Relé s necitlivosťou (obmedzenie M = 8, necitlivosť d = 0,8) je sériovo zapojené s lineárnym dynamickým systémom **3. rádu** $G(s) = \dfrac{K}{(Ts+1)^3}$, kde K = 10 a T = 1s. Aká bude frekvencia ω a amplitúda A oscilácií 1. harmonickej na výstupe nelinearity?

Lineárna prenosová funkcia (K = 10, T = 1s) $$G(s) = \frac{K}{(Ts+1)^3}$$

Relé s necitlivosťou (M = 8, d = 0,8) $$a_1 = \frac{4M}{\pi}\sqrt{1-\frac{d^2}{A^2}} \qquad b_1 = 0$$

Analytické riešenie metódou harmonickej rovnováhy: (na tabuli)

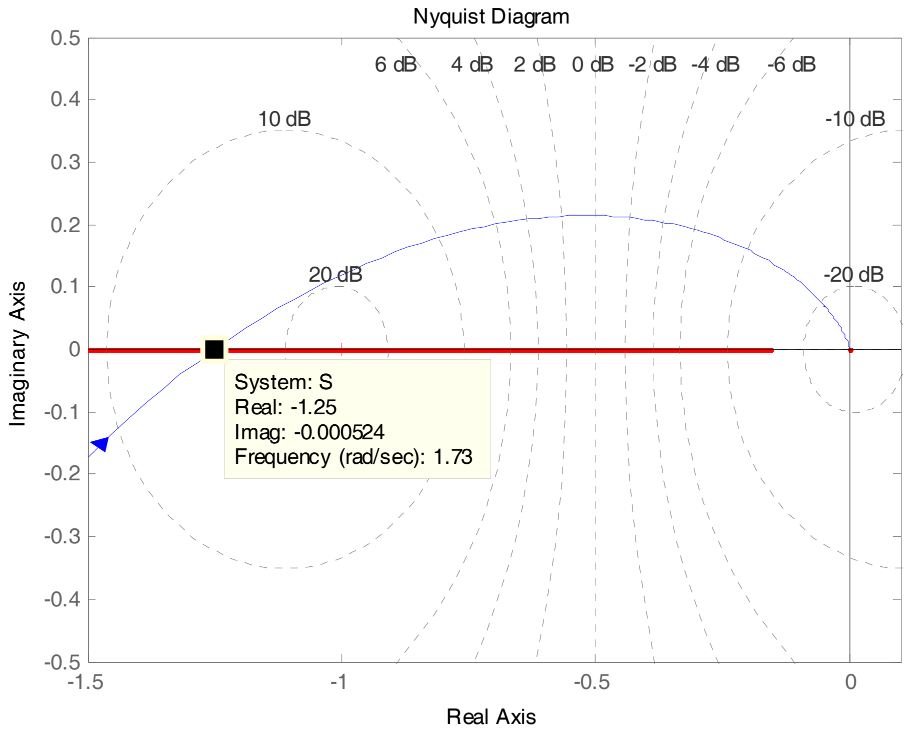
$\omega = \sqrt{3}\ \mathrm{rad\,s^{-1}} = 1{,}7321\ \mathrm{rad\,s^{-1}}$ $\qquad$ $A = 12{,}7071$

Graficko-analytické riešenie: (Matlab)

$$Re = -\frac{\pi}{4M}\frac{A^2}{\sqrt{A^2-d^2}} \qquad 4M\,Re\sqrt{A^2-d^2} = -\pi A^2 \qquad 16M^2\,Re^2(A^2-d^2) = \pi^2 A^4$$

$$\pi^2 A^4 - 16M^2\,Re^2\,A^2 + 16M^2\,Re^2\,d^2) = 0$$

```
% Rele s necitlivostou
clear;

K = 10;
T = 1;

M = 8;
d = 0.8;

% Analyticke riesenie
x = roots([4*pi*pi -K*K*M*M K*K*M*M*d*d]);
A = sqrt(x(1));

% Graficko-analyticke riesenie
s = tf('s');
S = K/(T*s+1)/(T*s+1)/(T*s+1);
w = logspace(-3,2,1000);
nyquist(S,w); grid on; hold on;

axis([-1.5 0.1 -0.5 0.5]);
plot([0 0],[-0.5 0.5],'k');
plot([-1.5 0.1],[0 0],'k');

i = 0;
for A=d:.01:20
    i = i+1;
    regn(i) = -A/(4*M/pi*sqrt(1-d*d/A/A));
    imgn(i) = 0;
end
plot(regn,imgn,'r')

Re = -1.25;
x1 = roots([pi*pi -16*M*M*Re*Re 16*M*M*Re*Re*d*d]);
A = sqrt(x1(1));
```

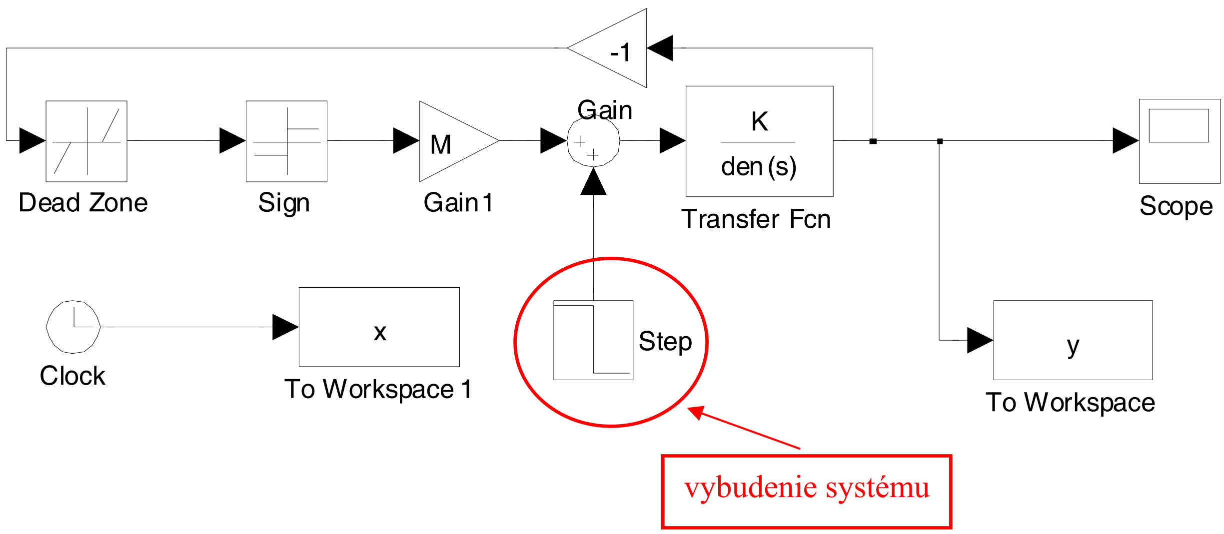


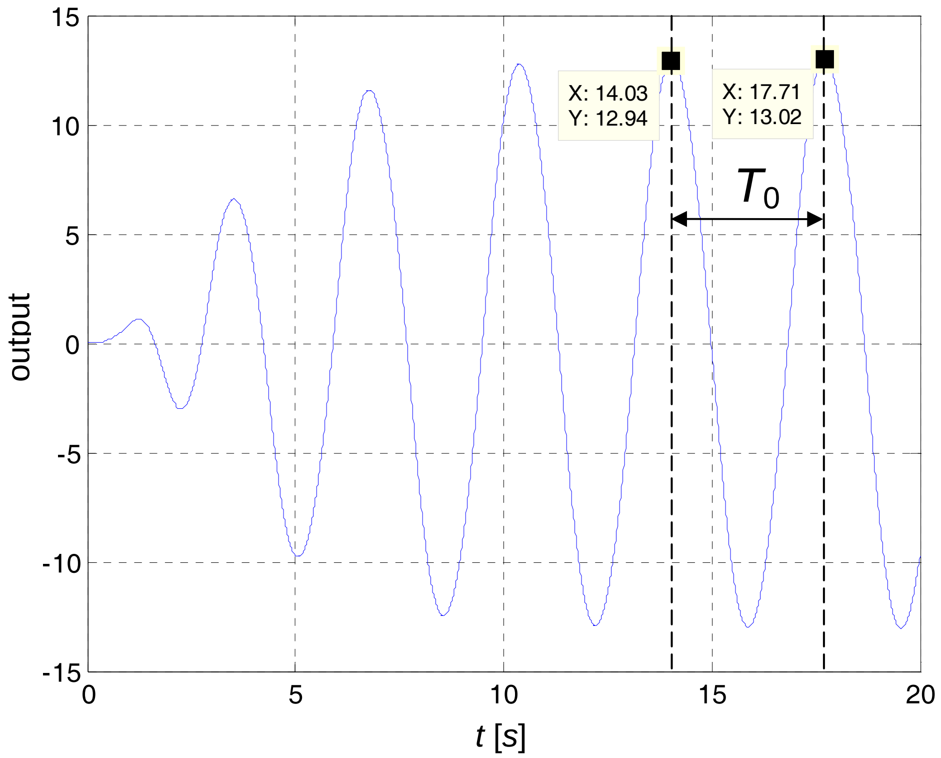
$$\pi^2 A^4 - 16M^2 \operatorname{Re}^2 A^2 + 16M^2 \operatorname{Re}^2 d^2 = 0$$

Z grafu: ω=1,73 rads^{-1} Re = - 1,25

Riešenie (roots): A = 12,7071

Simulácie v Matlabe





Vypočítané hodnoty:
$A = 12{,}7071$
$\omega = 1{,}7321$ rads^{-1}

Modelované hodnoty
$A = 13{,}02$
$\omega = 1{,}707$ rads^{-1}
$T_0 = 3{,}68$ s

Opačná úloha:

Relé s necitlivosťou (obmedzenie $M = 8$, necitlivosť $d = 0{,}8$) je sériovo zapojené s lineárnym dynamickým systémom **3. rádu** $G(s) = \dfrac{K}{(Ts+1)^3}$. Aké má byť zosilnenie K a časová konštanta T systému, ak požadujeme na výstupe systému oscilácie 1. harmonickej s amplitúdou $A = 10$ a frekvenciou $\omega = 1$ rads^{-1}?
(perióda oscilácií je $T_0 = 2\pi/\omega = 6{,}283$ s)

Lineárna prenosová funkcia: $$G(s) = \frac{K}{(Ts+1)^3}$$

Relé s necitlivosťou ($M = 8$, $d = 0{,}8$) $$a_1 = \frac{4M}{\pi}\sqrt{1-\frac{d^2}{A^2}} \qquad b_1 = 0$$

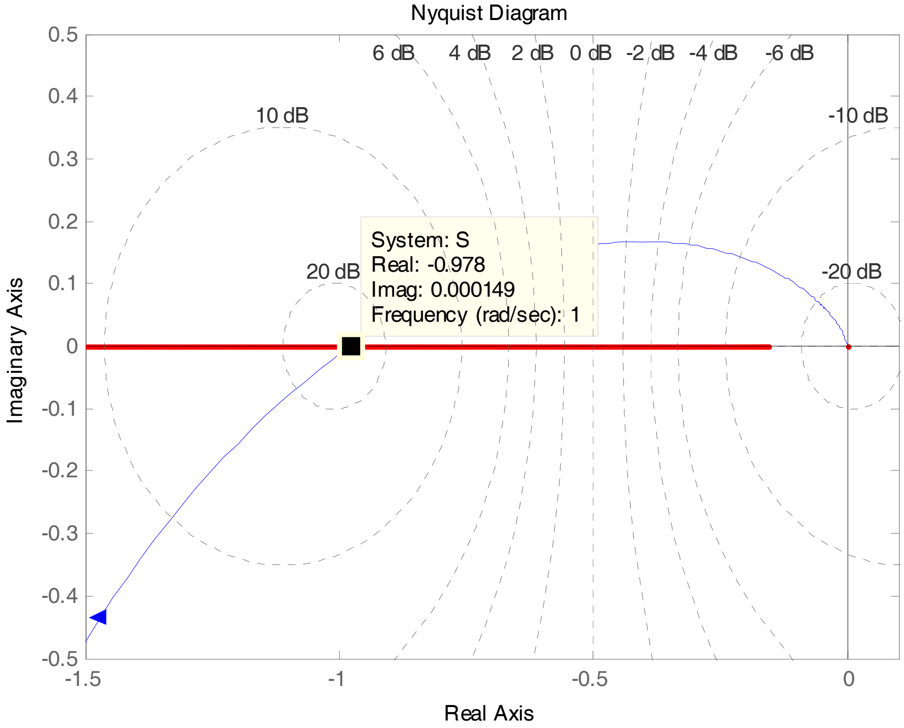
Analytické riešenie:

$$T = \frac{\sqrt{3}}{\omega} = 1{,}7321$$

$$K = \frac{2\pi A^2}{M\sqrt{A^2-d^2}} = 7{,}8290$$

Graficko-analytické riešenie (overenie): (Matlab)

```
% Rele s necitlivostou
clear;

A = 10;
w = 1;

M = 8;
d = 0.8;

T = sqrt(3)/w;
K = 2*pi*A*A/M/sqrt(A*A+d*d);

s = tf('s');
S = K/(T*s+1)/(T*s+1)/(T*s+1);
w = logspace(-3,2,1000);
nyquist(S,w); grid on; hold on;
axis([-1.5 0.1 -0.5 0.5]);
plot([0 0],[-0.5 0.5],'k');
plot([-1.5 0.1],[0 0],'k');

i = 0;
for A=0:.01:20
    i = i+1;
    regn(i) = -A/(4*M/pi*sqrt(1-d*d/A/A));
    imgn(i) = 0;
end
plot(regn,imgn,'r')

% Graficko-analyticke riesenie
Re = -0.978;
x1 = roots([pi*pi -16*M*M*Re*Re 16*M*M*Re*Re*d*d]);
A = sqrt(x1(1));
```



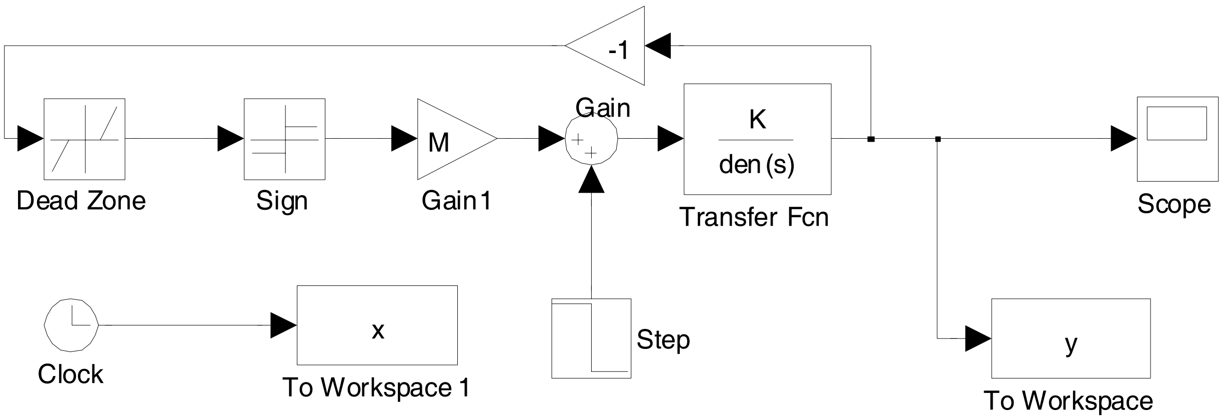
$$\mathrm{Re}\left\{-\frac{1}{G_N(A)}\right\} = -\frac{\pi}{4M}\sqrt{A^2-d^2} \qquad A=\sqrt{d^2+\left(\frac{4M\,\mathrm{Re}}{\pi}\right)^2}$$

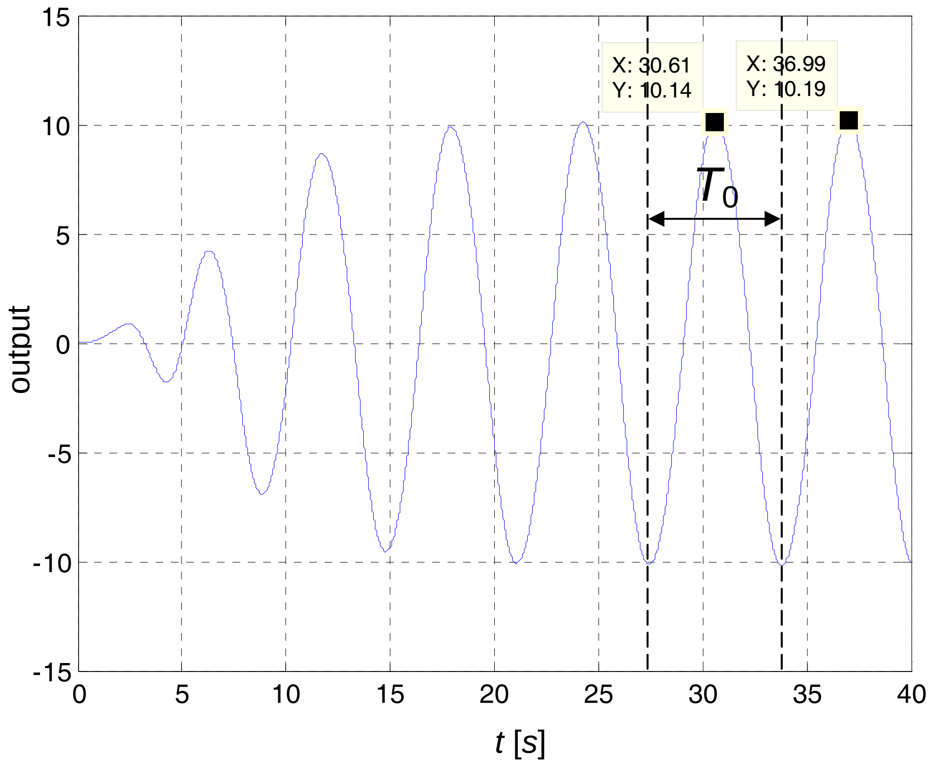
Z grafu: $\omega = 1 \mathrm{rads}^{-1}$ Re = - 0,978

$$T=\frac{\sqrt{3}}{\omega}=1{,}7321$$

$$K=-8\,\mathrm{Re}=7{,}824$$

Simulácie v Matlabe (overenie)





Modelované hodnoty
A = 10,14
ω = 0,9848 rads^{-1}
T_0 = 6,38 s

Požadované hodnoty
A = 10
ω = 1 rads^{-1}
T_0 = 6,283 s