ME 161/162 BASIC MECHANICS

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Unit Contents

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2. Course Content
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5. Definitions and Scope of Mechanics
6. Laws and Theories of Mechanics (Solid Rigid Body Mechanics)
7. Units of Measurement
8. Prefixes of SI Unit System
9. Principle of Transmissibility

Recommended Text Books

Main Textbook

Basic Engineering Mechanics by J. Antonio

Other Textbooks

1. Basic Engineering Mechanics by J. Antonio (Main Handout, (**Statics & Dynamics**)
2. Engineering Mechanics Statics by A. Pytel & J. Kiusalaas (**Statics only**)
3. Vector Mechanics for Engineers: Statics and Dynamics by F. P. Beer & E. R. Johnston (**Statics & Dynamics)**.
4. Statics and Mechanics of Materials: An Integrated Approach by W. F. Riley, et al. (**Statics only**).
5. Vector Mechanics for Engineers: Dynamics by F. P. Beer, et al. (**Dynamics only)**.
6. Principles of Dynamics by R. C. Hibbeler (**Dynamics only**)

Course Contents

FUNDAMENTALS

- Fundamental Concepts
  - Definition of Mechanics
  - Idealization of Mechanics
  - Systems of Units
  - Newton’s Laws of Motion
  - Newton’s Laws of Universal Gravitation
- Forces and Moments
  - Force Systems and Characteristics of Forces
  - Resultant of Concurrent Coplanar Forces
  - Resolution of Forces- 2D & 3D
  - Equilibrium of a Particle
  - Equilibrium of Rigid Bodies in 2D and 3D

STATICS

- Structural Analysis
  - Simple Trusses
    - Method of Joints
    - Method of Sections
  - Frames
  - Machines
- Dry Friction
  - Definitions and laws of dry friction
  - Problems involving dry friction
- Simple Machines
  - Mechanical Advantage, Velocity Ratio & Efficiency
  - Types of Simple Machines

Course Contents

STATICS

- Method of Virtual Work
  - The Principle of Virtual Work
  - Application of Virtual Work to Multi-degree-of-freedom systems
  - Application of Virtual Work to Completely Constrained systems
  - The Principle of Minimum Potential Energy

DYNAMICS

- Basic Dynamics of Particles
  - Rectilinear, curvilinear & rotational motions
  - Types of Motion: Continuous and Erratic Motions, Projectiles, Dependent Motions and Relative Motions
  - Equation of Motion
  - Work, Energy & Power
  - Momentum & Impulse of particles

- Basic Dynamics of Rigid Bodies
  - Translation of a rigid body
  - Rotation of a rigid body about a fixed axis
  - Moment of inertia
  - General plane motion of a rigid body
  - Potential & Kinetic energies of a rigid body
  - Momentum of a rigid body
  - Momentum & Impulse of Rigid bodies

- Simple Harmonic Motion (SHM)
  - Equation for SHM
  - Examples of SHM
  - Equivalent stiffness of combinations of springs
  - Energy method for conservative systems
  - Allowance for mass of spring

Course Objectives

Upon successful completion of this course, students should be able to:

1. Understand and apply Newton's laws of motion and other basic theories and laws of Newtonian mechanics to particles and rigid bodies.
2. Understand and use appropriate units of measurement, and SI unit prefixes.
3. Understand the basis of force and moments, and draw free body diagrams.
4. Analyze 2-D and 3-D equilibrium of system of forces for tensions in ropes/cables, forces in links, and support and contact reactions.
5. Determine centroids and centre of gravity of single and composite bodies.
6. Find support reactions and internal forces of two-dimensional determinant structures.
7. Solve static and dynamic problems involving dry friction.
8. Evaluate mechanical advantage, velocity ratio and efficiency of simple Machines.
9. Understand and Solve two-dimensional problems involving equation of motion, momentum, impulse and energy.
10. Solve simple one degree-of-freedom conservative vibration problems.
11. Solve simple applied mechanics problems involving combinations of items 1 to 10.

Assessment

- Continuous Assessment (30% of Total)
  1. Mid-Semester Exams (1/3 of Conti. Assess.)
  2. Homework/Assignment (1/3 of Conti. Assess.)
  3. Quiz (1/6 of Conti. Assess.)
  4. Attendance (1/6 of Conti. Assess.)
  5. Contribution to Class Discussions (Bonus, max 10%, only included if sum of items 1 to 4 is less than 30%)
- End of Semester Exams (70% of Total)
  - Multiple Choice/Fill-in Spaces (1/2 of End of Semester Exams)
  - Statics (1/4 of End of Semester Exams)
  - Dynamics (1/4 of End of Semester Exams)

Note

- You will NOT be allowed to write the End of Semester Exams if you miss at least four lectures without permission.
- All homework /assignments are due exactly a week after the assigned day. No excuse will be tolerated.

Unit 1

FUNDAMENTAL CONCEPTS

Mechanics

- *Mechanics* is a branch of physics that deals with the state of rest or motion of bodies under the action of forces.
- Categories of Mechanics:
  - Mechanics of Rigid bodies
    - *Statics-concerns with the equilibrium of bodies under the action of balanced forces*
    - ***Dynamics***-deals with motions of bodies under action of unbalanced forces. It is divided into
      - ***Kinematics-*** it deals with motion of bodies without referring to the forces that cause the motion.
      - ***Kinetics-*** it relates motion of bodies to the forces which cause the motion
  - Mechanics of Materials or Strength of Materials
    - Theory of elasticity
    - Theory of plasticity
  - Fluid Mechanics
    - Mechanics of Compressible fluids
    - Mechanics of incompressible fluids

Scope of Basic Mechanics

- ME 161/162 Basic mechanics is limited to Newtonian mechanics of particles and rigid-bodies. Other branches of mechanics which are not newtonian mechanics include:

- Relativistic mechanics, which addresses phenomena that occur at a cosmic scale (velocities approaching the speed of light, and strong gravitational fields, etc)

- Quantum mechanics, which is concerned with particles at on the atomic or subatomic scale.

Particle and Rigid Body

- A particle is a body (or an object) which is assumed to be so small that it may be regarded as geometric point. Thus, a particle has mass but its size is (assumed to be) negligible.
- When a body is idealised as a particle, the principles of mechanics reduce to a simplified form, since the geometry of the body will not be concerned in the analysis of the problem.
- All the forces acting on a particle are assumed to be applied at the same point, that is the forces are assumed to be concurrent.
- A rigid body is a collection of particles connected together in such a way that the distance between each pair of particles remains constant under all circumstances. That is, the size and the shape of rigid bodies (are assumed to) remain constant at all times.
- This is an ideal situation since all real bodies change in shape and/or size to some extent under a system of forces.

Scalar and Vector Quantities

- A scalar quantity can be described completely by magnitude. e.g. length, area, volume, mass, time, etc.

- A vector quantity is described by its magnitude, direction, line-of-action and sometimes point of application. E.g. force, moment, velocity, acceleration, momentum, etc.

- Note that some vector quantities are completely described by only magnitude and direction. E.g velocity, acceleration, momentum. But, it is inadequate to describe a force by only magnitude and direction. Similarly, moment has no line-of-action and is completely described by magnitude, direction and point of application.

Newton's Laws of Motion

The entire subject of rigid-body mechanics is formulated on the basis of the Newton's three laws of motion, which are:

1st Law: A particle at rest or moving in a straight line with constant velocity will remain in this state except compelled by an external force to act otherwise.

2nd Law: The rate of change of momentum is proportional to the applied external force and it occurs in the direction of the force.

3rd Law: For every force acting on a particle, the particle exerts an equal, opposite and collinear reactive force.

Significances and Deductions from Newton's Laws

❑ From the 1st Law

- For a particle to change its state of rest or motion (i.e to accelerate), it must be subjected to an external force. Only force can change the state of rest or motion of a body.
- However, if the particle is at rest or is moving in a straight line with constant velocity, the external forces acting on it, if any, must be balanced. That is the sum of the forces equals zero.

❑ From the 2nd Law

$$F = k\frac{d}{dt}(mv) = km\frac{d}{dt}(v) + kv\frac{d}{dt}(m)$$

If mass m = constant (for rigid bodies),

$$\frac{d}{dt}(m) = 0 \qquad \Rightarrow F = km\frac{d}{dt}(v)$$

The unit of force in SI unit, Newton, is defined such that k = unity = 1, and dv/dt = a , acceleration

$$F = km\frac{d}{dt}(v) = kma$$

$$F = ma$$

❑ From 3rd Law

- Force is due to interaction between two or more different bodies.

Relation between Newton's 1^{st} and 2^{nd} Laws

- From the 2^{nd} Law (Rigid body), $F = ma$
- If force $F = 0$ then

  either $m = 0$ or $a = 0$

  Since $m \neq 0$ for any matter, $a = 0$
- If $a=0$, then

  either $v = 0$: the body is at rest

  or $dv/dt = 0$: the body is moving in a straight line with a constant velocity.

  Note that dv = change in velocity, which is equal to sum of change in magnitude and direction. If $dv/dt = 0$, then both the magnitude and direction of the velocity are not changing.

  Hence: (a) constant velocity means no change in magnitude

  (b) straight line means no change in direction
- Hence, the 1^{st} Law is special case of the 2^{nd} Law.

Basic Definitions

- **Space** is a geometric region in which the physical events of interest in mechanics occur and it is given in terms of three coordinates measured from a reference point or origin.
- **Length** is used to specify the position of a point in space or size of a body
- **Time** is in an interval between two events or duration of an event
- **Matter** is anything that occupies space.
- **Inertia** is a property that cause a body to resist motion.
- **Mass** is a measure of inertia.
- **Force** is an action of a body upon another body.
- In Newtonian Mechanics, space (or length), time, and mass are absolute concepts, independent of each other. Force, however, is dependent of the other three. The force acting on a body is related to the mass of the body and the variation of its velocity with time.

Example 1-1

A body of mass 50 kg is acted upon by external force whose magnitude is 100 N. What is the acceleration of the body?

$$\mathit{Solution}$$

$$Mass = m = 50kg;\ Force = F = 100N$$

$$Acceleration = a = ?$$

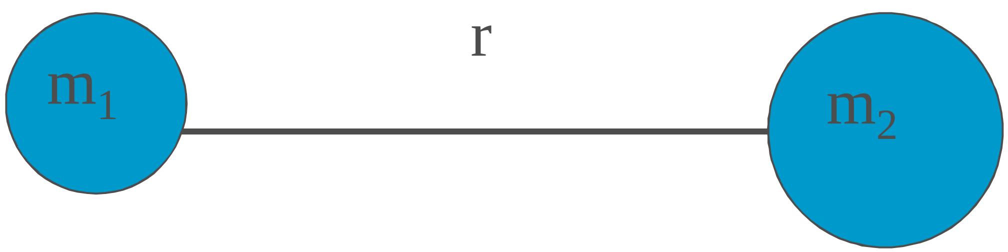
$$From: \quad F = ma$$

$$\Rightarrow\ 100N = 50kg \times a$$

$$\therefore a = \frac{100N}{50kg} = 2\,{}^{m}\!/\!{}_{s^2}$$

Law of Universal Gravitation (by Kepler)

For any two bodies separated by a distance r, the force of interaction between them is proportional to the product of their masses and inversely proportion to square of the separation distance r.



Mathematically :

$$F = G\frac{m_1 m_2}{r^2}$$

$$G = 6.673(10^{-11})\, {m^3}/{kg.s}\,[SI\ unit]$$

Mass and Weight

- Mass (m) of a body is the quantity of matter in the body and it is independent of geographical location and surroundings in which the body is placed.

- Weight (W) is the product of mass and acceleration due to gravity. Thus, weight depends on the geographical location or/and position of the body relative to some other bodies.

$$Thus\ on\ earth's\ surface$$

$$W = G\frac{m_e m}{{r_e}^2} = mg;$$

$$m_e = mass\ of\ earth$$

$$r_e = radius\ of\ earth$$

$$Therefore,\ \ g = G\frac{m_e}{{r_e}^2}$$

Example 1-2

Calculate the weight W of a body of mass 675 kg at a location on Earth where $g = 9.81\ m/s^2$.

Solution

Mass $m = 675$ kg; $g = 9.81\ m/s^2$

$$\begin{aligned}\text{Weight } W &= mg \\ &= 675 \text{ kg} \times 9.81 m/s^2 \\ &= 6.62 \times 10^3 \text{ N}\end{aligned}$$

Units of Measurement

1. The building blocks of mechanics are the physical quantities used to express the laws of mechanics.
2. Some of these quantities are mass, length, force and time.
3. Physical quantities are often divided into fundamental quantities and derived quantities.
4. Fundamental quantities cannot be defined in terms of other physical quantities. E.g time, length, mass
5. Derived quantities are those whose defining operations are based on measurement of other physical quantities. E.g. area, volume, velocity, acceleration.
6. In 1960, the Eleventh General Conference on Weights and Measures adopted a system of units of measurement based on metre, kilogram and second (abreviated as MKS) as the international standard . This international standard is known as Systèm International d'Unitès (International System of Units), for which the abbreviation is SI in all languages.
7. The SI units adopted by the conference includes three classes of units: (1) base units, (2) supplementary units, and (3) derived units. The supplementary units may be regarded as either base or derived units.

Units of Measurement

8. Derived units are expressed algebraically in terms of base units and/or supplementary units.
9. Symbols of derived units are obtained by means of the mathematical operations of multiplication and division. Eg. SI unit of velocity is metre per second (m/s).
10. Some derived SI units have special names and symbols eg. SI unit of force (which is derived from mass, time and length) is newton.
11. In SI units, all symbols bearing people's name begins with a capital letter. Eg N for newton.
12. Prefixes are used to form names and symbols for decimal multiples and submultiples of SI units. The multiple should be chosen so that numerical values of the quantity will be between 0.1 and 1000.
13. Only one prefix should be used in forming a multiple of a unit.
14. A prefix cannot be denominator.

Base SI Units and their Symbols

Base Units and Their Symbols

Quantity	Name of SI Unit	Symbol
Length	meter	m
Mass	kilogram	kg
Time	second	s
Electric Current	Ampere	A
Temperature	Kelvin	K
Amount of Substance	mole	mol
Luminous intensity	candela	cd

Supplementary Units and Their Symbols

Quantity	Name of SI Unit	Symbol
Plane Angle	radian	rad
Solid Angle	steradian	sr

Derived SI Units

Quantity	Derived SI Unit	Symbol	Special Name
Area	square meter	m^2	
Volume	cubic meter	m^3	
Linear Velocity	meter per second	m/s	
Linear Acceleration	meter per second squared	m/s^2	
Frequency	(cycle) per second	Hz	Hertz
Density	kilogram per cubic meter	Kg/m^3	
Force	Kilogram meter per second squared	N	Newton
Pressure and Stress	Newton per meter squared	Pa	Pascal
Work and Energy	Newton.meter	J	Joule
Power	Joule per second	W	Watt
Moment of Force	Newton-meter	N.m	

Some Approved Prefixes of SI Units

Factor by which unit is multiplied	Name of prefix	Symbol of prefix	Example
10^{12}	tera	T	1.23 TJ = 1 230 000 000 000 J
10^{9}	giga	G	4.53 GPa = 4 530 000 000 Pa
10^{6}	mega	M	7.68 MW = 7 680 000 W
10^{3}	kilo	k	5.46 kg = 5 460 g
10^{-2}	centi	c	3.34 cm = 0.0334 m
10^{-3}	milli	m	395 mm = 0.395 m
10^{-6}	micro	µ	65 µm = 0.000 065 m
10^{-9}	nano	n	34 nm = 0.000 000 034 m

Note: Always use an appropriate prefix to state numerical value of a quantity between 0.1 and 1000.

Example 1-3

Two balls of masses 5 kg and 10 kg are separated by a distance of 1 m. Calculate the force of attraction between them.

Solution

$$F = G\frac{m_1 m_2}{d^2} = 6.673 \text{ x } 10^{-11} \frac{5 \text{ x } 10}{1^2}$$

$$F = 3.3 \text{ x } 10^{-9} \text{ N}$$

F= 3.3 nN

Dimensional Considerations and Homogeneity

- All physical quantities encountered in mechanics can be expressed dimensionally in terms of the three fundamental quantities: mass, length and time, denoted respectively by M, L and T.
- Dimensions of derived quantities are derived from definitions or physical laws. E.g Dimension of velocity is L/T, which follows from the the definition of velocity: rate of change of position with time.
- An equation is said to be dimensionally homogeneous if the form of the equation does not depend on the units of measurement.

Dimensions of Physical Quantities of Mechanics

Physical Quantity	Dimension	Common SI Units
Length	L	m, cm, mm
Area	L^2	m^2, cm^2, mm^2
Angle	1(L/L)	rad, degree
Time	T	s
Linear velocity	L/T or LT^{-1}	m/s or ms^{-1}
Linear acceleration	L/T^2 or LT^{-2}	m/s^2 or ms^{-2}
Angular velocity	$1/T$ or T^{-1}	rad/s
Angular acceleration	$1/T^2$ or T^{-2}	rad/s^2
Mass	M	kg
Force	ML/T^2 or MLT^{-2}	N
Moment of a force	ML^2/T^2 or ML^2T^{-2}	N.m or N-m
Pressure, Stress	M/LT^2 or $ML^{-1}T^{-2}$	Pa, kPa, MPa
Work and Energy	ML^2/T^2 or ML^2T^{-2}	J, kJ
Power	ML^2/T^3 or ML^2T^{-3}	W, kW
Momentum and linear impulse	ML/Tor MLT^{-1}	N.s or N-s

Example 1-4

Determine the dimensions of I, R, w, M and C in the dimensionally homogeneous equation

$$EIy = Rx^3 - P(x-a)^3 - wx^4 + Mx^2 + C$$

in which x and y are lengths, P is a force, and E is a force per unit area.

Solution

The equation can be written dimensionally as

$$\left(\frac{F}{L^2}\right)(I)(L) = R(L)^3 - (F)(L-a)^3 - (w)(L)^4 + (M)(L)^2 + C$$

For the above equation to be homogeneous, a must be a length; hence

$$\left(\frac{F}{L^2}\right)(I)(L) = R(L)^3 = (F)(L)^3 = (w)(L)^4 = (M)(L)^2 = C$$

The dimensions of each of the unknown quantities are obtained as follows:

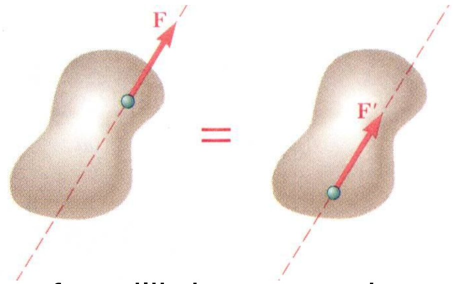
$$I = \left(\frac{L}{F}\right)(FL^3) \qquad I = L^4 \qquad M = \left(\frac{1}{L^2}\right)(FL^3) \qquad M = FL$$

$$R = \left(\frac{1}{L^3}\right)(FL^3) \qquad R = F \qquad C = FL^3$$

$$w = \left(\frac{1}{L^4}\right)(FL^3) \qquad w = \frac{F}{L}$$

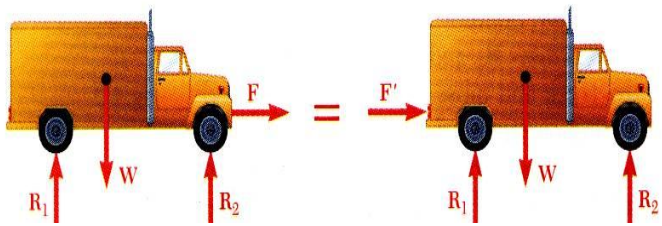
Principle of Transmissibility

The *principle of transmissibility* states that the point of application a force on a rigid body may be placed anywhere along its line of action without changing the conditions of equilibrium or motion of the body

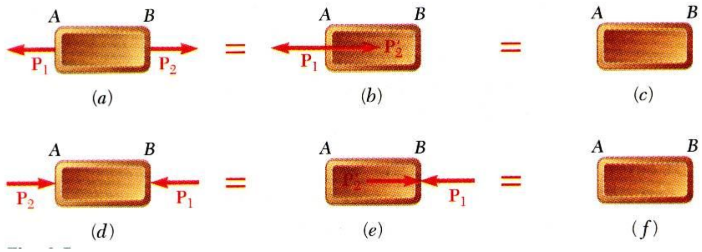


Conditions of equilibrium or motion are not affected by *transmitting* a force along its line of action. NOTE: **F** and **F**' are equivalent forces.

Moving the point of application of the force **F** to the rear bumper does not affect the motion or the other forces acting on the truck.



Principle of transmissibility may not always apply in determining internal forces and deformations.



ME 161/162 Basic Mechanics

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Unit 2

FORCES AND MOMENTS

Contents

Force Systems

- A *force* is any effect that may change the state of rest or motion of a body (Refer to Newton's 1st Law).

- A *force system* comprises of two or more forces acting on a body or a group of related bodies.

- Force systems are classified according to the arraignment of constituent forces.

- A body is in equilibrium when all the resultant forces acting on the body is zero.

- Note: No method exists for directly measuring a force. In mechanics, we use cause-and-effect relationship to measure/determine force e.g. spring scale uses deflection of spring to measure weight (force) and beam scale uses balancing of moment to measure weight (force).

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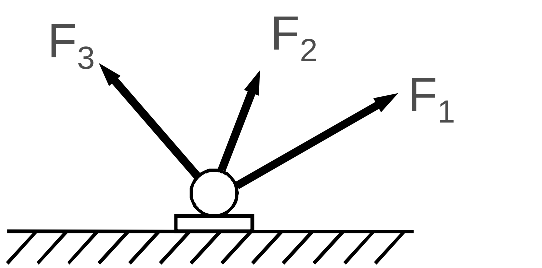
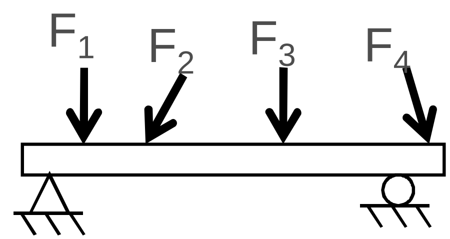
Types of Force System

A force system may be one-, two- (planar) or three-dimensional (spatial). A force system is said to be

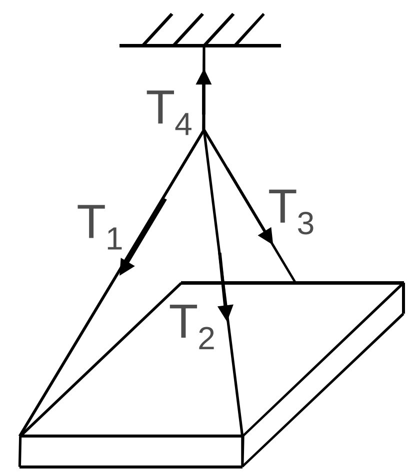
- *collinear* if the forces have a common line of action.
- *parallel* if the lines of action of the forces are parallel.
- *concurrent* if the lines of action of the forces intersect at a common point
- Coplanar when all the forces lie in the same plane.
- Spatial when all the forces do not lie in the same plane.



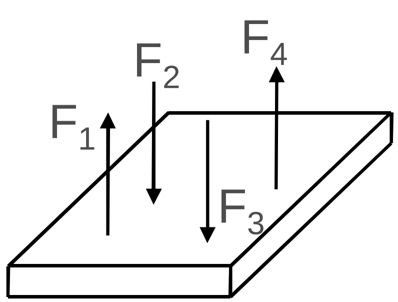
Concurrent Coplanar Force System



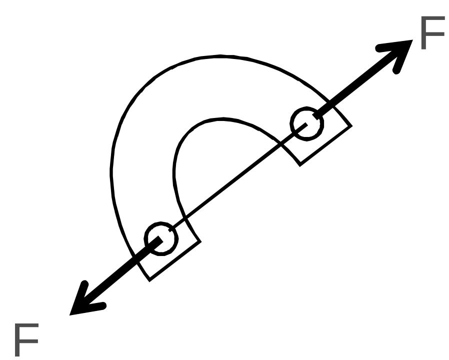
Non-Concurrent Coplanar Force System



Concurrent Spatial Force System

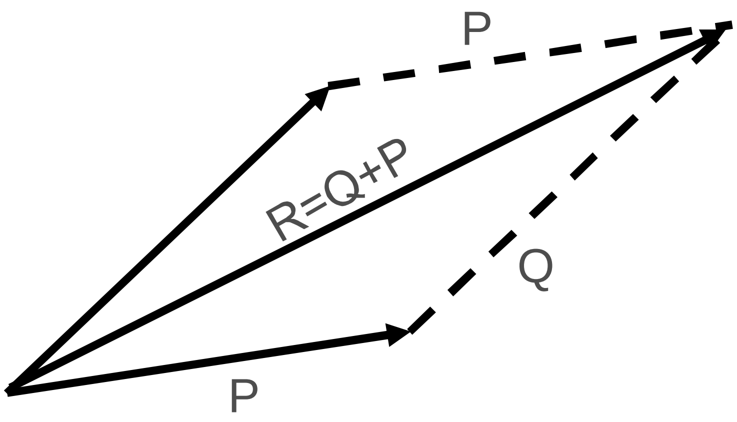


Parallel spatial Force System

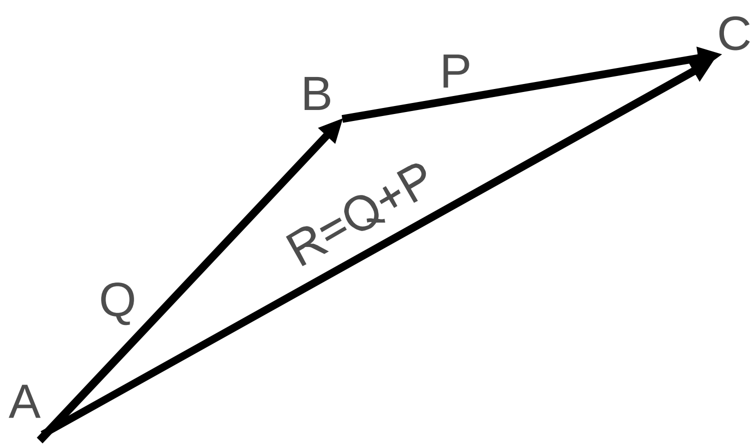


Collinear Force System

Addition of Vectors



▪Parallelogram Law



where A, B and C are angles

- Cosine Law (or Cosine rule)

$$R^2 = P^2 + Q^2 - 2PQ\cos B$$

- Sine Law (or sin rule)

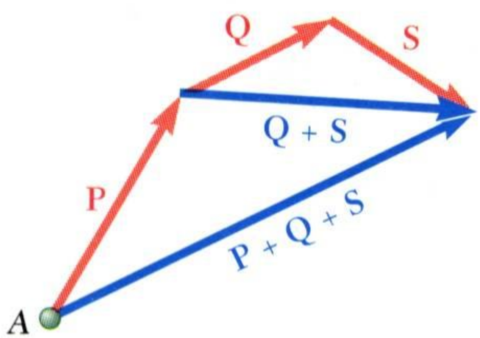
$$\frac{\sin A}{P} = \frac{\sin B}{R} = \frac{\sin C}{Q}$$

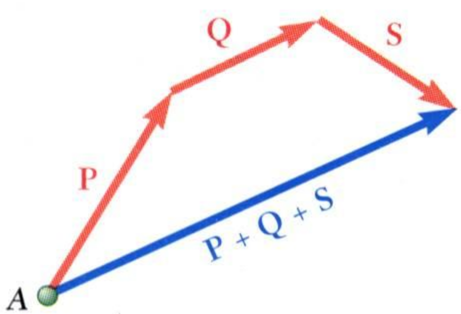
- Vector addition is commutative

$$\vec{P} + \vec{Q} = \vec{Q} + \vec{P}$$

- Note : P, Q and R are magnitudes of vectors $\vec{P}, \vec{Q}$ and $\vec{R}$ respectively. A, B and C are interior angles of the vector triangle.

Addition of Vectors



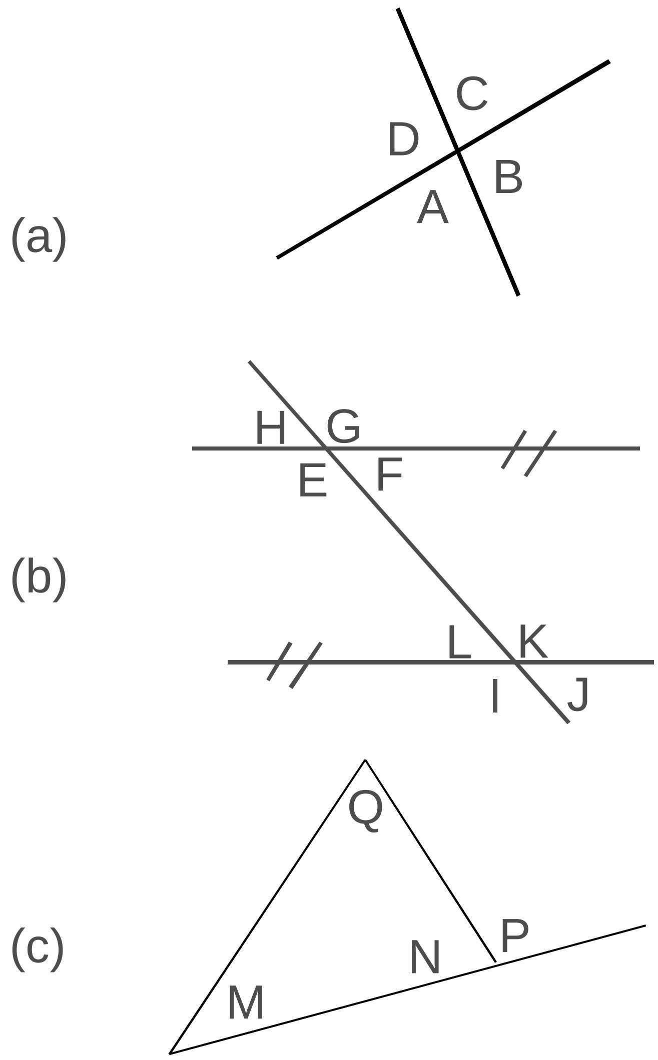


- Addition of three or more vectors through repeated application of the triangle rule

- The polygon rule for the addition of three or more vectors.

$$\vec{P}+\vec{Q}+\vec{S}=\left(\vec{P}+\vec{Q}\right)+\vec{S}=\vec{P}+\left(\vec{Q}+\vec{S}\right)$$

Brief Review of Geometry

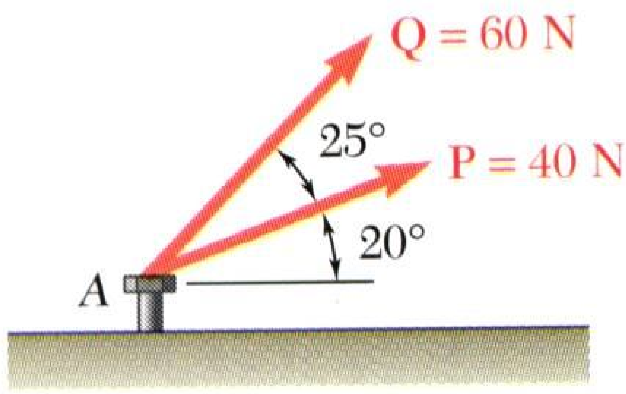


- From Diagram (a),
  D + A = 180º
  D + C = 180º
  ∴ A = C and B = D

From Diagram (b),
G = E=K=I
H =F=L=J

From Diagram (c),
M + N + Q =180
N =180 - P

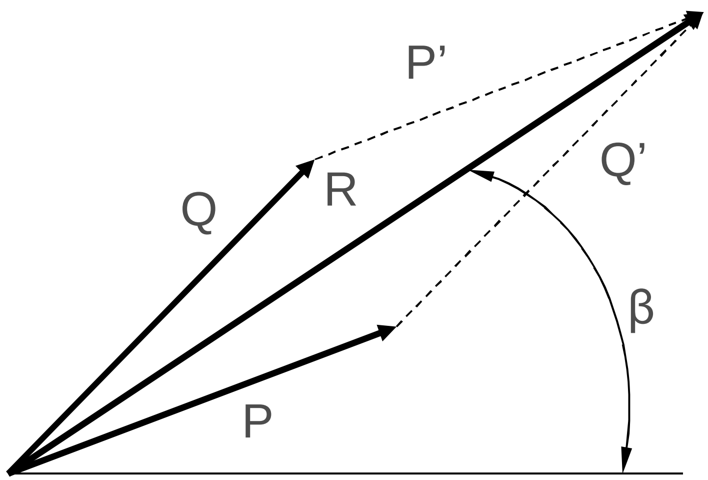
Example 2-1



Determine resultant of forces Q and P acting on the bolt shown above and angle the resultant makes with the horizontal axis.

Graphical Solution

Graphical solution - A half or full parallelogram with sides equal to **P** and **Q** is drawn to scale.

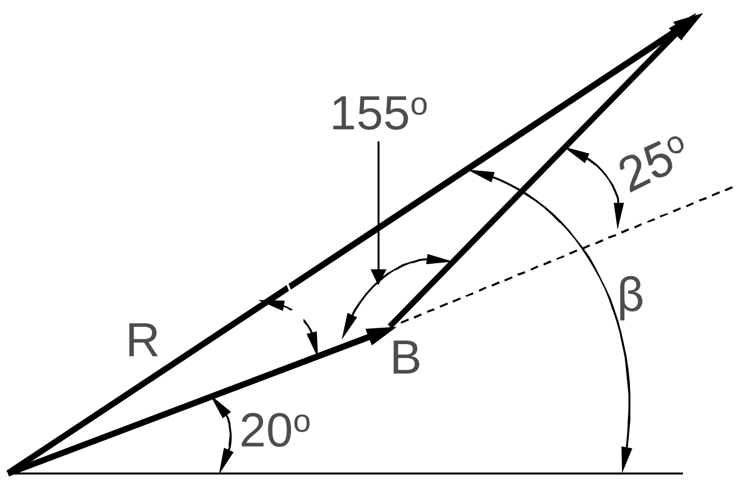


The magnitude and direction of the resultant or of the diagonal to the parallelogram are measured,

$R = 98$ N $\beta = 35^o$

Example 2-1, Cont;

- Parallelogram solution



From the Law of Cosines,

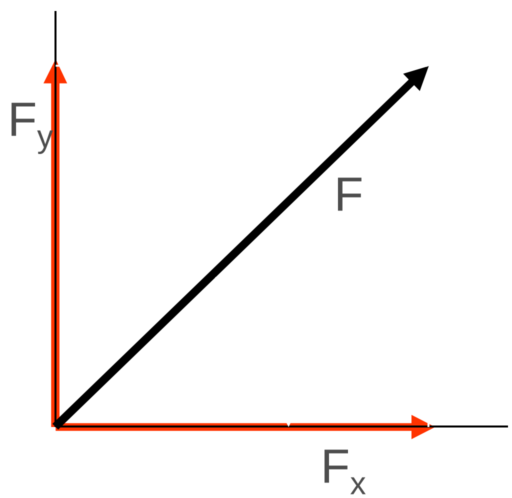
$$R^2 = P^2 + Q^2 - 2PQ\cos B$$
$$= (40\text{N})^2 + (60\text{N})^2 - 2(40\text{N})(60\text{N})\cos 155°$$

R = 97.73 N

From the Law of Sines,

$$\frac{\sin A}{Q} = \frac{\sin B}{R}$$
$$\sin\left(\beta - 20^o\right) = \sin B \frac{Q}{R}$$
$$= \sin 155° \frac{60\text{N}}{97.73\text{N}}$$
$$\beta - 20^o = 15°$$
$$\beta = 35^o$$

Rectangular Components of Force



A force vector may be resolved into perpendicular components so that the resulting parallelogram is a rectangle. The resulting x and y components are referred to as *rectangular vector components* and

$$\vec{F} = \vec{F}_x + \vec{F}_y$$

Vector components may be expressed as products of the unit vectors with the scalar magnitudes of the vector components.

$$\vec{F} = \vec{F}_x i + \vec{F}_y j$$

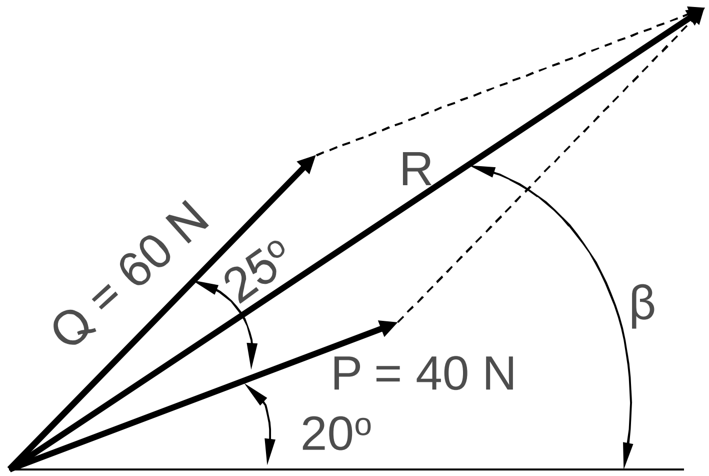
F_x and F_y are referred to as the *scalar components*

$$F = \sqrt{\left(F_x^{\ 2} + F_y^{\ 2}\right)}$$

$$\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right)$$

Example 2-2

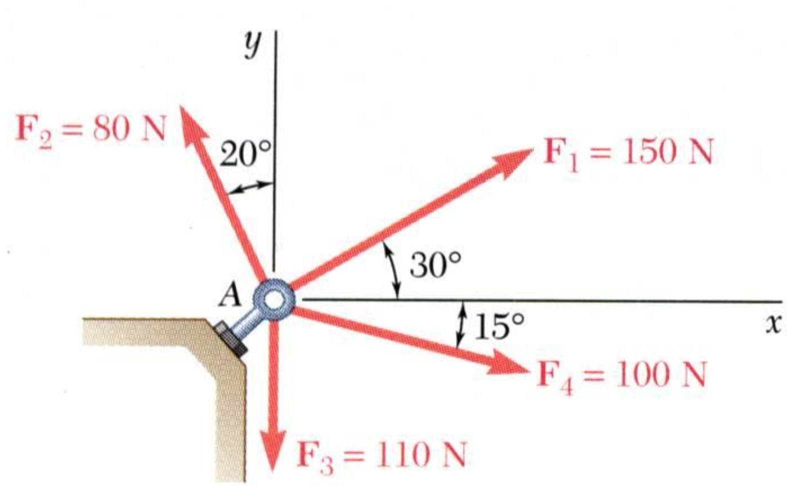
- Solve Example 3 using Rectangular Components solution



- $R_x = \Sigma F_x = P\cos 20^o + Q\cos(20^o + 25^o)$
  $= 40\cos 20^o + 60\cos 45^o$
  $= 80.014$ N

- $R_y = \Sigma F_y = P\sin 20^o + Q\sin(20^o + 25^o)$
  $= 40\sin 20^o + 60\sin 45^o$
  $= 57.107$ N

Example 2-3

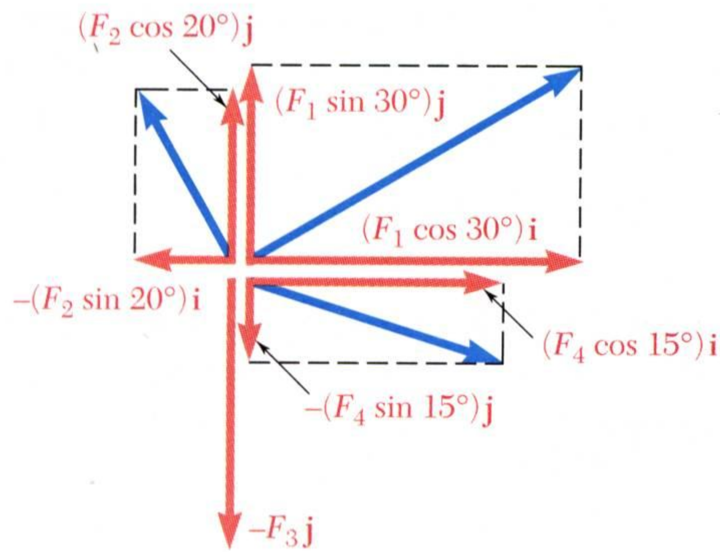


Four forces act on bolt *A* as shown. Determine the resultant of the force on the bolt.

SOLUTION:

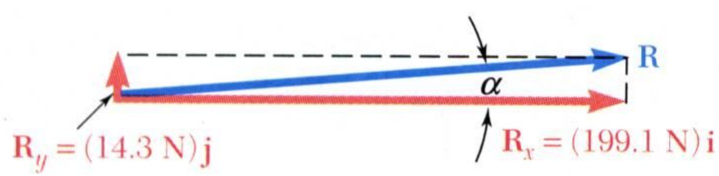
- Resolve each force into rectangular components.
- Determine the components of the resultant by adding the corresponding force components.
- Calculate the magnitude and direction of the resultant.

Example 2-3, Cont;



- Resolve each force into rectangular components.

force	*mag*	$x-comp$	$y-comp$
$\vec{F}_1$	150	$+129.9$	$+75.0$
$\vec{F}_2$	80	-27.4	$+75.2$
$\vec{F}_3$	110	0	-110.0
$\vec{F}_4$	100	$+96.6$	-25.9
		$R_x = +199.1$	$R_y = +14.3$



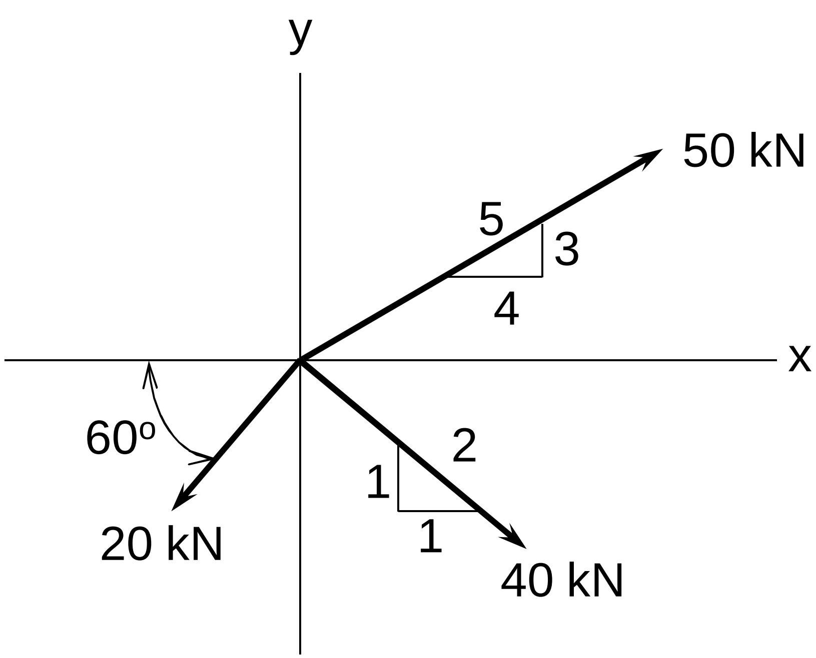
- Determine the components of the resultant by adding the corresponding force components.

$$R = \sqrt{199.1^2 + 14.3^2} \qquad \boxed{R = 199.6\mathrm{N}}$$

$$\tan\alpha = \frac{14.3\,\mathrm{N}}{199.1\,\mathrm{N}} \qquad \boxed{\alpha = 4.1°}$$

Example 2-4

- Determine the magnitude of the resultant force and its direction measured from the positive x axis.



Solution

$$R_x = \sum_{i=1}^{n} F_i Cos\theta_i$$

$$R_x = 50(4/5) - 20Cos60 + 40(1/\sqrt{2})$$

$$R_x = 58.284N$$

$$R_y = \sum_{i=1}^{n} F_i Sin\theta_i$$

$$R_y = 50(3/5) - 20Sin60 - 40(1/\sqrt{2})$$

$$R_y = -15.604N$$

$$R = \sqrt{\left(R_x^{\;2} + R_y^{\;2}\right)}$$

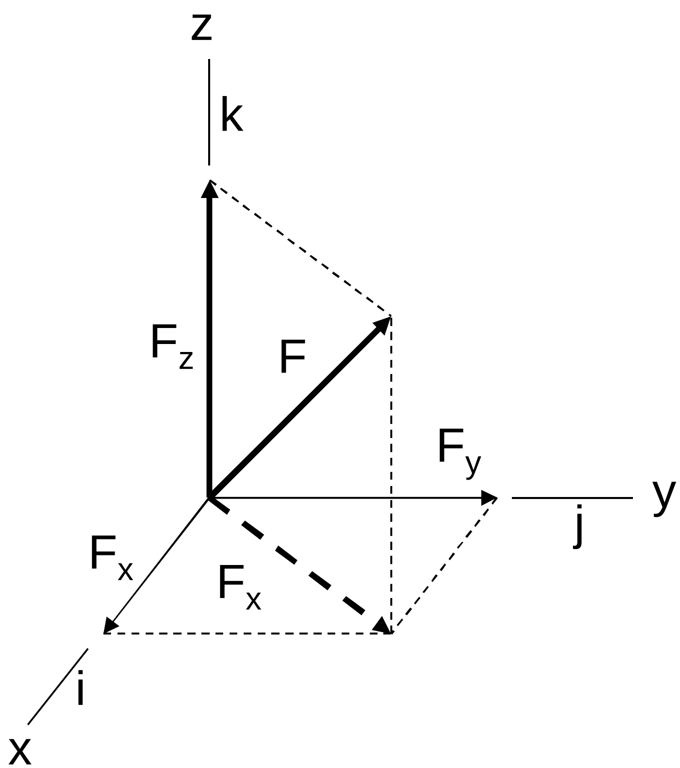
$$R = \sqrt{\left(58.284^2 + (-15.604)^2\right)}$$

$$R = 60.34N$$

$$\beta = \tan^{-1}\left(\frac{R_y}{R_x}\right)$$

$$\beta = \tan^{-1}\left(\frac{-15.604}{58.284}\right) = -15^o \text{ or } 345^o$$

Cartesian Vectors



- In right-handed Cartesian coordinated system, the right thumb points in the positive z direction when the right hand figures are curled from positive x direction to positive y direction about the z axis.
- The three axes x, y and z are right angle to each other.
- The unit vectors I, j and k are the unit vectors along the x, y and z axes, respectively.

Vector Representation

- Vectors are represented by the magnitudes and directions of the three components using the unit vectors I, j and k. e.g.

$$F = F_x i + F_y j + F_z k$$

- The magnitude of the vector is given as

$$|F| = \sqrt{\left(F_x^2 + F_y^2 + F_z^2\right)}$$

- A unit vector U is vector with unit (1) magnitude.

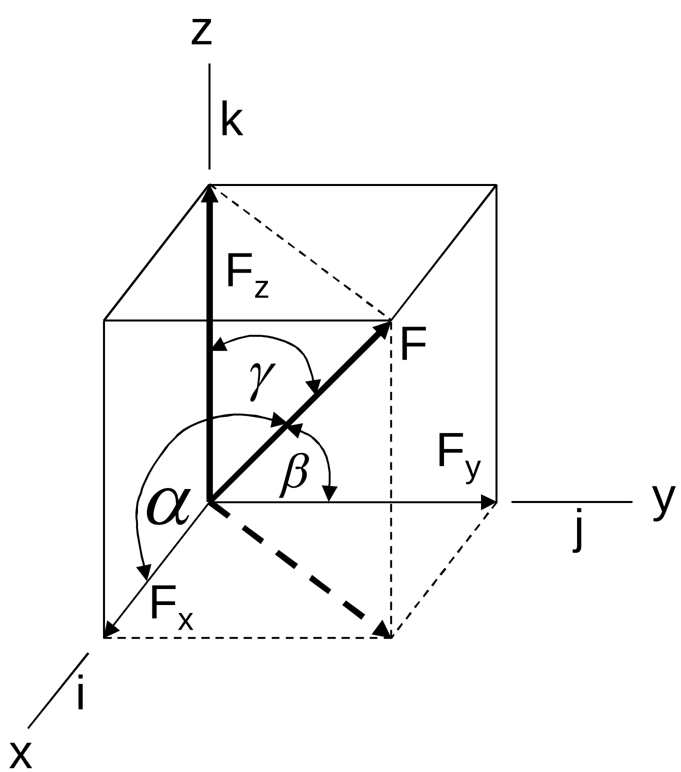
- If $|F| \neq 0$

$$U = \frac{1}{|F|}\left(F_x i + F_y j + F_z k\right)$$

$$U = \frac{F_x}{|F|} i + \frac{F_y}{|F|} j + \frac{F_z}{|F|} k$$

Direction Cosines

The direction cosines are the cosine of angles between the tail of a vector and the positive x, y and z axes.



Direction Cosines

$$\cos\alpha = \frac{F_x}{|F|} \qquad \cos\beta = \frac{F_y}{|F|} \qquad \cos\gamma = \frac{F_z}{|F|}$$

$$U_F = \cos\alpha i + \cos\beta j + \cos\gamma k$$

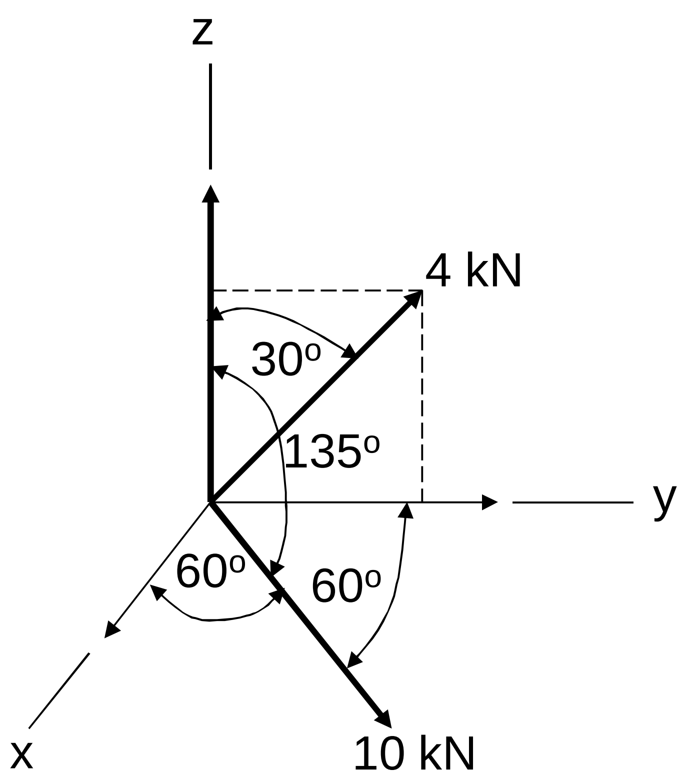
$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

If a force of magnitude $|F|$ is directed along a line r, then the force vector is defined as

$$F = |F| \frac{r}{|r|}$$

Example 2-5

Determine the magnitude and coordinate direction angles of the resultant force.



Solution

$$F_1 = 4\left(0i + \sin 30^o\, j + \cos 30^o\, k\right) kN$$

$$F_1 = \left(2j + 3.464k\right) kN$$

$$F_2 = 10\left(\cos 60^o\, i + \cos 60^o\, j + \cos 135^o\, k\right) kN$$

$$F_1 = \left(5i + 5j - 7.0711k\right) kN$$

$$F = F_1 + F_2$$

$$F = \left(5i + 7j - 3.607k\right) kN$$

$$\left|F\right| = \sqrt{\left(5^2 + 7^2 + \left(-3.607\right)^2\right)} = 9.328$$

$$U_F = \frac{F}{\left|F\right|} = 0.536i + 0.7504j - 0.3867k$$

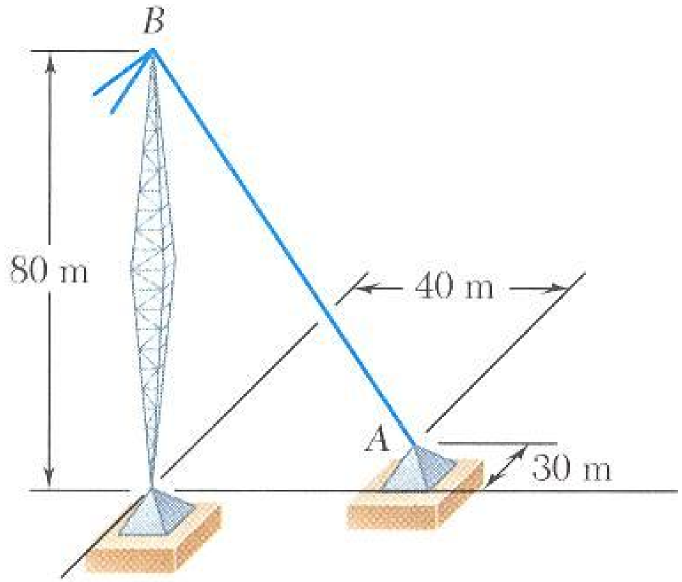
$$\text{Direction Cosine} = \cos^{-1}\left(U_F\right)$$

$$\alpha = 57.59^o$$

$$\beta = 41.37^o$$

$$\gamma = 112.70^o$$

Example 2-6



The tension in the guy wire is 2500 N. Determine:

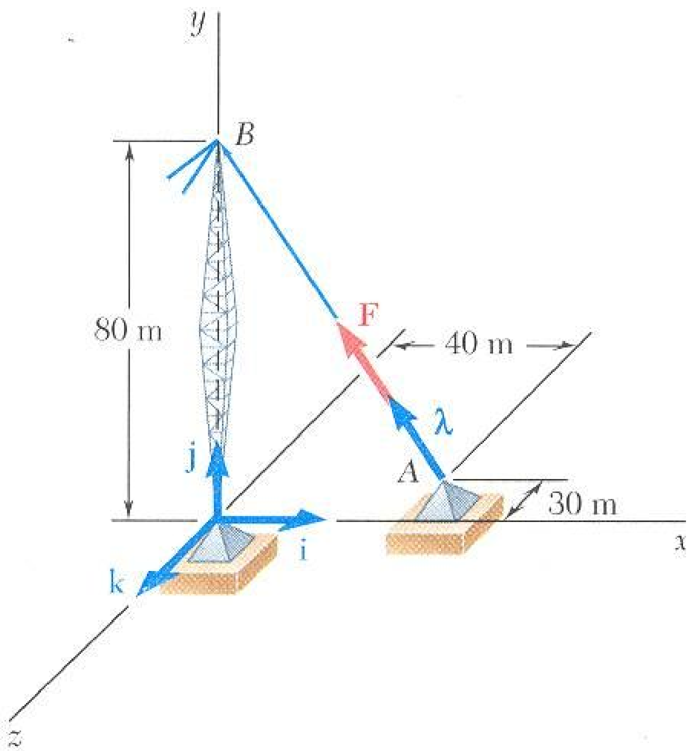
a) components F_x, F_y, F_z of the force acting on the bolt at *A*,

b) the angles θ_x, θ_y, θ_z defining the direction of the force

SOLUTION:

- Based on the relative locations of the points *A* and *B*, determine the unit vector pointing from *A* towards *B*.
- Apply the unit vector to determine the components of the force
- Noting that the components of the unit vector are the direction cosines for the vector, calculate the corresponding angles.

Example 2-6



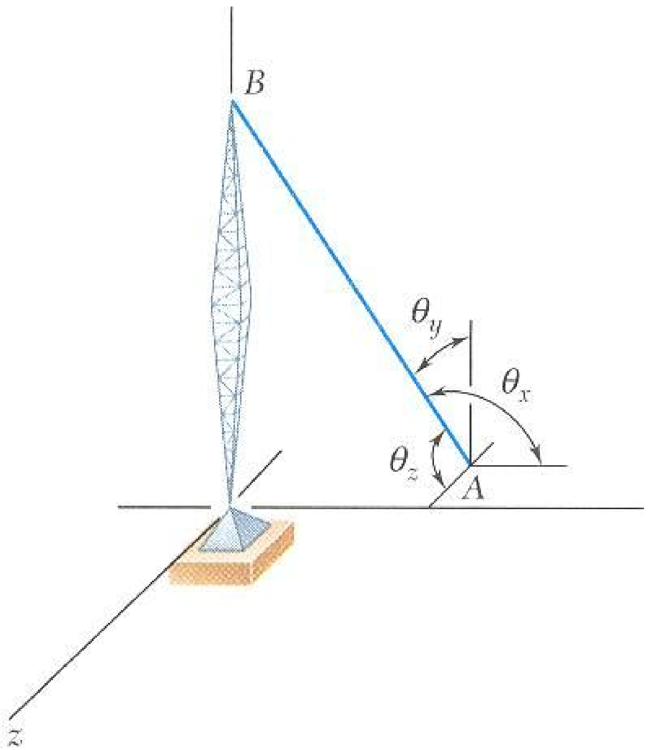
$$\overrightarrow{AB} = (-40\,\text{m})\vec{i} + (80\,\text{m})\vec{j} + (30\,\text{m})\vec{k}$$

$$AB = \sqrt{(-40\,\text{m})^2 + (80\,\text{m})^2 + (30\,\text{m})^2}$$
$$= 94.3\,\text{m}$$

$$\vec{\lambda} = \left(\frac{-40}{94.3}\right)\vec{i} + \left(\frac{80}{94.3}\right)\vec{j} + \left(\frac{30}{94.3}\right)\vec{k}$$
$$= -0.424\vec{i} + 0.848\vec{j} + 0.318\vec{k}$$

$$\vec{F} = F\vec{\lambda}$$
$$= (2500\,\text{N})\left(-0.424\vec{i} + 0.848\vec{j} + 0.318\vec{k}\right)$$
$$= (-1060\,\text{N})\vec{i} + (2120\,\text{N})\vec{j} + (795\,\text{N})\vec{k}$$

Example 2-6



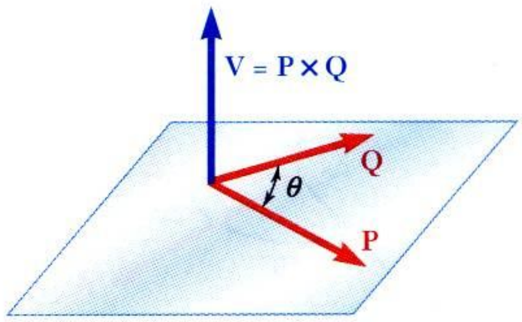
$$\vec{\lambda} = \cos\theta_x \vec{i} + \cos\theta_y \vec{j} + \cos\theta_z \vec{k}$$

$$= -0.424\vec{i} + 0.848\vec{j} + 0.318\vec{k}$$

$$\boxed{\begin{aligned} \theta_x &= 115.1^\circ \\ \theta_y &= 32.0^\circ \\ \theta_z &= 71.5^\circ \end{aligned}}$$

Product of Two Vectors

- Concept of the moment of a force about a point is more easily understood through applications of the *vector product* or *cross product*.
- Vector product of two vectors $\boldsymbol{P}$ and $\boldsymbol{Q}$ is defined as the vector $\boldsymbol{V}$ which satisfies the following conditions:
  1. Line of action of $\boldsymbol{V}$ is perpendicular to plane containing $\boldsymbol{P}$ and $\boldsymbol{Q}$.
  2. Magnitude of $\boldsymbol{V}$ is $V = PQ \sin\theta$
  3. Direction of $\boldsymbol{V}$ is obtained from the right-hand rule.



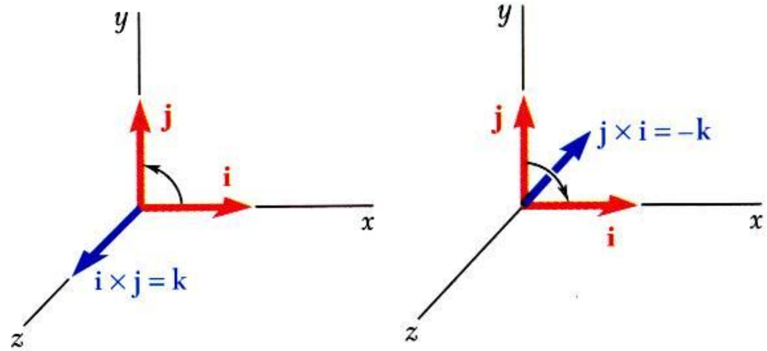
(a)

(b)

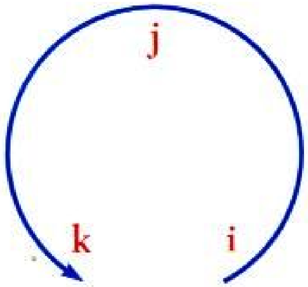
- Vector products:
  - are not commutative, $\boldsymbol{Q} \times \boldsymbol{P} = -(\boldsymbol{P} \times \boldsymbol{Q})$
  - are distributive, $\boldsymbol{P} \times (\boldsymbol{Q}_1 + \boldsymbol{Q}_2) = \boldsymbol{P} \times \boldsymbol{Q}_1 + \boldsymbol{P} \times \boldsymbol{Q}_2$
  - are not associative, $(\boldsymbol{P} \times \boldsymbol{Q}) \times \boldsymbol{S} \neq \boldsymbol{P} \times (\boldsymbol{Q} \times \boldsymbol{S})$

Vector Products

$$\vec{i}\times\vec{i}=0 \qquad \vec{j}\times\vec{i}=-\vec{k} \qquad \vec{k}\times\vec{i}=\vec{j}$$
$$\vec{i}\times\vec{j}=\vec{k} \qquad \vec{j}\times\vec{j}=0 \qquad \vec{k}\times\vec{j}=-\vec{i}$$
$$\vec{i}\times\vec{k}=-\vec{j} \qquad \vec{j}\times\vec{k}=\vec{i} \qquad \vec{k}\times\vec{k}=0$$



$$\vec{V}=\left(P_x\vec{i}+P_y\vec{j}+P_z\vec{k}\right)\times\left(Q_x\vec{i}+Q_y\vec{j}+Q_z\vec{k}\right)$$
$$=\left(P_yQ_z-P_zQ_y\right)\vec{i}+\left(P_zQ_x-P_xQ_z\right)\vec{j}$$
$$+\left(P_xQ_y-P_yQ_x\right)\vec{k}$$



$$=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ P_x & P_y & P_z\\ Q_x & Q_y & Q_z\end{vmatrix}$$

Moment of a Force

- The *moment* of $\boldsymbol{F}$ about O is defined as

$$\boldsymbol{M_O} = \boldsymbol{r} \times \boldsymbol{F}$$

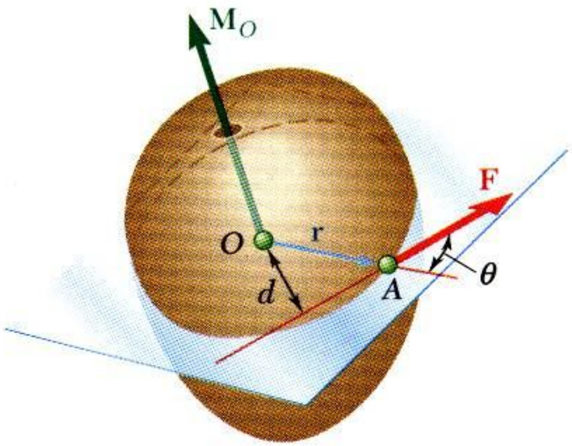
- The moment vector $\boldsymbol{M_O}$ is perpendicular to the plane containing O and the force $\boldsymbol{F}$.

- Magnitude of $\boldsymbol{M_O}$ measures the tendency of the force to cause rotation of the body about an axis along $\boldsymbol{M_O}$.

$$M_O = rF \sin\theta = Fd$$

  The sense of the moment may be determined by the right-hand rule.

- Any force $\boldsymbol{F'}$ that has the same magnitude and direction as $\boldsymbol{F}$, is *equivalent* if it also has the same line of action and therefore, produces the same moment.

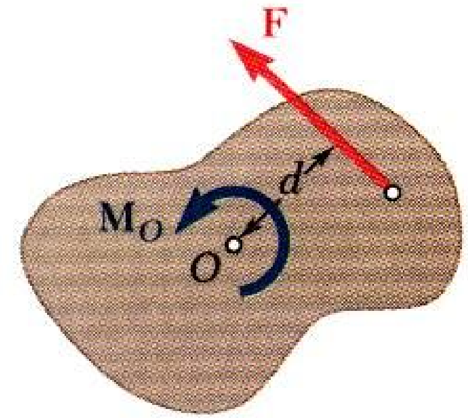


(a)

(b)

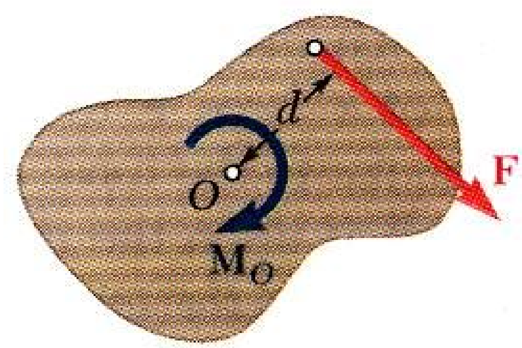
Moment of a Force

- *Two-dimensional structures* have length and breadth but negligible depth and are subjected to forces contained in the plane of the structure.
- The plane of the structure contains the point O and the force $\boldsymbol{F}$. $\boldsymbol{M_O}$, the moment of the force about O is perpendicular to the plane.



(a) $\mathbf{M}_O = +Fd$

- If the force tends to rotate the structure clockwise, the sense of the moment vector is out of the plane of the structure and the magnitude of the moment is positive
- If the force tends to rotate the structure counterclockwise, the sense of the moment vector is into the plane of the structure and the magnitude of the moment is negative.



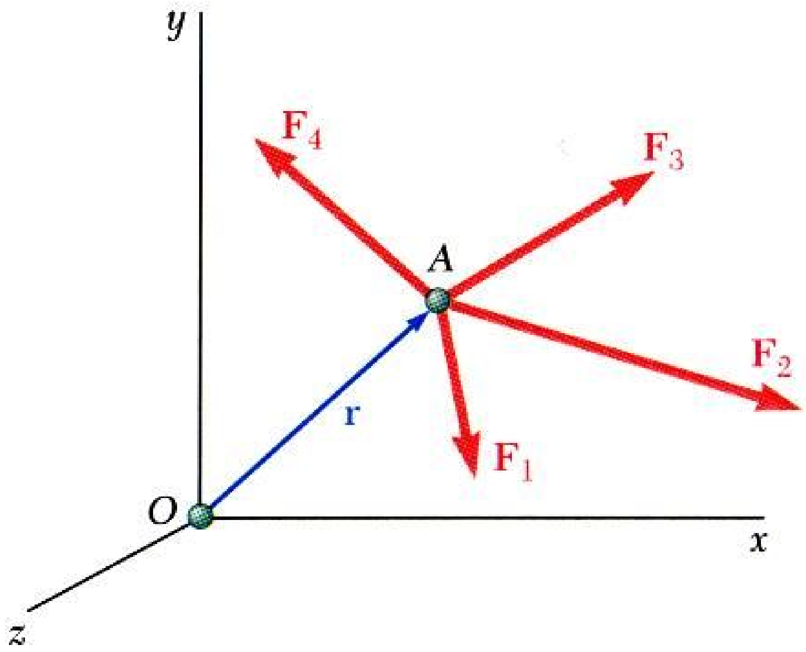
(b) $\mathbf{M}_O = -Fd$

Varignon's Theorem

- The moment about a give point O of the resultant of several concurrent forces is equal to the sum of the moments of the various moments about the same point O.

$$\vec{r} \times \left(\vec{F}_1 + \vec{F}_2 + \cdots\right) = \vec{r} \times \vec{F}_1 + \vec{r} \times \vec{F}_2 + \cdots$$

- Varignon's Theorem makes it possible to replace the direct determination of the moment of a force ***F*** by the moments of two or more component forces of ***F***.



Rectangular Components of Moment of a Force

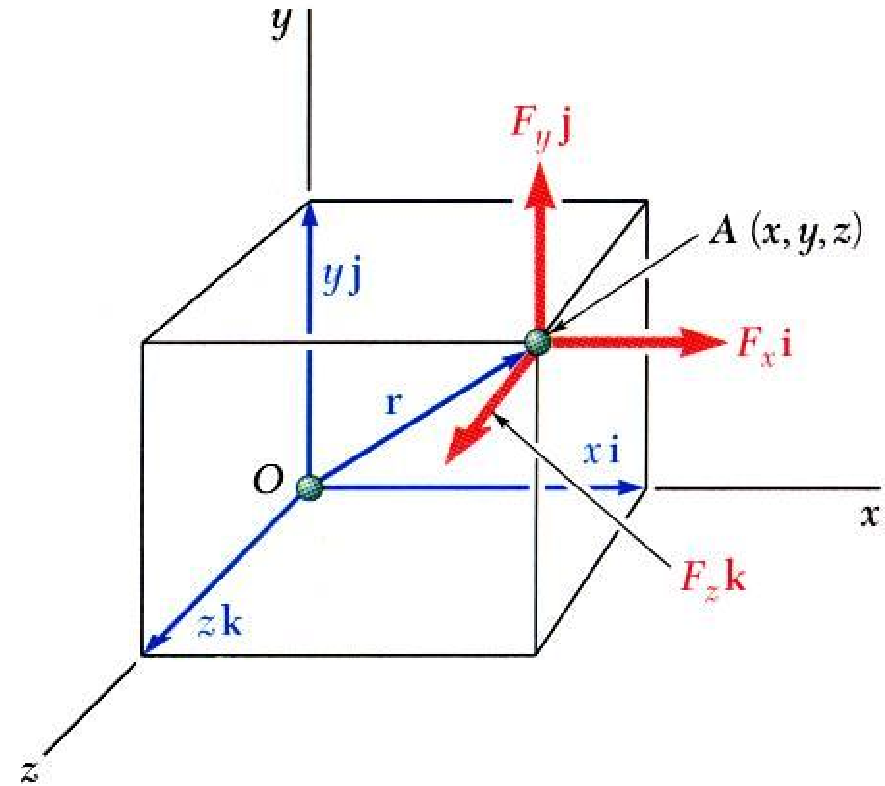
$$\vec{M}_O = \vec{r} \times \vec{F}, \qquad \vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$$

$$\vec{F} = F_x\vec{i} + F_y\vec{j} + F_z\vec{k}$$

$$\vec{M}_O = M_x\vec{i} + M_y\vec{j} + M_z\vec{k}$$

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ x & y & z \\ F_x & F_y & F_z \end{vmatrix}$$

$$= \left(yF_z - zF_y\right)\vec{i} + \left(zF_x - xF_z\right)\vec{j} + \left(xF_y - yF_x\right)\vec{k}$$



Rectangular Components of Moment of a Force

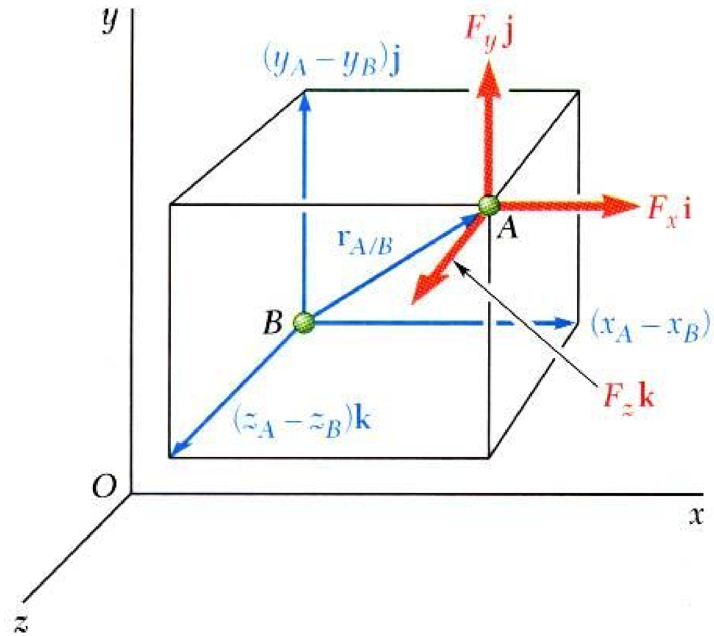
$$\vec{M}_B = \vec{r}_{A/B} \times \vec{F}$$

$$\vec{r}_{A/B} = \vec{r}_A - \vec{r}_B$$

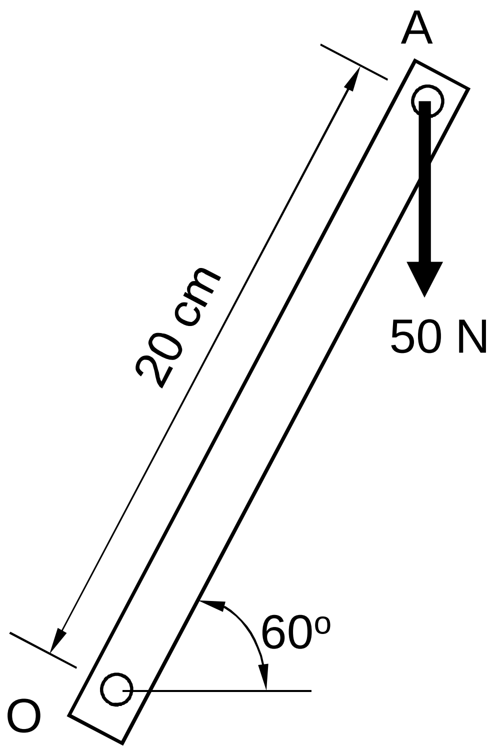
$$= (x_A - x_B)\vec{i} + (y_A - y_B)\vec{j} + (z_A - z_B)\vec{k}$$

$$\vec{F} = F_x\vec{i} + F_y\vec{j} + F_z\vec{k}$$

$$\vec{M}_B = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ (x_A - x_B) & (y_A - y_B) & (z_A - z_B) \\ F_x & F_y & F_z \end{vmatrix}$$



Example 2-7



50-N vertical force is applied to the end of a lever which is attached to a shaft at *O*.

Determine:

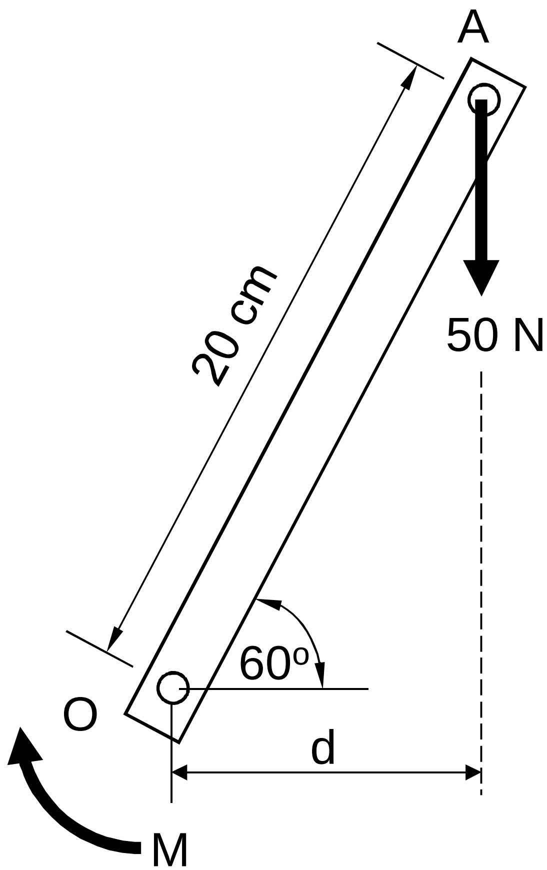
a) moment about *O*,

b) horizontal force at *A* which creates the same moment,

c) smallest force at A which produces the same moment,

d) location for a 125-N vertical force to produce the same moment,

Example 2-7



a) Moment about *O* is equal to the product of the force and the perpendicular distance between the line of action of the force and *O*. Since the force tends to rotate the lever clockwise, the moment vector is into the plane of the paper.

(a)

$$M_O = Fd$$

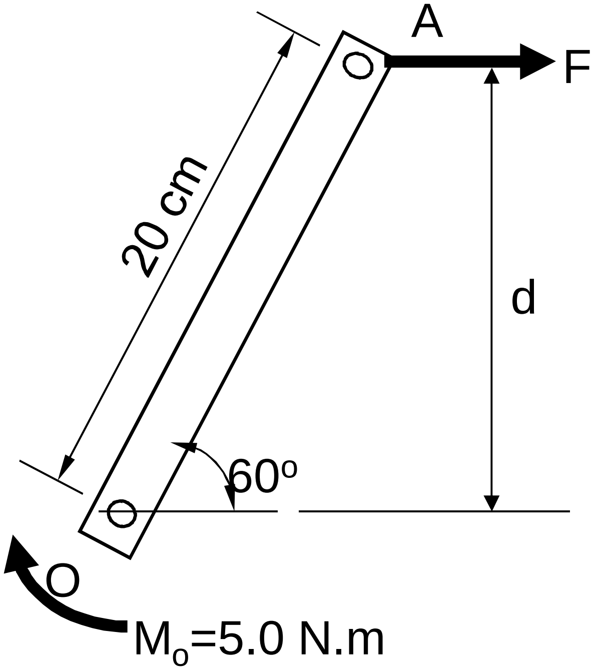
$$d = (0.20\,\mathrm{m})\cos 60° = 0.10\,\mathrm{m}.$$

$$M_O = (50)(0.10)$$

$$\boxed{M_O = 5.0\ N.\mathrm{m}}$$

Example 2-7

(b) horizontal force at *A* which creates the same moment



$$M_O = Fd$$

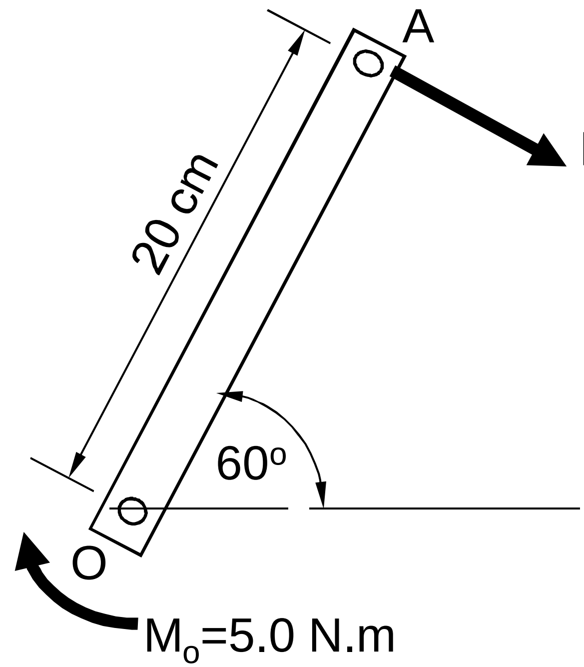
$$M_O = 5.0 N.m$$

$$d = (0.20\,\text{m})\sin 60° = 0.1732\,\text{m}.$$

$$F = \frac{M_O}{d} = \frac{5.0}{0.1732}$$

$$\boxed{F = 28.9\ N.\text{m}}$$

(c) smallest force at A which produc



$$M_O = Fd$$

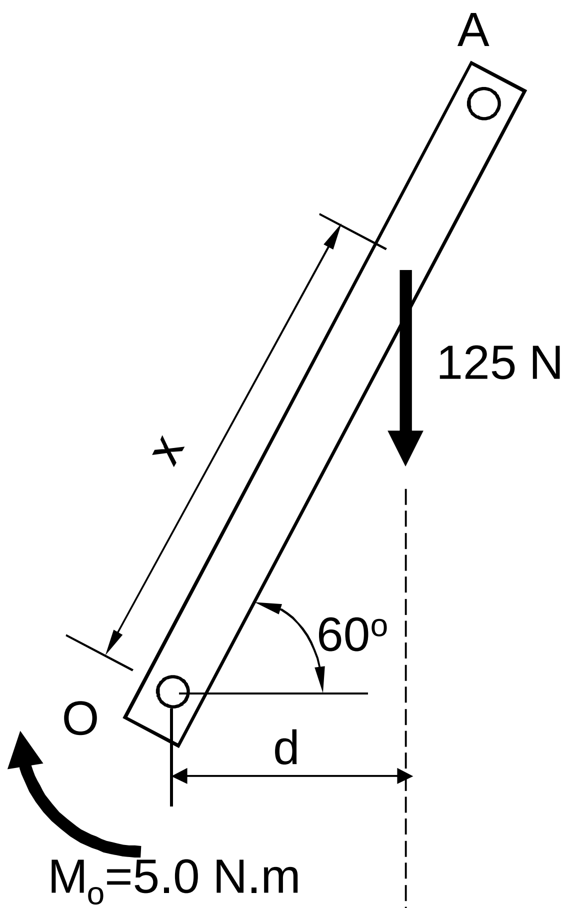
$$M_O = 5.0 N.m$$

$$d = 0.20\,\text{m}$$

$$F = \frac{M_O}{d} = \frac{5.0}{0.20}$$

$$\boxed{F = 25.0\ N.\text{m}}$$

Example 2-7



(d) location for a 125-N vertical force that produce the same moment,

$$M_O = Fd$$
$$M_O = 5.0\,N.m$$
$$F = 125\,N$$
$$d = \frac{M_O}{F} = \frac{5.0}{125} = 0.04m$$
$$x\cos(60^o) = d = 0.04$$
$$x = \frac{d}{\cos(60^o)} = \frac{0.04}{\cos(60^o)} = 0.08m$$

$$\boxed{x = 8\ \text{cm}}$$

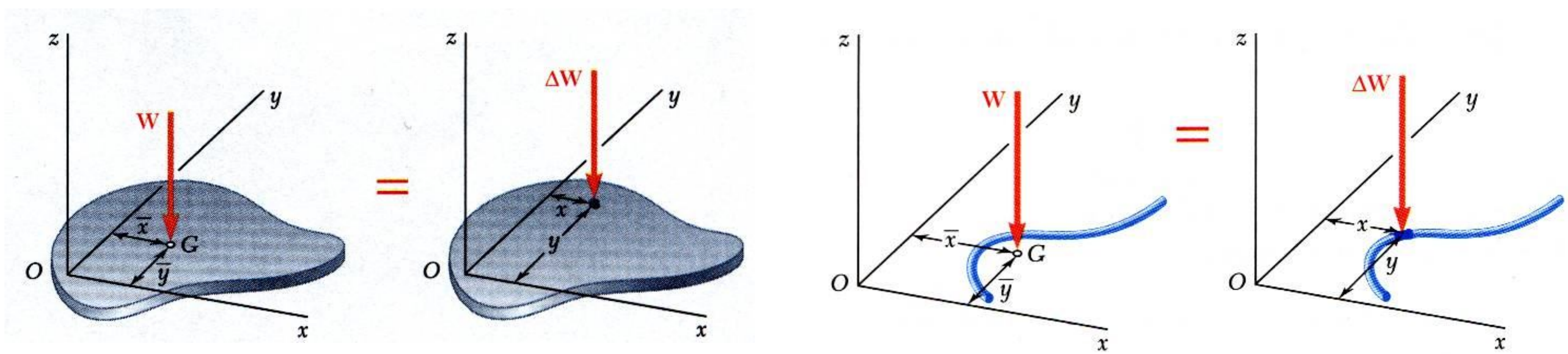
Centroids and Centres of Gravity

- The earth exerts a gravitational force on each of the particles forming a body. These forces can be replace by a single equivalent force equal to the weight of the body and applied at the *center of gravity* for the body.

- The *centroid of an area* is analogous to the center of gravity of a body. The concept of the *first moment of an area* is used to locate the centroid.

- Determination of the area of a *surface of revolution* and the volume of a *body of revolution* are accomplished with the *Theorems of Pappus-Guldinus*.

Center of Gravity of a 2D Body



$$\sum M_y \quad \bar{x}W = \sum x\Delta W$$ By Summation

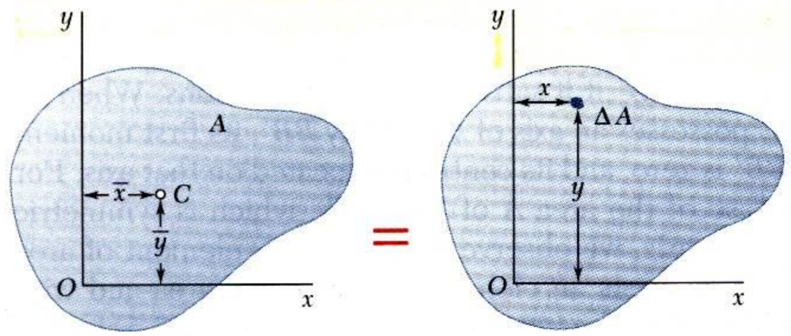
$$= \int x\,dW$$ By Integration

$$\sum M_y \quad \bar{y}W = \sum y\Delta W$$ By Summation

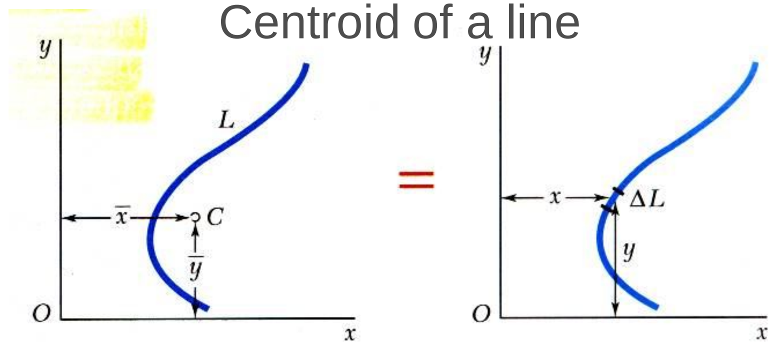
$$= \int y\,dW$$ By Integration

Centroids and First Moments of Areas and Lines

Centroid of an area



Centroid of a line



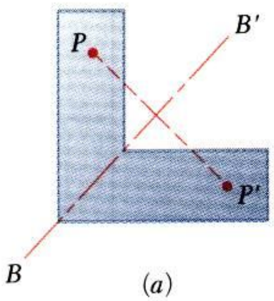
$$\bar{x}W = \int x\,dW$$
$$\bar{x}(\gamma At) = \int x(\gamma t)dA$$
$$\bar{x}A = \int x\,dA = Q_y$$
$$= \text{first moment with respect to } y$$
$$\bar{y}A = \int y\,dA = Q_x$$
$$= \text{first moment with respect to } x$$

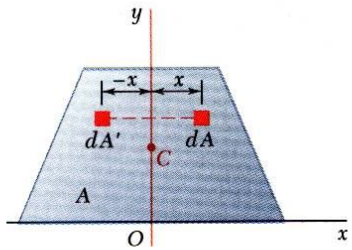
$$\bar{x}W = \int x\,dW$$
$$\bar{x}(\gamma La) = \int x(\gamma a)dL$$
$$\bar{x}L = \int x\,dL$$
$$\bar{y}L = \int y\,dL$$

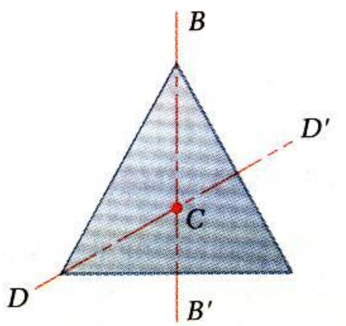
where γ is density and t is thickness

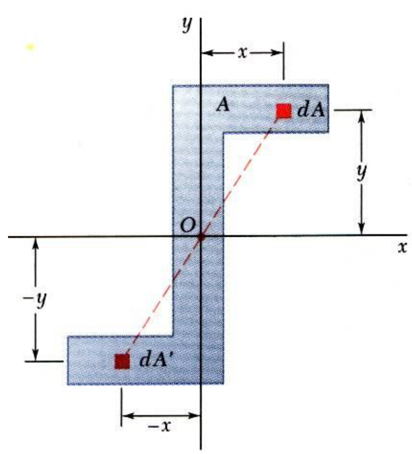
The centroid is located at $(\bar{x}, \bar{y})$

First Moments of Areas and Lines



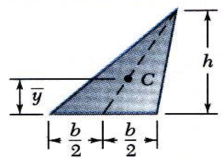
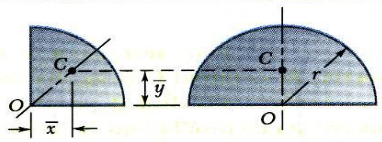
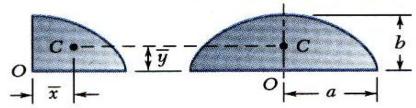
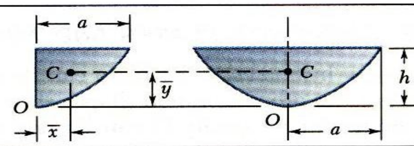
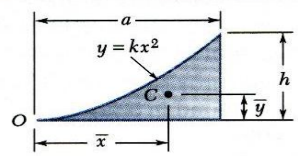
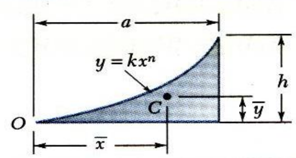
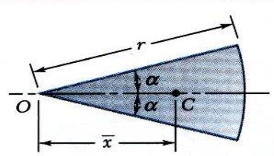




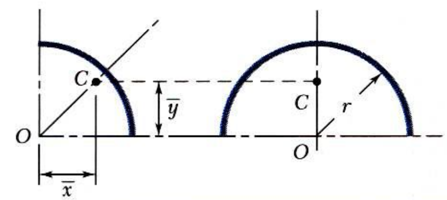
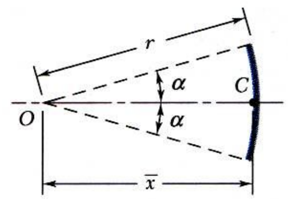


- An area is symmetric with respect to an axis *BB'* if for every point *P* there exists a point *P'* such that *PP'* is perpendicular to *BB'* and is divided into two equal parts by *BB'.*
- The first moment of an area with respect to a line of symmetry is zero.
- If an area possesses a line of symmetry, its centroid lies on that axis
- If an area possesses two lines of symmetry, its centroid lies at their intersection.
- An area is symmetric with respect to a center *O* if for every element *dA* at (*x*,*y*) there exists an area *dA'* of equal area at (-*x*,-*y*).
- The centroid of the area coincides with the center of symmetry.

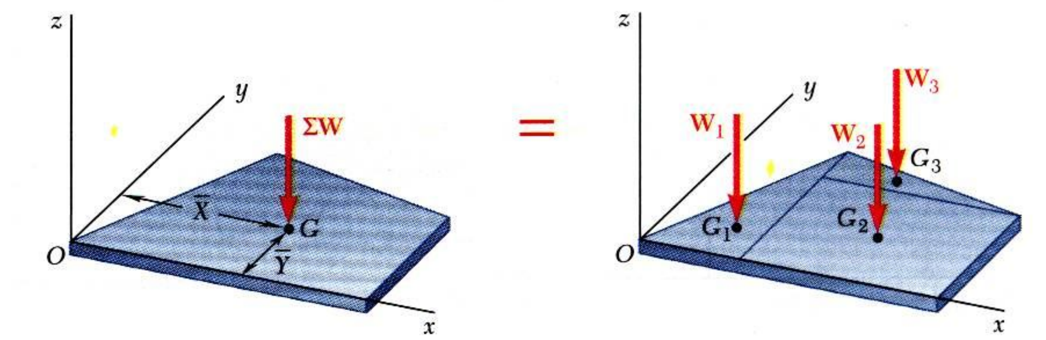
Centroids of Common Shapes of Areas

Shape		$\bar{x}$	$\bar{y}$	Area
Triangular area			$\frac{h}{3}$	$\frac{bh}{2}$
Quarter-circular area		$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{4}$
Semicircular area		0	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{2}$
Quarter-elliptical area		$\frac{4a}{3\pi}$	$\frac{4b}{3\pi}$	$\frac{\pi ab}{4}$
Semielliptical area		0	$\frac{4b}{3\pi}$	$\frac{\pi ab}{2}$
Semiparabolic area		$\frac{3a}{8}$	$\frac{3h}{5}$	$\frac{2ah}{3}$
Parabolic area		0	$\frac{3h}{5}$	$\frac{4ah}{3}$
Parabolic spandrel		$\frac{3a}{4}$	$\frac{3h}{10}$	$\frac{ah}{3}$
General spandrel		$\frac{n+1}{n+2}a$	$\frac{n+1}{4n+2}h$	$\frac{ah}{n+1}$
Circular sector		$\frac{2r \sin \alpha}{3\alpha}$	0	αr^2

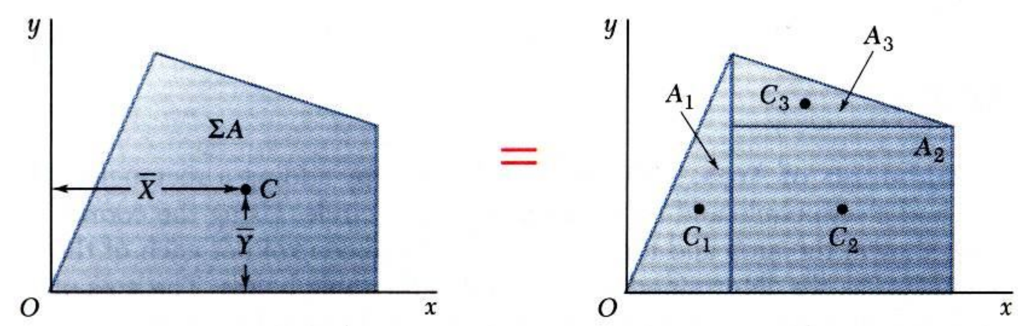
Centroids of Common Shapes of Lines

Shape		$\bar{x}$	$\bar{y}$	Length
Quarter-circular arc		$\frac{2r}{\pi}$	$\frac{2r}{\pi}$	$\frac{\pi r}{2}$
Semicircular arc		0	$\frac{2r}{\pi}$	πr
Arc of circle		$\frac{r \sin \alpha}{\alpha}$	0	$2\alpha r$

Composite Plates and Areas

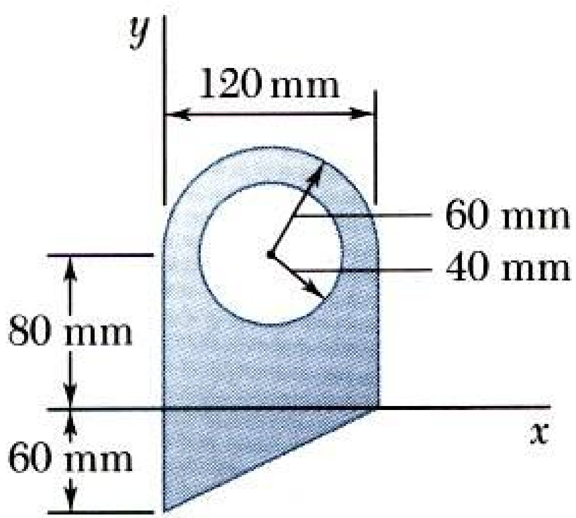


$$\bar{X}\sum W = \sum \bar{x} W$$
$$\bar{Y}\sum W = \sum \bar{y} W$$



$$\bar{X}\sum A = \sum \bar{x} A$$
$$\bar{Y}\sum A = \sum \bar{y} A$$

Example 2-8

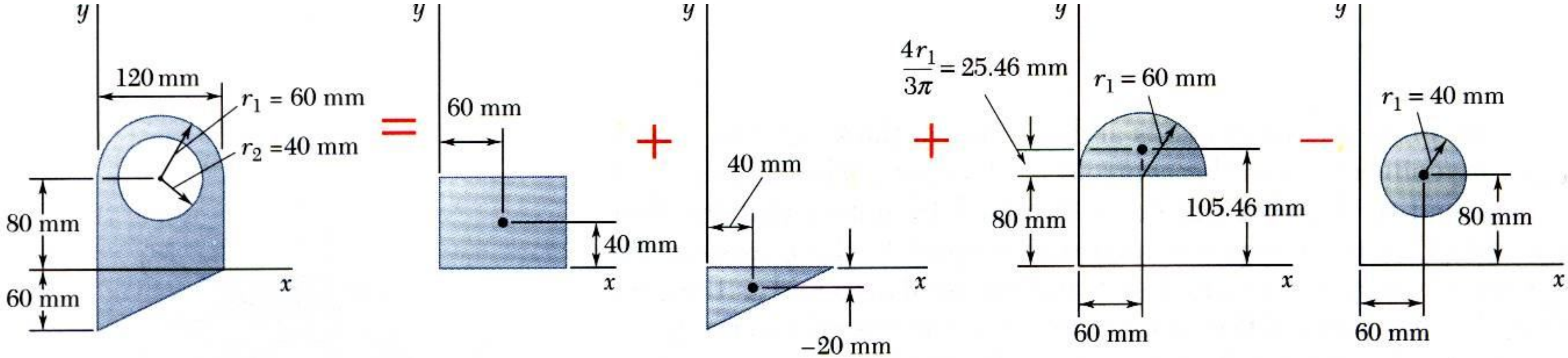


For the plane area shown, determine the first moments with respect to the *x* and *y* axes and the location of the centroid.

SOLUTION:

- Divide the area into a triangle, rectangle, and semicircle with a circular cutout.
- Calculate the first moments of each area with respect to the axes.
- Find the total area and first moments of the triangle, rectangle, and semicircle. Subtract the area and first moment of the circular cutout.
- Compute the coordinates of the area centroid by dividing the first moments by the total area.

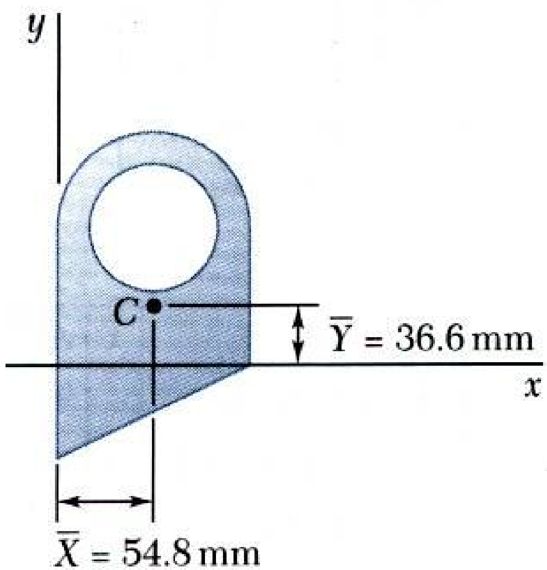
Example 2-8



Component	A, mm²	$\bar{x}$, mm	$\bar{y}$, mm	$\bar{x}A$, mm³	$\bar{y}A$, mm³
Rectangle	$(120)(80) = 9.6 \times 10^3$	60	40	$+576 \times 10^3$	$+384 \times 10^3$
Triangle	$\frac{1}{2}(120)(60) = 3.6 \times 10^3$	40	−20	$+144 \times 10^3$	-72×10^3
Semicircle	$\frac{1}{2}\pi(60)^2 = 5.655 \times 10^3$	60	105.46	$+339.3 \times 10^3$	$+596.4 \times 10^3$
Circle	$-\pi(40)^2 = -5.027 \times 10^3$	60	80	-301.6×10^3	-402.2×10^3
	$\Sigma A = 13.828 \times 10^3$			$\Sigma\bar{x}A = +757.7 \times 10^3$	$\Sigma\bar{y}A = +506.2 \times 10^3$

$$\boxed{\begin{aligned} Q_x &= +506.2 \times 10^3 \text{ mm}^3 \\ Q_y &= +757.7 \times 10^3 \text{ mm}^3 \end{aligned}}$$

Example 2-8



$$\bar{X} = \frac{\sum \bar{x} A}{\sum A} = \frac{+757.7 \times 10^3\,\text{mm}^3}{13.828 \times 10^3\,\text{mm}^2}$$

$$\boxed{\bar{X} = 54.8\,\text{mm}}$$

$$\bar{Y} = \frac{\sum \bar{y} A}{\sum A} = \frac{+506.2 \times 10^3\,\text{mm}^3}{13.828 \times 10^3\,\text{mm}^2}$$

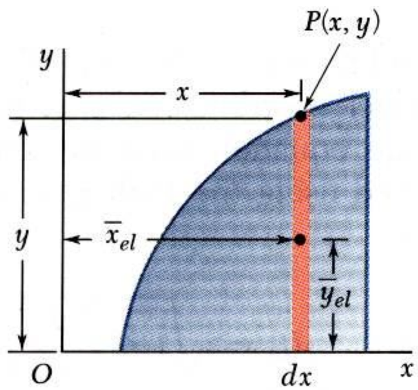
$$\boxed{\bar{Y} = 36.6\,\text{mm}}$$

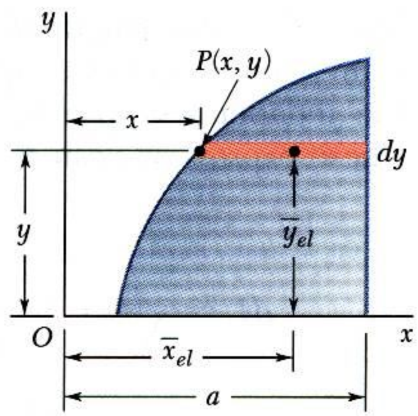
Determination of Centroids by Integration

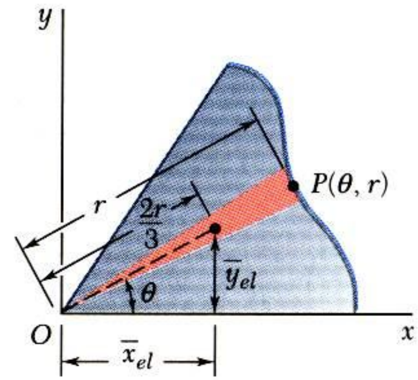
$$\bar{x}A = \int x\,dA = \iint x\,dxdy = \int \bar{x}_{el}\,dA$$

$$\bar{y}A = \int y\,dA = \iint y\,dxdy = \int \bar{y}_{el}\,dA$$

- Double integration to find the first moment may be avoided by defining *dA* as a thin rectangle or strip.







$$\bar{x}A = \int \bar{x}_{el}\,dA = \int x\left(ydx\right)$$

$$\bar{y}A = \int \bar{y}_{el}\,dA = \int \frac{y}{2}\left(ydx\right)$$

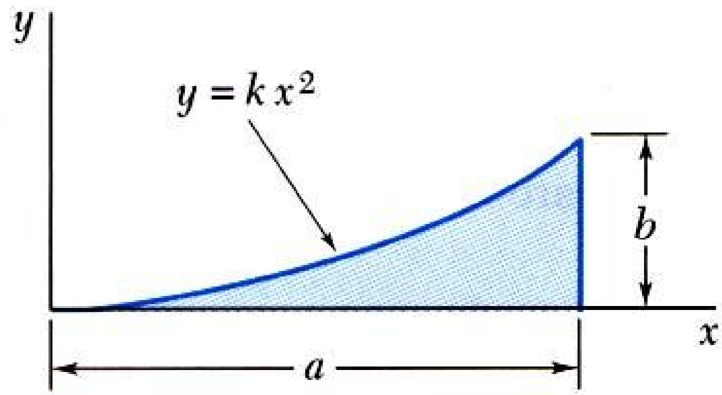
$$\bar{x}A = \int \bar{x}_{el}\,dA = \int \frac{a+x}{2}\left[\left(a-x\right)dy\right]$$

$$\bar{y}A = \int \bar{y}_{el}\,dA = \int y\left[\left(a-x\right)dy\right]$$

$$\bar{x}A = \int \bar{x}_{el}\,dA = \int \frac{2r}{3}\cos\theta\left(\frac{1}{2}r^2 d\theta\right)$$

$$\bar{y}A = \int \bar{y}_{el}\,dA = \int \frac{2r}{3}\sin\theta\left(\frac{1}{2}r^2 d\theta\right)$$

Example 2-9

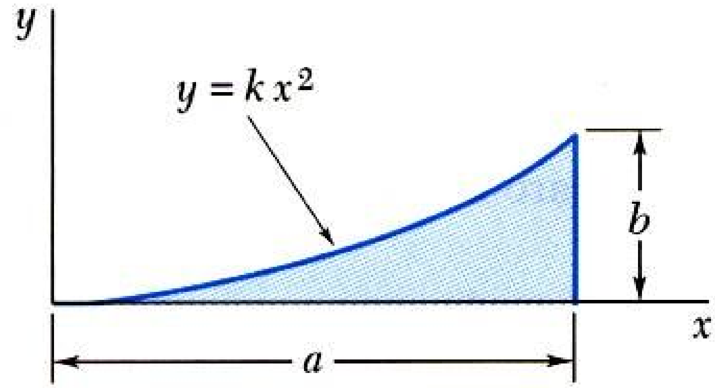


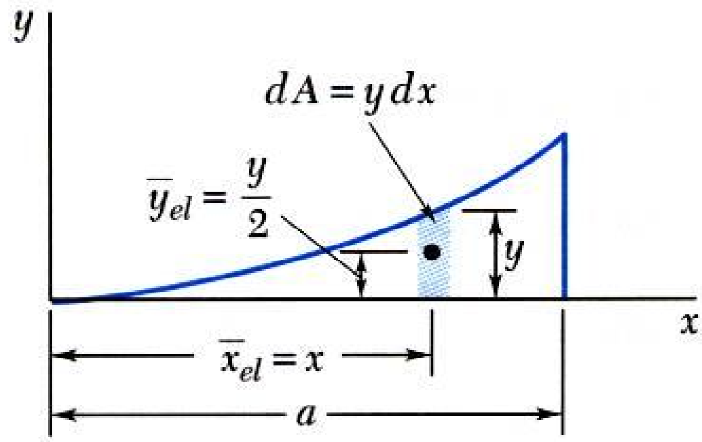
Determine by direct integration the location of the centroid of a parabolic spandrel.

SOLUTION:

- Determine the constant k.
- Evaluate the total area.
- Using either vertical or horizontal strips, perform a single integration to find the first moments.
- Evaluate the centroid coordinates.

Example 2-9





$$y = k\,x^2$$

$$b = k\,a^2 \quad \Rightarrow k = \frac{b}{a^2}$$

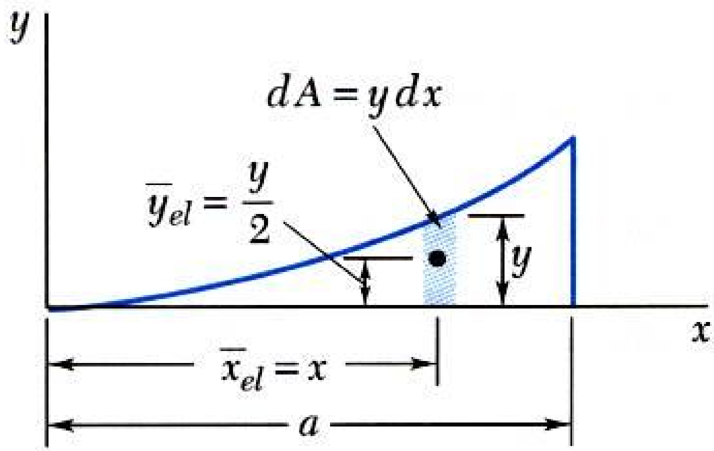
$$y = \frac{b}{a^2}x^2 \quad or \quad x = \frac{a}{b^{1/2}}y^{1/2}$$

$$A = \int dA$$

$$= \int y\,dx = \int_0^a \frac{b}{a^2}x^2 dx = \left[\frac{b}{a^2}\frac{x^3}{3}\right]_0^a$$

$$= \frac{ab}{3}$$

Example 2-9



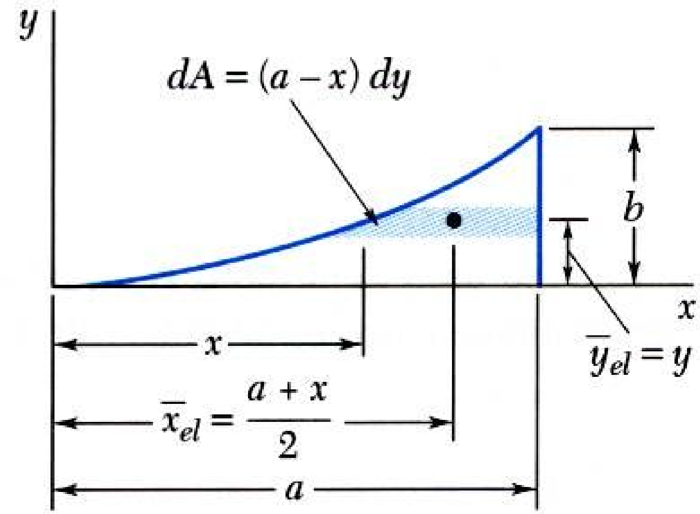
$$Q_y = \int \bar{x}_{el} dA = \int xy dx = \int_0^a x\left(\frac{b}{a^2}x^2\right)dx$$

$$= \left[\frac{b}{a^2}\frac{x^4}{4}\right]_0^a = \frac{a^2 b}{4}$$

$$Q_x = \int \bar{y}_{el} dA = \int \frac{y}{2} y dx = \int_0^a \frac{1}{2}\left(\frac{b}{a^2}x^2\right)^2 dx$$

$$= \left[\frac{b^2}{2a^4}\frac{x^5}{5}\right]_0^a = \frac{ab^2}{10}$$

Example 2-9



- Or, using horizontal strips, perform a single integration to find the first moments.

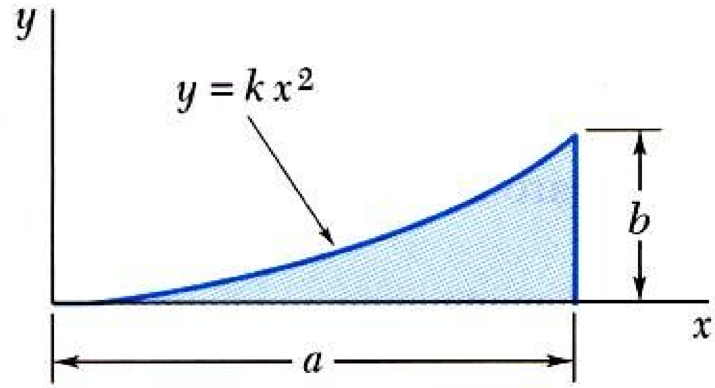
$$Q_y = \int \bar{x}_{el}\, dA = \int \frac{a+x}{2}(a-x)dy = \int_0^b \frac{a^2 - x^2}{2} dy$$

$$= \frac{1}{2}\int_0^b \left(a^2 - \frac{a^2}{b} y \right) dy = \frac{a^2 b}{4}$$

$$Q_x = \int \bar{y}_{el}\, dA = \int y(a-x)dy = \int y\left(a - \frac{a}{b^{1/2}} y^{1/2} \right) dy$$

$$= \int_0^b \left(ay - \frac{a}{b^{1/2}} y^{3/2} \right) dy = \frac{ab^2}{10}$$

Example 2-9



$$\bar{x}A = Q_y$$

$$\bar{x}\frac{ab}{3} = \frac{a^2b}{4}$$

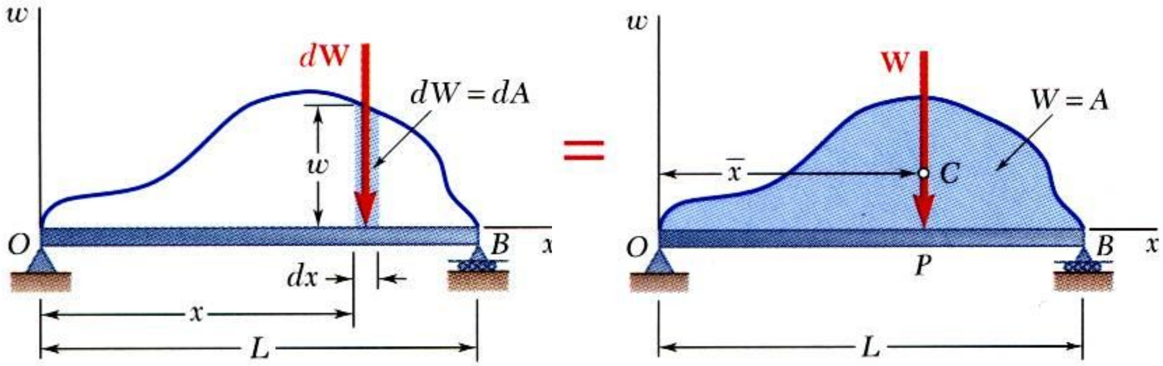
$$\boxed{\bar{x} = \frac{3}{4}a}$$

$$\bar{y}A = Q_x$$

$$\bar{y}\frac{ab}{3} = \frac{ab^2}{10}$$

$$\boxed{\bar{y} = \frac{3}{10}b}$$

Distributed Loads on Beams



$$W = \int_0^L w\,dx = \int dA = A$$

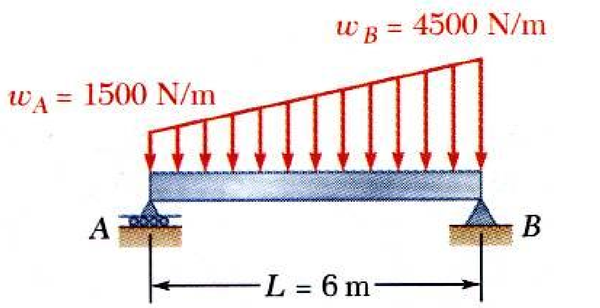
- A distributed load is represented by plotting the load per unit length, *w* (N/m) . The total load is equal to the area under the load curve.

$$(OP)W = \int x\,dW$$

$$(OP)A = \int_0^L x\,dA = \bar{x}A$$

- A distributed load can be replace by a concentrated load with a magnitude equal to the area under the load curve and a line of action passing through the area centroid.

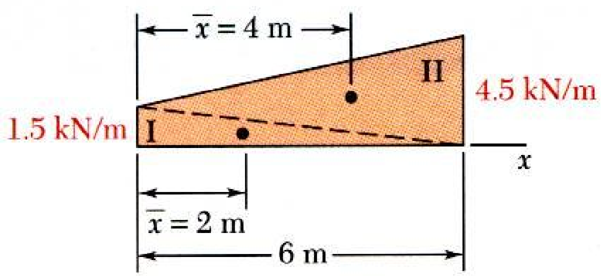
Example 2-10



SOLUTION:

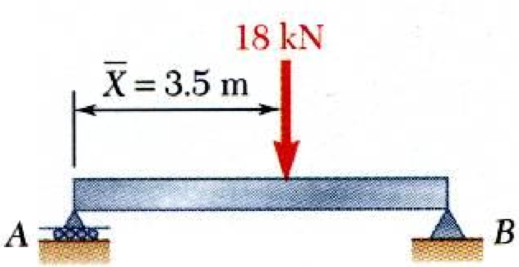
- The magnitude of the concentrated load is equal to the total load or the area under the curve

$$\boxed{F = 18.0\,\mathrm{kN}}$$



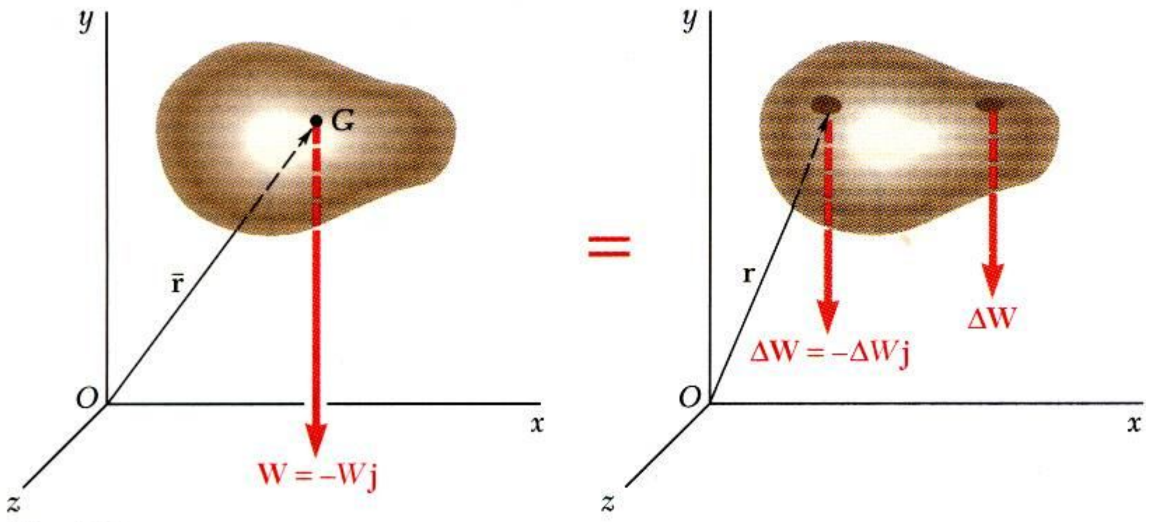
- The line of action of the concentrated load passes through the centroid of the area under the curve.

$$\bar{X} = \frac{63\,\mathrm{kN \cdot m}}{18\,\mathrm{kN}} \qquad \boxed{\bar{X} = 3.5\,\mathrm{m}}$$



Component	A, kN	$\bar{x}$, m	$\bar{x}A$, kN · m
Triangle I	4.5	2	9
Triangle II	13.5	4	54
	$\Sigma A = 18.0$		$\Sigma \bar{x}A = 63$

Center of Gravity of a 3D Body: Centroid of a Volume



- Center of gravity *G*

$$-W\vec{j} = \sum\left(-\Delta W\vec{j}\right)$$

$$\vec{r}_G \times \left(-W\vec{j}\right) = \sum\left[\vec{r} \times \left(-\Delta W\vec{j}\right)\right]$$
$$\vec{r}_G W \times \left(-\vec{j}\right) = \left(\sum \vec{r}\Delta W\right) \times \left(-\vec{j}\right)$$

$$W = \int dW \qquad \vec{r}_G W = \int \vec{r}\,dW$$

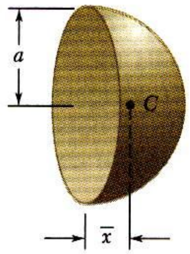
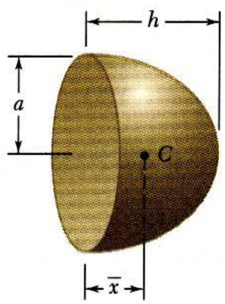
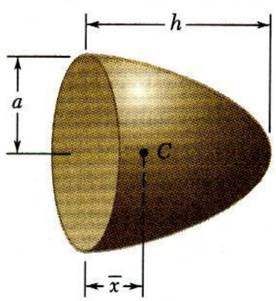
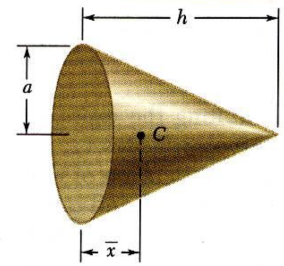
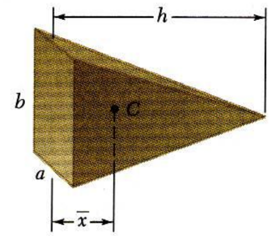
- Results are independent of body orientation,

$$\bar{x}W = \int x\,dW \quad \bar{y}W = \int y\,dW \quad \bar{z}W = \int z\,dW$$

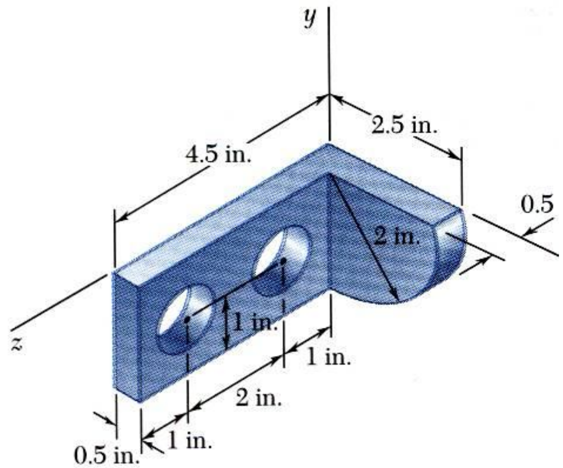
$$W = \gamma V \text{ and } dW = \gamma\,dV$$

$$\bar{x}V = \int x\,dV \quad \bar{y}V = \int y\,dV \quad \bar{z}V = \int z\,dV$$

Centroids of Common 3D Shapes

Shape		$\bar{x}$	Volume
Hemisphere		$\frac{3a}{8}$	$\frac{2}{3}\pi a^3$
Semiellipsoid of revolution		$\frac{3h}{8}$	$\frac{2}{3}\pi a^2 h$
Paraboloid of revolution		$\frac{h}{3}$	$\frac{1}{2}\pi a^2 h$
Cone		$\frac{h}{4}$	$\frac{1}{3}\pi a^2 h$
Pyramid		$\frac{h}{4}$	$\frac{1}{3}abh$

Composite 3D Bodies

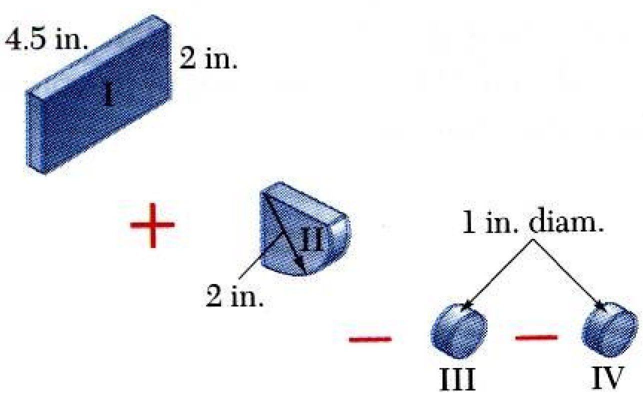


- Moment of the total weight concentrated at the center of gravity G is equal to the sum of the moments of the weights of the component parts.

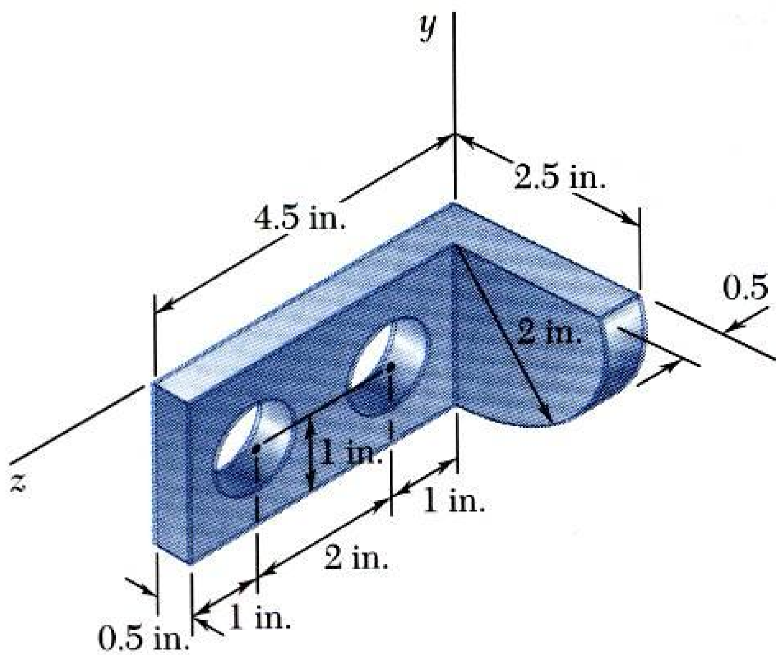
$$\bar{X}\sum W = \sum \bar{x} W \quad \bar{Y}\sum W = \sum \bar{y} W \quad \bar{Z}\sum W = \sum \bar{z} W$$

- For homogeneous bodies,

$$\bar{X}\sum V = \sum \bar{x} V \quad \bar{Y}\sum V = \sum \bar{y} V \quad \bar{Z}\sum V = \sum \bar{z} V$$



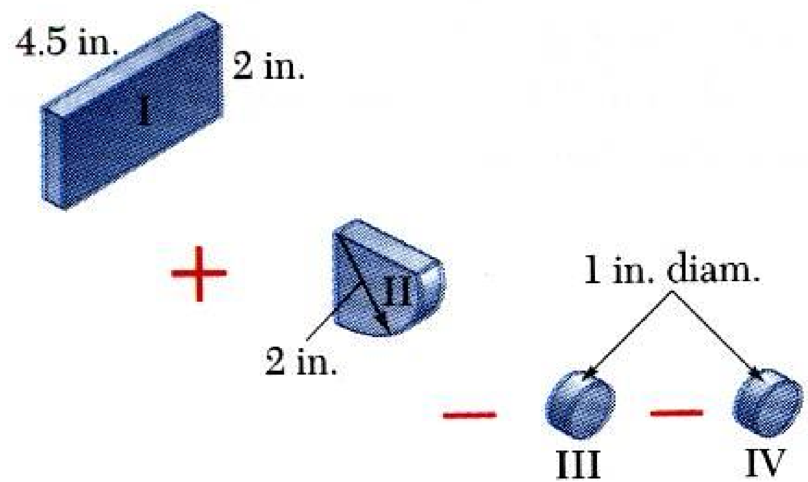
Example 2-11



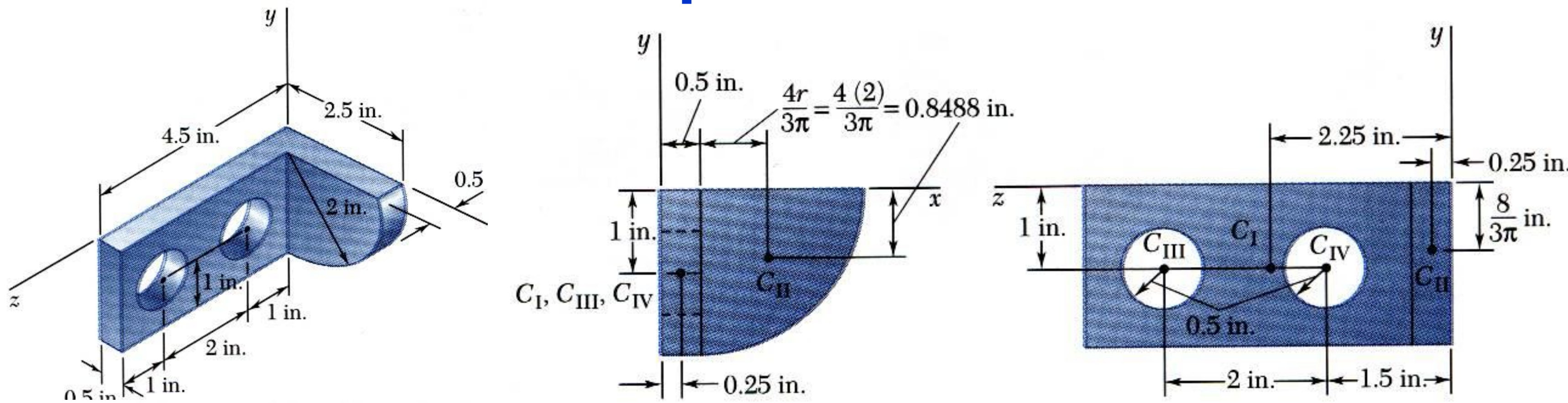
Locate the center of gravity of the steel machine element. The diameter of each hole is 1 in.

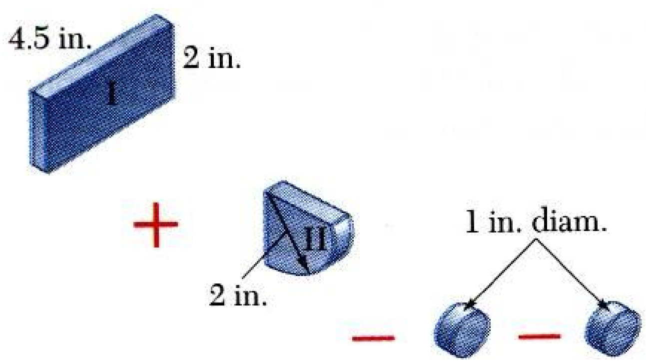
SOLUTION:

- Form the machine element from a rectangular parallelepiped and a quarter cylinder and then subtracting two 1-in. diameter cylinders.



Example 2-11

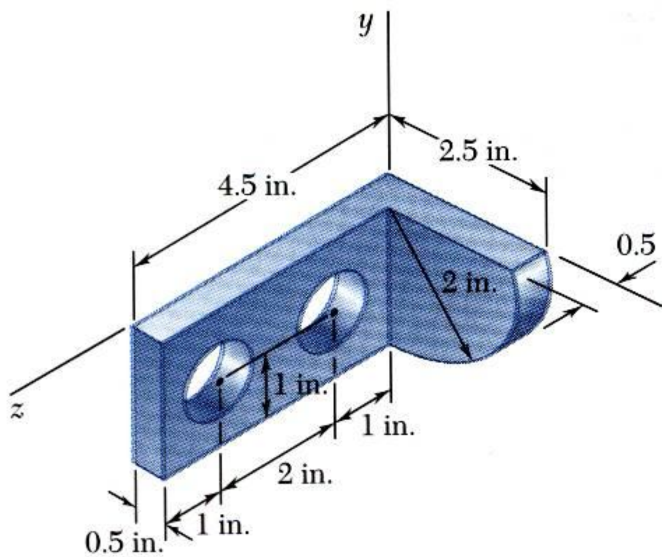




	V, in³	$\bar{x}$, in.	$\bar{y}$, in.	$\bar{z}$, in.	$\bar{x}V$, in⁴	$\bar{y}V$, in⁴	$\bar{z}V$, in⁴
I	$(4.5)(2)(0.5) = 4.5$	0.25	-1	2.25	1.125	-4.5	10.125
II	$\frac{1}{4}\pi(2)^2(0.5) = 1.571$	1.3488	-0.8488	0.25	2.119	-1.333	0.393
III	$-\pi(0.5)^2(0.5) = -0.3927$	0.25	-1	3.5	-0.098	0.393	-1.374
IV	$-\pi(0.5)^2(0.5) = -0.3927$	0.25	-1	1.5	-0.098	0.393	-0.589
	$\Sigma V = 5.286$				$\Sigma\bar{x}V = 3.048$	$\Sigma\bar{y}V = -5.047$	$\Sigma\bar{z}V = 8.555$

Example 2-11

	V, in^3	$\bar{x}$, in.	$\bar{y}$, in.	$\bar{z}$, in.	$\bar{x}V$, in^4	$\bar{y}V$, in^4	$\bar{z}V$, in^4
I	$(4.5)(2)(0.5) = 4.5$	0.25	−1	2.25	1.125	−4.5	10.125
II	$\frac{1}{4}\pi(2)^2(0.5) = 1.571$	1.3488	−0.8488	0.25	2.119	−1.333	0.393
III	$-\pi(0.5)^2(0.5) = -0.3927$	0.25	−1	3.5	−0.098	0.393	−1.374
IV	$-\pi(0.5)^2(0.5) = -0.3927$	0.25	−1	1.5	−0.098	0.393	−0.589
	$\Sigma V = 5.286$				$\Sigma\bar{x}V = 3.048$	$\Sigma\bar{y}V = -5.047$	$\Sigma\bar{z}V = 8.555$



$$\bar{X} = \sum \bar{x}V / \sum V = \left(3.08\,\text{in}^4\right) / \left(5.286\,\text{in}^3\right)$$

$$\boxed{\bar{X} = 0.577\,\text{in.}}$$

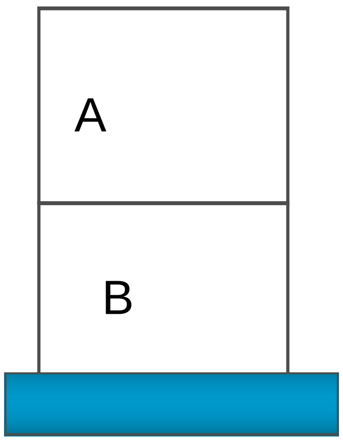
$$\bar{Y} = \sum \bar{y}V / \sum V = \left(-5.047\,\text{in}^4\right) / \left(5.286\,\text{in}^3\right)$$

$$\boxed{\bar{Y} = 0.577\,\text{in.}}$$

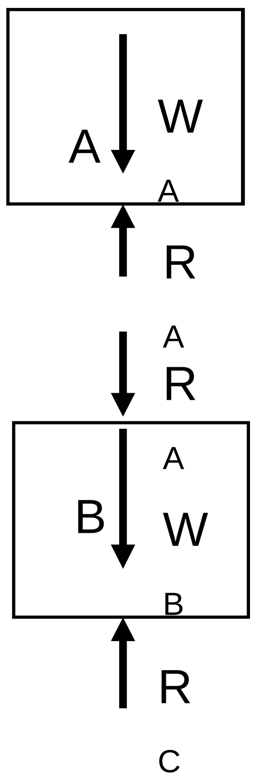
$$\bar{Z} = \sum \bar{z}V / \sum V = \left(1.618\,\text{in}^4\right) / \left(5.286\,\text{in}^3\right)$$

$$\boxed{\bar{Z} = 0.577\,\text{in.}}$$

Free-Body Diagram



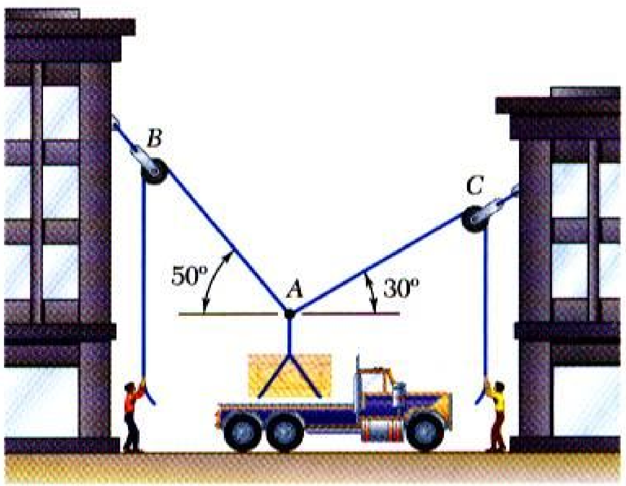
Two blocks of wood

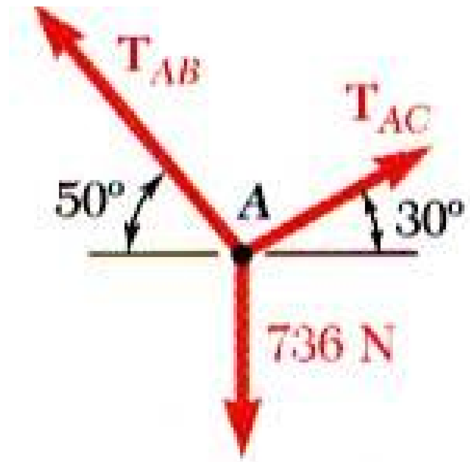


Steps for Drawing a Free-body Diagram

- Decide which body (or group of bodies) is under consideration , imagine it to be isolated from all other bodies, and sketch the outlined shape of the body.
- Indicate by means of arrows all external forces and moments acting on the body. This should include (a) the weight of the body, (b) all external forces (c) reactions at supports and other contacts with other bodies.
- For each unknown force, indicate its point of application and assumed a direction, if it is unknown.
- Include the dimensions and angles needed for computing moments of forces and resolving forces.
- The weight of a body always acts vertically downward through the centre of gravity of the body.
- Forces acting at joints should considered as internal forces and should not be shown if the joint is not separated. Once the joint is separated, the forces acting at the joints must consider as external forces and indicated on the free-body diagram

Free-Body Diagrams

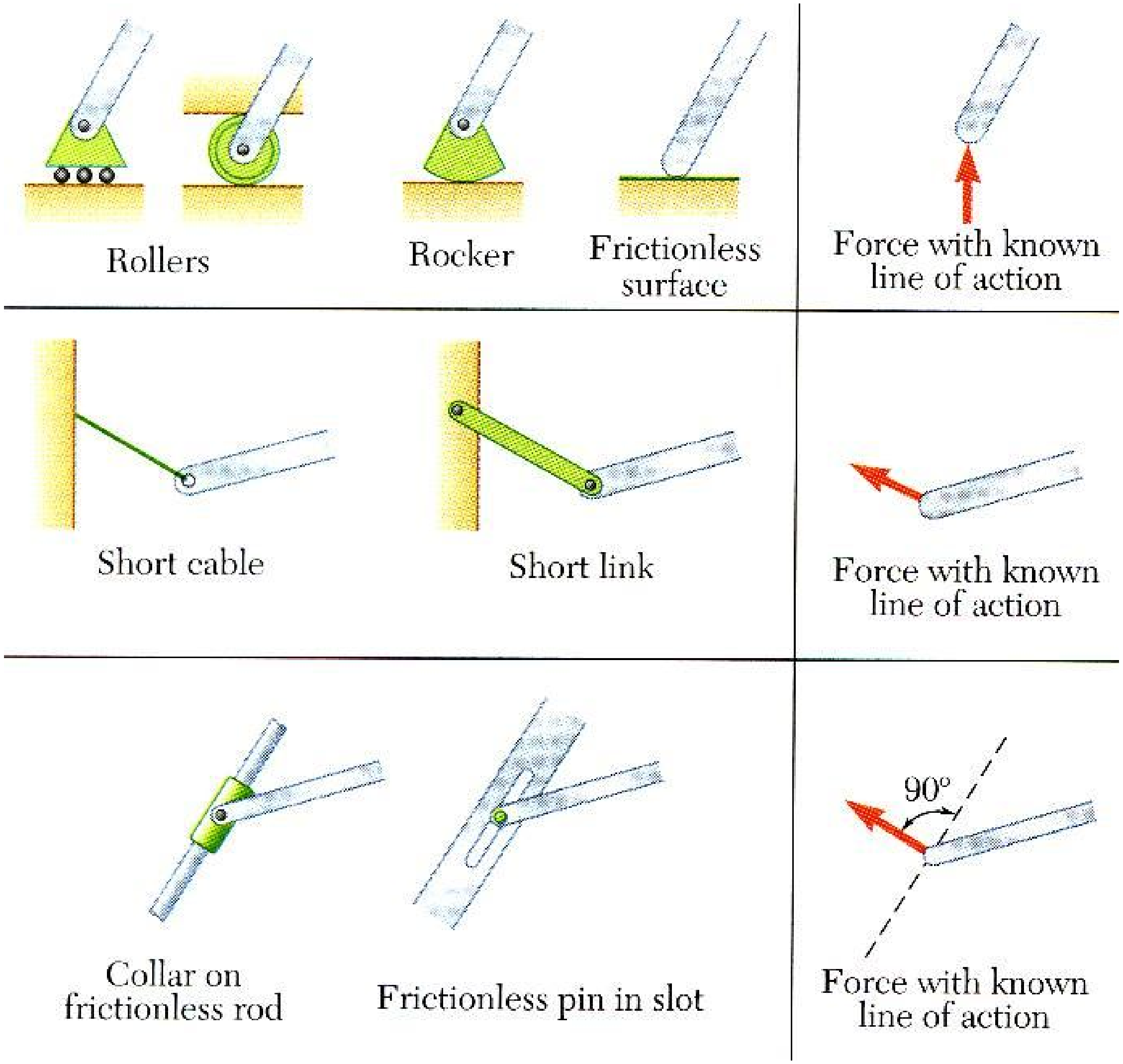




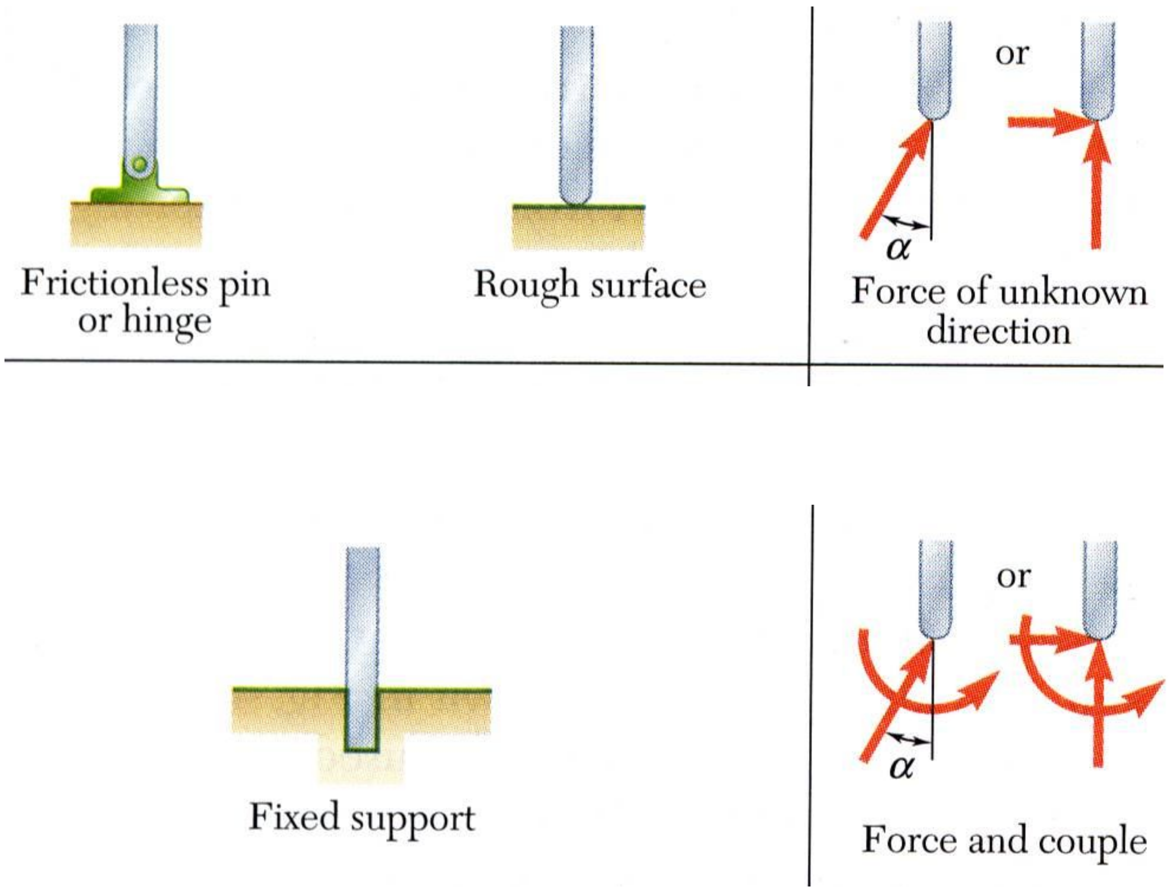
Space Diagram: A sketch showing the physical conditions of the problem.

Free-Body Diagram: A sketch showing only the forces on the selected particle

Reactions at Supports and Connections for a 2D Structure



Reactions at Supports and Connections for a 2D Structure



Equilibrium Conditions for a Particle

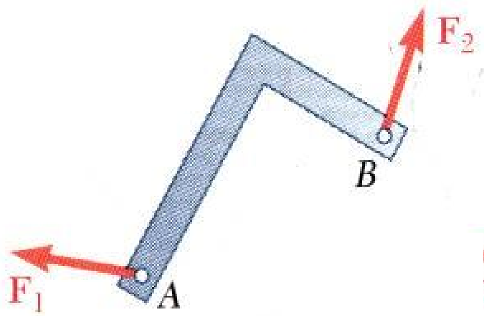
A particle is in equilibrium if

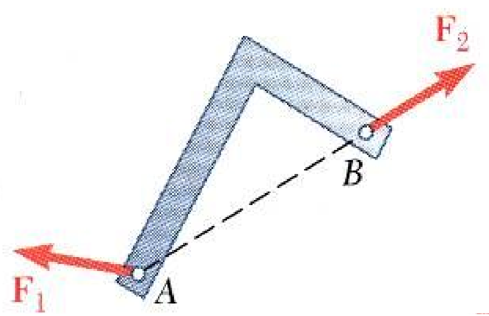
- it is at rest relative to an initial reference frame
- the body moves with constant velocity along a straight line relative to an initial frame

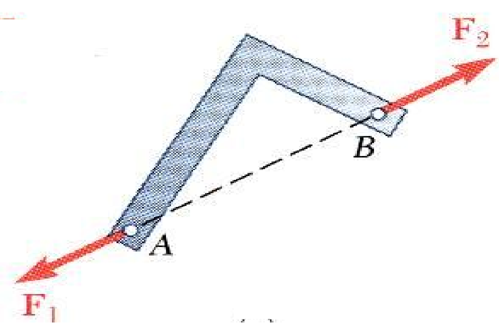
The conditions are :

$$\sum F_x = 0 \qquad \sum F_y = 0 \qquad \sum F_z = 0$$

Equilibrium of a Two-Force Body





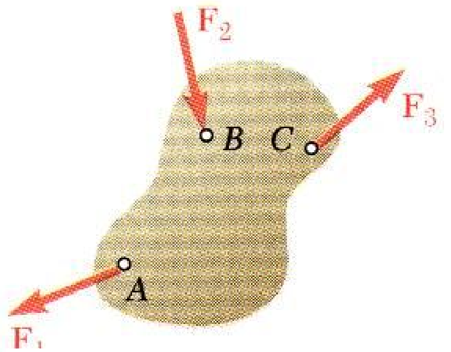


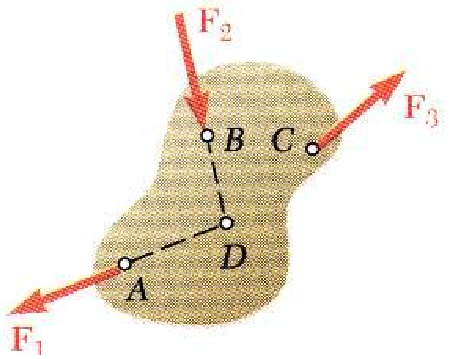
- For static equilibrium, the sum of moments about *A* must be zero. The moment of $\boldsymbol{F_2}$ must be zero. It follows that the line of action of $\boldsymbol{F_2}$ must pass through *A*.

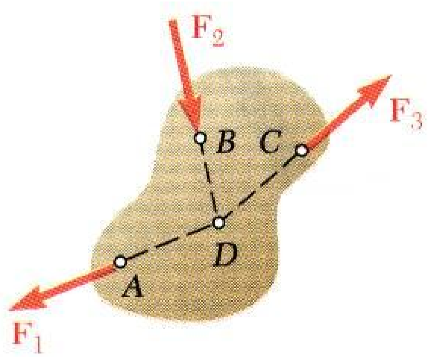
  - Similarly, the line of action of $\boldsymbol{F_1}$ must pass through *B* for the sum of moments about *B* to be zero.

  - Requiring that the sum of forces in any direction be zero leads to the conclusion that $\boldsymbol{F_1}$ and $\boldsymbol{F_2}$ must have equal magnitude but opposite sense.

Equilibrium of a Three-Force Body

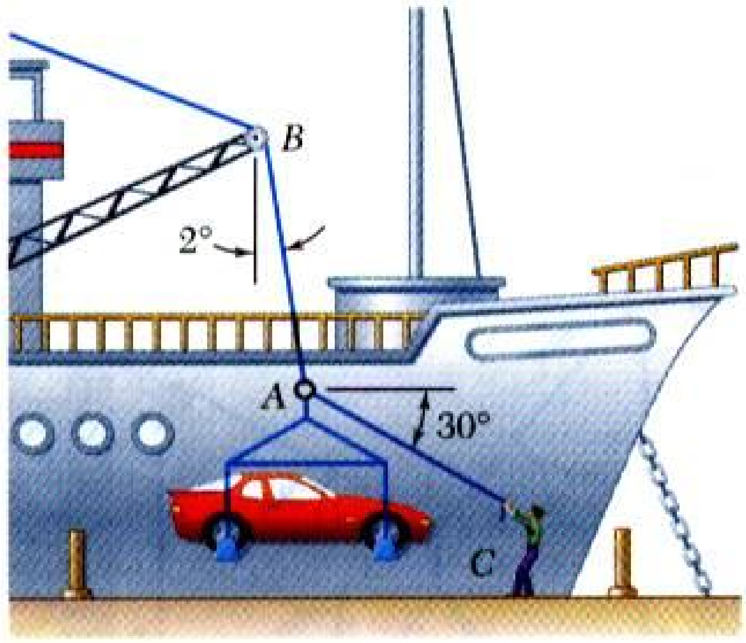






- Consider a rigid body subjected to forces acting at only 3 points.
- Assuming that their lines of action intersect, the moment of $\boldsymbol{F_1}$ and $\boldsymbol{F_2}$ about the point of intersection represented by D is zero.
- Since the rigid body is in equilibrium, the sum of the moments of $\boldsymbol{F_1}$, $\boldsymbol{F_2}$, and $\boldsymbol{F_3}$ about any axis must be zero. It follows that the moment of $\boldsymbol{F_3}$ about D must be zero as well and that the line of action of $\boldsymbol{F_3}$ must pass through D.
- The lines of action of the three forces must be concurrent or parallel.

Example 2-7

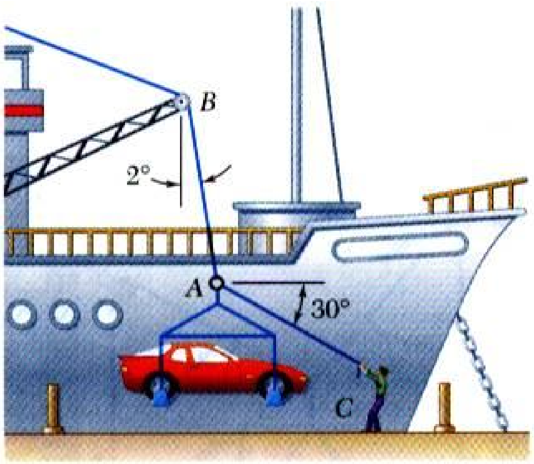


In a ship-unloading operation, a 3500-kg automobile is supported by a cable. A rope is tied to the cable and pulled to center the automobile over its intended position. What is the tension in the rope?

SOLUTION:

- Construct a free-body diagram for the particle at the junction of the rope and cable.
- Apply the conditions for equilibrium by creating a closed polygon from the forces applied to the particle.
- Apply trigonometric relations to determine the unknown force magnitudes.

Example 2-7

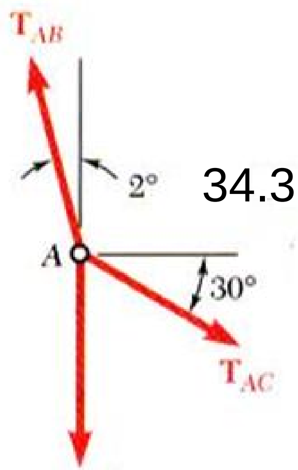


SOLUTION:

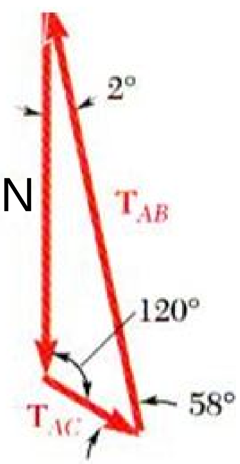
- Construct a free-body diagram for the particle at *A*.

- Apply the conditions for equilibrium.

- Solve for the unknown force magnitudes.



34.3 kN



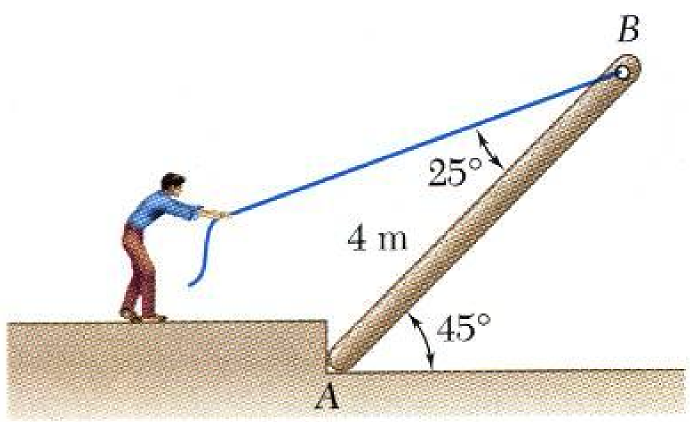
3500 x 9.81

$$\frac{T_{AB}}{\sin 120°} = \frac{T_{AC}}{\sin 2°} = \frac{34.3x1000}{\sin 58°}$$

$$T_{AB} = 35.1\,\mathrm{kN}$$

$$\boxed{T_{AC} = 1.41\,\mathrm{kN}}$$

Example 2-8



A man raises a 10 kg joist, of length 4 m, by pulling on a rope.

Find the tension in the rope and the reaction at *A*.

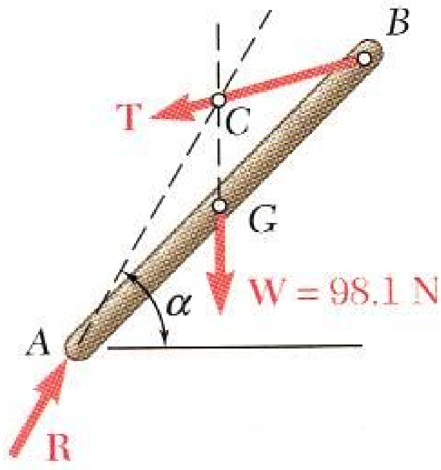
SOLUTION:

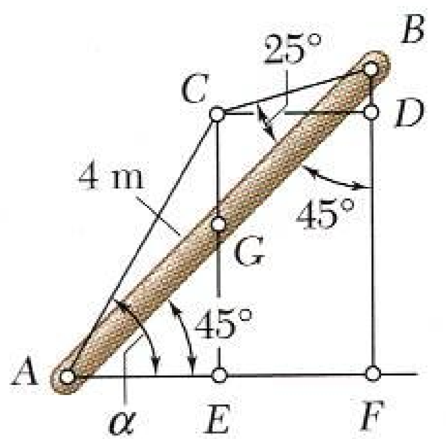
- Create a free-body diagram of the joist. Note that the joist is a 3 force body acted upon by the rope, its weight, and the reaction at *A*.

- The three forces must be concurrent for static equilibrium. Therefore, the reaction ***R*** must pass through the intersection of the lines of action of the weight and rope forces. Determine the direction of the reaction force ***R***.

- Utilize a force triangle to determine the magnitude of the reaction force ***R***.

Example 2-8





- Create a free-body diagram of the joist.
- Determine the direction of the reaction force ***R***.

$$AF = AB\cos 45 = (4\,\text{m})\cos 45 = 2.828\,\text{m}$$

$$CD = AE = \tfrac{1}{2}AF = 1.414\,\text{m}$$

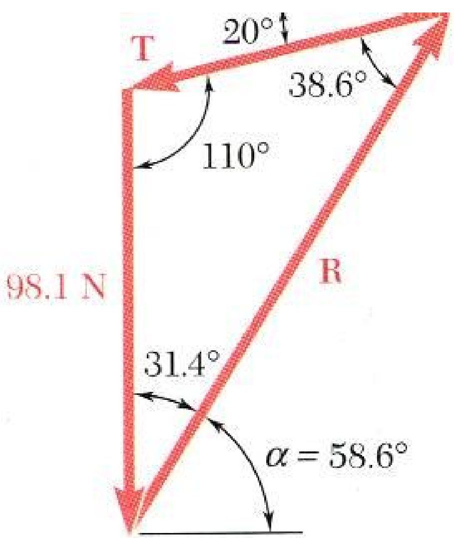
$$BD = CD\cot(45+20) = (1.414\,\text{m})\tan 20 = 0.515\,\text{m}$$

$$CE = BF - BD = (2.828 - 0.515)\,\text{m} = 2.313\,\text{m}$$

$$\tan\alpha = \frac{CE}{AE} = \frac{2.313}{1.414} = 1.636$$

$$\boxed{\alpha = 58.6^\circ}$$

Example 2-8



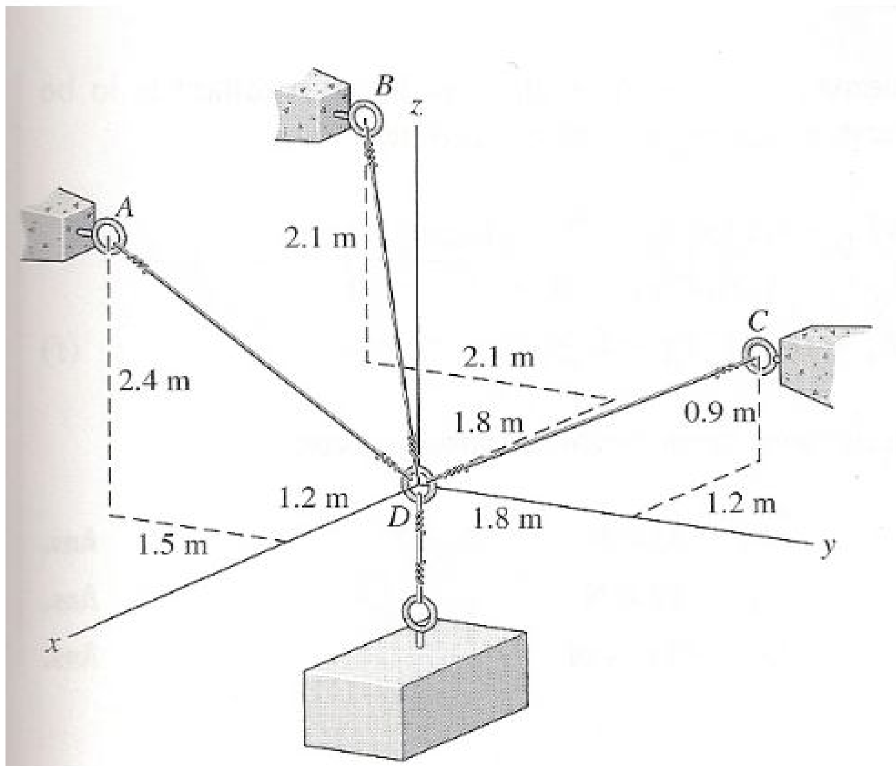
- Determine the magnitude of the reaction force ***R***.

$$\frac{T}{\sin 31.4^\circ} = \frac{R}{\sin 110^\circ} = \frac{98.1\,\mathrm{N}}{\sin 38.6^\circ}$$

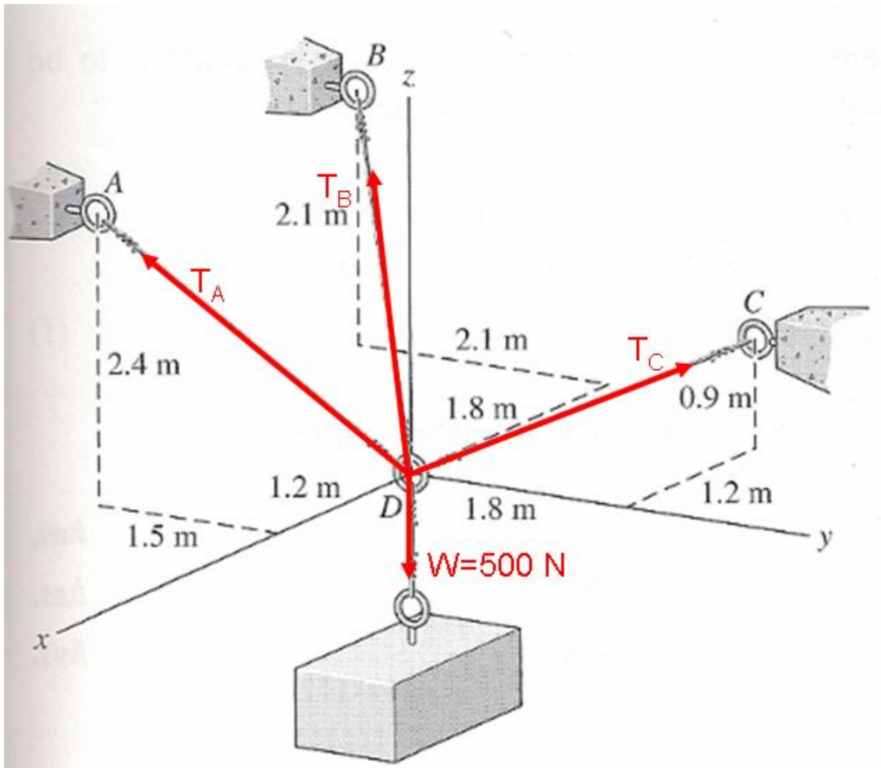
$$\boxed{\begin{aligned} T &= 81.9\,\mathrm{N} \\ R &= 147.8\,\mathrm{N} \end{aligned}}$$

Example 2-9

A 500-N block is supported by a system of cables as shown in .
Determine the tension in the cables A, B and C.



Example 2-9



$$r_A = (1.2-0)i + (-1.5-0)j + (2.4-0)k$$

$$r_A = 1.2i - 1.5j + 2.4k$$

$$|r_A| = \sqrt{\left(1.2^2 + (-1.5)^2 + 2.4^2\right)}$$

$$|r_A| = 2.773$$

$$\hat{r}_A = \frac{r_A}{|r_A|} = 0.4327i - 0.5409j + 0.8655k$$

$$r_B = -1.8i - 2.1j + 2.1k$$

$$|r_B| = \sqrt{\left((-1.8)^2 + (-2.1)^2 + 2.1^2\right)}$$

$$|r_B| = 3.4727$$

$$\hat{r}_B = \frac{r_B}{|r_B|} = -0.5183i - 0.6047j - 0.6047k$$

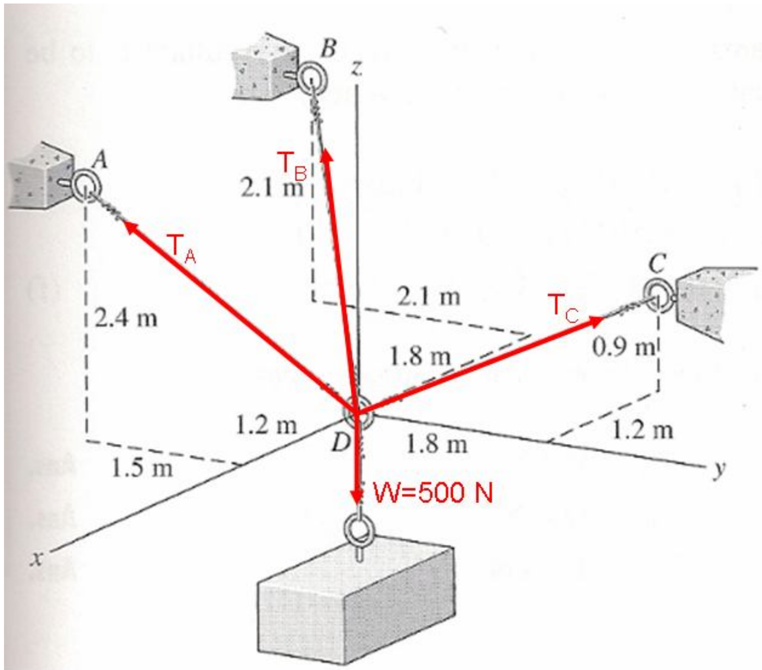
$$r_C = -1.2i + 1.8j + 0.9k$$

$$|r_C| = \sqrt{\left((-1.2)^2 + 1.8^2 + 0.9^2\right)}$$

$$|r_C| = 2.3431$$

$$\hat{r}_C = \frac{r_C}{|r_C|} = -0.5121i + 0.7682j + 0.3841k$$

Example 2-9



$$\sum F = 0$$

$$T_A(0.4327i - 0.5409j + 0.8655k) +$$
$$T_B(-0.5183i - 0.6047j - 0.6047k) +$$
$$T_C(-0.5121i + 0.7682j + 0.3841k) - 500k = 0$$

$$0.4327T_A - 0.5183T_B - 0.5121T_C = 0 \quad (1)$$

$$-0.5409T_A - 0.6047T_B + 0.7682T_C = 0 \quad (2)$$

$$0.8655T_A - 0.6047T_B + 0.3841T_C - 500 = 0 \quad (3)$$

Equilibrium of a Rigid Body in 3D

- Six scalar equations are required to express the conditions for the equilibrium of a rigid body in the general three dimensional case.

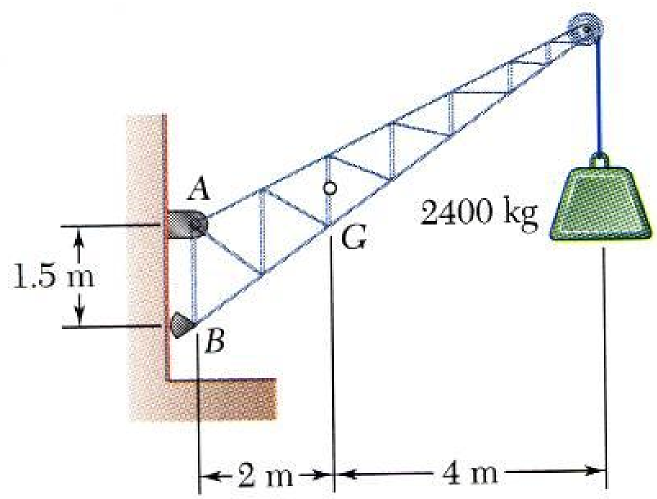
$$\sum F_x = 0 \quad \sum F_y = 0 \quad \sum F_z = 0$$
$$\sum M_x = 0 \quad \sum M_y = 0 \quad \sum M_z = 0$$

- These equations can be solved for no more than 6 unknowns which generally represent reactions at supports or connections.

- The scalar equations are conveniently obtained by applying the vector forms of the conditions for equilibrium,

$$\sum \vec{F} = 0 \quad \sum \vec{M}_O = \sum \left(\vec{r} \times \vec{F} \right) = 0$$

Example 2-11



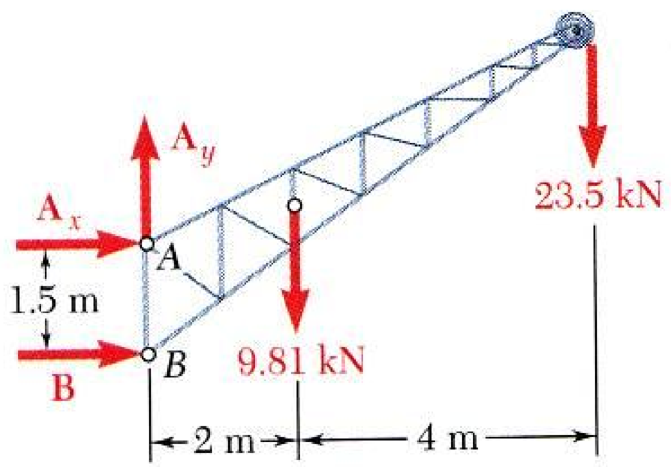
A fixed crane has a mass of 1000 kg and is used to lift a 2400 kg crate. It is held in place by a pin at *A* and a rocker at *B*. The center of gravity of the crane is located at *G*.

Determine the components of the

SOLUTION:

- Create a free-body diagram for the crane.
- Determine *B* by solving the equation for the sum of the moments of all forces about *A*. Note there will be no contribution from the unknown reactions at *A*.
- Determine the reactions at *A* by solving the equations for the sum of all horizontal force components and all vertical force components.
- Check the values obtained for the reactions by verifying that the sum of the moments about *B* of all forces is zero.

Example 2-11



- Create the free-body diagram.

- Determine B by solving the equation for the sum of the moments of all forces about A.

$$\sum M_A = 0: \quad +B(1.5\text{m}) - 9.81\,\text{kN}(2\text{m}) - 23.5\,\text{kN}(6\text{m}) = 0$$

$$\boxed{B = +107.1\,\text{kN}}$$

- Determine the reactions at A by solving the equations for the sum of all horizontal forces and all vertical forces.

$$\sum F_x = 0: \quad A_x + B = 0$$

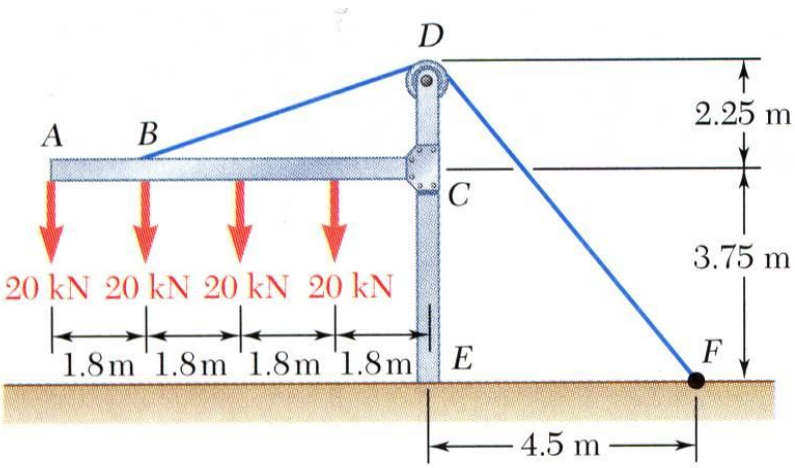
$$\boxed{A_x = -107.1\,\text{kN}}$$

$$\sum F_y = 0: \quad A_y - 9.81\,\text{kN} - 23.5\,\text{kN} = 0$$

$$\boxed{A_y = +33.3\,\text{kN}}$$

- Check the values obtained.

Example 2-12



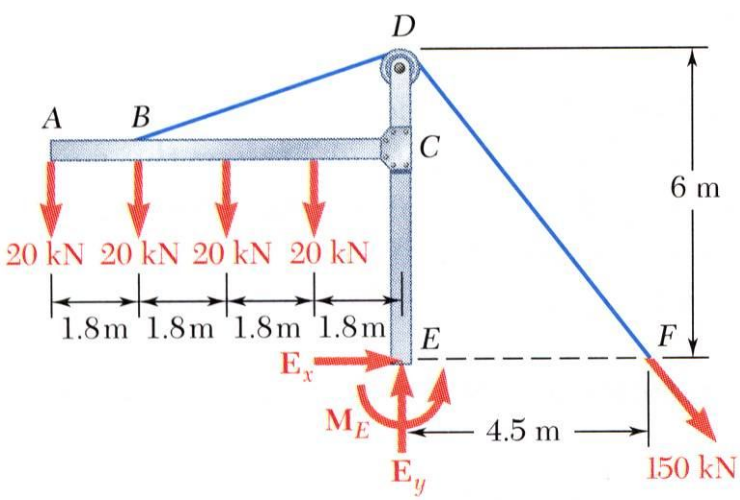
The frame supports part of the roof of a small building. The tension in the cable is 150 kN.

Determine the reaction at the fixed end *E*.

SOLUTION:

- Create a free-body diagram for the frame and cable.
- Solve 3 equilibrium equations for the reaction force components and couple at *E*.

Example 2-12



- Free-body diagram for the frame and cable.

- Solve 3 equilibrium equations for the reaction force components and couple.

$$\sum F_x = 0: \quad E_x + \frac{4.5}{7.5}(150\,\text{kN}) = 0$$

$$\boxed{E_x = -90.0\,\text{kN}}$$

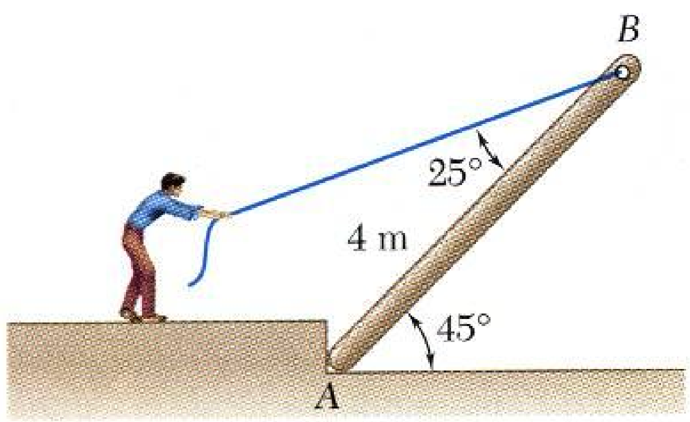
$$\sum F_y = 0: \quad E_y - 4(20\,\text{kN}) - \frac{6}{7.5}(150\,\text{kN}) = 0$$

$$\boxed{E_y = +200\,\text{kN}}$$

$$\sum M_E = 0: \quad +20\,\text{kN}(7.2\,\text{m}) + 20\,\text{kN}(5.4\,\text{m})$$
$$+20\,\text{kN}(3.6\,\text{m}) + 20\,\text{kN}(1.8\,\text{m})$$
$$-\frac{6}{7.5}(150\,\text{kN})4.5\,\text{m} + M_E = 0$$

$$\boxed{M_E = 180.0\,\text{kN}\cdot\text{m}}$$

Example 2-13



Solve Example 2-8 by summing moment.

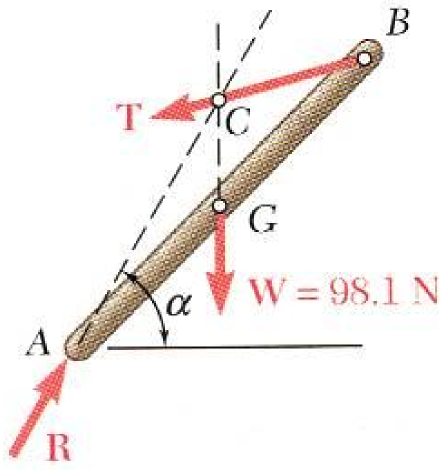
SOLUTION:

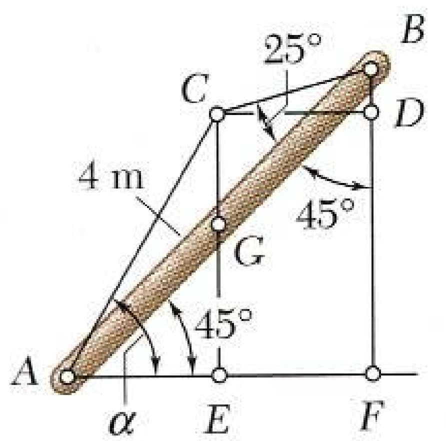
- Create a free-body diagram of the joist. Note that the joist is a 3 force body acted upon by the rope, its weight, and the reaction at *A*.

- The three forces must be concurrent for static equilibrium. Therefore, the reaction ***R*** must pass through the intersection of the lines of action of the weight and rope forces. Determine the direction of the reaction force ***R***.

- Utilize a force triangle to determine the magnitude of the reaction force ***R***.

Example 2-13





- Create a free-body diagram of the joist.
- Determine the tension in the rope by taking moment about A

$$T\cos 20.L\sin 45 - T\sin 20.L\cos 45 - 98.1\text{x}\tfrac{1}{2}L\cos 45 = 0$$

$$\boxed{T = 82.1\,\text{N}}$$

- Solve equilibrium equations for the reaction force components

$$\sum F_x = 0: \quad \text{R}\cos\alpha - (82.1)\cos 20 = 0$$

$$\text{R}\cos\alpha = 77.15\,\text{N} \qquad (1)$$

$$\sum F_y = 0: \quad \text{R}\sin\alpha - (82.1)\sin 20 - 98.1 = 0$$

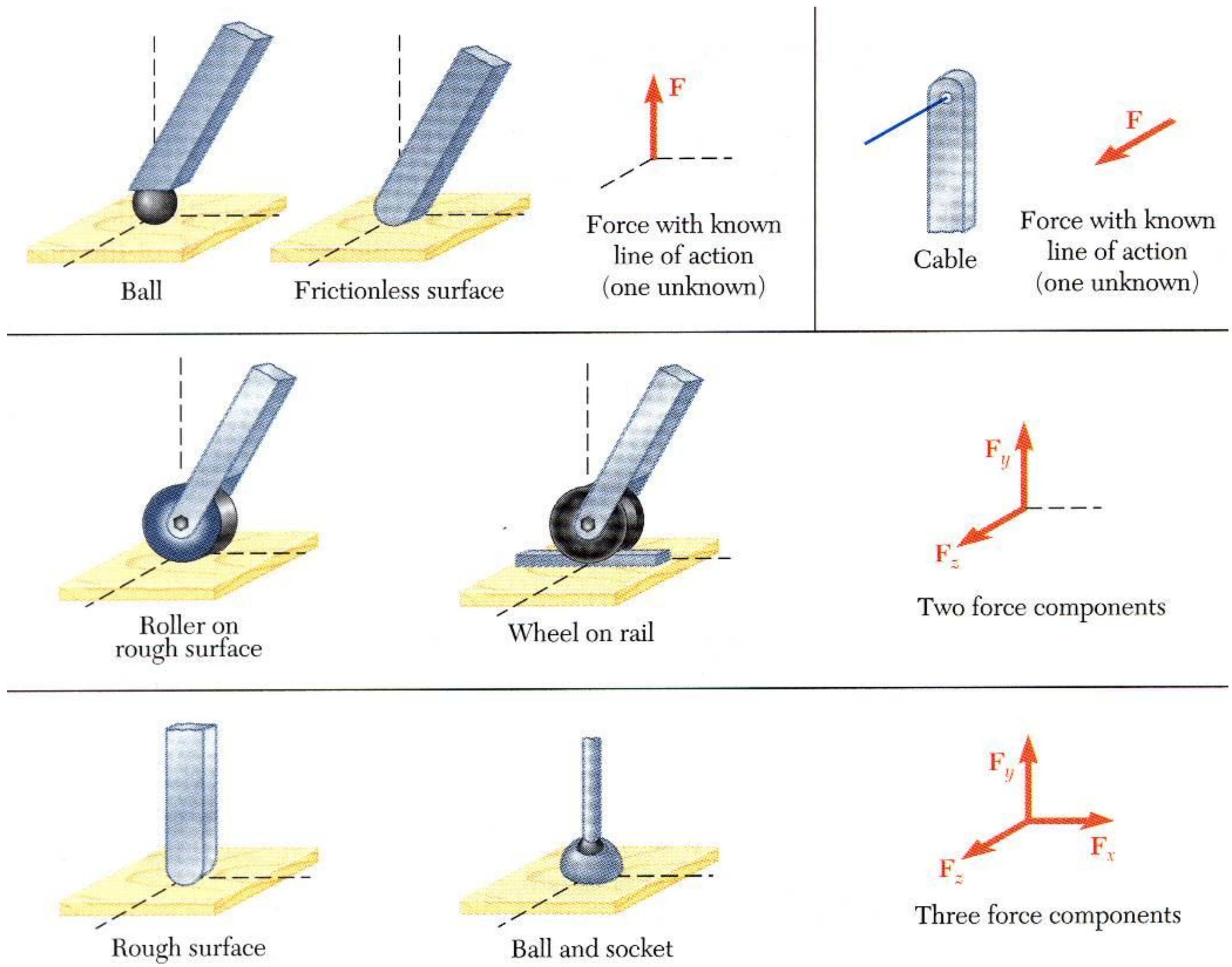
$$\text{R}\sin\alpha = 126.2\,\text{N} \qquad (2)$$

- Solving (1) and (2)

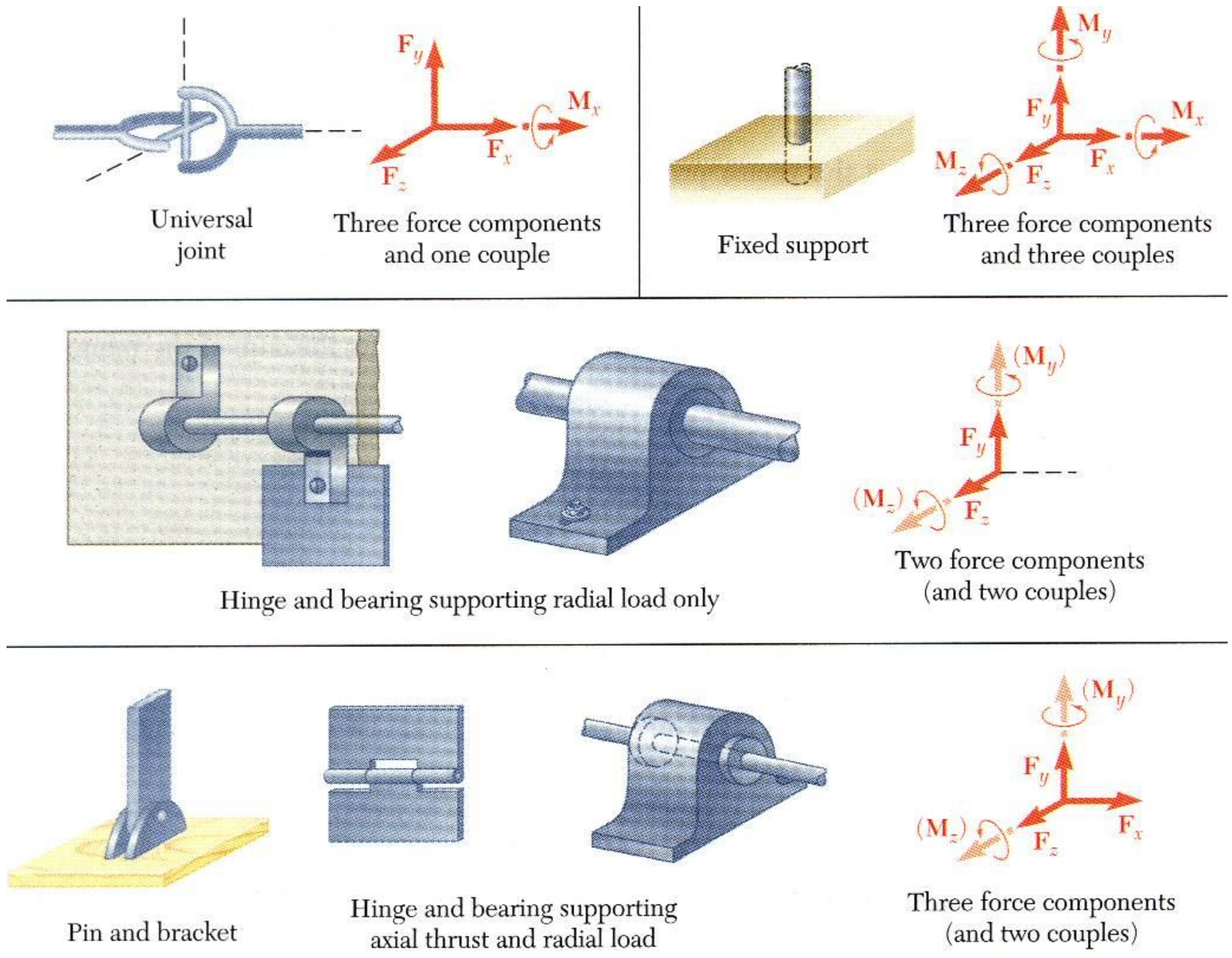
$$\boxed{R = 147.9\,\text{N}}$$

$$\alpha = 58.6^{\circ}$$

Reactions at Supports and Connections for a 3D Structure



Reactions at Supports and Connections for a 3D Structure



ME 161/162 BASIC MECHANICS

UNIT 4
STRUCTURAL ANALYSIS

Instructors: **Dr. Josh Ampofo**
Mr. P. O. Tawiah
Mr. F. W. Adams

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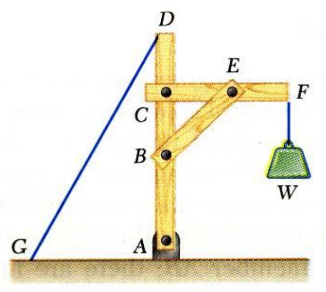
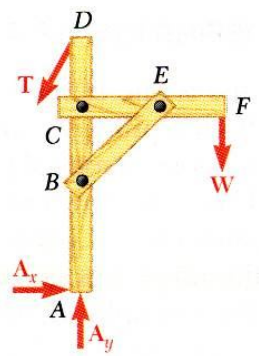
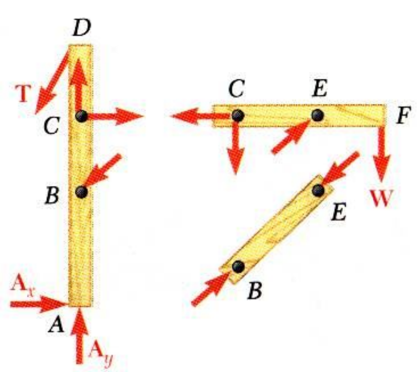
Kumasi, Ghana

Introduction

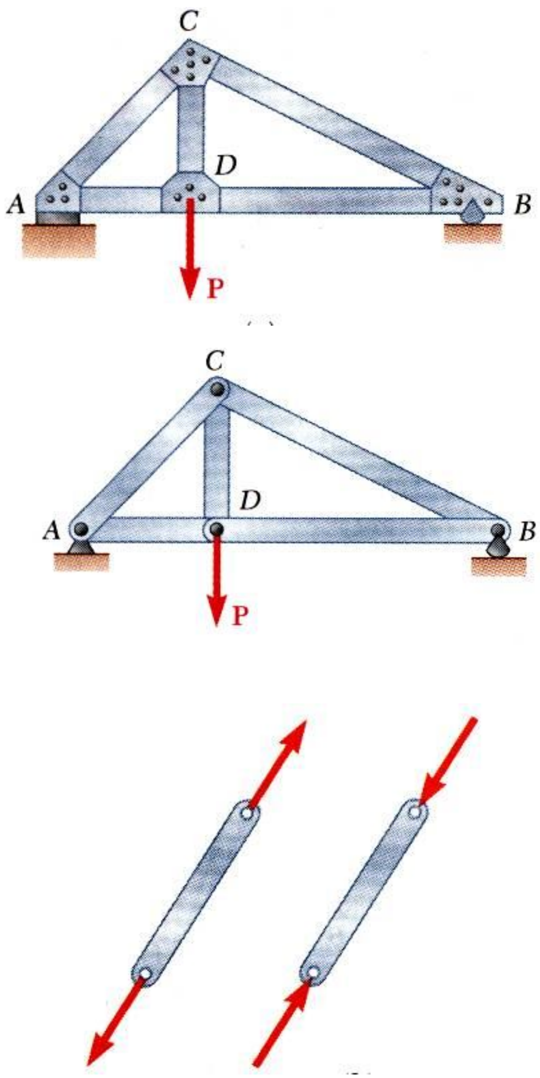






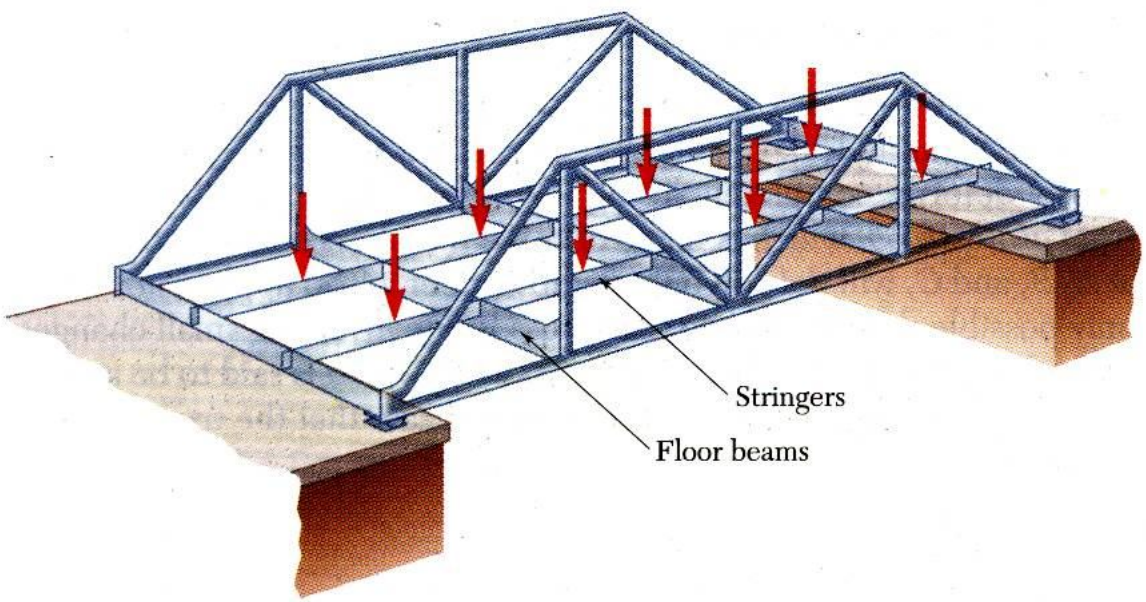
- For the equilibrium of structures made of several connected parts, the *internal forces* as well the *external forces* are considered.
- In the interaction between connected parts, Newton's 3rd Law states that the *forces of action and reaction* between bodies in contact have the same magnitude, same line of action, and opposite sense.
- Three categories of engineering structures are considered:
  a) *Frames*: contain at least one multi-force member, i.e., member acted upon by 3 or more forces.
  b) *Trusses*: formed from *two-force members*, i.e., straight members with end point connections
  c) *Machines*: structures containing moving parts designed to transmit and modify forces

Definition of a Truss



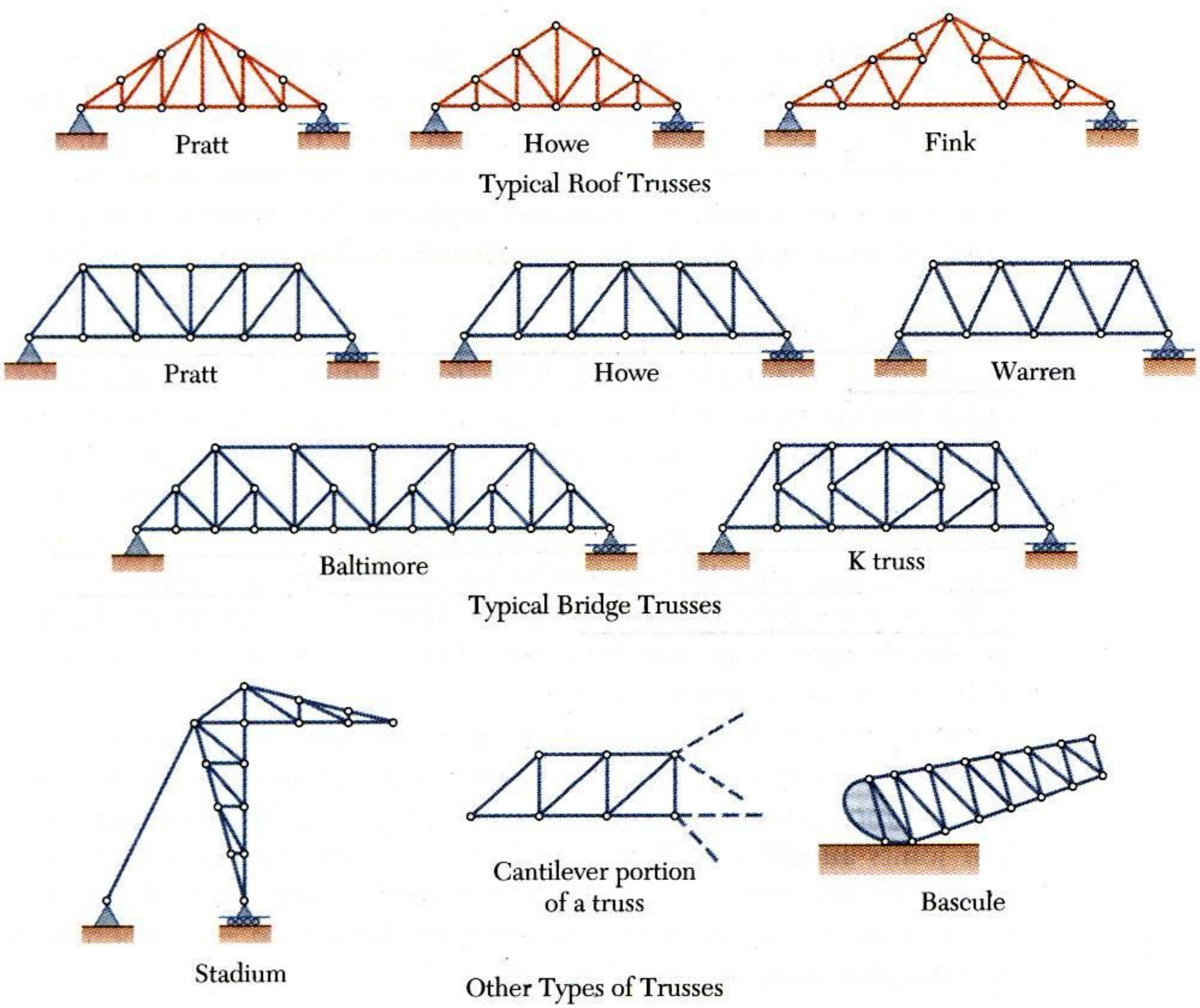
- A truss consists of straight members connected at joints. No member is continuous through a joint
- Most structures are made of several trusses joined together to form a space framework. Each truss carries those loads which act in its plane and may be treated as a two-dimensional structure.
- Bolted or welded connections are assumed to be pinned together. Forces acting at the member ends reduce to a single force and no couple. Only *two-force members* are considered.
- When forces tend to pull the member apart, it is in *tension*. When the forces tend to compress the member, it is in *compression*.

Definition of a Truss

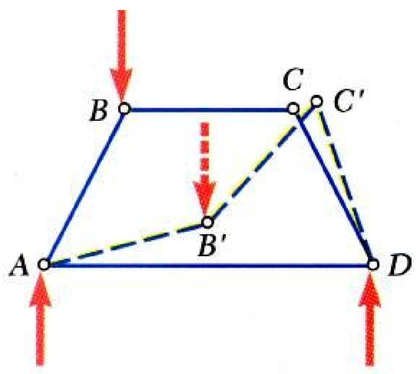


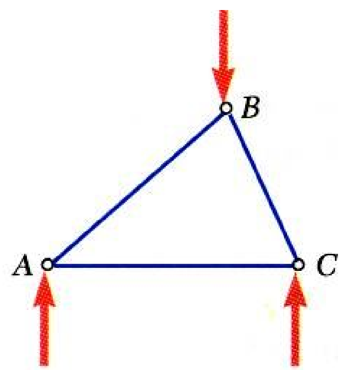
Members of a truss are slender and not capable of supporting large lateral loads. Loads must be applied at the joints.

Definition of a Truss

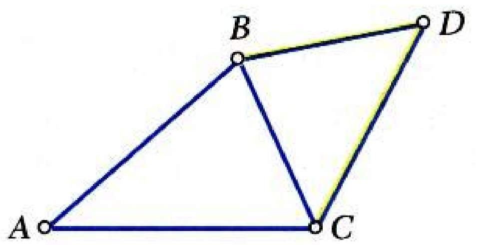


Simple Trusses

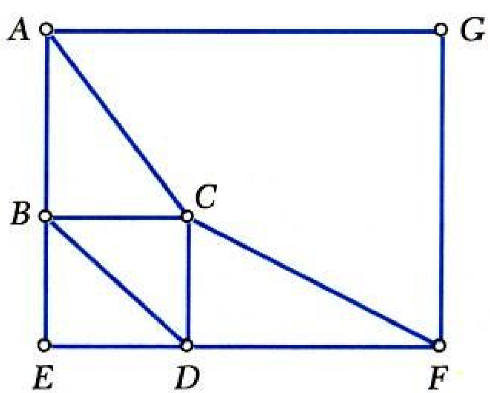




- A *rigid truss* will not collapse under the application of loads.

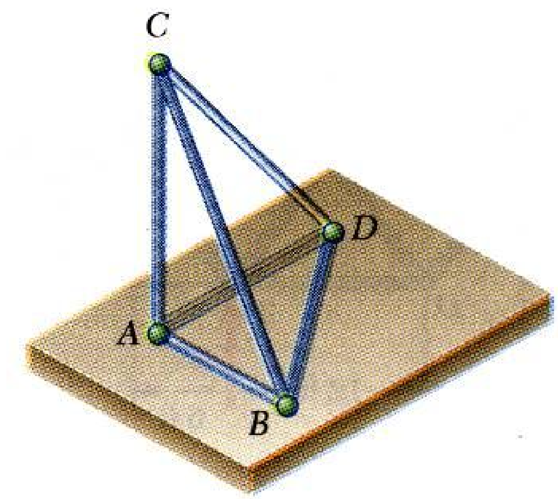


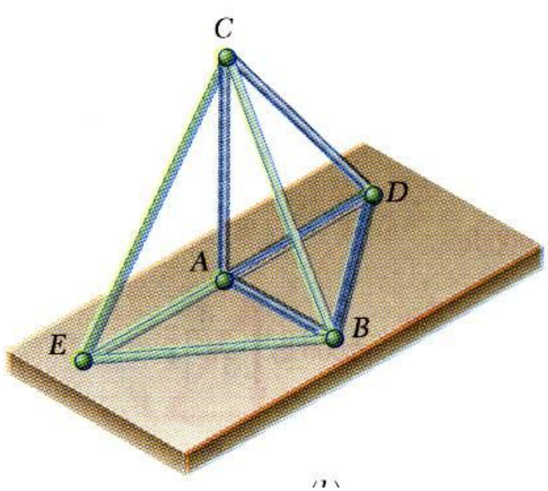
- A *simple truss* is constructed by successively adding two members and one connection to the basic triangular truss.



- In a simple truss, $m = 2n - 3$ where m is the total number of members and n is the number of joints.

Space (or 3D) Trusses

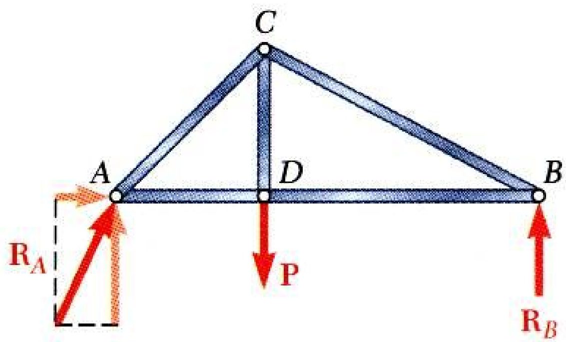


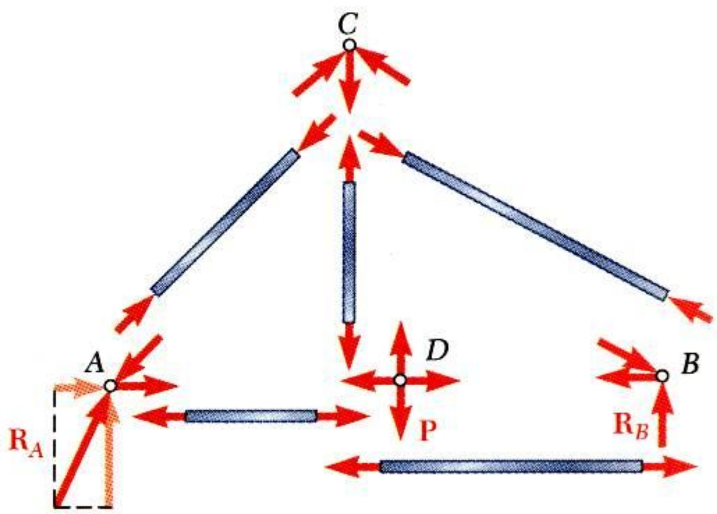


connected at 4 joints to form a tetrahedron.

- A *simple space truss* is formed and can be extended when 3 new members and 1 joint are added at the same time.
- In a simple space truss, $m = 3n - 6$ where m is the number of members and n is the number of joints.
- Conditions of equilibrium for the joints provide $3n$ equations. For a simple truss, $3n = m + 6$ and the equations can be solved for m member forces and 6 support reactions.
- Equilibrium for the entire truss provides 6 additional equations which are not independent of the joint equations.
- The analysis of space trusses is an advanced problem, which is not covered in ME 161 Basic Mechanics.

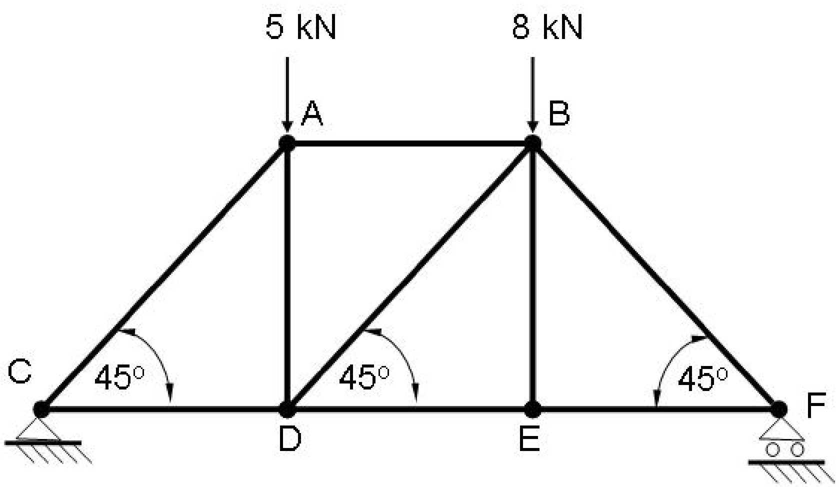
Analysis of Trusses by the Method of Joints





- Dismember the truss and create a freebody diagram for each member and pin.
- The two forces exerted on each member are equal, have the same line of action, and opposite sense.
- Forces exerted by a member on the pins or joints at its ends are directed along the member and equal and opposite.
- Conditions of equilibrium on the pins provide $2n$ equations for $2n$ unknowns. For a simple truss, $2n = m + 3$. May solve for m member forces and 3 reaction forces
- Conditions for equilibrium for the entire truss provide 3 additional equations which are not independent of the pin equations.

Sample Problem 4.1

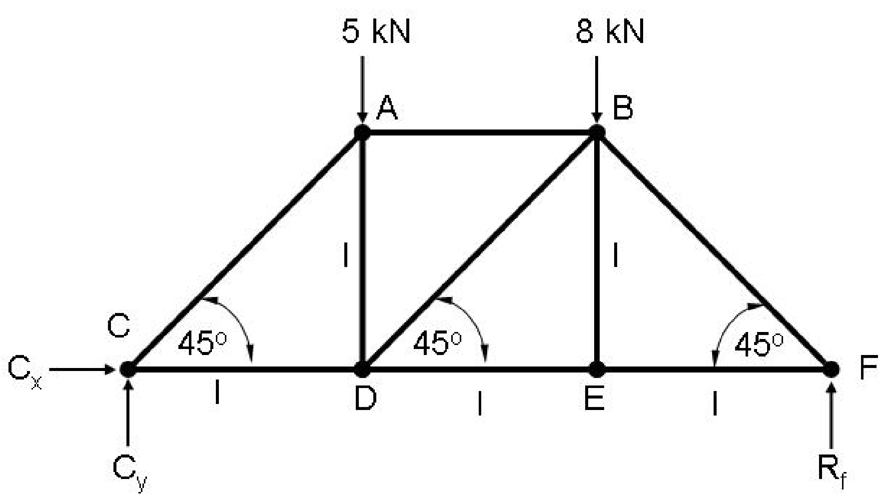


Using the method of joints, determine the force in each member of the truss.

SOLUTION:

- Based on the free-body diagram of the entire truss, solve the 3 equilibrium equations for the reactions at *C* and *F*.
- Joints *C and F are* subjected to only two unknown member forces. Determine these from the joints equilibrium requirements.
- In succession, determine unknown member forces at joints *A*, *D*, B and *E* from joints equilibrium requirements.
- All member forces and support reactions are known at joint *F*. However, the joint's equilibrium requirements may be applied to check the results.

Problem 4.1, Conti;



SOLUTION

- The free-body diagram of the entire truss is shown on the left. The reaction forces are found

$$\circlearrowleft \sum M_F = 0$$

$$5l + 8(2l) - R_f(3l) = 0$$

$$\boxed{R_f = 7\text{ kN}}$$

$$+\uparrow \sum F_y = 0$$

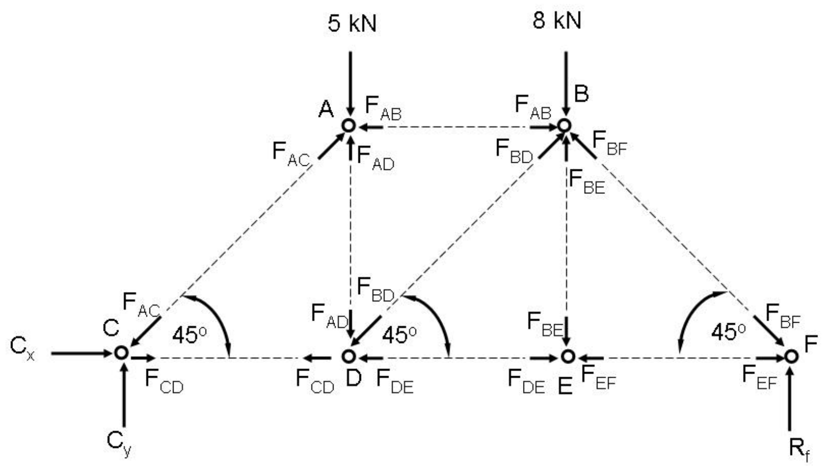
$$C_y + R_f - 5 - 8 = 0$$

$$\boxed{C_y = 6\text{ kN}}$$

$$+\rightarrow \sum F_x = 0$$

$$\boxed{C_x = 0}$$

Problem 4.1, Conti;



Joint C is subjected to two unknown member forces. From the free-body diagram of joint C, the forces in members AC and CD are determined as

$$+\downarrow \sum F_y = 0$$

$$F_{AC} \sin 45 - C_y = 0$$

$$F_{AC} = \frac{C_y}{\sin 45} = \frac{6}{\sin 45}$$

$$\boxed{F_{AC} = 8.5 \text{ kN}}$$

$$+\rightarrow \sum F_x = 0$$

$$F_{CD} - F_{AC} \cos 45 + C_x = 0$$

$$F_{CD} = 8.5 \cos 45 + 0 = 0$$

$$\boxed{F_{CD} = 6.0 \text{ kN}}$$

Similarly, from the free-body diagram of joint A, the forces in members AD and AB are determined as

$$+\uparrow \sum F_y = 0$$

$$F_{AD} + F_{AC} \sin 45 - 5 = 0$$

$$F_{AD} = -F_{AC} \sin 45 + 5$$

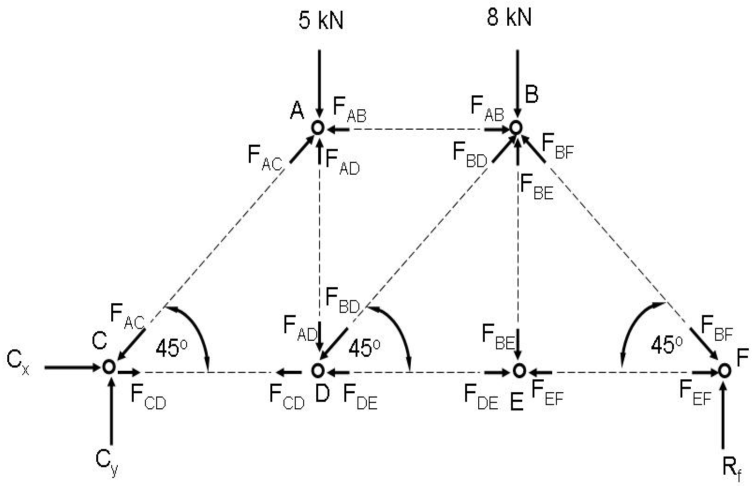
$$\boxed{F_{AD} = -1.0 \text{ kN}}$$

$$+\rightarrow \sum F_x = 0$$

$$F_{AB} - F_{AC} \cos 45 = 0$$

$$\boxed{F_{AB} = 6.0 \text{ kN}}$$

Problem 4.1, Conti;



From the free-body diagram of joint F, the forces in members EF and BF are

$$+\rightarrow \sum F_x = 0$$

$$F_{BF} \sin 45 - R_y = 0$$

$$F_{BF} = \frac{R_y}{\sin 45} = \frac{7}{\sin 45}$$

$$\boxed{F_{BF} = 9.9 \text{ kN}}$$

$$F_{EF} - F_{BF} \cos 45 = 0$$

$$\boxed{F_{EF} = 7.0 \text{ kN}}$$

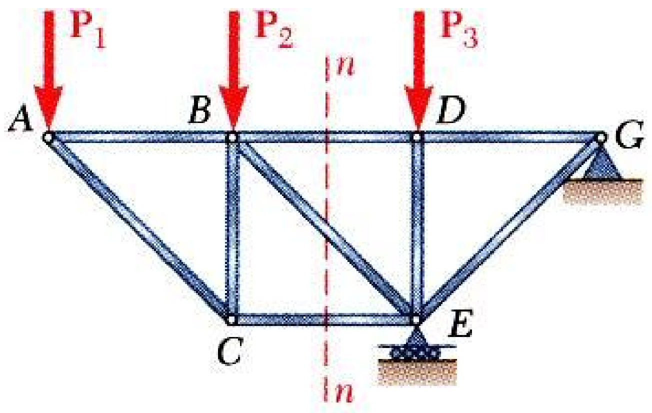
From the free-body diagram of joint B and D, the forces in members BD, BE and BF are

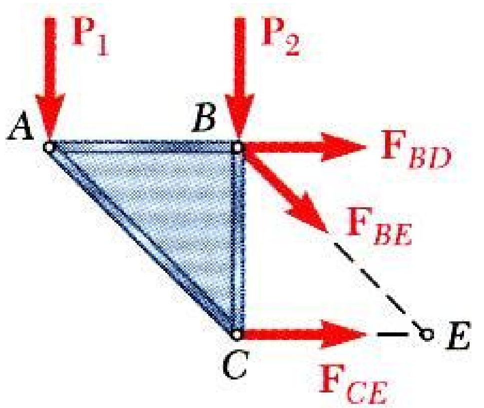
$$\boxed{\begin{aligned} F_{BD} &= 1.4 \text{ kN} \\ F_{DE} &= -7.0 \text{ kN} \\ F_{BE} &= 0 \text{ kN} \end{aligned}}$$

Alternatively, from joint E, BF is a zero-member. Therefore,

$$\boxed{F_{BE} = 0 \text{ kN}}$$

Analysis of Trusses by the Method of Sections

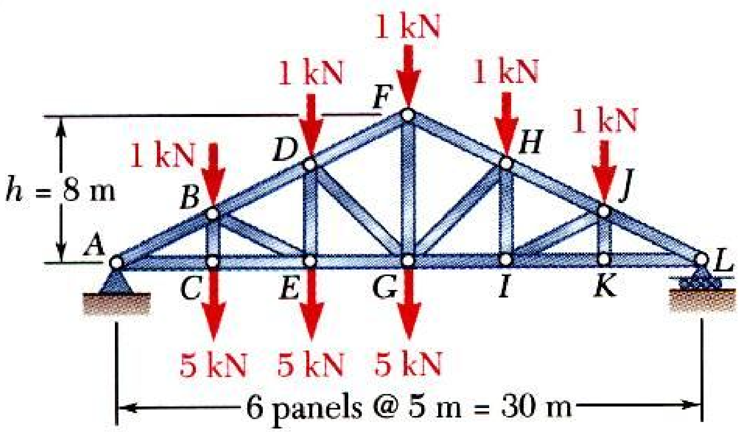




- When the force in only one member or the forces in a very few members are desired, the *method of sections* works well.
- To determine the force in member *BD*, *pass a section* through the truss as shown and create a free body diagram for the left side.
- With only three members cut by the section, the equations for static equilibrium may be applied to determine the unknown member forces, including F_{BD}.

Sample Problem 4.2

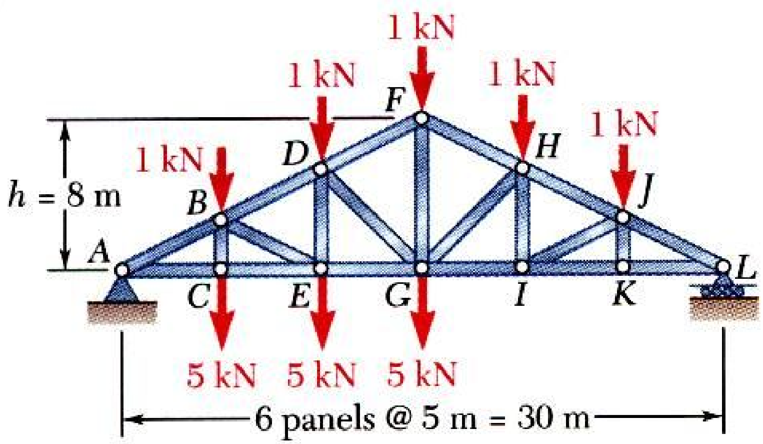
Determine the forces in members *FH*, *GH*, and *GI*.



<u>SOLUTION</u>:

- Take the entire truss as a free body. Apply the conditions for static equilibrium to solve for the reactions at *A* and *L*.
- Pass a section through members *FH*, *GH*, and *GI* and take the right-hand section as a free body.
- Apply the conditions for static equilibrium to determine the desired member forces.

Sample Problem 4.2, Conti;



$$\sum M_A = 0 = -(5\text{ m})(6\text{ kN}) - (10\text{ m})(6\text{ kN}) - (15\text{ m})(6\text{ kN})$$

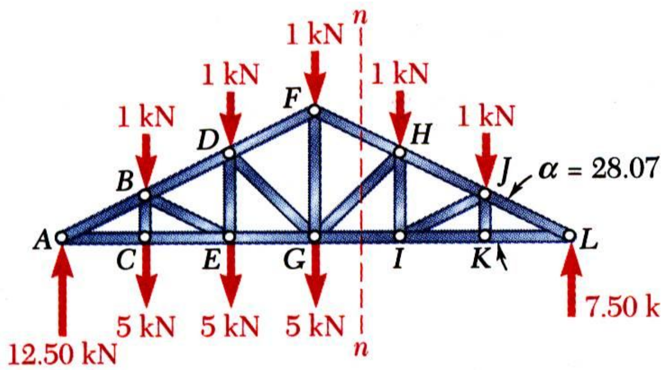
$$-(20\text{ m})(1\text{ kN}) - (25\text{ m})(1\text{ kN}) + (25\text{ m})L$$

$$L = 7.5\text{ kN} \uparrow$$

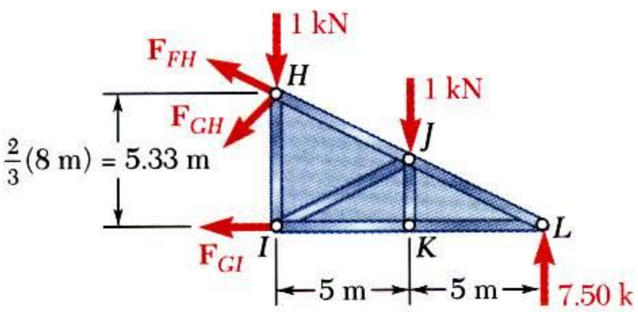
$$\sum F_y = 0 = -20\text{ kN} + L + A$$

$$A = 12.5\text{ kN} \uparrow$$

Sample Problem 4.2, Conti;



- Pass a section through members *FH*, *GH*, and *GI* and take the right-hand section as a free body.



- Apply the conditions for static equilibrium to determine the desired member forces.

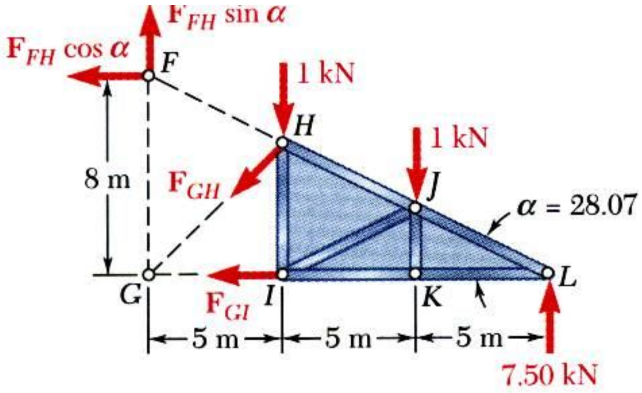
$$\sum M_H = 0$$

$$(7.50\,\mathrm{kN})(10\,\mathrm{m}) - (1\,\mathrm{kN})(5\,\mathrm{m}) - F_{GI}(5.33\,\mathrm{m}) = 0$$

$$F_{GI} = +13.13\,\mathrm{kN}$$

$$\boxed{F_{GI} = 13.13\,\mathrm{kN}\;\; T}$$

Sample Problem 4.2, Conti;



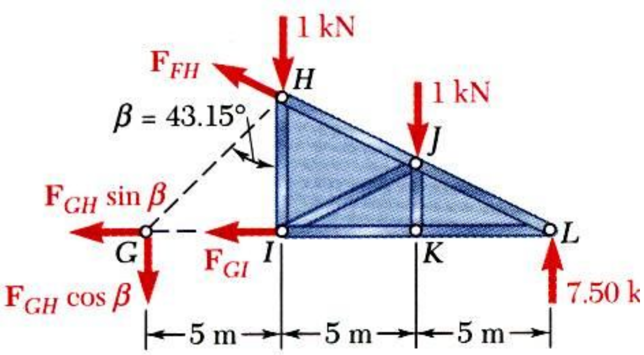
$$\tan\alpha = \frac{FG}{GL} = \frac{8\text{ m}}{15\text{ m}} = 0.5333 \qquad \alpha = 28.07°$$

$$\sum M_G = 0$$

$$(7.5\text{ kN})(15\text{ m}) - (1\text{ kN})(10\text{ m}) - (1\text{ kN})(5\text{ m}) + (F_{FH}\cos\alpha)(8\text{ m}) = 0$$

$$F_{FH} = -13.82\text{ kN}$$

$$\boxed{F_{FH} = 13.82\text{ kN}\ \ C}$$



$$\tan\beta = \frac{GI}{HI} = \frac{5\text{ m}}{\frac{2}{3}(8\text{ m})} = 0.9375 \qquad \beta = 43.15°$$

$$\sum M_L = 0$$

$$(1\text{ kN})(10\text{ m}) + (1\text{ kN})(5\text{ m}) + (F_{GH}\cos\beta)(10\text{ m}) = 0$$

$$F_{GH} = -1.371\text{ kN}$$

$$\boxed{F_{GH} = 1.371\text{ kN}\ \ C}$$

Sample Problem 4.3

Using the method of sections, determine the forces in member AB, BD and DE of Sample problem 4.1

SOLUTION

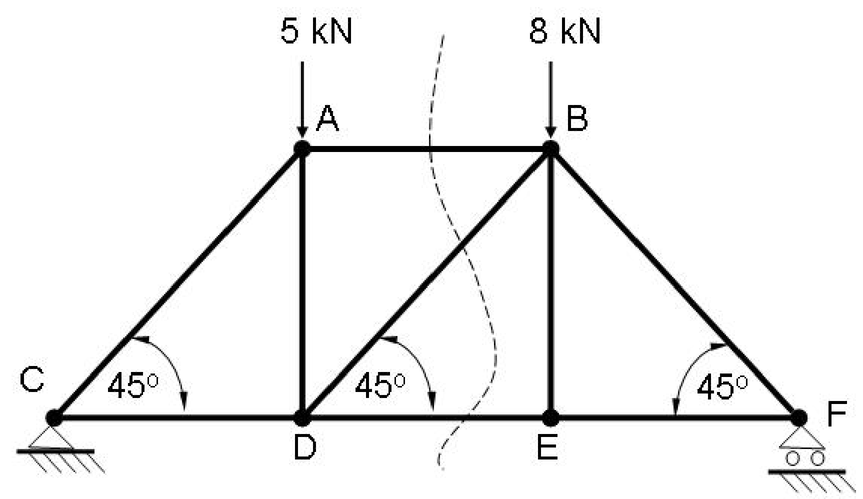
- The free-body diagram of the entire truss is shown on the left. From Problem 4.1, the reaction forces are

$$\boxed{R_f = 7\,\text{kN}}$$

$$\boxed{C_y = 6\,\text{kN}}$$

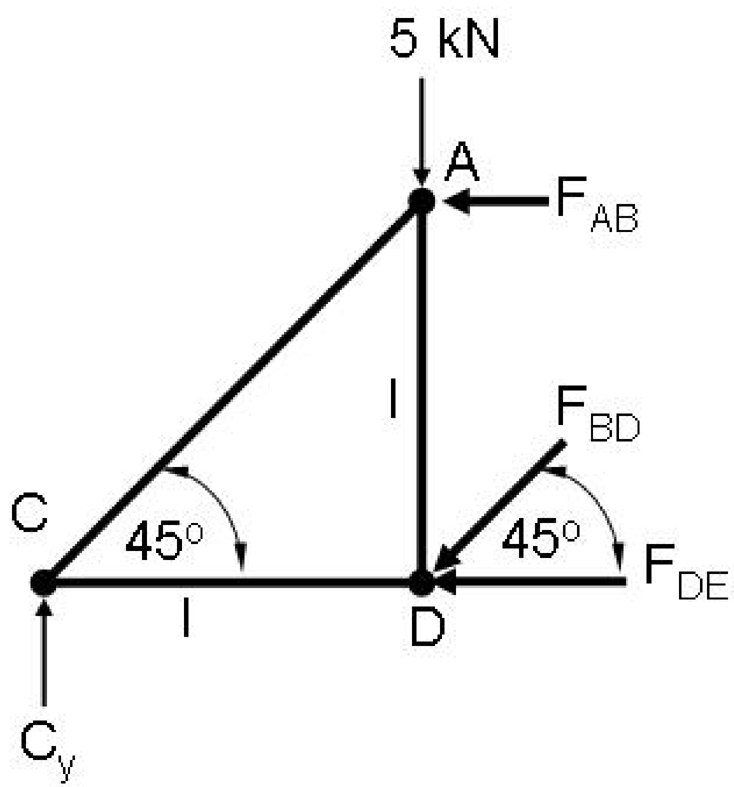
$$\boxed{C_x = 0}$$

To determine the forces in members AB, BD and DE, an imagined cutting section, shown by the dashed line, is passed through the truss such a way as to "cut" the three members. The left-hand portion is arbitrarily chosen for the analysis.



Sample Problem 4.3, Conti;

The free-body diagram of the left-hand portion is truss is shown below.



$$\circlearrowleft \sum M_D = 0$$

$$F_{AB} l - C_y l = 0 \qquad \Rightarrow F_{AB} = C_y$$

$$\boxed{F_{AB} = 6.0 \text{ kN}}$$

$$+\downarrow \sum F_y = 0$$

$$F_{BD} \sin 45 - C_y + 5 = 0$$

$$\boxed{F_{BD} = 1.4 \text{ kN}}$$

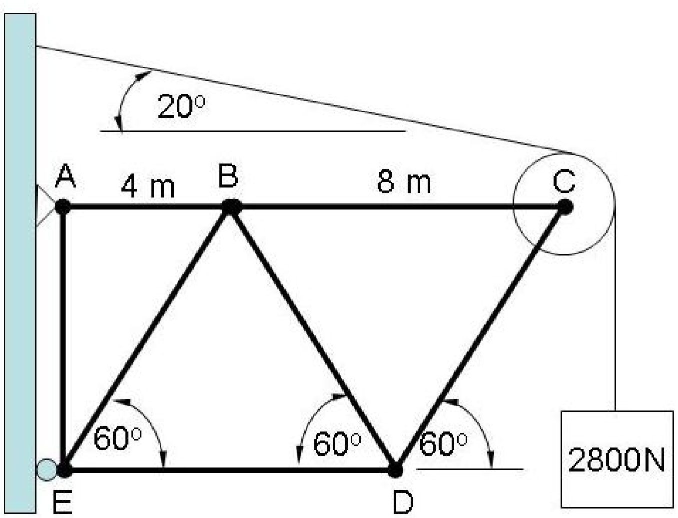
$$+\leftarrow \sum F_x = 0$$

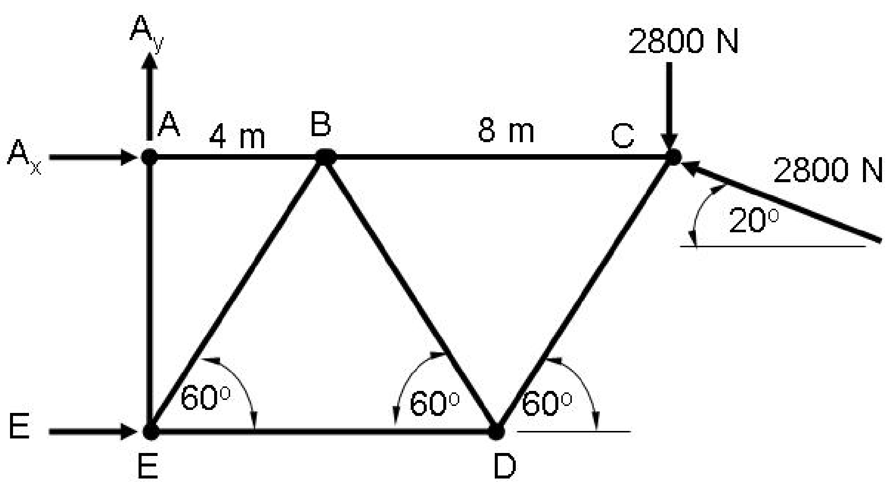
$$F_{DE} + F_{BD} \sin 45 + F_{AB} = 0$$

$$\boxed{F_{DE} = -7.0 \text{ kN}}$$

Pulley or Sheave Connections to Truss Joints

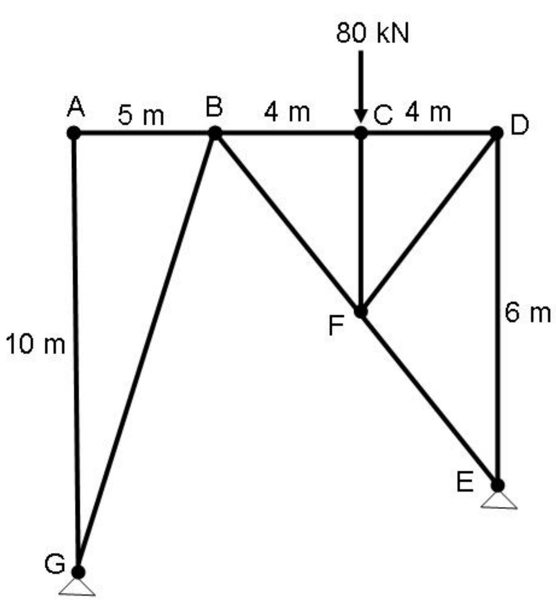
- Force in cable of pulley system is constant throughout the length of the cable.
- The weight of pulleys and cables are normally neglected.
- The reactions due to the tensions in cable act along the direction of the cable, as shown in the figure below.

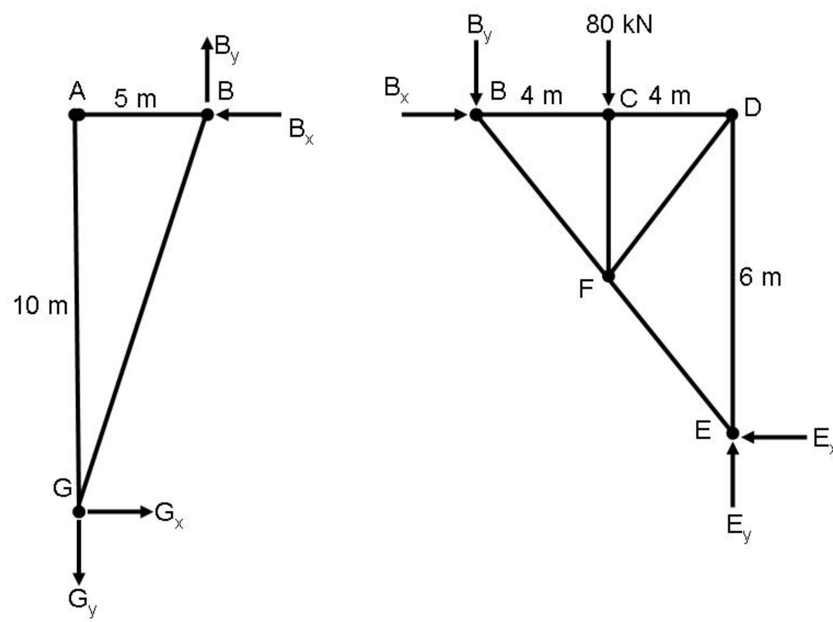




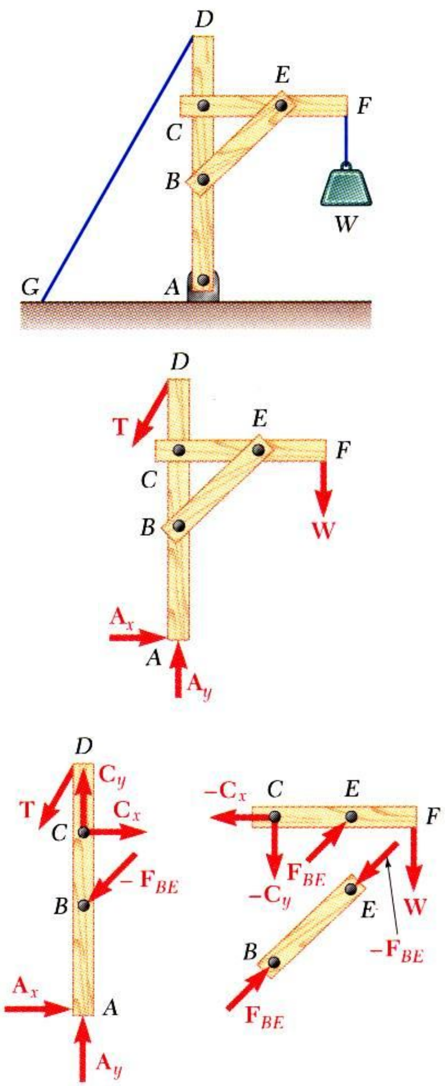
Connected Trusses

- In certain truss configuration, the free-body diagram of the entire truss contains more than three unknowns.
- The truss may be separated into two separate trusses, which share a common joint as shown below.
- By Newton's 3rd law, forces at the common must opposite senses when drawn on the free-body diagram.



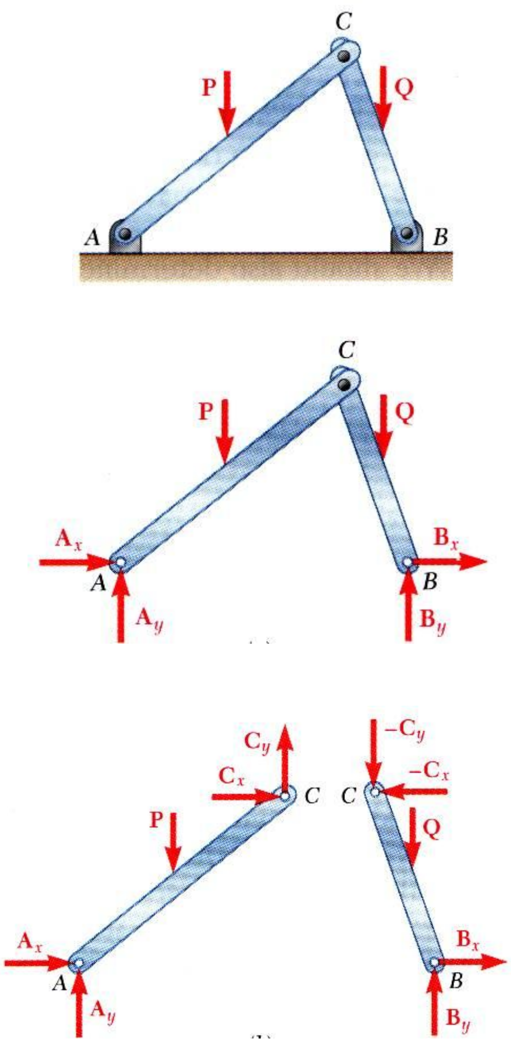


Analysis of Frames



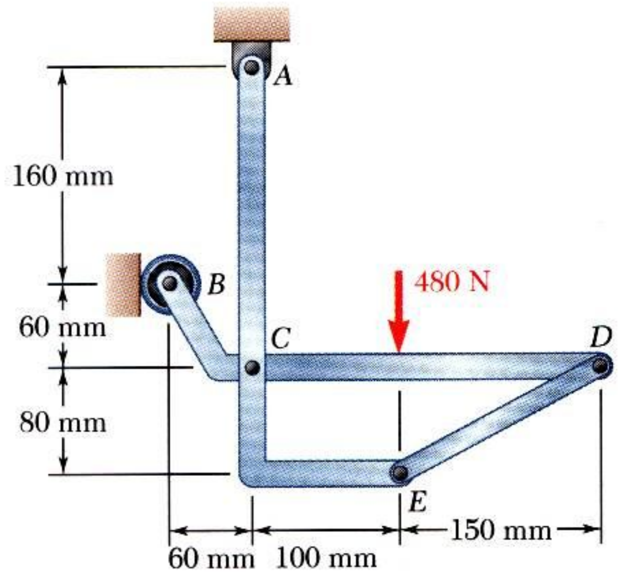
- *Frames* and *machines* are structures with at least one *multiforce* member. Frames are designed to support loads and are usually stationary. Machines contain moving parts and are designed to transmit and modify forces.
- A free body diagram of the complete frame is used to determine the external forces acting on the frame.
- Internal forces are determined by dismembering the frame and creating free-body diagrams for each component.
- Forces on two force members have known lines of action but unknown magnitude and sense.
- Forces on multiforce members have unknown magnitude and line of action. They must be represented with two unknown components.
- Forces between connected components are equal, have the same line of action, and opposite sense.

Analysis of Frames



- Some frames may collapse if removed from their supports. Such frames can not be treated as rigid bodies.
- A free-body diagram of the complete frame indicates four unknown force components which can not be determined from the three equilibrium conditions.
- The frame must be considered as two distinct, but related, rigid bodies.
- With equal and opposite reactions at the contact point between members, the two free-body diagrams indicate 6 unknown force components.
- Equilibrium requirements for the two rigid bodies yield 6 independent equations.

Sample Problem 4.4



Members *ACE* and *BCD* are connected by a pin at *C* and by the link *DE*. For the loading shown, determine the force in link *DE* and the components of the force exerted at *C* on member *BCD*.

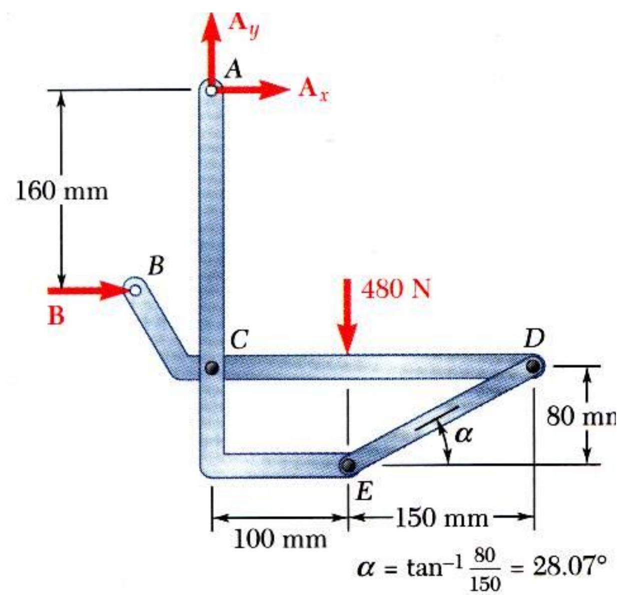
SOLUTION:

- Create a free-body diagram for the complete frame and solve for the support reactions.

  BCD. The force exerted by the link *DE* has a known line of action but unknown magnitude. It is determined by summing moments about *C*.

- With the force on the link *DE* known, the sum of forces in the *x* and *y* directions may be used to find the force components at
- With member *ACE* as a free-body, check the solution by summing moments about *A*.

Sample Problem 4.4, Cont;



SOLUTION:

- Create a free-body diagram for the complete frame and solve for the support reactions.

$$\sum F_y = 0 = A_y - 480\text{ N}$$

$$\boxed{A_y = 480\text{ N} \uparrow}$$

$$\sum M_A = 0 = -(480\text{ N})(100\text{ mm}) + B(160\text{ mm})$$

$$\boxed{B = 300\text{ N} \rightarrow}$$

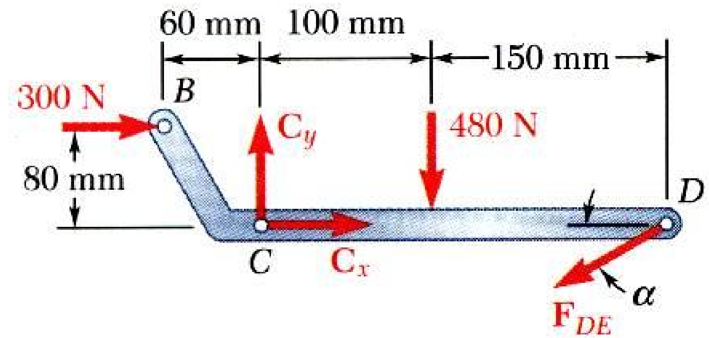
$$\sum F_x = 0 = B + A_x$$

$$\boxed{A_x = -300\text{ N} \leftarrow}$$

$$\alpha = \tan^{-1}\frac{80}{150} = 28.07°$$

Sample Problem 4.4, Cont;

- Define a free-body diagram for member *BCD*. The force exerted by the link *DE* has a known line of action but unknown magnitude. It is determined by summing



$$\sum M_C = 0 = (F_{DE}\sin\alpha)(250\text{ mm}) + (300\text{ N})(60\text{ mm}) + (480\text{ N})(100\text{ mm})$$

$$F_{DE} = -561\text{ N}$$

$$\boxed{F_{DE} = 561\text{ N}\ C}$$

$$\sum F_x = 0 = C_x - F_{DE}\cos\alpha + 300\text{ N}$$

$$0 = C_x - (-561\text{ N})\cos\alpha + 300\text{ N}$$

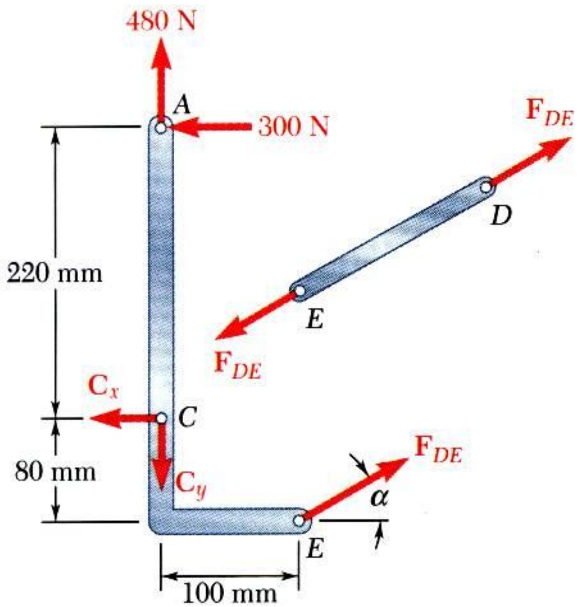
$$\boxed{C_x = -795\text{ N}}$$

$$\sum F_y = 0 = C_y - F_{DE}\sin\alpha - 480\text{ N}$$

$$0 = C_y - (-561\text{ N})\sin\alpha - 480\text{ N}$$

$$\boxed{C_y = 216\text{ N}}$$

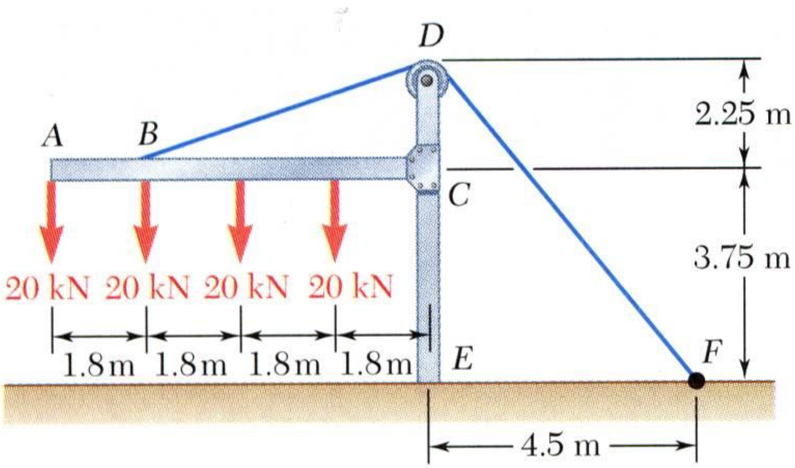
Sample Problem 4.4, Cont;



- With member *ACE* as a free-body, check the solution by summing moments about *A*.

$$\sum M_A = (F_{DE}\cos\alpha)(300\text{ mm}) + (F_{DE}\sin\alpha)(100\text{ mm}) - C_x(220\text{ mm})$$
$$= (-561\cos\alpha)(300\text{ mm}) + (-561\sin\alpha)(100\text{ mm}) - (-795)(220\text{ mm}) = 0$$

Example 4-3



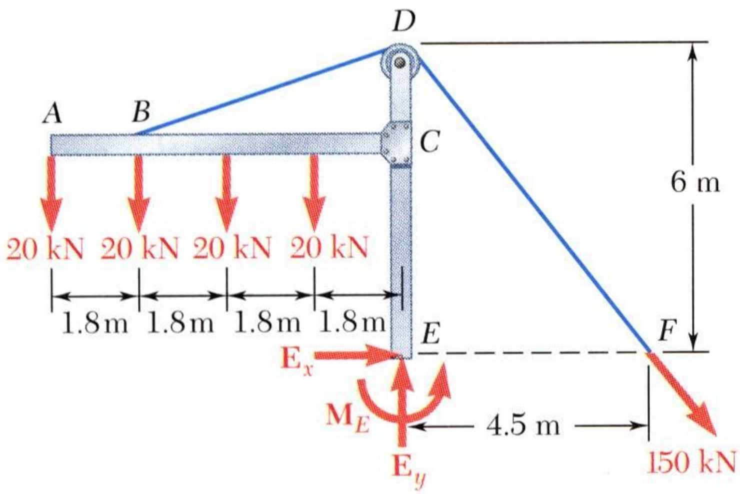
The frame supports part of the roof of a small building. The tension in the cable is 150 kN.

Determine the reaction at the fixed end *E*.

SOLUTION:

- Create a free-body diagram for the frame and cable.
- Solve 3 equilibrium equations for the reaction force components and couple at *E*.

Example 4-3, Cont;



- Free-body diagram for the frame and cable.

$$\sum F_x = 0: \quad E_x + \frac{4.5}{7.5}(150\,\text{kN}) = 0$$

$$\boxed{E_x = -90.0\,\text{kN}}$$

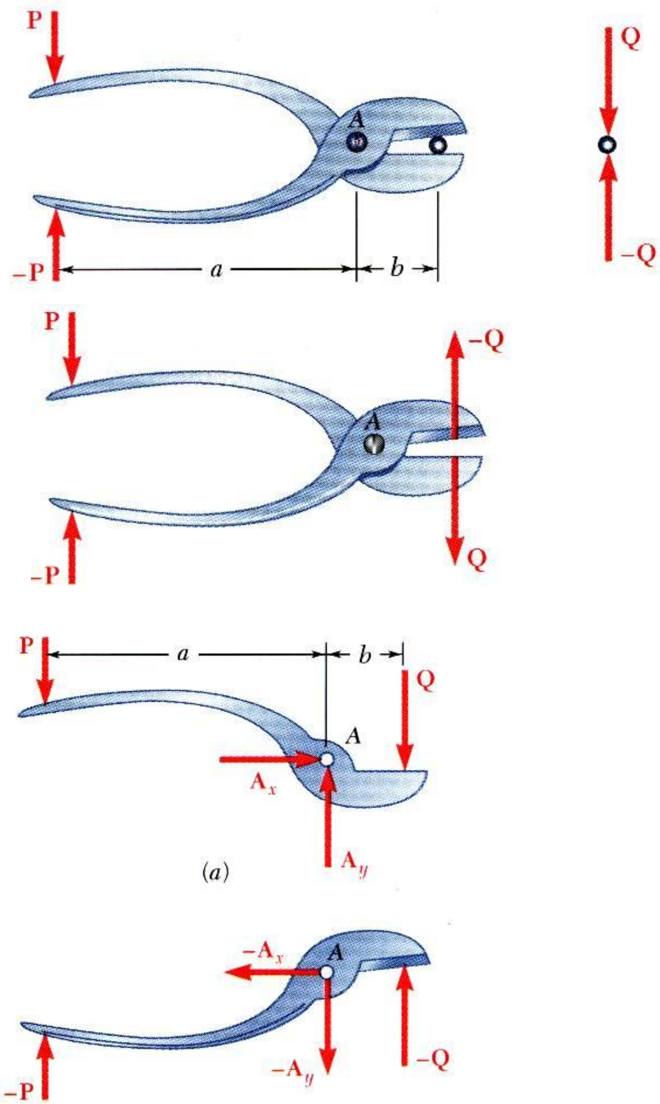
$$\sum F_y = 0: \quad E_y - 4(20\,\text{kN}) - \frac{6}{7.5}(150\,\text{kN}) = 0$$

$$\boxed{E_y = +200\,\text{kN}}$$

$$\begin{aligned}\sum M_E = 0: \quad & +20\,\text{kN}(7.2\,\text{m}) + 20\,\text{kN}(5.4\,\text{m}) \\ & +20\,\text{kN}(3.6\,\text{m}) + 20\,\text{kN}(1.8\,\text{m}) \\ & -\frac{6}{7.5}(150\,\text{kN})4.5\,\text{m} + M_E = 0\end{aligned}$$

$$\boxed{M_E = 180.0\,\text{kN}\cdot\text{m}}$$

Machines



- Machines are structures designed to transmit and modify forces. Their main purpose is to transform *input forces* into *output forces*

- Create a free-body diagram of the complete machine, including the reaction that the wire exerts.

- The machine is a nonrigid structure. Use one of the components as a free-body.

- Taking moments about *A*,

$$\sum M_A = 0 = aP - bQ \qquad Q = \frac{a}{b}P$$

ME 161/162 BASIC MECHANICS

UNIT 5 FRICTION

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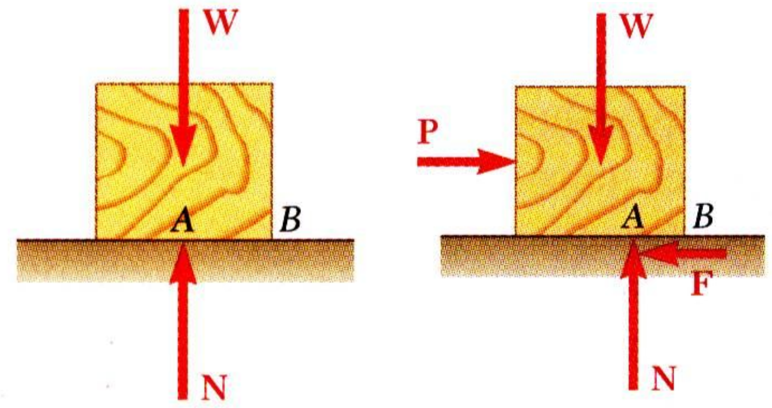
Introduction

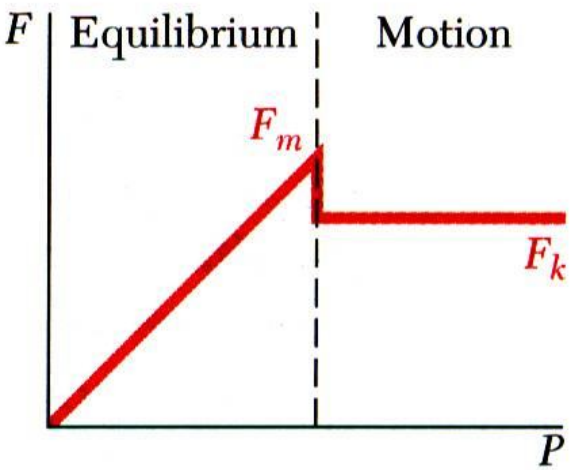
- Whenever two bodies are in contact, a tangential force, called *friction force*, at the surface of contact is developed that tends to oppose or prevent any relative sliding motion of the bodies.
- However, the friction forces are limited in magnitude and will not prevent motion if sufficiently large forces are applied.
- The distinction between frictionless and rough is, therefore, a matter of degree.
- There are two types of friction: *dry* or *Coulomb friction* and *fluid friction*. Fluid friction applies to lubricated mechanisms. The present discussion is limited to dry friction between nonlubricated surfaces.
- When there is no relative motion between the bodies, the frictional force is called static friction. The frictional force is called kinetic friction or dynamic friction when there is relative motion.

The Laws of Dry Friction



surface. Forces acting on block are its weight and reaction of surface *N*.

block to remain stationary, in equilibrium, a horizontal component *F* of the surface reaction is required. *F* is a *static-friction force.*



$$F_m = \mu_s N$$

$$F_k = \mu_k N$$

- The constants μ_s and μ_k are called the static and kinetic coefficient of friction.

The Laws of Dry Friction

- Both frictional resistances F_r (static and kinetic friction) are proportional to the normal reaction between the surfaces.

$$F_k = \mu_k N \qquad F_s = \mu_s N$$

- The frictional force is independent of the (apparent) area of contact. i.e. $F_m > F_m \qquad \mu_s > \mu_k$

- The limiting value of static friction force is greater than the kinetic friction force.

- At low velocity, the kinetic friction force is virtually independent of the velocity.

Coefficients of Friction

Table 8.1. Approximate Values of Coefficient of Static Friction for Dry Surfaces

Metal on metal	0.15–0.60
Metal on wood	0.20–0.60
Metal on stone	0.30–0.70
Metal on leather	0.30–0.60
Wood on wood	0.25–0.50
Wood on leather	0.25–0.50
Stone on stone	0.40–0.70
Earth on earth	0.20–1.00
Rubber on concrete	0.60–0.90

$$F_m = \mu_s N$$

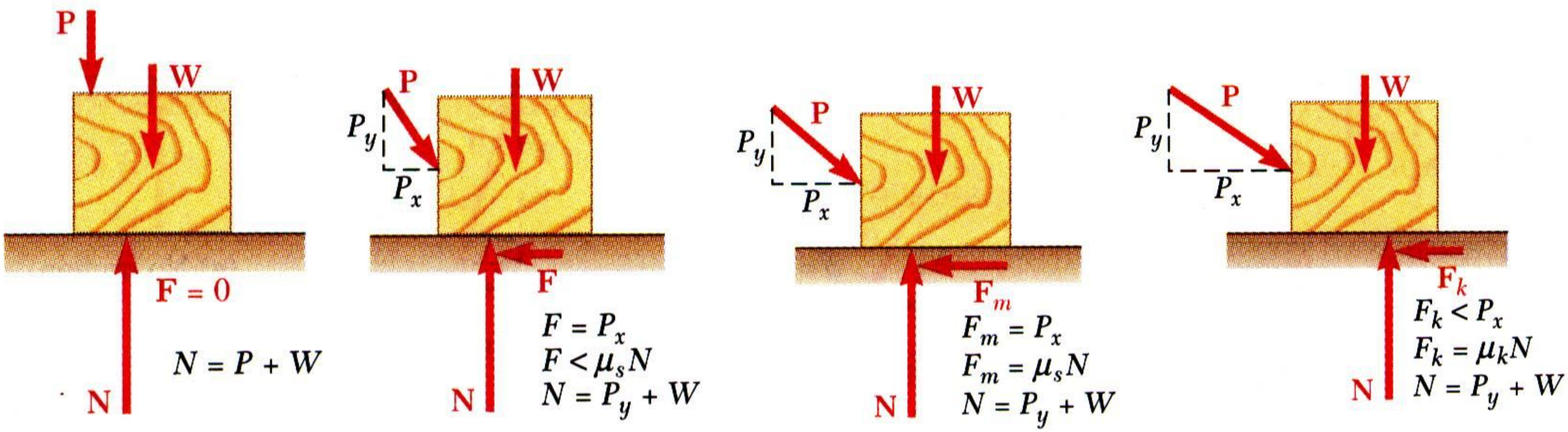
$$F_k = \mu_k N$$

$$\mu_k \cong 0.75 \mu_s$$

- Maximum static-friction force and kinetic-friction force are:
  - proportional to normal force
  - dependent on type and condition of contact surfaces
  - independent of contact area

Four Situations of Contact

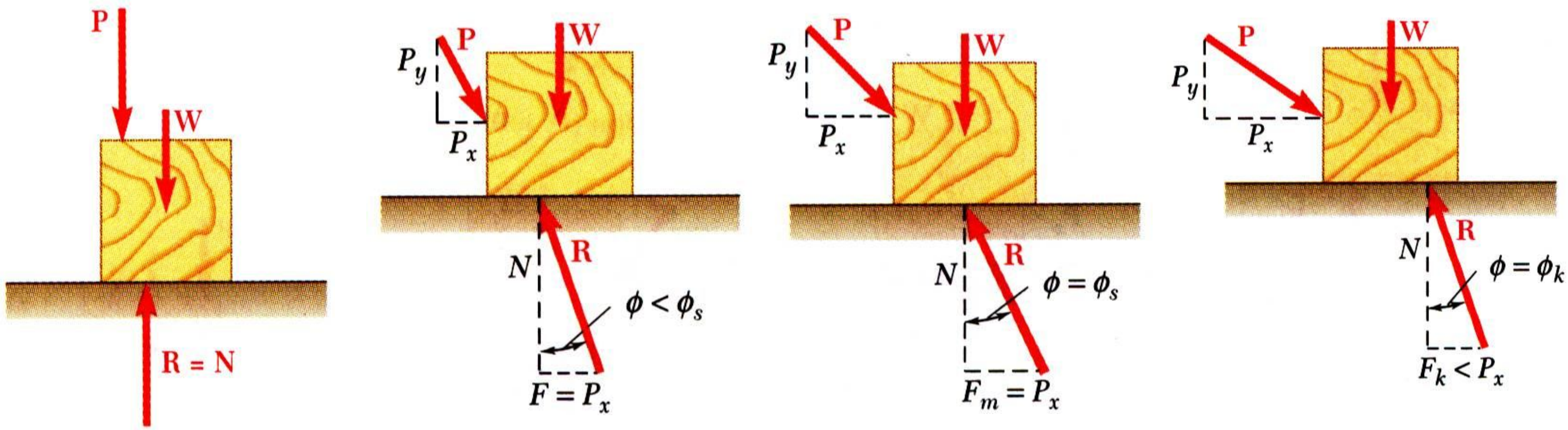
- Four situations can occur when a rigid body is in contact with a horizontal surface:



- No friction, ($P_x = 0$)
- No motion, ($P_x < F_m$)
- Motion impending, ($P_x = F_m$)
- Motion, ($P_x > F_m$)

Angles of Friction

- It is sometimes convenient to replace normal force N and friction force F by their resultant $\boldsymbol{R}$:



$$\tan\phi_s = \frac{F_m}{N} = \frac{\mu_s N}{N}$$

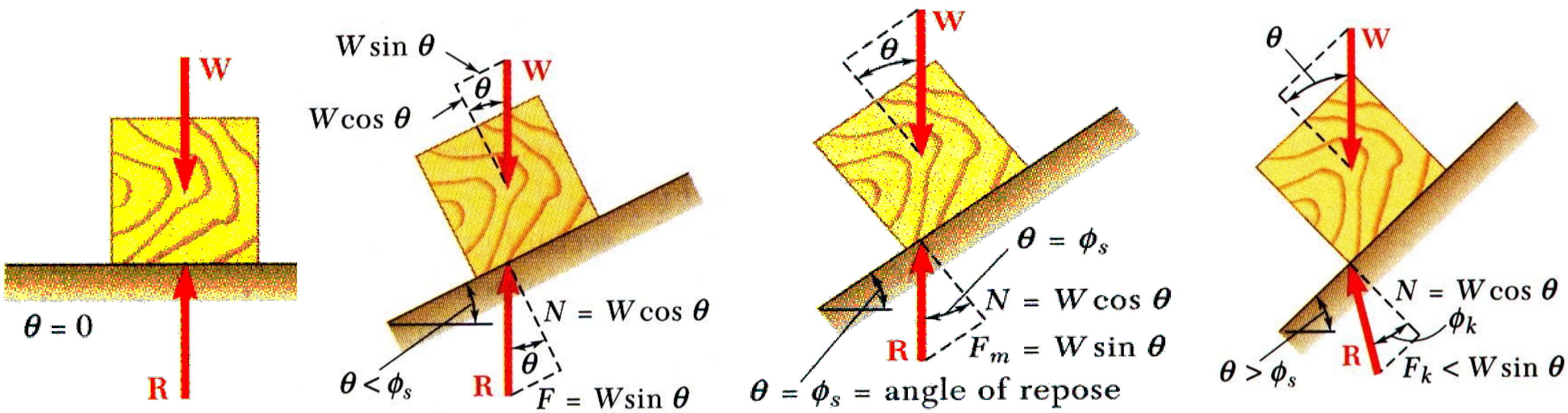
$$\tan\phi_s = \mu_s$$

$$\tan\phi_k = \frac{F_k}{N} = \frac{\mu_k N}{N}$$

$$\tan\phi_k = \mu_k$$

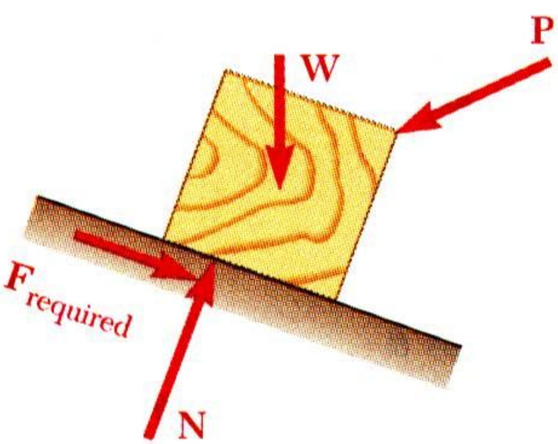
Angles of Friction

- Consider block of weight *W* resting on board with variable inclination angle *θ*.

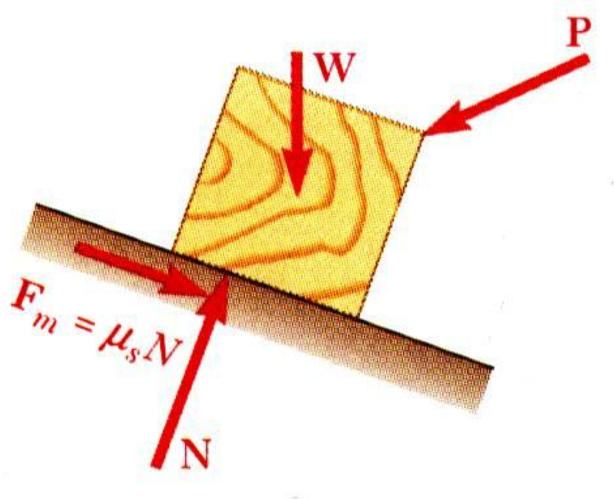


- No friction
- No motion
- Motion impending
- Motion

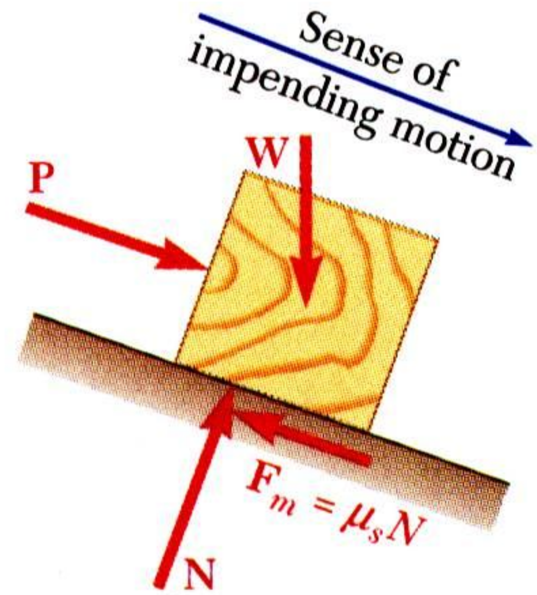
Problems Involving Dry Friction



- All applied forces known
- Coefficient of static friction is known
- Determine whether body will remain at rest or slide

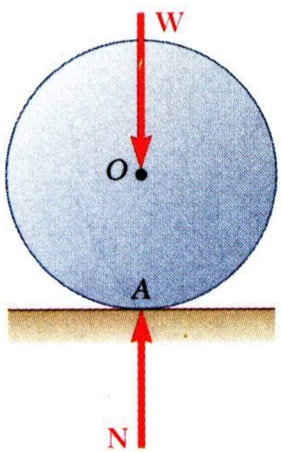


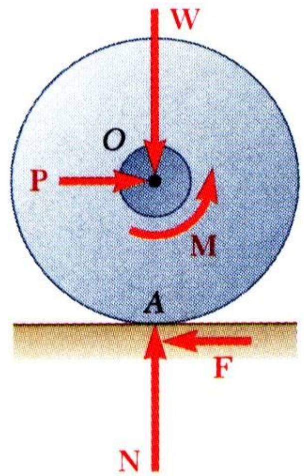
- All applied forces known
- Motion is impending
- Determine value of coefficient of static friction.

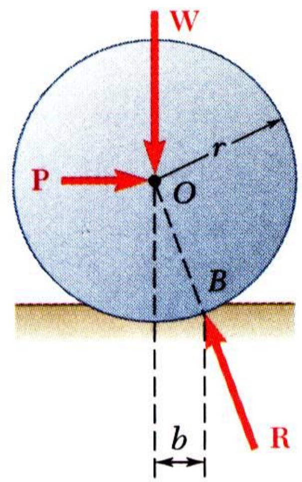


- Coefficient of static friction is known
- Motion is impending
- Determine magnitude or direction of one of the applied forces

Wheel Friction. Rolling Resistance







- Point of wheel in contact with ground has no relative motion with respect to ground.

  Ideally, no friction.

- Moment *M* due to frictional resistance of axle bearing requires couple produced by equal and opposite *P* and *F*.

  Without friction at rim, wheel would slide.

- Deformations of wheel and ground cause resultant of ground reaction to be applied at *B*. *P* is required to balance moment of *W* about *B*.

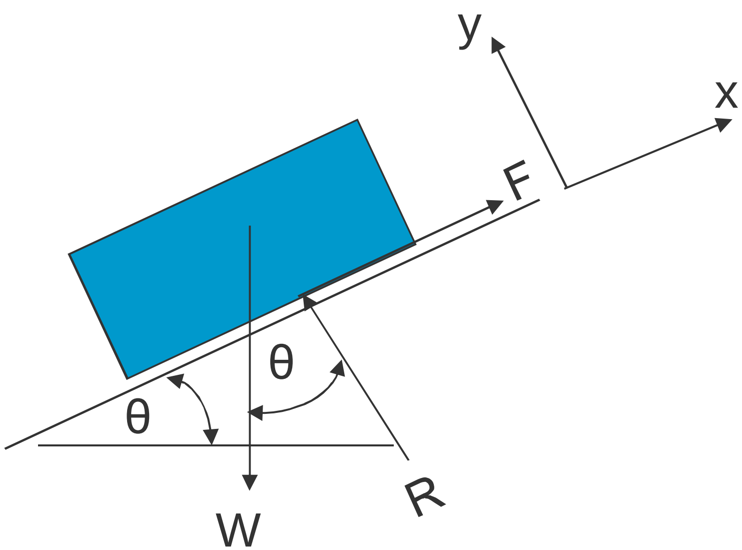
  $Pr = Wb$
  b = coefficient of rolling resistance

Sample Problem 5.1

At what angle θ must a surface be tilted to just cause an object resting on it to slide if the coefficient of static friction between the surface and the body is 0.35.

SOLUTION

Free-diagram of the object is shown below.



Free-body diagram of problem 5.1

$$\sum F_x = 0: \quad F - W\sin\theta = 0$$

$$F = \mu_s R = W\sin\theta$$

$$\sum F_y = 0: \quad R - W\cos\theta = 0$$

$$R = W\cos\theta$$

$$\mu_s W\cos\theta = W\sin\theta$$

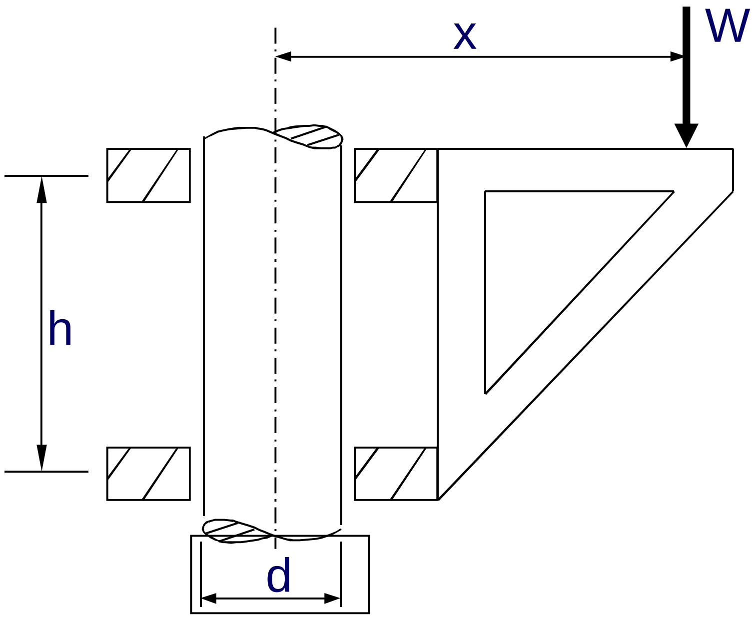
$$\mu_s = \frac{\sin\theta}{\cos\theta} = \tan\theta$$

$$\theta = \tan^{-1}\mu_s = \tan^{-1}0.35$$

$$\theta = 19.3^o$$

Sample Problem 5.2

The moveable bracket shown may be placed at any height on the 3-in. diameter pipe. If the coefficient of friction between the pipe and bracket is μ_s, determine the minimum distance x at which the load can be supported. Neglect the weight of the bracket.

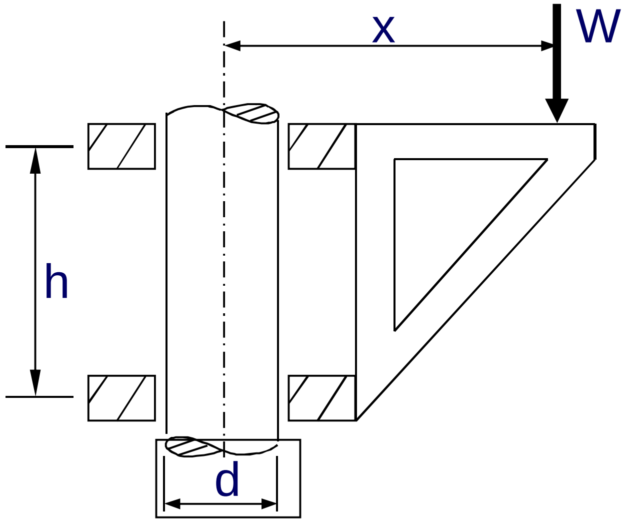


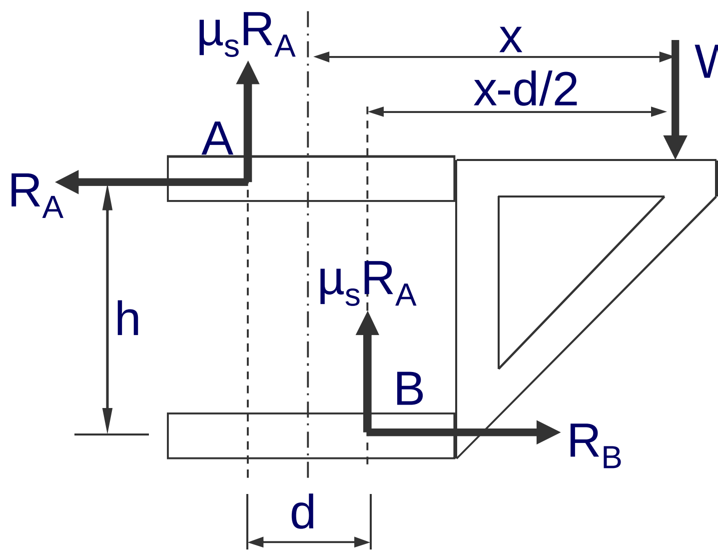
SOLUTION

- When W is placed at minimum x, the bracket is about to slip and friction forces in upper and lower collars are at maximum value.

- Apply conditions for static equilibrium to find minimum x.

Sample Problem 5.2, Cont;





- Apply conditions for static equilibrium to find minimum *x*.

$$\sum F_x = 0: \quad R_B - R_A = 0 \qquad R_B = R_A$$

$$\sum F_y = 0: \quad \mu_s R_A + \mu_s R_B - W = 0$$

$$2\mu_s R_A = W \quad \Rightarrow R_A = \frac{W}{2\mu_s}$$

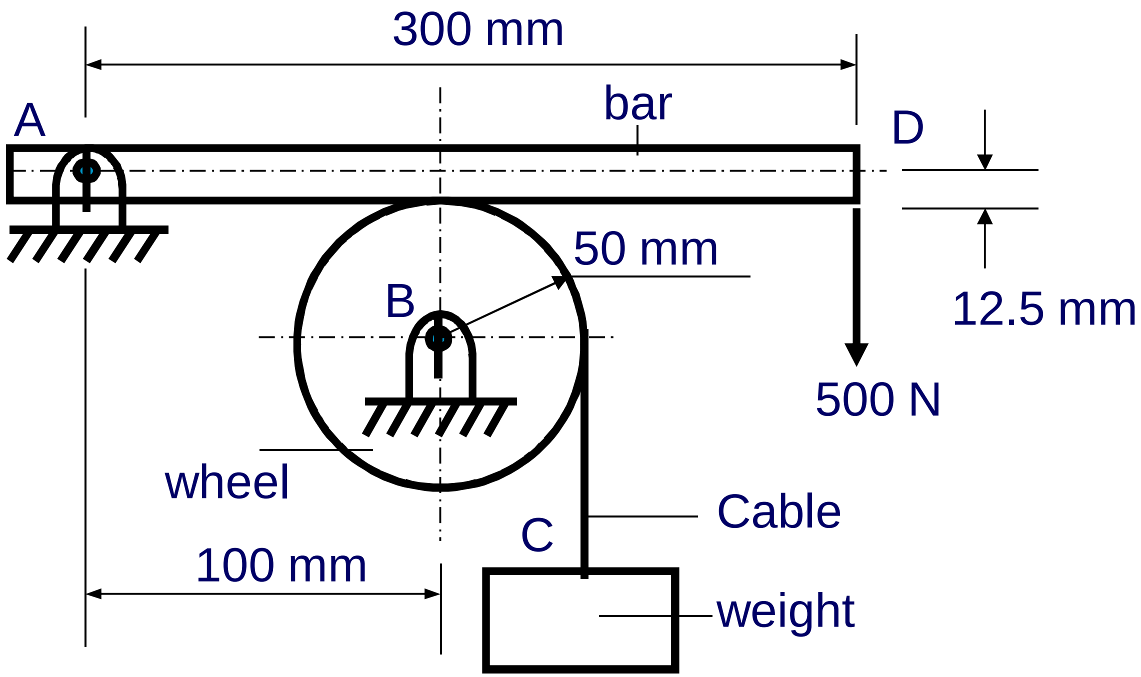
$$\sum M_B = 0: \quad R_A h - \mu_s R_A d - W\left(x - \frac{d}{2}\right) = 0$$

$$\left(\frac{W}{2\mu_s}\right)h - \mu_s\left(\frac{W}{2\mu_s}\right)d - W\left(x - \frac{d}{2}\right) = 0$$

$$\boxed{x = \frac{h}{2\mu_s}}$$

Sample Problem 5.3

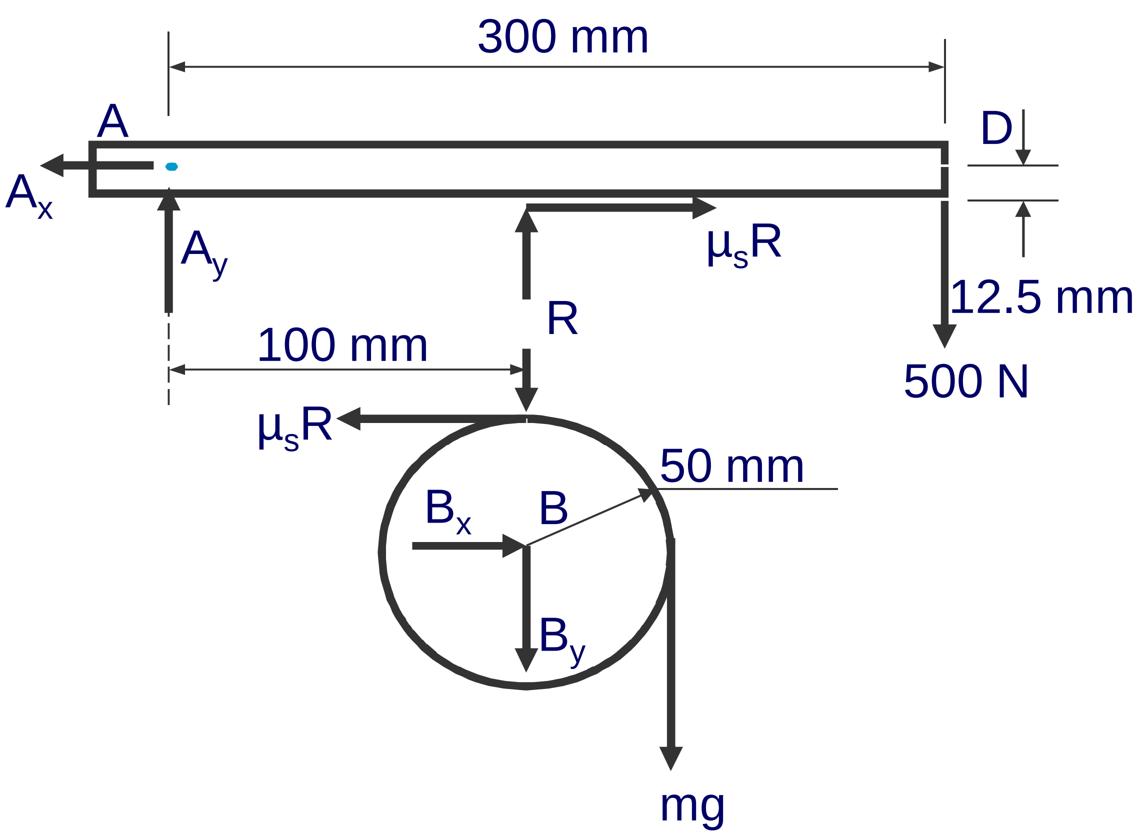
Determine the mass of the weight C to just turn the wheel B if the coefficient of static friction between the bar AD and the wheel is 0.4. Neglect friction in the wheel bearings and the weights of the bar and cable.



Sample Problem 5.3, Cont;

SOLUTION

Free-diagram of the bar and the wheel are shown below.



From the free-diagram of the bar, and taking moment about joint A

$$\sum M_A = 0:$$

$$R(100) + \mu_s R(12.5) - 500(300) = 0$$

$$R = \frac{500(300)}{100 + 12.5\mu_s} = \frac{500(300)}{100 + 12.5(0.35)}$$

$$R = 1.4371\text{x}10^3 \text{ N}$$

From the free-diagram of the bar, and taking moment about joint B

$$\sum M_B = 0: \quad mg(50) - \mu_s R(50) = 0$$

$$m = \frac{\mu_s R}{g} = \frac{(0.35)(1.4371\text{x}10^3)}{9.81}$$

$$m = 51.3\,\text{kg}$$

ME 161/162 BASIC MECHANICS

CHAPTER 6
SIMPLE MACHINES

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Introduction

- A *simple* machine is a device used to increase force, to increase speed or to change the direction of a force.
- In mechanics, there are only six simple machines: lever, inclined plane, wheel and axle, wedge and pulley.
- *Mechanical Advantage* (MA) of a machine is defined as the ratio of load, W, to applied effort, P. ie

$$Mechanical\ Advantage = \frac{W}{P}$$

- *Velocity Ratio* (VR) of a machine is defined as the ratio of distance (x) through which applied effort P moves to distance (y) through which load W moves , P.

$$Velocity\,Ratio = \frac{D_P}{D_W}$$

- Efficiency, η = Useful Work / Work input = MA / VR

$$\eta = \frac{\mathrm{W.y}}{P.x} = \frac{\left(\frac{\mathrm{W}}{\mathrm{P}}\right)}{\left(\frac{x}{y}\right)} = \frac{\mathrm{MA}}{VR}$$

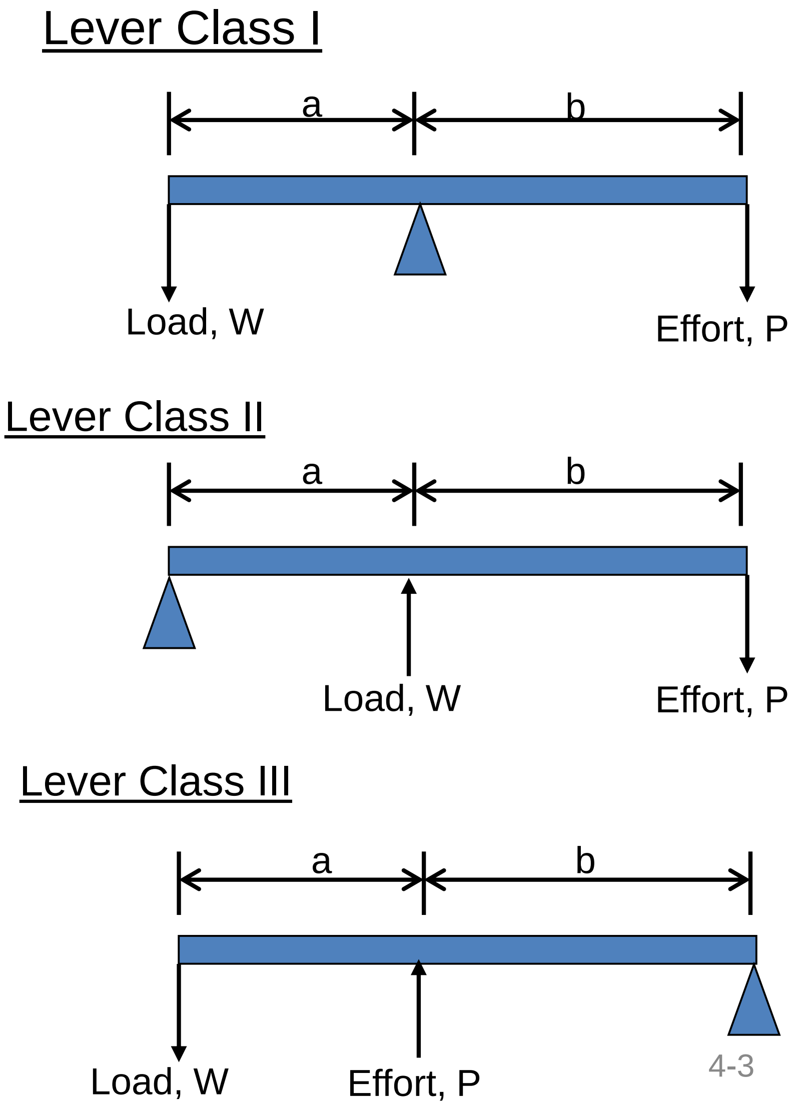
Lever

- A lever is a rigid bar, straight or curved, that is capable of turning about pivot or fulcrum along its length such that an effort acting at one point can support or resist load at a different point

- Levers are frequently are mainly used to move heavy loads or operate mechanisms

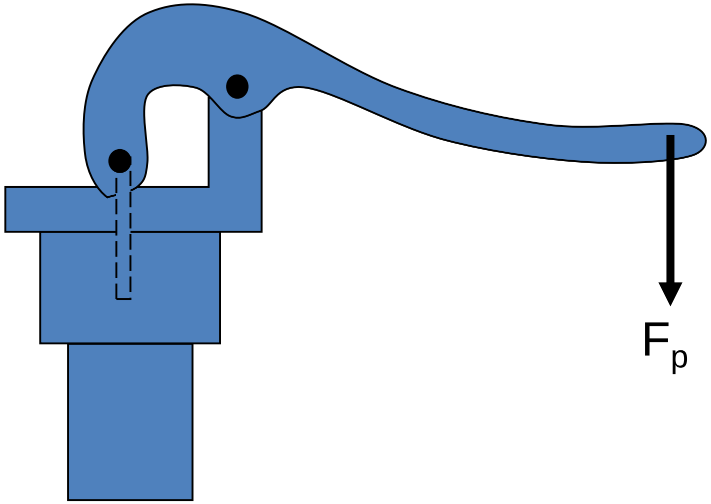
- There are three classes of lever, which are based on the location of the fulcrum to the load the load

The relationship load and effort for each class of lever is W x a = P x b



Sample Problem 6.1

If the mechanical advantage of the curved lever shown in the figure below is 2, what force F_p is required to raise 10 kg of water?



<u>Solution</u>

Load, W = 10 x 9.81 = 98.1 N

$$MA = \frac{W}{P}$$

$$P = \frac{W}{MA} = \frac{98.1}{2}$$

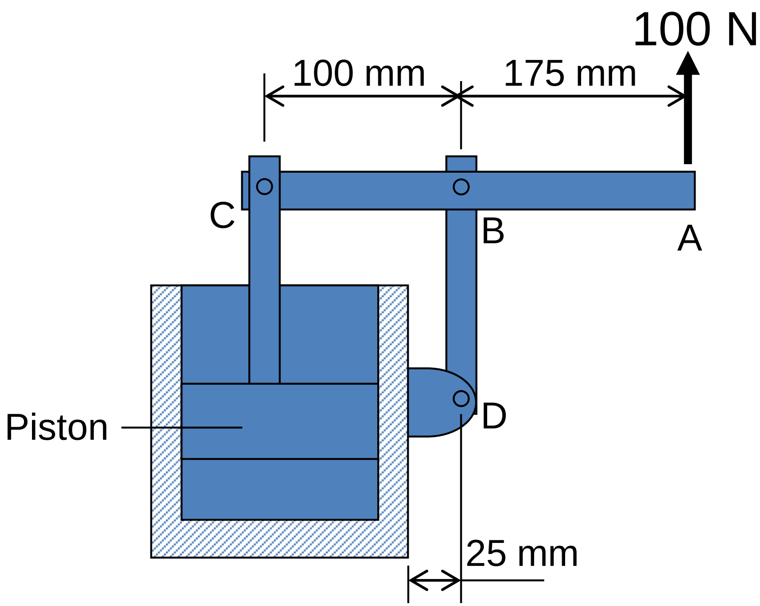
$$\boxed{P = 49.05\ \text{N}}$$

Sample Problem 6.2

A force of 150 N is applied to the handle AC of the pump shown in the figure below.

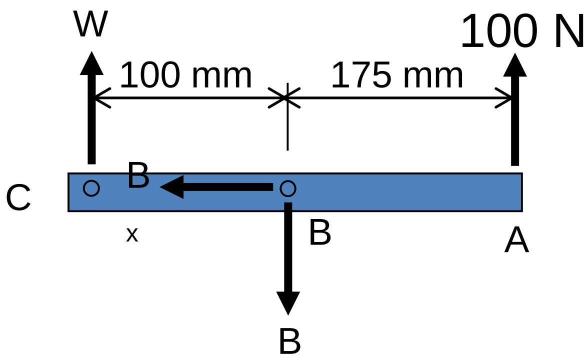
(a) Determine the force transmitted to the piston for the position shown.

(b) What is the mechanical advantage of the system



Solution

Shown below is the free-body diagram of the handle AC



(a) Taking moment about point B gives

W (100) = 100 (175)

$\boxed{W = 175 \text{ N}}$

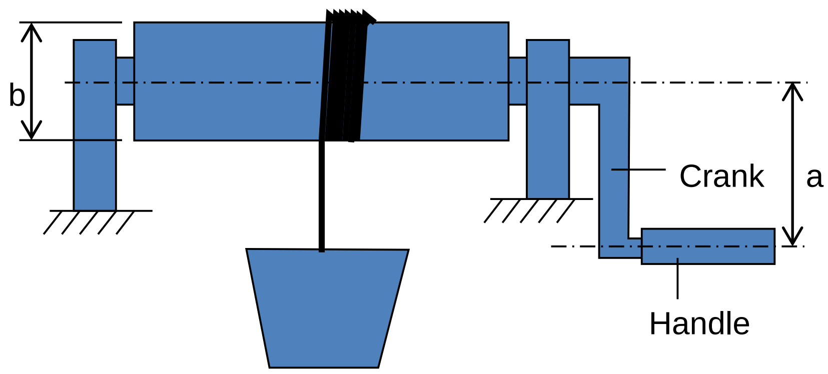
(b) MA = W / P

MA = 175 / 100

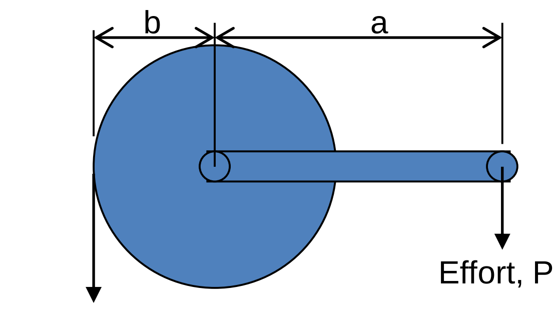
$\boxed{MA = 1.75}$

Wheel and Axle

A wheel and axle consists of wheel that is rigidly attached to a smaller-diameter shaft or axle. Examples of a wheel and axle are door knob, steering wheel and screw driver bit.



Geometry



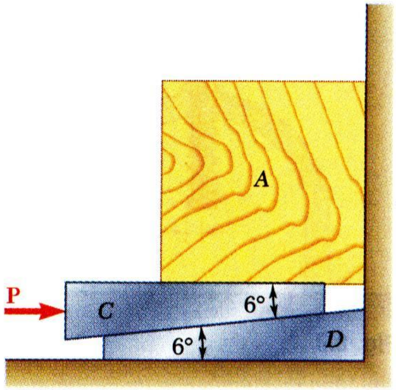
Free-body diagram

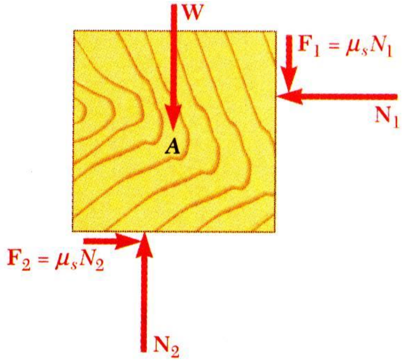
The figure of right is a typical wheel and axle and its free –body diagram. Summing up moments about the axis of the wheel and axles, we have

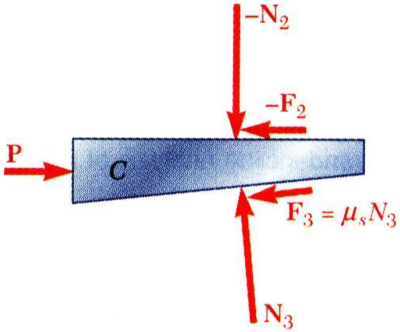
$$Wb = Pa$$

$$MA = \frac{W}{P} = \frac{a}{b}$$

Wedges







- *Wedges* - simple machines used to raise heavy loads.
- Force required to lift block is significantly less than block weight.
- Friction prevents wedge from sliding out.
- Want to find minimum force P to raise block.

- Block as free-body

$$\sum F_x = 0:$$
$$-N_1 + \mu_s N_2 = 0$$
$$\sum F_y = 0:$$
$$-W - \mu_s N_1 + N_2 = 0$$

or

$$\vec{R}_1 + \vec{R}_2 + \vec{W} = 0$$

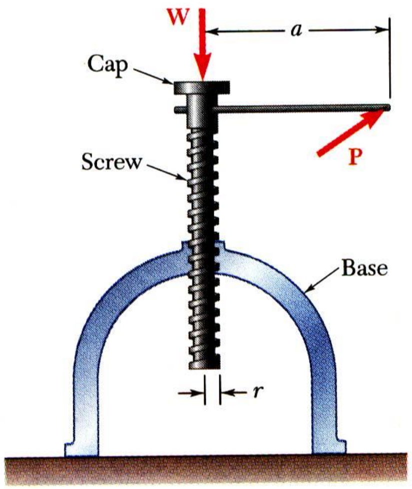
- Wedge as free-body

$$\sum F_x = 0:$$
$$-\mu_s N_2 - N_3(\mu_s \cos 6° - \sin 6°) + P = 0$$
$$\sum F_y = 0:$$
$$-N_2 + N_3(\cos 6° - \mu_s \sin 6°) = 0$$

or

$$\vec{P} - \vec{R}_2 + \vec{R}_3 = 0$$

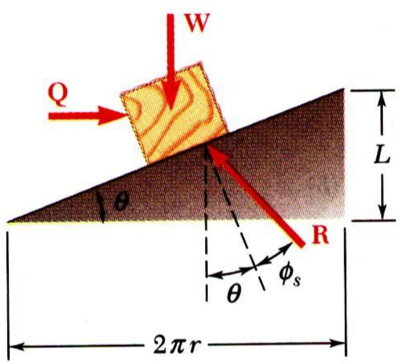
Square-Threaded Screws

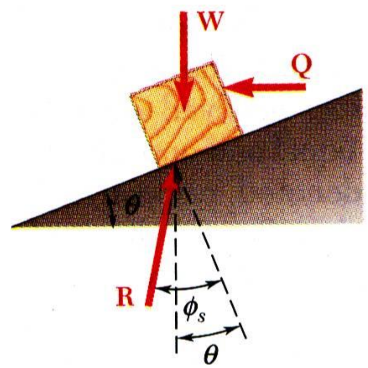


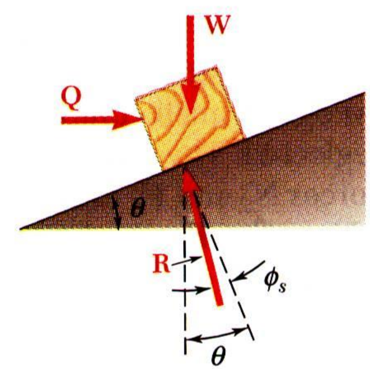
- Square-threaded screws frequently used in jacks, presses, etc. Analysis similar to block on inclined plane. Recall friction force does not depend on area of contact.

- Thread of base has been “unwrapped” and shown as straight line. Slope is $2\pi r$ horizontally and lead L vertically.

- Moment of force Q is equal to moment of force P. $Q = Pa/r$





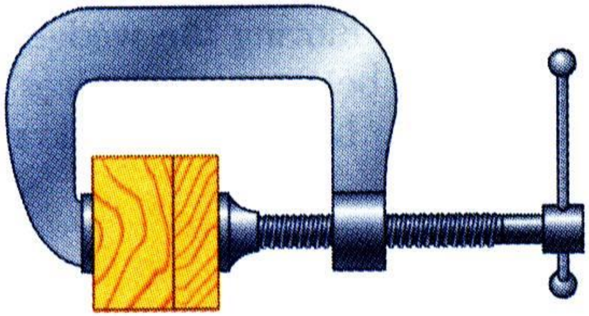


- Impending motion upwards. Solve for Q.

- $\phi_s > \theta$, Self-locking, solve for Q to lower load.

- $\phi_s > \theta$, Non-locking, solve for Q to hold load.

Sample Problem 8.5



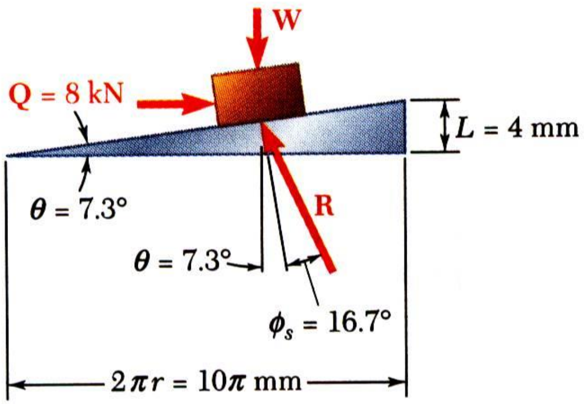
A clamp is used to hold two pieces of wood together as shown. The clamp has a double square thread of mean diameter equal to 10 mm with a pitch of 2 mm. The coefficient of friction between threads is $\mu_s = 0.30$.

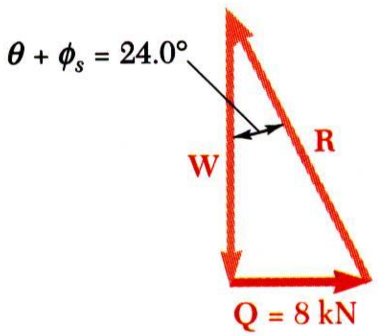
If a maximum torque of 40 N*m is applied in tightening the clamp, determine (a) the force exerted on the pieces of wood, and (b) the torque required to loosen the clamp.

SOLUTION

- Calculate lead angle and pitch angle.
- Using block and plane analogy with impending motion up the plane, calculate the clamping force with a force triangle.
- With impending motion down the plane, calculate the force and torque required to loosen the clamp.

Sample Problem 8.5





SOLUTION

- Calculate lead angle and pitch angle. For the double threaded screw, the lead L is equal to twice the pitch.

$$\tan\theta = \frac{L}{2\pi r} = \frac{2(2\text{ mm})}{10\pi\text{ mm}} = 0.1273 \qquad \theta = 7.3°$$

$$\tan\phi_s = \mu_s = 0.30 \qquad \phi_s = 16.7°$$

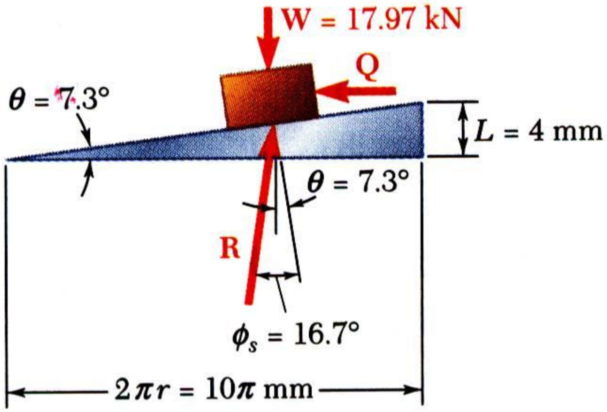
- Using block and plane analogy with impending motion up the plane, calculate clamping force with force triangle.

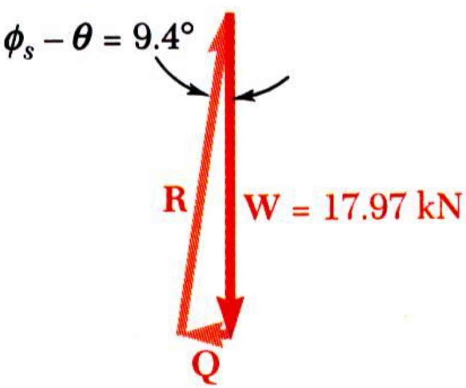
$$Qr = 40\text{ N}\cdot\text{m} \qquad Q = \frac{40\text{ N}\cdot\text{m}}{5\text{mm}} = 8\text{kN}$$

$$\tan(\theta+\phi_s) = \frac{Q}{W} \qquad W = \frac{8\text{kN}}{\tan 24°}$$

$$\boxed{W = 17.97\text{kN}}$$

Sample Problem 8.5





- With impending motion down the plane, calculate the force and torque required to loosen the clamp.

$$\tan(\phi_s - \theta) = \frac{Q}{W} \qquad Q = (17.97\,\text{kN})\tan 9.4^\circ$$

$$Q = 2.975\,\text{kN}$$

$$Torque = Q\,r = (2.975\,\text{kN})(5\,\text{mm})$$
$$= \left(2.975\times 10^3\,\text{N}\right)\left(5\times 10^{-3}\,\text{m}\right)$$

$$\boxed{Torque = 14.87\,\text{N}\cdot\text{m}}$$