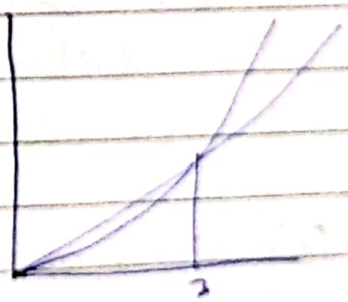
Tushar Singh
11912035
IT Branch

DS Assignment

Q1 $O(n^2)$
$t = 5$ sec
input $n = 10$
Time complexity is $O(n^2)$
$t \propto n^2$
$\frac{t_1}{t_2} = \left(\frac{n_1}{n_2}\right)^2 = \left(\frac{10}{50}\right)^2$

$\frac{5}{t_2} = \frac{100}{25} \Rightarrow t_2 = 125$ sec

Q2 $T_A(n) = n^3$
$T_B(n) = 2n^2$
$n^3 = 2n^2$
$n^3 - 2n^2 = 0$
$n^2(n-2) = 0$
~~$n = 0$~~, $n = 2$,
Break point



Q3 $n2^n = O(4^n)$
let $f(n) = n2^n$
$\lim_{n \to \infty} \frac{f(x)}{g(x)} \to \lim_{n \to \infty} \frac{n2^n}{4^n} \to \lim_{n \to \infty} \frac{n2^n}{2^{2n}} = \frac{n}{2^n}$

Q5 Θ = avg case
O = worst case

Θ : asymptotically bounded by upper side,
O = " " " lower side

Everything ~~...~~ above Θ is O but ~~...~~ opposite not true.
$\Theta \rightarrow$ give avg time of algorithm hence more informative

b) O is bounded from above until Ω is bounded from below.
O tell worst case ~~...~~ for which algorithm must not take more time than this.

Ω take more time as describe for algorithm
Ex $\rightarrow$ $O(n^2)$ vs $\Omega(n^2)$
$O(n^2) \Rightarrow$ take time till n^2.
$\Omega(n^2) \Rightarrow$ take time more than n^2

Q6 $n^4 + \log n + 17$ is $O(n^4)$

n	n^4	$\log n$	17	$f(n)$
1	1	0	17	17
2	8	$\log 2$	17	≈ 256
10	$(10)^4$	$4\log 10$	17	≈ 10017
100	$(100)^4$	$4\log 100$	17	$\approx (100)^4 + 216$
10000	$(10000)^4$	$4\log 10000$	17	$\approx (10000)^4 + 36.21$

classmate

Date
Page

so we can say n^4 dominate, therefore we can ignore n, $\log n$, 17 & denote eq. ~~by~~ $f(n)$ by $O(n^4)$

Q) a) $K = 1$

while $(K \le n)$

$K = K+1$

End while

$n = 0$

while $(K \le n)$	$K = K+1$	$K = K+1$
1	2	1
2	3	2
3	4	3
4	5	4
5	6	5
6	7	6
7		

no of step $= n+1$ Total $2(n+1)$

b) for $i = 1$ to $n-1$

for $j = 1+i$ to n

swap

end for

end for

Soln first loop $i = 1$

take 1 step

J will execute $(n-1)$ time

For outer loop $i = 1$

$i < n-1$ execute n time

Scanned with CamScanner

$J = i+1$ execute $(n-1)$ tim

i	j	$j \le n$	$j++$	swap
		n	$n-1$	1
1	2		⋮	1
2	3	$n-1$	⋮	1
3	4	$n-2$	⋮	1
⋮	⋮	⋮	⋮	1
$n-1$	n	2	1	

$1+2+3 \cdots n-1$

$J++ = \frac{n(n-1)}{2}$ swap $\frac{n(n-1)}{2}$

Total steps:

$1+n+(n-1)+(n-1)+\frac{n(n+1)}{2}-1$

$\cancel{3n-2} + \frac{n(n+1)}{2} + n(n-1)$

$2(n-1) + \frac{n}{2} + \frac{3n^2}{2}$

$\lim_{n\to\infty} \frac{T_A}{T_B} \quad \lim_{n\to\infty} \frac{100^n}{n^4}$

$\lim_{n\to\infty} \frac{n\,100^{n-1}}{4n^3}$

$\lim_{n\to\infty} \frac{n(n-1)\,100^{n-2}}{12n^2}$

$\Rightarrow \lim \frac{n(n-1)(n-2)\,100^{n-3}}{24n}$

$\lim_{n\to\infty} \frac{n(n-1)(n-2)(n-3)\,100^{n-4}}{24}$

n^n grows faster than 100^n

Q4 log fun. grow at slower rate but not slowest

let fn be n^α

$\lim_{n\to\infty} \frac{\log n}{n^\alpha} = \lim_{n\to\infty} \frac{1}{n\alpha n^{\alpha-1}} = \lim_{n\to\infty} \frac{1}{\alpha n^\alpha} = 0$

$\Rightarrow$ for $n\to\infty$ fn grows slow but fast than $\sqrt{\log x}$

$\Rightarrow$ take $\log_x x$ & $\log_e x$

$\lim_{n\to\infty} \frac{\log_x n}{\log_e n}$ as $\frac{\frac{1}{x\log x}}{\frac{1}{\log_e x}} = \frac{1}{x}$

So we can say log fun. are equal

Scanned with CamScanner

classmate
Date
Page

Q10 $\lim_{n \to \infty} \frac{\log(n!)}{n \log n}$

$\log n! = \log 1 + \log 2 + \ldots \log n$

$\sum_{K=1}^{n} = \log K$

$\int_1^K \ln x \, dx = (x \log x - x) \Big]_1^n$

$\lim_{n \to \infty} \frac{n \log x - x + 1}{n \log x} = 1 - \lim_{n \to \infty} \frac{1}{\ln x} - \lim_{n \to \infty}$

$\Rightarrow \frac{1}{\ln x}$

Q12 $2^{n-1} + 4^{n-1}$

θ is the avg of algorithm which bound the fun.

let 2^{n-1} & 4^{n+1} be two fn f & g

θ of fn ln b/w f & g

$f = 2^{n-1}$ $g = 4^{n+1}$ $= 2^{2(n+1)}$

$f = 2^{n-1}$ $g = 2^{2n+2}$

$\theta = 2^K$ where K, $n-1 < K < 2n+2$

Q11 $T_A = n^2$

$T_B = n+2$

Break pt :

$n^2 = n+2$

$n^2 - n - 2 = 0$

$n = 2, -1$

Break pt