Contents

1. Introduction

This report is the first in a series detailing the design of an autonomous line following buggy. An integral part of this are the systems by which it is driven, and this report describes the experimentation and calculations involved, as well as determining an appropriate gearbox. This investigation involves quantifying the torque generated by the motor, calculating the values of K_t and K_E, and examining the force needed to initiate and maintain motion on both a flat surface and an 18° incline.

The primary goal is to find the gearbox ratio, or torque multiplier, necessary for the buggy to navigate the test track successfully. The supplied motors have a maximum current of 1.4 A [1], which is insufficient to generate the required torque for initiating and maintaining motion. Therefore, it is crucial to select a gearbox that can multiply the torque to a sufficient level at the expense of angular velocity. This experiment explores various electrical and mechanical aspects of the motor and considers the relevant properties of the buggy.

Examining the electrical side, the speed and torque of the buggy are primarily controlled by voltage and current, with a maximum current of 1.4 A. However, the microcontroller cannot directly power the motor due to its limited current supply. Therefore, a motor driver board is necessary to act as an interface between the microcontroller and the motor. The driver board will enhance wheel rotation, enabling the buggy to move faster. The microcontroller will communicate with the motor by providing a Pulse Width Modulation (PWM) signal, which will be converted into an analogue voltage by the motor driver board [1]. Modulating the duty cycle of the PWM signal allows dynamic interaction between the microcontroller and the motor driver, enabling the adjustment of motor speed in response to programmed situational variations.

On the mechanical side, the advantages of using a gearbox are immediately evident, as it allows the buggy to overcome the inclined surface by multiplying the torque. However, this increased torque comes with trade-offs in both angular velocity and overall efficiency. Notably, the efficiency of the gearbox is estimated at 85% [1] for each stage. In the case of a two-stage gearbox, this efficiency further decreases to approximately 72.25%.

2. Motor characterisation

This section aims to calculate the constants K_T and K_E, the relationship between electrical parameters - voltage and current - and mechanical parameters - angular speed and torque. These values make it possible to predict how the motor will respond to different electrical signals, which is integral to the control of the buggy. K_T and K_E can be calculated using the motor torque equation (1) and the back EMF equation (2)[1]:

$$T = K_T I \tag{1}$$

$$E_{emf} = K_E \omega \tag{2}$$

where T is torque, K_T is the torque constant, I is current, E_{emf} is back EMF, K_E is EMF constant, and ω is the angular speed of rotation.

Equations (1) and (2) are derived from (3) and (4) that relate back EMF and force to flux density (B), length of armature (l), and velocity (v) [1].

$$E_{emf} = Blv \tag{3}$$

$$F = BIl \tag{4}$$

The magnetic flux density is fixed as the magnets used in these motors are permanent, and the length of armature and distance between windings are also fixed values; therefore, K_T and K_E are just used to combine these constants. K_T specifically relates the motor's torque to the current going through it whereas K_E relates the back EMF to the angular velocity of the motor shaft. Ideally, the values of K_T and K_E would be the same [1].

The first experiment measured the voltage and current when the motor was stalled. This means that the back EMF will equal zero as there was no angular velocity so the voltage across the motor is equal to the brush voltage (V_b) and motor resistance (R) multiplied by the current. Taking readings at different voltage values and plotting the results shows the motor resistance in the form of the gradient.

$$V = IR + V_b \tag{5}$$

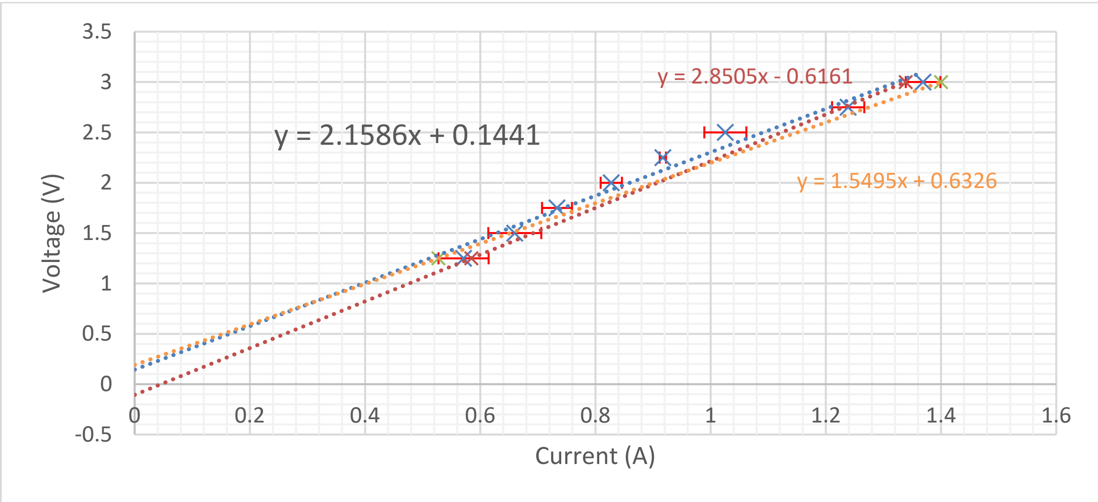

Fig. 1. Graph showing voltage against current for the stalled motor and the maximum and minimum gradients.

From Fig.1, the motor resistance is 2.2 ± 0.7 Ω and the maximum voltage at a current of 1.4 A is 3 V.

For the next experiment, a constant voltage of 5 V was applied across the motor, and a piece of string attached to two strain gauges. Overcurrent protection was set at 1.4 A and the friction torque of the string on the motor shaft was increased until the current through the motor reached 1.4 A. The torque produced by the motor was the difference between the strain gauges multiplied by the radius of the motor shaft. The gradient of Fig. 2. is the estimate for K_T which is 0.0074.

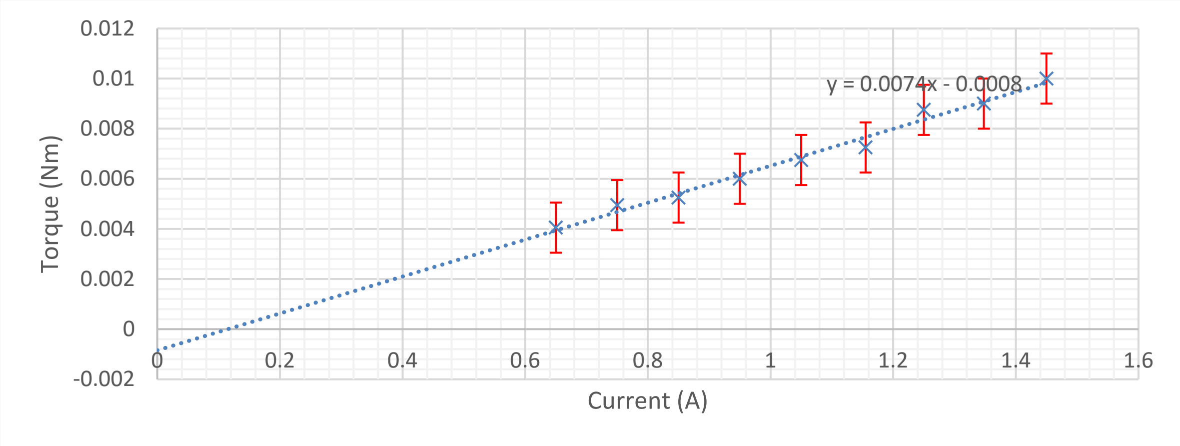

Fig. 2. Graph showing torque against current for constant motor voltage.

The voltage drop across motor resistance and brush voltage calculated earlier, along with the voltage dropped across the power supply leads, can be subtracted from the voltage across the motor to produce a value for the back EMF. The power lead resistance was measured to be an average of 50 mΩ As stated in equation (2), angular velocity is required to calculate K_E, so during the experiment, the motor shaft was measured by a tachometer. Plotting the back EMF against this gives us a value for K_E of 0.0094.

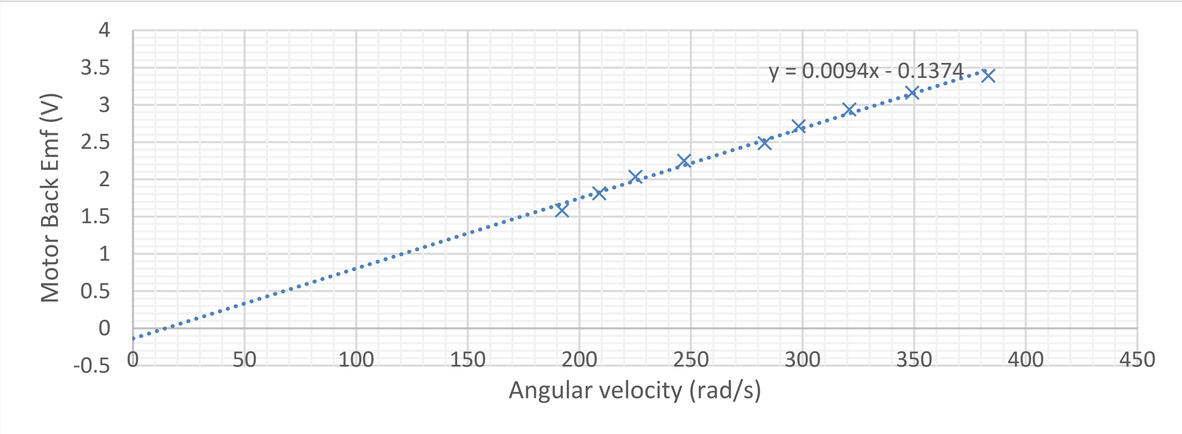

Fig. 3. Graph showing back EMF against angular velocity for constant voltage test.

To get another estimate for K_T, the values measured previously were reproduced but with the friction torque holding the motor stalled. The voltage was dropped to reduce the current below the overcurrent threshold. Plotting torque against current again gave a K_T value of 0.0063. The voltage - current graph was also plotted again to get new estimates for back voltage and motor resistance, which were 0.32 V and 2.6 Ω, respectively.

K_E depends on the value of motor resistance, which suffered from sizeable random error. Calculating K_E also relied on more measured values than the K_T calculations, amplifying the error in the estimate. This error could explain why K_E was much larger than the K_T values and would mean that the value calculated for K_T will be more accurate. By the time the values used to calculate the second estimate for K_T were obtained, the motor was getting very hot due to the high currents being passed through it. This would make the K_T value lower which is also reflected in the second estimate for motor resistance which is higher. Therefore, the first value of K_T (0.0074) will be the most appropriate value of the motor constant in further calculation.

3. Load measurements

The gearbox selected at the end of this design report should produce enough torque to drive the buggy up a gradient of up to 18°, but also attempt to maximise its speed. Therefore, this section aims to estimate the greatest amount of force opposing the motion of the buggy, so that it can be compared to the output torque of the three gearbox options. The gearbox with the lowest gear ratio while still producing enough force will be selected, as a lower gear ratio will increase the velocity of the buggy on the flat runs of the track.

The estimated weight of the parts of the buggy combined was 986 g [1]:

Table 1 Estimated weight of the parts of the buggy

Test Chassis	Batteries	Motor Drive Board	Microcontroller Breakout board	Microcontroller board with stacking mount	Total
560 g	265 g	53 g	38 g	70 g	986 g

However, the chassis weight can easily vary, and additional components may be added. Therefore, force tests were conducted with buggy weights of 1.3 kg, 1.5 kg and 1.7 kg to provide flexibility in the buggy design. One such test was attaching a force gauge to the test buggy and pulling it up a slope. This test was flawed as it was difficult to move the buggy at a constant speed due to the uneven slope, resulting in the cart accelerating and decelerating and the force measurement fluctuating. Additionally, the force gauge used was only precise to the nearest 40 g and at points the track exceeded the 18° specification, reaching a 21° incline. This test was repeated on a flat surface, and while this test suffered some of the inaccuracies of the slope test, the flat surface made it easier to pull the buggy at a constant speed, and the lower force required to pull the buggy meant that a more precise force gauge could be used. The results from these tests are shown in the Table 2.

Table 2 Static and rolling forces on flat surface and on slope for different masses

Mass of Buggy (kg)	Static Force on Flat Surface (N)	Rolling Force on Flat Surface (N)	Rolling Force on 18° Slope (N)
1.304	1.0	0.7	4.7
1.509	1.4	1.1	5.6
1.713	1.5	1.3	6.1

From these values, the coefficients of rolling and static friction, C_f, can be calculated using the following equations [1]:

$$C_f = \frac{F}{mg} \tag{6}$$

$$C_f = \frac{F - mg\sin(\theta)}{mg\cos(\theta)} \tag{7}$$

Where m is mass of buggy, g is gravitational constant, θ is angle of slope.

Using these equations produces the results shown in the Table 3.

Table 3 Coefficient of friction of different masses for static and rolling forces on flat surface and on slope

Mass of Buggy (kg)	C_f on Flat Surface (Static)	C_f on Flat Surface (Rolling)	C_f on 18° Slope (Rolling)
1.304	0.078	0.055	0.061
1.509	0.095	0.074	0.073
1.713	0.089	0.077	0.057

The situation that produces the largest amount of force opposing forward motion is if the buggy is static on a slope, the free body diagram is shown in Fig. 4.

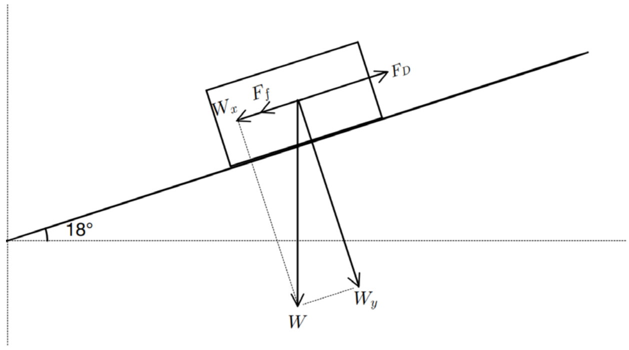

Fig. 4. A free body diagram showing the forces acting on the buggy.

The calculated values for these forces are shown for the different weights in the Table 4.

Table 4 Component of forces for different masses

Mass of Buggy (kg)	Weight force acting down the slope, W_x(N)	Normal Force, -W_y (N)	Static Friction Force, F_f (N)	Total Force opposing motion:
1.3	3.94	12.12	0.95	4.89
1.5	4.54	13.98	1.33	5.87
1.7	5.15	15.84	1.41	6.56

Air resistance is negligible as the cross-sectional area and velocity are small. To find the required torque, the equation relating force and distance from pivot to torque can be used [1]:

$$T = \frac{Fr}{2} \tag{8}$$

Using this equation and considering a wheel radius of 35 mm, the required torque from each gearbox would be 0.086 Nm, 0.102 Nm and 0.115 Nm for 1.3 kg, 1.5 kg and 1.7 kg, respectively.

4. Gear ratio selection

A critical design decision in achieving an autonomous line following buggy is the gearbox, as specific characteristics must be prioritised over others. This section explores those decisions and how the selected gearbox best works to achieve the project's aims.

As previous sections of this report have derived K_T and the maximum required torque, it is possible to calculate the current required to drive the buggy up an incline without a gearbox.

Equation (9) shows the trend line of Fig. 2 and can be used to calculate the maximum required current as 11.71 A.

$$T = 0.0074I - 0.0008 \tag{9}$$

Where T is torque, and I is current.

Similarly, the relationship between current and torque is described by the trend line of Fig. 1, equation (10), giving a maximum required voltage of 25.43 V.

$$V = 2.159I + 0.1441 \tag{10}$$

This is problematic for several reasons, most concerningly that the required voltage is higher than the batteries can supply, and the required current is higher than the maximum safe current, i.e., the motors would overheat. As the motor cannot safely or practically drive the buggy up an incline, a gearbox is necessary. The maximum torque that can be safely provided is calculated by substituting the maximum safe current, 1.4 A, into (9). These calculations give a maximum safe torque of 0.00956 Nm.

A gearbox will increase the torque from the motors at the expense of motor speed. As one of the design requirements for the project is to complete tasks in the fastest possible time, this must be weighed against providing the required torque. Furthermore, and the available gearboxes are assumed only to be 85% efficient per stage [1]. The gearbox must have a ratio of at least 9.00, assuming overall efficiency of 72.25% and a buggy mass of 1.3 kg.

Table 5 Gear ratios, wheel torque and gears of the available gearboxes [1][2]

Gearbox	Gear 1	Gear 2	Gear 3	Ideal Gear Ratio	Practical Gear Ratio	Wheel Torque
1	16	48/12	48	$\frac{48 \times 48}{12 \times 16} = 12.0$	$12 \times 72.3\% = 8.67$	0.0829 Nm
2	16	50/10	48	$\frac{48 \times 50}{10 \times 16} = 15.0$	$15 \times 72.3\% = 10.8$	0.104 Nm
3	16	50/10	60	$\frac{60 \times 50}{10 \times 16} = 18.75$	$18.75 \times 72.3\% = 13.5$	0.130 Nm

Gearbox 1 does not provide adequate torque to get the buggy up an 18° incline; thus, it can be ruled out. As both gearboxes 2 and 3 provide sufficient torque, either could be used for the project, with gearbox 3 increasing the maximum practical weight of the buggy and gearbox 2 optimising for speed.

The first step to ensure the gears are the correct distance apart is calculating the pitch circle diameter, which can be calculated using (11) [1]

$$PCD = MOD \times Number\ of\ teeth \tag{11}$$

Where PCD is the pitch circle diameter, MOD is the module of the gears, in this case 0.5 mm. This gives the gears a pitch circle diameter of 8 mm, 25/5 mm, and 24 mm, respectively. To calculate the distance in millimetres between the centres of any two gears the (12) can be employed [1].

$$centre\ distance = \frac{PCD(A) + PCD(B)}{2} + 0.1 \tag{12}$$

Where PCD(A) and PCD(B) are the pitch circle diameter of the 2 gears to be meshed, and 0.1 mm is clearance. Equation (12) gives distances of 16.6 mm and 14.6 mm. As the distance between the motor shaft and wheel shaft is fixed at 31.1 mm [2], a triangle can be constructed to work out the intermediate shaft, shown in Fig. 5.

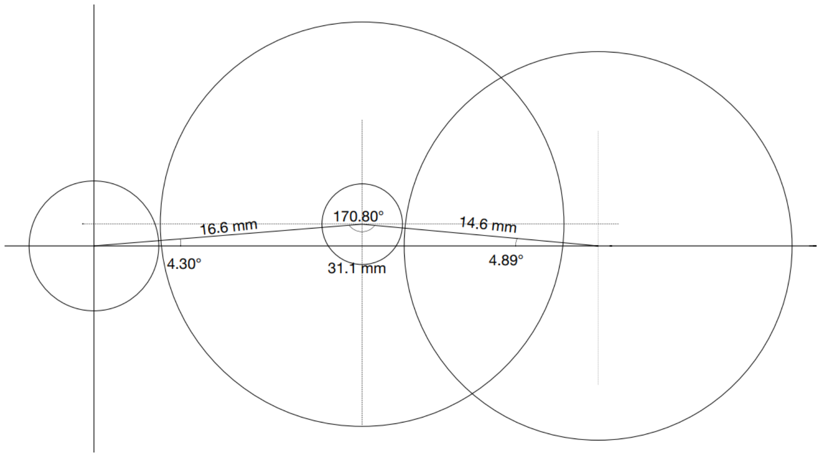

Fig. 5. The design of the gearbox plotted showing the PCD between the gears.

Using basic trigonometry, it is possible to calculate position of the intermediate shaft; If the motor shaft is at the origin, the intermediate shaft must be at the point (16.56,1.16) to 2 decimal places.

The speed of the buggy is related to the forces resisting the motor as shown in Fig. 6:

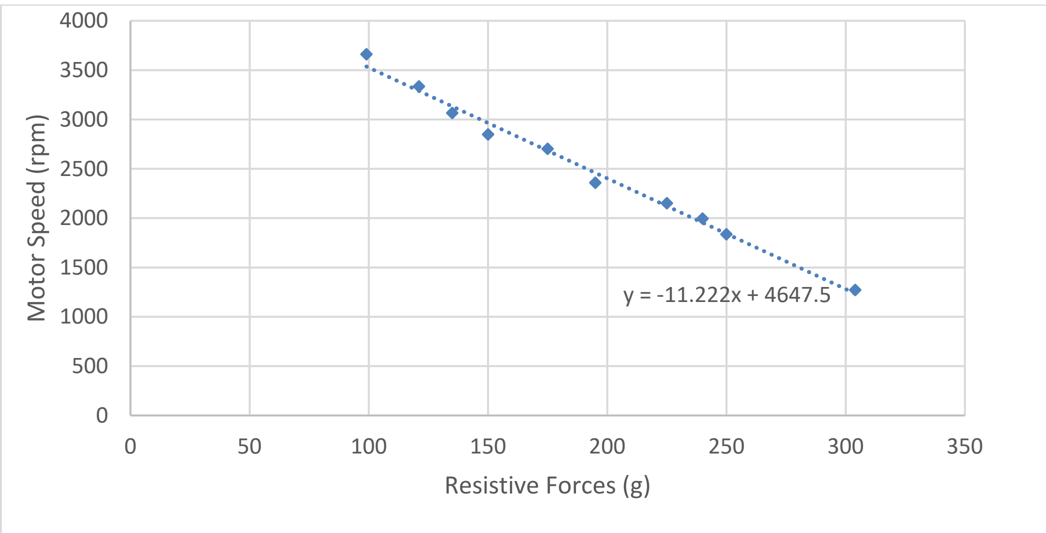

Fig. 6. Graph showing motor speed against opposing forces.

From the trend line, the motor speed on a level and inclined surface is 3806 RPM and 1904 RPM. The gearbox multiplier for gearbox 2 is 10.8 The wheels have a radius of 35 mm, and therefore a circumference of 220 mm. From that can calculate the following.

Table 6 Opposing forces and other values used to calculate the speed of the buggy

	Opposing forces (g)	Motor Speed (RPM)	Axle Speed (RPM)	Buggy speed (mm/min)	Buggy Speed (m/s)
Level	75	3806	352	77530	1.3
Incline	245	1904	176	38789	0.646

5. Summary

Based on experimentation and calculations, the values of K_T and K_E, 0.0074 and 0.0094, respectively, were obtained. Using these values, it was possible to calculate the maximum torque that the motors could produce at 1.4 A, which is 0.00956 Nm. The force and torque required by the buggy is calculated by determining the maximum force that will oppose motors on the incline. Then, by comparing the torque needed and the torque a motor can produce, the minimum required gearbox ratio can be calculated as 9.0. Thus, gearbox 2, which has a gear ratio of 10.8, would be the most appropriate for this project. However, it is important to consider weight. The buggy must weigh less than 1.5 kg to climb the incline.

It is important to note that some assumptions may affect the outcome due to the nature of the experiments. One major assumption was that the motor's operating temperature did not affect performance. Due to the time constraints of the experiment, the motor did not have enough time to return to room temperature between each test, which impacted the values and was especially pronounced when calculating the value for K_E. Another assumption was that the weight of the actual buggy will be comparable to the test buggy.

It is also important to note that the lead used to connect the motor to the power supply has an average resistance of 50.675 mΩ per lead. Although this value is small, it will impact the results.

Lastly, the incline of the test ramp was not 18°, but when measured, it had a maximum incline of 21°. This discrepancy had a definite impact on the coefficient of friction measurements.

6. References

[1] University of Manchester, *Embedded Systems Project EEEN21000 Technical Handbook*, 2023/24 ed.

[2] University of Manchester, *Embedded Systems Project EEEN21000 Procedures Handbook*, Version 2023.1 (September 2023).