Riešenie preurčeného systému rovníc

Prepokladajme preurčený systém rovníc v tvare

$$-14{,}16\,\hat{\theta}_0 + \hat{\theta}_1 = -2{,}04$$
$$-12{,}6\,\hat{\theta}_0 + \hat{\theta}_1 = 0$$
$$-7{,}08\,\hat{\theta}_0 + \hat{\theta}_1 = 4{,}08$$
$$-4{,}92\,\hat{\theta}_0 + \hat{\theta}_1 = 7{,}92$$

Ak by sme riešili iba 2 rovnice o 2 neznámych dostaneme rôzne riešenia:

(dôvod: namerané výstupy sú ovplyvnené šumom)

Prvé dve rovnice:

$$\hat{\boldsymbol{\theta}}_A^* = \begin{pmatrix} -14{,}16...1 \\ -12{,}6 \quad 1 \end{pmatrix}^{-1} \begin{pmatrix} -2{,}04 \\ 0 \end{pmatrix} = \begin{pmatrix} 1{,}3077 \\ 16{,}4769 \end{pmatrix}$$

$$Q_A = 0^2 + 0^2 + 3{,}1385^2 + 2{,}1231^2 = 14{,}3575$$

Druhé dve rovnice:

$$\hat{\boldsymbol{\theta}}_B^* = \begin{pmatrix} -7{,}08...1 \\ -4{,}92 \quad 1 \end{pmatrix}^{-1} \begin{pmatrix} 4{,}08 \\ 7{,}92 \end{pmatrix} = \begin{pmatrix} 1{,}7778 \\ 16{,}6667 \end{pmatrix}$$

$$Q_B = 6{,}4663^2 + 5{,}7336^2 + 0^2 + 0^2 = 74{,}6954$$

Preurčený systém rovníc $\mathbf{H}\hat{\boldsymbol{\theta}} = \mathbf{y}$

1. **Gaussov vzťah**

$$\mathbf{H} = \begin{pmatrix} -14.16 & 1 \\ -12.6 & 1 \\ -7.08 & 1 \\ -4.92 & 1 \end{pmatrix} \qquad \mathbf{y} = \begin{pmatrix} -2.04 \\ 0 \\ 4.08 \\ 7.92 \end{pmatrix}$$

$$\hat{\boldsymbol{\theta}}^* = \left(\mathbf{H}^{\mathsf{T}}\mathbf{H}\right)^{-1}\mathbf{H}^{\mathsf{T}}\mathbf{y} \qquad \hat{\boldsymbol{\theta}}^* = \begin{pmatrix} 0.9919 \\ 12.1018 \end{pmatrix}$$

$$\hat{\mathbf{y}} = \mathbf{H}\hat{\boldsymbol{\theta}}^* = \begin{pmatrix} -1.9435 \\ -0.3961 \\ 5.0791 \\ 7.2217 \end{pmatrix} \qquad \mathbf{e} = \mathbf{y} - \hat{\mathbf{y}} = \begin{pmatrix} -2.04 \\ 0 \\ 4.08 \\ 7.92 \end{pmatrix} - \begin{pmatrix} -1.94 \\ -0.3961 \\ 5.0789 \\ 7.2217 \end{pmatrix} = \begin{pmatrix} -0.0965 \\ 0.3961 \\ -0.9991 \\ 0.6983 \end{pmatrix}$$

$$Q = \mathbf{e}^{\mathsf{T}}\mathbf{e} = 0.0965^2 + 0.3961^2 + 0.9991^2 + 0.6983^2 = 1.6522$$

2. QR rozklad matice H

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[Q,R]=qr(H)        (Matlab)
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$$Q = \begin{pmatrix} -0.6800 & -0.3633 & -0.4735 & -0.4259 \\ -0.6051 & -0.1727 & 0.4048 & 0.6635 \\ -0.3400 & 0.5018 & 0.5864 & -0.5373 \\ -0.2363 & 0.7657 & -0.5177 & 0.2997 \end{pmatrix}$$

$$R = \begin{pmatrix} 20.8230 & -1.8614 \\ 0 & 0.7316 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$Q^T y = \begin{pmatrix} -1.8713 \\ 8.8533 \\ -0.7414 \\ 1.05 \end{pmatrix}$$

$$R.\hat{\theta}^* = Q^T y$$

$$\Downarrow \quad \Downarrow \quad \Downarrow$$

$$\begin{pmatrix} 20.8230 & -1.8614 \\ 0 & 0.7316 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \hat{\theta}_0 \\ \hat{\theta}_1 \end{pmatrix} = \begin{pmatrix} -1.8713 \\ 8.8533 \\ -0.7414 \\ 1.05 \end{pmatrix}$$

$$20.823\,\hat{\theta}_0 - 1.8614\,\hat{\theta}_1 = -1.8713$$

$$0.7316\,\hat{\theta}_1 = 8.8533$$

$$\Rightarrow$$

$$\hat{\theta}_1 = \frac{8.8533}{0.7316} = 12.1018$$

$$\hat{\theta}_0 = \frac{-1.8713 + 1.8614\,\hat{\theta}_1}{20.823} = 0.9919$$