Chapter 3 Exercise

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3.1) 2n+1

3.3) Assume for a base case of n = m+1 indicating that there is more than 1 object in the set of holes we can then use induction step showing that n = k(m + 1) where the k is a positive integer. Then we can use induction hypothesis for n = k(m+1)-m. Now if we add m + 1 to the existing configurations the k boxes will contain more then the m boxes. Finally distribute the m + 1 into the m boxes indicating that the extended pigeonhole principle is true showing there is more than one pigeon in the hole.

3.5) Assume $\sum_{i=0}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$ if we set the base case of P(0) which would result as true making both sides equal to 0. Then we can show that P(n+1) while also return true proving that for n is greater than or equal to 0.

3.7) Given $\sum_{i=0}^{n} i^3 = (\sum_{i=0}^{n})^2$ assume n is 0 it will results in both being equal to 0 thus, it's true. In addition, we can set the equal as $1^2 + 2^2 + \cdots + (n+1)^2 = (1{+}2{+}3{+}\cdots{+}(n{+}1))^3$ and then we can prove that n is greater than or equal to 0.

3.9)For subset A = h1, h2, ... , hn, the induction hypothesis indeed applies because it's a set of size n, and you assume that all horses in a set of size n are the same color. In addition. subset B = h2, h3, ... , hn+1, it's not a set of size n but rather a set of size n + 1. Therefore, the induction hypothesis does not directly apply to this subset.