Random Variables

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January 12, 2025

Outline

1. Introduction
2. Example:
3. Activities (Classify and Apply)

☞ Definition

Random Variables

The real-valued functions defined on the sample space, are known as **random variable**.

There are two types of random variables:

- **Discrete Random Variable**: Takes on a finite or countable number of values.
- **Continuous Random Variable**: Takes on an infinite number of values within an interval.

Formally, if S is the sample space of an experiment, a random variable X is a function:

$$X : S \to \mathbb{R}$$

where $X(s)$ is the value of the random variable corresponding to the outcome $s \in S$.

☞Discrete Random Variable

Discrete Random Variable

Consider a random experiment of rolling a fair six-sided die. The sample space is:

$$S = \{1, 2, 3, 4, 5, 6\}.$$

Define the random variable X as the result of the die roll. Here, X is a discrete random variable that can take on the values:

$$X \in \{1, 2, 3, 4, 5, 6\}.$$

☞Continuous Random Variable

Continuous Random Variable

Consider a random experiment where a dart is thrown at a circular target with a radius of 1 unit. The distance X from the center of the circle to the dart's landing point is a continuous random variable, as it can take any value between 0 and 1.

☞Activity 1

Problem: Flipping Two Coins

Consider a random experiment where two fair coins are flipped simultaneously. The sample space S for this experiment is:

$$S = \{HH, HT, TH, TT\},$$

where H represents heads and T represents tails.

☞Activity 1

Solution:

☞Activity 1

Solution:

Define a random variable X as the number of heads observed in the outcome. The random variable X assigns a numerical value to each outcome in the sample space as follows:

$$X(HH) = 2,$$
$$X(HT) = 1,$$
$$X(TH) = 1,$$
$$X(TT) = 0.$$

The range of X is:

$$\{0, 1, 2\}.$$

☞Activity 1

Solution:

The probabilities for each value of X can be calculated based on the outcomes:

$$P(X = 0) = P(\{TT\}) = \tfrac{1}{4},$$
$$P(X = 1) = P(\{HT, TH\}) = \tfrac{2}{4} = \tfrac{1}{2},$$
$$P(X = 2) = P(\{HH\}) = \tfrac{1}{4}.$$

☞ Activity 2

Rolling Two Dice

Consider the random experiment of rolling two six-sided dice simultaneously. The sample space S consists of all possible ordered pairs of outcomes:

$$S = \{(1,1),(1,2),\ldots,(6,6)\},$$

where each outcome represents the numbers appearing on the two dice.

☞ Activity 2

Rolling Two Dice

Solution: Define the random variable X as the **sum of the numbers rolled on the two dice**. For each outcome $(a, b) \in S$, the random variable X is given by:

$$X(a, b) = a + b,$$

where a and b are the numbers rolled on the first and second dice, respectively.
The possible values of X range from 2 (when both dice show 1) to 12 (when both dice show 6). Thus:

$$\text{Range of } X = \{2, 3, 4, \ldots, 12\}.$$

☞ Activity 2

Rolling Two Dice

☞ Activity 2

Rolling Two Dice

Solution: The probability of each value of X depends on the number of outcomes that result in that value. Since there are $6 \times 6 = 36$ equally likely outcomes, the probabilities are calculated as follows:

$$
\begin{aligned}
P(X = 2) &= \frac{1}{36} \quad \text{(only } (1,1)\text{)}, \\
P(X = 3) &= \frac{2}{36} = \frac{1}{18} \quad \text{(outcomes: } (1,2), (2,1)\text{)}, \\
P(X = 4) &= \frac{3}{36} = \frac{1}{12} \quad \text{(outcomes: } (1,3), (2,2), (3,1)\text{)}, \\
&\vdots \\
P(X = 12) &= \frac{1}{36} \quad \text{(only } (6,6)\text{)}.
\end{aligned}
$$

THANK YOU