# ICPC Cheatsheet - We are trying to have

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# 1 Info

### 1.1 Template

```
#include <bits/stdc++.h>

void test(){
    //read
    //reset
}

int main(){
    //initialize
    int T;
    std::cin >> T;
    while(T--) test();
}
```

# 1.2 Debug compile options

g++ -Wall -Wextra -pedantic -g -std=c++11 -02 -Wshadow -Wformat=2 -Wfloat-equal -Wconversion -Wlogical-op -Wcast-qual -Wcast-align -D\_GLIBCXX\_DEBUG -D\_GLIBCXX\_DEBUG\_PEDANTIC -fsanitize=address -fsanitize=undefined -fstack-protector

# 1.3 Complexity

Value	Complexity
$n < 10^{18}$	O(1), O(polylog(N))
$n < 10^9$	$O(\operatorname{sqrt}(N))$
$n < 10^6$	$O(N), O(N \log(N))$
$n < 10^3$	$O(N^2)$ , $O(N^2 \log(N))$
n < 100	$O(N^3)$
n < 20	$O(2^N)$
n < 16	$O(N 2^N), O(N^2 2^N)$
n < 10	O(N!)

### 1.4 Common errors

- Loop bounds (especially with DP)
- Array bounds (make slightly bigger)
- Initialization (init and reset)
- Incorrect output format
- Wrong nesting
- Precision (use epsilon)
- Overflow (use 64-bit when in doubt)
- Invalid expressions (divide by zero, segfault)
- Index offset

# Algorithms

Binary search, Functions (math)
Prime check, factorization
Greedy, Sorting, Binary search + Greedy, Divide and conquer, 1D Dynamic programming
2D Dynamic programming, All-pair shortest path
Max-flow, Various unoptimized traversals
Combinations
Bitmask dynamic programming
Permutations

- Rounding (floor, ceil or normal)
  Read complete input
  Boundary cases
  Wrong variable, copying mistakes, etc

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### 2 STL

#### 2.1 I/O

```
std::ios::sync_with_stdio(false); //speedup IO (dont combine with printf/scanf)

std::cin.ignore(n); //ignore n characters before continue

std::getline(std::cin, s); //reads whole line into string

std::cin >> std::noskipws; //dont skip whitespace

std::cout << std::fixed; //use fixed-point notation

std::cout << std::setprecision(n); //set the precision to n decimals

std::cout << std::setw(n); //set the length each output will contain

std::cout << std::setfill(c); //set the char to fill the remaining chars from above
```

### 2.2 String

```
//read a line
    std::string s;
    std::getline(std::cin, s);
    //string split
    std::vector<std::string> split(const std::string &s, char delim) {
        std::vector<std::string> elems;
        std::stringstream ss(s);
        std::string item;
        while (std::getline(ss, item, delim)) {
9
             elems.push_back(item);
10
11
        return elems;
12
13
    //substring
14
    str.substr(pos, LENGTH); //till end if LENGTH is empty
15
    //string find
    str.find("substring") //return std::string::npos if not found, else position first letter
```

#### 2.3 Containers

```
std::vector<int> vec;
    //sort from small to large
    bool cmp(int a, int b){
        return a < b;
    std::sort(vec.begin(), vec.end(), cmp);
    std::sort(vec.begin(), vec.end(), std::less<int>());
    → //use std::greater<int>() for large to small and std::less<int>() for small to large
    std::find(vec.begin(), vec.end(), NEEDLE); //use .find() if avaible (map/set)
    std::count(vec.begin(), vec.end(), NEEDLE); //use .find() if available (map/set)
    //find union/intersection/difference -- both need to be SORTED already
10
    std::vector<int> tmp(vec.size()+vec2.size());
11
    std::set_intersection(vec.begin(), vec2.begin(), vec2.end(), tmp.begin());
    std::set_union(vec.begin(), vec2.end(), vec2.end(), tmp.begin());
13
    std::set_difference(vec.begin(), vec.end(), vec2.begin(), vec2.end(). tmp.begin());
14
    //filter unique elements
15
    auto last = std::unique(vec.begin(), vec.end());
16
    vec.erase(last, v.end());
17
```

### 2.4 String streams

```
//c-style for complex parsing
int in;
sscanf(str.c_str(),"%d",&in)
```

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```
//convert number to string -- itoa()
std::ostringstream os;
os << (int) i;
os.str();
//convert string to number(s) -- atoi() or atol()
std::istringstream is(str);
is >> i;
```

#### 2.5 Constants

```
INT_MIN
INT_MAX
LLONG_MIN
LLONG_MAX
PI //defined in header
EPS //defined in header
```

#### 2.6 Math

```
ceil(a); //round up
floor(a); //round down
round(a); //round nearest
atan2(a, b); //atan with two parameters
frac_part = modf(d, intpart); //split up in parts (int_part is a double!)
std::__gcd(a, b); //greatest common divisor a, b
```

#### 2.7 Permutations

```
std::vector arr;
std::sort(arr.begin(), arr.end());
do {
//work with permutation
} while (next_permutation(arr.begin(),arr.end));
```

#### 2.8 Binary search

```
std::multimap<int, int> m;
m.lower_bound(i); //first element equal or higher then i
m.higher_bound(i); //first element higher then i
std::pair<auto iter, auto iter> p = m.equal_range(i); //get all with equal value
```

# 2.9 Bit twiddling hacks

```
in |= (1 << a); //enable bit at position a
in ^= (1 << a); //toggle bit at position a
if(in & (1 << a)); //check bit at position a

//get last bit set (least signifcant)
c = (b & -b)
//ll for the long long versions
__builtin_popcount(in); //count the amount of bits set
__builtin_ffs(in); //give the least signifcant index + 1 in binary representation
__builtin_clz(in); //returns the number of leading zeros
__builtin_ctz; //give number of trailing zeros</pre>
```

#### 3 Datastructures

### 3.1 BIT

```
_{-} O(log(N))
    //use a std::map to save very large tables or for simple 2D (faster below)
    int lst[MAXN];
    int sum(int b){
         int sum = 0; b+=1;
         for(; b; b=(b&(-b))){
6
             sum += lst[b];
        return sum;
    }
10
11
    int update(int b, int v){
12
         b+=1;
13
         for(; b < MAXN; b + = (b & (-b))){
14
             lst[b] += v;
16
    }
17
```

#### 3.2 2D-BIT

```
_{-} O(N log(N))
     //online O(N (log(N)^2) is also possible using std::map<int, int>
    std::pair<int, int> rs[MAXN]; //list of points (y,x)
    std::vector<int> lst2[MAXN];
3
    std::vector<int> order[MAXN];
    int sum(int b, int c){
         int sum = 0; b+=1; c+=1;
         for(; b; b-=(b&(-b))){
             int h = upper_bound(order[b].begin(), order[b].end(), c) - order[b].begin();
             for(int d = h; d; d-=(d\&(-d))){
10
                  sum += lst2[b][d];
11
             }
12
         }
13
         return sum;
14
15
16
    void update(int b, int c, int v){
17
         b+=1; c+=1;
18
         int unt = 0;
19
         for(; b<MAXN; b+=(b&(-b))){
20
             int h = upper_bound(order[b].begin(), order[b].end(), c) - order[b].begin();
21
             for(int d = h; d<1st2[b].size(); d+=(d&(-d))){
22
                  lst2[b][d] += v;
23
             }
24
         }
25
    }
26
27
    void init(){
28
         std::sort(rs, rs+MAXN);
29
         for(int i=0; i<MAXN; ++i){</pre>
30
             for(int j=rs[i].second+1; j<MAXN; j+=(j&(-j))){
31
                  order[j].push_back(rs[i].first+1);
32
                  lst2[j].push_back(0);
33
             }
34
         }
35
         for(int i=0; i<MAXN; ++i) lst2[i].push_back(0);</pre>
36
```

|}

# 3.3 Segment tree

```
_{-} O(log(N)) _{-}
    // inclusion segment tree (a node includes both it endpoints!)
    struct Node{
    Node(): li(0), ri(0), l(0), r(0) {}
         int li;
         int ri;
         Node *1;
         Node *r;
         int val;
10
         bool hupd;
11
12
         int upd;
    };
13
14
    Node *r;
15
16
    Node *build(int li, int ri) {
17
         Node *n = new Node;
18
         n->li = li;
         n->ri = ri;
20
         n->upd = 0;
21
         if(li == ri){
22
             n->val = 0; //init
23
         }else{
24
             int mi = (li+ri)/2;
25
             n->r = build(li, mi);
             n->l = build(mi+1, ri);
27
28
             n-val = std::min(n-v-val, n-val); // merge
29
         }
30
         return n;
31
    }
32
33
    void pushd(Node *n){
34
         if(!n->hupd) return;
35
         n->l->val = n->upd; //update
36
         n->r->val = n->upd; //update
37
         n->1->upd = n->upd; //split (move old updates)
38
         n->r->upd = n->upd; //split (move old updates)
         n->1->hupd = true;
40
         n->r->hupd = true;
41
         n->hupd = false;
42
         n->upd = 0;
43
    }
44
45
     int query(Node *n, int li, int ri){
46
         if(ri < n->li || n->ri < li){
47
             //outside
48
             return INT_MAX;
         }else if(li <= n->li && n->ri <= ri){
50
             //inside
51
             return n->val;
52
         }
53
         pushd(n);
54
55
         int la = query(n->1, li, ri);
```

```
int ra = query(n->r, li, ri);
57
         return std::min(la, ra); // merge
58
    }
59
60
    void update(Node *n, int li, int ri, int v){
61
         if(ri < n->li || n->ri < li){
62
             //outside
63
             return;
64
         }else if(li <= n->li && n->ri <= ri){</pre>
65
             //inside
66
             if(n->ri != n->li){
67
                  pushd(n);
68
                  n->upd = v; //split
69
                  n->hupd = true;
70
71
             n->val = v; //update
             return;
73
74
         pushd(n);
75
         update(n->1, li, ri, v);
77
         update(n->r, li, ri, v);
78
         n-val = std::min(n-v-val, n-val); // merge
79
80
```

#### 3.4 Union-Find

```
_____O(lpha) _
    int pr[MAXN];
1
    int sz[MAXN];
2
    int find(int k){
        if(pr[k] == k) return k;
        else return pr[k] = find(pr[k]);
    void merge(int a, int b){
        a = find(a); b = find(b);
        if(a == b) return;
10
        if(sz[a] > sz[b]) std::swap(a, b);
11
12
        pr[a] = b;
13
        sz[b] += sz[a];
14
        sz[a] = 0;
15
    }
```

# 4 Math

# 4.1 GCD/LCM

```
int gcd(int a, int b){
    if(b == 0) return a;
    return gcd(b, a%b);
}

int lcm(int a, int b){
    return a*b/gcd(a,b);
}
```

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### 4.2 Extended Euclidean

```
_{-} O(log(K)) _{-}
    //determines a and b satisfying s*a+t*b == gcd(a, b)
    std::pair<int, int> extgcd(int a, int b){
        int s = 0, old_s = 1;
        int t = 1, old_t = 0;
        int r = b, old_r = a;
        while(r){
            int q = old_r/r;
            int sv;
            sv = r;
            r = old_r - q*r; old_r = sv;
10
            sv = s;
11
             s = old_s - q*s; old_s = sv;
12
            sv = t;
13
            t = old_t - q*t; old_t = sv;
14
        }
15
        return std::make_pair(old_s, old_t);
16
        //return s, t if you want with result = zero
17
```

# 4.3 Multiplicative inverse

# 4.4 Modular exponentiation

```
0(log(K))

//trick also works with matrices
long long expmod(long long a, long long b, long long m){
long long res = 1;
a = a%m;
while(b > 0){
if((b%2) == 1) res = (res*a)%m;
b >>= 1;
a = (a*a) % m;
}
return res;
}
```

### 4.5 Fibonacci

```
int fib(int n){
    int a = 0, b = 1;
    for(int i=0; i<n; ++i){
        int t = b;
        b = a+b;
        a = t;
    }
}</pre>
```

```
s return a;
9 }
```

#### 4.6 Combinations

```
int cmb[MAXN] [MAXN];
int comb(int i, int j){
    if(i < 0) return 0;
    else if(j == 0) return 1;
    else if(cmb[i][j] != -1) return cmb[i][j];
    else return cmb[i][j] = comb(i-1,j-1)+comb(i-1,j);
}</pre>
```

#### 4.7 Gaussian elimination

```
oldsymbol{oldsymbol{\bot}} O(N^3) .
    int N; //amount of rows (columns = rows + 1)
    double aug[MAXN] [MAXN];
    double ans[MAXN];
    void gaussian_elimination() {
         // the forward elimination phase
         for (int i = 0; i < N - 1; i++) {
             int 1 = i;
             // which row has larest column value
             for (int j = i + 1; j < N; j++) if (fabs(aug[j][i]) > fabs(aug[l][i])) l = j;
10
             // swap this pivot row, reason: minimize floating point error
11
             for (int k = i; k \le N; k++){
12
                      double t = aug[i][k];
13
                      aug[i][k] = aug[l][k];
                      aug[1][k] = t;
15
16
             // the actual forward elimination phase
17
             for (int j = i + 1; j < N; j++)
                 for (int k = N; k >= i; k--) aug[j][k] -= aug[i][k] * aug[j][i] / aug[i][i];
19
20
         }
21
22
         // the back substitution phase
23
         for (int j = N - 1; j >= 0; j--) {
24
             double t = 0.0;
             for (int k = j + 1; k < N; k++) t += aug[j][k] * ans[k];
26
             ans[j] = (aug[j][N] - t) / aug[j][j];
27
28
29
```

### 4.8 Prime generation

```
std::vector<long long> prms;
long long is_prm[MAXN];

//generate primes needed for functions below

void init_prms(){

for(long long i=0; i<MAXN; ++i) is_prm[i] = true;

for(long long i=2; i<MAXN; ++i){

if(!is_prm[i]) continue;

prms.push_back(i);

for(long long j=2*i; j<MAXN; j+=i) is_prm[j] = false;

}
```

|}

### 4.9 Is prime

```
| Dool is_prime(long long n){
| for(size_t i=0; i < prms.size(); ++i) {
| if(prms[i] * prms[i] > n) return true;
| if((n % prms[i]) == 0) return false;
| }
| return true;
| }
```

#### 4.10 Factorize

```
\_ O(N log(log(N))) \_
    //returns list of factors and how many times they occur
    std::vector<std::pair<int, int> > factors(long long n){
        std::vector<std::pair<int, int> > v;
        for(size_t i=0; iorms.size(); ++i){
            long long k = 0;
            while((n % prms[i]) == 0){
                     ++k;
                     n/=prms[i];
            }
            if(k) v.push_back(std::make_pair(prms[i], k));
10
11
        if(n!=1) v.push_back(std::make_pair(n, 1));
12
        return v;
13
14
```

### 4.11 Is prime (fast)

```
_{-} O(polylog(N)) .
    //deterministic until 2^64 -- needs long long expmod probably!
    bool miller_rabin(long long n){
        long long prms[12] = {2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37};
3
        long long s = 0, d = n-1;
        while((d \% 2) == 0){
             d/=2;
             s++;
        }
         for(int i=0; i<12; ++i){
10
             if(prms[i] >= n) break;
11
             if(expmod(prms[i], d, n) == 1) continue;
             long long c = 1;
13
             int j = 0;
             for(j=0; j<s; ++j){
15
                 if(expmod(prms[i], c*d, n) == n-1) break;
16
17
18
             if(j == s) return false;
19
        }
20
        return true;
21
22
```

# 5 Graph

### 5.1 Header

```
struct Edge{
         //add constructor if convenient
         int f; //from
         int t; //to
         int d; //distance
         int cap; //capacity (max flow)
         int flw; //flow (max flow)
         Edge *rev; //reverse edge (in case undirected)
10
         bool use; //edge is used (euler-tour)
11
    };
12
13
    struct Node{
14
         int n; //index
15
         std::vector<Edge*> ed; //adjadency list
16
    };
17
18
    Node nd[MAXN]; //nodes
19
     int dis[MAXN]; //distance to node
20
21
     int dpt[MAXN]; //depth node (SCC/bridge)
22
     int low[MAXN]; //low-link node (SCC/bridge)
23
     //next edge of a node, used to keep track of next path in a non-recursive dfs
    std::vector<Edge*>::iterator eit[MAXN];
25
26
    Edge *frm[MAXN]; //edge used to node
27
    bool vis[MAXN]; //visited
29
    int incnt[MAXN]; //active indegree count (topo-sort)
30
31
     void init(){
32
         for(int i=0; i<MAXN; ++i){</pre>
33
             nd[i].n = i;
             for(size_t j=0; j<nd[i].ed.size(); ++j) delete nd[i].ed[j];</pre>
35
             nd[i].ed.clear();
36
             dis[i] = INT_MAX;
             dpt[i] = -1;
38
             low[i] = INT_MAX;
39
             frm[i] = 0;
40
             vis[i] = false;
             incnt[i] = 0;
42
         }
43
```

#### 5.2 BFS

```
void bfs(int F){
    for(int i=0; i < MAXN; ++i) dis[i] = INT_MAX;

    std::queue < int > q; //replace by stack for dfs

dis[F] = 0;
q.push(F);
while(!q.empty()){
    int p = q.front();
q.pop();
for(auto iter = nd[p].ed.begin(); iter != nd[p].ed.end(); ++iter){
```

### 5.3 Dijkstra

```
______O((E+V)log(V)) _____
    int phi[MAXN];
    #define POT(u,v) (phi[u] - phi[v])
    void dijkstra(int F){
4
        for(int i=0; i<MAXN; ++i) dis[i] = INT_MAX, phi[i] = 0;</pre>
        std::priority_queue<std::pair<int,</pre>
         int>, std::yector<std::pair<int, int > >, std::greater<std::pair<int, int > > pq;
        dis[F] = 0;
        pq.push(std::make_pair(0, F));
        while(!pq.empty()){
10
            std::pair<int, int> p = pq.top();
            pq.pop();
12
            if(dis[p.second] != p.first) continue;
13
            for(auto iter = nd[p.second].ed.begin(); iter != nd[p.second].ed.end(); ++iter){
14
                 Edge *e = *iter;
15
                 if(p.first + e->d + POT(p.first, e->t) < dis[e->t]){
16
                     dis[e->t] = p.first + e->d + POT(p.first, e->t);
17
                     pq.push(std::make_pair(dis[e->t], e->t));
                 }
19
            }
20
        }
21
22
        //addition if we want to work with negative-weight paths (min-cost max-flow)
23
        //for(int i=0; i<MAXN; ++i) if(phi[i] < INT_MAX) phi[i] += dis[i];
24
    }
25
```

# 5.4 Floyd-Warshall

#### 5.5 Bellman-Ford

```
bool bellman_ford(int F){

//compute shortest path

for(int i=0; i<MAXN; ++i) dis[i] = INT_MAX;

dis[F] = 0;
```

```
for(int k=0; k<MAXN-1; ++k){</pre>
             for(int i=0; i<MAXN; ++i){</pre>
                  for(auto iter = nd[i].ed.begin(); iter != nd[i].ed.end(); ++iter){
                      Edge *e = *iter;
                      if(dis[i] + e->d < dis[e->t]){
                           dis[e->t] = dis[i] + e->d;
10
                  }
12
             }
13
         }
14
15
         //check for negative weight path
16
         for(int i=0; i<MAXN; ++i){</pre>
17
             for(auto iter = nd[i].ed.begin(); iter != nd[i].ed.end(); ++iter){
18
                  Edge *e = *iter;
19
                  if(dis[i] + e->d < dis[e->t]) return false;
20
             }
21
         }
22
         return true;
23
```

### 5.6 Minimum spanning tree

```
\_ O(E log(V)) _{	extstyle -}
    //needs union-find
    bool cmp(Edge *e1, Edge *e2) { return e1->d < e2->d; }
    std::vector<Edge*> kruskal(std::vector<Edge*> &edg){
         std::vector<Edge*> vec;
         std::sort(edg.begin(), edg.end(), cmp);
         for(size_t i=0; i<edg.size(); ++i){</pre>
             int a = find(edg[i]->f);
             int b = find(edg[i]->t);
             if(a != b){
11
                  vec.push_back(edg[i]);
12
                  merge(a, b);
13
             }
14
15
16
17
         return vec;
18
```

# 5.7 Biconnected components

```
_____ O(E+V) _
    std::vector<int> biconnected(int r){
        for(int i=0; i<MAXN; ++i) eit[i] = nd[i].ed.begin();</pre>
        std::vector<int> res; //result nodes
        std::stack<int> st; //stack
        st.push(r);
        dpt[r] = 0;
        while(!st.empty()){
             int c = st.top();
             st.pop();
10
11
             //add to result
             if(eit[c] == nd[c].ed.end()){
13
                 int cnt = 0;
14
                 for(auto iter = nd[c].ed.begin(); iter != nd[c].ed.end(); ++iter){
15
                     int n = (*iter) -> t;
16
```

```
if(dpt[c] != 0 \&\& low[n] >= dpt[c] \&\& dpt[n] == dpt[c]+1){ //other edges}
17
                          res.push_back(c);
                          break;
                      }else if(dpt[c] == 0 && dpt[n] == dpt[c]+1){ //root
20
                          dpt[n] = 0;
21
                          ++cnt;
22
                      }
                 if(dpt[c] == 0 \&\& cnt >= 2) res.push_back(c); //root
25
26
                 continue;
             }
27
28
             //init
29
             if(low[c] == INT_MAX) low[c] = dpt[c];
30
31
             //loop through children (non-recursive so we should come back to this node)
32
             st.push(c);
33
             auto iter = eit[c];
34
             for(; iter!=nd[c].ed.end(); ++iter){
35
                 int n = (*iter) -> t;
                 if(dpt[n] != -1){
37
                      if(dpt[n] < dpt[c]-1) low[c] = std::min(low[c], dpt[n]); // back edge
38
                      else if(dpt[n] > dpt[c]) low[c] = std::min(low[c], low[n]); // forward edge
39
40
                      dpt[n] = dpt[c]+1;
41
                      st.push(n);
42
                      break;
             }
             eit[c] = iter;
47
         return res;
48
```

### 5.8 Strongly connected components

```
_ O(E+V)
    int tarjan_ind = 0;
    std::vector<std::vector<int> > tarjan(int r){
        for(int i=0; i<MAXN; ++i) eit[i] = nd[i].ed.begin();</pre>
        std::vector<std::vector<int> > res; //result components
        std::stack<int> st; //stack
        std::stack<int> cp; //current component
        st.push(r);
        dpt[r] = ++tarjan_ind;
        while(!st.empty()){
             int c = st.top();
11
             st.pop();
12
13
             //init
             if(low[c] == INT_MAX){
15
                 cp.push(c);
                 low[c] = dpt[c];
             }
18
19
             //add to result
             if(eit[c] == nd[c].ed.end()){
21
                 if(low[c] == dpt[c]){
22
                     res.push_back(std::vector<int>());
23
                     while(true){
```

```
int n = cp.top();
                           dpt[n] = INT_MAX; low[n] = INT_MAX;
26
                           res.back().push_back(n);
                           cp.pop();
28
                           if(n == c) break;
29
                      }
30
                  }
31
                  continue;
32
             }
33
34
             //loop through children (non-recursive so we should come back to this node)
35
             st.push(c);
36
37
             auto iter = eit[c];
             for(; iter!=nd[c].ed.end(); ++iter){
38
                  int n = (*iter)->t;
39
                  if(dpt[n] != -1){
                      if(dpt[n] < dpt[c]) low[c] = std::min(low[c], dpt[n]); // back edge
41
                      else if(dpt[n] > dpt[c]) low[c] = std::min(low[c], low[n]); // forward edge
42
                  }else{
43
                      dpt[n] = ++tarjan_ind;
                      st.push(n);
45
                      break;
46
                  }
48
             eit[c] = iter;
49
50
         return res;
51
    }
52
```

# 5.9 2-SAT

```
O(E+V)
     //needs tarjan (MAXN = 2x the amount of variables)
    int asgn[MAXN]; //start from 2 all even indexes contain normal var
     //give terms with negative as negotiation (make sure no zero used!)
    int comp[MAXN];
     int casg[MAXN];
    bool two_sat(std::vector<std::pair<int, int> > terms){
         for(int i=0; i<MAXN; ++i) comp[i] = casg[i] = -1;</pre>
         //build graph
10
         for(size_t i=0; i<terms.size(); ++i){</pre>
11
             std::pair<int, int> p = terms[i];
             Edge *e = new Edge();
13
             if(p.first < 0) e \rightarrow f = -2*p.first;
             else e->f = 2*p.first+1;
             if(p.second < 0) e->t = -2*p.second+1;
16
             else e->t = 2*p.second;
17
             nd[e->f].ed.push_back(e);
18
             e = new Edge();
19
             if(p.second < 0) e->f = -2*p.second;
20
             else e->f = 2*p.second+1;
21
             if(p.first < 0) e \rightarrow t = -2*p.first+1;
             else e->t = 2*p.first;
23
             nd[e->f].ed.push_back(e);
24
         }
25
26
         //apply tarjan
27
         std::vector<std::vector<int> > all;
         for(int k=0; k<MAXN; ++k){</pre>
29
```

```
if(dpt[k] != -1) continue;
30
             std::vector<std::vector<int> > vec = tarjan(k);
31
             for(size_t i=0;
              i<vec.size(); ++i) for(size_t j=0; j<vec[i].size(); ++j) comp[vec[i][j]] = i+all.size();</pre>
             all.insert(all.end(), vec.begin(), vec.end());
33
         }
34
35
         //reverse topological traverse
36
         for(size_t i=0; i<all.size(); ++i){</pre>
37
             if(casg[i] == -1) casg[i] = true;
38
39
             for(size_t j=0; j<all[i].size(); ++j){</pre>
40
                  int chk = comp[all[i][j]]/2;
41
                  if(comp[chk*2] == comp[chk*2+1]) return false;
42
43
                  if(all[i][j] % 2) casg[comp[all[i][j]-1]] = !casg[i];
                  else casg[comp[all[i][j]+1]] = !casg[i];
45
             }
46
         }
47
         //set assignment (2 contains assignment var 1, 4 assignment var 2 etc...)
49
50

    i=0; i<all.size(); ++i) for(size_t j=0; j<all[i].size(); ++j) asgn[all[i][j]/2] = casg[i];</pre>
         return true;
51
52
```

#### 5.10 Max Flow

```
_{-} O(V^{2}E)
     //bfs is safer (unless path length is limited and integer)
     //for min-cost replace by Bellman-Ford or Dijkstra with potentials
    int dfs(int a, int b){
         if(a == b) return INT_MAX;
         if(vis[a]) return 0;
         vis[a] = true;
         for(size_t i=0; i<nd[a].ed.size(); ++i){</pre>
             Edge *e = nd[a].ed[i];
10
             int cap = (e->cap-e->flw)+e->rev->flw;
11
             if(cap == 0) continue;
12
             else{
13
                  int k = dfs(e->t, b);
14
                  if(k == 0) continue;
                  frm[e->t] = e;
16
                  return std::min(cap, k);
17
             }
         }
19
         return 0;
20
21
22
    int max_flow(int a, int b){
23
         int mf = 0;
24
         frm[a] = 0;
26
         while(true){
27
             for(int i=0; i<2000; ++i) vis[i] = false;</pre>
             int f = dfs(a, b);
29
             if(f == 0) break;
30
             mf += f;
31
32
```

```
int lst = b;
33
              while(frm[lst]){
34
                  Edge *nr = frm[lst];
36
                  int rf = std::min(nr->cap-nr->flw, f);
37
                  nr->flw += rf;
38
                  nr->rev->flw -= f-rf;
39
40
                  lst = frm[lst]->f;
41
              }
42
43
         return mf;
44
45
```

# 5.11 Bipartite matching

```
\_ O(V^2E)
    int mtch[MAXN]; //only the size of one side
    bool aug(int n) {
         if (vis[n]) return false;
         vis[n] = true;
         for (size_t i = 0; i < nd[n].ed.size(); i++) {</pre>
             Edge *e = nd[n].ed[i];
             //try match with edge (if available or previous can be rematched)
             if (mtch[i] == -1 \mid \mid aug(mtch[e->t])) {
                  mtch[e->t] = n;
10
                  return true;
11
12
         }
13
         return false;
    }
15
16
    void bipartite_matching() {
17
         for (int i = 0; i < MAXN; i++) mtch[i] = -1;
18
19
         int M = 0; //contains the maximum matching
20
         for (int i = 0; i < MAXN; i++) { //try to start match from i
21
             for (int i = 0; i < MAXN; i++) vis[i] = false;</pre>
22
             if (aug(i)) M++;
23
         }
24
25
```

### 5.12 Euler tour

```
= O(V^2E)
    //start from one of the two non-even edges (if trail)
1
    std::list<int> euler_tour(int s){
        std::list<int> ans;
        for(int i=0; i<MAXN; ++i) eit[i] = nd[i].ed.begin();</pre>
        std::stack<int> st;
        st.push(s);
        while(!st.empty()){
             int c = st.top();
             st.pop();
10
11
             auto iter = eit[c];
12
             if(iter == nd[c].ed.end()){
13
                 ans.push_front(c);
14
                 continue;
15
             }
16
```

```
17
             st.push(c);
             for(; iter != nd[c].ed.end(); ++iter){
                  Edge *e = *iter;
20
                  if(e->use) continue;
21
                  e->use = e->rev->use = true;
22
                  st.push(e->t);
                  break;
25
26
             eit[c] = iter;
27
28
29
         return ans;
    }
30
```

# 5.13 Topological sort

```
__ O(E+V)
    int ecnt[MAXN];
    std::vector<int> toposort(){
         std::vector<int> res;
         std::queue<int> q;
         for(int i=0; i<MAXN; ++i){</pre>
             if(incnt[i] == 0) q.push(i);
         while(!q.empty()){
10
             int c = q.front();
11
             q.pop();
12
             res.push_back(c);
             for(auto iter = nd[c].ed.begin(); iter != nd[c].ed.end(); ++iter){
                 Edge *e = (*iter);
                 --incnt[e->t];
                 if(incnt[e->t] == 0) q.push(e->t);
17
             }
18
         }
19
         return res;
20
21
```

### 5.14 LCA

```
_ O(E+V) -
    //set MAXK to the log2 of MAXN
    int par[MAXN] [MAXK]; //example of sparse table idea
    void construct(int k, int p = -1){
         if(dpt[k] != -1) return;
         if(p == -1) dpt[k] = 0;
         else dpt[k] = dpt[p]+1;
         //compute parents
         par[k][0] = p;
10
         for(int i=1; i<MAXK; ++i){</pre>
11
             if(par[k][i-1] == -1) break;
             par[k][i] = par[par[k][i-1]][i-1];
13
         }
14
         //dfs children
16
         for(size_t i=0; i<nd[k].ed.size(); ++i){</pre>
17
             int t = nd[k].ed[i]->t;
18
19
```

```
construct(t, k);
         }
21
     }
22
23
     int query(int a, int b){
24
         if(dpt[a] < dpt[b]) std::swap(a, b);</pre>
25
         //level out
27
         int k = 0;
28
         for(; k<MAXK; ++k){</pre>
29
              if(par[a][k] == -1 \mid \mid dpt[par[a][k]] < dpt[b]) break;
30
31
         --k;
32
         while(dpt[a] != dpt[b]){
33
              a = par[a][k];
34
              while (k>=0 \&\& (par[a][k] == -1 || dpt[par[a][k]] < dpt[b])) --k;
         }
36
37
         //go up
38
         for(k=0; k<MAXK; ++k){
              if(par[a][k] == par[b][k]) break;
40
41
         --k;
         while(k \ge 0){
43
              a = par[a][k];
44
              b = par[b][k];
45
              while(k \ge 0 \&\& par[a][k] == par[b][k]) --k;
47
         //do last jump if necessary
         if(a != b){
              k++;
50
51
              a = par[a][k];
              b = par[b][k];
53
54
55
         return a;
56
57
```

#### 6 Geometry

# 6.1 Headers

```
#define PI (2*std::acos(0.0))
    #define EPS 1e-9
    struct Vec{
         Vec(): x(0), y(0), z(0) {}
         Vec(Vec p1, Vec p2): x(p2.x-p1.x), y(p2.y-p1.y), z(p2.z-p1.z) {}
         Vec(double i, double j, double k = 0): x(i), y(j), z(k) {}
         double x;
         double y;
         double z;
10
         bool operator<(const Vec &o){</pre>
12
             if(fabs(x - o.x) > EPS) return x < o.x;
13
             else if(fabs(y - o.y) > EPS) return y < o.y;</pre>
14
             else return z < o.z;</pre>
15
16
         bool operator==(const Vec &o){
17
             return (fabs(x-o.x) < EPS) \&\& (fabs(y-o.y) < EPS) \&\& (fabs(z-o.z) < EPS);
18
```

```
}
19
    };
20
    Vec operator+(const Vec &a, const Vec &b){
21
         return Vec(a.x+b.x, a.y+b.y, a.z+b.z);
22
23
    Vec operator*(const double d, Vec b){
24
         return Vec(d*b.x, d*b.y, d*b.z);
26
    }
27
28
    struct Line{
         Line(Vec i, Vec j): b(i), d(j) {}
29
         Vec b; //base
30
         Vec d; //direction
31
    };
32
    Line fromPoints(Vec i, Vec j){
33
         return Line(i, Vec(i, j));
34
35
36
    double dot(Vec a, Vec b){ //dot product
37
         return a.x*b.x+a.y*b.y+a.z*b.z;
39
    double cross(Vec a, Vec b){ //cross product (2D)
40
         return a.x*b.y-a.y*b.x;
41
42
    Vec cross_vec(Vec a, Vec b){
43
         return Vec(a.y*b.z-b.y*a.z, a.z*b.x-b.z*a.x, a.x*b.y-a.y*b.x);
44
45
    double len_sq(Vec a){ //give the squared length of a vector
46
         return dot(a, a);
47
    double len(Vec a){ //squares the squared length
49
         return std::sqrt(len_sq(a));
50
51
52
    double angle(Vec p, Line 1){ //return the angle between the line and the point
53
         Vec c(1.b, p);
54
         return acos(dot(c, 1.d)/std::sqrt(len_sq(c)*len_sq(1.d)));
55
56
    bool colineair(Vec p, Line 1){ //returns if a point is on a line
57
         return fabs(cross(Vec(1.b, p), 1.d)) < EPS;</pre>
58
59
    double ccw(Vec p, Line 1){ //return true if p is left of l
60
         return cross(Vec(1.b, p), 1.d) < 0;
61
62
```

### 6.2 Distances

```
double dist(Vec p1, Vec p2){
    return hypot(p1.x-p2.x, p1.y-p2.y);
}

double dist(Vec p, Line 1){
    Vec c(1.b, p);
    double u = dot(c, 1.d)/len_sq(1.d);
    if(u < 0.0) return dist(1.b, p);
    else if(u > 1.0) return dist(1.b+1.d, p);
    return dist(p, 1.b+u*1.d);
}
```

### 6.3 Intersect

```
0(1)
    std::pair<bool, Vec> intersect(Line 11, Line 12){
        Vec v = Vec(11.b, 12.b);
        double c = cross(11.d, 12.d);
        if(fabs(c) < EPS){</pre>
            if((-EPS <= dot(v, 11.d) && dot(v, 11.d) <= len_sq(11.d)+EPS) ||
                 (-EPS \le dot(v, 12.d) \&\& dot(v, 12.d) \le len_sq(12.d)+EPS))
                 return std::make_pair(true, Vec()); //colinear and touching
                 else return std::make_pair(false, Vec()); //either colineair but not touching or parallel
        }else{
            double t = cross(v, l1.d)/c;
10
            double u = cross(v, 12.d)/c;
11
            if(-EPS <= t && t

← <= 1+EPS && -EPS <= u && u <= 1+EPS) return std::make_pair(true, 11.b+u*11.d); //intersects
</p>
            else return std::make_pair(false, Vec()); //not intersecting
13
        }
15
```

#### 6.4 Area

```
__ O(N) _
    //returns signed area
    double areaPolygon(const std::vector<Vec> &v){ //ADD FIRST POINT AGAIN!
2
        double res = 0.0;
        for(size_t i=0; i<v.size()-1; ++i){
            res += v[i].x*v[i+1].y-v[i+1].x*v[i].y;
5
        }
        return res/2.0;
    }
    double areaCircle(double ab, double bc, double ca){
10
        double pm = ab+bc+ca;
11
        double sp = pm/2;
12
        return std::sqrt(sp*(sp-ab)*(sp-bc)*(sp-ca));
13
    }
14
```

#### 6.5 Miscellaneous

```
__ O(N) .
    /* TEST THESE */
    bool inPolygon(Vec pt, const std::vector<Vec> &v){//ADD FIRST POINT AGAIN!
3
        double sum = 0.0;
        for(size_t i=0; i<v.size()-1; ++i){</pre>
             if(ccw(v[i+1], fromPoints(pt, v[i]))){ //extend this to handle border
                 sum += angle(v[i+1], fromPoints(pt, v[i]));
             }else sum -= angle(v[i+1], fromPoints(pt, v[i]));
        return fabs(fabs(sum) - 2*PI) < EPS;
10
11
    }
12
    Vec centroid(const std::vector<Vec> &v){//ADD FIRST POINT AGAIN!
13
14
         for(size_t i=0; i<v.size()-1; ++i){</pre>
15
             double spc = v[i].x*v[i+1].y-v[i+1].x*v[i].y;
16
             ans.x += (v[i].x+v[i+1].x)*spc;
17
             ans.y += (v[i].y+v[i+1].y)*spc;
18
        }
19
        ans = (1/(6.0*areaPolygon(v)))*ans;
20
        return ans;
21
    }
```

```
23
    double radiusInCircle(double ab, double bc, double ca){
24
        double sp = 0.5*(ab+bc+ca);
        return areaCircle(ab, bc, ca)/sp;
26
27
28
    double radiusCircumCircle(double ab, double bc, double ca){
29
        return ab*bc*ca/(4.0*areaCircle(ab, bc, ca));
30
    }
31
32
    std::pair<int, Vec> inCircleTriangle(Vec pa, Vec pb, Vec pc){
33
         double r = radiusInCircle(len(Vec(pa, pb)), len(Vec(pb, pc)), len(Vec(pa, pc)));
34
        if(fabs(r) < EPS) return std::make_pair(r, Vec());</pre>
35
36
        double ratio = len(Vec(pa, pb))/len(Vec(pa, pc));
37
        Vec pt = pb + (ratio/(1+ratio))*Vec(pb, pc);
        Line 11 = fromPoints(pa, pt); //check this line
39
40
        ratio = len(Vec(pb, pa))/len(Vec(pb, pc));
41
        pt = pa + (ratio/(1+ratio))*Vec(pa, pc);
42
        Line 12 = fromPoints(pb, pt); //check this line
43
44
        return std::make_pair(r, intersect(11, 12).second);
45
46
```

### 6.6 Convex-hull

```
_{-} O(N log(N)) .
    Vec pivot(0, 0); //will contain the point that is used as pivot
1
2
    bool angle_cmp(Vec p1, Vec p2){
         Line 1 = Line(pivot, Vec(pivot, p2));
         if(colineair(p1, 1)) return dist(pivot, p1) < dist(pivot, p2);</pre>
         return !ccw(p1, 1);
    std::vector<Vec> convex_hull(std::vector<Vec> v){ //DONT ADD FIRST POINT AGAIN
         std::vector<Vec> ans;
10
         if(v.size() <= 3) return v;</pre>
11
12
         int s = 0;
13
         for(size_t i=1; i<v.size(); ++i){</pre>
14
             if(v[i].y+EPS < v[s].y \mid | (fabs(v[i].y - v[s].y) < EPS \&\& v[i].x < v[s].x)) s = i;
15
17
         std::swap(v[0], v[s]);
         pivot = v[0];
         sort(++v.begin(), v.end(), angle_cmp);
20
21
         ans.push_back(v.back()); ans.push_back(v[0]); ans.push_back(v[1]);
22
         size_t i = 2;
23
         while(i < v.size()){</pre>
24
             size_t j = ans.size()-1;
25
             if(ccw(ans[j-1], Line(ans[j], v[i]))) ans.push_back(v[i++]);
26
             else ans.pop_back();
27
28
         return ans;
29
30
```

### 7 String

### 7.1 Edit distance

```
int dp[MAXN] [MAXN];
    inline int edit_distance(std::string a, std::string b){
        dp[0][0] = 0;
        for(size_t i=1; i<=a.size(); ++i) dp[i][0] = i;
        for(size_t j=1; j<=b.size(); ++j) dp[0][j] = j;</pre>
        for(size_t i=1; i<=a.size(); ++i){</pre>
            for(size_t j=1; j<=b.size(); ++j){</pre>
                dp[i][j] = std::min(dp[i][j-1], dp[i-1][j])+1; //add character: +1
                if(a[i-1] == b[j-1]) dp[i][j] = std::min(dp[i][j], dp[i-1][j-1]); //same character: no cost
                else dp[i][j] = std::min(dp[i][j], dp[i-1][j-1]+1); //replace character +1
10
            }
        }
12
        return dp[a.size()][b.size()];
13
```

#### 7.2 KMP

```
___ O(N) .
     int bt[MAXN]; //back table created preprocessing pattern
    void kmpPreprocess(std::string P) {
         size_t i = 0, j = -1; bt[0] = -1;
         while (i < P.size()) {</pre>
             while (j \ge 0 \&\& P[i] != P[j]) j = bt[j];
             i++; j++;
             bt[i] = j;
         }
    }
10
    void kmpSearch(std::string T, std::string P) {
11
         kmpPreprocess(P); //preprocess first
12
13
         size_t i = 0, j = 0;
         while (i < T.size()) {</pre>
15
             while (j \ge 0 \&\& T[i] != P[j]) j = bt[j];
16
             i++; j++;
17
             if (j == P.size()) {
18
                  //pattern is found in text at index i-j -- DO STUFF HERE
19
                  j = bt[j];
20
             }
         }
22
    }
23
```

#### 7.3 Suffix array

```
O(N log(N)²)

//MAXN should be 2x normal
int SA[MAXN];

int tempRa[MAXN];
int tempRa[MAXN];
int SK;
bool cmp(int a, int b){
   if(RA[a] == RA[b]) return RA[a+SK] < RA[b+SK];
   else return RA[a] < RA[b];
}

void constructSA(std::string T){
   T += '$';</pre>
```

```
int N = T.size();
14
         for(int i=0; i<2*N; ++i) {</pre>
15
              if(i < N) RA[i] = T[i];
              else RA[i] = 0;
17
              SA[i] = i;
18
         }
19
         int r = 0;
         for(SK=1; SK<N; SK<<=1){
21
              std::sort(SA, SA+N, cmp); //can also use 2x counting sort
22
23
              tempRA[SA[O]] = r = 0;
24
              for(int i=0; i<N; ++i){</pre>
25
                  if(RA[SA[i]] != RA[SA[i-1]] || RA[SA[i]+SK] != RA[SA[i-1]+SK]) ++r;
26
                  tempRA[SA[i]] = r;
27
              }
28
29
              for(int i=0; i<N; ++i) RA[i] = tempRA[i];</pre>
30
              if(RA[SA[N-1]] == N-1) break;
31
         }
32
     }
```

# 7.4 Longest common subsequence

```
_{-} O(N)
     int LCP[MAXN];
     int PLCP[MAXN];
     int PHI[MAXN];
     void constructLCP(std::string T){
         T += '$';
         int N = T.size();
         PHI[SA[O]] = -1;
         for(int i=1; i<N; ++i) PHI[SA[i]] = SA[i-1];</pre>
11
         int L = 0;
12
         for(int i=0; i<N; ++i){</pre>
13
              if(PHI[i] == -1) {
14
                  PLCP[i] = 0;
15
                  continue;
16
17
              while(T[i+L] == T[PHI[i] + L]) L++;
18
              PLCP[i] = L;
19
              L = std::max(L-1, 0);
20
21
         for(int i=0; i<N; ++i) LCP[i] = PLCP[SA[i]];</pre>
22
    }
23
```

#### 8 Other

### 8.1 Binary search (integer)

```
bool can(double f){
    //add implementation here
    return true;
}

int binsearch(int lo, int hi){
    while(lo < hi){
    int mid = (lo+hi)/2;
}</pre>
```

```
if (can(mid)) lo = mid+1;
else hi = mid;
}
return lo;
}
```

#### 8.2 Binary search (floating-point)

```
double binsearch(double lo, double hi){
    double mid, ans;
    while(std::fabs(hi-lo) > EPS){
        mid = (lo+hi)/2.0;
        if(can(mid)) {
            hi = mid;
            ans = mid;
        }else lo = mid;
    }
    return ans;
}
```

### 8.3 Counting sort

```
_{-} O(N)
     const int MAXNUM = 100;
     int ind[MAXNUM];
     int tmp[MAXN];
     int arr[MAXN];
     void counting_sort() { //stable counting sort
         for(int i=0; i<MAXNUM; ++i) ind[i] = 0;</pre>
         for(int i=0; i<MAXN; ++i){</pre>
              ind[arr[i]]++;
10
         }
         int sum = 0;
12
         for(int i=0; i<MAXNUM; ++i){</pre>
13
              int t = ind[i];
              ind[i] = sum;
15
              sum += t;
16
         }
17
18
         for(int i=0; i<MAXN; ++i){</pre>
              tmp[ind[arr[i]]++] = arr[i];
19
         }
20
         for(int i=0; i<MAXN; ++i){</pre>
21
              arr[i] = tmp[i];
22
         }
23
     }
```

# 8.4 Generate permutations

```
std::list<std::vector<int> > gen_permutation(std::vector<int> &opts, int sz){

std::list<std::vector<int> > all;

if(sz == 0){

for(size_t i=0; i<opts.size(); ++i) all.push_back({opts[i]});

return all;

}

for(size_t i=0; i<opts.size(); ++i){

std::list<std::vector<int> > lst = gen_permutation(opts, sz-1);

for(auto iter = lst.begin(); iter != lst.end(); ++iter) iter->push_back(opts[i]);

all.insert(all.end(), lst.begin(), lst.end());
```

#### 8.5 Generate combinations

```
- \mathsf{O}(N\binom{N}{K})
    std::list<std::vector<int> > gen_combinations(std::vector<int> &opts, int k){
         std::vector<bool> has(opts.size());
         std::fill(has.begin() + k, has.end(), true);
         std::list<std::vector<int> > all;
         do {
             std::vector<int> comb;
             for (size_t i = 0; i < has.size(); ++i) {</pre>
                  if (!has[i]) comb.push_back(opts[i]);
10
             all.push_back(comb);
         } while (std::next_permutation(has.begin(), has.end()));
12
13
         return all;
14
15
```

# 8.6 LIS

```
- O(N\binom{N}{K})
    int ldp[MAXN]; //lis dp array
    int lin[MAXN]; //lin[inp.size()-1] contains last index
    int frm[MAXN]; //contains index number came from
    int lis(std::vector<int> inp){ //non-decreasing lis
        for(int i=0; i<MAXN; ++i) ldp[i] = INT_MAX;</pre>
        for(size_t i=0; i<inp.size(); ++i){</pre>
             int k = std::upper_bound(ldp, ldp+MAXN, inp[i])-ldp;
             ldp[k] = inp[i];
             lin[k] = i;
10
             if(k == 0) frm[i] = -1;
11
             else frm[i] = lin[k-1];
        }
13
14
        return std::upper_bound(ldp, ldp+MAXN, INT_MAX-1)-ldp;
15
    }
```