 0;
}
ROBDD parse(string expression)
{
    cout << "\nComputing " << expression << "..." << endl;
    if (!expression.empty())
    {
        int index = 0;
        while ((index = expression.find(' ', index)) != string::npos)
        {
            expression.erase(index, 1);
        }
    }
    int pos = expression.find('(', 0);
    if (pos == string::npos)
    {
        return robdds[sym_to_graph[expression]];
    }
    string op = expression.substr(0, pos);
    if (op == "") return parse(expression.substr(1, expression.length() - 2));
    if (op == "and" || op == "AND")
    {
        string remainder = expression.substr(pos, expression.length() - pos);
        int comma = remainder.find(',', 0);
        string expr1 = remainder.substr(0, comma);
        string expr2 = remainder.substr(comma, remainder.length() - comma);
        return AND(parse(expr1), parse(expr2));
    }
    else if (op == "or" || op == "OR")
    {
        string remainder = expression.substr(pos, expression.length() - pos);
        int comma = remainder.find(',', 0);
        string expr1 = remainder.substr(0, comma);
        string expr2 = remainder.substr(comma, remainder.length() - comma);
        return OR(parse(expr1), parse(expr2));
    }
    else if (op == "imply" || op == "IMPLY")
    {
        string remainder = expression.substr(pos, expression.length() - pos);
        int comma = remainder.find(',', 0);
        string expr1 = remainder.substr(0, comma);
        string expr2 = remainder.substr(comma, remainder.length() - comma);
        return IMPLY(parse(expr1), parse(expr2));
    }
    else if (op == "ex" || op == "EX")
    {
        string remainder = expression.substr(pos, expression.length() - pos);
        return EX(total_graph, parse(remainder));
    }
    else if (op == "eg" || op == "EG")
    {
        string remainder = expression.substr(pos, expression.length() - pos);
```

```
        return EG(total_graph, parse(remainder));
    }
    else if (op == "eu" || op == "EU")
    {
        string remainder = expression.substr(pos, expression.length() - pos);
        int comma = remainder.find(',', 0);
        string expr1 = remainder.substr(0, comma);
        string expr2 = remainder.substr(comma, remainder.length() - comma);
        return EU(total_graph, parse(expr1), parse(expr2));
    }
    else if (op == "not" || op == "NOT")
    {
        string remainder = expression.substr(pos, expression.length() - pos);
        return NOT(parse(remainder));
    }
    else if (op == "af" || op == "AF") //AF p=~EG~p
    {
        string remainder = expression.substr(pos, expression.length() - pos);
        cout << "AF(" << remainder << ")=NOT(EG(NOT" << remainder << ")))" << endl;
        return NOT(EG(total_graph,NOT(parse(remainder))));
    }
    else if (op == "ax" || op == "AX") //AX p=~EX~p
    {
        string remainder = expression.substr(pos, expression.length() - pos);
        cout << "AX(" << remainder << ")=NOT(EX(NOT" << remainder << ")))" << endl;
        return NOT(EX(total_graph, NOT(parse(remainder))));
    }
    else if (op == "ef" || op == "EF") //EF ϕ ≡ E[T U ϕ]
    {
        ROBDD robdd_true;
        ROBDDNode* NewNode = new ROBDDNode;
        NewNode->label = -1;
        NewNode->value.value = 0;
        robdd_true.nodes.push_back(NewNode);
        robdd_true.root = NewNode;
        string remainder = expression.substr(pos, expression.length() - pos);
        cout << "EF(" << remainder << ")=E(T U " << remainder << ")" << endl;
        return EU(total_graph, robdd_true, parse(remainder));
    }
    else if (op == "ag" || op == "AG") //AG ϕ ≡ ~E[T U ~ϕ]
    {
        ROBDD robdd_true;
        ROBDDNode* NewNode = new ROBDDNode;
        NewNode->label = -1;
        NewNode->value.value = 0;
        robdd_true.nodes.push_back(NewNode);
        robdd_true.root = NewNode;
        string remainder = expression.substr(pos, expression.length() - pos);
        cout << "AG(" << remainder << ")=NOT(E(T U NOT(" << remainder << ")))" <<
endl;
        return NOT(EU(total_graph, robdd_true, NOT(parse(remainder))));
    }
}
```

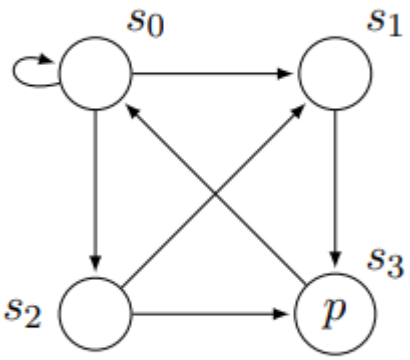
此外，还有用于将十进制数转化为二进制向量的函数。它们写在MathFunc中。

```
//MathFunc.h
#pragma once
#include<vector>
using namespace std;
vector<bool> IntToBinVec(int num, int len); //len represents the length of returned
vector
```

```
//MathFunc.cpp
#include "MathFunc.h"

vector<bool> IntToBinVec(int num, int len)
{
    vector<bool> ret; //vector to be returned
    while(ret.size()<len)
    {
        ret.insert(ret.begin(), num % 2);
        num /= 2;
    }
    return ret;
}
```

程序运行

考虑下面的一幅图:



程序的输入输出如下:

```
Input number of symbols:1
Input these symbols,separated by blank:
p
Input total number of vertices:4
Input total number of edges:7
Input the source node and destination node of each edge respectively:
0 0
0 1
0 2
1 3
2 1
2 3
3 0
Input true vertices for symbol p, -1 indicates end3
-1
```

```
Input your expression(input exit to quit):
AF(p)

Computing AF(p)...
AF((p))=NOT(EG(NOT(p))))

Computing (p)...

Computing p...

Implementing EG...

Epoch 0

t0:
0(tests x0)  ----False---->  3(True)
0(tests x0)  ----True---->  1
1(tests x1)  ----False---->  3(True)
1(tests x1)  ----True---->  2(False)
2  stands for False
3  stands for True

P1:
0(tests x0)  ----False---->  7
0(tests x0)  ----True---->  1
1(tests x1)  ----False---->  6
1(tests x1)  ----True---->  2
2(tests x2)  ----False---->  4
2(tests x2)  ----True---->  3(False)
3  stands for False
4(tests x3)  ----False---->  5(True)
4(tests x3)  ----True---->  3(False)
5  stands for True
6(tests x3)  ----False---->  3(False)
6(tests x3)  ----True---->  5(True)
7(tests x1)  ----False---->  9
7(tests x1)  ----True---->  8
8(tests x2)  ----False---->  3(False)
8(tests x2)  ----True---->  6
9(tests x2)  ----False---->  5(True)
9(tests x2)  ----True---->  4

P2:
0(tests x2)  ----False---->  3(True)
0(tests x2)  ----True---->  1
1(tests x3)  ----False---->  3(True)
1(tests x3)  ----True---->  2(False)
2  stands for False
3  stands for True

P:
0(tests x0)  ----False---->  8
0(tests x0)  ----True---->  1
1(tests x1)  ----False---->  6
1(tests x1)  ----True---->  2
```

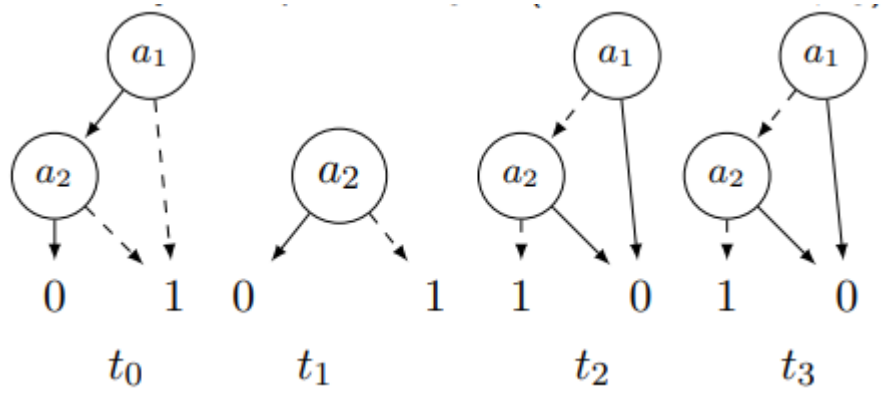
```
2(tests x2)  ----False---->  4
2(tests x2)  ----True---->  3(False)
3  stands for False
4(tests x3)  ----False---->  5(True)
4(tests x3)  ----True---->  3(False)
5  stands for True
6(tests x2)  ----False---->  7
6(tests x2)  ----True---->  3(False)
7(tests x3)  ----False---->  3(False)
7(tests x3)  ----True---->  5(True)
8(tests x1)  ----False---->  9
8(tests x1)  ----True---->  3(False)
9(tests x2)  ----False---->  5(True)
9(tests x2)  ----True---->  4

SPe:
0(tests x0)  ----False---->  2
0(tests x0)  ----True---->  1(True)
1  stands for True
2(tests x1)  ----False---->  1(True)
2(tests x1)  ----True---->  3(False)
3  stands for False

T:
0(tests x0)  ----False---->  3(True)
0(tests x0)  ----True---->  1
1(tests x1)  ----False---->  3(True)
1(tests x1)  ----True---->  2(False)
2  stands for False
3  stands for True

V:
0(tests x1)  ----False---->  2(True)
0(tests x1)  ----True---->  1(False)
1  stands for False
2  stands for True

Epoch 1

t1:
0(tests x1)  ----False---->  2(True)
0(tests x1)  ----True---->  1(False)
1  stands for False
2  stands for True

P1:
0(tests x0)  ----False---->  7
0(tests x0)  ----True---->  1
1(tests x1)  ----False---->  6
1(tests x1)  ----True---->  2
2(tests x2)  ----False---->  4
2(tests x2)  ----True---->  3(False)
3  stands for False
4(tests x3)  ----False---->  5(True)
4(tests x3)  ----True---->  3(False)
```

```
5  stands for True
6(tests x3)  ----False---->  3(False)
6(tests x3)  ----True---->  5(True)
7(tests x1)  ----False---->  9
7(tests x1)  ----True---->  8
8(tests x2)  ----False---->  3(False)
8(tests x2)  ----True---->  6
9(tests x2)  ----False---->  5(True)
9(tests x2)  ----True---->  4

P2:
0(tests x3)  ----False---->  2(True)
0(tests x3)  ----True---->  1(False)
1  stands for False
2  stands for True

P:
0(tests x0)  ----False---->  2
0(tests x0)  ----True---->  1
1(tests x1)  ----False---->  3(False)
1(tests x1)  ----True---->  2
2(tests x1)  ----False---->  4
2(tests x1)  ----True---->  3(False)
3  stands for False
4(tests x3)  ----False---->  5(True)
4(tests x3)  ----True---->  3(False)
5  stands for True

SPe:
0(tests x0)  ----False---->  4
0(tests x0)  ----True---->  1
1(tests x1)  ----False---->  3(False)
1(tests x1)  ----True---->  2(True)
2  stands for True
3  stands for False
4(tests x1)  ----False---->  2(True)
4(tests x1)  ----True---->  3(False)

T:
0(tests x0)  ----False---->  3(True)
0(tests x0)  ----True---->  1
1(tests x1)  ----False---->  3(True)
1(tests x1)  ----True---->  2(False)
2  stands for False
3  stands for True

V:
0(tests x0)  ----False---->  2
0(tests x0)  ----True---->  1(False)
1  stands for False
2(tests x1)  ----False---->  3(True)
2(tests x1)  ----True---->  1(False)
3  stands for True

Epoch 2
```

```
t2:
0(tests x0)  ----False---->  2
0(tests x0)  ----True---->  1(False)
1  stands for False
2(tests x1)  ----False---->  3(True)
2(tests x1)  ----True---->  1(False)
3  stands for True

P1:
0(tests x0)  ----False---->  7
0(tests x0)  ----True---->  1
1(tests x1)  ----False---->  6
1(tests x1)  ----True---->  2
2(tests x2)  ----False---->  4
2(tests x2)  ----True---->  3(False)
3  stands for False
4(tests x3)  ----False---->  5(True)
4(tests x3)  ----True---->  3(False)
5  stands for True
6(tests x3)  ----False---->  3(False)
6(tests x3)  ----True---->  5(True)
7(tests x1)  ----False---->  9
7(tests x1)  ----True---->  8
8(tests x2)  ----False---->  3(False)
8(tests x2)  ----True---->  6
9(tests x2)  ----False---->  5(True)
9(tests x2)  ----True---->  4

P2:
0(tests x2)  ----False---->  2
0(tests x2)  ----True---->  1(False)
1  stands for False
2(tests x3)  ----False---->  3(True)
2(tests x3)  ----True---->  1(False)
3  stands for True

P:
0(tests x0)  ----False---->  6
0(tests x0)  ----True---->  1
1(tests x1)  ----False---->  3(False)
1(tests x1)  ----True---->  2
2(tests x2)  ----False---->  4
2(tests x2)  ----True---->  3(False)
3  stands for False
4(tests x3)  ----False---->  5(True)
4(tests x3)  ----True---->  3(False)
5  stands for True
6(tests x1)  ----False---->  2
6(tests x1)  ----True---->  3(False)

SPe:
0(tests x0)  ----False---->  4
0(tests x0)  ----True---->  1
1(tests x1)  ----False---->  3(False)
```

```
1(tests x1)  ----True---->  2(True)
2  stands for True
3  stands for False
4(tests x1)  ----False---->  2(True)
4(tests x1)  ----True---->  3(False)

T:
0(tests x0)  ----False---->  3(True)
0(tests x0)  ----True---->  1
1(tests x1)  ----False---->  3(True)
1(tests x1)  ----True---->  2(False)
2  stands for False
3  stands for True

V:
0(tests x0)  ----False---->  2
0(tests x0)  ----True---->  1(False)
1  stands for False
2(tests x1)  ----False---->  3(True)
2(tests x1)  ----True---->  1(False)
3  stands for True

tn=tn-1

Result:
0(tests x0)  ----False---->  2
0(tests x0)  ----True---->  1(True)
1  stands for True
2(tests x1)  ----False---->  3(False)
2(tests x1)  ----True---->  1(True)
3  stands for False
```

这和ROBDD每一步迭代结果相吻合。



其中$t_3 = t_2$。

注

完整代码详见https://github.com/Kobe972/ROBDD