Príklad: Majme preurčený systém rovníc

$$-14{,}16\hat{\theta}_0 + \hat{\theta}_1 = -2{,}04$$
$$-12{,}6\hat{\theta}_0 + \hat{\theta}_1 = 0$$
$$-7{,}08\hat{\theta}_0 + \hat{\theta}_1 = 4{,}08$$
$$-4{,}92\hat{\theta}_0 + \hat{\theta}_1 = 7{,}92$$

1. Rekurzívna metóda najmenších štvorcov

$$\begin{pmatrix} -14{,}16 & 1 \\ -12{,}6 & 1 \\ -7{,}08 & 1 \\ -4{,}92 & 1 \end{pmatrix} \begin{pmatrix} \hat{\theta}_0 \\ \hat{\theta}_1 \end{pmatrix} = \begin{pmatrix} -2{,}04 \\ 0 \\ 4{,}08 \\ 7{,}92 \end{pmatrix} \qquad n = 2$$

Štart $\boxed{\mathbf{P}_N = \left(\mathbf{H}_N^T \mathbf{H}_N\right)^{-1}}$ $\boxed{\hat{\boldsymbol{\theta}}_N^* = \mathbf{P}_N \mathbf{H}_N^T \mathbf{y}_N}$

$$N \overset{!}{=} 2 \qquad H_2 = \begin{pmatrix} -14{,}16 & 1 \\ -12{,}6 & 1 \end{pmatrix}$$

$$R_2 = H_2^T H_2 = \begin{pmatrix} -14{,}16 & -12{,}6 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} -14{,}16 & 1 \\ -12{,}6 & 1 \end{pmatrix} = \begin{pmatrix} 359{,}2656 & -26{,}76 \\ -26{,}76 & 2 \end{pmatrix}$$

$$P_2 = R_2^{-1} = \begin{pmatrix} 359{,}2656 & -26{,}76 \\ -26{,}76 & 2 \end{pmatrix}^{-1} = \begin{pmatrix} 0{,}8218 & 10{,}9961 \\ 10{,}9961 & 147{,}6272 \end{pmatrix}$$

$$\hat{\theta}_2^* = \begin{pmatrix} 0{,}8218 & 10{,}9961 \\ 10{,}9961 & 147{,}6272 \end{pmatrix} \begin{pmatrix} -14{,}16 & -12{,}6 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} -2{,}04 \\ 0 \end{pmatrix} = \begin{pmatrix} 1{,}3077 \\ 16{,}4769 \end{pmatrix}$$

Rekurzívny výpočet

$$\boxed{\begin{array}{ll} \mathbf{d}_{N+1} = \mathbf{P}_N \mathbf{h}_{N+1} & \rho_{N+1} = \left(1 + \mathbf{h}_{N+1}^T \mathbf{d}_{N+1}\right)^{-1} \\ e_{N+1} = y_{N+1} - \mathbf{h}_{N+1}^T \hat{\boldsymbol{\theta}}_N^* & Q\left(\hat{\boldsymbol{\theta}}_{N+1}^*\right) = Q\left(\hat{\boldsymbol{\theta}}_N^*\right) + \rho_{N+1} e_{N+1}^2 \\ \hat{\boldsymbol{\theta}}_{N+1}^* = \hat{\boldsymbol{\theta}}_N^* + \rho_{N+1} e_{N+1} \mathbf{d}_{N+1} & P_{N+1} = \left(I - \rho_{N+1} d_{N+1} h_{N+1}^T\right) \mathbf{P}_N \end{array}}$$

$$d_3 = \begin{pmatrix} 0{,}8218 & 10{,}9961 \\ 10{,}9961 & 147{,}6272 \end{pmatrix} \begin{pmatrix} -7{,}08 \\ 1 \end{pmatrix} = \begin{pmatrix} 5{,}1775 \\ 69{,}7752 \end{pmatrix}$$

$$\rho_3 = \left[1 + \begin{pmatrix} -7{,}08 & 1 \end{pmatrix} \begin{pmatrix} 5{,}1775 \\ 69{,}7752 \end{pmatrix}\right]^{-1} = 0{,}0293$$

$$e_3 = 4{,}08 - \begin{pmatrix} -7{,}08 & 1 \end{pmatrix} \begin{pmatrix} 1{,}3077 \\ 16{,}4763 \end{pmatrix} = -3{,}1384$$

$$Q_3 = 0 + 0{,}0293 . 3{,}1384^2 = 0{,}2886$$

$$\hat{\boldsymbol{\theta}}_3^* = \begin{pmatrix} 1{,}3077 \\ 16{,}4769 \end{pmatrix} - 0{,}0293\,.\,3{,}1384 \begin{pmatrix} 5{,}1775 \\ 69{,}7752 \end{pmatrix} = \begin{pmatrix} 0{,}8314 \\ 10{,}0585 \end{pmatrix}$$

$$\mathbf{P}_3 = \left[\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - 0{,}0293\,.\begin{pmatrix} 5{,}1775 \\ 69{,}7752 \end{pmatrix} (-7{,}08 \quad 1) \right] \mathbf{P}_2 = \begin{pmatrix} 0{,}0362 & 0{,}4076 \\ 0{,}4076 & 4{,}9309 \end{pmatrix}$$

$$\mathbf{d}_4 = \begin{pmatrix} 0{,}0362 & 0{,}4076 \\ 0{,}4076 & 4{,}9309 \end{pmatrix} \begin{pmatrix} -4{,}92 \\ 1 \end{pmatrix} = \begin{pmatrix} 0{,}2299 \\ 2{,}9256 \end{pmatrix}$$

$$\rho_4 = \left[1 + \left(-4{,}92 \quad 1 \right) \begin{pmatrix} 0{,}2299 \\ 2{,}9256 \end{pmatrix} \right]^{-1} = 0{,}3579$$

$$e_4 = 7{,}92 - \left(-4{,}92 \quad 1 \right) \begin{pmatrix} 0{,}8314 \\ 10{,}0585 \end{pmatrix} = -1{,}9520$$

$$Q_4 = Q_3 + 0{,}3579\,.\,1{,}9520^2 = 1{,}6522$$

$$\hat{\boldsymbol{\theta}}_4^* = \begin{pmatrix} 0{,}8314 \\ 10{,}0585 \end{pmatrix} - 0{,}3579\,.\,1{,}9520 \begin{pmatrix} 0{,}2299 \\ 2{,}9256 \end{pmatrix} = \begin{pmatrix} 0{,}9920 \\ 12{,}1020 \end{pmatrix}$$

2. Odmocninová verzia RMNŠ

$$P_2 = \left(H_2^T H_2 \right)^{-1} = \begin{pmatrix} 0{,}8218 & 10{,}9961 \\ 10{,}9961 & 147{,}6272 \end{pmatrix}$$

Štart: $N \overset{!}{=} 2$ Určenie $\mathbf{G}_2$ Choleskeho algoritmom

$$g_{11} = \pm\sqrt{p_{11}}$$

$$g_{1j} = \frac{1}{g_{11}} p_{1j} \qquad j = 2, 3, \ldots, n$$

$$g_{ii} = \pm\sqrt{p_{ii} - \sum_{k=1}^{i-1} g_{ki}^2} \qquad i = 2, 3, \ldots, n$$

$$g_{ij} = \frac{1}{g_{ii}} \left(p_{ij} - \sum_{k=1}^{i-1} g_{ki} g_{kj} \right) \qquad j = i + 1, \ldots, n$$

$$g_{11} = \sqrt{0{,}8218} = 0{,}9066 \qquad g_{12} = \frac{10{,}9961}{0{,}9066} = 12{,}1296$$

$$g_{22} = \sqrt{147{,}6272 - 12{,}1296^2} = 0{,}7071$$

$$G_2 = \begin{pmatrix} 0{,}9066 & 12{,}1296 \\ 0 & 0{,}7071 \end{pmatrix}$$

$$\hat{\boldsymbol{\theta}}_2^* = \mathbf{G}_2^T \mathbf{G}_2 \mathbf{H}_2^T \mathbf{y}_2 = \begin{pmatrix} 1{,}3077 \\ 16{,}4769 \end{pmatrix}$$

Rekurzívny výpočet

$$e_{N+1} = y_{N+1} - \mathbf{h}_{N+1}^T \hat{\boldsymbol{\theta}}_N^* \qquad \mathbf{z}_{N+1} = \mathbf{G}_N \mathbf{h}_{N+1}$$

$$\rho_{N+1} = \left(1 + \mathbf{z}_{N+1}^T \mathbf{z}_{N+1}\right)^{-1} \qquad Q\left(\hat{\boldsymbol{\theta}}_{N+1}^*\right) = Q\left(\hat{\boldsymbol{\theta}}_N^*\right) + \rho_{N+1} e_{N+1}^2$$

$$\hat{\boldsymbol{\theta}}_{N+1}^* = \hat{\boldsymbol{\theta}}_N^* + \rho_{N+1} e_{N+1} \mathbf{G}_N^T \mathbf{z}_{N+1} \qquad \mathrm{G}_{N+1} = \left(\mathrm{I} - \frac{\rho_{N+1}}{1+\sqrt{\rho_{N+1}}} \mathrm{z}_{N+1} \mathrm{z}_{N+1}^T\right) \mathrm{G}_N$$

$$e_3 = 4{,}08 - \left(\begin{pmatrix} -7{,}08 & 1 \end{pmatrix} \begin{pmatrix} 1{,}3077 \\ 16{,}4763 \end{pmatrix}\right) = -3{,}1384$$

$$\mathbf{z}_3 = \begin{pmatrix} 0{,}9066 & 12{,}1296 \\ 0 & 0{,}7071 \end{pmatrix} \begin{pmatrix} -7{,}08 \\ 1 \end{pmatrix} = \begin{pmatrix} 5{,}7112 \\ 0{,}7071 \end{pmatrix}$$

$$\rho_3 = \left[1 + \left(\begin{pmatrix} 5{,}7112 & 0{,}7071 \end{pmatrix} \begin{pmatrix} 5{,}7112 \\ 0{,}7071 \end{pmatrix}\right]^{-1} = 0{,}0293$$

$$Q_3 = 0 + 0{,}0293\,.\,3{,}1384^2 = 0{,}2886$$

$$\hat{\boldsymbol{\theta}}_3^* = \begin{pmatrix} 1{,}3077 \\ 16{,}4769 \end{pmatrix} - 0{,}0293\,.\,3{,}1384 \begin{pmatrix} 0{,}9066 & 0 \\ 12{,}1296 & 0{,}7071 \end{pmatrix} \begin{pmatrix} 5{,}7112 \\ 0{,}7071 \end{pmatrix} = \begin{pmatrix} 0{,}8314 \\ 10{,}0584 \end{pmatrix}$$

$$\mathrm{G}_3 = \left[\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \frac{0{,}0293}{1+\sqrt{0{,}0293}} . \begin{pmatrix} 5{,}7112 \\ 0{,}7071 \end{pmatrix} (5{,}7112 ... 0{,}7071)\right] \mathrm{G}_2 = \begin{pmatrix} 0{,}1665 & 2{,}1569 \\ -0{,}0961 & -0{,}5276 \end{pmatrix}$$

$$e_4 = 7{,}92 - \left(\begin{pmatrix} -4{,}92 & 1 \end{pmatrix} \begin{pmatrix} 0{,}8314 \\ 10{,}0585 \end{pmatrix}\right) = -1{,}9520$$

$$\mathrm{z}_4 = \mathrm{G}_3 \begin{pmatrix} -4{,}92 \\ 1 \end{pmatrix} = \begin{pmatrix} 1{,}3375 \\ -0{,}0768 \end{pmatrix}$$

$$\rho_4 = \left[1 + \left(\begin{pmatrix} 1{,}3375 & -0{,}0768 \end{pmatrix} \begin{pmatrix} 1{,}3375 \\ -0{,}0768 \end{pmatrix}\right]^{-1} = 0{,}3578$$

$$Q_4 = Q_3 + 0{,}3578\,.\,1{,}9520^2 = 1{,}6522$$

$$\hat{\boldsymbol{\theta}}_4^* = \begin{pmatrix} 0{,}8314 \\ 10{,}0585 \end{pmatrix} - 0{,}3578\,.\,1{,}9520 \begin{pmatrix} 0{,}1665 & -0{,}0961 \\ 2{,}1569 & -0{,}5276 \end{pmatrix} \begin{pmatrix} 1{,}3375 \\ -0{,}0768 \end{pmatrix} = \begin{pmatrix} 0{,}9919 \\ 12{,}1018 \end{pmatrix}$$

3. Rekurzívne algoritmy Peterku

$$\begin{pmatrix} -14{,}16 & 1 \\ -12{,}6 & 1 \\ -7{,}08 & 1 \\ -4{,}92 & 1 \end{pmatrix} \begin{pmatrix} \hat{\theta}_0 \\ \hat{\theta}_1 \end{pmatrix} = \begin{pmatrix} -2{,}04 \\ 0 \\ 4{,}08 \\ 7{,}92 \end{pmatrix} \qquad \lambda \overset{!}{=} 1 \qquad \mathbf{Z}_4 = \begin{pmatrix} -14{,}16 & 1 & -2{,}04 \\ -12{,}6 & 1 & 0 \\ -7{,}08 & 1 & 4{,}08 \\ -4{,}92 & 1 & 7{,}92 \end{pmatrix}$$

Štart: Nech $N \overset{!}{=} 3$

$$\mathbf{V}_3 = \mathbf{Z}_3^T \mathbf{Z}_3 = \begin{pmatrix} -14.16 & -12.6 & -7.08 \\ 1 & 1 & 1 \\ -2.04 & 0 & 4.08 \end{pmatrix} \begin{pmatrix} -14.16 & 1 & -2.04 \\ -12.6 & 1 & 0 \\ -7.08 & 1 & 4.08 \end{pmatrix} = \begin{pmatrix} 409.39 & -33.84 & 0 \\ -33.84 & 3 & 2.04 \\ 0 & 2.04 & 20.81 \end{pmatrix}$$

$$\mathbf{R}_3 = \mathbf{V}_3^{-1} = \begin{pmatrix} 2.431 & 29.375 & -2.880 \\ 29.375 & 355.375 & -34.841 \\ -2.88 & -34.841 & 3.464 \end{pmatrix}$$

3.1 REFIL (Odmocninový rozklad)

$$g_{nn} = \sqrt{r_{nn}}$$

$$g_{nj} = \frac{1}{g_{nn}} r_{nj} \qquad j = 1, \ldots, n-1$$

$$g_{ii} = \sqrt{r_{ii} - \sum_{k=i+1}^{n} g_{ki}^2} \qquad i = n-1, \ldots, 1$$

$$g_{ij} = \frac{1}{g_{ii}} \left(r_{ij} - \sum_{k=i+1}^{n} g_{ki} g_{kj} \right) \qquad j = i-1, \ldots, 1$$

Štart: Na ilustráciu vypočítame maticu $\mathbf{G}_3$ z matice $\mathbf{R}_3$:

$$g_{33} = \sqrt{r_{33}} = \sqrt{3{,}4638} = 1{,}8611 \qquad g_{32} = \frac{r_{32}}{g_{33}} = \frac{-34{,}8407}{1{,}8611} = -18{,}7202;$$

$$g_{31} = \frac{r_{31}}{g_{33}} = \frac{-2{,}8799}{1{,}8611} = -1{,}5474 \qquad g_{22} = \sqrt{r_{22} - g_{32}^2} = \sqrt{355{,}375 - 18{,}7202^2} = 2{,}2205$$

$$g_{21} = \frac{r_{21} - g_{31} g_{32}}{g_{22}} = \frac{29{,}375 - 1{,}5474 . 18{,}7202}{2{,}2205} = 0{,}1835$$

$$g_{11} = \sqrt{r_{11} - g_{21}^2 - g_{31}^2} = \sqrt{2{,}4306 - 0{,}1835^2 - 1{,}5474^2} = 0{,}0494$$

$$\mathbf{G}_3 = \begin{pmatrix} 0.0494 & 0 & 0 \\ 0.1835 & 2.2205 & 0 \\ -1.5474 & -18.7202 & 1.8611 \end{pmatrix} = \begin{pmatrix} \mathbf{\Gamma} & \mathbf{0} \\ \mathbf{g}^T & \gamma \end{pmatrix}$$

Rekurzívny výpočet

$$\boxed{\begin{array}{lll} \hat{\boldsymbol{\theta}}_k^* = -\dfrac{\mathbf{g}}{\gamma} & & Q\left(\hat{\boldsymbol{\theta}}_k\right) = \dfrac{1}{\gamma^2} \\ \mathbf{f}_{k+1} = \mathbf{G}_k \mathbf{z}_{k+1} & & \\ s_0^2 = \lambda^2 & s_q^2 = s_{q-1}^2 + f_q^2 & q = 1, 2, \ldots, k \\ {}^{k+1}g_{ij} = \dfrac{1}{\lambda}\dfrac{s_{i-1}}{s_i}\left({}^k g_{ij} - \dfrac{f_i}{s_{i-1}^2}\displaystyle\sum_{m=j}^{i-1} f_m \, {}^k g_{mj} \right) & & \end{array}}$$

$$\hat{\boldsymbol{\theta}}_3^* = \frac{1}{1{,}8611}\begin{pmatrix} 1{,}5474 \\ 18{,}7202 \end{pmatrix} = \begin{pmatrix} 0{,}831 \\ 10{,}058 \end{pmatrix}$$

$$Q(\hat{\boldsymbol{\theta}}_3^*) = \frac{1}{1.8611^2} = 0{,}2887$$

$$f_4 = G_3 z_4 = \begin{pmatrix} 0.0494 & 0 & 0 \\ 0.1835 & 2.2205 & 0 \\ -1.5474 & -18.7202 & 1.8611 \end{pmatrix}\begin{pmatrix} -4.92 \\ 1 \\ 7.92 \end{pmatrix} = \begin{pmatrix} -0.2432 \\ 1.3175 \\ 3.6332 \end{pmatrix} = \begin{pmatrix} f_1 \\ f_2 \\ f_3 \end{pmatrix}$$

$$s_0^2 = 1$$

$$s_1^2 = s_0^2 + f_1^2 = 1 + 0{,}2432^2 = 1{,}0591$$

$$s_2^2 = s_1^2 + f_2^2 = 1{,}0591^2 + 1{,}3175^2 = 2{,}7948$$

$$s_3^2 = s_2^2 + f_3^2 = 2{,}7948^2 + 3{,}6332^2 = 15{,}9948$$

$$s_0 = 1$$
$$s_1 = 1{,}0291$$
$$s_2 = 1{,}6718$$
$$s_3 = 3{,}9994$$

$$g_{11} = \frac{s_0}{s_1} g_{11} = \frac{0{,}0494}{1{,}0291} = 0{,}0480$$

$$g_{22} = \frac{s_1}{s_2} g_{22} = \frac{1{,}0291}{1{,}6718} 2{,}2205 = 1{,}3669$$

$$g_{33} = \frac{s_2}{s_3} g_{33} = \frac{1{,}6718}{3{,}9994} 1{,}8611 = 0{,}7780$$

$$g_{21} = \frac{s_1}{s_2}\left(g_{21} - \frac{f_2}{s_1^2} f_1 g_{11} \right) = \frac{1{,}0291}{1{,}6718}\left[0{,}1835 - \frac{1{,}3175}{1{,}0291^2}.(-0{,}2432).0{,}0494 \right] = 0{,}1222$$

$$g_{31} = \frac{s_2}{s_3}\left(g_{31} - \frac{f_3}{s_2^2}(f_1 g_{11} + f_2 g_{21}) \right) =$$

$$= \frac{1{,}6718}{3{,}9994}\left[-1{,}5474 - \frac{3{,}6332}{1{,}6718^2}(1{,}3175.0{,}1835 - 0{,}2432.0{,}0494) \right] = -0.7718$$

$$g_{32} = \frac{s_2}{s_3}\left(g_{32} - \frac{f_3}{s_2^2} f_2 g_{22} \right) = \frac{1{,}6718}{3{,}9994}\left[-18{,}7202 - \frac{3{,}6332}{1{,}6718^2} 1{,}3175.2{,}2205 \right] = -9.4149$$

$$\mathbf{G}_4 = \begin{pmatrix} 0.048 & 0 & 0 \\ 0.1222 & 1.3669 & 0 \\ -0.7717 & -9.4149 & 0.778 \end{pmatrix} = \begin{pmatrix} \mathbf{\Gamma} & \mathbf{0} \\ \mathbf{g}^T & \gamma \end{pmatrix}$$

$$\hat{\boldsymbol{\theta}}_4^* = \frac{1}{0{,}7780}\begin{pmatrix} 0{,}7717 \\ 9{,}4149 \end{pmatrix} = \begin{pmatrix} 0{,}9919 \\ 12{,}1018 \end{pmatrix}$$

$$Q(\hat{\boldsymbol{\theta}}_4^*) = \frac{1}{0{,}7780^2} = 1{,}6522$$

3.2 LDFIL (LDL rozklad)

$$\boxed{\begin{array}{ll} d_i = r_{ii} - \sum\limits_{k=i+1}^{n} d_k l_{ki}^2 & i = n,\ldots,1 \\ l_{ij} = \dfrac{1}{d_i}\left(r_{ij} - \sum\limits_{k=i+1}^{n} l_{ki} d_k l_{kj}\right) & j = i-1,\ldots,1 \\ l_{ii} = 1 & \end{array}}$$

Štart: Na ilustráciu vypočítame rozklad matice $\mathbf{R}_3$:

$$\mathbf{R}_3 = \begin{pmatrix} 2.4306 & 29.375 & -2.8799 \\ 29.375 & 355.375 & -34.8407 \\ -2.8799 & -34.8407 & 3.4638 \end{pmatrix} \overset{!}{=} \mathbf{R} = \begin{pmatrix} \mathbf{\Lambda}^T & \boldsymbol{\lambda} \\ \mathbf{0}^T & 1 \end{pmatrix}\begin{pmatrix} \mathbf{\Delta} & \mathbf{0} \\ \mathbf{0}^T & d \end{pmatrix}\begin{pmatrix} \mathbf{\Lambda} & \mathbf{0} \\ \boldsymbol{\lambda}^T & 1 \end{pmatrix}$$

$$d_3 = R_{33} = 3{,}4638$$

$$l_{32} = \frac{R_{32}}{d_3} = \frac{-34{,}8407}{3{,}4638} = -10{,}0525$$

$$l_{31} = \frac{R_{31}}{d_3} = \frac{-2{,}8799}{3{,}4638} = -0{,}8314$$

$$d_2 = R_{22} - d_3 l_{32}^2 = 355{,}375 - 3{,}4638.10{,}0585^2 = 4{,}9306$$

$$l_{21} = \frac{R_{21} - d_3 l_{32} l_{31}}{d_2} = \frac{29{,}375 - 3{,}4638.10{,}0585.0{,}8314}{4{,}9306} = 0{,}0827$$

$$d_1 = R_{11} - d_3 l_{31}^2 - d_2 l_{21}^2 = 2{,}4306 - 3{,}4638.0{,}8314^2 - 4{,}9306.0{,}0827^2 = 0{,}0024$$

$$\mathbf{R}_3 = \begin{pmatrix} 1 & 0.0827 & -0.8314 \\ 0 & 1 & -10.0585 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 0.0024 & 0 & 0 \\ 0 & 4.9306 & 0 \\ 0 & 0 & 3.4638 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0.0827 & 1 & 0 \\ -0.8314 & -10.0585 & 1 \end{pmatrix}$$

Rekurzívny výpočet

$$\hat{\boldsymbol{\theta}}_k^* = -\boldsymbol{\lambda} \qquad Q\left(\hat{\boldsymbol{\theta}}_k^*\right) = \frac{1}{d}$$

$$\mathbf{f}_{k+1} = \mathbf{L}_k \mathbf{z}_k$$

$$s_0^2 = \lambda^2$$

$$s_q^2 = \lambda^2 + \sum_{k=1}^{q} d_k f_k^2 = s_{q-1}^2 + d_q f_q^2 \qquad q = 1, \ldots, n$$

$$^{k+1}d_i = {}^k d_i \frac{1}{\lambda^2} \frac{s_{i-1}^2}{s_i^2} \qquad ^{k+1}l_{ij} = {}^k l_{ij} - \frac{f_i}{s_{i-1}^2} \sum_{m=j}^{i-1} f_m \; {}^k d_m \; {}^k l_{mj}$$

$$\hat{\boldsymbol{\theta}}_3^* = -\boldsymbol{\lambda} = [0.8314 \; 10.0585]^T$$

$$Q(\hat{\boldsymbol{\theta}}_3^*) = \frac{1}{d} = \frac{1}{3.4638} = 0.2887$$

$$\mathbf{f}_4 = \mathbf{L}_3 \mathbf{z}_4 = \begin{pmatrix} 1 & 0 & 0 \\ 0.0827 & 1 & 0 \\ -0.8314 & -10.0585 & 1 \end{pmatrix} \begin{pmatrix} -4.92 \\ 1 \\ 7.92 \end{pmatrix} = \begin{pmatrix} -4.92 \\ 0.5933 \\ 1.9521 \end{pmatrix}$$

$$s_0^2 = 1$$

$$s_1^2 = s_0^2 + d_1 f_1^2 = 1 + 0{,}0024.4{,}92^2 = 1{,}0591$$

$$s_2^2 = s_1^2 + d_2 f_2^2 = 1{,}0591 + 4{,}9306.0{,}5933^2 = 2{,}7948$$

$$s_3^2 = s_2^2 + d_3 f_3^2 = 2{,}7948 + 3{,}4638.1{,}9521^2 = 15{,}9948 = \lambda^2 + \underline{f}^T \underline{D}\,\underline{f}$$

$$d_1 = \frac{s_0^2}{s_1^2} d_1 = \frac{1}{1{,}0591} 0{,}0024 = 0{,}0023 \qquad l_{21} = l_{21} - \frac{f_1 f_2}{s_1^2} d_1 = 0{,}0894$$

$$d_2 = \frac{s_1^2}{s_2^2} d_2 = \frac{1{,}0591}{2{,}7948} 4{,}9306 = 1{,}8685 \qquad l_{32} = l_{32} - \frac{f_2 f_3}{s_2^2} d_2 = -12{,}1018$$

$$d_3 = \frac{s_2^2}{s_3^2} d_3 = \frac{2{,}7948}{15{,}9948} 3{,}4638 = 0{,}6052 \qquad l_{31} = l_{31} - \frac{f_3}{s_3^2}\left[l_{21} f_1 d_1 + f_2 d\;\right] = -0{,}9919$$

$$\mathbf{D}_4 = \begin{pmatrix} 0.0023 & 0 & 0 \\ 0 & 1.8685 & 0 \\ 0 & 0 & 0.6052 \end{pmatrix} \qquad \mathbf{L}_4 = \begin{pmatrix} 1 & 0 & 0 \\ 0.0894 & 1 & 0 \\ -0.9919 & -12.1018 & 1 \end{pmatrix}$$

$$\hat{\boldsymbol{\theta}}_4 = -\boldsymbol{\lambda} = [0.9919 \quad 12.1018]^T$$

$$Q(\hat{\boldsymbol{\theta}}_4^*) = \frac{1}{d} = \frac{1}{0.6052} = 1.6522$$