



Book of ABSTRACTS

**5-6 September
2022**

Bangsaen Heritage Hotel,
Chonburi, Thailand

International Workshop on *Applied Nonlinear Analysis*

Organized by: Working Group on Applied Nonlinear Analysis
Supported by: Center of Excellence in Theoretical and
Computational Science

Book of Abstracts

IWANA2022

The 3rd International Workshop on
Applied Nonlinear Analysis

Bangsaen Heritage Hotel, Bangsean, Chonburi, Thailand

September 5–6, 2022

Contents

Contents	1
Forward	3
Organizing Committee Members	5
General Information	8
Schedule	10
Abstracts of Keynote Talks	19
Abstracts of Invited Talks	28
Abstracts of Contributed Talks	39
Organizing Staffs	90





Forward

In 2022, the International Workshop on Applied Nonlinear Analysis (IWANA) has come into its third iterate. The first of its series was named differently (IWNAA) and was organized in Granada, Spain. The second workshop took place in Chonburi, Thailand, and was renamed into IWANA.

The IWANA 2022 is organized again in Chonburi, Thailand, on 5–6 September 2022. Due to the COVID-19 situation worldwide, the event is organized in a hybrid platform – limited number of selected participants are allowed on site and the remaining are asked to enjoy the workshop remotely. This year, we are honored to host several talks by renowned mathematicians from all over the world, namely

- Professor Somyot Plubtieng from Naresuan University, Thailand.
- Professor Yeol Je Cho from Gyeongsang National University, South Korea.
- Professor Mohamed Amine Khamsi from Khalifa University, United Arab Emirates.
- Professor Vasile Berinde from Technical University of Baia Mare, North University Center at Baia Mare, Baia Mare, Romania.
- Professor Jong Kyu Kim from Kyungnam University, South Korea.
- Professor Suthep Suantai from Chiangmai University, Thailand.

There are also two very special celebrations at the IWANA 2022 – (1) The 100 years of the Banach Contraction Principle, and (2) The 60th birthday and retirement of Professor Somyot Plubtieng.

Banach Contraction Principle is the founding theorem of what we know today as the metric fixed point theory. It simply states that *every contraction map defined on a complete metric space has a unique fixed point*. Apart from that, the theorem also provides an iterative scheme to reach that unique fixed point as well as the error estimation. In the original work of Banach, the fixed point equation is an abstract realization of the Cauchy problem. Afterwards, many extensions of the Banach Contraction Principle are obtained and several more applications were found until today.

Professor Somyot Plubtieng is one of the *BIG THREE S.'s*, alongside with Professor Sompong Dhompangsa and Professor Suthep Suantai, who have laid foundation of fixed point theory, as well as modern mathematical research, in Thailand. He has been inventing several important results in the area of fixed point theory, variational



inequality, variational analysis, and optimization. Moreover, he had trained several young mathematicians to be able to produce meaningful researches and writing successful research proposals.

We wish everyone would have a great time enjoying mathematics and Thai beach at the IWANA 2022.

Best regards,
The organizers of IWANA2022.



Organizing Committee Members

Organizing Committee

- Juan Martínez-Moreno (Universidad de Jaén, Spain)
- Antonio Francisco Roldán-López-de-Hierro (University of Granada, Spain)
- Dhananjay Gopal (Guru Ghasidas Vishwavidyalaya, India)
- Ovidiu Bagdasar (University of Derby, United Kingdom)
- Poom Kumam (King Mongkut's University of Technology Thonburi, Thailand)
- Parin Chaipanya (King Mongkut's University of Technology Thonburi, Thailand)
- Supak Phiangsungnoen (Rajamangala University of Technology Rattanakosin, Thailand)
- Nantaporn Chuensupantharat (Bansomdejchaopraya Rajabhat University, Thailand)

Local Organizing Committee

- Poom Kumam (King Mongkut's University of Technology Thonburi, Thailand)
- Parin Chaipunya (King Mongkut's University of Technology Thonburi, Thailand)
- Supak Phiangsungnoen (Rajamangala University of Technology Rattanakosin, Thailand)
- Konrawut Khammahawong (Rajamangala University of Technology Thanyaburi, Thailand)
- Wiyada Kumam (Rajamangala University of Technology Thanyaburi, Thailand)
- Kamonrat Sombut (Rajamangala University of Technology Thanyaburi, Thailand)
- Thongchai Botmart (Khon Kaen University, Thailand)
- Kanit Mukdasai (Khon Kaen University, Thailand)
- Wajaree Weera (Khon Kaen University, Thailand)
- Nimit Nimana (Khon Kaen University, Thailand)



- Thidaporn Seangwattana (King Mongkut's University of Technology North Bangkok, Thailand)
- Kanokwan Sitthithakerngkiet (King Mongkut's University of Technology North Bangkok, Thailand)
- Kasamsuk Ungchittrakool (Naresuan University, Thailand)
- Nantaporn Chuensupantharat (Bansomdejchaopraya Rajabhat University, Thailand)
- Phumin Sumalai (Muban Chombueng Rajabhat University, Thailand)
- Issara Inchan (Uttaradit Rajabhat University, Thailand)
- Thanatporn Bantaojai (Valaya Alongkorn Rajabhat University under the Royal Patronage, Thailand)
- Areerat Arunchai (Nakhon Sawan Rajabhat University, Thailand)

Scientific Committee

- Ravi P. Agarwal (Texas A&M University-Kingsville, USA)
- Nutefe Kwami Agbeko (Institute of Mathematics, University of Miskolc, Hungary)
- Shigeo Akashi (Tokyo University of Science, Japan)
- Vasil Angelov (University of Mining and Geology, Bulgaria)
- Vasile Berinde (Universitatea de Nord din Baia Mare, Romania)
- Janusz Brzdek (Pedagogical University of Cracow, Poland)
- Theodore Burton (Northwest Research Institute, USA)
- Jinde Cao (South East University, China)
- Yeol Je Cho (Gyeongsang National University, Korea)
- Dragan Djordjevic (University of Nis, Serbia)
- Tomás Domínguez Benavides (Universidad de Sevilla, Spain)
- Wei-Shih Du (National Kaohsiung Normal University, Taiwan)
- Inci M. Erhan (Atılım University, Turkey)



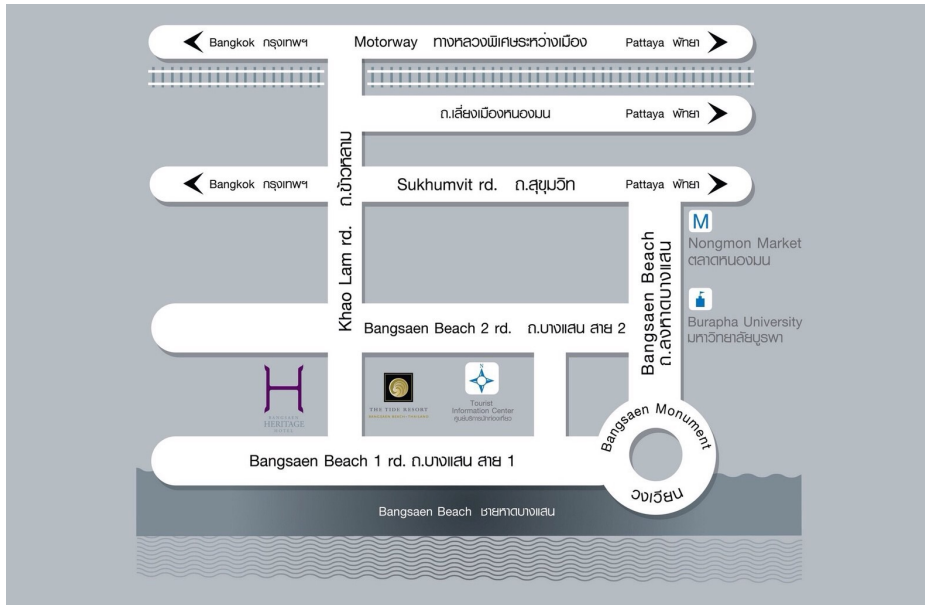
- Marlene Frigon (Université de Montréal, Canada)
- C. Masood Khalique (North-West University, South Africa)
- Yasunori Kimura (Toho University, Japan)
- Poom Kumam (King Mongkut's University of Technology Thonburi, Thailand)
- Anthony To-Ming Lau (University of Alberta, Canada)
- Giuseppe Marino (Università della Calabria, Italy)
- Juan Martínez-Moreno (Universidad de Jaén, Spain)
- Sehie Park (Seoul National University, Republic of Korea)
- Narin Petrot (Naresuan University)
- Adrian Petruşel (Babeş-Bolyai University, Romania)
- Ariana Pitea (Polytechnic University of Bucharest, Romania)
- Vladimir Rakocevic (University of Nis, Serbia)
- Simeon Reich (Israel Institute of Technology, Israel)
- Antonio Francisco Roldán-López-de-Hierro (University of Granada, Spain)
- Ramachandra Raja (Alagappa University, India)
- Salvador Romaguera Bonilla (Instituto Universitario de Matematica Pura y Aplicada (IUMPA), Universitat Politecnica de Valencia, Spain)
- Brailey Sims (The University of Newcastle, Australia)
- Naseer Shahzad (King Abdul Aziz University, Saudi Arabia)
- Wataru Takahashi (Kaohsiung Medical University, Taiwan)
- Jen-Chih Yao (China Medical University, Kaohsiung, Taiwan)
- Jong Kyu Kim (Kyungnam University, South Korea)

General Information

Conference Venue Bangsaen Heritage Hotel, 50 54 Bangsaen Sai 1, Saen Suk, Chon Buri District, Chon Buri 20130

Phone: +66 3839-9899

Website: <https://www.bs-heritagehotel.com>



Official Language The official language of the conference is English. All the presentations will be made in English. No arrangements are available for simultaneous translation.

Registration Regular and Student participants who made registration will be provided with a conference kit. Lunches and refreshments during the conference are also available. All the participants can attend the conference banquet without additional fees.

Name Tag Badges All the participants will be issued with a name tag badge at registration. You are always requested to wear the badge; it is your entrance ticket to all conference sessions and other arrangements. For loss of badge, contact the Information Desk.



Messages A message board will be located near the Information Desk.

Presentation Detail

- There are four kinds of lectures in the technical program; keynote lectures, invited lectures and contributed talks in the regular sessions. Organized sessions focusing on specific topics are located in regular sessions.
- A keynote lecture has 30 min each, an invited lecture have 20 min each and other talks have 15 min each including preparation time, questions and answers.
- If you find your name as a chairperson of a session, you are requested to go to the room and preside the session.
- A video projector (for PC) will be provided in each session. VGA connection is available. The speakers need to bring their own PC if necessary. For further requirements, please ask the Information Desk.

Lunch Lunch tickets will be provided at the registration. You can use them for the exchange of the meal at the Sila restaurant.

Smoking Policy Participants are kindly requested to refrain from smoking inside the building including all the session rooms. Smoking is only allowed in the designated smoking area.

Dietary Requirements/Disability Assistance If you have special dietary requirements and/or you require disability assistance, please contact the Information Desk staff at any time.

Wi-Fi Connection Free Wi-Fi connection is available at the conference rooms.

Banquet The banquet venue for IWANA2022 is “Saensuk 1–2” in the Bangsean Heritage Hotel. All participants can attend the banquet without additional charge.



Schedule

IWANA2022 Program Timetable

Monday, September 5, 2022.

Venue: Bangsaen Heritage Hotel

Times	Events
08.00 – 09.00	Registration
09.00 – 09.10	Opening Ceremony
09.10 – 09.40	Honorary Lecture (S. Plubtieng)
09.40 – 09.55	Coffee Break
09.55 – 12.00	Parallel Session 1
12.00 – 13.00	Lunch
13.00 – 13.30	Keynote Lecture (J.K. Kim)
13.30 – 15.05	Parallel Session 2
15.05 – 15.20	Coffee Break
15.20 – 16.40	Parallel Session 3
16.45 – 17.15	Keynote Lecture (V. Berinde)
18.00 – 18.30	Special Discussion (S. Dhompongsa S. Suantai S. Plubtieng)
18:30	Banquet Party

Tuesday, September 6, 2022.

Venue: Bangsaen Heritage Hotel

Times	Events
09.00 – 09.30	Keynote Lecture (S. Suantai)
09.30 – 10.50	Parallel Session 4
10.50 – 11.00	Coffee Break
11.00 – 12.20	Parallel Session 5
12.20 – 13.30	Lunch
13.30 – 14.00	Keynote Lecture (Y.J. Cho)
14.00 – 14.15	Best Presentation Award Ceremony
14.15 – 14.45	Keynote Lecture (M.A. Khamsi)
14.45 – 15.00	Closing Ceremony



Monday, September 5, 2022.

Honorary Lecture | 09.10 – 09.40

Chair : Suthep Suantai

Somyot Plubtieng

▷ Modified inertial subgradient extragradient algorithm for solving bilevel system of equilibrium problems

Parallel Session 1 | 09.55 – 12.00

Room 1 : Differential Equation I

Chair : Thongchai Botmart

- 09.55 – 10.15 **THONGCHAI BOTMART (INVITED)**
▷ Pinning sampled-data control for function projective synchronization of neural networks with hybrid couplings and time-varying delays
- 10.15 – 10.30 **Arthit Hongsri**
▷ Extended dissipative analysis for sampled-data synchronization of complex dynamical networks with coupling time-varying delays
- 10.30 – 10.45 **Saqib Murtaza**
▷ Numerical analysis of fractal fractional non-linear electro osmotic flow with cadmium telluride nanoparticles
- 10.45 – 11.00 **Sunisa Luemsai**
▷ Improved extended dissipativity results for T-S fuzzy neural networks with interval time-varying delay
- 11.00 – 11.15 **Phichsinee Khongja**
▷ New guaranteed cost control for exponential stability of nonlinear system with mixed time-varying delays via feedback control
- 11.15 – 11.30 **Peerapongpat Singkibud**
▷ New exponential passivity analysis of neutral-type neural networks with distributed time-varying delays
- 11.30 – 11.45 **Jitsin Piyawatthanachot**
▷ New stability criterion for linear system with mixed time-varying delays and nonlinear perturbations



Room 2 : Fixed Point Theory I

Chair : Wutiphol Sintunavarat

- 09.55 – 10.15 **WUTIPHOL SINTUNAVARAT (INVITED)**
▷ On Thai mathematician's approaches for solving linear/nonlinear problems with the fixed point method
- 10.15 – 10.30 **Ajay Kumar Chaudhary**
▷ Fixed point result by using compatible mapping of type P in menger space
- 10.30 – 10.45 **Sunisa Somsit**
▷ Quasi-nonexpansive selections and convergence theorems for multivalued nonexpansive mappings in Banach spaces
- 10.45 – 11.00 **Kasamsuk Ungchittrakool**
▷ Accelerated Mann-type iterative algorithm for solving image restoration problems
- 11.00 – 11.15 **Thanatporn Bantaojai**
▷ Sadovski-Darbo fixed point theorem for ψ -Riemann-Liouville fractional differential equation with Riemann-Stieltjes integral conditions

Keynote Lecture | 13.00 – 13.30

Chair : Yeol Je Cho

Jong Kyu Kim

▷ Gap functions for general set-valued nonlinear variational-hemivariational inequalities

Parallel Session 2 | 13.30 – 15.05

Room 1 : Applied Mathematics I

Chair : Rabian Wangkeeree

- 13.30 – 13.50 **ANTONIO FRANCISCO ROLDAN LOPEZ DE HIERRO (INVITED)**
▷ On the approximation of fuzzy sets by fuzzy numbers
- 13.50 – 14.05 **Vipavee Damminsed**
▷ Laplacian twin support vector machine with Pinball loss for semi-supervised classification
- 14.05 – 14.20 **Shigeo Akashi**
▷ A Relation between Hilbert's 13th problem and data compression problem



- 14.20 – 14.35 **Harish Chandra Brhaandari**
▷ Graph interpretation and feature extraction strategy for electroencephalogram signal classification

Room 2 : Differential Equation II
Chair : Jessada Tariboon

- 13.30 – 13.50 **JESSADA TARIBOON (INVITED)**
▷ Fractional (p, q) -difference equations and boundary value problems
- 13.50 – 14.05 **Janejira Tranthi**
▷ A novel criteria on exponentially passive analysis for Takagi-Sugeno fuzzy of neutral dynamic system with various time-varying delays
- 14.05 – 14.20 **Issaraporn Khonchaiyaphum**
▷ Finite-time passivity analysis of neutral-type neural networks with mixed time-varying delays
- 14.20 – 14.35 **Chalida Phanlert**
▷ Finite-time mixed H_∞ /passivity criteria for generalized neural networks with mixed interval time-varying delays
- 14.35 – 14.50 **Chantapish Zamart**
▷ Finite-time stability analysis of generalized neural networks with mixed interval time-varying delays via new integral inequality
- 14.50 – 15.05 **Charuwat Chantawat**
▷ Finite-time synchronization control for coronary artery chaos system with state and input time-varying delays

Parallel Session 3 | 15.20 – 16.40

Room 1 : Fixed Point Theory II
Chair : Dhananjay Gopal

- 15.20 – 15.40 **DHANANJAY GOPAL (INVITED)**
▷ Search of minimal metric structure in the context of fixed point theorem
- 15.40 – 15.55 **Sachiko Atsushiba**
▷ Convergence theorems for monotone nonexpansive mappings in ordered uniformly convex Banach spaces
- 15.55 – 16.10 **Dang-Khoa Nguyen**
▷ Fast Krasnosel'skiĭ-Mann algorithm with a convergence rate of the fixed point iteration of $o(1/k)$



- 16.10 – 16.25 **Jayesh D Savaliya**
 ▷ Some discussion on generalizations of metric spaces in fixed point perspective
- 16.25 – 16.40 **Sani Salisu**
 ▷ Properties of enriched nonexpansive mappings in Hadamard spaces

Room 2 : Optimization I

Chair : Narin Petrot

- 15.20 – 15.40 **OVIDIU BAGDASARL (INVITED)**
 ▷ One certain generalised convexity concepts and related local-global type properties
- 15.40 – 15.55 **Salihu Nasiru**
 ▷ A hybrid Hestenes-Stiefel and Dai-Yuan minimization with application in three degrees of freedom real-time motion control robotic model
- 15.55 – 16.10 **Ibrahim Arzuka**
 ▷ Three-term conjugate gradient method for solving variational inequality problems over the fixed point set of nonexpansive mapping

Keynote Lecture | 16.45 – 17.15

Chair : Mohamed Amine Khamssi

Vasile Berinde

- ▷ From Banach contractions to almost contractions, enriched contractions and beyond

Special Discussion | 18.00 – 18.30

Chair : Poom Kumam

Sompong Dhompongsa || Suthep Suantai || Somyot Plubtieng





Tuesday, September 6, 2022.

Keynote Lecture | 09.00 – 09.30

Chair : Jamnian Nantadilok

Suthep Suantai

▷ An accelerated convex optimization algorithm with line search and applications in machine learning

Parallel Session 4 | 09.30 – 10.50

Room 1 : Fixed Point Theory III

Chair : Juan Martínez-Moreno

09.30 – 09.50 **JUAN MARTINEZ-MORENO (INVITED)**

▷ Progressive iterative approximation methods

09.50 – 10.05 **Thanittha Kowan**

▷ Modified inertial algorithms for inclusion problems

10.05 – 10.20 **Kiattisak Rattanaseeha**

▷ A new iterative methods for a finite family of the split generalized equilibrium problem and fixed point problem

10.20 – 10.35 **Dawrawee Makmuang**

▷ A regularized stochastic Nesterov's accelerated quasi-newton method with applications

10.35 – 10.50 **Abubakar Bala Abubakar**

▷ A hybrid approach for finding approximate solutions to constrained nonlinear monotone operator equations with applications

Room 2 : Optimization II

Chair : Kanit Mukdasai

09.30 – 09.50 **KANIT MUKDASAI (INVITED)**

▷ Robust stability analysis for discrete-time neural networks with mixed time-varying delays via a new summation inequality

09.50 – 10.05 **Muhammad Ramzan**

▷ Heat transfer analysis of the mixed convective flow of magneto-hydrodynamic hybrid nanofluid past a stretching sheet with velocity and thermal slip conditions



- 10.05 – 10.20 **Muhammad Arif**
▷ Heat transfer analysis of radiator using different shaped nanoparticles water-based ternary hybrid nanofluids with applications: a fractional model
- 10.20 – 10.35 **Boonyachart Meesuptong**
▷ H_∞ performance for uncertain neutral system with mixed time-varying delays

Parallel Session 5 | 11.00 – 12.20

Room 1 : Geodesic Spaces

Chair : Bancha Panyanak

- 11.00 – 11.20 **BANCHA PANYANAK (INVITED)**
▷ Endpoint theorems for some generalized multivalued nonexpansive mappings
- 11.20 – 11.35 **Jamnian Nantadilok**
▷ On the existence of fixed points of quasi-nonexpansive multimaps in a $CAT(0)$ space
- 11.35 – 11.50 **Kazuya Sasaki**
▷ Conditions for perturbations to define the resolvent of the equilibrium problem on geodesic spaces
- 11.50 – 12.05 **Adamu Yusuf Inuwa**
▷ Equilibrium problems and proximal algorithm in monotone vector field
- 12.05 – 12.20 **Yasunori Kimura**
▷ Iterative schemes converging to a fixed point of a mapping on a complete geodesic space

Room 2 : Applied Mathematics II

Chair : Kamsing Nonlaopon

- 11.00 – 11.20 **OSVALDO MENDEZ (INVITED)**
▷ Physical applications of variable exponent spaces: Kirchoff's equation for the elastic string
- 11.20 – 11.35 **Bharat Raj Wagle**
▷ Closed queueing network analysis of vehicle sharing in a city
- 11.35 – 11.50 **Patcharapa Srichok**
▷ Levitin-Polyak well-posedness for generalized (η, g, φ) -mixed vector variational-type inequality problem



Keynote Lecture | 13.30 – 14.00

Chair : Jong Kyu Kim

Yeol Je Cho

▷ Projection methods for solving variational Inequality problems in Hilbert spaces

Keynote Lecture | 14.15 – 14.45

Chair : Somyot Plubtieng

Mohamed Amine Khamsi

▷ Linear functional analysis in: metric spaces

Abstracts of Keynote Talks



Gap functions for general set-valued nonlinear variational-hemivariational inequalities

Jong Kyu Kim

*Department of Mathematics Education, Kyungnam University,
Changwon, Gyeongnam, 51767, Republic of Korea.
e-mail address: jongkyuk@kyungnam.ac.kr*

Abstract

The objective of this talk is to study the general set-valued nonlinear variational-hemivariational inequalities and investigate the gap function, regularized gap function and Moreau-Yosida type regularized gap functions for the general set-valued nonlinear variational-hemivariational inequalities. And also, we discuss the error bounds for such inequalities using the characteristic of the Clarke generalized gradient, locally Lipschitz continuity, inverse strong monotonicity and Hausdorff Lipschitz continuous mappings.

Acknowledgements: Thanks for the invitation me as a keynote speaker at The 2nd International Workshop on Applied Nonlinear Analysis 2022^{*} (IWANA 2022) which will be held as a hybrid (onsite + online) conference at Bang San Beach, to the Organizing committee of IWANA 2022.

References

- [1] Fukushima, M.: A class of gap functions for quasi-variational inequality problems. *J. Ind. Manag. Optim.* **3**, 165–171 (2007).
- [2] Yamashita, N., Fukushima, M.: Equivalent unconstrained minimization and global error bounds for variational inequality problems. *SIAM J. Control Optim.* **35**, 273–284 (1997).
- [3] Anh, L.Q., Bantaojai, T., Hung, N.V., Tam, V.M., Wangkeeree, R.: Painleve–Kuratowski convergences of the solution sets for generalized vector quasi-equilibrium problems. *Comput. Appl. Math.* **37**, 3832–3845 (2018).
- [4] Kim, J.K., Salahuddin, Dar, A.H.: Existence solution for the generalized relaxed pseudomonotone variational inequalities. *Nonlinear Funct. Anal. Appl.* **25**(1) 25–34 (2020).
- [5] Kim, J.K., Alesemi, M. and Salahuddin: Convergence theorem of relaxed quasimonotone variational inequality problems, *Nonlinear and Convex Analysis*, **22**(12)(2021), 2672–2678



Linear functional analysis in: metric spaces

Mohamed Amine Khamsi

*Department of Applied Mathematics and Sciences,
Khalifa University, Abu Dhabi, UAE.*

email: mohamed.khamsi@ku.ac.ae

Abstract

As for the linear case, compactness for the strong topology is very restrictive. Since the beginning of the fixed point theory, weak-compactness offered an acceptable alternative in Banach spaces. But when we deal with metric spaces, this natural extension is no longer easy to implement. One has to go back to the linear case and investigate the weak-topology with a new eye. In this talk, I will share some of the ideas of how to extend concepts of linear nature to nonlinear spaces, i.e., metric spaces.

References

- [1] N. Aronszajn and P. Panitchpakdi, *Extension of uniformly continuous transformations and hyperconvex metric spaces*, Pacific J. Math. **6** (1956), 405-439.
- [2] M. A. Khamsi, M. Pouzet, *A fixed point theorem for commuting families of relational homomorphisms. Applications to metric spaces, ordered sets and oriented graphs*, Topology and its Applications, 273 (2020) 106970
- [3] K. Goebel, T. Sekowski, and A. Stachura, *Uniform convexity of the hyperbolic metric and fixed points of holomorphic mappings in the Hilbert ball*, Nonlinear Analysis, (**4** (1980), 1011-1021.

Short-bio: Dr. Mohamed A. Khamsi graduated from the École Polytechnique, Paris, France. He completed his PhD at the Pierre-et-Marie-Curie University, Paris, France. His research interests include Nonlinear Functional Analysis, Fixed Point Theory, Logic Programming, Discrete Mathematics, and Internet Mathematics Education. Dr. Khamsi is considered one of the world experts on Metric Fixed Point Theory.

Modified inertial subgradient extragradient algorithm for solving bilevel system of equilibrium problems

T. Yuying¹, B.V. Dinh² and S. Plubtieng^{3,*}

¹*Department of Mathematics, Uttaradit Rajabhat University, Uttaradit, Thailand*

²*Faculty of Information Technology, Le Quy Don Technical University, Hanoi, Vietnam*

³*Department of Mathematics, Faculty of Science, Naresuan University, Phitsanulok, Thailand.*

** presenting author.*

email: somyotp@nu.ac.th

Abstract

In this talk, we propose the inertial subgradient extragradient algorithms for solving bilevel system equilibrium problems in real Hilbert spaces. The algorithm can be used without the prior knowledge of the Lipschitz constant of the involving bifunction and only compute the minimization of strongly bifunctions onto the feasible set is required. Under suitable conditions, we obtain strong convergence theorems of the our algorithms. Finally, some numerical examples are presented to show the efficiency of the suggested algorithms.

References

- [1] Moudafi, A.: Proximal methods for a class of bilevel monotone equilibrium problems, J. Global Optim. 47, 287-292 (2010).
- [2] Quy, N.V.: An algorithm for a bilevel problem with equilibrium and fixed point constraints. Optimization 64, 1-17 (2014).
- [3] Thuy, L.Q., Hai, T.N.: A projected subgradient algorithm for bilevel equilibrium problems and applications, J. Optim. Theory Appl., doi 10.1007/s10957-017-1176-2.
- [4] Bento, G.C., Cruz Neto, J.X., Lopes, J.O., Soares Jr, P.A., Soubeyran, A.: Generalized proximal distances for bilevel equilibrium problems. SIAM J. Optim 26, 810-830 (2016).
- [5] Chbani, Z., Riahi, H.: Weak and strong convergence of proximal penalization and proximal splitting algorithms for two-level hierarchical Ky Fan minimax inequalities, Optimization 64, 1285-1303 (2015).
- [6] Mastroeni, G.: On auxiliary principle for equilibrium problems. Publicatione del Dipartimento di Matematica dell'Universita di Pisa 3, 12441258 (2000).
- [7] Muu, L.D., Quoc, T.D.: Regularization Algorithms for Solving Monotone Ky Fan Inequalities with Application to a Nash-Cournot Equilibrium Model. J Optim Theory Appl 142, 185204 (2009).
- [8] Dinh, B.V., Muu, L.D.: Aprojection algorithm for solving pseudomonotone equilibrium problems and it's application to a class of bilevel equilibria, Optimization, (2013).
- [9] Duca, P.M., Le D.M.: A splitting algorithm for a class of bilevel equilibrium problems involving non-expansive mappings, Optimization, (2016).

- [10] Munkong, J., Dinh, B.V., Ungchittrakool, K.: An inertial extragradient method for solving bilevel equilibrium problems, *Carpathian Journal of Mathematics* 36, 91-107 (2020).
- [11] Yuying, T., Dinh, B.V., Kim, D.S., Plubtieng, S.: Extragradient subgradient methods for solving bilevel equilibrium problems, *J. Inequal. Appl.* 327, (2018).
- [12] Anh, P.N., Thanh D.D., Linh N.K., Tu H.P.: New Explicit Extragradient Methods for Solving a Class of Bilevel Equilibrium Problems, *Bull. Malays. Math. Sci. Soc.* 44, 32853305 (2021).
- [13] Duc, M.H., Thanh, H.N.T., Huyen, T.T.T., Dinh, B.V.: Ishikawa Subgradient Extragradient Method for Equilibrium Problems and Fixed Point Problems in Hilbert Spaces, *Numer. Funct. Anal. Optim.* 41, 1065-1088 (2020).
- [14] Muu, L. D., Oettli, W. (1992). Convergence of an adaptive penalty scheme for finding constrained equilibria. *Nonlinear Anal.: TMA.* 18(12):11591166. DOI: 10.1016/ 0362-546X(92)90159-C
- [15] Quoc, T.D., Muu, L.D., Nguyen, V.H.: Extragradient algorithms extended to equilibrium problems. *Optimization* 57, 749776 (2008).
- [16] Chbani, Z., Riahi, H.: Weak and strong convergence of an inertial proximal method for solving Ky Fan minimax inequalities. *Optim. Lett.* 7, 185206 (2013).
- [17] Vinh, N.T., Muu, L.D.: Inertial extragradient algorithms for solving equilibrium problems. *Acta Math. Vietnam.* 44, 639663 (2019).
- [18] Thong, D.V., Cholamjiak, P., Rassias, M.T., Cho, Y.J. : Strong convergence of inertial subgradient extragradient algorithm for solving pseudomonotone equilibrium problems, *Optim. Lett.* (2021).
- [19] Moudafi, A.: Proximal methods for a class of bilevel monotone equilibrium problems, *J. Global Optim.* 47, 287-292 (2010).
- [20] Quy, N.V.: An algorithm for a bilevel problem with equilibrium and fixed point constraints. *Optimization* 64, 1-17 (2014).

An accelerated convex optimization algorithm with line search and applications in machine learning

Dawan Chumpungam¹, Panitarn Sarnmeta² and Suthep Suantai^{1,*}

¹*Department of Mathematics, Faculty of Science, Chiang Mai University, Chiang Mai 50200, Thailand*

²*KOSEN-KMITL, Bangkok 10520, Thailand.*

**presenting author.*

email: suthep.s@cmu.ac.th

Abstract

In this paper, we introduce a new line search technique, then employ it to construct a novel accelerated forward-backward algorithm for solving convex minimization problems of the form of the summation of two convex functions in which one of these functions is smooth in a real Hilbert space. We establish a weak convergence to a solution of the proposed algorithm without the Lipschitz assumption on the gradient of the objective function. Furthermore, we analyze its performance by applying the proposed algorithm to solving classification problems on various data sets and compare with other line search algorithms. Based on the experiments, the proposed algorithm performs better than other line search algorithms.

Acknowledgements This research has received funding support from the NSRF via the Program Management Unit for Human Resources & Institutional Development, Research and Innovation (Grant Number B05F640183). This research was also supported by Chiang Mai University.

References

- [1] Beck, A.; Teboulle, M. A fast iterative shrinkage-thresholding algorithm for linear inverse problems. *SIAM J. Imaging Sci.* 2009, 2, 183–202.
- [2] Kankam, K.; Pholasa, N.; Cholanjiak, C. On convergence and complexity of the modified forward-backward method involving new line searches for convex minimization. *Math. Meth. Appl. Sci.* 2019, 1352–1362.
- [3] Hanjing, A.; Suantai, S. A fast image restoration algorithm based on a fixed point and optimization method. *Mathematics* 2020, 8, 378.

From Banach contractions to almost contractions, enriched contractions and beyond

Vasile Berinde^{*,1,2}, Mădălina Păcurar³

¹ *Department of Mathematics and Computer Science, Technical University of Baia Mare, North University Center at Baia Mare, Baia Mare, Romania*

² *Academy of Romanian Scientists, Bucharest, Romania*

³ *Department of Economics and Business Administration in German Language Faculty of Economics and Business Administration Babeş-Bolyai University of Cluj-Napoca, Cluj-Napoca, Romania.*

** presenting author.*

email: vasile.berinde@mi.utcluj.ro

Abstract

One hundred years ago, in 1922, Banach published his seminal paper [1], where, amongst many other fundamental results, it has been stated the first version of the contraction mapping principle, commonly called the Picard-Banach or Banach fixed point theorem. This moment marked the inception of the metric fixed point theory which afterwards developed in an extraordinary impressive way in many theoretical and applicative directions.

The main aim of the present paper is to survey some fixed point results for some classes of contractive type mappings originating from Banach fixed point theorem, including *almost contractions* ([2], [3], [4], [5], [7]), *enriched contractions* ([8], [9], [10], [11], [12], [13], [6]) and many other related ones.

Acknowledgements

We dedicate this paper to Professor Emeritus Ioan A. Rus, our PhD supervisor, on the occasion of his 86th anniversary, with love and gratefulness for introducing and guiding us in the wonderful world of FIXED POINT THEORY.

References

- [1] Banach, S. Sur les opérations dans les ensembles abstraits et leurs applications aux équations intégrales. *Fundam. Math.* **3** (1922), 133–181.
- [2] Berinde, V. Approximating fixed points of weak contractions using the Picard iteration. *Nonlinear Anal. Forum* **9** (2004), no. 1, 43–53.
- [3] Berinde, V. *Iterative Approximation of Fixed Points*, Springer, 2007.
- [4] Berinde, V. General constructive fixed point theorems for Ćirić-type almost contractions in metric spaces. *Carpathian J. Math.* **24** (2008), no. 2, 10–19.

- [5] Berinde, V. Approximating fixed points of implicit almost contractions. *Hacettepe J. Math. Stat.* **41** (2012), no. 1, 93–102.
- [6] Berinde, V. Maia type fixed point theorems for some classes of enriched contractive mappings in Banach spaces. *Carpathian J. Math.* **38** (2022), no. 1, 35–46.
- [7] Berinde, V.; Păcurar, M. *Iterative approximation of fixed points of single-valued almost contractions*, in Fixed Point Theory and Graph Theory, 29–97, Elsevier/Academic Press, Amsterdam, 2016.
- [8] Berinde, V.; Păcurar, M. Approximating fixed points of enriched contractions in Banach spaces. *J. Fixed Point Theory Appl.* **22** (2020), no. 2, Paper No. 38, 10 pp.
- [9] Berinde, V.; Păcurar, M. Kannan’s fixed point approximation for solving split feasibility and variational inequality problems. *J. Comput. Appl. Math.* **386** (2021), Paper No. 113217, 9 pp.
- [10] Berinde, V.; Păcurar, M. Fixed point theorems for enriched Ćirić-Reich-Rus contractions in Banach spaces and convex metric spaces. *Carpathian J. Math.* **37** (2021), no. 2, 173–184.
- [11] Berinde, V.; Păcurar, M. Approximating fixed points of enriched Chatterjea contractions by Krasnoselskij iterative algorithm in Banach spaces. *J. Fixed Point Theory Appl.* **23** (2021), no. 4, Paper No. 66, 16 pp.
- [12] Berinde, V.; Păcurar, M. Fixed points theorems for unsaturated and saturated classes of contractive mappings in Banach spaces. *Symmetry* **13** (2021), Article Number 713.
- [13] Berinde, V.; Păcurar, M. Approximating fixed points of enriched contractive type mappings in convex metric spaces. *Symmetry* **13** (2021), Article Number 498.



Projection methods for solving variational inequality problems in Hilbert spaces

Yeol Je Cho*

*Graduate School, Gyeongsang National University, Jinju, Korea.
e-mail: yjchomath@gmail.com*

Abstract

In this talk, we consider the following:

- (1) new projection-type methods for solving variational inequality problems involving a monotone and L-Lipschitzian continuous operator in Hilbert spaces. Only one projection in the method is used per each iteration.
- (2) some strong convergence theorems for the proposed method under suitable conditions on control parameters by using the regularization technique.
- (3) some numerical experiments comparing the proposed method with some other methods given by some authors.
- (4) some remarks on the convergence rate of some algorithms given by some authors.

Abstracts of Invited Talks

On the approximation of fuzzy sets by fuzzy numbers

Antonio Francisco Roldán López de Hierro^{*,1}, Miguel Ángel Tíscar²,
Concepción Roldán¹ & Humberto Bustince³

¹ *Department of Statistics and Operations Research, University of Granada, Granada, Spain,*

² *Castillo de la Yedra, Cazorla, Jaén, Spain,*

³ *Departamento de Estadística, Informática y Matemáticas, Universidad Pública de Navarra, Spain.*

**presenting author.
email: aroldan@ugr.es*

Abstract

A fuzzy set is a mapping that associates to each element of a classical set X a real number in the interval $[0, 1]$ which is interpreted as its membership degree for belonging to the fuzzy set. A fuzzy number of the real line is a fuzzy set on the family of all real numbers satisfying certain properties. Fuzzy numbers are algebraic generalizations of the notion of real number to a uncertain setting. Many techniques are successfully applied by employing fuzzy numbers, but such techniques cannot be applied for general fuzzy sets, unless they are very similar to fuzzy numbers. Then, it is interesting to consider a procedure in which fuzzy sets are approached by fuzzy numbers, especially in the cases in which the fuzzy sets are reasonably similar to fuzzy numbers. In this work we introduce a novel methodology to carry out such approximation, and we describe some of their main properties.

Acknowledgements

This work has been partially supported by Junta de Andalucía by Project FQM-365 of the Andalusian CI-CYE, and also by Projects PID2020-119478GB-I00 and PID2019-108392GB-I00 (AEI/10.13039/501100011033, Ministerio de Ciencia e Innovación).

References

- [1] H. Bustince, E. Barrenechea, M. Pagola, J. Fernández, Z. Xu, B. Bedregal, J. Montero, H. Hagra, F. Herrera, B. de Baets. *A historical account of types of fuzzy sets and their relationships*. IEEE Trans. Fuzzy Syst. 24 (2016), 179–194.
- [2] A.F. Roldán López de Hierro, C. Roldán, F. Herrera. *On a new methodology for ranking fuzzy numbers and its application to real economic data*. Fuzzy Sets Syst. 353 (2018), 86–110.
- [3] A.F. Roldán López de Hierro, M.Á. Tíscar, C. Roldán, H. Bustince. *A fuzzy methodology for approaching fuzzy sets of the real line by fuzzy numbers*. Fuzzy Sets Syst. 435 (2022), 55–77.



Endpoint theorems for some generalized multivalued nonexpansive mappings

Bancha Panyanak*

Department of Mathematics, Faculty of Science, Chiang Mai University, Chiang Mai 50200, Thailand.

**presenting author.
email: bancha.p@cmu.ac.th*

Abstract

The concept of endpoints for multi-valued mappings is an important concept which lies between the concept of fixed points for single-valued mappings and the concept of fixed points for multi-valued mappings. The existence of endpoints for some generalized nonexpansive mappings in Banach spaces is studied. Weak and strong convergence theorems of several iterations for such kinds of mappings are also discussed.



Search of minimal metric structure in the context of fixed point theorem

Dhananjay Gopal*

*Department of Mathematics Guru Ghasidas Vishwavidyalaya,
(A Central University) Bilaspur, India.*

**presenting author.
email: gopaldhananjay@yahoo.in*

Abstract

In this talk, we discuss about some recently introduced metric spaces and corresponding topological properties in the realm of fixed point. Finally, we demonstrate what could be the minimal metric structure to prove fixed point theorem for contractive type mappings.

Acknowledgements

I would like to thank Administration of GGV Bilaspur and the working group of IWANA for the support.

References

- [1] A. Deshmukh, D. Gopal, Topology of non-triangular metric spaces and related fixed point results, *Filomat* 35 (11), (2021), 3557-3570.
- [2] F. Khojasteh, H. Khandani, Scrutiny of some fixed point results by S-operators without triangular inequality, *Mathematica Slovaca* 70 (2), pp. 467-476, 2020.



Fractional (p, q) -difference equations and boundary value problems

Jessada Tariboon

*Intelligent and Nonlinear Dynamic Innovations Research Center,
Department of Mathematics, Faculty of Applied Science,
King Mongkut's University of Technology North Bangkok,
Bangkok, 10800 Thailand.*

email: jessada.t@sci.kmutnb.ac.th

Abstract

In this talk, some properties of fractional (p, q) -calculus are discussed. Fractional (p, q) -difference equations with boundary conditions are presented. Some examples are also shown the advantage of this study.

Acknowledgements

This research was supported by the Thailand Research Fund under the project RSA618005.

References

- [1] N. Kamsrisuk, C. Promsakon, S.K. Ntouyas, J. Tariboon, Nonlocal boundary value problems for (p, q) -difference equations, *Differential Equations and Applications*, Vol. 10, No. 2 (2018), 183-195.
- [2] T. Nuntigrangjana, S. Putjuso, S.K. Ntouyas, J. Tariboon, Impulsive (p, q) - difference equations, *Advances in Difference Equations*, (2020) 2020:98.



Progressive iterative approximation methods

J. Martinez-Moreno^{1,*}, S. Channark², P. Kumam²

¹ *Universidad de Jaen, Spain*

² *King Mongkut's University of Technology Thonburi KMUTT, Thailand.*

**presenting author.*

email: jmmoreno@ujaen.es

Abstract

Geometric iteration methods constitute a class of iterative methods for solving linear equations in linear algebra. Since their origin, they have been widely used in academic research and engineering practices in geometric design and related fields, especially for their excellent shape and geometric properties. These iteration techniques achieved better results in addressing traditional geometric design problems such as offset curves, degree reduction, and approximating polynomials to rational curves and surfaces and others.

Among these techniques is the progressive iterative approximation method (PIA) proposed by Lin et al. The PIA is a linear iterative interpolation algorithm, which can be viewed as a dynamic algorithm that finds the closest points to the data points to be interpolated on the iterative curve or surface. The main objective of this presentation is to describe the PIA method and its variants.

Robust stability analysis for discrete-time neural networks with mixed time-varying delays via a new summation inequality

Kanit Mukdasai*

Department of Mathematics, Faculty of Science, Khon Kaen University, Khon Kaen 40002, Thailand.

**presenting author.
email: kanit@kku.ac.th*

Abstract

This paper uses our rebuilt summation inequality to investigate the robust stability analysis issue for discrete-time neural networks that incorporate interval time-varying leakage, discrete and distributed delays. The summation inequality plays an important role in the creation of delay-dependent criteria for discrete-time systems with time-varying delay, as well as in the development of other delay-dependent criteria. It is a novelty of this study to consider a new inequality, which makes it less conservative than the well-known Jensen inequality, and use it in the context of discrete-time delay systems. Using a combination of Lyapunov-Krasovskii stability theory, coefficient matrix decomposition technique, mobilization of zero equation, mixed model transformation, and reciprocally convex combination, new stability criteria that depend on the time delay range are obtained in terms of linear matrix inequalities (LMIs) for computing the allowable maximum admissible upper bound of the delay-range. With the assistance of the LMI Control toolbox in MATLAB, numerical examples are provided to demonstrate the validity and efficiency of the theoretical findings of this research.

Acknowledgements

This research has received funding support from the NSRF via the Program Management Unit for Human Resources & Institutional Development, Research and Innovation [grant number B05F640204].

References

- [1] Shan, Y.; Zhong, S.; Cui, J.; Hou, L.; Li, Y. Improved criteria of delay-dependent stability for discrete-time neural networks with leakage delay. *Neurocomputing* **2017**, *266*, 409–419.
- [2] Wu, M.; Peng, C.; Zhang, J.; Fei, M.; Tian, Y. Further results on delay-dependent stability criteria of discrete systems with an interval time-varying delay. *J. Franklin Inst.* **2017**, *354*, 4955–4965.
- [3] Zhang, X.M.; Han, Q.L.; Ge, X.; Zhang, B.L. Delay-variation-dependent criteria on extended dissipativity for discrete-time neural networks with time-varying delay. *IEEE Trans. Neural. Netw. Learn. Syst.* **2021**, 1–10.



Physical applications of variable exponent spaces: Kirchoff's equation for the elastic string

Osvaldo Méndez

*University of Texas,
El Paso, TX,
USA.*

** Osvaldo Méndez.
email: osmendez@utep.edu*

Abstract

We present examples of the use of variable exponent Lebesgue spaces in the solution of concrete physical problems. Special attention will be given to Kirchhoff's equation for an elastic string.

Acknowledgements

Your acknowledgements goes here.

References

- [1] On the eigenvalue problem for a class of Kirchhoff-type equations, J. Math. Anal. Appl., 494, Issue 2, 15 February 2021, 124671.

One certain generalised convexity concepts and related local-global type properties

Ovidiu Bagdasar

School of Computing and Engineering, University of Derby, United Kingdom.

** presenting author.*

email: o.bagdasar@derby.ac.uk

Abstract

Many types of generalized convex functions are known in the literature, each having useful properties. For example, for real-valued functions we know that:

- the semistrict quasiconvexity ensures the “*local min - global min*” property, i.e., every local minimum point is a global minimum point;
- the explicit quasiconvexity ensures a “*local max - global min*” property, namely every local maximum point is a global minimum point if it belongs to the intrinsic core of the function’s domain.

We show that such “*local min - global min*” and “*local max - global min*” type properties can be extended and unified by a single general local-global extremality principle for certain generalized convex vector-valued functions with respect to two proper subsets of the outcome space. In particular, we recover several local-global properties concerning classical vector/multicriteria optimization, and then explore the relationships between various classes of generalized convex functions.

Acknowledgements

I would like to thank Professor Nicolae Popovici for introducing me to this subject.

References

- [1] Bagdasar, O., Popovici, N.: Local maximum points of explicitly quasiconvex functions. *Optim. Lett.* **9**, 769–777 (2015)
- [2] Bagdasar, O., Popovici, N.: Local maximizers of generalized convex vector-valued functions. *J. Non-linear Convex Anal.* **18**, 2229–2250 (2017)
- [3] Bagdasar, O., Popovici, N.: Unifying local-global type properties in vector optimization. *J. Global Optim.* **72**, 155–179 (2018)
- [4] Luc, D.T., Schaible, S.: Efficiency and generalized concavity. *J. Optim. Theory Appl.* **94**, 147–153 (1997)
- [5] Ponstein, J.: Seven kinds of convexity. *SIAM Review* **9**, 115–119 (1967)

Pinning sampled-data control for function projective synchronization of neural networks with hybrid couplings and time-varying delays

Thongchai Botmart*

Department of Mathematics, Faculty of Science, Khon Kaen University, Khon Kaen, 40002, Thailand

**presenting author.
email:thongbo@kku.ac.th*

Abstract

This paper is concerned with the dissipative problem based pinning sampled-data control scheme. We investigate the problem for function projective synchronization of neural networks with hybrid couplings and time-varying delays. The main purpose is focused on designing a pinning sampled-data function projective synchronization controller such that the resulting function projective synchronization neural networks are stable and satisfy a strictly H_∞ , $L_2 - L_\infty$, passivity and dissipativity performance by setting parameters in the general performance index. It is assumed that the parameter uncertainties are norm-bounded. By construction of an appropriate Lyapunov-Krasovskii containing single, double and triple integrals, which fully utilize information of the neuron activation function and use refined Jensen's inequality for checking the passivity of the addressed neural networks are established in linear matrix inequalities (LMIs). This result is less conservative than the existing results in literature. It can be checked numerically using the effective LMI toolbox in MATLAB. Numerical examples are provided to demonstrate the effectiveness and the merits of the proposed methods

Acknowledgements This research received funding support from the NSRF via the Program Management Unit for Human Resources and Institutional Development, Research and Innovation.

References

- [1] D. Zeng, R. Zhang, X. Liu, S. Zhong, and K. Shi, Pinning stochastic sampled-data control for exponential synchronization of directed complex dynamical networks with sampled-data communications, *Appl. Math. Comput.*, vol. 337, pp. 102–118, Nov. 2018.
- [2] J. Wang, L. Su, H. Shen, Z. G. Wu, and J. H. Park, Mixed H_∞ /passive sampled-data synchronization control of complex dynamical networks with distributed coupling delay, *J. Franklin Inst.*, vol. 354, no. 3, pp. 1302–1320, Feb. 2017



On Thai mathematician's approaches for solving linear/nonlinear problems with the fixed point method

Wutiphol Sintunavarat

Department of Mathematics and Statistics, Faculty of Science and Technology, Thammasat University Rangsit Center, 99 Moo 18, Khlong Nueng, Khlong Luang, Pathum Thani 12120, Thailand

**presenting author.*

email: wutiphol@mathstat.sci.tu.ac.th

Abstract

Nowadays, there are various methods for solving linear/nonlinear problems. However, due to the limitation of each method, they cannot be applied for solving all linear/nonlinear problems. This talk aims to present new techniques of Thai mathematicians to solve linear/nonlinear problems using various fixed point results.

Abstracts of Contributed Talks

A hybrid approach for finding approximate solutions to constrained nonlinear monotone operator equations with applications

Auwal Bala Abubakar*, Poom Kumam

King Mongkut's University of Technology Thonburi.

**presenting author.*

email: ababubakar.mth@buk.edu.ng

Abstract

In this work, a hybrid approach technique incorporated with three-term conjugate gradient (CG) method is proposed to solve constrained nonlinear monotone operator equations. The search direction is defined such that it is close to the one obtained by the memoryless Broyden-Fletcher-Goldfarb-Shanno (BFGS) method. Independent of the line search, the search direction possess the sufficient descent and trust region properties. Furthermore, the sequence of iterates generated converge globally under some appropriate assumptions. In addition, numerical experiments is carried out to test the efficiency of the proposed method in contrast with existing methods. Finally, the applicability of the proposed method in compressive sensing is shown.

Acknowledgements

The authors acknowledge the financial support provided by the Center of Excellence in Theoretical and Computational Science (TaCS-CoE), KMUTT. Also, the first author would like to thank the Postdoctoral Fellowship from King Mongkut's University of Technology Thonburi (KMUTT), Thailand. Moreover, this research project is supported by Thailand Science Research and Innovation (TSRI) Basic Research Fund: Fiscal year 2021 under project number 64A306000005.

References

- [1] A. B. Abubakar, A. H. Ibrahim, A. B. Muhammad, and C Tammer. A modified descent dai-yuan conjugate gradient method for constraint nonlinear monotone operator equations. *Applied Analysis and Optimization*, 4:1–24, 2020
- [2] A. B. Abubakar, P. Kumam, H. Mohammad, and A. M. Awwal. A Barzilai-Borwein gradient projection method for sparse signal and blurred image restoration. *Journal of the Franklin Institute*, 357(11):7266–7285, 2020.
- [3] A. H. Ibrahim, P. Kumam, A. B. Abubakar, W. Jirakitpuwapat, and J. Abubakar. A hybrid conjugate gradient algorithm for constrained monotone equations with application in compressive sensing. *Heliyon*, 6(3):e03466, 2020

Equilibrium problems and proximal algorithm in monotone vector field

Adamu Yusuf Inuwa^{1,2,*}, Poom Kumam^{1,3}, Parin Chaipunya³.

¹*Center of Excellence in Theoretical and Computational Science (TaCS-CoE) & KMUTT Fixed Point Research Laboratory, Room SCL 802, Fixed Point Laboratory, Science Laboratory Building, Departments of Mathematics, Faculty of Science, King Mongkut's University of Technology Thonburi (KMUTT), 126 Pracha-Uthit Road, Bang Mod, Thung Khru, Bangkok 10140, Thailand,*

²*Bayero University Kano Faculty of Physical Sciences Department of Mathematics Kabuga, 700241, Kano, Nigeria,*

³*Department of Medical Research, China Medical University Hospital, China Medical University, Taichung 40402, Taiwan.*

**presenting author.*

email: ibnyusufkoki@gmail.com

Abstract

In this work, equilibrium problems and their regularized problems were studied under the setting of monotone vector field. The solution to the regularized problem is represented in terms of resolvent operators. We construct the proximal algorithm based on this regularization and give convergence analysis adequately.

Acknowledgements

The authors acknowledge the financial support provided by the Center of Excellence in Theoretical and Computational Science (TaCS-CoE) and that of Petchra Pra Jom Klao Ph.D. Research Scholarship from King Mongkut's University of Technology Thonburi.

References

- [1] Chaipunya, P., Kohsaka, F., Kumam, P.: Monotone vector fields and generation of nonexpansive semigroups in complete cat(0) spaces. arXiv:1906.05984
- [2] Kumam, P., & Chaipunya, P. (2017). Equilibrium Problems and Proximal Algorithms in Hadamard Spaces. *Journal of Nonlinear Analysis and Optimization: Theory & Applications*, 8(2), 155-172.

Extended dissipative analysis for sampled-data synchronization of complex dynamical networks with coupling time-varying delays

Arthit Hongsri*, Thongchai Botmart

Department of Mathematics, Khon Kaen University, Khon Kaen 40002, Thailand.

**presenting author.
email: arthith@kkumail.com*

Abstract

This research addresses the problem of extended dissipative analysis for complex dynamical networks with coupling delays with a sampled-data control scheme. Firstly, we use the input delay method and combine it with a time-dependent Lyapunov functional, which can fully use the sampling period's information. Secondly, novel sufficient synchronization criteria are formulated by applying Jensen's inequality, Wirtinger's integral inequality, new integral inequalities, free-weighting matrix technique, and convex combination method. Moreover, we extend the criteria to the extended dissipative analysis, which encompasses $L_2 - L_\infty$, H_∞ , passivity, and dissipativity in a unified framework. These conditions can exhibit in Linear matrix inequalities (LMIs), which can solve with Matlab software. Finally, two numerical examples demonstrate our method's effectiveness and less conservatism.

Acknowledgements

The first author was supported by the Science Achievement Scholarship of Thailand (SAST)

References

- [1] Li, N., Zhang, Y., Hu, J., & Nie, Z. (2011). Synchronization for general complex dynamical networks with sampled-data. *Neurocomputing*, 74(5), 805-811.
- [2] Yang, H., Shu, L., Zhong, S., & Wang, X. (2016). Extended dissipative exponential synchronization of complex dynamical systems with coupling delay and sampled-data control. *Journal of the Franklin Institute*, 353(8), 1829-1847.
- [3] Hongsri, A., Botmart, T., Weera, W., & Junsawang, P. (2021). New delay-dependent synchronization criteria of complex dynamical networks with time-varying coupling delay based on sampled-data control via new integral inequality. *IEEE Access*, 9, 64958-64971.

Closed queueing network analysis of vehicle sharing in a city

Bharat Raj Wagle*, Ram Prasad Ghimire

School of Business, Pokhara University, Nepal, Department of Mathematics, School of Sciences, Kathmandu University, Nepal.

**presenting author.
email: bharatwagle@pusob.edu.np*

Abstract

Vehicle sharing is an immense issue in the world from the point of view of environmental protection, traffic management, and economics. Our study deals with the construction of a closed queueing network with a finite number of nodes and vehicles that provide the service to the riders. The customer's average arrival and service rate are provisioned to be heterogeneous on a first-come-first-served basis. Arrivals of the customers in the nodes are taken to be Poisson and the customer's service in exponential fashion. With the help of the transition diagram under study, finite difference equations have been set up, which have been solved explicitly to obtain the probability of individual state conditions of vehicles. Moreover, the performance of the network's product form is obtained by using the Gordon and Newell theorem of closed queueing networks. Some numerical results with the help of MATLAB software have also been computed to show the validity and the applicability of the model under study.

Acknowledgements

I would like to offer my special thanks to the Department of Mathematics, School of Sciences, Kathmandu University where I got enrollment for PhD study, similarly I would like to express my very great appreciation to Pokhara University, Nepal who provides me with study leave and I am particularly grateful to my supervisor Prof Dr. R P Ghimire for his regular advice and motivation in my study.

References

- [1] Sherif I. Ammar, Transient analysis of a two-heterogeneous servers queue with impatient behavior, Journal of the Egyptian Mathematical Society. 22 (2014), 90 – 95
- [2] Madhu Jain and Mayank Singh, Transien Analysis of a Markov Queueing Model with Feedback, Discouragement and Diseaster, Int. J. Appl. Comput. Math. 6 (2020)
- [3] Adan, I, Lecture Notes: Closed Queueing Network Models. Eindhoven University. (2015)
- [4] Akyildiz, I.F., Exact Product Form Solutions for Queueing Networks with Blocking, IEEE Trans. Computers C-36. 1 (1987), 122 – 125
- [5] Chesoong Kim, Sergei Dudin and Olga Dudina Queueing Network with Moving Servers as a Model of Car Sharing Systems, Mathematics. 7 (2019), DOI 10.3390/math7090825

H_∞ performance for uncertain neutral system with mixed time-varying delays

Boonyachart Meesuptong*, Kanit Mukdasai

Department of Mathematics, Khon Kaen University, Khon Kaen 40002, Thailand.

** presenting author.*

email: boonyachart_m@kkumail.com

Abstract

This research investigates the exponential stability analysis and H_∞ performance for a neutral system with interval time-varying discrete, neutral, and distributed delays, and nonlinear uncertainties. The uncertainties under consideration are nonlinear time-varying parameter perturbations. Based on Jensen's integral inequality, Wirtinger-base integral inequality, Leibniz-Newton formula, Peng-Park's integral inequality, decomposition matrix technique, utilization of zero equation and the appropriate Lyapunov-Krasovskii functional (LKF), new delay-range-dependent sufficient conditions for the H_∞ performance with exponential stability of the system are established in terms of linear matrix inequalities. Moreover, we present the improved delay-range-dependent exponential stability criterion of the system with discrete, neutral, and distributed time-varying delays, and nonlinear uncertainties. A Numerical example is given to illustrate the effectiveness of the proposed method.

Acknowledgements

The first author was supported by the Science Achievement Scholarship of Thailand (SAST).

References

- [1] Ali, M. S. (2012). On exponential stability of neutral delay differential system with nonlinear uncertainties. **Communications in Nonlinear Science and Numerical Simulation**, **17**(6), 2595–2601.
- [2] Meesuptong, B., Mukdasai, K., & Khonchaiyaphum, I. (2020). New exponential stability criterion for neutral system with interval time-varying mixed delays and nonlinear uncertainties. **Thai Journal of Mathematics**, **18**(1), 333–349.
- [3] Singkibud, P., & Mukdasai, K. (2018). On robust stability for uncertain neutral systems with non-differentiable interval time-varying discrete delay and nonlinear perturbations. **Asian-European Journal of Mathematics**, **11**(01), 1–30.

Finite-time mixed H_∞ /passivity criteria for generalized neural networks with mixed interval time-varying delays

Chalida Phanlert*, Thongchai Botmart

¹*Department of Mathematics, Khon Kaen University, Khon Kaen 40002, Thailand.*

**presenting author.
email: chalida_p@kkumail.com*

Abstract

In this research, we investigate the finite-time mixed H_∞ /passivity for generalized neural networks with interval discrete and distributed time-varying delays. It is noted that this is the first time for studying in the combination of H_∞ , passivity, and finite-time boundedness. Based on an appropriate Lyapunov-Krasovskii function (LKF) and estimating the bound of time derivative in LKF with the use of Jensen's integral inequality, an extended single and double Wirtinger's integral inequality, and a new triple integral inequality are derived in the form of linear matrix inequalities (LMIs). Finally, five numerical examples are given to illustrate the effectiveness of the obtained results.

Acknowledgements

The first author was supported by the Science Achievement Scholarship of Thailand (SAST).

References

- [1] K. Mathiyalagan, J. H. Park, and R. Sakthivel, "Novel results on robust finite-time passivity for discrete-time delayed neural networks," *Neurocomputing*, vol. 177, pp. 585–593, Feb. 2016.
- [2] C. Hua, Y. Wang, and S. Wu, "Stability analysis of neural networks with time-varying delay using a new augmented Lyapunov–Krasovskii functional," *Neurocomputing*, vol. 332, pp. 1–9, Mar. 2019.
- [3] Z. Feng, H. Shao, and L. Shao, "Further improved stability results for generalized neural networks with time-varying delays," *Neurocomputing*, vol. 367, pp. 308–318, Jul. 2019.
- [4] S. Saravanan, M. S. Ali, A. Alsaedi, and B. Ahmad, "Finite-time passivity for neutral-type neural networks with time-varying delays—via auxiliary function-based integral inequalities," *Nonlinear Anal.*, vol. 25, no. 2, pp. 206–224, Mar. 2020.

Finite-time stability analysis of generalized neural networks with mixed interval time-varying delays via new integral inequality

Chantapish Zamart*, Thongchai Botmart

Department of Mathematics, Khon Kaen University, Khon Kaen 40002, Thailand.

**presenting author.*

email: za.chantapish@kkumail.com

Abstract

This research presents new sufficient conditions of finite-time stability for generalized neural networks (GNNs) with mixed interval time-varying delays to achieve the improved stability criterion. We also propose a new integral inequality with an exponential function to estimate the derivative of the Lyapunov-Krasovskii functional (LKF). Moreover, the new integral inequality covers the well-known Wirtinger's inequality. Finally, we present a numerical example to show the advantages of our obtained methods. The example can apply continuous time-varying delays that do not need to be differentiable.

Acknowledgements

The first author was supported by the Science Achievement Scholarship of Thailand (SAST).

References

- [1] Seuret, A., & Gouaisbaut, F. (2013). Wirtinger-based integral inequality: Application to time-delay systems. *Automatica*, 49(9), 2860-2866.
- [2] Rajavel, S., Samidurai, R., Cao, J., Alsaedi, A., & Ahmad, B. (2017). Finite-time non-fragile passivity control for neural networks with time-varying delay. *Applied Mathematics and Computation*, 297, 145-158.
- [3] Zamart, C., & Rojsiraphisal, T. (2019). Finite-time stabilization of linear systems with time-varying delays using new integral inequalities. *Thai Journal of Mathematics*, 17(1), 173-191.
- [4] Prasertsang, P., & Botmart, T. (2021). Improvement of finite-time stability for delayed neural networks via a new Lyapunov-Krasovskii functional. *AIMS Mathematics*, 6(1), 998-1023.

Finite-time synchronization control for coronary artery chaos system with state and input time-varying delays

Charuwat Chantawat*, Thongchai Botmart

Department of Mathematics, Faculty of Science, Khon Kaen University, Khon Kaen 40002, Thailand.

**presenting author.
email: charuwat_c@kkumail.com*

Abstract

This is the first time for studying the issue of finite-time synchronization control for the coronary artery chaos system (CACS) with state and input time-varying delays. By constructing the Lyapunov-Krasovskii functional (LKF) is derived for finite-time stability criteria of CACS with interval and continuous differential time-varying delays. We use Wirtinger-based integral inequality to evaluate the upper bound of the time derivative of the LKF. We apply the single integral form and the double integral form of the integral inequality, according to Wirtinger-based integral inequality, to ensure that the feedback controller for synchronization has good performance with disturbance and time-varying delay. The new sufficient finite-time stability conditions have appeared in the form of linear matrix inequalities (LMIs). Numerical checks can be performed using the LMI toolbox in MATLAB. A numerical example is presented to demonstrate the success of the proposed methods. This resultant is less conservative than the resultants available in the previous works.

Acknowledgements

The first author was financially supported by the Science Achievement Scholarship of Thailand (SAST). The second author was financially supported by Khon Kaen University.

References

- [1] Zhang J, Li SS, Zhao Z, Sun J. Improved synchronization criteria for coronary artery input time-delay system. *IEEE Access*. 2018;6:68221-68232.
- [2] Li SS, Zhao ZS, Zhang J, Sun J, Sun LK. H_∞ control of coronary artery input time-delay system via the free-matrix-based integral inequality. *Math Probl Eng*. 2018;2018:1-12.
- [3] Li B, Zhao Z, Wang R, Ding G. Synchronization control design based on Wirtinger inequality for uncertain coronary artery time-delay system with input saturation. *IEEE Access*. 2019;7:76611-76619.
- [4] Li XM, Zhao ZS, Zhang J, Sun LK. H_∞ synchronization of the coronary artery system with input time-varying delay. *Chin Phys B*. 2016;25:1-9.

- [5] Harshavarthini S, Sakthivel R, Kong F. Finite-time synchronization of chaotic coronary artery system with input time-varying delay. *Chaos Solitons Fractals*. 2020;134:1-8.
- [6] Li W. Tracking control of chaotic coronary artery system. *Int J Syst Sci*. 2012;43:21-30.
- [7] Guo J, Zhao ZS. Adaptive observation control for synchronization of coronary artery time-delay systems. *Mod Phys Lett B*. 2019;33(36):1-21.
- [8] Wang R, Li B, Zhao ZS, Guo J, Zhu Z. Synchronization of fuzzy control design based on Bessel–Legendre inequality for coronary artery state time-delay system. *IEEE Access*. 2019;7:181933-181941.
- [9] Zhao ZS, Du Y, Zhang J, Sun LK. Observer-based H_∞ synchronization control for input and output time-delays coronary artery system. *Asian J Control*. 2019;21(3):1-11.
- [10] Guo J, Zhao ZS, Shi FD, Wang RK, Li SS. Observer-based synchronization control for coronary artery time-delay chaotic system. *IEEE Access*. 2019;7:51222-51235.
- [11] Wu WS, Zhao ZS, Zhang J, Sun LK. State feedback synchronization control of coronary artery chaos system with interval time-varying delay. *Nonlinear Dyn*. 2017;87:1773-1783.

Fixed point result by using compatible mapping of type P in menger space

Ajay Kumar Chaudhary*, K.B. Manandhar, Kanhaiya Jha

*Asst. Professor of Mathematics, Tribhuvan University and PhD Scholar, Kathmandu University,
Nepal,*

*Asst. Professor of Mathematics, School of Science, Kathmandu University, Professor of
Mathematics, School of Science, Kathmandu University, Nepal.*

** presenting author.*

email:akcsaurya81@gmail.com

Abstract

Menger Space is one of the important generalizations of metric space introduced by Austrian Mathematician Karl Menger [3] in 1942. The study of this space expanded rapidly after pioneer work of B. Schweizer and A. Sklar [6] in 1960. In 1972, V.M. Sehgal and A.T. Bharucha Reid [5] had introduced contraction mapping in probabilistic version as generalization of Banach contraction principle that was given by S. Banach [1] in 1922 in metric space. In 1991, S.N. Mishra [4] first extended the notion of compatibility in probabilistic metric space as G. Jungck [2] introduced it in Metric space. After that so many researchers have introduced different compatible mappings in Menger space for fixed point results. The objective of paper is to discuss on Compatible Mapping of type P and a common fixed point theorem under this condition in Menger space with suitable example.

Acknowledgements

First author is highly thankful to University Grant Commission, Nepal for providing UGC PhD fellowship award PhD-76/77-S and T-3.

References

- [1] Banach, S., Sur les Operations dans les ensembles abstraits et leur applications aux equations integral, Fund Math. (1922), no.3, 133-181.
- [2] Jungck, G., Compatible Mapping and common fixed points, Int. J. Math. Sci. 9(4) (1986), 771-779.
- [3] Menger, K., Statistical Matrices, Proceedings of National Academy of Sciences of USA, 28 (1942), 535-537.
- [4] Mishra S.N., Common fixed points of Compatible Mappings in probabilistic Metric Space, Math. Japon. 36 (1991) 283-289.
- [5] Sehgal, V.M. and Bharucha-Reid A.T., Fixed Point contraction mapping in Probabilistic Metric Space. Math System Theory, 6 (1972), 97-102.
- [6] Sklar A. and Schweizer B. (2005) Probabilistic Metric space. Dover Publications, INC, Mineola, New York.

Fast Krasnosel'skiĭ-Mann algorithm with a convergence rate of the fixed point iteration of $\mathcal{O}(1/k)$

Radu Ioan Boţ, Ernő Robert Csetnek, Dang-Khoa Nguyen*

*Faculty of Mathematics, University of Vienna,
Oskar-Morgenstern-Platz 1, 1090 Vienna,
Austria.*

**presenting author.*

email: dang-khoa.nguyen@univie.ac.at

Abstract

The Krasnosel'skiĭ-Mann (KM) algorithm is the most fundamental iterative scheme designed to find a fixed point of an averaged operator in the framework of a real Hilbert space, since it lies at the heart of various numerical algorithms for solving monotone inclusions and convex optimization problems. We enhance the Krasnosel'skiĭ-Mann algorithm with Nesterov's momentum updates and show that the resulting numerical method exhibits a convergence rate for the fixed point residual of $\mathcal{O}(\frac{1}{k})$ while preserving the weak convergence of the iterates to a fixed point of the operator. Numerical experiments illustrate the superiority of the resulting so-called Fast KM algorithm over various fixed point iterative schemes, and also its oscillatory behavior, which is a specific of Nesterov's momentum optimization algorithms.

Acknowledgements

The research of R.I.B. is partially supported by FWF (Austrian Science Fund), projects W 1260 and P 34922-N. The research of E.R.C. is partially supported by FWF (Austrian Science Fund), project P 29809-N32. The research of D.-K.N. is supported by FWF (Austrian Science Fund), project P 34922-N.

References

- [1] R. I. Boţ, D.-K. Nguyen, *Fast Krasnosel'skiĭ-Mann algorithm with a convergence rate of the fixed point iteration of $\mathcal{O}(1/k)$* . [arXiv:2206.09462](#)
- [2] R. I. Boţ, E. R. Csetnek, D.-K. Nguyen, *Fast OGDA in continuous and discrete time*. [arXiv:2203.10947](#)

A regularized stochastic Nesterov's accelerated quasi-newton method with applications

Dawrawee Makmuang*, Siwakon Suppalap, Rabian Wangkeeree

*Department of Mathematics, Faculty of Science, Naresuan University, Phitsanulok 65000 Thailand,
Research center for Academic Excellence in Mathematics, Naresuan University, Phitsanulok 65000
Thailand.*

**presenting author.
email: dawrawee@gmail.com*

Abstract

The stochastic Broyden-Fletcher-Goldfarb-Shanno (BFGS) method has effectively solved strongly convex optimization problems. However, this method frequently encounters the near-singularity problem of the Hessian. Additionally, obtaining the optimal solution necessitates a long convergence time. In this paper, we present a Regularized Stochastic Nesterov's Accelerated Quasi-Newton method that combines Nesterov acceleration with a novel momentum coefficient to effectively accelerate convergence speed and avoid the near-singularity problem of the Hessian update in the stochastic BFGS method. Moreover, we show the almost sure convergence of the generated subsequence of iterates to an optimal solution of the strongly convex optimization problems. Finally, experiments on various data sets are conducted to verify the efficacy of the proposed method.

Key words

Strongly convex optimization, Nesterovs accelerated gradient, Quasi-Newton method, Momentum coefficient, Support vector machine.

References

- [1] Sutskever, I., Martens, J., Dahl, G., Hinton, G.: On the importance of initialization and momentum in deep learning. Proceedings of the 30th International Conference on International Conference on Machine Learning. 28, 11391147, 2013.
- [2] Schraudolph, N.N., Yu, J., Günter, S.: A Stochastic Quasi-Newton Method for Online Convex Optimization. Proceedings of The 11th International Conference on Artificial Intelligence and Statistics (AISTATS 2007). 2, 436-443, 2017.
- [3] Mokhtari, A., Ribeiro, A.: RES: Regularized Stochastic BFGS Algorithm. IEEE Transactions on Signal Processing. 62(23), 60896104, 2014.
- [4] Indrapriyadarsini, S., Mahboubi, S., Ninomiya, H., Asai, H.: A Stochastic Quasi-Newton Method with Nesterovs accelerated gradient. Machine Learning and Knowledge Discovery in Databases. ECML PKDD 2019. Lecture Notes in Computer Science. (11906). <https://doi.org/10.1007/978-3-030-46150-8-43>

Graph interpretation and feature extraction strategy for electroencephalogram signal classification

Harish Chandra Bhandari*, Kanhaiya Jha, Bishesh Khanal

*Department of Mathematics, School of Science, Kathmandu University, Nepal,
Nepal Applied Mathematics & Informatics Institute for Research (NAAMII), Kathmandu, Nepal.*

**presenting author.*

email: harish.bhandari@ku.edu.np

Abstract

Electroencephalogram (EEG) is a medical test to record brain activities through electrodes in the scalp. Doctors and medical experts use these signals to diagnose different Neurological Disorders (NDs). According to WHO, every single person in a six people are suffering from some sorts of NDs. However, as the EEG signals have non-linear nature, the experts have only about 50ND patients. That is why recent research has shown tremendous interest in graphs and Graph Neural Networks (GNNs) to generalize deep learning methods in non-linear and non-Euclidean data like EEG signal. On the other hand, graph interpretation is one of the main issues in GNNs. Thus, we develop an appropriate graph interpretation and feature extraction strategy from EEG signals before sending it to GNNs for signal classification. We construct visibility graphs on real world electroencephalogram dataset for signals classification and the effectiveness of our proposed method is validated on this dataset.

References

- [1] Lacasa, L., Luque, B., Ballesteros, F., Luque, J., & Nuno, J. C. (2008). From time series to complex networks: The visibility graph. *Proceedings of the National Academy of Sciences*, 105(13), 4972-4975.
- [2] Wagh, N., & Varatharajah, Y. (2020, November). Eeg-gcnn: Augmenting electroencephalogrambased neurological disease diagnosis using a domain-guided graph convolutional neural network. In *Machine Learning for Health* (pp. 367-378). PMLR.
- [3] Nunez, P. L., & Srinivasan, R. (2007). Electroencephalogram. *Scholarpedia*, 2(2), 1348.
- [4] World Health Organization. (2006). Neurological disorders: public health challenges. World Health Organization.



Three-term conjugate gradient method for solving variational inequality problems over the fixed point set of nonexpansive mapping

Ibrahim Arzuka*, Parin Chiapunya, Poom Kumam, Jamilu Abubakar

Center of Excellence in Theoretical and Computational Science (TaCSCoE), Fixed Point Research Laboratory, Fixed Point Theory and Applications Research Group, Faculty of Science, King Mongkut's University of Technology Thonburi (KMUTT), Bangkok 10140, Thailand, KMUTT-Fixed Point Research Laboratory, Room SCL 802, Science Laboratory Building, Department of Mathematics, Faculty of Science, King Mongkut's University of Technology Thonburi (KMUTT), Bangkok 10140, Thailand.

** presenting author.
email: arzuka2000@gmail.com*

Abstract

In this study, we propose an iterative scheme via Three-term Conjugate Gradient Method for solving variational inequality problems which converges strongly to the solution of the given problem over the fixed point set of a nonexpansive mapping. The convergence was established and the numerical results indicate the effectiveness and robustness of the proposed method over the existing methods.

Acknowledgements

The authors acknowledge the financial support provided by the Petchra Pra Jom Klao Scholarship of King Mongkut's University of Technology Thonburi (KMUTT) and Center of Excellence in Theoretical and Computational Science (TaCS-CoE), KMUTT.

Finite-time passivity analysis of neutral-type neural networks with mixed time-varying delays

Issaraporn Khonchaiyaphum*, Kanit Mukdasai

Department of Mathematics, Faculty of Science, Khon Kaen University, Khon Kaen 40002, Thailand.

**presenting author.*

email: k.issaraporn@kkumail.com

Abstract

This work focuses on the problem of finite-time passivity analysis of neutraltype neural networks with mixed time-varying delays. The time-varying delays are distributed, discrete and neutral in that the upper bounds for the delays are available. We are investigating the creation of sufficient conditions for finite boundness, finite-time stability and finite-time passivity, which has never been performed before. First, we create a new LyapunovKrasovskii functional, PengParks integral inequality, descriptor model transformation and zero equation use, and then we use Wirtingers integral inequality technique. New finite-time stability necessary conditions are constructed in terms of linear matrix inequalities in order to guarantee finite-time stability for the system. Finally, numerical examples are presented to demonstrate the results effectiveness. Moreover, our proposed criteria are less conservative than prior studies in terms of larger timedelay bounds.

Acknowledgements

This work is supported by the Faculty of Engineering, Rajamangala University of Technology Isan Khon Kaen Campus and Research and Graduate Studies, Khon Kaen University.

References

- [1] Syed Ali, M.; Narayanana, G.; Sevgenb, S.; Shekher. V.; Arik, S. Global stability analysis of fractional-order fuzzy BAM neural networks with time delay and impulsive effects. *Communications in Non-linear Science and Numerical Simulation* 2019, 78.
- [2] Samorn, N.; Yotha, N.; Srisilp, P.; Mukdasai, K. LMI-based results on robust exponential passivity of uncertain neutral-type neural networks with mixed interval time-varying delays via the reciprocally convex combination technique. *Computation* 2021, 9.
- [3] Meesuptong, B.; Mukdasai, K.; Khonchaiyaphum, I. New exponential stability criterion for neutral system with interval time-varying mixed delays and nonlinear uncertainties. *Thai Journal of Mathematics* 2020, 18, 333–349.
- [4] Saravanan, S.; Syed Ali, M.; Alsaedib, A.; Ahmadb, B. finite-time passivity for neutraltype neural networks with time-varying delays. *Nonlinear Analysis: Modelling and Control* 2020, 25, 206–224.



- [5] Syed Ali, M.; Meenakshi, K.; Gunasekaran, N. Finite-time H_∞ boundedness of discretetime neural networks normbounded disturbances with time-varying delay. *International Journal of Control, Automation and Systems* 2017, 15, 2681-2689.
- [6] Phanlert, C.; Botmart, T.; Weera, W.; Junsawang, P. Finite-Time Mixed H_∞ / passivity for neural networks with mixed interval time-varying delays using the multiple integral Lyapunov-Krasovskii functional. *IEEE Access* 2021, 9, 89461–89475.

On the existence of fixed points of quasi-nonexpansive multimaps in a CAT(0) space

Pitchay Kingkam, Jamnian Nantadilok*

Department of Mathematics, Faculty of Science, Lampang rajabhat University, Thailand.

**presenting author.*

email: Jamnian2010@gmail.com

Abstract

In this manuscript, we establish strong and weak convergence theorems for fixed points of multivalued quasi-nonexpansive mappings in the setting of CAT(0) space. Our results extend and unify some of the related results in the literature.

Acknowledgements

The authors would like to thank the Faculty of Science, Lampang Rajabhat University, for financial support in writing this research work.

References

- [1] K. Amnuaykarn, P. Kumam, and J. Nantadilok, On the existence of best proximity points of multi-valued mappings in CAT(0) spaces, *J. Nonlinear Funct. Anal.* 2021(2021), Article ID 25, 1-14, 2021.
- [2] M. Abbas, T. Nazir, A new faster iteration process applied to constrained minimization and feasibility problems, *Mathematicki Vesnik*, 106 (2013), 1-12, 2013.
- [3] M. Bridson, A. Haefliger, *Metric Spaces of Non-Positive Curvature*, Springer, Berlin, 1999.
- [4] K.S. Brown, *Buildings*, Springer, New York (1989).
- [5] F. Bruhat, J. Tits, Groupes réductifs sur un corps local. I. Données radicielles valuées. *Inst. Hautes Études Sci. Publ. Math.* 41, 5-251, 1972. (in French)
- [6] C.E. Chidume, C.C.O Chidume, N. Djitte, M.S. Minjibir, Krasnoselskii-type algorithm for fixed points of multi-valued strictly pseudo-contractive mappings, *Fixed Point Theory and Applications* 2013, 2013:58, 2013.
- [7] S. Dhompongsa, W.A.Kirk, and B. Panyanak, Nonexpansive set-valued mappings in metric and Banach spaces, *J. Nonlinear Convex Anal.* 8, 35-45, 2007.
- [8] S. Dhompongsa, B. Panyanak, On Δ -convergence theorems in CAT(0) spaces, *Comput. Math. Appl.* 56 (2008), 2572-2579, 2008.

A novel criteria on exponentially passive analysis for Takagi-Sugeno fuzzy of neutral dynamic system with various time-varying delays

Janejira Tranthi*, Thongchai Botmart

Department of Mathematics, Khon Kaen University, Khon Kaen 40002, Thailand.

** presenting author.*

email: Janejira.t@kkumail.com

Abstract

This study is relevant to designing reliable exponentially passive analysis for T-S fuzzy of dynamic systems with various time-varying delays such as neutral, discrete, and distributed time-varying delays. By using the Lyapunov-Krasovskii function, The NewtonâLeibniz theory, the zero equations, and the matrix inequality technique. The multiple delay-dependent criterion with assure exponentially passive on the discussed T-S fuzzy system, are defined in respect of linear matrix inequalities (LMIs) that can be checked easily using LMI toolbox of MATLAB. A less conservative exponentially passive criterion given for special cases of general stability of a neutral differential system, by those approaches. The results of this study are delay-dependent, which depend on the lower bound and the upper bound with the time-varying delay. Lastly, the examples of Numerical are presented the performance of our criteria with the results obtained summarize some of the previous results as well.

Acknowledgements

This research has received funding support from the NSRF via the Program Management Unit for Human Resources & Institutional Development, Research and Innovation [grant number B05F640088].

References

- [1] Takagi T, Sugeno M. Fuzzy identification of systems and its application to modeling and control. IEEE Trans. Syst. Man Cybern. 1985; 15(1) : 116–132.
- [2] Zhang H, Yang D, Chai T. Guaranteed cost networked control for T-S fuzzy systems with time delays. IEEE Trans. Syst. Man Cybern. 2007; 37(2) : 160–172.
- [3] Li Y, Deng F, Xie F, Jiao L. Robust exponential stability of uncertain fuzzy stochastic neutral neural networks with mixed time-varying delays. Int. J. Innov. Comput. 2018; 14(2) : 615–627.
- [4] Li J, Huang X, Li Z. Exponential stabilization for fuzzy sampled-data system based on a unified framework and its application. J. Franklin Inst. 2017; 354(13) : 5302–5327.
- [5] Xu S, Song B, Lu J, Lam J. Robust stability of uncertain discrete-time singular fuzzy systems. Fuzzy Sets Syst. 2007; 158(20) : 2306–2316.

- [6] Zhu XL, Yang GH. Jensen integral inequality approach to stability analysis of continuous time systems with time-varying delay. IET Control Theory and Applic. 2008; 2(6) : 524–534.
- [7] Lien CH. Delay-dependent and delay-independent guaranteed cost control for uncertain neutral systems with time-varying delays via LMI approach. Chaos Solit. Fractals. 2007; 33(3) : 1017–1027.
- [8] Chen B, Liu X, Tong S, Lin C. Guaranteed cost control of T-S fuzzy systems with state and input delays. Fuzzy Sets Syst. 2007; 158(20) : 2251–2267.
- [9] Lien CH, Yu KW, Chen WD, Wan ZL, Chung YJ. Stability criteria for uncertain Takagi- Sugeno fuzzy systems with interval time-varying delay. IET Control Theory Appl. 2007; 1(3) : 764–769.

Some discussion on generalizations of metric spaces in fixed point perspective

Jayesh Savaliya^{1,*}, Dhananjay Gopal², Shailesh Kumar Srivastava¹

¹ *Department of Mathematics and Humanities, Sardar Vallabhbhai National Institute of Technology, Surat, Gujarat-395007, India,* ² *Department of Mathematics, Guru Ghasidas Vishwavidyalaya, Bilaspur, Chhattisgarh-495009, India.*

**presenting author.*

email: jdsavaliya555@gmail.com

Abstract

The motive of this paper is to provide an understanding on the role of generalizations of metric spaces in fixed point perspective. For this purpose the concept of pseudo non-triangular metric space is introduced which become a minimal required metric structure to establish new fixed point theorem for contractive type mappings.

Acknowledgements

Sardar Vallabhbhai National Institute of Technology, Surat, India has funded this work in the form of fellowship (Admission number- DS19MA005) to the first author.

References

- [1] S. Banach, Sur les operations dans les ensembles abstraits et leur application aux equations integrales, *Fund. Math.* 3(1922) 133-181.
- [2] J. Caristi, Fixed point theorems for mappings satisfying inwardness conditions, *Trans. Amer. Math. Soc.* 215 (1976) 241-251.
- [3] L. Ciric, A generalization of Banach's contraction principle, *Proc. Amer. Math. Soc.* 45 (1974) 267-273.
- [4] A. Deshmukh and D. Gopal, Topology of non-triangular metric spaces and related fixed point results, *Filomat* 3(11) (2021) 3557-3570.
- [5] L. Gajic and N. Ralevic, A common fixed point result in strong JS-metric space, *Bull. Cl. Sci. Math. Nat. Sci. Math.* 43 (2018) 113-121.
- [6] D. Gopal, P. Kumam and P. Agarwal, *Metric structures and fixed point theory*. CRC Press, Florida (2021). doi.org/10.1201/9781003139607.
- [7] M. Jleli and B. Samet, A generalized metric space and related fixed point theorems, *J. Fixed Point Theory Appl.* 2015(1) (2015) 1-14.



- [8] E. Karapinar, F.Khojasteh and Z.D. Mitrovic, A proposal for revisiting Banach and Caristi type theorems in b-metric Spaces, *Mathematics* 7(4) (2019) 308-311.
- [9] F. Khojasteh and H. Khandani, Scrutiny of some fixed point results by S-operators without triangular inequality, *Math. Slovaca* 70(2) (2020) 467-476. 1(3) : 764–769.

New stability criterion for linear system with mixed time-varying delays and nonlinear perturbations

Jitsin Piyawatthanachot*, Kanit Mukdasai

Department of Mathematics, Faculty of Science, Khon Kaen University, Khon Kaen 40002, Thailand.

**presenting author.
email: jitsinp@kkumail.com*

Abstract

This work investigates delay-dependent stability criterion for linear system with mixed time-varying delays and nonlinear perturbations. The less conservative stability criteria are obtained for the systems by constructing new augmented Lyapunov-Krasovskii functional, Leibniz-Newton formula, descriptor model transformation and various inequalities, which are presented in terms of linear matrix inequalities. Finally a numerical examples have been given to show the effectiveness of the proposed results.

Acknowledgements

The authors thank Development and Promotion of Science and Technology Talents Project and Department of Mathematics, Faculty of Science, Khon Kaen University for all the support.

References

- [1] Chen, W., Gao, F., & Liu, G. (2020). New results on delay-dependent stability for nonlinear systems with two additive time-varying delays. *European Journal of Control*, 58(March), 123–130
- [2] Liu, H., & Liu, F. (2020). New stability analysis results for linear system with two additive time-varying delay components. *Complexity*, 2020(April), 1–12.
- [3] Tian, J.K., & Liu, Y.M. (2020). A new stability criterion for systems with distributed time-varying delays via mixed inequalities method. *Complexity*, 2020(April), 1–6.

Accelerated Mann-type iterative algorithm for solving image restoration problems

Purit Thammasiri¹, Vasile Berinde², Narin Petrot^{1,3}, Somyot Plubtieng^{1,3},
Kasamsuk Ungchitrakool^{1,3,*}

¹*Department of Mathematics, Faculty of Science, Naresuan University, Phitsanulok 65000, Thailand,*

²*Department of Mathematics and Computer Science, North University Center at Baia Mare, Technical University of Cluj-Napoca, Victoriei 76, 430122 Baia Mare, Romania,*

³*Research Center for Academic Excellence in Nonlinear Analysis and Optimization, Faculty of Science, Naresuan University, Phitsanulok 65000, Thailand.*

** presenting author.*

email: kasamsuku@nu.ac.th

Abstract

In this paper, an accelerated Mann-type iterative algorithm generated by some inertial terms and three update vectors is introduced and studied for solving a fixed point problem of a nonexpansive mapping in Hilbert spaces. Under some usable properties of the considered mappings together with some nice properties on Hilbert spaces and the suitable conditions on scalar controls, these yield us to get a strong convergence result of an accelerated Mann-type iterative algorithm for finding a fixed point of a nonexpansive mapping which is associated to a zero point problem of some monotone mappings. Moreover, in order to demonstrate the benefits and applications of this new knowledge, we use an accelerated Mann-type iterative algorithm to solve image restoration problems. Finally, numerical results in different cases are shown in order to show the efficiency of the new algorithm, it was found that the new algorithm illustrates through numerical results that it has better performance in many ways than the previous existing results.

Acknowledgements

The fifth author is extremely grateful to Prof. Berinde for the ERASMUS+ grant awarded to MR. Purit Thammasiri for his three-month research visit at the Faculty of Sciences, Technical University of Cluj-Napoca, North University Centre of Baia Mare, Romania. Moreover, this research project is also supported by Thailand Science Research and Innovation (TSRI) Basic Research Fund: Fiscal year 2022 under project number R2565B071.

References

- [1] N. Artsawang, K. Ungchitrakool, *Inertial Mann-type algorithm for a nonexpansive mapping to solve monotone inclusion and image restoration problems*, Symmetry **12** (5: 750) (2020), 1-17.



- [2] R. I. Boţ, E. R. Csetnek, D. Meier, *Inducing strong convergence into the asymptotic behaviour of proximal splitting algorithms in Hilbert spaces*, Optim. Methods Software **34 (3)** (2018), 489 - 514.
- [3] W. R. Mann, *Mean value methods in iteration*, Proc. Amer. Math. Soci. **4** (1953), 506 - 510.
- [4] B. T. Polyak, *Some methods of speeding up the convergence of iterative methods*, Zh. Vychisl. Mat. Mat. **4 (5)** (1964), 1 - 17.

Conditions for perturbations to define the resolvent of the equilibrium problem on geodesic spaces

Yasunori Kimura, Kazuya Sasaki*

Department of Information Science, Toho University, Japan.

**presenting author.*

email: 7521001s@st.toho-u.jp

Abstract

Equilibrium problem [1] on a geodesic space X is defined as to find $z \in K$ such that $f(z, y) \geq 0$ for all $y \in K$, where $f: K \times K \rightarrow \mathbb{R}$ and K is a closed convex subset of X . Equilibrium problems have been studied as it relates to several problems such as fixed point problems, convex minimization problems, Nash equilibria, and so on.

Recently, in 2018, Kimura and Kishi [2] proposed a resolvent operator of equilibrium problems defined by a solution of an equilibrium problem on $CAT(0)$ spaces. A $CAT(0)$ space (X, d) is one of the geodesic spaces which have generally nonlinear structure, and the class of $CAT(0)$ spaces contains Hilbert spaces. Its resolvent is used a square of the metric d^2 as a perturbation function. In 2021, Kimura [3] showed that a resolvent of an equilibrium problem can be defined using $-\log \cos d$ as the perturbation function on $CAT(1)$ spaces.

In this talk, we consider sufficient conditions for perturbation functions to define the resolvent of an equilibrium problem on $CAT(\kappa)$ space.

Acknowledgements

This work was partially supported by JSPS KAKENHI Grant Number JP21K03316.

References

- [1] E. Blum, W. Oettli, *From optimization and variational inequalities to equilibrium problems*, Math. Student **63** (1994), 123–145.
- [2] Y. Kimura and Y. Kishi, *Equilibrium problems and their resolvents in Hadamard spaces*, J. Nonlinear Convex Anal. **19** (2018), 1503–1513.
- [3] Y. Kimura, *Resolvents of equilibrium problems on a complete geodesic space*, Carpathian J. Math. **37** (2021), 463–476.

A new iterative methods for a finite family of the split generalized equilibrium problem and fixed point problem

Kiattisak Rattanaseeha*

**presenting author.*
email: kiattisakrat@live.com

Abstract

In this paper, we introduce a new iterative method for finding a common element of the set of solutions of a finite family of split generalized equilibrium problems, finite variational inequality problems and the set of common fixed points of a countable family of nonexpansive mapping in Hilbert spaces. Under appropriate conditions imposed on the parameters, the strong convergence theorems is obtained.

Acknowledgements

The author would like to thank the referees for their comments and suggestions. This work was supported by Faculty of Science and Technology, Loei Rajabhat University, Thailand.

References

- [1] S. Plubtieng, R. Punpaeng, A new iterative method for equilibrium problems and fixed point problems of nonexpansive mappings and monotone mappings, *Appl. Math. Comput.* 197 (2008) 548-558.
- [2] S. Takahashi, W. Takahashi, Strong convergence theorem for a generalized equilibrium problem and a nonexpansive mappings in a Hilbert spaces, *Nonlinear Anal.* 69 (2008) 1025-1033.
- [3] P. Kumam and C. Jaiboon, Approximation of common solutions to system of mixed equilibrium problems, variational inequality problem, and strict pseudo-contractive mappings, *Fixed Point Theory and Applications*, 30 (2011) doi:10.1155/2011/347204.
- [4] S. Takahashi, W. Takahashi, Viscosity approximation methods for equilibrium problems and fixed point problems in Hilbert spaces, *J. Math. Anal. Appl.* 331 (2007) 506-515.
- [5] K.R. Kazmi, S. H. Rizvi, Iterative approximation of a common solution of a split equilibrium problem, a variational inequality problem and fixed point problem. *J. Egypt. Math. Soc.*, 21, (2013) 44-51.
- [6] F. Cianciaruso, G. Marino, L. Muglia, Y. Yao, A hybrid projection algorithm for finding solutions of mixed equilibrium problem and variational inequality problem, *Fixed Point Theory and Applications*, 2010(1), 383740(2009).
- [7] A. Moudafi, Split monotone variational inclusions. *J. Optim. Theory* 150(2), (2011) 275-283.
- [8] S. Wang, X. Gong, A.A. Abdou, Y.J. Cho, Iterative algorithm for a family of split equilibrium problems and fixed point problems in Hilbert spaces with applications. *Fixed Point Theory Appl.* 2016(1), 4(2016).



- [9] Q. Cheng, Parallel hybrid viscosity method for fixed point problems, variational inequality problems and split generalized equilibrium problems. *J. Inequal. Appl.* 169 (2019).
- [10] G. Marino, H.K. Xu, A general iterative method for nonexpansive mapping in Hilbert spaces, *J. Math. Anal. Appl.* 318 (2006) 43-52.
- [11] K. Rattanaseeha, The general iterative methods for equilibrium problems and fixed point problems of a countable family of nonexpansive mappings in Hilbert spaces, *J. Inequal. Appl.* 153 (2013).
- [12] R. Wangkeeree, K. Rattanaseeha, and R. Wangkeeree. The general iterative methods for split variational inclusion problem and fixed point problem in Hilbert spaces, *Comput Anal Appl.* 25(1) (2018) 19-31.

Heat transfer analysis of radiator using different shaped nanoparticles water-based ternary hybrid nanofluids with applications: a fractional model

Muhammad Arif^{1,2,*} and Poom Kumam^{1,2}

¹ Fixed Point Research Laboratory, Fixed Point Theory and Applications Research Group, Center of Excellence in Theoretical and Computational Science (TaCS-CoE), Faculty of Science, King Mongkut's University of Technology Thonburi (KMUTT), 126 Pracha Uthit Rd., Bang Mod, Thung Khru, Bangkok 10140, Thailand,

² Center of Excellence in Theoretical and Computational Science (TaCS-CoE), Faculty of Science, King Mongkut's University of Technology Thonburi (KMUTT), 126 Pracha Uthit Rd., Bang Mod, Thung Khru, Bangkok 10140, Thailand.

* presenting author.

email: arifmaths667@gmail.com

Abstract

The suspension of nanoparticles in the conventional base fluids getting more attention of the scholars and researchers due to its unique thermal performance in different field of engineering sciences. Nanofluid performed well and showed satisfactory results in the heat transport phenomena which attracted the scientists to suspend different combinations of nanoparticles which named as "hybrid nanofluid". From the experimental investigations it is found that the rate of heat transfer is higher for hybrid nanofluid as compared to unitary nanofluid. Based on the above motivation the present study is focused to consider water-based ternary hybrid nanofluid with three different shaped nanoparticles i.e, spherical shaped aluminum oxide(Al_2O_3), cylindrical carbon nanotubes(CNT), and platelet shaped(Graphene) for the advance cooling process of radiator. From the present analysis it is found that this advance water-based ternary hybrid nanofluid showed promising enhancement in the heat transfer rate as compared to hybrid and unitary nanofluid. The present problem is formulated in the form of momentum and energy equations in terms of partial differential equations along with physical initial and boundary conditions. Furthermore, we have considered water-based ternary hybrid nanofluid with different shaped nanoparticles in channel. For the exact solutions the Laplace and Fourier transforms are applied. The influence of all the flow parameters is highlighted using the computational software MATHCAD. Using water-based ternary hybrid nanofluid enhances the rate of heat transfer up-to 33.67% which shows a promising thermal performance in the heat transfer rate. Furthermore, we have used nanoparticles in different ratios and found some interesting results which can be applied in different engineering problems specially, in cooling process.

Acknowledgements

The authors acknowledge the financial support provided by the Center of Excellence 767 in Theoretical and Computational Science (TaCS-CoE), KMUTT. Saqib Murtaza is supported by the Petchra Pra Jom



Klao Ph.D. Research Scholarship from King Mongkut's University of Technology Thonburi (Grant no. 37/2561).

References

- [1] Elnaqeeb, T., Animasaun, I. L., & Shah, N. A. (2021). Ternary-hybrid nanofluids: significance of suction and dual-stretching on three-dimensional flow of water conveying nanoparticles with various shapes and densities. *Zeitschrift für Naturforschung A*, 76(3), 231-243.
- [2] Sahoo, R. R., & Kumar, V. (2020). Development of a new correlation to determine the viscosity of ternary hybrid nanofluid. *International Communications in Heat and Mass Transfer*, 111, 104451.

Heat transfer analysis of the mixed convective flow of magnetohydrodynamic hybrid nanofluid past a stretching sheet with velocity and thermal slip conditions

Muhammad Ramzan*, Poom Kumam

Department of Mathematics, KMUTT Fixed Point Research Laboratory, Room SCL 802 Fixed Point Laboratory, Science Laboratory Building, Faculty of Science, King Mongkut's University of Technology Thonburi (KMUTT), Bangkok, Thailand.

** presenting author.*

email: ramzanmaths785@gmail.com

Abstract

The present study is related to the analytical investigation of the magnetohydrodynamic flow of water hybrid nanofluid with slip conditions via an extending surface. The thermal radiation and Joule heating effects are incorporated within the existing hybrid nanofluid model. The system of higher-order partial differential equations is converted to the nonlinear system of ordinary differential equations by interpreting the similarity transformations. With the implementation of a strong analytical method called HAM, the solution of resulting higher-order ordinary differential equations is obtained. The results of the skin friction coefficient, Nusselt number, velocity profile, and temperature profile of the hybrid nanofluid for varying different flow parameters are attained in the form of graphs and tables. Some important outcomes showed that the Nusselt number and skin friction are increased with the enhancement in Eckert number, stretching parameter, heat generation parameter, and radiation parameter for both slip and no-slip conditions. The thermal profile of the hybrid nanofluid is higher for the suction effect but lower for the Eckert number, stretching parameter, magnetic field, heat generation, and radiation parameter. For both slip and no-slip conditions, the hybrid nanofluid velocity shows an upward trend for both the stretching and mixed convection parameters.

Acknowledgements

The authors acknowledge the financial support provided by the Center of Excellence 767 in Theoretical and Computational Science (TaCS-CoE), KMUTT. Muhammad Ramzan is supported by the Petchra Pra Jom Klao Ph.D. Research Scholarship from King Mongkut's University of Technology Thonburi (Grant no. 37/2561).

References

- [1] Shoaib, M., Raja, M. A. Z., Sabir, M. T., Islam, S., Shah, Z., Kumam, P., & Alrabaiah, H. (2020). Numerical investigation for rotating flow of MHD hybrid nanofluid with thermal radiation over a stretching sheet. *Scientific Reports*, 10(1), 1-15.



- [2] Al-Hanaya, A. M., Sajid, F., Abbas, N., & Nadeem, S. (2020). Effect of SWCNT and MWCNT on the flow of micropolar hybrid nanofluid over a curved stretching surface with induced magnetic field. Scientific Reports, 10(1), 1-18.

Levitin-Polyak well-posedness for generalized (η, g, φ) —mixed vector variational-type inequality problem

Patcharapa Srichok*, Panu Yimmuang

School of Mathematics, Institute of Science, Suranaree University of Technology, Nakhon Ratchasima 30000, Thailand.

**presenting author.
email: pp.phoar@gmail.com*

Abstract

We introduce and analyze the notion of Levitin-Polyak (LP) well-posedness for generalized (η, g, φ) —mixed vector variational-type inequality problem. We establish some sufficient conditions for verifying these LP well-posedness and generalized LP well-posedness properties. Moreover, we give example to explain the determined assumptions.

Acknowledgements

I would like to express special thanks to the Development and Promotion of Science and Technology Talents Project (DPST) for the financial support to this work to complete.

References

- [1] Chang, S. S., Wang, L., Wang, X. R., & Zhao, L. C. (2019). Well-posedness for generalized (η, g, φ) —mixed vector variational-type inequality and optimization problems. *Journal of Inequalities and Applications*, 2019(1), 1-16.
- [2] Levitin, E.S. and Polyak, B.T. (1966). Convergence of minimizing sequences in conditional extremum problems. *Soviet Math. Dokl.*, 7, 764-767.
- [3] Wangkeeree, R., & Yimmuang, P. (2015). Levitin-Polyak Well-posedness for parametric generalized quasivariational inequality problem of the minty type. *Journal of Nonlinear and Convex Analysis*, 16(12), 2401-2417
- [4] Huang, X. X., Yang, X. Q., & Zhu, D. L. (2009). Levitin-Polyak well-posedness of variational inequality problems with functional constraints. *Journal of Global Optimization*, 44(2), 159.

New exponential passivity analysis of neutral-type neural networks with distributed time-varying delays

Peerapongpat Singkibud^{1,*}, Narongsak Yotha¹ and Kanit Mukdasai²

¹ *Department of Applied Mathematics and Statistics, Faculty of Science and Liberal Arts, Rajamangala University of Technology Isan, Nai-Muang Sub-district, Muang District, Nakhon Ratchasima 30000, Thailand,*

² *Department of Mathematics, Faculty of Science, Khon Kaen University, Khon Kaen 40002, Thailand.*

** presenting author.*

email: peerapongpat.si@rmuti.ac.th

Abstract

The problem of exponential passivity analysis for neutral-type neural networks with distributed time-varying delays is studied. The discrete, neutral and distributed time-varying delays that the upper bounds for the delays are available. By constructing novel augmented Lyapunov-Krasovskii functional and various inequalities, new delay-dependent criteria are established to ensure the exponential passivity of the neutral-type neural networks with mixed time-varying delays. The exponential passivity criteria are presented in terms of linear matrix inequalities (LMIs). Finally, numerical examples are shown to demonstrate the benefits and effectiveness of the derived theoretical results. The method given in this paper is less conservative and more general than the others.

Acknowledgements

The authors thank anonymous reviewers for their valuable comments and suggestions. Moreover, this research was supported by Rajamangala University of Technology Isan.

References

- [1] C. Peng and M.R. Fei, "An improved result on the stability of uncertain T-S fuzzy systems with interval time-varying delay," *Fuzzy Sets and Systems*, Vol. 212, pp. 97-109, 2013.
- [2] O.M. Kwon, M.J. Park, J.H. Park, S.M. Lee and E.J. Cha, "Analysis on robust H_∞ performance and stability for linear systems with interval time-varying state delays via some new augmented Lyapunov-Krasovskii functional," *Applied Mathematics and Computation*, vol. 224, pp. 108-122, 2013.
- [3] P.G. Park, J. W. Ko and C.K. Jeong, "Reciprocally convex approach to stability of systems with time-varying delays," *Automatica*, Vol.47, no. 1, pp. 235-238, 2011.
- [4] A. Seuret and F. Gouaisbaut, "Wirtinger-based integral inequality: application to timedelay system," *Automatica*, vol. 49, no. 9, pp. 2860-2866, 2013.

- [5] P. Singkibud, P. Niamsup and K. Mukdasai, "Improved results on delay-range-dependent robust stability criteria of uncertain neutral systems with mixed interval time-varying delays," *IAENG International Journal of Applied Mathematics*, vol. 47, no. 2, pp. 209-222, 2017.
- [6] S. Zhu, Y. Shen, G. Chen, Exponential passivity of neural networks with time-varying delay and uncertainty, *Physics Letters A* 375 (2010) 136142.

New guaranteed cost control for exponential stability of nonlinear system with mixed time-varying delays via feedback control

Phichsinee Khongja^{*}, Thongchai Botmart

Department of Mathematics, Faculty of Science, Khon Kaen University, Khon Kaen 40002, Thailand.

^{}presenting author.
email: Nuchira.k@kkumail.com*

Abstract

This paper presents a new guaranteed cost control for exponential stability of nonlinear system with mixed time-delays in state and feedback control. The mixed time-delays are considered both the discrete and the distributed time-varying delays, but not necessary to be differentiable. The proposed conditions allow us to design the state feedback controllers which stabilize the closed-loop system. By constructing an appropriate Lyapunov-Krasovskii functional, new delay-dependent sufficient conditions for the existence of guaranteed cost control are given in terms of linear matrix inequalities (LMIs). Moreover, we design new quadratic cost functions and minimize upper bound of them. Finally, numerical examples are given to illustrate the effectiveness and improve over some existing results in the literature.

Acknowledgements

The first author was financially supported by the Research Fund for Supporting Lecturer to Admit High Potential Student to Study and Research on His Expert Program Year 2019. The second author was financially supported by Khon Kaen University.

References

- [1] Botmart T, Weera W. Guaranteed cost control for exponential synchronization of cellular neural networks with mixed time - varying delays via hybrid feedback control. *Abstract and Applied Analysis* 2013. 2013;Article ID 175796:1-12.
- [2] Dong Y, Liu J. Exponential stabilization of uncertain nonlinear time-delay systems. *Advances in Differential Equations*. 2012;180:1-15.
- [3] El-Morshedy H.A. Global attractivity in a population model with nonlinear death rate and distributed delays. *Journal of Mathematical Analysis and Applications*. 2014;410:642-658.
- [4] He H, Yan L, Tu J. Guaranteed cost stabilization of time-varying delay cellular neural networks via riccati inequality approach. *Neural Processing Letters*. 2012;35:151-158.

- [5] He P, Wang PL, Li Y. Guaranteed cost synchronization of complex networks with uncertainties and time-varying delays. *complexity*. 2015;21:381-395.
- [6] Lee TH, Ji DH, Park JH, Jung HY. Decentralized guaranteed cost dynamic control for synchronization of a complex dynamical network with randomly switching topology. *Applied Mathematics and Computation*. 2012;219:996-1010.
- [7] Lee TH, Park JH, Ji DH, Kwon OM, Lee SM. Guaranteed cost synchronization of a complex dynamical network via dynamic feedback control. *Applied Mathematics and Computation*. 2012;218:6469-6481.
- [8] Merola A, Cosentino C, Colacino D, Amato F. Optimal control of uncertain nonlinear quadratic systems. *Automatica*. 2017;83:345-350.
- [9] Niamsup P, Botmart T, Weera W. Modified function projective synchronization of complex dynamical networks with mixed time-varying and asymmetric coupling delays via new hybrid pinning adaptive control. *Advances in Differential Equations*. 2017;124:1-31.
- [10] Prasertsang P, Botmart T. Novel delay - dependent exponential stabilization criteria of a nonlinear system with mixed time-varying delays via hybrid intermittent feedback control. *Advances in Differential Equations*. 2017;199:1-21.
- [11] Zhang B, Deng F, Peng S, Xie S. Stabilization and destabilization of nonlinear systems via intermittent stochastic noise with application to memristor-based system. *Journal of the Franklin Institute*. 2018;355:3829-3852.
- [12] Zhao Y, Zhang W, Guo W, Yu S, Song F. Exponential state observers for nonlinear systems with incremental quadratic constraints and output nonlinearities. *Journal of Control, Automation and Electrical Systems*. 2018;29:127-135.



Convergence theorems for monotone nonexpansive mappings in ordered uniformly convex Banach spaces

Sachiko Atsushiba*

*Department of Mathematics, Tokyo Woman's Christian University,
2-6-1 Zempukuji, Suginami-ku, Tokyo 167-8585, Japan.*

** presenting author.
email: s-atsushiba@lab.twcu.ac.jp*

Abstract

In this talk, we prove nonlinear ergodic theorems for a family of monotone nonexpansive mappings in ordered uniformly convex Banach spaces. We also weak and strong convergence theorems for the mappings.

Acknowledgements

The author is supported by Grant-in-Aid for Scientific Research No. 19K03582 from Japan Society for the Promotion of Science.

A hybrid Hestenes-Stiefel and Dai-Yuan minimization with application in three degrees of freedom real-time motion control robotic model

Nasiru Salihu*, Poom Kumam, Aliyu Muhammed Awwal, Mathew Remilekun Odekunle

Center of Excellence in Theoretical and Computational Science (TaCS-CoE), Fixed Point Research Laboratory, Fixed Point Theory and Applications Research Group, Faculty of Science, King Mongkut's University of Technology Thonburi (KMUTT), Bangkok 10140, Thailand, KMUTT-Fixed Point Research Laboratory, Room SCL 802, Science Laboratory Building, Department of Mathematics, Faculty of Science, King Mongkut's University of Technology Thonburi (KMUTT), Bangkok 10140, Thailand, Department of Mathematics, Faculty of Science, Gombe State University, Department of Mathematics, Faculty of Sciences, Modibbo Adama University, Yola.

** presenting author.
email: nsalihu@mautech.edu.ng*

Abstract

This article presents a hybrid Conjugate Gradient (CG) minimization algorithm from optimal choice of the modulating non-negative parameter of Dai-Liao conjugacy condition. The new hybrid parameter is selected in such away that a convex combination of Hestenes-Stiefel and Dai-Yuan algorithms is fulfilled. The numerical computation adopts inexact line search which shows that the algorithm is robust and efficient compared with some known algorithms in literature equally the method is applicable to solve real life three degree of freedom motion control robotic model. The theoretical analysis shows the that the hybrid method proposed converges globally.

Acknowledgements

The authors acknowledge the financial support provided by the Petchra Pra Jom Klao Scholarship of King Mongkut's University of Technology Thonburi (KMUTT) and Center of Excellence in Theoretical and Computational Science (TaCS-CoE), KMUTT.

Properties of enriched nonexpansive mappings in Hadamard spaces

Sani Salisu^{1,2,*}, Poom Kumam^{1,3}, Songpon Sriwongsa¹

¹*Center of Excellence in Theoretical and Computational Science (TaCS-CoE) & KMUTT Fixed Point Research Laboratory, Room SCL 802, Fixed Point Laboratory, Science Laboratory Building, Departments of Mathematics, Faculty of Science, King Mongkut's University of Technology Thonburi (KMUTT), 126 Pracha-Uthit Road, Bang Mod, Thung Khru, Bangkok 10140, Thailand*

²*Department of Mathematics, Sule Lamido University Kafin Hausa, P.M.B 048, Jigawa, Nigeria.*

³*Department of Medical Research, China Medical University Hospital, China Medical University, Taichung 40402, Taiwan*

** presenting author.*

email: sani.salisu@mail.kmutt.ac.th

Abstract

In this work, we study some important properties of enriched nonexpansive mappings in the framework of Hadamard spaces and propose a viscosity-type algorithm for approximating fixed points of such mappings. Under appropriate assumptions, we obtain strong convergence of the sequences generated therefrom. As an application of our results, we establish the existence of solution of the variational inequality problem involving the mapping. Finally, we deduce an iterative scheme that converges strongly to a solution of the variational inequality problem.

Acknowledgements

The authors acknowledge the financial support provided by the Center of Excellence in Theoretical and Computational Science (TaCS-CoE) and that of Petchra Pra Jom Klao Ph.D. Research Scholarship from King Mongkut's University of Technology Thonburi.

Numerical analysis of fractal fractional non-linear electro osmotic flow with cadmium telluride nanoparticles

Saqib Murtaza*, Poom Kumam

Department of Mathematics, Faculty of Science, King Mongkut's University of Technology Thonburi (KMUTT), 126 Pracha Uthit Rd., Bang Mod, Thung Khru, Bangkok 10140, Thailand.

**presenting author.*

email: saqibali6997@gmail.com

Abstract

Numerical simulations of non-linear Casson nanofluid flow were carried out in a microchannel using the fractal-fractional flow model. The nano-liquid is prepared by dispersing Cadmium Telluride nanoparticles in regular engine oil. Using relative constitutive equations, the system of mathematical governing equations has been formulated along with initial and boundary conditions. Dimensionless variables have been used to obtain non-dimensional form of the governing equations. Using Caputo-Fabrizio fractal-fractional operator, the dimensionless ordinary mathematical model was transformed into a fractal-fractional model. As the exact solution of non-linear fractal-fractional model is very tough to find therefore, the generalized formulated model has been solved numerically via Crank-Nicolson scheme. Various plots are generated in relation to the inserted parameters. From the analysis it has been observed that greater magnitude of electro-kinetic parameter slows down the fluid's velocity. It is also worth noting that the fractional and classical model can also be deduced from the fractal-fractional model by taking the parameters $\alpha \rightarrow 0$, and $\alpha, \beta \rightarrow 0$ respectively.

Acknowledgements

The authors acknowledge the financial support provided by the Center of Excellence 767 in Theoretical and Computational Science (TaCS-CoE), KMUTT. Saqib Murtaza is supported by the Petchra Pra Jom Klao Ph.D. Research Scholarship from King Mongkut's University of Technology Thonburi (Grant no. 37/2561).

References

- [1] Murtaza, S., Kumam, P., Ahmad, Z., Seangwattana, T., & Ali, I. E. (2022). Numerical Analysis of Newly Developed Fractal-Fractional Model of Casson Fluid with Exponential Memory. *Fractals*.
- [2] Murtaza, S., Kumam, P., Ahmad, Z., Sitthithakerngkiet, K., & Ali, I. E. (2022). Finite Difference Simulation of Fractal-Fractional Model of Electro-Osmotic Flow of Casson Fluid in a Micro Channel. *IEEE Access*, 10, 26681-26692.



A Relation between Hilbert's 13th problem and data compression problem

Shigeo Akashi*

Tokyo University of Science. Cisco Networking Academy

** presenting author.*

email: akashi@is.noda.tus.ac.jp

Abstract

In 1957, Kolmogorov and Arnold solved negatively Hilbert's 13th problem asking if all continuous functions of several real variables can be represented as appropriate superpositions constructed from several continuous functions of fewer real variables.

In this talk, we discuss a relation of their solution to Hilbert's 13th problem and the data scientific problem asking if all higher dimensional numerical tables can be represented as appropriate superpositions constructed from several lower dimensional numerical ones.

Acknowledgements

The presenter would like to express his hearty gratitude to Professor Dhompongsa, Professor Suantai, Professor Plubtieng for their great and long-term encouragement to Japanese research activity.

References

- [1] S.Akashi, A version of Hilbert's 13th problem for analytic functions, The Bulletin of the London Mathematical Society, 35(2003), 8-14.
- [2] A.N.Kolmogorov, On the representation of continuous functions of several variables by superpositions of continuous functions of one variable and addition, Dokl., 114(1957), 679-681.
- [3] G.G.Lorentz, Approximation of Functions, Holt, Rinehart and Winston Inc., New York, 1966.

Improved extended dissipativity results for T-S fuzzy neural networks with interval time-varying delay

Sunisa Luemsai*, Thongchai Botmart

Department of Mathematics, Faculty of Science, Khon Kaen University, Khon Kaen 40002.

**presenting author.*

email: L_sunisa@kkumail.com

Abstract

The asymptotic stability and extended dissipativity performance of T-S fuzzy neural networks (NNs) with interval time-varying delay are investigated in this work. To obtain the improved results, we construct the Lyapunov-Krasovskii functional (LKF), which consists of single, double, triple, and quadruple integral terms containing full information of the delays and a state variable. Moreover, an improved Wirtinger inequality, a new triple integral inequality, and zero equation, along with a convex combination approach, are used to deal with the derivative of the LKF. By using Matlab's LMI toolbox and the above methods, the new asymptotic stability and extended dissipativity conditions are gained in the form of linear matrix inequalities (LMIs), which include passivity, $L_2 - L_\infty$, H_∞ , and dissipativity performance. Finally, numerical examples that are less conservative than previous results are presented. Furthermore, we give numerical examples to demonstrate the correctness and efficacy of the proposed method for asymptotic stability and extended dissipativity performance of the T-S fuzzy NNs.

Acknowledgements

The first author was supported by the Science Achievement Scholarship of Thailand (SAST).

References

- [1] Jiang, F.J., Yu, G.Z., Song, W.B., Zhu, K.Y., and Cai, K. (2017). Some improved methods to analysis stability of recurrent neural networks with interval time-varying delays. **International Journal of Computer Mathematics**, 94(6), 1228-1251.
- [2] Kwon, O.M., Park, M.J., Park, J.H., Lee, S.M., and Cha, E.J. (2014). New and improved results on stability of static neural networks with interval time-varying delays. **Applied Mathematics and Computation**, 239, 346-357.
- [3] Manivannan, R., Mahendrakumar, G., Samidurai, R., Cao, J., and Alsaedi, A. (2017). Exponential stability and extended dissipativity criteria for generalized neural networks with interval time-varying delay signals. **Journal of the Franklin Institute**, 354(11), 4353-4376.
- [4] Senthilraj, S., Raja, R., Zhu, Q., Samidurai, R., and Yao, Z. (2016). New delay-interval-dependent stability criteria for static neural networks with time-varying delays. **Neurocomputing**, 186, 1-7.

- [5] Sun, J., and Chen, J. (2013). Stability analysis of static recurrent neural networks with interval time-varying delay. **Applied Mathematics and Computation**, **221**, 111-120.
- [6] Wu, Z.G., Shi, P., Su, H., and Chu, J. (2011). Delay-dependent stability analysis for switched neural networks with time-varying delay. **IEEE Transactions on Systems, Man, and Cybernetics, Part B (Cybernetics)**, **41**(6), 1522-1530.
- [7] Zeng, H.B., Park, J.H., Zhang, C.F., and Wang, W. (2015). Stability and dissipativity analysis of static neural networks with interval time-varying delay. **Journal of the Franklin Institute**, **352**(3), 1284-1295.
- [8] Zuo, Z., Yang, C., and Wang, Y. (2010). A new method for stability analysis of recurrent neural networks with interval time-varying delay. **IEEE Transactions on Neural Networks**, **21**(2), 339-344.



Quasi-nonexpansive selections and convergence theorems for multivalued nonexpansive mappings in Banach spaces

Sunisa Somsit*, Attapol Kaewkhao

*Graduate Master's Degree Program in Mathematics, Department of Mathematics, Faculty of Science, Chiang Mai University, Chiang Mai 50200, Thailand,
Data Science Research Center, Department of Mathematics, Faculty of Science, Chiang Mai University, Chiang Mai 50200, Thailand.*

**presenting author.*

email: sunisa_som@cmu.ac.th

Abstract

In this talk, we investigate selections of multivalued mappings and prove that a multivalued nonexpansive mapping with endpoint property in a Banach space has a quasi-nonexpansive selection. We apply our results to prove weak convergence theorems of two multivalued nonexpansive mappings in uniformly convex Banach spaces.

Acknowledgements

I would like to express my sincere thanks to my thesis advisor, Assoc. Prof. Dr. Attapol Kaewkhao and my thesis co-advisor, Assoc. Prof. Dr. Bantha Panyanak for their guidance, kindness, invaluable help and constant encouragement throughout the course of this research. I am most grateful for their teaching and advice, not only the research methodologies but also many other methodologies in life. I would not have achieved this far and this thesis would not have been completed without all the support that I have always received from them.

I would like to express my sincere thank to Asst. Prof. Dr. Kamonrat Nammanee and Assoc. Prof. Dr. Warunun Inthakon for their comments and suggestions.

I would like to thank my friends in major Mathematics, Chiang Mai University for help, advice and encouragement.

Finally, I most gratefully acknowledge my parents for all their giving encourage and support every thing throughout the period of this research.

References

- [1] Agarwal, R. P., O'Regan, D., and Sahu, D. R. *Fixed point theory for Lipschitzian-type mappings with applications*, Vol. 6, New York: Springer, 2009, pp. x+368.
- [2] Aksoy, A. G. and Khamsi, M. A., "A Selection Theorems in Metric Spaces," American Mathematical Society, Vol. 134, No. 10, 2006, pp. 2957–2966.

- [3] Ali, J. and Ali, F. "Approximation of common fixed points and the solution of image recovery problem," *Results in Mathematics*, Vol. 74, NO. 4, 2019, pp. 1–22.
- [4] Amini- Harandi, A. "Endpoints of set-valued constructions in metric spaces," *Nonlinear Analysis*, Vol. 73, 2010, pp. 132–134.
- [5] Bartle, Robert G., and Donald R. Sherbert. *Introduction to real analysis*, Vol. 2. New York: Wiley, 2000.
- [6] Clarkson, J.A., "Uniformly convex spaces," *Trans. Amer. Math. Soc.*, Vol. 40, No. 3, 1936, 369–414.
- [7] Dhompongsa, S., Kaewkhao, A. and Panyanak, B., "Browder's convergence theorem for multivalued mapping without endpoint condition," *Topology Appl*, Vol. 159, 2012, pp. 2757–2763.
- [8] Espinola, R. Hosseini, M. and Nourouzi, K. "On stationary points of nonexpansive set-valued mappings," *Fixed Point Theory and Applications*, Vol. 236, 2015.
- [9] Hutchinson, John E., and Richard J. Loy. "Introduction To Mathematical Analysis," School of Mathematical Sciences, 1995.
- [10] Ishikawa, S., "Fixed points by a new iteration method," *Proceedings of the American Mathematical Society*, Vol. 44, No. 1, 1974, pp. 147–150.
- [11] Khamsi, M., W. Kirk, and Carlos Martinez Yañez. "Fixed point and selection theorems in hyperconvex spaces," *Proceedings of the American Mathematical Society*, Vol. 128, No. 11, 2000, pp. 3275–3283.
- [12] Mann, W.R., "Mean value method in iteration," *Proceedings of the American Mathematical Society*, Vol. 4. No. 3, 1953, pp. 506–510.
- [13] Neammanee, K., and Kaewkhao, A., "On multi-valued weak contraction mappings," *Journal of Mathematics Research*, Vol. 3, No. 2, 2011, pp. 151-156.
- [14] Niyamosot, N., "Common Fixed Point Theorems and Attractive Point Theorems with the Split Equilibrium Problem in Banach Spaces," Graduate School, Chiang Mai University, Chiang Mai. 2020.
- [15] Panyanak, B. "Endpoint of multivalued nonexpansive mappings in geodesic spaces," *Fixed Point Theory and Applications*, Vol. 147, 2015.
- [16] Picard, E., "Memoire sur la theorie des equations aux derivees partielles et la methode des approximations successives," *Journal de Mathématiques pures et appliquées*, Vol. 6, 1890, pp. 145–210.
- [17] Puiwong, J., and Satit S., "On convergence theorems for single-valued and multi-valued mappings in p -uniformly convex metric spaces," *Carpathian Journal of Mathematics*, Vol. 37, No. 3, 2021, pp. 513-527.
- [18] Takahashi, W. and Tamura, T., "Convergence theorems for a pair of non-expansive mappings," *J. Convex Anal.* Vol. 5, 1998, pp. 45–58.
- [19] Takahashi, W., "Introduction to nonlinear and convex analysis," Yokohama: Yokohama Publishers, 2009, pp. iv+-234.
- [20] Thongpaen, P. A. Kaewkhao, N. Phudolsitthiphat, S. Suantai and W. Intrakon, "Weak and Strong Convergence Theorems for Common Attractive Points of Widely More Generalized Hybrid Mappings in Hilbert Spaces," *Mathematics*, Vol. 9, No. 19, 2021.
- [21] Yost, D., "There can be no Lipschitz version of Michael's selection theorem," *Proceedings of the analysis conference, Singapore, North-Holland, Amsterdam*, Vol. 150, 1988, pp. 295–299.
- [22] Wasan, S.K. and Prakash, R., *Real Analysis*. Tata McGraw-Hill, 1985.

Sadovski-Darbo fixed point theorem for ψ - Riemann-Liouville fractional differential equation with Riemann-Stieltjes integral conditions

Piyachat Borisut, Thanatporn Bantaojai*

*Faculty of Liberal Arts, Rajamangala University of Technology Rattanakosin (RMUTR),
Samphanthawong, Bangkok 10100, Thailand,
Department of Mathematics (English Program), Faculty of Education, Valaya Alongkorn Rajabhat
University under the Royal Potronage, Pathumthani 13180, Thailand.*

**presenting author.
email: thanatporn.ban@vru.ac.th*

Abstract

In this paper, we study and consider the existence, uniqueness and stability solution of ψ - Riemann-Liouville Fractional Differential Equation with Riemann-Stieltjes integral Conditions. Via fixed point theorem of Sadovski-Darbo and using Boyd-Wong find uniqueness solution, apart from this we find condition of generalized Ulam-Hyers stability. An example is included to illustrate the applicability of our results.

Acknowledgements

The first author would like to thank Rajamangala University of Technology Rattanakosin (RMUTR) and the second author would like to thank Valaya Alongkorn Rajabhat University under the Royal Potronage (VRU) for giving us the opportunity to do research. Also, the authors are grateful to the referees for many useful comments and suggestions which have improved the presentation of this paper.

References

- [1] Wongcharoen, A., Ntouyas, S.K., and Tariboon, J. (1905). Boundary Value Problems for Hilfer Fractional Differential Inclusions with Nonlocal Integral Boundary Conditions, *Mathematics* 2020, 8(11), 1905; <https://doi.org/10.3390/math8111905>.
- [2] Qiao, Y., Zhou, Z. (2017). Existence of positive solutions of singular fractional differential equations with infinite-point boundary conditions, *Advances in Difference Equations*, Vol. 8.
- [3] Borisut, P., Kumam, P., Ahmed I., and Jirakitpuwapat W. (2019). Existence and uniqueness for ψ -Hilfer fractional differential equation with nonlocal multi-point condition, *Mathematical Methods in the Applied Sciences*, Vol. 2020, 1-15.

Modified inertial algorithms for inclusion problems

Thanittha Kowan^{*,1}, Chirasak Mongkolkeha¹, Thanyarat Jitpeera²

¹*Department of Mathematics Statistics and Computer, Faculty of Liberal Arts and Science,
Kasetsart University, Kampaeng Saen Campus,*

²*Program in Mathematics, Department of Science, Faculty of Science and Agriculture,
Rajamangala University of Technology Lanna.*

** presenting author.
email: Thanittha.ko@ku.th*

Abstract

We introduce some new inertial algorithms for finding the common solution of an inclusion problem and a fixed point problem of a nonexpansive mapping in real Hilbert space. We prove some weak and strong convergence theorems of the modified inertial forward-backward algorithms and give some numerical examples and comparison of other related works to show advantages and applicability of our algorithms.

Acknowledgements

The first author was supported by Kasetsart University Research and Development Institute, KURDI and Department of Mathematics Statistics and Computer, Faculty of Liberal Arts and Science, Kasetsart University, Kampaeng Saen Campus.

References

- [1] Agarwal, R. P., Meehan, M., and O'regan, D. (2001). Fixed point theory and applications (Vol. 141). Cambridge university press.
- [2] Alvarez, F., and Attouch, H. (2001). An inertial proximal method for maximal monotone operators via discretization of a nonlinear oscillator with damping. *Set-Valued Analysis*, 9(1), 3-11.
- [3] Attouch, H., Peypouquet, J., and Redont, P. (2014). A dynamical approach to an inertial forward-backward algorithm for convex minimization. *SIAM Journal on Optimization*, 24(1), 232-256.
- [4] Berinde, V., and Takens, F. (2007). Iterative approximation of fixed points (Vol. 1912, pp. xvi+322). Berlin: Springer.
- [5] Bello Cruz, J. Y., and Díaz Millán, R. (2015). A variant of forward-backward splitting method for the sum of two monotone operators with a new search strategy. *Optimization*, 64(7), 1471-1486.
- [6] Bot, R. I., and Csetnek, E. R. (2017). Second-order dynamical systems associated to variational inequalities. *Applicable Analysis*, 96(5), 799-809.
- [7] Combettes, P. L. (2004). Solving monotone inclusions via compositions of nonexpansive averaged operators. *Optimization*, 53(5-6), 475-504.

- [8] Van Hieu, D., Anh, P. K., and Muu, L. D. (2021). Modified forward–backward splitting method for variational inclusions. *4OR*, 19(1), 127-151.
- [9] Guo, K. (2021). On the linear convergence rate of a relaxed forward–backward splitting method. *Optimization*, 70(5-6), 1161-1170.
- [10] Lorenz, D. A., and Pock, T. (2015). An inertial forward-backward algorithm for monotone inclusions. *Journal of Mathematical Imaging and Vision*, 51(2), 311-325.
- [11] Minty, G. J. (1962). Monotone (nonlinear) operators in Hilbert space. *Duke Mathematical Journal*, 29(3), 341-346.
- [12] Moudafi, A., and Shehu, Y. (2019). Convergence of the forward–backward method for split null-point problems beyond coerciveness. *J Nonlinear Convex Anal*, 20, 1659-1672.
- [13] Takahashi, W. (2000). *Nonlinear functional analysis*, Yokohama Publishere. Yokohama Japan.
- [14] Tang, Y., Lin, H., Gibali, A., and Cho, Y. J. (2022). Convergence analysis and applications of the inertial algorithm solving inclusion problems. *Applied Numerical Mathematics*
- [15] Tseng, P. (2000). A modified forward-backward splitting method for maximal monotone mappings. *SIAM Journal on Control and Optimization*, 38(2), 431-446.
- [16] Thong, D. V., and Van Hieu, D. (2017). An inertial method for solving split common fixed point problems. *Journal of Fixed Point Theory and Applications*, 19(4), 3029-3051.
- [17] Shehu, Y., Liu, L., Mu, X., and Dong, Q. L. (2021). Analysis of versions of relaxed inertial projection and contraction method. *Applied Numerical Mathematics*, 165, 1-21.
- [18] Shehu, Y., and Iyiola, O. S. (2020). Projection methods with alternating inertial steps for variational inequalities: weak and linear convergence. *Applied Numerical Mathematics*, 157, 315-337.

Laplacian twin support vector machine with Pinball loss for semi-supervised classification

Vipavee Damminsed*, Wanida Panup, Rabian Wangkeeree

Department of Mathematics, Faculty of Science, Naresuan University, Phitsanulok, Thailand.

**presenting author.
email: vipaveed61@nu.ac.th*

Abstract

Semi-supervised learning utilizes labeled data and the geometric information in the unlabeled data embedded in the marginal distribution to construct a model whereas supervised learning makes use of the only labeled data. So, semisupervised learning establishes a more reasonable classifier. In recent years, the Laplacian support vector machine (Lap-SVM) for semi-supervised classification has received a lot of interest. To develop the performance of Lap-SVM, Laplacian twin support vector machine (Lap-TSVM) for semi-supervised classification has shown exceptional performance as an addition to improve the computational complexity. However, dealing with noise sensitivity and instability for resampling is still a challenge. In this paper, we provide a Laplacian twin support vector machine with the pinball loss function (Lap-PTSVM) for semi-supervised classification to effectively handle the aforementioned problems. As a result, it improves a better generalization ability of the classifier. Several experiments have been performed on artificial and UCI datasets. Linear and nonlinear situations are discussed. Also, the results show that our proposed Lap-PTSVM has noise insensitivity comparable to the Lap-TSVM and has a great generalization performance.

Acknowledgements

This research was supported by the Science Achievement Scholarship of Thailand, and Development and Promotion of the Gifted in Science and Technology Project.

References

- [1] Jayadeva, Khemchandani R, Chandra S (2007) Twin support vector machines for pattern classification. *IEEE Trans. Pattern Anal. Mach. Intell.* 29(5): 905-910.
- [2] Huang X, Shi L, Suykens JAK (2014) Support vector machine classifier with pinball loss. *IEEE Trans. Pattern Anal. Mach. Intell.* 36: 984-997.
- [3] Qi Z, Tian Y, Shi Y (2012) Laplacian twin support vector machine for semisupervised classification. *Neural Netw.* 35:46-53.

Iterative schemes converging to a fixed point of a mapping on a complete geodesic space

Yasunori Kimura*

Department of Information Science, Toho University, Japan.

**presenting author.*

email: yasunori@is.sci.toho-u.ac.jp

Abstract

The study of a nonexpansive mapping defined on a metric space has been considered by many researchers in the setting of Hilbert and Banach spaces. In 2004, Kirk [2] proved a fixed point theorem for Hadamard spaces, defined as a complete $CAT(0)$ space. Since then, various kinds of existence and approximation results for mappings on such spaces have been proposed. This work mainly focuses on approximation methods to a fixed point of a mapping defined on a complete geodesic space. The Halpern iterative scheme is a crucial method converging to a solution to our problem. In this scheme, we use a convex combination with an anchor point to generate an iterative sequence. The Mann iterative scheme is another method, and it is also generated by using a convex combination, but it does not use an anchor point. For convergence results for these schemes on Hadamard spaces, see [3,1]. In this work, we consider yet another type of approximation method called a projection method. We compare these different types of approximation methods and obtain several new convergence results with recent developments.

Acknowledgements

This work was partially supported by JSPS KAKENHI Grant Number JP21K03316.

References

- [1] J. S. He, D. H. Fang, G. López, and C. Li, Mann's algorithm for nonexpansive mappings in $CAT(\kappa)$ spaces, *Nonlinear Anal.* 75 (2012), 445–452.
- [2] W. A. Kirk, Fixed point theorems in $CAT(0)$ spaces and R-trees, *Fixed Point Theory Appl.* (2004), 309–316.
- [3] S. Saejung, Halpern's iteration in $CAT(0)$ spaces, *Fixed Point Theory Appl.* (2010), Art. ID 471781, 13.



Conference Organizers

- Working Group on Applied Nonlinear Analysis

Sponsors

- Center of Excellence in Theoretical and Computational Science (TaCS-CoE)
- Applied Mathematics for Science and Engineering Research Unit (AMSERU)
- Program Management Unit for Human Resources & Institutional Development, Research and Innovation (PMU-B)
- Nakhon Sawan Rajabhat University
- Muban Chombueng Rajabhat University

Editors of the Book of Abstracts

- Professor Dr. Poom Kumam
- Dr. Parin Chaipunya
- Dr. Konrawut Khammahawong

Graphic Designers

- Patcharin Boonron
- Pawicha Phairatchatniyom
- Urairat Deepan

Secretarial Contacts

- Dr. Parin Chaipunya (parin.cha@mail.kmutt.ac.th)
- Dr. Nantaporn Chuensupantharat (jo_y_yaya@hotmail.com)



Book *of*
ABSTRACTS

5-6 September
2022

Bangsaen Heritage Hotel,
Chonburi, Thailand

**For more details
about registration,
please visit**



<https://iwana2022.wordpress.com>