Classification

OneR – Naïve Bayes

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Classification

Outlook	Temp	Humidity	Windy	Play
Sunny	Hot	High	False	No
Sunny	Hot	High	True	No
Overcast	Hot	High	False	Yes
Rainy	Mild	High	False	Yes
Rainy	Cool	Normal	False	Yes
Rainy	Cool	Normal	True	No
Overcast	Cool	Normal	True	Yes
Sunny	Mild	High	False	No
Sunny	Cool	Normal	False	Yes
Rainy	Mild	Normal	False	Yes
Sunny	Mild	Normal	True	Yes
Overcast	Mild	High	True	Yes
Overcast	Hot	Normal	False	Yes
Rainy	Mild	High	True	No

Outlook	Temp.	Humidity	Windy	Play
Sunny	Cool	High	True	?

Classification

- Classification:
  - predicts categorical class labels
  - classifies data (constructs a model) based on the training set and the values (class labels) in a classifying attribute and uses it in classifying new data

- Typical Applications
  - credit approval
  - target marketing
  - medical diagnosis
  - treatment effectiveness analysis

Classification—A Two-Step Process

- Model construction: describing a set of predetermined classes
  - Each tuple/sample is assumed to belong to a predefined class, as determined by the class label attribute
  - The set of tuples used for model construction: training set
  - The model is represented as classification rules, decision trees, or mathematical formulae
- Model usage: for classifying future or unknown objects
  - Estimate accuracy of the model
    - The known label of test sample is compared with the classified result from the model
    - Accuracy rate is the percentage of test set samples that are correctly classified by the model
    - Test set is independent of training set, otherwise over-fitting will occur

Classification Process (1): Model Construction

Training Data

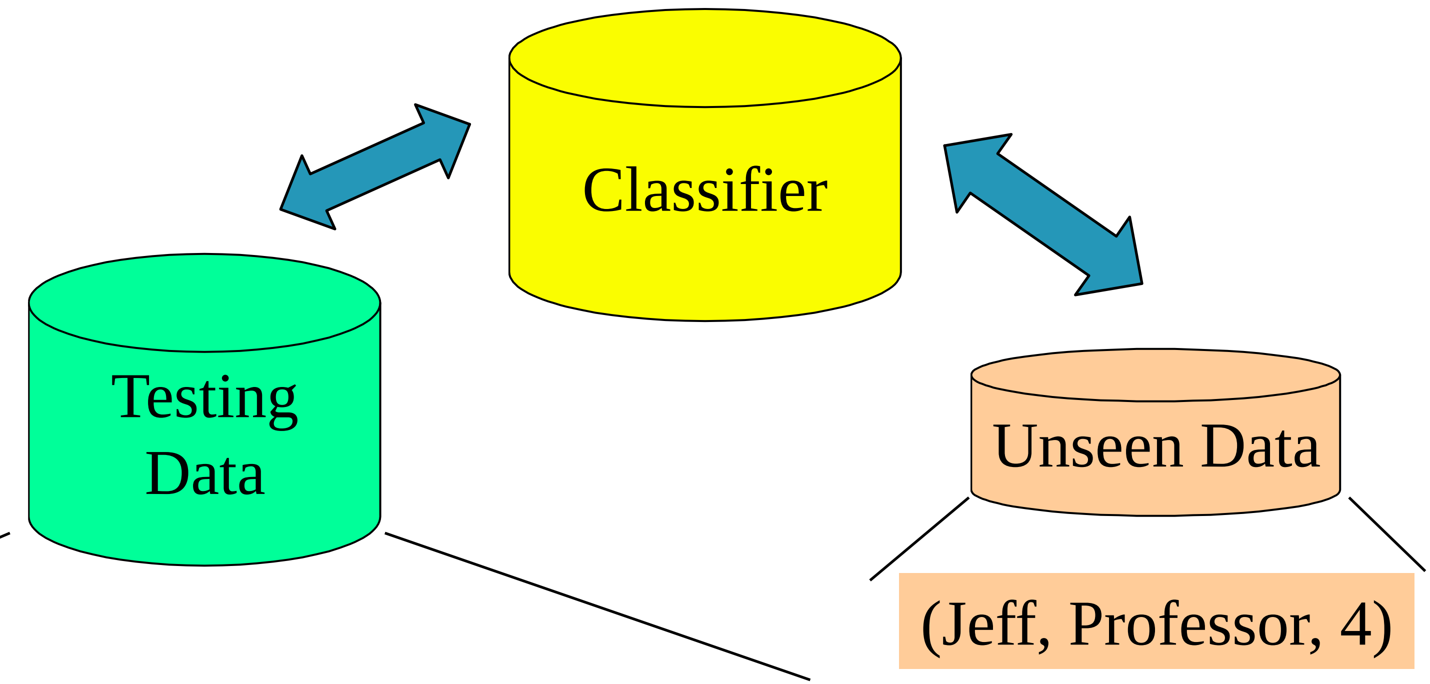
Classification Algorithms

NAME	RANK	YEARS	TENURED
Mike	Assistant Prof	3	no
Mary	Assistant Prof	7	yes
Bill	Professor	2	yes
Jim	Associate Prof	7	yes
Dave	Assistant Prof	6	no
Anne	Associate Prof	3	no

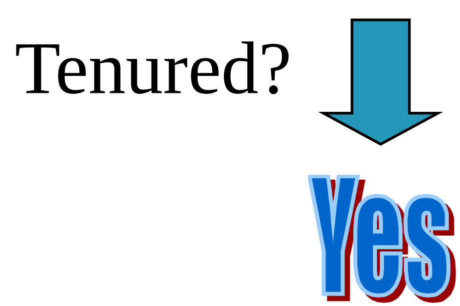
Classifier (Model)

IF rank = 'professor'
OR years > 6
THEN tenured = 'yes'

Classification Process (2): Use the Model in Prediction



NAME	RANK	YEARS	TENURED
Tom	Assistant Prof	2	no
Merlisa	Associate Prof	7	no
George	Professor	5	yes
Joseph	Assistant Prof	7	yes



Supervised vs. Unsupervised Learning

- Supervised learning (classification)
  - Supervision: The training data (observations, measurements, etc.) are accompanied by labels indicating the class of the observations
  - New data is classified based on the training set
- Unsupervised learning (clustering)
  - The class labels of training data is unknown
  - Given a set of measurements, observations, etc. with the aim of establishing the existence of classes or clusters in the data

Issues regarding classification and prediction (1): Data Preparation

- Data cleaning
  - Preprocess data in order to reduce noise and handle missing values
- Relevance analysis (feature selection)
  - Remove the irrelevant or redundant attributes
- Data transformation
  - Generalize and/or normalize data

Issues regarding classification and prediction (2): Evaluating Classification Methods

- Predictive accuracy
- Speed and scalability
  - time to construct the model
  - time to use the model
- Robustness
  - handling noise and missing values
- Scalability
  - efficiency in disk-resident databases
- Interpretability:
  - understanding and insight provided by the model
- Goodness of rules
  - decision tree size
  - compactness of classification rules

Simplicity first: 1R

- Simple algorithms often work very well!
- There are many kinds of simple structure, eg:
  - One attribute does all the work
  - All attributes contribute equally & independently
  - A weighted linear combination might do
  - Instance-based: use a few prototypes
  - Use simple logical rules

- Success of method depends on the domain

Inferring rudimentary rules

- 1R: learns a 1-level decision tree
  - I.e., rules that all test one particular attribute
- Basic version
  - One branch for each value
  - Each branch assigns most frequent class
  - Error rate: proportion of instances that don't belong to the majority class of their corresponding branch
  - Choose attribute with lowest error rate

(*assumes nominal attributes*)

Pseudo-code for 1R

For each attribute,

For each value of the attribute, make a rule as follows:

count how often each class appears

find the most frequent class

make the rule assign that class to this attribute-value

Calculate the error rate of the rules

Choose the rules with the smallest error rate

- Note: “missing” is treated as a separate attribute value

Evaluating the weather attributes

Outlook	Temp	Humidity	Windy	Play
Sunny	Hot	High	False	No
Sunny	Hot	High	True	No
Overcast	Hot	High	False	Yes
Rainy	Mild	High	False	Yes
Rainy	Cool	Normal	False	Yes
Rainy	Cool	Normal	True	No
Overcast	Cool	Normal	True	Yes
Sunny	Mild	High	False	No
Sunny	Cool	Normal	False	Yes
Rainy	Mild	Normal	False	Yes
Sunny	Mild	Normal	True	Yes
Overcast	Mild	High	True	Yes
Overcast	Hot	Normal	False	Yes
Rainy	Mild	High	True	No

Attribute	Rules	Errors	Total errors
Outlook	Sunny → No	2/5	4/14
	Overcast → Yes	0/4	
	Rainy → Yes	2/5	
Temp	Hot → No*	2/4	5/14
	Mild → Yes	2/6	
	Cool → Yes	1/4	
Humidity	High → No	3/7	4/14
	Normal → Yes	1/7	
Windy	False → Yes	2/8	5/14
	True → No*	3/6	

* indicates a tie

Using Rules

Attribute	Rules
Outlook	Sunny $\rightarrow$ No
	Overcast $\rightarrow$ Yes
	Rainy $\rightarrow$ Yes

- A new day:

Outlook	Temp.	Humidity	Windy	Play
Sunny	Cool	High	True	?

Using Rules

Attribute	Rules
Outlook	Sunny $\rightarrow$ No
	Overcast $\rightarrow$ Yes
	Rainy $\rightarrow$ Yes

- A new day:

Outlook	Temp.	Humidity	Windy	Play
Sunny	Cool	High	True	No

Dealing with numeric attributes

Outlook	Temperature	Humidity	Windy	Play
Sunny	85	85	False	No
Sunny	80	90	True	No
Overcast	83	86	False	Yes
Rainy	75	80	False	Yes
…	…	…	…	…

- Discretize numeric attributes
- Divide each attribute's range into intervals
  - Sort instances according to attribute's values
  - Place breakpoints where the class changes (the majority class)
  - This minimizes the total error

- Example: *temperature* from weather data

```
64    65    68   69   70    71   72    72   75   75    80    81    83    85
Yes | No | Yes  Yes  Yes | No   No | Yes  Yes  Yes | No | Yes   Yes | No
```

The problem of overfitting

- This procedure is very sensitive to noise
  - One instance with an incorrect class label will probably produce a separate interval

- Simple solution:
  enforce minimum number of instances in majority class per interval

Discretization example

- Example (with min = 3):

64		65		68	69	70		71	72		72	75	75		80		81	83		85
Yes	~~|~~	No	~~|~~	Yes	Yes	Yes	|	No	No	~~|~~	Yes	Yes	Yes	|	No	~~|~~	Yes	Yes	~~|~~	No

- Final result for temperature attribute

64	65	68	69	70		71	72	72	75	75		80	81	83	85
Yes	No	Yes	Yes	Yes	~~|~~	No	No	Yes	Yes	Yes	|	No	Yes	Yes	No

Yes | No

With overfitting avoidance

- Resulting rule set:

Attribute	Rules	Errors	Total errors
Outlook	Sunny → No	2/5	4/14
	Overcast → Yes	0/4	
	Rainy → Yes	2/5	
Temperature	≤ 77.5 → Yes	3/10	5/14
	> 77.5 → No*	2/4	
Humidity	≤ 82.5 → Yes	1/7	3/14
	> 82.5 and ≤ 95.5 → No	2/6	
	> 95.5 → Yes	0/1	
Windy	False → Yes	2/8	5/14
	True → No*	3/6	

Bayesian (Statistical) modeling

- "Opposite" of 1R: use all the attributes
- Two assumptions: Attributes are
  - *equally important*
  - *statistically independent* (given the class value)
    - I.e., knowing the value of one attribute says nothing about the value of another
      (if the class is known)
- Independence assumption is almost never correct!
- But … this scheme works well in practice

Probabilities for weather data

Outlook			Temperature			Humidity			Windy			Play	
	Yes	*No*		*Yes*	*No*		*Yes*	*No*		*Yes*	*No*	*Yes*	*No*
Sunny			Hot			High			False				
Overcast			Mild			Normal			True				
Rainy			Cool										
Sunny			Hot			High			False				
Overcast			Mild			Normal			True				
Rainy			Cool										

Outlook	Temp	Humidity	Windy	Play
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Sunny	Hot	High	True	No
Overcast	Hot	High	False	Yes
Rainy	Mild	High	False	Yes
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Sunny	Mild	High	False	No
Sunny	Cool	Normal	False	Yes
Rainy	Mild	Normal	False	Yes
Sunny	Mild	Normal	True	Yes
Overcast	Mild	High	True	Yes
Overcast	Hot	Normal	False	Yes
Rainy	Mild	High	True	No

Probabilities for weather data

Outlook			Temperature			Humidity			Windy			Play	
	Yes	*No*		*Yes*	*No*		*Yes*	*No*		*Yes*	*No*	*Yes*	*No*
Sunny	2	3	Hot	2	2	High	3	4	False	6	2	9	5
Overcast	4	0	Mild	4	2	Normal	6	1	True	3	3		
Rainy	3	2	Cool	3	1								
Sunny	2/9	3/5	Hot	2/9	2/5	High	3/9	4/5	False	6/9	2/5	9/14	5/14
Overcast	4/9	0/5	Mild	4/9	2/5	Normal	6/9	1/5	True	3/9	3/5		
Rainy	3/9	2/5	Cool	3/9	1/5								

Outlook	Temp	Humidity	Windy	Play
Sunny	Hot	High	False	No
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Rainy	Cool	Normal	False	Yes
Rainy	Cool	Normal	True	No
Overcast	Cool	Normal	True	Yes
Sunny	Mild	High	False	No
Sunny	Cool	Normal	False	Yes
Rainy	Mild	Normal	False	Yes
Sunny	Mild	Normal	True	Yes
Overcast	Mild	High	True	Yes
Overcast	Hot	Normal	False	Yes
Rainy	Mild	High	True	No

Probabilities for weather data

Outlook			Temperature			Humidity			Windy			Play	
	Yes	*No*		*Yes*	*No*		*Yes*	*No*		*Yes*	*No*	*Yes*	*No*
Sunny	2	3	Hot	2	2	High	3	4	False	6	2	9	5
Overcast	4	0	Mild	4	2	Normal	6	1	True	3	3		
Rainy	3	2	Cool	3	1								
Sunny	2/9	3/5	Hot	2/9	2/5	High	3/9	4/5	False	6/9	2/5	9/14	5/14
Overcast	4/9	0/5	Mild	4/9	2/5	Normal	6/9	1/5	True	3/9	3/5		
Rainy	3/9	2/5	Cool	3/9	1/5								

- A new day:

Outlook	Temp.	Humidity	Windy	Play
Sunny	Cool	High	True	?

Bayes's rule

- Probability of event *H* given evidence *E* :

$$\Pr[H \mid E] = \frac{\Pr[E \mid H]\Pr[H]}{\Pr[E]}$$

- *A priori* probability of *H* :
  - Probability of event *before* evidence is seen $\Pr[H]$
- *A posteriori* probability of *H* :
  - Probability of event *after* evidence is seen $\Pr[H \mid E]$

from Bayes "Essay towards solving a problem in the doctrine of chances" (1763)

Thomas Bayes

Born: 1702 in London, England
Died: 1761 in Tunbridge Wells, Kent, England

Naïve Bayes for classification

- Classification learning: what's the probability of the class given an instance?
  - Evidence E = instance
  - Event H = class value for instance
- Naïve assumption: evidence splits into parts (i.e. attributes) that are *independent*

$$\Pr[H \mid E] = \frac{\Pr[E_1 \mid H]\Pr[E_2 \mid H]\ldots\Pr[E_n \mid H]\Pr[H]}{\Pr[E]}$$

Weather data example

Outlook	Temp.	Humidity	Windy	Play
Sunny	Cool	High	True	?

Evidence E

$$\Pr[yes \mid E] = \Pr[Outlook = Sunny \mid yes]$$
$$\times \Pr[Temperature = Cool \mid yes]$$
$$\times \Pr[Humidity = High \mid yes]$$
$$\times \Pr[Windy = True \mid yes]$$
$$\times \frac{\Pr[yes]}{\Pr[E]}$$

Probability of class "yes"

$$= \frac{\frac{2}{9} \times \frac{3}{9} \times \frac{3}{9} \times \frac{3}{9} \times \frac{9}{14}}{\Pr[E]}$$

Probabilities for weather data

Outlook			Temperature			Humidity			Windy			Play	
	Yes	*No*		*Yes*	*No*		*Yes*	*No*		*Yes*	*No*	*Yes*	*No*
Sunny	2	3	Hot	2	2	High	3	4	False	6	2	9	5
Overcast	4	0	Mild	4	2	Normal	6	1	True	3	3		
Rainy	3	2	Cool	3	1								
Sunny	2/9	3/5	Hot	2/9	2/5	High	3/9	4/5	False	6/9	2/5	9/14	5/14
Overcast	4/9	0/5	Mild	4/9	2/5	Normal	6/9	1/5	True	3/9	3/5		
Rainy	3/9	2/5	Cool	3/9	1/5								

- A new day:

Outlook	Temp.	Humidity	Windy	Play
Sunny	Cool	High	True	?

Likelihood of the two classes

For "yes" = $2/9 \times 3/9 \times 3/9 \times 3/9 \times 9/14 = 0.0053$

For "no" = $3/5 \times 1/5 \times 4/5 \times 3/5 \times 5/14 = 0.0206$

Conversion into a probability by normalization:

P("yes") = 0.0053 / (0.0053 + 0.0206) = 0.205

P("no") = 0.0206 / (0.0053 + 0.0206) = 0.795

The "zero-frequency problem"

- What if an attribute value doesn't occur with every class value?
(e.g. "Outlook = Overcast" for class "no")
  - Probability will be zero! $\Pr[Outlook = Overcast \mid no] = 0$
  - *A posteriori* probability will also be zero!
(No matter how likely the other values are!) $\Pr[yes \mid E] = 0$

- Remedy: add 1 to the count for every attribute value-class combination (*Laplace estimator)*

- Result: probabilities will never be zero!
(also: stabilizes probability estimates)

*Modified probability estimates

- In some cases adding a constant different from 1 might be more appropriate
- Example: attribute *outlook* for class *yes*

$$\frac{2+\mu/3}{9+\mu} \qquad \frac{4+\mu/3}{9+\mu} \qquad \frac{3+\mu/3}{9+\mu}$$

Sunny ***Overcast*** ***Rainy***

- Weights don't need to be equal (but they must sum to 1)

$$\frac{2+\mu p_1}{9+\mu} \qquad \frac{4+\mu p_2}{9+\mu} \qquad \frac{3+\mu p_3}{9+\mu}$$

Missing values

- Training: instance is not included in frequency count for attribute value-class combination
- Classification: attribute will be omitted from calculation
- Example:

Outlook	Temp.	Humidity	Windy	Play
Null	Cool	High	True	?

Likelihood of "yes" = $3/9 \times 3/9 \times 3/9 \times 9/14 = 0.0238$

Likelihood of "no" = $1/5 \times 4/5 \times 3/5 \times 5/14 = 0.0343$

P("yes") = $0.0238 / (0.0238 + 0.0343) = 41\%$

P("no") = $0.0343 / (0.0238 + 0.0343) = 59\%$

Numeric attributes

- Usual assumption: attributes have a *normal* or *Gaussian* probability distribution (given the class)
- The *probability density function* for the normal distribution is defined by two parameters:
  - *Sample mean* μ

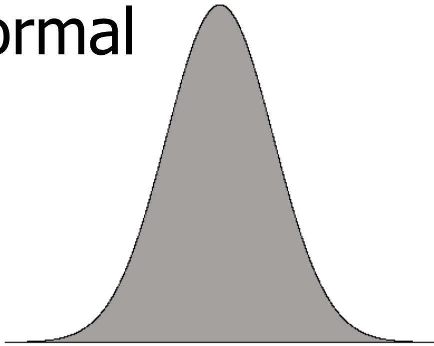
$$\mu = \frac{1}{n}\sum_{i=1}^{n} x_i$$

  - *Standard deviation* σ

$$\sigma^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \mu)^2$$

  - Then the density function *f(x) is*

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$



Karl Gauss, 1777-1855
great German
mathematician

Statistics for weather data

Outlook			Temperature		Humidity		Windy			Play	
	Yes	*No*	*Yes*	*No*	*Yes*	*No*		*Yes*	*No*	*Yes*	*No*
Sunny	2	3	64, 68,	65, 71,	65, 70,	70, 85,	False	6	2	9	5
Overcast	4	0	69, 70,	72, 80,	70, 75,	90, 91,	True	3	3		
Rainy	3	2	72, …	85, …	80, …	95, …					
Sunny	2/9	3/5	μ =73	μ =75	μ =79	μ =86	False	6/9	2/5	9/14	5/14
Overcast	4/9	0/5	σ =6.2	σ =7.9	σ =10.2	σ =9.7	True	3/9	3/5		
Rainy	3/9	2/5									

- Example density value:

$$f(temperature = 66 \mid yes) = \frac{1}{\sqrt{2\pi}6.2} e^{-\frac{(66-73)^2}{2*6.2^2}} = 0.0340$$

Classifying a new day

- A new day:

Outlook	Temp.	Humidity	Windy	Play
Sunny	66	90	true	?

Likelihood of "yes" = $2/9 \times 0.0340 \times 0.0221 \times 3/9 \times 9/14 = 0.000036$

Likelihood of "no" = $3/5 \times 0.0291 \times 0.0380 \times 3/5 \times 5/14 = 0.000142$

P("yes") = 0.000036 / (0.000036 + 0. 000142) = 20.25%

P("no") = 0.000142 / (0.000036 + 0. 000142) = 79.75%

- Missing values during training are not included in calculation of mean and standard deviation

Naïve Bayes: discussion

- Naïve Bayes works surprisingly well (even if independence assumption is clearly violated)
- Why? Because classification doesn't require accurate probability estimates *as long as maximum probability is assigned to correct class*
- However: adding too many redundant attributes will cause problems (e.g. identical attributes)
- Note also: many numeric attributes are not normally distributed .

Naïve Bayes Extensions

- Improvements:
  - select best attributes (e.g. with greedy search)
  - often works as well or better with just a fraction of all attributes

Summary

- OneR – uses rules based on just one attribute
- Naïve Bayes – use all attributes and Bayes rules to estimate probability of the class given an instance.
- Simple methods frequently work well, but …
  - Complex methods can be better (as we will see)