Lab2

January 27, 2017

1 CS178 HW 2 WINTER 2017

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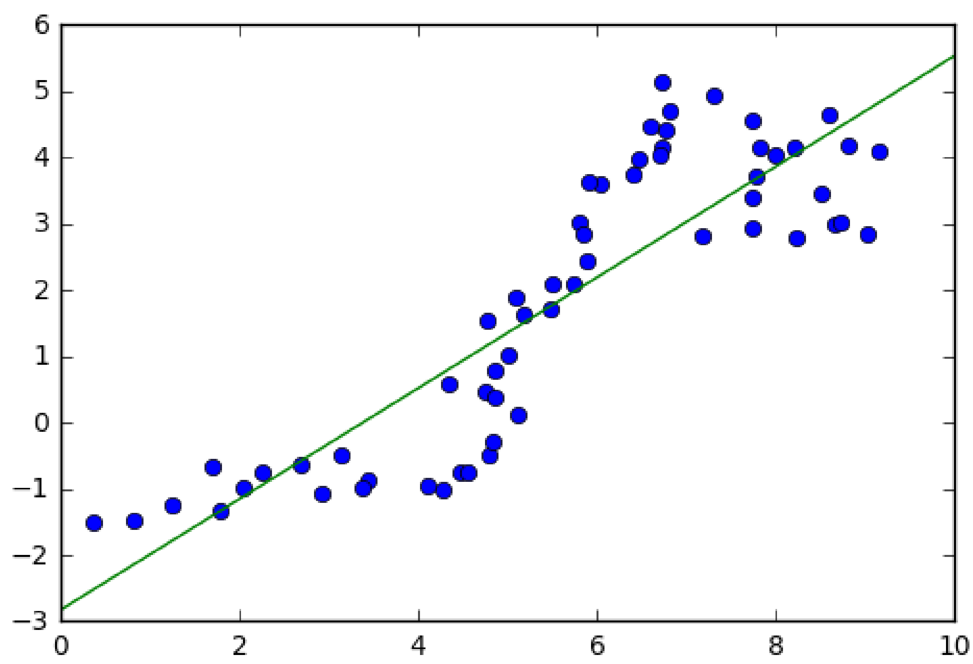
2.0.1 Problem 1A

```
In [12]: import numpy as np
         import matplotlib.pyplot as plt
         import mltools as ml

         # Linear Regression Data Points
         data = np.genfromtxt("C:\Python35\CS178\Lab2\data\curve80.txt",delimiter=N

         # 1A Splitting 2 column data/curve80.txt into 75/25%
         X = data[:,0]
         Y = data[:,1]

         X = X[:,np.newaxis]  # code expects (M,N), newaxis

         Xtr, Xte, Ytr, Yte = ml.splitData(X,Y,0.75)
```

2.0.2 Problem 1B

```
In [13]: # 1B Creating linear regression predictor of y given x

         lr = ml.linear.linearRegress( Xtr, Ytr ); # creating and training model
         xs = np.linspace(0,10,200);               # sample large number of possibl
         xs = xs[:,np.newaxis]                     # force it to be an single colum
         ys = lr.predict( xs );                    # make predictions on it


         # Plot the training data along with your prediction function.
         plt.plot(Xtr, Ytr,'o')    # Graph points
         plt.plot(xs,ys)           # Linear Regression Line
         AXES = plt.axis()         # Save axis
         plt.show()

         # Print the regression coefficicents y = (theta0)x + (theta1) (lr.theta[1]
```

```
print("Y = {}x + {}\n\n".format(lr.theta[:,1][0], lr.theta[:,0][0]))
# Check they match the plot

# Calculate and report the mean squared error for both data
# MSE = sum(Ytr - (mx + b)).mean()


TrainingTargets = []
for each in range(0,len(Xtr)):
    target = (lr.theta[:,1][0] * Xtr[each] + lr.theta[:,0][0])
    TrainingTargets.append((Ytr[each] - target)**2)

Training_MSE = np.mean(TrainingTargets)
print("Training MSE: {}\n".format(Training_MSE))


TestTargets = []
for each in range(0,len(Xte)):
    target = (lr.theta[:,1][0] * Xte[each] + lr.theta[:,0][0])
    TestTargets.append((Yte[each] - target) **  2)

Test_MSE = np.mean(TestTargets)
print("Test MSE: {}".format(Test_MSE))
```




```
Y = 0.8360691602619539x + -2.827650487664813
```

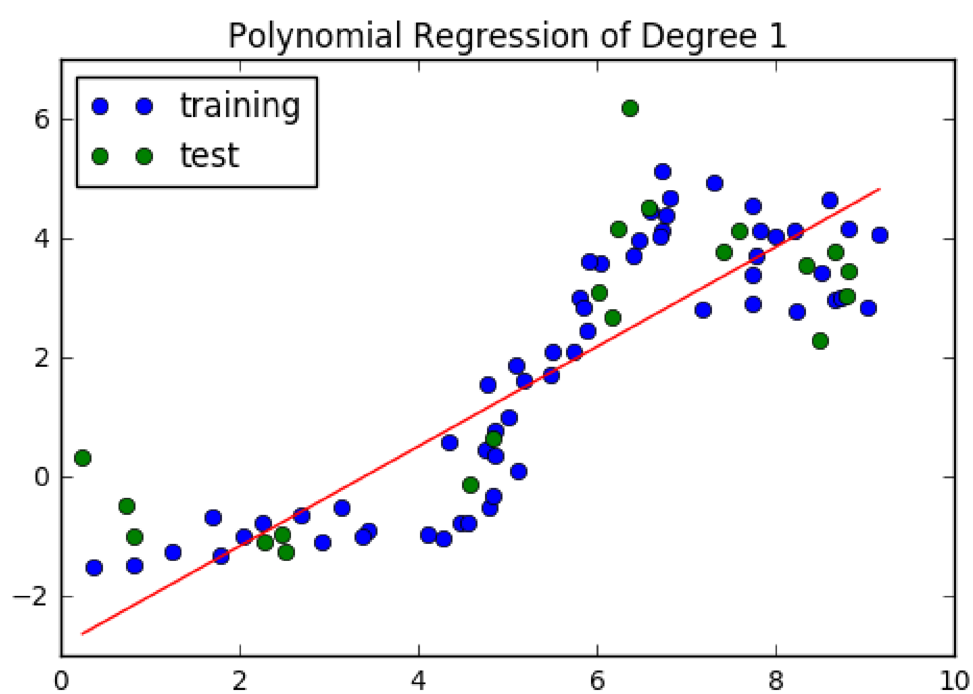
```
Training MSE: 1.127711955609391

Test MSE: 2.242349203010126
```

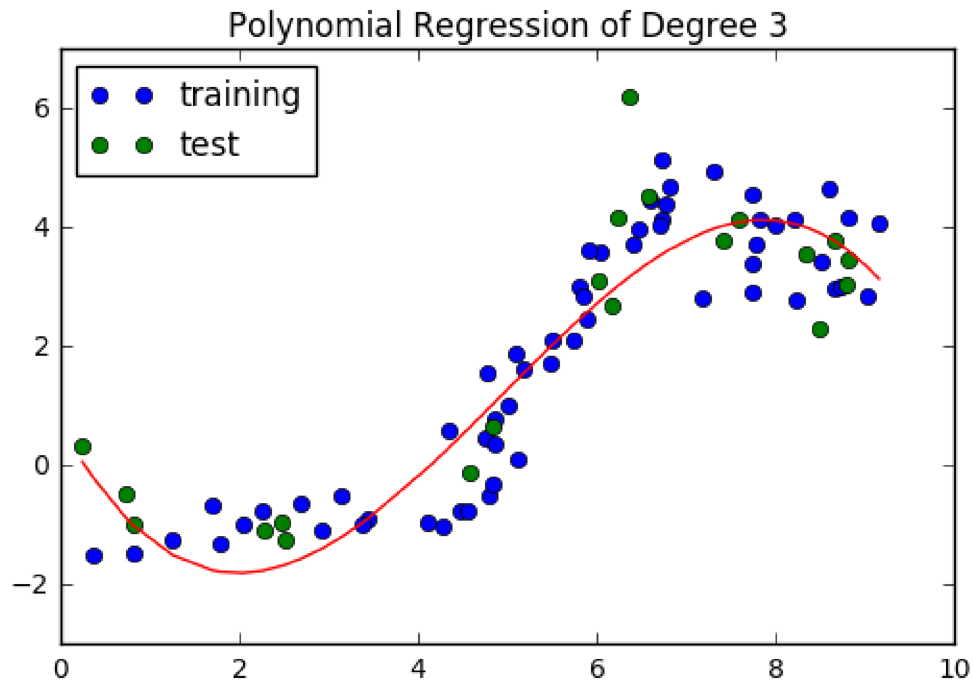
2.0.3 Problem 1C

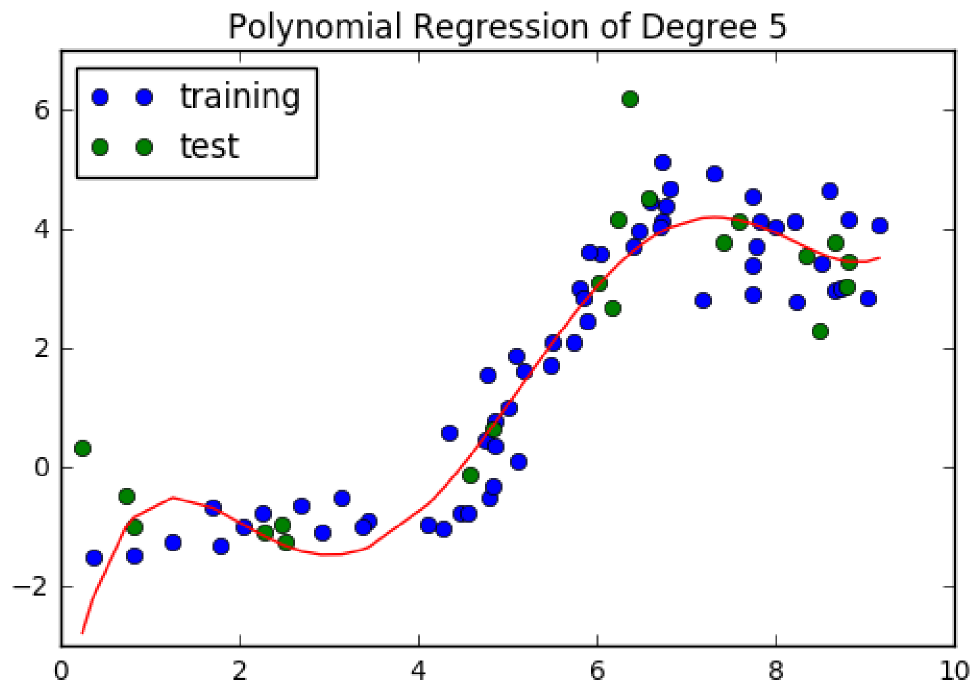
```
In [14]: # 1C fitting y = f(x) increasing polynomial order

         # Xtr2 = np.zeros( (Xtr.shape[0], 2) )      # M x 2 (m rows,2 columns) mat
         # Xtr2[:,0] = Xtr[:,0]                       # place original X feature as
         # Xtr2[:,1] = Xtr[:,0]**2                    # place x^2 feature as column

         # Create polynomial features up to degree; don't make it constant
         # (the linear regression learner will add the constant feature automatical
         ##XtrP = ml.transforms.fpoly(Xtr, degree, bias=False)

         # Rescale the data matrix so that the features have similar ranges / varia
         ##XtrP, params = ml.transforms.rescale(XtrP);
         # "params" returns the transformation parameters (shift & scale) (mu & sig

         # Then we can train the model on the scaled feature matrix
         ##lr = ml.linear.linearRegress( XtrP, Ytr );

         # Now, apply the same polynomial expansion & scaling transformation to Xte
         ##XteP = ml.transforms.rescale( ml.transforms.fpoly(Xte, degree, false), p

         # # Create polynomial features up to "degree"; don't create constant featu
         # # (the linear regression learner will add the constant feature automatic
         # XtrP = ml.transforms.fpoly(Xtr, degree, bias=False);

         # # Rescale the data matrix so that the features have similar ranges / var
         # XtrP,params = ml.transforms.rescale(XtrP);
         # # "params" returns the transformation parameters (shift & scale)

         # # Then we can train the model on the scaled feature matrix:
         # lr = ml.linear.linearRegress( XtrP, Ytr ); # create and train model

         # # Now, apply the same polynomial expansion & scaling transformation to X
         # XteP,_ = ml.transforms.rescale( ml.transforms.fpoly(Xte,degree,false), p

         # # Define a function "Phi(X)" which outputs the expanded and scaled featu

         # Phi = lambda X: ml.transforms.rescale( ml.transforms.fpoly(X, degree,Fal

         # # the parameters "degree" and "params" are memorized at the function de
```

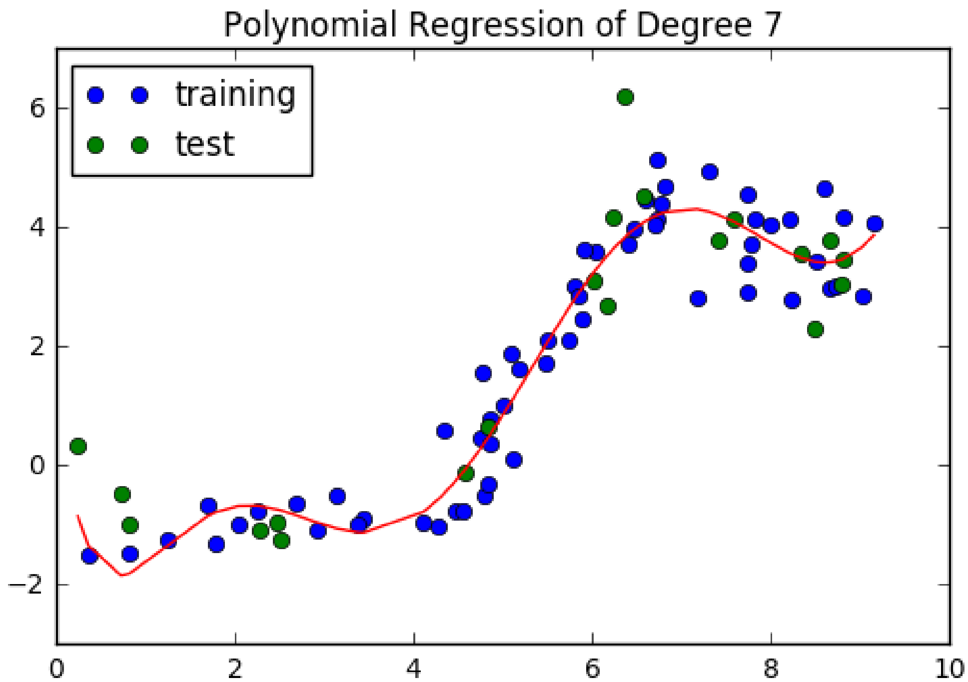
```
# # Now, Phi will do the required feature expansion and rescaling:

# YhatTrain = lr.predict( Phi(Xtr) ); # predict on training data

# YhatTest = lr.predict( Phi(Xte) ); # predict on test data

degree = [1,3,5,7,10,18]
TestError = []
TrainingError = []

for d in degree:

    XtrP = ml.transforms.fpoly(Xtr, d, bias=False) # features with d colum
    XtrP, params = ml.transforms.rescale(XtrP)     # rescaled features wit
                                                   # params are shifted an
    lr = ml.linear.linearRegress( XtrP, Ytr )      # regress on new shift

    XteP,_ = ml.transforms.rescale( ml.transforms.fpoly(Xte, d, False), pa


    Phi = lambda X: ml.transforms.rescale( ml.transforms.fpoly(X, d, False

    YhatTrain = lr.predict( Phi(Xtr) ); # predict on training data

    YhatTest = lr.predict( Phi(Xte) ); # predict on test data

    XXX = np.append(Phi(Xtr)[:,-1], Phi(Xte)[:,-1])
    #print(XXX)
    YYY = np.append(YhatTrain,YhatTest)
    #print(YYY)

    order = np.argsort(Phi(Xtr)[:,-1]) # order x axis chronologically
    orders = np.argsort(Phi(Xte)[:,-1])
    orderz = np.argsort(XXX)



    plt.plot(Xtr,Ytr,'o',label = 'training') # training
    plt.plot(Xte,Yte,'o',label = 'test') # test


    plt.plot(X[orderz],YYY[orderz])
#     plt.plot(np.array(Xtr)[order], np.array(YhatTrain)[order]) # regress
#     plt.plot(np.array(Xte)[orders],np.array(YhatTest)[orders])


    trainErr = zip(Ytr[order], YhatTrain[order])
    total = []
```

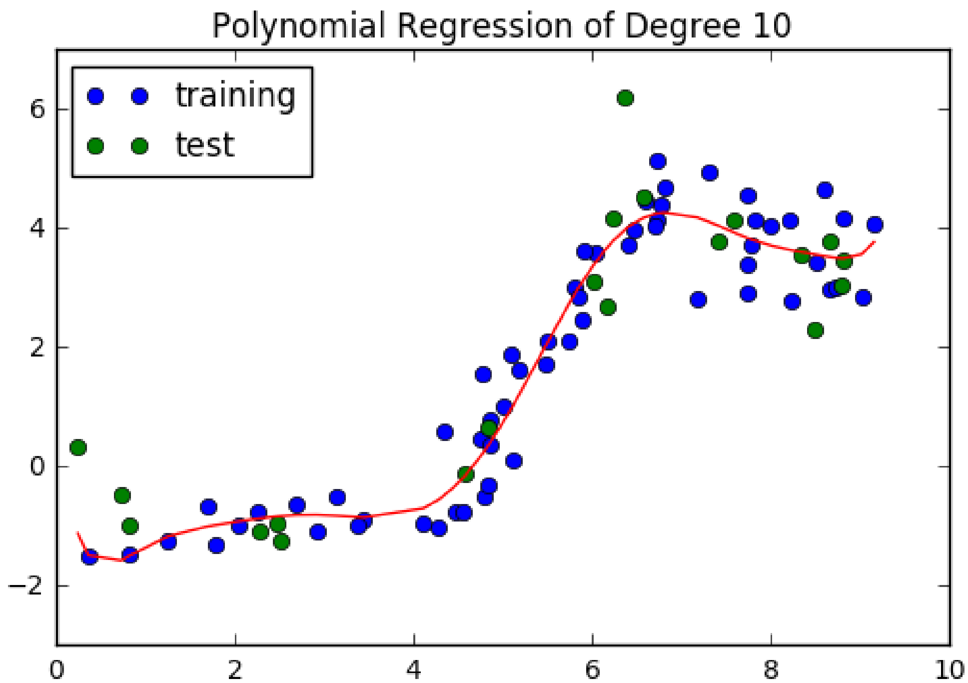
```
    for i,j in trainErr:
        total.append((i-j[0])**2)
    TrainingError.append(np.mean(total))


    testErr = zip(Yte[orders], YhatTest[orders])
    total2 = []
    for i,j in testErr:
        total2.append((i-j[0])**2)
    TestError.append(np.mean(total2))

    if d == 1:
        AXES = plt.axis()
    else:
        plt.axis(AXES)

    plt.title("Polynomial Regression of Degree {}".format(d))
    plt.legend(loc="upper left")
    plt.show()


plt.semilogy(degree,TrainingError,label="Training Error")
plt.semilogy(degree,TestError,label = "Test Error")
plt.xlabel("Degree")
plt.ylabel("MSE")
plt.title("Mean Squared Errors")
plt.show()
```

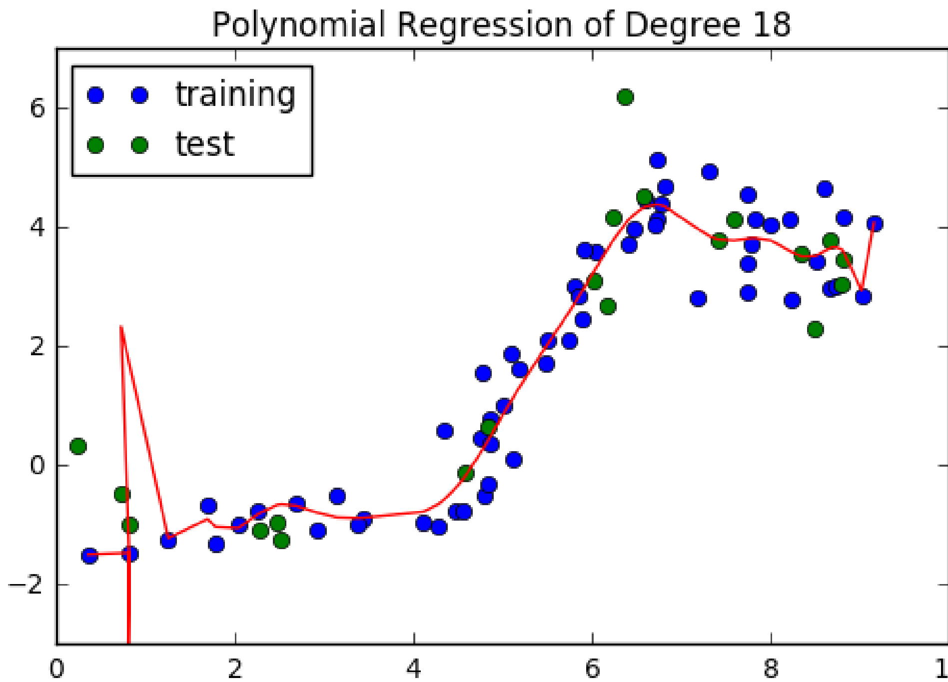


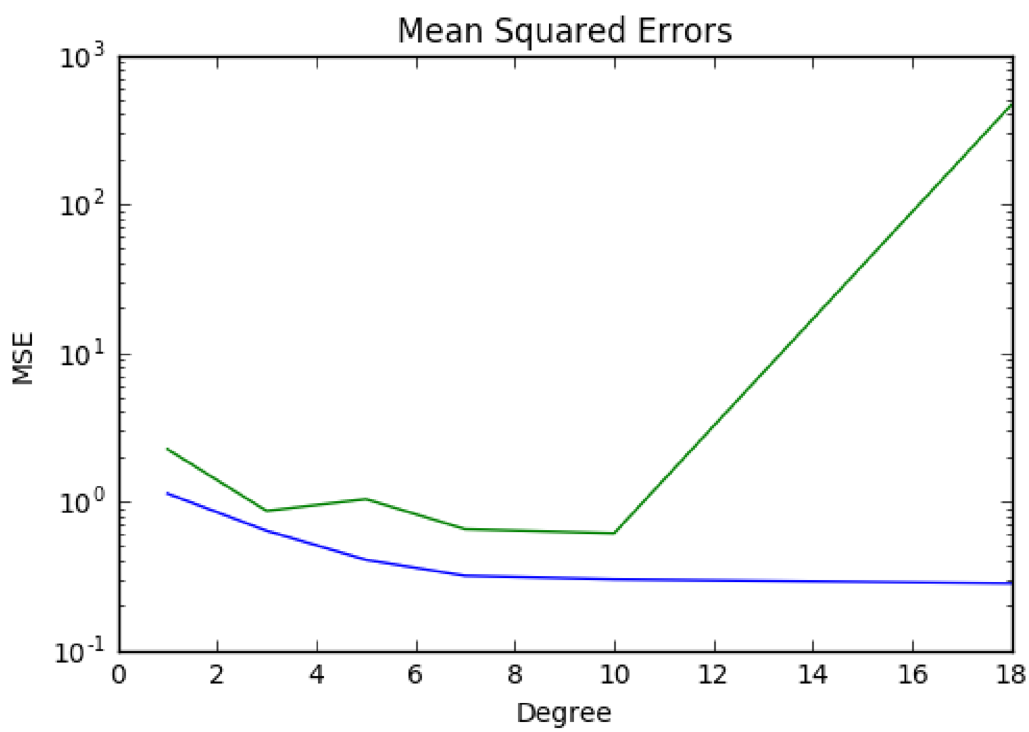

Polynomial Regression of Degree 3
training
test
6
4
2
0
−2
0
2
4
6
8
10


Polynomial Regression of Degree 5
training
test
6
4
2
0
−2
0
2
4
6
8
10


Polynomial Regression of Degree 7
training
test
6
4
2
0
−2
0
2
4
6
8
10

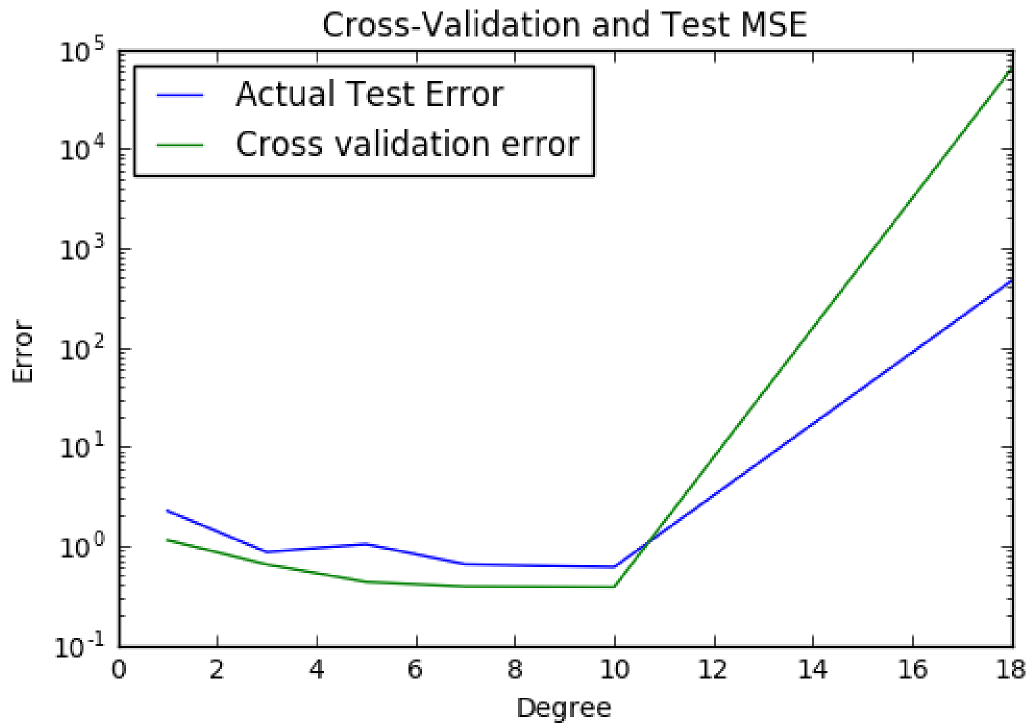

Polynomial Regression of Degree 10
training
test
6
4
2
0
−2
0
2
4
6
8
10


Polynomial Regression of Degree 18
training
test
6
4
2
0
−2
0
2
4
6
8
10


Mean Squared Errors
MSE
10^3
10^2
10^1
10^0
10^{-1}
0
2
4
6
8
10
12
14
16
18
Degree

2.0.4 Problem 2

```
In [15]: # Cross Validation by creating 5 fold validation test

         from sklearn.metrics import mean_squared_error

         degree = [1,3,5,7,10,18]
         cross_validation_for_each_degree = []

         for d in degree:

             XtrP = ml.transforms.fpoly(Xtr, d, False)

             XtrP, params = ml.transforms.rescale(XtrP)

             J = []

             nFolds = 5;

             for iFold in range(nFolds):

                 Xti,Xvi,Yti,Yvi = ml.crossValidate(XtrP,Ytr,nFolds,iFold); # take

                 learner = ml.linear.linearRegress(Xti, Yti)  # TODO: train on Xti,

                 Yhat = learner.predict( XtrP )

                 #print(Xti,Xvi,Yti,Yvi)

                 J.append(mean_squared_error(Ytr,Yhat)) # TODO: now compute the MSE

             # the overall estimated validation performance is the average of the p

             cross_validation_for_each_degree.append(np.mean(J))

         plt.semilogy(degree,TestError,label = "Actual Test Error")
         plt.semilogy(degree, cross_validation_for_each_degree, label = "Cross vali
         plt.legend(loc = "upper left")
         plt.title("Cross-Validation and Test MSE")
         plt.xlabel("Degree")
         plt.ylabel("Error")
         plt.show()
```



From this graph we can see that a degree of 10 has the lowest cross-validation error, and all the cross-validation errors are smaller than the actual test MSE except for degree 18, which is higher.

```
In [ ]:
```